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Volume 2



DC Pandey

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PREFACE

Engineering offers the most exciting and fulfilling of careers. As a Engineer you can find satisfaction by serving the society through your knowledge of technology. Although the number of Engineering colleges imparting quality education and training has significantly increased after independence in the country, but simultaneous increase in the number of serious aspirants has made the competition difficult, it is no longer easy to get a seat in a prestigious Engineering college today.

For success, you require an objective approach of the study. This does not mean you 'prepare' yourself for just 'objective questions'. Objective Approach means more than that. It could be defined as that approach through which a student is able to master the concepts of the subject and also the skills required to tackle the questions asked in different entrances such as JEE Main & Advanced, as well other regional Engineering entrances. These two-volume books on Physics 'Objective Physics (Vol.1 & 2)' fill the needs of such books in the market in Physics and are borne out of my experience of teaching Physics to Engineering aspirants.

The plan of the presentation of the subject matter in the books is as follows

- The whole chapter has been divided under logical topic heads to cover the syllabi of JEE Main & Advanced and various Engineering entrances in India.
- The Text develops the concepts in an easy going manner, taking the help of the examples from the day-to-day life.
- Important points of the topics have been highlighted in the text. Under Notes, some extra points regarding the topics have been given to enrich the students.
- The Solved Examples make the students learn the basic problem solving skills in Physics. In some of the example problems How to Proceed has been provided to make the students skilled in systematically tackling the problems.
- An Extra Knowledge Points frequently follows the discussion of few topics which includes the important theorems, results and formulae to increase the grasp of the subject matter.
- Additional Solved Examples given at the end of text part to make the students practice of the complete chapter as a whole.
- Assignments at the end of the chapters have been divided into two parts Objective Questions and Entrance Corner.
- The answers / solutions to all the assignment questions have been provided.
- The Objective Questions have been divided into two levels. Level 1 contains elementary MCQs while Level 2 has MCQs which are relatively tougher and take the students to a level required for various Engineering Entrances require in the present scenario.
- Entrance Gallery and includes the previous years' questions asked in various Engineering entrances. At the end of the book, JEE Main & Advanced & Other Regional Entrances Solved Papers have been given.

I am extremely thankful to Mrs. Sarita Pandey, Mr. Anoop Dhyani & Mr. Shubham Sharma of their endless effort during the Project. I would open-heartedly welcome the suggestions for the further improvements of this book (Vol.2) from the students and teachers.

DC Pandey

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DEDICATION

This book is dedicated to my honourable grandfather

(LATE) SH. PITAMBER PANDEY

a Kumaoni poet; resident of village Dhaura (Almora) Uttarakhand

18

Electrostatics

18.1 Electric Charge

The electrical nature of matter is inherent in atomic structure. An atom consists of a small, relatively massive nucleus that contains particles called protons and neutrons. A proton has a mass of 1.673×10^{-27} kg, while a neutron has a slightly greater mass 1.675×10^{-27} kg. Surrounding the nucleus there is a diffuse cloud of orbiting particles called electrons. An electron has a mass of 9.11×10^{-31} kg.

Like mass **electric charge** is an intrinsic property of protons and electrons and only two types of charge have been discovered namely positive and negative. A proton has a positive charge and an electron has a negative charge. A neutron has no net electric charge.The magnitude of the charge on the proton exactly equals the magnitude of the charge on the electrons. The proton carries a charge $+e$ and the electron carries a charge $-e$. The SI unit of charge is coulomb (C) and e has the value, $e = 1.6 \times 10^{-19}$ C.

Regarding charge, following points are worth noting:

1. Like charges repel each other and unlike charges attract each other.
2. Charge is a scalar and can be of two types positive or negative.
3. Charge is quantized. The quantum of charge is e. The charge on anybody will be some integral multiple of e, i.e.

$$q = \pm ne \text{ where, } n = 1, 2, 3\ldots$$

Charge on any body can never be $\left(\frac{1}{3}e\right)$, $1.5e$, etc.

Note Points

- Apart from charge, energy, angular momentum and mass are also quantized. The quantum of energy is $h\nu$ and that of angular momentum is $\frac{h}{2\pi}$. Quantum of mass is yet not known.
- The protons and neutrons are combination of other entities called **quarks,** which have charges $1/3e$ and $\pm 2/3e$. However, isolated quarks have not been observed, so quantum of charge is still e.

4. During any process, the net electric charge of an isolated system remains constant or we can say that charge is conserved. Pair production and pair annihilation are two examples of conservation of charge.

Chapter Snapshot

- Electric Charge
- Conductors and Insulators
- Charging of a Body
- Coulomb's Law
- Electric Field
- Electric Potential Energy
- Electric Potential
- Relation between Electric Field and Potential
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- Properties of a Conductor
- Electric Field and Potential due to Charged Spherical Shell of Solid Conducting Sphere
- Electric Field and Potential due to a Solid Sphere of Charge
- Capacitance
- Methods of Finding Equivalent Resistance and Capacitance

5. A charge particle at rest produces electric field. A charge particle in an unaccelerated motion produces both electric and magnetic fields but does not radiate energy. But an accelerated charged particle not only produces an electric and magnetic fields but also radiates energy in the form of electromagnetic waves.

6. 1 coulomb $= 3 \times 10^9$ esu $= \frac{1}{10}$ emu of charge.

Example 18.1 *How many electrons are there in 1 C negative charge?*

Sol. The negative charge is due to the presence of excess electrons, since they carry negative charge. Because an electron has a charge whose magnitude is $e = 1.6 \times 10^{-19}$ C, the number of electrons is equal to the charge q divided by the charge e on each electron. Therefore, the number n of electrons is

$$n = \frac{q}{e} = \frac{1.0}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$$

18.2 Conductors and Insulators

For the purpose of electrostatic theory all substances can be divided into two main groups — conductors and insulators. In conductors, electric charges are free to move from one place to another, whereas in insulators they are tightly bound to their respective atoms. In an uncharged body there are equal number of positive and negative charges.

The examples of conductors of electricity are the metals, human body and the earth and that of insulators are glass, hard rubber and plastics. In metals, the free charges are free electrons known as **conduction electrons**.

Semiconductors are a third class of materials, and their electrical properties are somewhere between those of insulators and conductors. Silicon and germanium are well known examples of semiconductors.

18.3 Charging of a Body

Mainly there are following three methods of charging a body:

Charging by Rubbing

The simplest way to experience electric charges is to rub certain bodies against each other. When a glass rod is rubbed with a silk cloth, the glass rod acquires some positive charge and the silk cloth acquires negative charge by the same amount. The explanation of appearance of electric charge on rubbing is simple. All material bodies contain large number of electrons and equal number of protons in their normal state. When rubbed against each other, some electrons from one body pass onto the other body. The body that donates the electrons becomes positively charged while that which receives the electrons becomes negatively charged. For example when glass rod is rubbed with silk cloth, glass rod becomes positively charged because it donates the electrons while the silk cloth becomes negatively charged because it receives electrons. Electricity so obtained by rubbing two objects is also known as **frictional electricity**. The other places where the frictional electricity can be observed are when amber is rubbed with wool or a comb is passed through a dry hair. Clouds also become charged by friction.

Charging by Contact

When a negatively charged ebonite rod is rubbed on a metal object such as a sphere, some of the excess electrons from the rod are transferred to the sphere. Once the electrons are on the metal sphere where they can move readily, they repel one another and spread out over the sphere's surface.

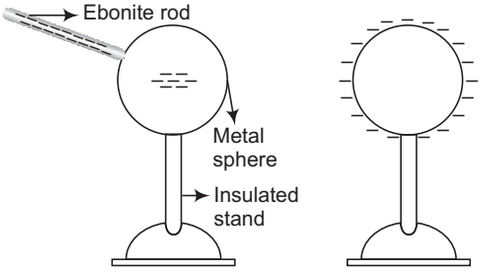


Fig. 18.1

The insulated stand prevents them from flowing to the earth. When the rod is removed, the sphere is left with a negative charge distributed over its surface. In a similar manner, the sphere will be left with a positive charge after being rubbed with a positively charged rod. In this case, electrons from the sphere would be transferred to the rod. The process of giving one object a net electric charge by placing it in contact with another object that is already charged is known as **charging by contact.**

Charging by Induction

It is also possible to charge a conductor in a way that does not involve contact.

In Fig. (a), a negatively charged rod brought close to (but does not touch) a metal sphere. In the sphere, the free electrons close to the rod move to the other side (by repulsion). As a result, the part of the sphere nearer to the rod becomes positively charged and the part farthest from the rod negatively charged.

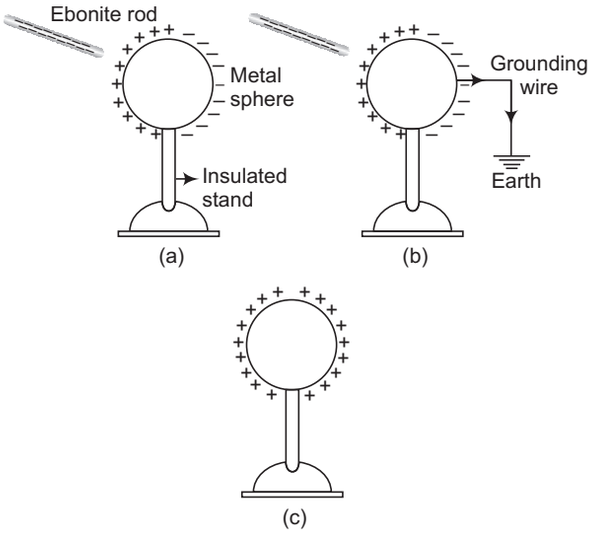


Fig. 18.2

This phenomenon is called induction. Now, if the rod is removed, the free electrons return to their original places and the charged regions disappear. Under most conditions the earth is a good electric conductor. So, when a metal wire is attached between the sphere and the ground as in Fig. (b) some of the free electrons leave the sphere and distribute themselves on the much larger earth. If the grounding wire is then removed, followed by the ebonite rod, the sphere is left with a net positive charge. The process of giving one object a net electric charge without touching the object to a second charged object is called **charging by induction**. The process could also be used to give the sphere a net negative charge, if a positively charged rod were used. Then, electrons would be drawn up from the ground through the grounding wire and onto the sphere.

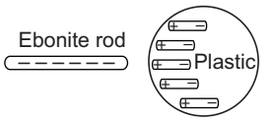


Fig. 18.3

If the sphere were made from an insulating material like plastic, instead of metal, the method of producing a net charge by induction would not work because very little charge would flow through the insulating material and down the grounding wire. However, the electric force of the charged rod would have some effect as shown in figure.

The electric force would cause the positive and negative charges in the molecules of the insulating material to separate slightly, with the negative charges being pushed away from the negative rod. The surface of the plastic sphere does acquire a slight induced positive charge, although no net charge is created.

Example 18.2 *If we comb our hair on a dry day and bring the comb near small pieces of paper, the comb attracts the pieces, why?*

Sol. This is an example of frictional electricity and induction. When we comb our hair, it gets positively charged by rubbing. When the comb is brought near the pieces of paper, some of the electrons accumulate at the edge of the paper piece which is closer to the comb. At the farther end of the piece, there is deficiency of electrons and hence positive charge appears there. Such a redistribution of charge in a material, due to presence of a nearby charged body is called induction. The comb exerts larger attraction on the negative charges of the paper piece as compared to the repulsion on the positive charge. This is because the negative charges are closer to the comb. Hence, there is a net comb and the paper piece.

Example 18.3 *Does the attraction between the comb and the piece of papers last for longer period of time?*

Sol. No, because the comb loses its net charge after some time. The excess charge of the comb transfers to earth through our body after some time.

Example 18.4 *Can two similarly charged bodies attract each other?*

Sol. Yes, when the charge on one body (q_1) is much greater than that on the other (q_2) and they are close enough to each other so that force of attraction between q_1 and induced charge on the other exceeds the force of repulsion between q_1 and q_2. However two similar point charges can never attract each other because no induction will take place here.

Example 18.5 *Does in charging the mass of a body change?*

Sol. Yes, as charging a body means addition or removal of electrons and electron has a mass.

Example 18.6 *Why a third hole in a socket provided for grounding?*

Sol. All electric appliances may end with some charge due to faulty connections. In such a situation, charge will be accumulated on the appliance. When the user touches the appliance, he may get a shock. By providing the third hole for grounding, all accumulated charge is discharged to the ground and the appliance is safe.

18.4 Coulomb's Law

The law that describes how charges interact with one another was discovered by Charles Augustin de Coulomb in 1785. With a sensitive torsion balance, Coulomb measured the electric force between charged spheres. In Coulomb's experiment the charged spheres were much smaller than the distance between them so that the charges could be treated as point charges. The results of the experiments of Coulomb and others are summarized in **Coulomb's law**.

The electric force $\mathbf{F}_e$ exerted by one point charge on another acts along the line between the charges. It varies

inversely as the square of the distance separating the charges and is proportional to the product of charges. The force is repulsive if the charges have the same signs and attractive if the charges have opposite signs.

The magnitude of the electric force exerted by a charge q_1 on another charge q_2 at a distance r away is thus, given by

$$F_e = \frac{k\,|q_1 q_2|}{r^2} \qquad ...(i)$$

The value of the proportionality constant k in Coulomb's law depends on the system of units used. In SI units the constant k is

$$k = 8.987551787 \times 10^9\ \text{N-m}^2/\text{C}^2$$
$$\approx 8.988 \times 10^9\ \text{N-m}^2/\text{C}^2$$

The value of k is known to such a large number of significant digits because this value is closely related to the speed of light in vacuum. This speed is defined to be exactly $c = 2.99792458 \times 10^8\ \text{m/s}$. The numerical value of k is defined in terms of c to be precisely.

$$k = (10^{-7}\ \text{N-s}^2\text{C}^2)\ c^2$$

This constant k is often written as $\frac{1}{4\pi\varepsilon_0}$, where ε_0 (epsilon-nought) is another constant. This appears to complicate matters, but it actually simplifies many formulae that we will encounter in later chapters. Thus, Eq. (i) can be written as,

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2} \qquad ...(ii)$$

Here, $\frac{1}{4\pi\varepsilon_0} = (10^{-7}\ \text{N-s}^2\text{C}^2)\,c^2$

Substituting the value of

$c = 2.99792458 \times 10^8$ m/s, we get

$$\frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\ \text{N-m}^2/\text{C}^2$$

In examples and problems, we will often use the approximate value,

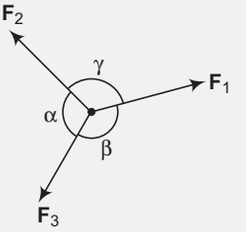
$$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\ \text{N-m}^2/\text{C}^2$$

Here, the quantity ε_0 is called the **permittivity of free space**. It has the value,

$$\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2/\text{N-m}^2$$

Extra Knowledge Points

- In few problems of electrostatics Lami's theorem is very useful.



According to this theorem, 'if three concurrent forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ as shown in figure are in equilibrium or if $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_2 = 0$, then

$$\frac{\mathbf{F}_1}{\sin\alpha} = \frac{\mathbf{F}_2}{\sin\beta} = \frac{\mathbf{F}_3}{\sin\gamma}$$

- Suppose the position vectors of two charges q_1 and q_2 are $\mathbf{r}_1$ and $\mathbf{r}_2$, then, electric force on charge q_1 due to charge q_2 is,

$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}(\mathbf{r}_1 - \mathbf{r}_1)$$

Similarly, electric force on q_2 due to charge q_1 is

$$\mathbf{F}_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|^3}(\mathbf{r}_2 - \mathbf{r}_1)$$

Here q_1 and q_2 are to be substituted with sign.

$$\mathbf{r}_1 = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}}$$

and $$\mathbf{r}_2 = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}}$$

where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the co-ordinates of charges q_1 and q_2.

Regarding Coulomb's law, following points are worth noting:

1. Coulomb's law stated above described the interaction of two point charges. When two charges exert forces simultaneously on a third charge, the total force acting on that charge is the vector sum of the forces that the two charges would exert individually. This important property, called the **principle of superposition of forces,** holds for any number of charges. Thus,
$$\mathbf{F}_{\text{net}} = \mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_n$$
2. The electric force is an action reaction pair, i.e. the two charges exert equal and opposite forces on each other.
3. The electric force is conservative in nature.

4. Coulomb's law as we have stated above can be used for point charges in vacuum. If some dielectric (insulator) is present in the space between the charges, the net force acting on each charge is altered because charges are induced in the molecules of the intervening medium. We will describe this effect later. Here at this moment, it is enough to say that the force decreases K times if the medium extends till infinity. Here, K is a dimensionless constant which depends on the medium and called dielectric constant of the medium. Thus,

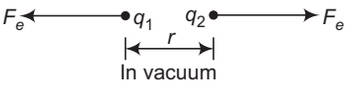


Fig. 18.4

$$F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2} \qquad \text{(in vacuum)}$$

$$F_e' = \frac{F_e}{K} = \frac{1}{4\pi\varepsilon_0 K}\cdot\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon}\cdot\frac{q_1 q_2}{r^2} \text{ (in medium)}$$

Here, $\varepsilon = \varepsilon_0 K$ is called permittivity of the medium.

Example 18.7 *What is the smallest electric force between two charges placed at a distance of* 1.0 *m?*

Sol. $F_e = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_1 q_2}{r^2}$ …(i)

For F_e to be minimum $q_1 q_2$ should be minimum. We know that

$$(q_1)_{min} = (q_2)_{min} = e = 1.6\times10^{-19}\ \text{C}$$

Substituting in Eq. (i), we have

$$(F_e)_{min} = \frac{(9.0\times10^9)\,(1.6\times10^{-19})\,(1.6\times10^{-19})}{(1.0)^2}$$
$$= 2.304\times10^{-28}\ \text{N}$$

Example 18.8 *Three charges $q_1 = 1\,\mu C$, $q_2 = -2\,\mu C$ and $q_3 = 3\,\mu C$ are placed on the vertices of an equilateral triangle of side* 1 *m. Find the net electric force acting on charge q_1.*

How to Proceed Charge q_2 will attract charge q_1 (along the line joining them) and charge q_3 will repel charge q_1. Therefore, two forces will act on q_1, one due to q_2 and another due to q_3. Since, the force is a vector quantity both of these forces (say $\mathbf{F}_1$ and $\mathbf{F}_2$) will be added by vector method.

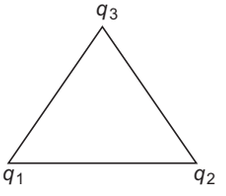


Fig. 18.5

Following are two methods of their addition:

Sol. In the figure,

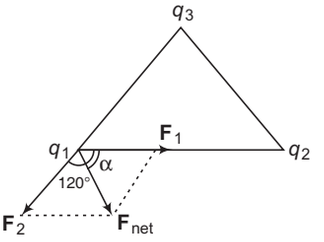


Fig. 18.6

$$|F_1| = F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}$$
$$= \text{magnitude of force between } q_1 \text{ and } q_2$$
$$= \frac{(9.0\times10^9)\,(1.0\times10^{-6})\,(2.0\times10^{-6})}{(1.0)^2}$$
$$= 1.8\times10^{-2}\ \text{N}$$

Similarly, $|F_2| = F_2 = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_1 q_3}{r^2}$

$$= \text{magnitude of force between } q_1 \text{ and } q_2$$
$$= \frac{(9.0\times10^9)\,(1.0\times10^{-6})\,(3.0\times10^{-6})}{(1.0)^2}$$
$$= 2.7\times10^{-2}\ \text{N}$$

Now, $|F_{net}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°}$

$$= \left(\sqrt{(1.8)^2 + (2.7)^2 + 2\,(1.8)\,(2.7)\left(-\frac{1}{2}\right)}\right)\times10^{-2}\ \text{N}$$
$$= 2.38\times10^{-2}\ \text{N}$$

and $\tan\alpha = \dfrac{F_2\sin 120°}{F_1 + F_2\cos 120°}$

$$= \frac{(2.7\times10^{-2})\,(0.87)}{(1.8\times10^{-2}) + (2.7\times10^{-2})\left(-\frac{1}{2}\right)}$$

or $\alpha = 79.2°$

Thus, the net force on charge q_1 is 2.38×10^{-2} N at an angle $\alpha = 79.2°$ with a line joining q_1 and q_2 as shown in the figure.

Method 2. In this method, let us assume a coordinate axis with q_1 at origin as shown in figure. The coordinates of q_1, q_2 and q_3 in this coordinate system are (0, 0, 0), (1 m, 0, 0) and (0.5 m, 0.87 m, 0), respectively. Now,

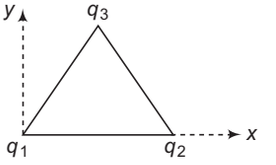


Fig. 18.7

F_1 = force on q_1 due to charge q_2

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{|r_1 - r_2|^3}(r_1 - r_2)$$
$$= \frac{(9.0\times10^9)\,(1.0\times10^{-6})\,(-2.0\times10^{-6})}{(1.0)^3}$$
$$\times[(0-1)\,\hat{i} + (0-0)\,\hat{j} + (0-0)\,\hat{k}]$$
$$= (1.8\times10^{-2}\,\hat{i})\ \text{N}$$

and F_2 = force on q_1 due to charge q_3

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{|r_1 - r_3|^3}(r_1 - r_3)$$

$$= \frac{(9.0\times 10^9)\,(1.0\times 10^{-6})\,(3.0\times 10^{-6})}{(1.0)^3} \times [(0-0.5)\hat{i} + (0-0.87)\,\hat{j} + (0-0)\,\hat{k}]$$

$$= (-1.35\,\hat{i} - 2.349\,\hat{j})\times 10^{-2}\ \text{N}$$

Therefore, net force on q_1 is

$$F = F_1 + F_2 = (0.45\,\hat{i} - 2.349\,\hat{j})\times 10^{-2}\ \text{N}$$

- Once you write a vector in terms of $\hat{i}$, $\hat{j}$ and $\hat{k}$, there is no need of writing the magnitude and direction of vector separately.

Example 18.9 *Two identical balls each having a density* ρ *are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium each string makes an angle* θ *with vertical. Now, both the balls are immersed in a liquid. As a result the angle* θ *does not change. The density of the liquid is* σ. *Find the dielectric constant of the liquid.*

Sol. Each ball is in equilibrium under the following three forces:

(i) tension

(ii) electric force

(iii) weight

So, Lami's theorem can be applied.

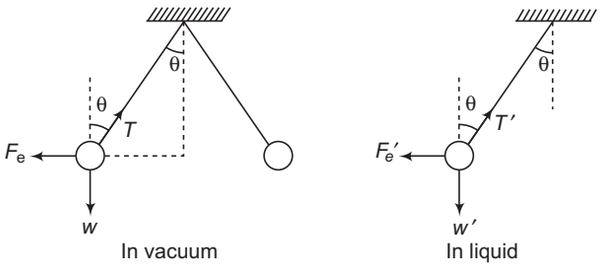


Fig. 18.8

In the liquid, $F_e' = \dfrac{F_e}{K}$

where, K = dielectric constant of liquid

and $w' = w$ – upthrust

Applying Lami's theorem in vacuum

$$\frac{w}{\sin(90^\circ + \theta)} = \frac{F_e}{\sin(180^\circ - \theta)}$$

or $$\frac{w}{\cos\theta} = \frac{F_e}{\sin\theta} \quad \ldots(\text{i})$$

Similarly in liquid,

$$\frac{w'}{\cos\theta} = \frac{F_e'}{\sin\theta} \quad \ldots(\text{ii})$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{w}{w'} = \frac{F_e}{F_e'}$$

or $$K = \frac{w}{w - \text{upthrust}} \qquad \left(\text{as } \frac{F_e}{F_e'} = K\right)$$

$$= \frac{V\rho g}{V\rho g - V\sigma g} \qquad (V = \text{volume of ball})$$

or $$K = \frac{\rho}{\rho - \sigma}$$

- In the liquid, F_e and w have been changed. Therefore, T will also change.

18.5 Electric Field

A charged particle cannot directly interact with another particle kept at a distance. A charge produces something called an electric field in the space around it and this electric field exerts a force on any other charge (except the source charge itself) placed in it.

Thus, the region surrounding a charge or distribution of charge in which its electrical effects can be observed is called the **electric field** of the charge or distribution of charge. Electric field at a point can be defined in terms of either a vector function **E** called **electric field strength** or a scalar function V called **electric potential**. The electric field can also be visualised graphically in terms of **lines of force**.

Electric Field Strength (E)

Like its gravitational counterpart, the electric field strength (often called electric field) at a point in an electric field is defined as the electrostatic force $\mathbf{F}_e$ per unit positive charge. Thus, if the electrostatic force experienced by a small test charge q_0 is $\mathbf{F}_e$, then field strength at that point is defined as,

$$\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}_e}{q_0}$$

The electric field is a vector quantity and its direction is the same as the direction of the force $\mathbf{F}_e$ on a positive test charge. The SI unit of electric field is N/C. Here, it should be noted that the test charge q_0 does not disturb other charges which produces **E**. With the concept of electric field, our description of electric interactions has two parts. First, a given charge distribution acts as a source of electric field. Second, the electric field exerts a force on any charge that is present in this field.

An Electric Field Leads to a Force

Suppose there is an electric field strength **E** at some point in an electric field, then the electrostatic force acting on a charge $+q$ is qE in the direction of **E**, while on the charge $-q$ it is qE in the opposite direction of **E**.

Example 18.10 *An electric field of* 10^5 *N/C points due west at a certain spot. What are the magnitude and direction of the force that acts on a charge of* $+2\,\mu C$ *and* $-5\,\mu C$ *at this spot?*

Sol. Force on $+2\,\mu C = qE = (2\times10^{-6})(10^5)$

$= 0.2$ N (due west)

Force on $-5\,\mu C = (5\times10^{-6})(10^5) = 0.5$ N (due east)

Electric Field Due to a Point Charge

The electric field produced by a point charge q can be obtained in general terms from Coulomb's law. First note that the magnitude of the force exerted by the charge q on a test charge q_0 is,

$$F_e = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{r^2}$$

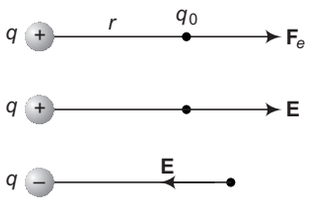


Fig. 18.9

then divide this value by q_0 to obtain the magnitude of the field.

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

If q is positive **E** is directed away from q. On the other hand if q is negative, then **E** is directed towards q.

The electric field at a point is a vector quantity. Suppose $\mathbf{E}_1$ is the field at a point due to a charge q_1 and $\mathbf{E}_2$ is the field at the same point due to a charge q_2. The resultant field when both the charges are present is

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

If the given charge distribution is continuous, we can use the technique of integration to find the resultant electric field at a point.

Example 18.11 *Two positive point charges* $q_1 = 16\,\mu C$ *and* $q_2 = 4\,\mu C$*, are separated in vacuum by a distance of* 3.0 *m. Find the point on the line between the charges where the net electric field is zero.*

Sol. Between the charges the two field contributions have opposite directions and the net electric field is zero at a point (say P), where the magnitudes of E_1 and E_2 are equal. However, since, $q_2 < q_1$, point P must be closer to q_2, in order that the field of the smaller charge can balance the field of the larger charge.

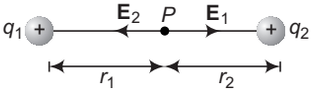


Fig. 18.10

At P, $E_1 = E_2$

or $$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_2^2}$$

$$\therefore \quad \frac{r_1}{r_2} = \sqrt{\frac{q_1}{q_2}}$$

$$= \sqrt{\frac{16}{4}} = 2 \qquad \ldots(i)$$

Also, $r_1 + r_2 = 3.0$ m $\qquad \ldots(ii)$

Solving these equations, we get

$r_1 = 2$ m

and $r_2 = 1$ m

Thus, the point P is at a distance of 2 m from q_1 and 1 m from q_2.

Electric Field of a Ring of Charge

Electric field at distance x from the centre of uniformly charged ring of total charge q on its axis is given by

$$E_x = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{qx}{(x^2+R^2)^{3/2}}$$

The direction of this electric field is along the axis and away from the ring in case of positively charged ring and towards the ring in case of negatively charged ring.

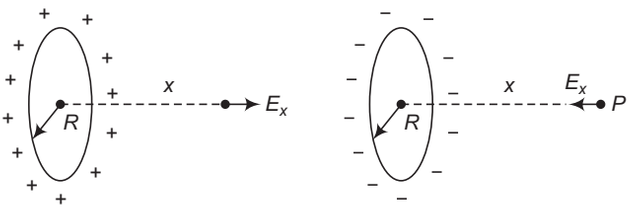


Fig. 18.11

From the above expression, we can see that

(i) $E_x = 0$ at $x = 0$, i.e. field is zero at the centre of the ring. We should expect this, charges on opposite sides of the ring would push in opposite directions on a test charge at the centre and the forces would add to zero.

(ii) $E_x = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{x^2}$ for $x \gg R$, i.e. when the point P is much farther from the ring, its field is the same as that of a point charge. To an observer far from the ring, the ring would appear like a point and the electric field reflects this.

(iii) E_x will be maximum where $\frac{dE_x}{dx}=0$. Differentiating E_x w.r.t. x and putting it equal to zero, we get $x=\frac{R}{\sqrt{2}}$ and E_{max} comes out to be $\frac{2}{\sqrt[3]{3}}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}\right)$.

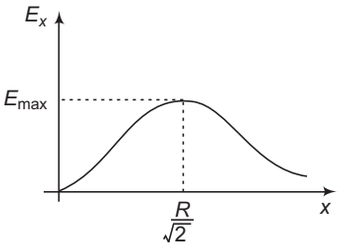


Fig. 18.12

Electric Field of an Infinitely Long Line Charge

Electric field at distance r from an infinitely long line charge is given by

$$E=\frac{\lambda}{2\pi\varepsilon_0 r}$$

Here λ is charge per unit length. Direction of this electric field is away from the line charge in case of positively charged line charge and towards the line charge in case of negatively charged line charge.

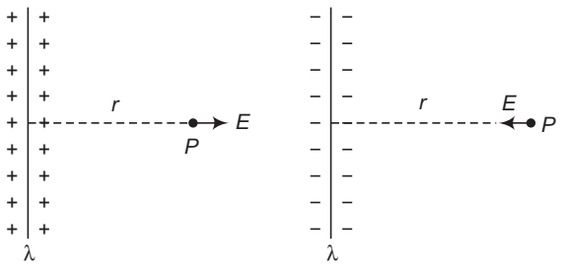


Fig. 18.13

or $$E\propto\frac{1}{r}$$

Thus, E-r graph is as shown in Fig. 18.14.

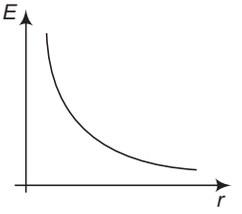


Fig. 18.14

The direction of **E** is radially outward from the line.

Extra Knowledge Points

- Suppose a charge q is placed at a point whose position vector is $\mathbf{r}_q$ and we want to find the electric field at a point P whose position vector is $\mathbf{r}_p$. Then, in vector form the electric field is given by

$$\mathbf{E}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{|\mathbf{r}_p-\mathbf{r}_q|^3}(\mathbf{r}_p-\mathbf{r}_q)$$

Here, $\mathbf{r}_p=x_p\hat{\mathbf{i}}+y_p\hat{\mathbf{j}}+z_p\hat{\mathbf{z}}$

and $\mathbf{r}_q=x_q\hat{\mathbf{i}}+y_q\hat{\mathbf{j}}+z_q\hat{\mathbf{k}}$

In this equation, q is to be substituted with sign.

Example 18.12 *A charge $q=1\mu C$ is placed at point $(1m, 2m, 4m)$. Find the electric field at point P $(0m, -4m, 3m)$.*

Sol. Here, $\mathbf{r}_q=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ and $\mathbf{r}_p=-4\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

$\therefore$ $\mathbf{r}_p-\mathbf{r}_q=-\hat{\mathbf{i}}-6\hat{\mathbf{j}}-\hat{\mathbf{k}}$

or $|\mathbf{r}_p-\mathbf{r}_q|=\sqrt{(-1)^2+(-6)^2+(-1)^2}=\sqrt{38}$ m

Now, $\mathbf{E}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{|\mathbf{r}_p-\mathbf{r}_q|^3}(\mathbf{r}_p-\mathbf{r}_q)$

Substituting the values, we have

$$\mathbf{E}=\frac{(9.0\times10^9)(1.0\times10^{-6})}{(38)^{3/2}}(-\hat{\mathbf{i}}-6\hat{\mathbf{j}}-\hat{\mathbf{k}})$$

$$=(-38.42\,\hat{\mathbf{i}}-230.52\,\hat{\mathbf{j}}-38.42\,\hat{\mathbf{k}})\ \text{N/C}$$

Electric Field Lines

As we have seen, electric charges create an electric field in the space surrounding them. It is useful to have a kind of map that gives the direction and indicates the strength of the field at various places. Field lines, a concept introduced by Michael Faraday, provide us with an easy way to visualize the electric field.

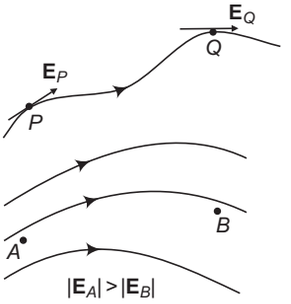


Fig. 18.15

An **electric field line** is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric field vector at that point. The relative closeness of the lines at some place give an idea about the intensity of electric field at that point.

The electric field lines have the following properties:

1. The tangent to a line at any point gives the direction of **E** at that point. This is also the path on which a positive test charge will tend to move if free to do so.
2. Electric field lines always begin on a positive charge and end on a negative charge and do not start or stop in midspace.
3. The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge. This means, for example that if 100 lines are drawn leaving a $+4\,\mu C$ charge, then 75 lines would have to end on a $-3\,\mu C$ charge.

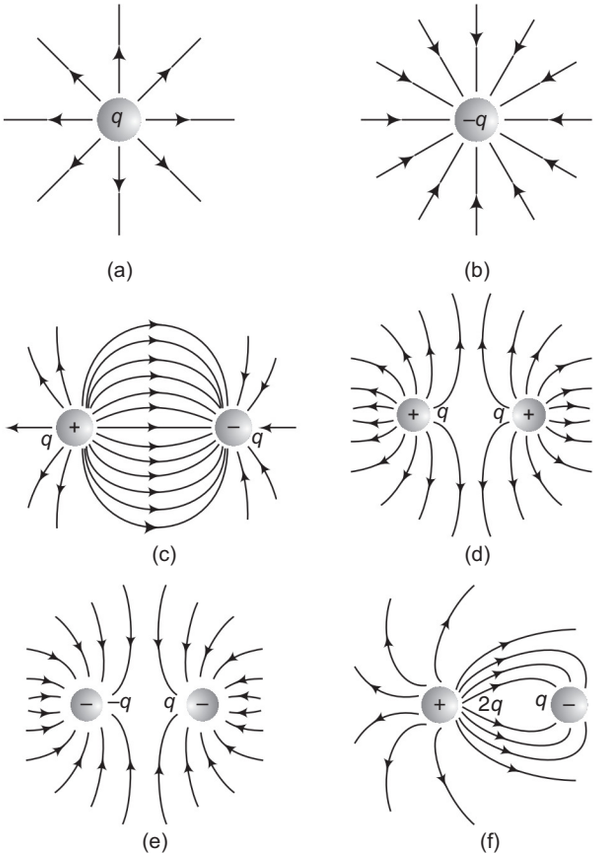


Fig. 18.16

4. Two lines can never intersect. If it happens then two tangents can be drawn at their point of intersection, i.e. intensity at that point will have two directions which is absurd.
5. In a uniform field, the field lines are straight parallel and uniformly spaced.
6. The electric field lines can never form closed loops as a line can never start and end on the same charge.
7. Electric field lines also give us an indication of the equipotential surface (surface which has the same potential)
8. Electric field lines always flow from higher potential to lower potential.
9. In a region where there is no electric field, lines are absent. This is why inside a conductor (where electric field is zero) there, cannot be any electric field line.
10. Electric lines of force ends or starts normally from the surface of a conductor.

18.6 Electric Potential Energy

If the force **F** is conservative, the work done by **F** can always be expressed in terms of a **potential energy** U. When the particle moves from a point where the potential energy is U_a to a point where it is U_b, the change in potential energy is, $\Delta U = U_b - U_a$. This is related by the work $W_{a \to b}$ as

$$W_{a\to b} = U_a - U_b$$
$$= -(U_b - U_a) = -\Delta U \qquad \ldots(i)$$

Here, $W_{a\to b}$ is the work done in displacing the particle from a to b by the conservative force (here electrostatic) not by us. Moreover, we can see from Eq. (i) that if $W_{a \to b}$ is positive, ΔU is negative and the potential energy decreases. So, **whenever the work done by a conservative force is positive, the potential energy of the system decreases and *vice-versa*.** That's what happens when a particle is thrown upwards, the work done by gravity is negative and the potential energy increases.

Example 18.13 *A uniform electric field E_0 is directed along positive y-direction. Find the change in electric potential energy of a positive test charge q_0, when it is displaced in this field from $y_i = a$ to $y_f = 2a$ along the y-axis.*

Sol. Electrostatic force on the test charge,

$$F_e = q_0 E_0 \qquad \text{(along positive } y\text{-direction)}$$

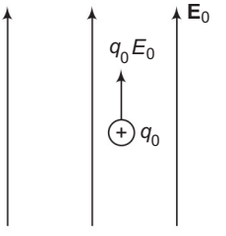


Fig. 18.17

$$\therefore \qquad W_{i-f} = -\Delta U$$

or
$$\Delta U = -W_{i-f}$$
$$= -[q_0 E_0 (2a - a)]$$
$$= -q_0 E_0 a$$

Here, work done by electrostatic force is positive. Hence, the potential energy is decreasing.

Electric Potential Energy of Two Charges

The idea of electric potential energy is not restricted to the special case of a uniform electric field as in example 18.13. Let us now calculate the work done on a test charge q_0 moving in a non-uniform electric field caused by a single, stationary point charge q.

The Coulomb's force on q_0 at a distance r from a fixed charge q is,

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{r^2}$$

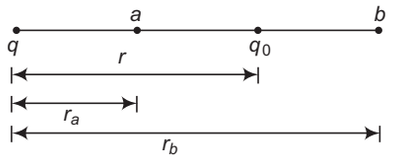


Fig. 18.18

If the two charges have same signs, the force is repulsive and if the two charges have opposite signs, the force is attractive. The force is not constant during the displacement, so we have to integrate to calculate the work $W_{a\to b}$ done on q_0 by this force as q_0 moves from a to b.

$$\therefore \quad W_{a\to b} = \int_{r_a}^{r_b} F\,dr = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{r^2}\,dr$$

$$= \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

Being a conservative force, this work is path independent. From the definition of potential energy,

$$U_b - U_a = -W_{a-b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

We choose the potential energy of the two charges system to be zero when they have infinite separation. This means $U_\infty = 0$. The potential when the separation is r is U_r.

$$\therefore \quad U_r - U_\infty = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right)$$

or $$U_r = \frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r}$$

This is the expression for electric potential energy of two point charges kept at a separation r. In this expression, both the charges q and q_0 are to be substituted with sign.

Electric Potential Energy of a System of Charges

To find electrical potential energy of a system of charges, we make pairs. If there are total n charges, then total number of pairs are $\{n(n-1)\}/2$.

Neither of the pair should be repeated nor should be left.

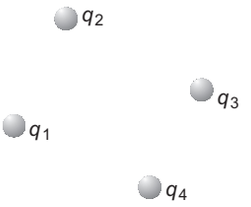


Fig. 18.19

For example, electric potential energy of four point charges q_1, q_2, q_3 and q_4 would be given by,

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_4q_3}{r_{43}} + \frac{q_4q_2}{r_{42}} + \frac{q_4q_1}{r_{41}} + \frac{q_3q_2}{r_{32}} + \frac{q_3q_1}{r_{31}} + \frac{q_2q_1}{r_{21}}\right] \quad \ldots(ii)$$

Here, all the charges are to be substituted with sign.

Example 18.14 *Four charges $q_1 = 1\,\mu C$, $q_2 = 2\,\mu C$, $q_3 = -3\,\mu C$ and $q_4 = 4\,\mu C$ are kept on the vertices of a square of side $1\,m$. Find the electric potential energy of this system of charges.*

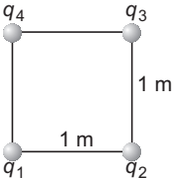


Fig. 18.20

Sol. In this problem, $r_{41} = r_{43} = r_{32} = r_{21} = 1$ m

and $$r_{42} = r_{31} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}\ \text{m}$$

Substituting the proper values with sign in Eq. (ii), we get

$$U = (9.0\times10^9)(10^{-6})(10^{-6})\left[\frac{(4)(-3)}{1} + \frac{(4)(2)}{\sqrt{2}} + \frac{(4)(1)}{1} + \frac{(-3)(2)}{1} + \frac{(-3)(1)}{\sqrt{2}} + \frac{(2)(1)}{1}\right]$$

$$= (9.0\times10^{-3})\left[-12 + \frac{5}{\sqrt{2}}\right] = -7.62\times10^{-2}\,\text{J}$$

Here negative sign of U implies that positive work has been done by electrostatic forces in assembling these charges at respective distances from infinity.

Example 18.15 *Two point charges are located on the x-axis, $q_1 = -1\,\mu C$ at $x = 0$ and $q_2 = +1\,\mu C$ at $x = 1\,m$.*

(a) Find the work that must be done by an external force to bring a third point charge $q_3 = +1\,\mu C$ from infinity to $x = 2\,m$.

(b) Find the total potential energy of the system of three charges.

Sol. (a) The work that must be done on q_3 by an external force is equal to the difference of potential energy U when the charge is at $x = 2$ m the potential energy when it is at infinity.

$$\therefore \quad W = U_f - U_i = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_f} + \frac{q_3q_1}{(r_{31})_f} + \frac{q_2q_1}{(r_{21})_f}\right] - \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_i} + \frac{q_3q_1}{(r_{31})_i} + \frac{q_2q_1}{(r_{21})_i}\right]$$

Here, $(r_{21})_i = (r_{21})_f$ and $(r_{32})_i = (r_{31})_i = \infty$

$$\therefore \quad W = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_f} + \frac{q_3q_1}{(r_{31})_f}\right]$$

Substituting the values, we have

$$W = (9.0\times 10^9)\,(10^{-12})\left[\frac{(1)\,(1)}{(1.0)} + \frac{(1)\,(-1)}{(2.0)}\right]$$

$$= 4.5\times 10^{-3}\ \text{J}$$

(b) The total potential energy of the three charges is given by,

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_3q_2}{r_{32}} + \frac{q_3q_1}{r_{31}} + \frac{q_2q_1}{r_{21}}\right)$$

$$= (9.0\times 10^9)\left[\frac{(1)\,(1)}{(1.0)} + \frac{(1)\,(-1)}{(2.0)} + \frac{(1)\,(-1)}{(1.0)}\right](10^{-12})$$

$$= -4.5\times 10^{-3}\ \text{J}$$

Example 18.16 *Two point charges $q_1 = q_2 = 2\,\mu C$ are fixed at $x_1 = +3\,m$ and $x_2 = -3\,m$ as shown in figure. A third particle of mass 1 g and charge $q_3 = -4\,\mu C$ are released from rest at $y = 4.0\,m$. Find the speed of the particle as it reaches the origin.*

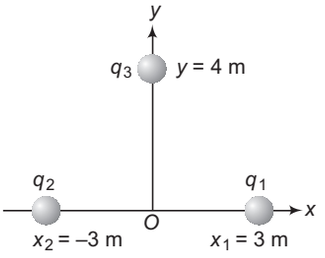


Fig. 18.21

How to Proceed Here the charge q_3 is attracted towards q_1 and q_2 both. So, the net force on q_3 is towards origin.

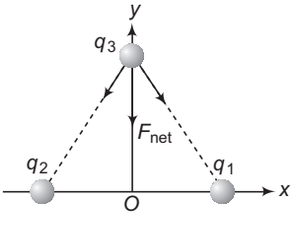


Fig. 18.22

By this force charge is accelerated towards origin, but this acceleration is not constant. So, to obtain the speed of particle at origin by kinematics, we will have to find first the acceleration at same intermediate position and then will have to integrate it with proper limits. On the other hand, it is easy to use energy conservation principle as the only forces are conservative.

Sol. Let v be the speed of particle at origin. From conservation of mechanical energy,

$$U_i + K_i = U_f + K_f$$

$$\text{or} \quad \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_i} + \frac{q_3q_1}{(r_{31})_i} + \frac{q_2q_1}{(r_{21})_i}\right] + 0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_3q_2}{(r_{32})_f} + \frac{q_3q_1}{(r_{31})_f} + \frac{q_2q_1}{(r_{21})_f}\right] + \frac{1}{2}mv^2$$

Here, $(r_{21})_i = (r_{21})_f$

Substituting the proper values, we have

$$(9.0\times 10^9)\left[\frac{(-4)\,(2)}{(5.0)} + \frac{(-4)\,(2)}{(5.0)}\right]\times 10^{-12}$$

$$= (9.0\times 10^9)\left[\frac{(-4)\,(2)}{(3.0)} + \frac{(-4)\,(2)}{(3.0)}\right]\times 10^{-12} + \frac{1}{2}\times 10^{-3}\times v^2$$

$$\therefore \quad (9\times 10^{-3})\left(-\frac{16}{5}\right) = (9\times 10^{-3})\left(-\frac{16}{3}\right) + \frac{1}{2}\times 10^{-3}\times v^2$$

$$(9\times 10^{-3})\,(16)\left(\frac{2}{15}\right) = \frac{1}{2}\times 10^{-3}\times v^2$$

$$\therefore \quad v = 6.2\ \text{m/s}$$

18.7 Electric Potential

As we have discussed in Article 18.5 that an electric field at any point can be defined in two different ways:

(i) by the field strength **E**.

(ii) by the electric potential V at the point under consideration.

Both **E** and V are functions of position and there is a fixed relationship between these two. Of these, the field strength **E** is a vector quantity while the electric potential V is a scalar quantity. In this article, we will discuss about the electric potential and in the next the relationship between **E** and V. **Potential** is the potential energy per unit charge. Electric potential at any point in an electric field is defined as the potential energy per unit charge, same as the field strength is defined as the force per unit charge. Thus,

$$V = \frac{U}{q_0} \quad \text{or} \quad U = q_0V$$

The SI unit of potential is **volt** (V), which is equal to joule per coulomb. So,

$$1\ \text{V} = 1\ \text{volt} = 1\ \text{J/C} = 1\ \text{joule/coulomb}$$

The work done by the electrostatic force in displacing a test charge q_0 from a to b in an electric field is defined as the negative of change in potential energy between them or

$$\Delta U = -W_{a-b}$$

$$\therefore \quad U_b - U_a = -W_{a-b}$$

We divide this equation by q_0,

$$\frac{U_b}{q_0} - \frac{U_a}{q_0} = -\frac{W_{a-b}}{q_0} \quad \text{or} \quad V_a - V_b = \frac{W_{a-b}}{q_0}$$

as

$$V = \frac{U}{q_0}$$

Thus, the work done per unit charge by the electric force, when a charged body moves from a to b is equal to the potential at a minus the potential at b. We sometimes abbreviate this difference as

$$V_{ab} = V_a - V_b.$$

Another way to interpret the potential difference V_{ab} is that the potential at a minus potential at b, equals the work that must be done to move a unit positive charge slowly from b to a against the electric force.

$$V_a - V_b = \frac{(W_{b-a})_{\text{external force}}}{q_0}$$

Absolute Potential at Some Points

Suppose we take the point b at infinity and as a reference point assign the value $V_b = 0$, the above equations can be written as

$$V_a - V_b = \frac{(W_{a-b})_{\text{electric force}}}{q_0} = \frac{(W_{b-a})_{\text{external force}}}{q_0}$$

or
$$V_a = \frac{(W_{a-\infty})_{\text{electric force}}}{q_0} = \frac{(W_{\infty-a})_{\text{external force}}}{q_0}$$

Thus, the absolute electric potential at point a in an electric field can be defined as the work done in displacing a unit positive charge from infinity to a by the external force or the work done per unit positive charge in displacing it from a to infinity.

Following three formulae are very useful in the problems related to work done in electric field.

$$(W_{a-b})_{\text{electric force}} = q_0 (V_a - V_b)$$
$$(W_{a-b})_{\text{external force}} = q_0 (V_b - V_a) = -(W_{a-b})_{\text{electric force}}$$
$$(W_{\infty-a})_{\text{external force}} = q_0 V_a$$

Here, q_0, V_a and V_b are to be substituted with sign.

Example 18.17 *The electric potential at point A is 20 V and at B is – 40 V. Find the work done by an external force and electrostatic force in moving an electron slowly from B to A.*

Sol. Here, the test charge is an electron, i.e.

$$q_0 = -1.6 \times 10^{-19} \text{ C}$$
$$V_A = 20 \text{ V and } V_B = -40 \text{ V}$$

Work done by external force

$$(W_{B-A})_{\text{external force}} = q_0 (V_A - V_B)$$
$$= (-1.6 \times 10^{-19}) [(20) - (-40)]$$
$$= -9.6 \times 10^{-18} \text{ J}$$

Work done by electric force

$$(W_{B-A})_{\text{electric force}} = -(W_{B-A})_{\text{external force}}$$
$$= -(-9.6 \times 10^{-18} \text{ J}) = 9.6 \times 10^{-18} \text{ J}$$

Here, we can see that the electron (a negative charge) moves from B (lower potential) to A (higher potential) and the work done by electric force is positive. Therefore, we may conclude that whenever a negative charge moves from a lower potential to higher potential work done by the electric force is positive or when a positive charge moves from lower potential to higher potential the work done by the electric force is negative.

Example 18.18 *Find the work done by some external force in moving a charge $q = 2\,\mu C$ from infinity to a point where electric potential is 10^4 V.*

Sol. Using the relation, $(W_{\infty-a})_{\text{external force}} = qV_a$

We have, $(W_{\infty-a})_{\text{external force}} = (2 \times 10^{-6})(10^4) = 2 \times 10^{-2}$ J

Electric Potential Due to a Point Charge q

From the definition of potential,

$$V = \frac{U}{q_0} = \frac{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,q_0}{r}}{q_0}$$

or
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

Here, r is the distance from the point charge q to the point at which the potential is evaluated.

If q is positive, the potential that it produces is positive at all points. If q is negative, it produces a potential that is negative everywhere. In either case, V is equal to zero at $r = \infty$.

Electric Potential Due to a System of Charges

Just as the electric field due to a collection of point charges is the vector sum of the fields produced by each charge, the electric potential due to a collection of point charges is the scalar sum of the potentials due to each charge.

$$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$$

Extra Knowledge Points

- In the equation $V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$ if the whole charge is at equal distance r_0 from the point where V is to be evaluated, then we can write

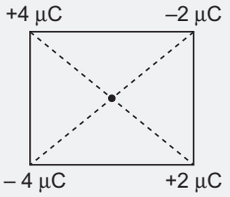
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_{\text{net}}}{r_0}$$

where, q_{net} is the algebraic sum of all the charges of which the system is made.

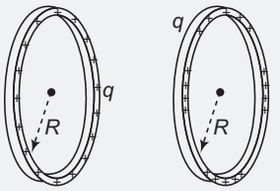
Here are few examples:

Example 1. Four charges are placed on the vertices of a square as shown in figure. The electric potential at centre of the square is zero as all the charges are at same distance from the centre and

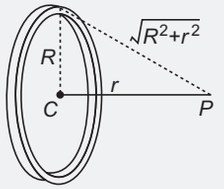
$q_{net} = 4\,\mu C - 2\,\mu C + 2\,\mu C - 4\,\mu C$
$= 0.$



Example 2. A charge q is uniformly distributed over the circumference of a ring in Fig. (a) and is non-uniformly distributed in Fig. (b).



The electric potential at the centre of the ring in both the cases is



$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$ (where, R = radius of ring)

and at a distance r from the centre of ring on its axis would be

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{R^2 + r^2}}$$

Example 18.19 *Three point charges $q_1 = 1\,\mu C$, $q_2 = -2\,\mu C$ and $q_3 = 3\,\mu C$ are placed at (1 m, 0, 0), (0, 2m, 0) and (0, 0, 3 m) respectively. Find the electric potential at origin.*

Sol. The net electric potential at origin is,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right]$$

Substituting the values, we have

$$V = (9.0\times 10^9)\left(\frac{1}{1.0} - \frac{2}{2.0} + \frac{3}{3.0}\right)\times 10^{-6}$$
$$= 9.0\times 10^3\ \text{V}$$

Example 18.20 *A charge $q = 10\,\mu C$ is distributed uniformly over the circumference of a ring of radius 3 m placed on x-y plane with its centre at origin. Find the electric potential at a point P(0, 0, 4 m).*

Sol. The electric potential at point P would be

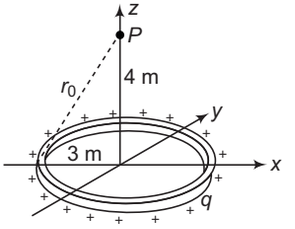


Fig. 18.23

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_0}$$

Here, r_0 = distance of point P from circumference ring
$= \sqrt{(3)^2 + (4)^2} = 5$ m and $q = 10\,\mu C = 10^{-5}$ C

Substituting the values, we have

$$V = \frac{(9.0\times 10^9)\,(10^{-5})}{(5.0)} = 1.8\times 10^4\ \text{V}$$

Variation of Electric Potential on the Axis of a Charged Ring

We have discussed in medical galaxy that the electric potential at the centre of a charged ring (whether charged uniformly or non-uniformly) is $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$ and at a distance r from the centre on the axis of the ring is $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{R^2 + r^2}}$.

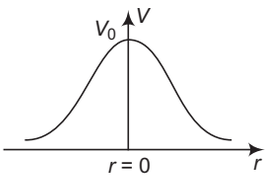


Fig. 18.24

From these expressions, we can see that electric potential is maximum at the centre and decreases as we move away from the centre on the axis. Thus, potential varies with distance r as shown in figure.

In the figure, $V_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$

Example 18.21 *Find out the points on the line joining two charges $+q$ and $-3q$ (kept at a distance of 1.0 m), where electric potential is zero.*

Sol. Let P be the point on the axis either to the left or to the right of charge $+q$ at a distance r where potential is zero. Hence,

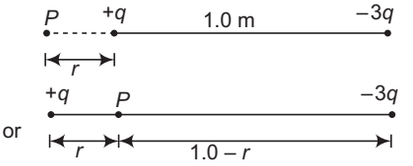


Fig. 18.25

$$V_P = \frac{q}{4\pi\varepsilon_0 r} - \frac{3q}{4\pi\varepsilon_0 (1+r)} = 0$$

Solving this, we get $r = 0.5$ m

Further, $$V_P = \frac{q}{4\pi\varepsilon_0 r} - \frac{3q}{4\pi\varepsilon_0 (1-r)} = 0,$$

which gives $r = 0.25$ m.

Thus, the potential will be zero at point P on the axis which is either 0.5 m to the left or 0.25 m to the right of charge $+q$.

18.8 Relation between Electric Field and Potential

Let us first consider the case when electric potential V is known and we want to calculate **E**. The relation is as under

In Case of Cartesian Coordinates

$$\mathbf{E} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}} + E_z \hat{\mathbf{k}}$$

Here, $E_x = -\frac{\partial V}{\partial x} = -$ (partial derivative of V w.r.t. x)

$$E_y = -\frac{\partial V}{\partial y} = -\text{ (partial derivative of } V \text{ w.r.t. } y)$$

$$E_z = -\frac{\partial V}{\partial z} = -\text{ (partial derivative of } V \text{ w.r.t. } z)$$

$$\therefore \quad \mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right]$$

This is also sometimes written as

$$\mathbf{E} = -\text{ gradient } V = -\text{ grad } V = -\nabla V$$

Example 18.22 *The electric potential in a region is represented as $V = 2x + 3y - z$. Obtain expression for electric field strength.*

Sol. $\mathrm{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathrm{i}} + \frac{\partial V}{\partial y}\hat{\mathrm{j}} + \frac{\partial V}{\partial z}\hat{\mathrm{k}}\right]$

Here, $$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(2x + 3y - z) = 2$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(2x + 3y - z) = 3$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(2x + 3y - z) = -1$$

$$\therefore \quad \mathrm{E} = -2\hat{\mathrm{i}} - 3\hat{\mathrm{j}} + \hat{\mathrm{k}}$$

We have determined how to calculate electric field **E** from the electrostatic potential V. Let us now consider how to calculate potential difference or absolute potential if electric field **E** is known. For this use the relation,

$$dV = -\mathrm{E}\cdot dr$$

or $$\int_A^B dV = -\int_A^B \mathrm{E}\cdot dr \quad \text{or} \quad V_B - V_A = -\int_A^B \mathrm{E}\cdot dr$$

Here, $$dr = dx\,\hat{\mathrm{i}} + dy\,\hat{\mathrm{j}} + dz\,\hat{\mathrm{k}}$$

When **E** is Uniform

Let us take this case with the help of an example.

Example 18.23 *Find V_{ab} in an electric field*

$$\mathbf{E} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\frac{N}{C}$$

where $\mathbf{r}_a = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})\ m$

and $\mathbf{r}_b = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})\ m.$

Sol. Here the given field is uniform (constant). So using,

$$dV = -\mathrm{E}\cdot dr$$

or $$V_{ab} = V_a - V_b = -\int_b^a \mathrm{E}\cdot dr$$

$$= -\int_{(2,\,1,-2)}^{(1,-2,\,1)} (2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + 4\hat{\mathrm{k}})\cdot(dx\,\hat{\mathrm{i}} + dy\,\hat{\mathrm{j}} + dz\,\hat{\mathrm{k}})$$

$$= -\int_{(2,\,1,-2)}^{(1,-2,\,1)} (2\,dx + 3\,dy + 4\,dz)$$

$$= -[2x + 3y + 4z]_{(2,\,1,\,-2)}^{(1,\,-2,\,1)} = -1\ \mathrm{V}$$

- In uniform electric field we can also apply, $V = Ed$

Here V is the potential difference between any two points, E the magnitude of electric field and d is the projection of the distance between two points along the electric field.

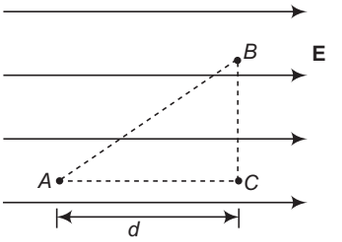


Fig. 18.26

For example, in the figure if we are interested in the potential difference between points A and B we will have to keep two points in mind,

(i) $V_A > V_B$ as electric lines always flow from higher potential to lower potential.

(ii) $d \neq AB$ but $d = AC$

Hence, in the above figure, $V_A - V_B = Ed$.

Example 18.24 *In uniform electric field* $\mathbf{E} = 10\ N/C$ *as shown in figure, find*

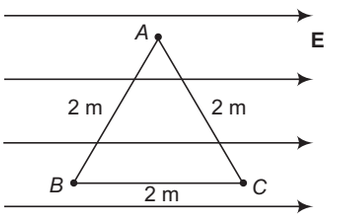


Fig. 18.27

(a) $V_A - V_B$ *(b)* $V_B - V_C$

Sol. (a) $V_B > V_A$, So, $V_A - V_B$ will be negative.

Further $d_{AB} = 2 \cos 60° = 1\,\text{m}$

$\therefore \quad V_A - V_B = -Ed_{AB} = (-10)\,(1) = -10\,\text{V}$

(b) $V_B > V_C$, so, $V_B - V_C$ will be positive

Further, $d_{BC} = 2.0\,\text{m}$

$\therefore \quad V_B - V_C = (10)\,(2) = 20\,\text{V}$

Example 18.25 *A uniform electric field of* 100 *V/m is directed at* 30° *with the positive x-axis as shown in figure. Find the potential difference* V_{BA} *if* $OA = 2$ *m and* $OB = 4$ *m.*

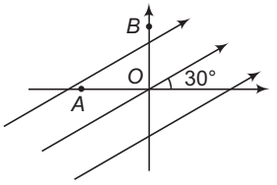


Fig. 18.28

Sol. This problem can be solved by both the methods as discussed above.

Method 1. Electric field in vector form can be written as,

$$\mathbf{E} = (100 \cos 30°\,\hat{\mathbf{i}} + 100 \sin 30°\,\hat{\mathbf{j}})\ \text{V/m}$$

$$= (50\sqrt{3}\,\hat{\mathbf{i}} + 50\hat{\mathbf{j}})\ \text{V/m}$$

$A \equiv (-2\,\text{m}, 0, 0)$ and $B \equiv (0, 4\,\text{m}, 0)$

$$\therefore \quad V_{BA} = V_B - V_A = -\int_A^B \mathbf{E}\cdot d\mathbf{r}$$

$$= -\int_{(-2\text{m},\,0,\,0)}^{(0,\,4\text{m},\,0)} (50\sqrt{3}\,\hat{\mathbf{i}} + 50\,\hat{\mathbf{j}})\cdot(dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}})$$

$$= -[50\sqrt{3}\,x + 50\,y]_{(-2\text{m},\,0,\,0)}^{(0,\,4\text{m},\,0)}$$

$$= -100\,(2 + \sqrt{3})\ \text{V}$$

Method 2. We can also use, $V = Ed$

With the view that $V_A > V_B$ or $V_B - V_A$ will be negative.

Here, $d_{AB} = OA \cos 30° + OB \sin 30°$

$$= 2 \times \frac{\sqrt{3}}{2} + 4 \times \frac{1}{2} = (\sqrt{3} + 2)$$

$$\therefore \quad V_B - V_A = -Ed_{AB} = -100\,(2 + \sqrt{3})$$

18.9 Equipotential Surfaces

The equipotential surfaces in an electric field have the same basic idea as topographic maps used by civil engineers or mountain climbers. On a topographic map, contour lines are drawn passing through the points having the same elevation. The potential energy of a mass m does not change along a contour line as the elevation is same everywhere.

By analogy to contour lines on a topographic map, an **equipotential surface** is a three dimensional surface on which the electric potential V is the same at every point on it. An equipotential surface has the following characteristics.

1. Potential difference between any two points in an equipotential surface is zero.
2. If a test charge q_0 is moved from one point to the other on such a surface, the electric potential energy q_0V remains constant.
3. No work is done by the electric force when the test charge is moved along this surface.
4. Two equipotential surfaces can never intersect each other because otherwise the point of intersection will have two potentials which is of course not acceptable.
5. As the work done by electric force is zero when a test charge is moved along the equipotential surface, it follows that **E** must be perpendicular to the surface at every point so that the electric force $q_0\,\mathbf{E}$ will always be perpendicular to the displacement of a charge moving on the surface. Thus, **field lines and equipotential surfaces are always mutually perpendicular.** Some equipotential surfaces are shown in Fig. 18.29.

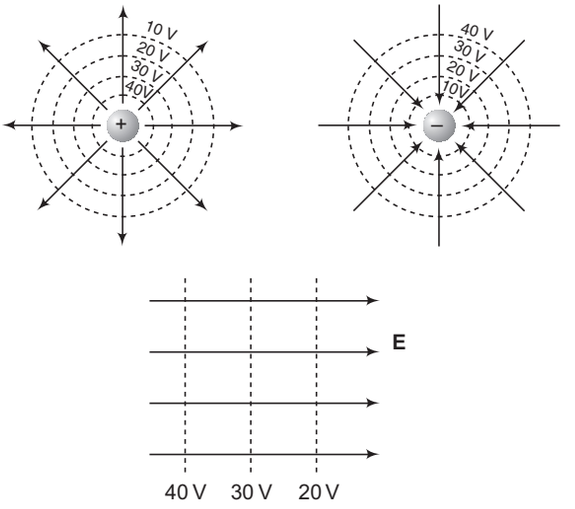


Fig. 18.29

The equipotential surfaces are a family of concentric spheres for a point charge or a sphere of charge and are a family of concentric cylinders for a line of charge or cylinder of charge. For a special case of a uniform field, where the field lines are straight, parallel and equally spaced the equipotentials are parallel planes perpendicular to the field lines.

Note Points

- While drawing the equipotential surfaces we should keep in mind the two main points.
  1. These are perpendicular to field lines at all places.
  2. Field lines always flow from higher potential to lower potential.

Example 18.26 *Equipotential spheres are drawn round a point charge. As we move away from the charge, will the spacing between two spheres having a constant potential difference decrease, increase or remain constant.*

Sol. $V_1 > V_2$

$$V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_1} \quad \text{and} \quad V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_2}$$

Now, $V_1 - V_2 = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$

$$\therefore \quad (r_2 - r_1) = \frac{(4\pi\varepsilon_0)(V_1 - V_2)}{q}(r_1\, r_2)$$

For a constant potential difference $(V_1 - V_2)$,

$$r_2 - r_1 \propto r_1 r_2$$

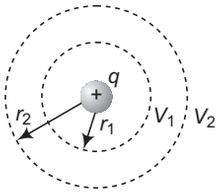


Fig. 18.30

i.e. the spacing between two spheres $(r_2 - r_1)$ increases as we move away from the charge, because the product $r_1 r_2$ will increase.

18.10 Electric Dipole

A pair of equal and opposite point charges $\pm q$, that are separated by a fixed distance is known as **electric dipole.** Electric dipole occurs in nature in a variety of situations. The hydrogen fluoride (HF) molecule is typical. When a hydrogen atom combines with a fluorine atom, the single electron of the former is strongly attracted to the latter and spends most of its time near the fluorine atom. As a result, the molecule consists of a strongly negative fluorine ion some (small) distance away from a strongly positive ion, though the molecule is electrically neutral overall.

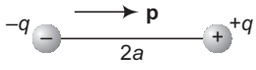


Fig. 18.31

Every electric dipole is characterized by its **electric dipole moment** which is a vector **p** directed from the negative to the positive charge.

The magnitude of dipole moment is $p = (2a)\, q$

Here, $2a$ is the distance between the two charges.

Electric Potential and Field Due to an Electric Dipole

Consider an electric dipole lying along positive y-direction with its centre at origin.

$$\mathbf{p} = 2aq\ \hat{\mathbf{j}}$$

The electric potential due to this dipole at point $A\ (x, y, z)$ as shown is simply the sum of the potentials due to the two charges. Thus,

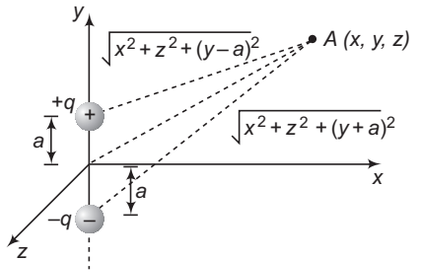


Fig. 18.32

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2 + (y-a)^2 + z^2}} - \frac{q}{\sqrt{x^2 + (y+a)^2 + z^2}}\right]$$

By differentiating this function, we obtain the electric field of the dipole.

$$E_x = -\frac{\partial V}{\partial x} = \frac{q}{4\pi\varepsilon_0}\left\{\frac{x}{[x^2 + (y-a)^2 + z^2]^{3/2}} - \frac{x}{[x^2 + (y+a)^2 + z^2]^{3/2}}\right\}$$

$$E_y = -\frac{\partial V}{\partial y} = \frac{q}{4\pi\varepsilon_0}\left\{\frac{y-a}{[x^2 + (y-a)^2 + z^2]^{3/2}} - \frac{y+a}{[x^2 + (y+a)^2 + z^2]^{3/2}}\right\}$$

$$E_z = -\frac{\partial V}{\partial z} = \frac{q}{4\pi\varepsilon_0}\left\{\frac{z}{[x^2 + (y-a)^2 + z^2]^{3/2}} - \frac{z}{[x^2 + (y+a)^2 + z^2]^{3/2}}\right\}$$

Special Cases

On the Axis of the Dipole (i.e. along y-axis)

$$x = 0, \quad z = 0$$

$$\therefore \quad V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{y-a} - \frac{1}{y+a}\right] = \frac{2aq}{4\pi\varepsilon_0\,(y^2 - a^2)}$$

or $$V = \frac{p}{4\pi\varepsilon_0\,(y^2 - a^2)} \qquad (\text{as } 2aq = p)$$

i.e. at a distance r from the centre of the dipole $(y = r)$

$$V = \frac{p}{4\pi\varepsilon_0\,(r^2 - a^2)} \text{ or } V_{\text{axis}} \approx \frac{p}{4\pi\varepsilon_0 r^2} \qquad (\text{for } r >> a)$$

V is positive when the point under consideration is towards positive charge and negative if it is towards negative charge.

Moreover the components of electric field are as under,

$$E_x = 0, \quad E_z = 0 \qquad (\text{as } x = 0, \ z = 0)$$

and
$$E_y = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(y-a)^2} - \frac{1}{(y+a)^2}\right]$$
$$= \frac{4ayq}{4\pi\varepsilon_0 (y^2 - a^2)^2}$$

or
$$E_y = \frac{1}{4\pi\varepsilon_0}\frac{2py}{(y^2 - a^2)^2}$$

- That E_y is along positive y-direction or parallel to **p**.

Further, at a distance r from the centre of the dipole ($y = r$).

$$E_y = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2 - a^2)^2}$$

or
$$E_{\text{axis}} \approx \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3} \qquad (\text{for } r >> a)$$

On the Perpendicular Bisector of Dipole

Say along x-axis (it may be along z-axis also).

$$y = 0, \quad z = 0$$

$$\therefore \qquad V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2 + a^2}} - \frac{q}{\sqrt{x^2 + a^2}}\right] = 0$$

or
$$V_{\perp \text{ bisector}} = 0$$

Moreover the components of electric field are as under,

$$E_x = 0, \qquad E_z = 0$$

and
$$E_y = \frac{q}{4\pi\varepsilon_0}\left\{\frac{-a}{(x^2 + a^2)^{3/2}} - \frac{a}{(x^2 + a^2)^{3/2}}\right\}$$
$$= \frac{-2aq}{4\pi\varepsilon_0 (x^2 + a^2)^{3/2}}$$

or
$$E_y = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{(x^2 + a^2)^{3/2}}$$

Here, negative sign implies that the electric field is along negative y-direction or antiparallel to **p**.

Further, at a distance r from the centre of dipole ($x = r$), the magnitude of electric field is,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2 + a^2)^{3/2}} \quad \text{or} \quad E_{\perp \text{ bisector}} \approx \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3}$$

(for $r >> a$)

Electric Dipole in Uniform Electric Field

As we have said earlier also a uniform electric field means, at every point the direction and magnitude of electric field is constant. A uniform electric field is shown by parallel equidistant lines. The field due to a point charge or due to an electric dipole is non-uniform in nature. Uniform electric field is found between the plates of a parallel plate capacitor. Now, let us discuss the behaviour of a dipole in uniform electric field.

Force on Dipole

Suppose an electric dipole of dipole moment $|\mathbf{p}| = 2aq$ is placed in a uniform electric field **E** at an angle θ. Here, θ is the angle between **p** and **E**. A force $\mathbf{F}_1 = q\mathbf{E}$ will act on positive charge and $\mathbf{F}_2 = -q\mathbf{E}$ on negative charge. Since, $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal in magnitude but opposite in direction. Hence,

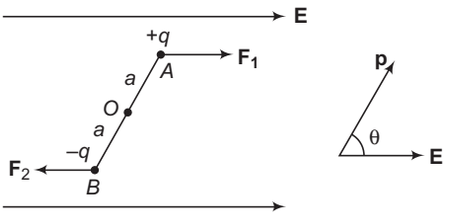


Fig. 18.33

$$\mathbf{F}_1 + \mathbf{F}_2 = 0 \quad \text{or} \quad \mathbf{F}_{\text{net}} = 0$$

Thus, net force on a dipole in uniform electric field is zero. While in a non-uniform electric field it may or may not be zero.

Torque on Dipole

The torque of $\mathbf{F}_1$ about O,

$$\tau_1 = \mathbf{OA} \times \mathbf{F}_1 = q\,(\mathbf{OA} \times \mathbf{E})$$

and torque of $\mathbf{F}_2$ about O is,

$$\tau_2 = \mathbf{OB} \times \mathbf{F}_2 = -q\,(\mathbf{OB} \times \mathbf{E}) = q\,(\mathbf{OB} \times \mathbf{E})$$

The net torque acting on the dipole is,

$$\tau = \tau_1 + \tau_2 = q\,(\mathbf{OA} \times \mathbf{E}) + q\,(\mathbf{BO} \times \mathbf{E})$$
$$= q\,(\mathbf{OA} + \mathbf{BO}) \times \mathbf{E} = q\,(\mathbf{BA} \times \mathbf{E})$$

or
$$\tau = \mathbf{p} \times \mathbf{E}$$

Thus, the magnitude of torque is $\tau = pE \sin\theta$. The direction of torque is perpendicular to the plane of paper inwards. Further this torque is zero at $\theta = 0°$ or $\theta = 180°$, i.e. when the dipole is parallel or antiparallel to **E** and maximum at $\theta = 90°$.

Potential Energy of Dipole

When an electric dipole is placed in an electric field **E**, a torque $\tau = \mathbf{p} \times \mathbf{E}$ acts on it. If we rotate the dipole through a small angle $d\theta$, the work done by the torque is

$$dW = \tau\, d\theta$$

$$dW = -\,pE \sin\theta\, d\theta$$

The work is negative as the rotation $d\theta$ is opposite to the torque. The change in electric potential energy of the dipole is therefore

$$dU = -\,dW = pE \sin\theta\, d\theta$$

Now at angle $\theta = 90°$, the electric potential energy of the dipole may be assumed to be zero as net work done by the electric forces in bringing the dipole from infinity to this position will be zero. Integrating,

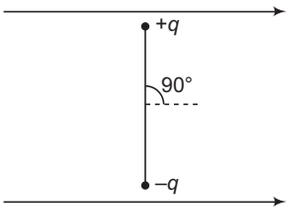


Fig. 18.34

$$dU = pE \sin\theta \, d\theta$$

From 90° to θ, we have

$$\int_{90°}^{\theta} dU = \int_{90°}^{\theta} pE \sin\theta \, d\theta$$

or $U(\theta) - U(90°) = pE\,[-\cos\theta]_{90°}^{\theta}$

$\therefore \quad U(\theta) = -pE\cos\theta = -\mathbf{p}\cdot\mathbf{E}$

If the dipole is rotated from an angle θ_1 to θ_2, then work done by external forces $= U(\theta_2) - U(\theta_1)$

or $W_{\text{external forces}} = -pE\cos\theta_2 - (-PE\cos\theta_1)$

or $W_{\text{external forces}} = pE(\cos\theta_1 - \cos\theta_2)$

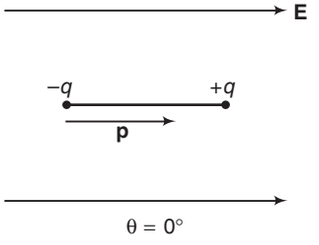
and work done by electric forces,

$$W_{\text{electric force}} = -W_{\text{external force}} = pE(\cos\theta_2 - \cos\theta_1)$$

Equilibrium of Dipole

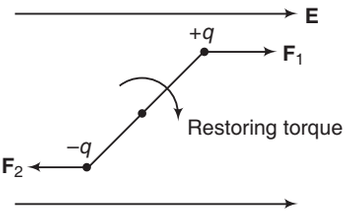
When an electric dipole is placed in a uniform electric field, net force on it is zero for any position of the dipole in the electric field. But torque acting on it is zero only at $\theta = 0°$ and 180°. Thus, we can say that at these two positions of the dipole, net force or torque on it is zero or the dipole is in equilibrium. Of this $\theta = 0°$ is the **stable equilibrium position** of the dipole because potential energy in this position is minimum ($U = -pE\cos 0° = -pE$) and when displaced from this position a torque starts acting on it which is restoring in nature and which has a tendency to bring the dipole back in its equilibrium position.

On the other hand at $\theta = 180°$, the potential energy of the dipole is maximum ($U = -pE\cos 180° = +pE$) and when it is displaced from this position, the torque has a tendency to rotate it in other direction. This torque is not restoring in nature. So this equilibrium is known as **unstable equilibrium position.**

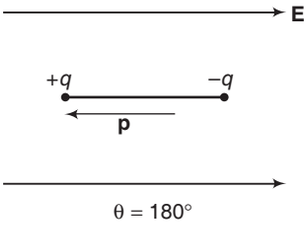


U = minimum $= -PE$

$F_{net} = 0, \tau = 0$

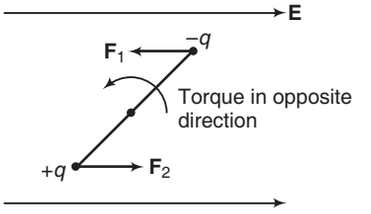


When displaced from mean position a restoring torque acts on the



U = maximum $= +pE$

$F_{\text{net}} = 0, \tau = 0$



When displaced from mean position, torque acts in opposite direction.

Fig. 18.35

Extra Knowledge Points

- **As there are too many formulae in electric dipole, we have summarised them as under**



$|\mathbf{p}| = (2a)q$

- **On the axis of dipole**

(i) $V = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{p}{r^2 - a^2}$ or $V_{\text{axis}} \approx \dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{r^2}$ if $r >> a$

(ii) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2pr}{(r^2 - a^2)^2}$ (along **p**)

or $E_{\text{axis}} \approx \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{2p}{r^3}$ for $r >> a$

- **On the perpendicular bisector of dipole**

(i) $V = 0$

(ii) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{(r^2 + a^2)^{3/2}}$ (opposite to **p**)

$E \approx \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{p}{r^3}$ (for $r >> a$)

▪ **Dipole in uniform electric field**

(i) $\mathbf{F}_{net} = 0$

(ii) $\tau = \mathbf{p} \times \mathbf{E}$ and $|\tau| = pE \sin\theta$

(iii) $U(\theta) = -\mathbf{pE} = -pE\cos\theta$ with $U(90°) = 0$

(iv) $(W_{\theta_1 \to \theta_2})_{\text{external force}} = pE(\cos\theta_2 - \cos\theta_1)$

(v) $(W_{\theta_1 \to \theta_2})_{\text{electric force}} = pE(\cos\theta_2 - \cos\theta_1)$
$= -(W_{\theta_1 \to \theta_2})_{\text{external force}}$

(vi) At $\theta = 0°$, $\mathbf{F}_{net} = 0$, $\tau_{net} = 0$, U = minimum
(stable equilibrium position)

(vii) At $\theta = 180°$, $\mathbf{F}_{net} = 0$, $\tau_{net} = 0$, U = maximum
(unstable equilibrium position)

Example 18.27 *An electric dipole of dipole moment p is placed in a uniform electric field E in stable equilibrium position. Its moment of inertia about the centroidal axis is I. If it is displaced slightly from its mean position, find the period of small oscillations.*

Sol. When displaced at an angle θ, from its mean position the magnitude of restoring torque is,

$$\tau = -pE\sin\theta$$

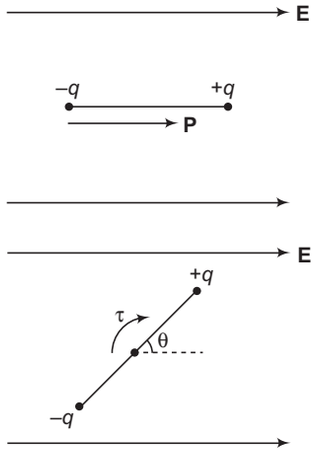


Fig. 18.36

For small angular displacement, $\sin\theta \approx \theta$

$$\tau = -pE\theta$$

The angular acceleration is,

$$\alpha = \frac{\tau}{I} = -\left(\frac{pE}{I}\right)\theta = -\omega^2\theta$$

where $\omega^2 = \dfrac{pE}{I}$

$\therefore$ $T = 2\pi\sqrt{\dfrac{I}{pE}}$

18.11 Gauss's Law

Gauss's law is a tool of simplifying electric field calculations where there is symmetrical distribution of charge. Gauss's law is a relation between the field at all the points on the surface and the total charge enclosed within the surface.

Electric Flux

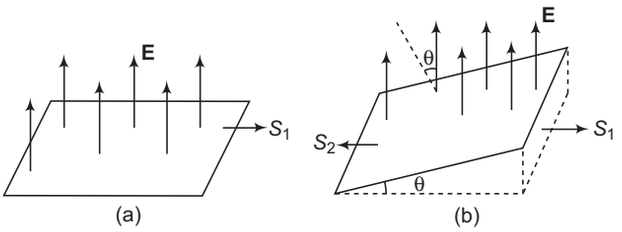


Fig. 18.37

Electric flux (ϕ_e) is a measure of the number of field lines crossing a surface.

Fig. (a) shows a planar surface of area S_1 that is perpendicular to a uniform electric field E. The number of field lines passing through per unit area (N/S_1) will be proportional to the electric field.

$$\frac{N}{S_1} \propto E$$

$\therefore$ $N \propto ES_1$ …(i)

The quantity ES_1 is the **electric flux** through S_1. It is a scalar quantity and has the SI unit N-m^2/C. Eq. (i) can also be written as

$$N \propto \phi_e$$

Now let us consider a planar surface that is not perpendicular to the field. How would the electric flux then be represented Fig. (b) shows a surface S_2 that is inclined at an angle θ to the electric field. Its projection perpendicular to **E** is S_1. The areas are related by

$$S_2\cos\theta = S_1$$

Because the same number of field lines cross both S_1 and S_2, the fluxes through both surfaces must be the same. The flux through S_2 is therefore,

$$\phi_e = ES_1 = ES_2\cos\theta$$

Designating $\hat{\mathbf{n}}_2$ as a unit vector normal to S_2, we obtain

$$\phi_e = \mathbf{E}\cdot\hat{\mathbf{n}}_2 S_2$$

This result can be easily generalized to the case of an arbitrary electric field **E** varying over an arbitrary surface S.

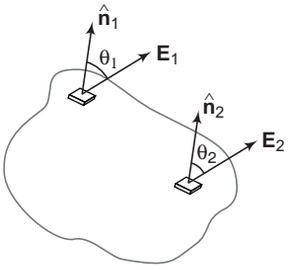


Fig. 18.38

We first divide S into infinitesimal elements with areas $dS_1, dS_2, \ldots$, etc. and unit normals $\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2, \ldots$, etc. Since, the elements are infinitesimal, they may be assumed to be planar and **E** may be taken as constant over any element. The flux $d\phi_e$ through an area dS is given by,

$$d\phi_e = E\, dS \cos\theta = \mathbf{E}\cdot\hat{\mathbf{n}}\, dS$$

The net flux is the sum of the infinitesimal flux elements over the entire surface.

$$\therefore \qquad \phi_e = \int_S \mathbf{E}\cdot\hat{\mathbf{n}}\, dS \qquad \text{(open surface)}$$

To distinguish between the flux through an open surface and the flux through a closed surface, we represent flux for the latter case by,

$$\phi_e = \oint_S \mathbf{E}\cdot\hat{\mathbf{n}}\, dS \qquad \text{(closed surface)}$$

Direction of $\hat{\mathbf{n}}$

Since, $\hat{\mathbf{n}}$ is a unit vector normal to a surface, it has two possible directions at every point on that surface. On a closed surface $\hat{\mathbf{n}}$ is usually chosen to be the outward normal at every point. For an open surface we can use either direction, as long as we are consistent over the entire surface. It is sometimes helpful to visualize electric flux as the flow of the field lines through a surface. When field lines leave (or flow out of) a closed surface, ϕ_e is positive, when they enter (or flow into) the surface, ϕ_e is negative. But remember that field lines are just a visual aid, these are just imaginary lines.

Here are two special cases for calculating the electric flux passing through a surface S of finite size (whether closed or open)

***Case* 1.** $\phi_e = ES$

If at every point on the surface, the magnitude of electric field is constant and perpendicular (to the surface).

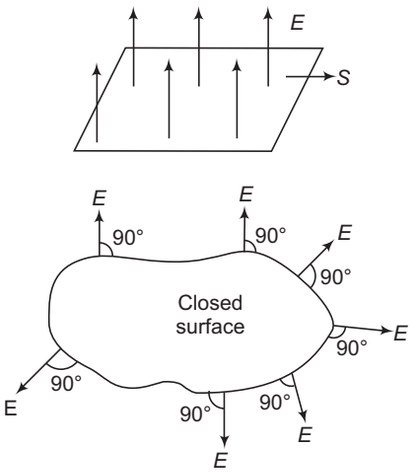


Fig. 18.39

***Case* 2.** $\phi_e = 0$

If at all points on the surface the electric field is tangential to the surface.

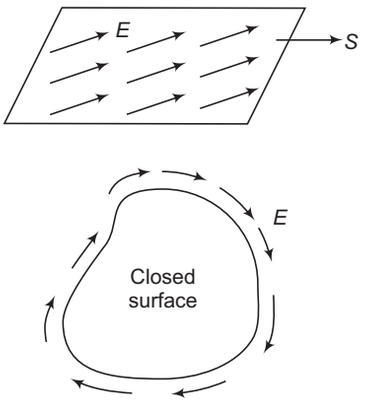


Fig. 18.40

Gauss's Law

This law gives a relation between the net electric flux through a closed surface and the charge enclosed by the surface.

According to this law, the net electric flux through any closed surface is equal to the net charge inside the surface divided by ε_0. In symbols it can be written as,

$$\phi_e = \oint_S \mathbf{E}\cdot\hat{\mathbf{n}}\, dS = \frac{q_{\text{in}}}{\varepsilon_0} \qquad \ldots\text{(i)}$$

where, q_{in} represents the net charge inside the surface and **E** represents the electric field at any point on the surface.

In principle Gauss's law is valid for the electric field of any system of charges or continuous distribution of charge. In practice however, the technique is useful for calculating the electric field only in situations where the degree of symmetry is high. Gauss's law can be used to evaluate the electric field for charge distributions that have spherical, cylindrical or plane symmetry.

Gauss's law in simplified form can be written as under,

$$ES = \frac{q_{\text{in}}}{\varepsilon_0} \qquad \ldots\text{(ii)}$$

but this form of Gauss's law is applicable only under following two conditions :

(i) the electric field at every point on the surface is either perpendicular or tangential.

(ii) magnitude of electric field at every point where it is perpendicular to the surface has a constant value (say E).

Here S is the area where electric field is perpendicular to the surface.

Applications of Gauss's Law

As Gauss's law does not provide expression for electric field but provides only for its flux through a closed surface. To calculate E we choose an imaginary closed surface (called Gaussian surface) in which Eq. (i) or (ii) can be applied easily. In most of the cases we will use Eq. (ii). Let us discuss few simple cases.

Electric Field due to a Point Charge

The electric field due to a point charge is every where radial. We wish to find the electric field at a distance r from the charge q. We select Gaussian surface, a sphere at distance r from the charge. At every point of this sphere the electric field has the same magnitude E and it is perpendicular to the surface itself. Hence, we can apply the simplified form of Gauss law,

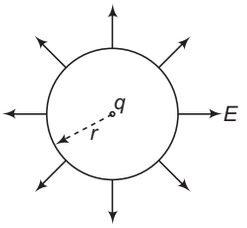


Fig. 18.41

$$ES = \frac{q_{in}}{\varepsilon_0}$$

Here, S = area of sphere = $4pr^2$ and q_{in} = net charge enclosing the Gaussian surface = q

$$\therefore \quad E\,(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

It is nothing but Coulomb's law.

Electric Field due to a Linear Charge Distribution

Consider a long line charge with a linear charge density (charge per unit length) λ. We have to calculate the electric field at a point, a distance r from the line charge. We construct a Gaussian surface, a cylinder of any arbitrary length l of radius r and its axis coinciding with the axis of the line charge. This cylinder have three surfaces. One is curved surface and the two plane parallel surfaces. Field lines at plane parallel surfaces are tangential (so flux passing through these surfaces is zero). The magnitude of electric field is having the same magnitude (say E) at curved surface and simultaneously the electric field is perpendicular at every point of this surface.

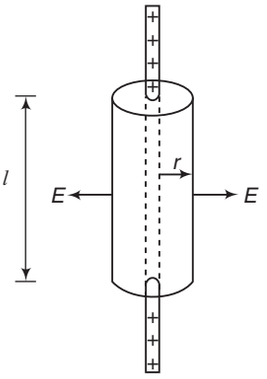
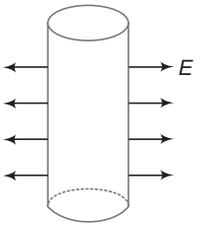


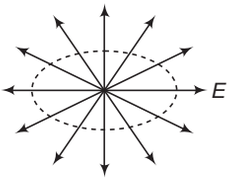
Fig. 18.42

Hence, we can apply the Gauss law as,

$$ES = \frac{q_{in}}{\varepsilon_0}$$



Curved surface



Plane surface

Fig. 18.43

Here, S = area of curved surface = $(2\pi rl)$
and q_{in} = net charge enclosing this cylinder = λl

$$\therefore \quad E\,(2\pi rl) = \frac{\lambda l}{\varepsilon_0}$$

$$\therefore \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

i.e. $$E \propto \frac{1}{r}$$

or E-r graph is a rectangular hyperbola as shown in Fig. 18.44

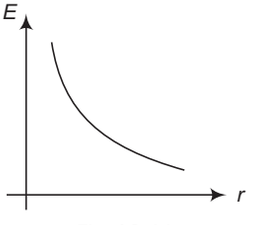


Fig. 18.44

Electric Field due to a Plane Sheet of Charge

Figure shows a portion of a flat thin sheet, infinite in size with constant surface charge density σ (charge per unit area). By symmetry, since, the sheet is infinite, the field must have the same magnitude and the opposite directions at two points equidistant from the sheet on opposite sides. Let us draw a Gaussian surface (a cylinder) with one end on one side and other end on the other side and of cross sectional area S_0. Field lines will be tangential to the curved surface, so flux passing through this surface is zero. At plane surfaces electric field has same magnitude and perpendicular to surface.

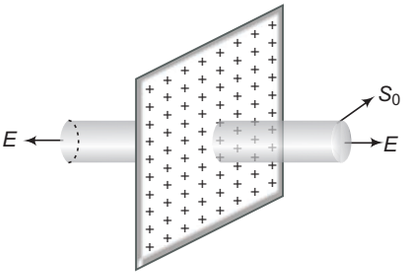


Fig. 18.45

Hence, using, $ES = \frac{q_{in}}{\varepsilon_0}$

$\therefore \quad E(2S_0) = \frac{(\sigma)(S_0)}{\varepsilon_0}$

$\therefore \quad E = \frac{\sigma}{2\varepsilon_0}$

Thus, we see that the magnitude of the field is independent of the distance from the sheet. Practically an infinite sheet of charge does not exist. This result is correct for real charge sheets if points under consideration are not near the edges and the distances from the sheet are small compared to the dimensions of sheet.

Electric Field near a Charged Conducting Surface

When a charge is given to a conducting plate, it distributes itself over the entire outer surface of the plate. The surface density σ is uniform and is the same on both surfaces if plate is of uniform thickness and of infinite size.

This is similar to the previous one the only difference is that this time charges are on both sides. Hence, applying,

$$ES = \frac{q_{in}}{\varepsilon_0}$$

Here, $S = 2S_0$ and $q_{in} = (\sigma)(2S_0)$

$\therefore \quad E(2S_0) = \frac{(\sigma)(2S_0)}{\varepsilon_0}$

$\therefore \quad E = \frac{\sigma}{\varepsilon_0}$

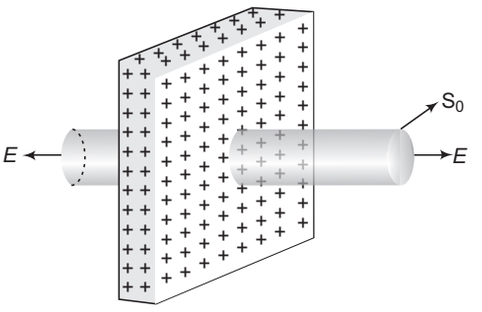


Fig. 18.46

Thus, field due to a charged conducting plate is twice the field due to plane sheet of charge. It also has same limitations.

Later we will see that the electric field near a charged conducting surface of any shape is $\sigma\varepsilon_0$ and it is normal to the surface.

Extra Knowledge Points

- In case of closed symmetrical body with charge q at its centre, the electric flux linked with each half will be $\frac{\phi_e}{2} = \frac{q}{2\varepsilon_0}$. If the symmetrical closed body has n identical faces with point charge at its centre, flux linked with each face will be $\frac{\phi_e}{n} = \frac{q}{n\,\varepsilon_0}$.

Example 18.28 *An electric dipole is placed at the centre of a sphere. Find the electric flux passing through the sphere.*

Sol. Net charge inside the sphere, $q_{in} = 0$. Therefore, according to Gauss's law net flux passing through the sphere is zero.

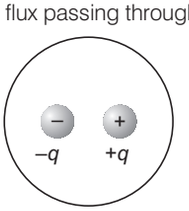


Fig. 18.47

Example 18.29 *A point charge q is placed at the centre of a cube. What is the flux linked,*
(a) with all the faces of the cube?
(b) with each face of the cube?
(c) if charge is not at the centre, then what will be the answers of parts (a) and (b)?

Sol. (a) According to Gauss,s law,

$$\phi_{total} = \frac{q_{in}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

(b) The cube is a symmetrical body with 6 faces and the point charge is at its centre, so electric flux linked with each face will be,

$$\phi_{\text{each face}} = \frac{\phi_{\text{total}}}{6} = \frac{q}{6\,\varepsilon_0}$$

(c) If charge is not at the centre, the answer of part (a) will remain same while that of part (b) will change.

18.12 Properties of a Conductor

Conductors (such as metals) possess free electrons. If a resultant electric field exists in the conductor these free charges will experience a force which will set a current flow. When no current flows, the resultant force and the electric field must be zero. Thus, under **electrostatic conditions** the value of **E** at all points within a conductor is zero. This idea, together with the Gauss's law can be used to prove several interesting facts regarding a conductor.

Excess Charge on a Conductor Resides on its Outer Surface

Consider a charged conductor carrying a charge q and no currents are flowing in it. Now, consider a Gaussian surface inside the conductor everywhere on which $\mathbf{E} = 0$. Thus, from Gauss's law,

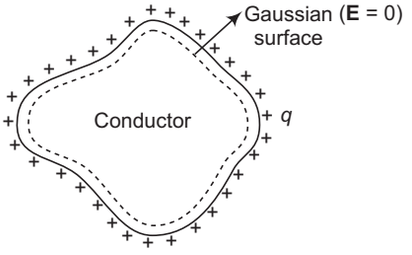


Fig. 18.48

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q_{\text{in}}}{\varepsilon_0}$$

we get, $q_{\text{in}} = 0$, as $\mathbf{E} = 0$

Thus, the sum of all charges inside the Gaussian surface is zero. This surface can be taken just inside the surface of the conductor, hence, any charge on the conductor must be on the surface of the conductor. In other words,

Under electrostatic conditions, the excess charge on a conductor resides on its outer surface.

Electric Field at any Point Close to the Charged Conductor is $\frac{\sigma}{\varepsilon_0}$

Consider a charged conductor of irregular shape. In general, surface charge density will vary from point to point. At a small surface ΔS, let us assume it to be a constant σ. Let us construct a Gaussian surface in the form of a cylinder of cross-section ΔS. One plane face of the cylinder is inside the conductor and other outside the conductor close to it. The surface inside the conductor does not contribute to the flux as **E** is zero everywhere inside the conductor. The curved surface outside the conductor also does not contribute to flux as **E** is always normal to the charged conductor and hence, parallel to the curved surface. Thus, the only contribution to the flux is through the plane face outside the conductor. Thus, from Gauss's law,

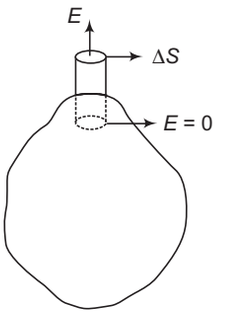


Fig. 18.49

$$\oint_S \mathbf{E} \cdot \mathbf{S} = \frac{q_{\text{in}}}{\varepsilon_0} \text{ or } E \cdot \Delta S = \frac{(\sigma)\,(\Delta S)}{\varepsilon_0} \text{ or } E = \frac{\sigma}{\varepsilon_0}$$

Note Points

- Electric field changes discontinuously at the surface of a conductor. Just inside the conductor, it is zero and just outside the conductor it is $\frac{\sigma}{\varepsilon_0}$. In fact, the field gradually decreases from $\frac{\sigma}{\varepsilon_0}$ to zero in a small thickness of about 4 to 5 atomic layers at the surface.
- For a non-uniform conductor the surface charge density (σ) varies inversely as the radius of curvature (ρ) of that part of the conductor, i.e.

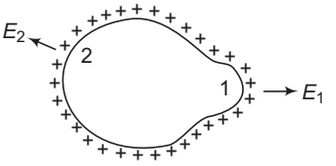


Fig. 18.50

$$\sigma \propto \frac{1}{\text{radius of curvature}\,(\rho)}$$

For example in the figure,

$$\rho_1 < \rho_2 \Rightarrow \therefore\ \sigma_1 > \sigma_2 \text{ or } E_1 > E_2 \text{ as } E = \frac{\sigma}{\varepsilon_0}.$$

Electric Field and Field Lines are Normal to the Surface of a Conductor

Net field inside a conductor is zero. It implies that no field lines enter a conductor. On the surface of a conductor, electric field and hence field lines are normal to the surface of the conductor.

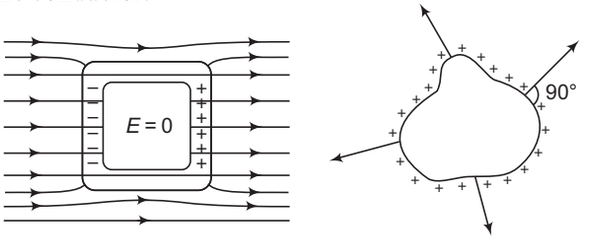


Fig. 18.51

If a conducting box is immersed in a uniform electric field, the field lines near the box are somewhat distorted. Similarly if a conductor is positively charged, the field lines originate from the surface and are normal at every point and if it is negatively charged the field lines terminate on the surface normally at every point.

Cavity Inside a Conductor

Consider a charge $+q$ suspended in a cavity in a conductor. Consider a Gaussian surface just outside the cavity and inside the conductor. $\mathbf{E} = 0$ on this Gaussian surface as it is inside the conductor. Hence, from Gauss's law,

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q_{\text{in}}}{\varepsilon_0} \qquad \text{gives } q_{\text{in}} = 0$$

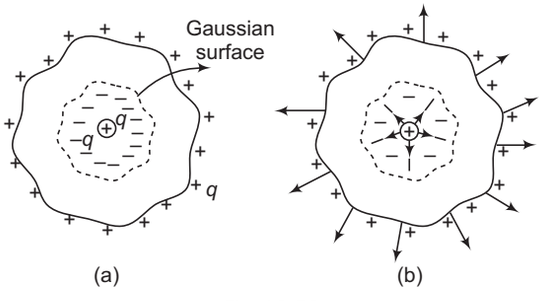


Fig. 18.52

This concludes that a charge of $-q$ must reside on the metal surface of the cavity so that the sum of this induced charge $-q$ and the original charge $+q$ within the Gaussian surface is zero. In other words, a charge q suspended inside a cavity in a conductor induces an equal and opposite charge $-q$ on the surface of the cavity. Further as the conductor is electrically neutral a charge $+q$ is induced on the outer surface of the conductor. As field inside the conductor is zero the field lines coming from q cannot penetrate into the conductor. The field lines will be as shown in Fig. (b).

The same line of approach can be used to show that the field inside the cavity of a conductor is zero when no charge is suspended in it.

Electrostatic shielding Suppose we have a very sensitive electronic instrument that we want to protect from external electric fields that might cause wrong measurements.

We surround the instrument with a conducting box or we keep the instrument inside the cavity of a conductor. By doing this charge in the conductor is so distributed that the net electric field inside the cavity becomes zero and the instrument is protected from the external fields. This is called electronic shielding.

The Potential of a Charged Conductor throughout its Volume is same

In any region in which $\mathbf{E} = 0$ at all points, such as the region vary far from all charges or the interior of a charged conductor, the line integral of $\mathbf{E}$ is zero along any path. It means that the potential difference between any two points in the conductor are at the same potential or the interior of a charged conductor is an equipotential region.

18.13 Electric Field and Potential due to Charged Spherical Shell of Solid Conducting Sphere

Electric Field

At all points inside the charged spherical conductor or hollow spherical shell, electric field $\mathbf{E} = 0$, as there is no charge inside such a sphere. In an isolated charged spherical conductor any excess charge on it is distributed uniformly over its outer surface same as that of charged spherical shell or hollow sphere.

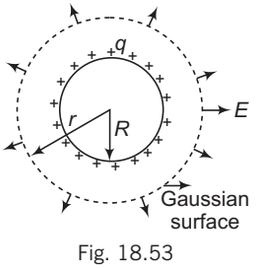


Fig. 18.53

The field at external points has the same symmetry as that of a point charge. We can construct a Gaussian surface (a sphere) of radius $r > R$. At all points of this sphere the magnitude of electric field is the same and its direction is perpendicular to the surface. Thus, we can apply

$$ES = \frac{q_{in}}{\varepsilon_0} \quad \text{or} \quad E\,(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\therefore \qquad E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

Hence, the electric field at any external point is the same as if the total charge is concentrated at centre.

At the surface of sphere $r = R$,

$$\therefore \qquad E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}$$

Thus, we can write, $E_{inside} = 0$

$$E_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

$$E_{outside} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

The variation of electric field (E) with the distance from the centre (r) is as shown in Fig. 18.54.

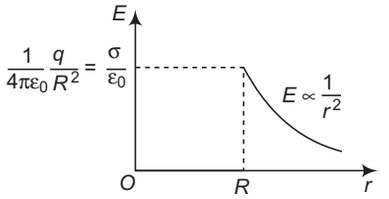


Fig. 18.54

Note Points

- At the surface graph is discontinuous
- $E_{surface} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} = \frac{q/4\pi R^2}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

Potential At external points the potential at any point is the same when the whole charge is assumed to be concentrated at the centre. At the surface of the sphere, $r = R$

$$\therefore \qquad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$

At some internal point electric field is zero everywhere, therefore, the potential is same at all points which is equal to the potential at surface. Thus, we can write,

$$V_{inside} = V_{surface} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R} \text{ and } V_{outside} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

The potential (V) varies with the distance from the centre (r) as shown in Fig. 18.55.

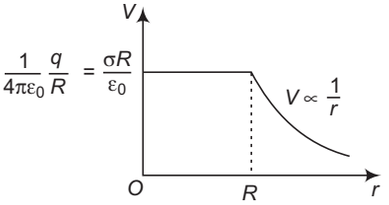


Fig. 18.55

18.14 Electric Field and Potential due to a Solid Sphere of Charge

Electric Field

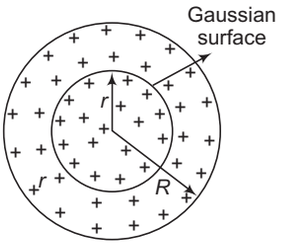


Fig. 18.56

Positive charge q is uniformly distributed throughout the volume of a solid sphere of radius R. For finding the electric field at a distance $r\,(< R)$ from the centre let us choose as our Gaussian surface a sphere of radius r, concentric with the charge distribution. From symmetry the magnitude E of electric field has the same value at every point on the Gaussian surface, and the direction of **E** is radial at every point on the surface. So, applying Gauss's law,

$$ES = \frac{q_{in}}{\varepsilon_0} \qquad \ldots(i)$$

Here, $S = 4\pi r^2$

and $q_{in} = (\rho)\left(\frac{4}{3}\pi r^3\right)$

Here, ρ = charge per unit volume $= \dfrac{q}{\frac{4}{3}\pi R^3}$

Substituting these values in Eq. (i)

We have, $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^3}\cdot r$ or $E \propto r$

At the centre $r = 0$, so $E = 0$

At surface $r = R$, so $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}$

To find the electric field outside the charged sphere, we use a spherical Gaussian surface of radius $r\,(> R)$. This surface encloses the entire charged sphere, so $q_{in} = q$, and Gauss's law gives

$$E\,(4\pi r^2) = \frac{q}{\varepsilon_0} \text{ or } E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} \text{ or } E \propto \frac{1}{r^2}$$

Note that if we set $r = R$ in either of the two expressions for E (outside and inside the sphere), we get the same result,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}$$

This is because E is **continuous function** of r in this case. By contrast, for the charged conducting sphere the magnitude of electric field is discontinuous at $r = R$ (it jumps from $E = 0$ to $E = \sigma/\varepsilon_0$).

Thus, for a uniformly charged solid sphere we have the following formulae for magnitude of electric field.

$$E_{\text{inside}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^3}\cdot r$$

$$E_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}$$

$$E_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

The variation of electric field (E) with the distance from the centre of the sphere (r) is shown in Fig. 18.57.

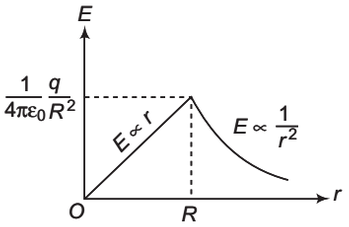


Fig. 18.57

Potential For a uniformly charged solid sphere we have the following formulae for potential.

$$V_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

$$V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$

and $$V_{\text{inside}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}\left[\frac{3}{2}-\frac{1}{2}\frac{r^2}{R^2}\right]$$

At centre potential will be, $V_c = \frac{3}{2}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}\right)$, which is equal to $\frac{3}{2}$ times the potential at surface. This can be obtained by putting $r = 0$ in the formula of V_{inside}.

The variation of potential (V) with distance from the centre (r) is as shown in Fig. 18.58.

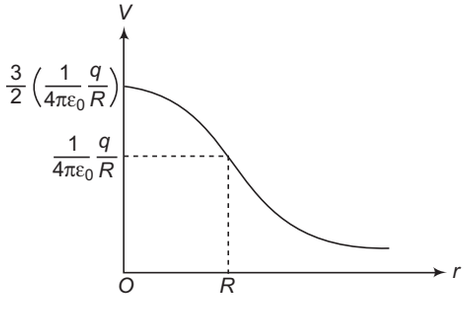


Fig. 18.58

Extra Knowledge Points

- To find the electric potential due to a conducting sphere (or shell) we should keep in mind the following two points :

  (i) Electric potential on the surface and at any point inside the sphere is

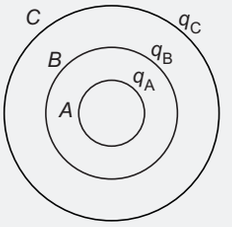
  $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$ (R = radius of sphere)

  (ii) Electric potential at any point outside the sphere is

  $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

  (r = distance of the point from the centre)

  For example, in the shown figure, potential at A is



$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_A}+\frac{q_B}{r_B}+\frac{q_C}{r_C}\right]$$

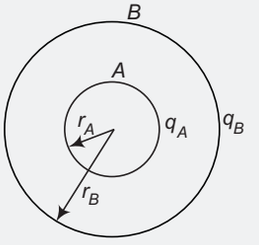
Similarly, potential at B is

$$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_B}+\frac{q_B}{r_B}+\frac{q_C}{r_C}\right]$$

and potential at C is,

$$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_C}+\frac{q_B}{r_C}+\frac{q_C}{r_C}\right]$$

- **Principle of a generator** A generator is an instrument for producing high voltages in the million volt region. Its design is based on the principle that if a charged conductor (say A) is brought into contact with a hollow conductor (say B), all of its charge transfers to the hollow conductor no matter how high the potential of the latter may be. This can be shown as under:



In figure, $$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_A}-\frac{q_B}{r_B}\right]$$

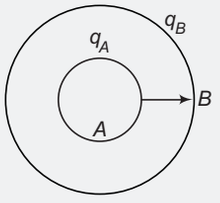
and $$V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_B}-\frac{q_B}{r_B}\right]$$

$\therefore$ $$V_A - V_B = \frac{q_A}{4\pi\varepsilon_0}\left[\frac{1}{r_A}-\frac{1}{r_B}\right]$$

From this expression, following conclusions can be drawn :

(i) the potential difference (PD) depends on q_A only. It does not depend on q_B.

(ii) if q_A is positive, then $V_A - V_B$ is positive (as $r_A < r_B$), i.e. $V_A > V_B$. So, if the two spheres are connected by a conducting wire charge flows from inner sphere to outer sphere (positive charge flows from higher potential to lower potential) till $V_A = V_B$ or $V_A - V_B = 0$. And potential difference will become zero only when $q_A = 0$, i.e. all charge q_A flows from inner sphere to outer sphere.



(iii) if q_A is negative, $V_A - V_B$ is negative, i.e. $V_A < V_B$. Hence, when the two spheres are connected by a thin wire all charge q_A will flow from inner sphere to the outer sphere. Because negative charge flows from lower potential to higher potential. Thus, we see that the whole charge q_A flows from inner sphere to outer sphere, no matter how high q_B is. Charge always flows from A to B, whether $q_A > q_B$ or $q_B > q_A$, $V_A > V_B$ or $V_B > V_A$.

- **Earthing a conductor** Potential of earth is often taken to be zero. If a conductor is connected to the earth, the potential of the conductor becomes equal to that of the earth, i.e. zero. If the conductor was at some other potential, charges will flow from it to the earth or from the earth to it to bring its potential to zero.

Example 18.30 *Figure shows two conducting thin concentric shells of radii r and 3r. The outer shell carries charge q. Inner shell is neutral. Find the charge that will flow from inner shell to earth after the switch S is closed.*

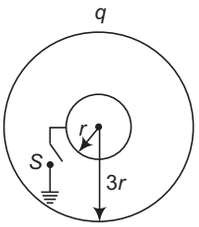


Fig. 18.59

Sol. Let q' be the charge on inner shell when it is earthed.

$$V_{\text{inner}} = 0 \Rightarrow \therefore \frac{1}{4\pi\varepsilon_0}\left[\frac{q'}{r} + \frac{q}{3r}\right] = 0$$

$$\therefore \quad q' = -\frac{q}{3}$$

i.e. $+\frac{q}{3}$ charge will flow from inner shell to earth.

Example 18.31 *Initially the spheres A and B are at potentials V_A and V_B. Find the potential of A when sphere B is earthed.*

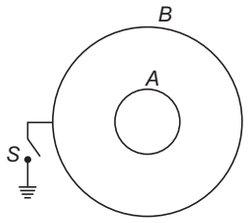


Fig. 18.60

Sol. As we have studied in Medical Galaxy the potential difference between these two spheres depends on the charge on the inner sphere only. Hence, the PD will remain unchanged because by earthing the sphere B charge on A remains constant. Let V_A' be the new potential at A. Then,

$$V_A - V_B = V_A' - V_B'$$

but, $V_B' = 0$ as it is earthed. Hence,

$$V_A' = V_A - V_B$$

18.15 Capacitance

In practice, we cannot handle free point charges or hold them fixed at desired positions. A practical way to handle a charge would be to put it on a conductor. Thus, one use of a conductor is to store electric charge (or electric potential energy). But every conductor has, its maximum limit of storing the electric charge or potential energy. Beyond that limit the dielectric (or insulator) in which the conductor is placed becomes ionized. A capacitor is a device which can store more electric charge or potential energy compared to an isolated conductor.

Capacitors have a tremendous number of applications. In the flash light used by photographers the energy and charge stored in a capacitor are recovered quickly. In other applications, the energy is released more slowly.

Capacitance of an Isolated Conductor

When a charge q is given to a conductor, it spreads over the outer surface of the conductor. The whole conductor comes to the same potential (say V). This potential V is directly proportional to the charge q, i.e.

$$V \propto q$$

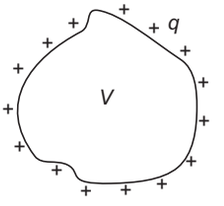


Fig. 18.61

When the proportionality sign is removed a constant of proportionality $\frac{1}{C}$ comes in picture.

Hence, $$V = \frac{q}{C}$$

or $$C = \frac{q}{V}$$

Here, C is called the **capacitance** of the conductor. The SI unit of capacitance is called one **farad** (1 F). One farad is equal to coulomb per volt (1 C/V)

$$\therefore \quad 1\ \text{F} = 1\ \text{farad} = 1\ \text{C/V}$$
$$= 1\ \text{coulomb/volt}$$

Method of Finding Capacitance of a Conductor

Give a charge q to the conductor. Find potential on it due to charge q. This potential V will obviously be a function of q and finally find q/V, which is the desired capacitance C.

Capacitance of a Spherical Conductor

When a charge q is given to a spherical conductor of radius R, the potential on it is,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

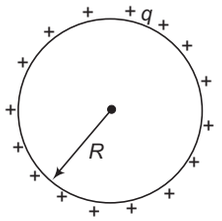


Fig. 18.62

From this expression we find that,

$$\frac{q}{V} = 4\pi\varepsilon_0 R = C$$

Thus, capacitance of the spherical conductor is,

$$C = 4\pi\varepsilon_0 R$$

From this expression, we can draw the following conclusions :

(i) $C \propto R$ or C depends on R only. Which we have already stated that C depends on the dimensions of the conductor. Moreover, if two conductors have radii R_1 and R_2 then,

$$\frac{C_1}{C_2} = \frac{R_1}{R_2}$$

(ii) Earth is also a spherical conductor of radius $R = 6.4 \times 10^6$ m. The capacity of earth is therefore,

$$C = \left(\frac{1}{9 \times 10^9}\right)(6.4 \times 10^6) \approx 711 \times 10^{-6}\ \text{F}$$

or $\quad C = 711\ \mu\text{F}$

From here, we can see that farad is a large unit. As capacity of such a huge conductor is only $711\ \mu$F.

Energy Stored in a Charged Conductor

Work has to be done in charging a conductor against the force of repulsion by the already existing charges on it. The work is stored as a potential energy in the electric field of the conductor. Suppose a conductor of capacity C is charged to a potential V_0 and let q_0 be the charge on the conductor at this instant. The potential of the conductor when (during charging) the charge on it was $q\ (< q_0)$ is,

$$V = \frac{q}{C}$$

Now work done in bringing a small charge dq at this potential is,

$$dW = V dq = \left(\frac{q}{C}\right) dq$$

$\therefore$ Total work done in charging it from 0 to q_0 is,

$$W = \int_0^{q_0} dW = \int_0^{q_0} \frac{q}{C}\, dq = \frac{1}{2}\frac{q_0^2}{C}$$

This work is stored as the potential energy,

$$\therefore \quad U = \frac{1}{2}\frac{q_0^2}{C}$$

Further by using $q_0 = CV_0$, we can write this expression also as,

$$U = \frac{1}{2} C V_0^2 = \frac{1}{2} q_0 V_0$$

In general, if a conductor of capacity C is charged to a potential V by giving it a charge q, then

$$U = \frac{1}{2} C V^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} qV$$

Redistribution of Charge

Let us take an analogous example. Some liquid is filled in two vessels of different sizes upto different heights. These are joined through a valve which was initially closed. When the valve is opened the level in both the vessels become equal but the volume of liquid in the right side vessel is more than the liquid in the left side vessel. This is because the base area (or capacity) of this vessel is greater.

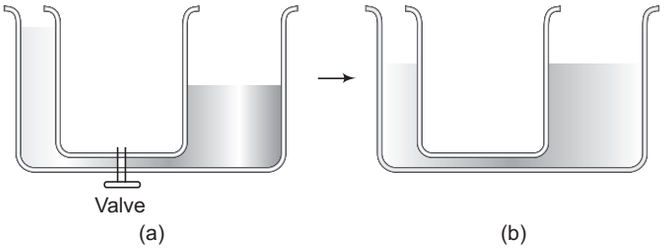


Fig. 18.63

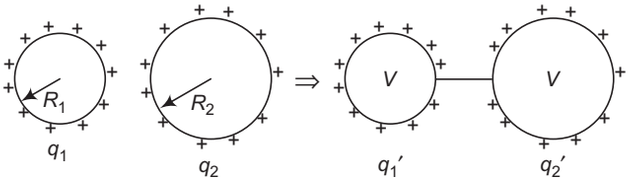


Fig. 18.64

Now, suppose two conductors of capacities C_1 and C_2 have charges q_1 and q_2 respectively when they are joined together by a conducting wire, charge redistributes in these conductors in the ratio of their capacities. Charge redistributes till potential of both the conductors become equal. Thus, let q_1' and q_2' be the final charges on them, then

$$q' \propto C \quad \text{or} \quad \frac{q_1'}{q_2'} = \frac{C_1}{C_2}$$

and if they are spherical conductors, then $\dfrac{C_1}{C_2} = \dfrac{R_1}{R_2}$

$$\therefore \qquad \frac{q_1'}{q_2'} = \frac{C_1}{C_2} = \frac{R_1}{R_2}$$

Since, the total charge is $(q_1 + q_2)$. Therefore,

$$q_1' = \left(\frac{C_1}{C_1 + C_2}\right)(q_1 + q_2) = \left(\frac{R_1}{R_1 + R_2}\right)(q_1 + q_2)$$

and $$q_2' = \left(\frac{C_2}{C_1 + C_2}\right)(q_1 + q_2) = \left(\frac{R_2}{R_1 + R_2}\right)(q_1 + q_2)$$

Loss of Energy During Redistribution of Charge

We can show that in redistribution of charge energy is always lost.

Initial potential energy,

$$U_i = \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2}$$

Final potential energy,

$$U_f = \frac{1}{2}\frac{(q_1 + q_2)^2}{C_1 + C_2}$$

$$\Delta U = U_i - U_f = \frac{1}{2}\left[\frac{q_1^2}{C_1} + \frac{q_2^2}{C_2} - \frac{(q_1 + q_2)^2}{C_1 + C_2}\right]$$

or $$\Delta U = \frac{1}{2C_1C_2\,(C_1 + C_2)} \times (q_1^2C_1C_2 + q_1^2C_2^2 + q_2^2C_1^2 + q_2^2C_1C_2 - q_1^2C_1C_2 - q_2^2C_1C_2 - 2q_1q_2C_1C_2)$$

$$= \frac{C_1^2C_2^2}{2C_1C_2\,(C_1 + C_2)}\left[\frac{q_1^2}{C_1^2} + \frac{q_2^2}{C_2^2} - \frac{2q_1q_2}{C_1C_2}\right]$$

$$= \frac{C_1C_2}{2\,(C_1 + C_2)}(V_1^2 + V_2^2 - 2V_1V_2)$$

or $$\Delta U = \frac{C_1C_2}{2\,(C_1 + C_2)}(V_1 - V_2)^2$$

Now as C_1, C_2 and $(V_1 - V_2)^2$ are always positive, $U_i > U_f$, i.e. there is decrease in energy. Hence, energy is always lost in redistribution of charge. Further,

$$\Delta U = 0 \qquad \text{if} \qquad V_1 = V_2$$

this is because no flow of charge takes place when both the conductors are at same potential.

Example 18.32 *Two isolated spherical conductors have radii* 5 *cm and* 10 *cm respectively. They have charges of* $12\,\mu C$ *and* $-3\,\mu C$. *Find the charges after they are connected by a conducting wire. Also find the common potential after redistribution.*

Sol. Net charge = (12 – 3) μC = 9 μC

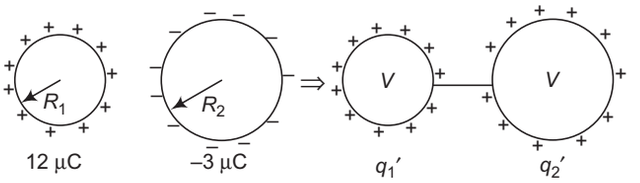


Fig. 18.65

Charge is distributed in the ratio of their capacities (or radii in case of spherical conductors), i.e.

$$\frac{q_1'}{q_2'} = \frac{R_1}{R_2} = \frac{5}{10} = \frac{1}{2}$$

$$\therefore \qquad q_1' = \left(\frac{1}{1+2}\right)(9) = 3\,\mu C$$

and $$q_2' = \left(\frac{2}{1+2}\right)(9) = 6\,\mu C$$

Common potential

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{(9 \times 10^{-6})}{4\pi\varepsilon_0(R_1 + R_2)}$$

$$= \frac{(9 \times 10^{-6})\,(9 \times 10^{9})}{(15 \times 10^{-2})} = 5.4 \times 10^5 \text{ V}$$

Example 18.33 *An insulated conductor initially free from charge is charged by repeated contacts with a plate which after each contact is replenished to a charge Q. If q is the charge on the conductor after first operation prove that the maximum charge which can be given to the conductor in this way is* $\dfrac{Qq}{Q - q}$.

Sol. Let C_1, be the capacity of plate and C_2 that of the conductor. After first contact charge on conductor is q. Therefore, charge on plate will remain $Q - q$. As the charge redistributes in the ratio of capacities.

$$\frac{Q-q}{q} = \frac{C_1}{C_2} \quad \ldots(i)$$

Let q_m be the maximum charge which can be given to the conductor. Then, flow of charge from the plate to the conductor will stop when,

$$V_{conductor} = V_{plate}$$

$$\therefore \quad \frac{q_m}{C_2} = \frac{Q}{C_1}$$

$$\therefore \quad q_m = \left(\frac{C_2}{C_1}\right) Q$$

Substituting $\frac{C_2}{C_1}$ from Eq. (i), we get

$$q_m = \frac{Qq}{Q-q}$$

Capacitors

Any two conductors separated by an insulator (or a vacuum) form a capacitor.

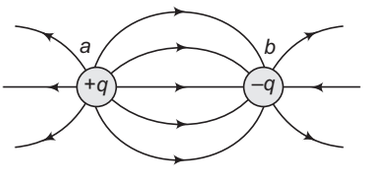


Fig. 18.66

In most practical applications, each conductor initially has zero net charge and electrons are transferred from one conductor to the other. This is called charging of the conductor. Then, the two conductors have charges with equal magnitude and opposite sign, and the net charge on the capacitor as a whole remains zero. When we say that a capacitor has charge q, we mean that the conductor at higher potential has charge $+q$ and the conductor at lower potential has charge $-q$. In circuit diagram, a capacitor is represented by two parallel lines as shown in Fig. 18.67.

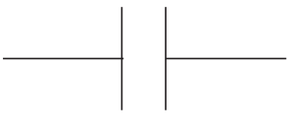


Fig. 18.67

One common way to charge a capacitor is to connect the two conductors to opposite terminals of a battery. This gives a fixed potential difference V_{ab} between the conductors, which is just equal to the voltage of the battery. The ratio $\frac{q}{V_{ab}}$ is called the capacitance of the capacitor. Hence,

$$C = \frac{q}{V_{ab}} \quad \text{(capacitance of a capacitor)}$$

Parallel Plate Capacitor

Two metallic parallel plates of any shape but of same size and separated by a small distance constitute parallel plate capacitor. Suppose the area of each plate is A and the separation between the two plates is d. Also assume that the space between the plates contains vacuum.

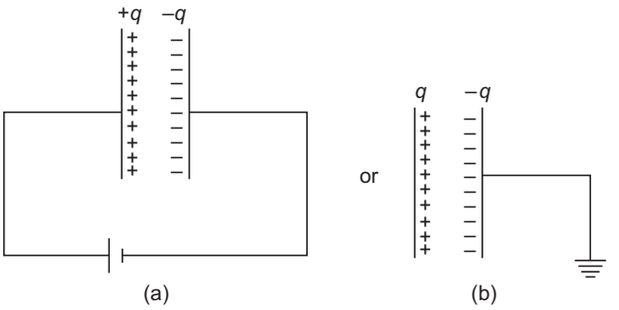


Fig. 18.68

We put a charge q on one plate and a charge $-q$ on the other. This can be done either by connecting one plate with the positive terminal and the other with negative plate of a battery [as shown in Fig. (a)] or by connecting one plate to the earth and by giving a charge $+q$ to the other plate only. This charge will induce a charge $-q$ on the earthed plate. The charges will appear on the facing surfaces. The charge density on each of these surfaces has a magnitude $\sigma = q/A$.

If the plates are large as compared to the separation between them, then the electric field between the plates (at point B) is uniform and perpendicular to the plates except for a small region near the edge. The magnitude of this uniform field E may be calculated by using the fact that both positive and negative plates produce the electric field in the same direction (from positive plate towards negative plate) of magnitude $\sigma / 2\varepsilon_0$ and therefore, the net electric field between the plates will be,

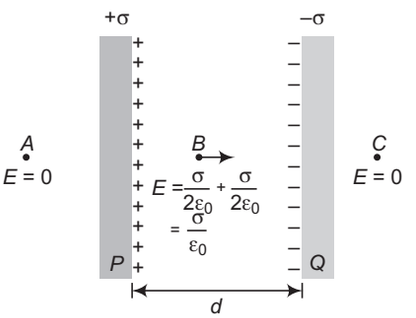


Fig. 18.69

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Outside the plates (at points A and C) the field due to positive sheet of charge and negative sheet of charge are in opposite directions. Therefore, net field at these points is zero.

The potential difference between the plates is,

$$\therefore \qquad V = E \cdot d = \left(\frac{\sigma}{\varepsilon_0}\right) d = \frac{qd}{A\varepsilon_0}$$

$\therefore$ The capacitance of the parallel plate capacitor is,

$$C = \frac{q}{V} = \frac{A\varepsilon_0}{d}$$

or $$C = \frac{\varepsilon_0 A}{d}$$

Note Points

- Instead of two plates if there are *n* similar plates at equal distances from each other and the alternate plates are connected together, the capacitance of the arrangement is given by,

$$C = \frac{(n-1)\,\varepsilon_0 A}{d}$$

- From the above relation, it is clear that the capacitance depends only on geometrical factors (*A* and *d*).

Effect of Dielectrics

Most capacitors have a non-conducting material or dielectric between their conducting plates. Placing a solid dielectric between the plates of a capacitor serves following three functions :

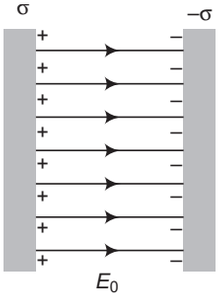


Fig. 18.70

(i) it solves the problem of maintaining two large metal sheets at a very small separation without actual contact.

(ii) it increases the maximum possible potential difference which can be applied between the plates of the capacitor without the dielectric breakdown. Many dielectric materials can tolerate stronger electric fields without breakdown than can air.

(iii) it increases the capacity of the capacitor. When a dielectric material is inserted between the plates (keeping the charge to be constant) the electric field and hence the potential difference decreases by a factor *K* (the dielectric constant of the dielectric).

$$\therefore \qquad E = \frac{E_0}{K}$$

and $$V = \frac{V_0}{K} \qquad (\text{when } q \text{ is constant})$$

Electric field is decreased because an induced charge of the opposite sign appears on each surface of the dielectric. This induced charge produces an electric field inside the dielectric in opposite direction and as a result net electric field is decreased. The induced charge in the dielectric can be calculated as under

$$E = E_0 - E_i$$

or $$\frac{E_0}{K} = E_0 - E_i$$

$$\therefore \qquad E_i = E_0\left(1 - \frac{1}{K}\right)$$

Therefore, $$\frac{\sigma_i}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}\left(1 - \frac{1}{K}\right)$$

or $$\sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$

or $$q_i = q\left(1 - \frac{1}{K}\right)$$

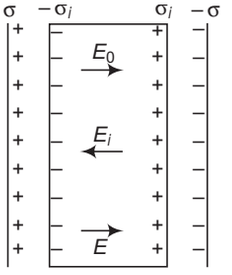


Fig. 18.71

For a conductor $K = \infty$. Hence,

$$q_i = q, \quad \sigma_i = \sigma$$

and $$E = 0$$

and otherwise $$q_i < q$$

Thus, $$q_i \le q, \quad \sigma_i \le \sigma$$

Hence, we may conclude the above discussion as under

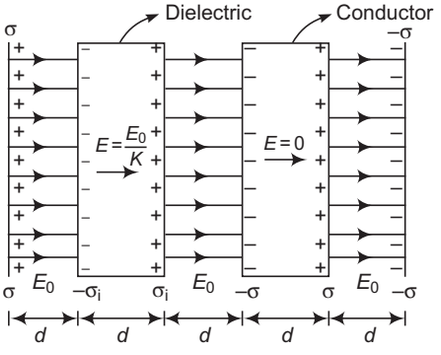


Fig. 18.72

(i) $E_{\text{vacuum}} = E_0 = \dfrac{\sigma}{\varepsilon_0} = \dfrac{q}{A\varepsilon_0}$

(ii) $E_{\text{dielectric}} = \dfrac{E_0}{K}$ (K = dielectric constant)

(iii) $E_{\text{conductor}} = 0$ (as $K = \infty$)

If we plot a graph between potential and distance from positive plate it will be as shown in Fig. 18.73

Slope of AB = slope of CD = Slope of $EF = E_0$

Slope of $BC = \dfrac{E_0}{K}$ and Slope of $DE = 0$

Further, the potential difference between positive and negative plate is

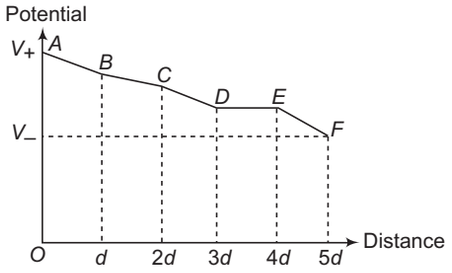


Fig. 18.73

$$V_+ - V_- = E_0 d + \frac{E_0}{K} \cdot d + E_0 d + 0 + E_0 d$$

$$= 3E_0 d + \frac{E_0}{K} \cdot d$$

Here, we have used

$$PD = Ed \qquad \text{(in a uniform electric field)}$$

Capacitance of a Capacitor Partially Filled with Dielectric

Suppose, a dielectric is partially filled with a dielectric (dielectric constant = K) as shown in figure. If a charge q is given to the capacitor, an induced charge q_i is developed on the dielectric.

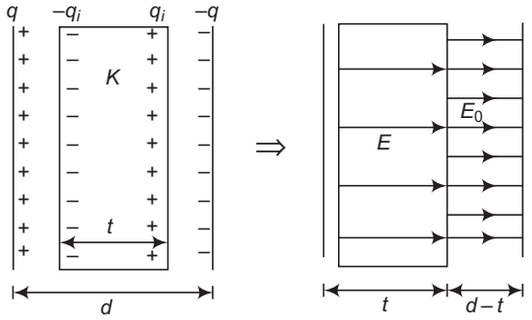


Fig. 18.74

where, $$q_i = q\left(1 - \frac{1}{K}\right)$$

Moreover, if E_0 is the electric field in the region where dielectric is absent, then electric field inside the dielectric will be $E = E_0/K$. The potential difference between the plates of the capacitor is,

$$V = V_+ - V_- = Et + E_0\,(d - t) = \frac{E_0}{K} t + E_0\,(d - t)$$

$$= E_0\left(d - t + \frac{t}{K}\right) = \frac{q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right)$$

Now, as per the definition of capacitance,

$$C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

Different Cases

(i) If more than one dielectric slabs are placed between the capacitors, then

$$C = \frac{\varepsilon_0 A}{d - t_1 - t_2 - \ldots - t_n + \frac{t_1}{K_1} + \frac{t_2}{K_2} + \ldots + \frac{t_n}{K_n}}$$

(ii) If the slab completely fills the space between the plates, then $t = d$, and therefore,

$$C = \frac{\varepsilon_0 A}{d/K} = \frac{K\varepsilon_0 A}{d}$$

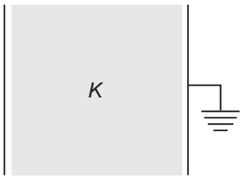


Fig. 18.75

(iii) If a conducting slab ($K = \infty$) is placed between the plates, then

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{\infty}} = \frac{\varepsilon_0 A}{d - t}$$

This can also be understood from the following figure:

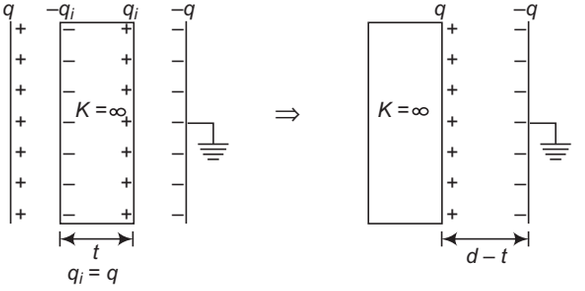


Fig. 18.76

(iv) If the space between the plates is completely filled with a conductor, then $t = d$ and $K = \infty$.

Then, $$C = \frac{\varepsilon_0 A}{d - d + \frac{d}{\infty}} = \infty$$

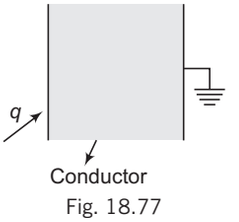


Fig. 18.77

The significance of infinite capacity can be understood as under

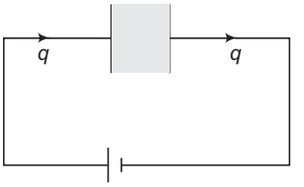


Fig. 18.78

If one of the plates of a capacitor is earthed and the second one is given a charge q, then the whole charge transfers to earth and as the capacity of earth is very large compared to the capacitor, we can say that the capacitance has become infinite.

Alternatively, if the plates of the capacitor are connected to a battery, the current starts flowing in the circuit, that is as much charge enters the positive plate of the capacitor, the same leaves the negative plate. So, we can say, the positive plate can accept infinite amount of charge or its capacitance has become infinite.

Energy Stored in Charged Capacitor

A charged capacitor stores an electric potential energy in it, which is equal to the work required to charge it. This energy can be recovered if the capacitor is allowed to discharge. If the charging is done by a battery, electrical energy is stored at the expense of chemical energy of battery.

Suppose at time t, a charge q is present on the capacitor and V is the potential of the capacitor. If dq amount of charge is brought against the forces of the field due to the charge already present on the capacitor, the additional work needed will be

$$dW = (dq)\,V = \left(\frac{q}{C}\right)\cdot dq \qquad (\text{as } V = q/C)$$

$\therefore$ Total work to charge a capacitor to a charge q_0,

$$W = \int dW = \int_0^{q_0} \left(\frac{q}{C}\right)\cdot dq = \frac{q_0^2}{2C}$$

$\therefore$ Energy stored by a charged capacitor,

$$U = W = \frac{q_0^2}{2C}$$

$$= \frac{1}{2}CV_0^2 = \frac{1}{2}q_0V_0$$

Thus, if a capacitor is given a charge q, the potential energy stored in it is,

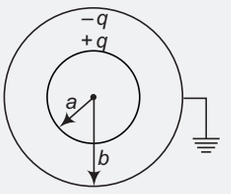
$$U = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

The above relation shows that the charged capacitor is the electrical analog of a stretched spring whose elastic potential energy is $\frac{1}{2}kx^2$. The charge q is analogous to the elongation x and $\frac{1}{C}$, i.e. the reciprocal of capacitance to the force constant k.

Extra Knowledge Points

- Capacity of a spherical conductor enclosed by an earthed concentric spherical shell. If a charge q is given to the inner spherical conductor it spreads over the outer surface of it and a charge $-q$ appears on the inner surface of the shell. The electric field is produced only between the two. From the principle of generator, the potential difference between the two will depend on the inner charge q only and is given by



$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

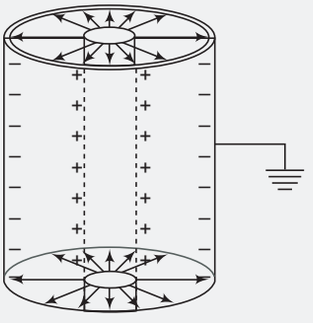
Hence, the capacitance of this system

$$C = \frac{q}{V} \quad \text{or} \quad C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$

From this expression, we see that if $b = \infty$, $C = 4\pi\varepsilon_0 a$, which corresponds to that of an isolated sphere, i.e. the charged sphere may be regarded as a capacitor in which the outer surface has been removed to infinity.

- **Capacity of a cylindrical capacitor** When a metallic cylinder of radius a is placed coaxially inside an earthed hollow metallic cylinder of radius $b\,(>a)$, we get cylindrical capacitor. If a charge q is given to the inner cylinder, induced charge $-q$ will reach to the inner surface of the outer cylinder. Assume that the capacitor is of very large length $(l >> b)$ so that the lines of force are radial. Using Gauss's law, we can prove that

$$E\,(r) = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad (\text{for } a \le r \le b)$$



Here, λ = charge per unit length.

The potential difference between the cylinders in this case comes out to be

$$V = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

$\therefore$

$$\frac{\lambda}{V} = \frac{\text{charge / length}}{\text{potential difference}}$$

$$= \frac{\text{capacity}}{\text{length}} = \frac{2\pi\varepsilon_0}{\ln(b/a)}$$

Hence, capacity per unit length $= \dfrac{2\pi\varepsilon_0}{\ln(b/a)}$

- Some problems may be asked when a capacitor is charged through a battery and then the capacity is changed :
  (i) either by inserting a dielectric slab or removing the slab (if it already exists) or
  (ii) by changing the distance between the plates of capacitor or
  (iii) by both.

The questions will be based on the change in electric field, potential, etc. In such problems two cases are possible.

Case 1. When battery is removed after charging

If the battery is removed after charging, then the charge stored in the capacitor remains constant.

$$q = \text{constant}$$

First of all find the change in capacitance and according to the formula find the change in other quantities.

For example, an air capacitor is first charged through a battery. The charging battery is then removed and a dielectric slab of dielectric constant $K = 4$ is inserted between the plates. Simultaneously the distance between the plates is reduced to half, then,

Change in capacitance

$$C = \frac{K\varepsilon_0 A}{d} \propto \frac{K}{d}$$

$\therefore$ Capacitance will become 8 times $\left(K = 4,\ d' = \frac{d}{2}\right)$

Change in electric field

$$\therefore \qquad E = E_0/K \quad \text{or} \quad E \propto \frac{1}{K} \quad (\text{if } q = \text{constant})$$

or the electric field will become 1/4 times its initial value.

Change in potential difference

$$V = \frac{q}{C} \quad \text{or} \quad V \propto \frac{1}{C} \qquad (\text{if } q = \text{constant})$$

Therefore, potential difference becomes 1/8 times of its initial value.

Alternate method $V = Ed$

Electric field has become 1/4 times its initial value and d is reduced to half. Hence, V becomes 1/8 times.

Change in stored potential energy

$$U = \frac{1}{2}\frac{q^2}{C} \quad \text{or} \quad U \propto \frac{1}{C} \quad (\text{if } q = \text{constant})$$

Capacitance has become 8 times. Therefore, the stored PE, U will become 1/8 times.

Case 2. When battery remains connected

If the battery remains connected, the potential difference V becomes constant. So, in the above example, capacitance will become 8 times.

The charge stored ($q = CV$ or $q \propto C$) will also increase to 8 times. The electric field $\left(E = \frac{V}{d} \text{ or } E \propto \frac{1}{d}\right)$ becomes twice and the stored

PE $\left(U = \frac{1}{2}CV^2 \text{ or } U \propto C\right)$ is 8 times.

Example 18.34 *Two parallel conducting plates of area A charge $+q$ and $-q$ are as shown. A dielectric slab of dielectric constant K and thickness d and a conducting plate of same thickness d is inserted between them. Taking $x = 0$ at positive plate and $x = 5d$ at negative plate, plot E-x and V-x graphs. Here, E is the electric field and V the potential.*

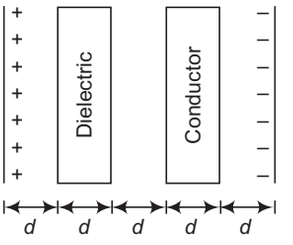


Fig. 18.79

Sol. In air electric field is $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$. In dielectric, field is $E = \frac{E_0}{K}$ and in conductor field is zero. Hence, the E-x graph is as shown in figure.

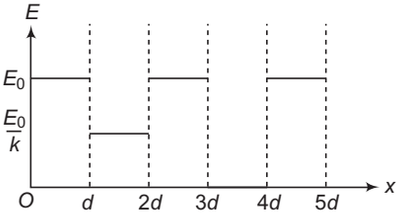


Fig. 18.80

Using $V = Ed$ (in uniform field) and assuming the potential at positive plate as V_0, the V-x graph is as shown

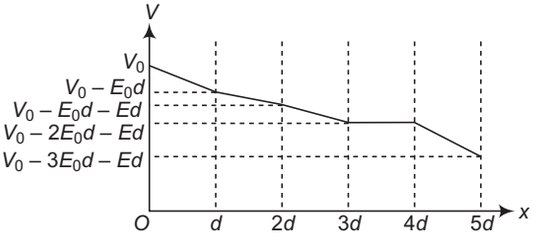


Fig. 18.81

Capacitors in Series and Parallel

In Series

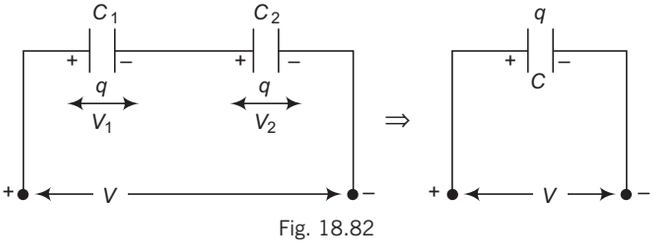


Fig. 18.82

In a series connection, the magnitude of charge on all plates is same. The potential is distributed in the inverse ratio of the capacity (as $V = q/C$ or $V \propto 1/C$). Thus, in the

figure, if a potential difference V is applied across the two capacitors C_1 and C_2, then

$$\frac{V_1}{V_2}=\frac{C_2}{C_1} \quad \text{or} \quad V_1=\left(\frac{C_2}{C_1+C_2}\right)V$$

and $$V_2=\left(\frac{C_1}{C_1+C_2}\right)V$$

Further, in the figure,

$$V=V_1+V_2 \quad \text{or} \quad \frac{q}{C}=\frac{q}{C_1}+\frac{q}{C_2} \quad \text{or} \quad \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$$

Here, C is the **equivalent capacitance.**

The equivalent capacitance of the series combination is defined as the capacitance of a single capacitor for which the charge q is the same as for the combination, when the same potential difference V is applied across it. In other words, the combination can be replaced by an equivalent capacitor of capacitance C. We can extend this analysis to any number of capacitors in series. We find the following result for the equivalent capacitance.

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots$$

Following points are important in case of series combination of capacitors :

(i) In a series connection, the equivalent capacitance is always less than any individual capacitance.

(ii) For the equivalent capacitance of two capacitors, it is better to remember the following form :

$$C=\frac{C_1C_2}{C_1+C_2}$$

For example, equivalent capacitance of two capacitors $C_1=6\,\mu\text{F}$ and $C_2=3\,\mu\text{F}$ is,

$$C=\frac{C_1C_2}{C_1+C_2}=\left(\frac{6\times3}{6+3}\right)\mu\text{F}=2\,\mu\text{F}$$

(iii) If n capacitors of equal capacity C are connected in series, then their equivalent capacitance is $\frac{C}{n}$.

Example 18.35 *In the circuit shown in figure, find*

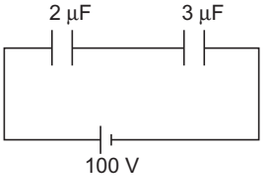


Fig. 18.83

(a) the equivalent capacitance,
(b) the charge stored in each capacitor and
(c) the potential difference across each capacitor.

Sol. (a) The equivalent capacitance

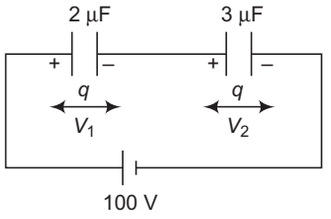


Fig. 18.84

$$C=\frac{C_1C_2}{C_1+C_2}$$

or $$C=\frac{(2)(3)}{2+3}=1.2\,\mu\text{F}$$

(b) The charge q, stored in each capacitor is,

$$q=CV=(1.2\times10^{-6})\,(100)\ \text{C}=120\,\mu\text{C}$$

(c) In series combination, $V\propto\frac{1}{C}$

or $$\frac{V_1}{V_2}=\frac{C_2}{C_1}$$

$\therefore$ $$V_1=\left(\frac{C_2}{C_1+C_2}\right)V$$

$$=\left(\frac{3}{2+3}\right)(100)=60\ \text{V}$$

and $$V_2=V-V_1=100-60$$

$$=40\ \text{V}$$

In Parallel

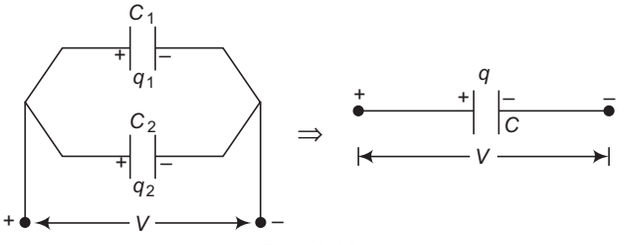


Fig. 18.85

The arrangement shown in figure is called a parallel connection. In a parallel combination the potential difference for all individual capacitors is the same and the total charge q is distributed in the ratio of their capacities. (as $q=CV$ or $q\propto C$ for same potential difference). Thus,

$$\frac{q_1}{q_2}=\frac{C_1}{C_2} \quad \text{or} \quad q_1=\left(\frac{C_1}{C_1+C_2}\right)q$$

and $$q_2=\left(\frac{C_2}{C_1+C_2}\right)q$$

The parallel combination is equivalent to a single capacitor with the same total charge $q=q_1+q_2$ and potential difference V.

Thus, $$q=q_1+q_2$$

or $$CV=C_1V+C_2V \quad \text{or} \quad C=C_1+C_2$$

In the same way, we can show that for any number of capacitors in parallel,

$$C = C_1 + C_2 + C_3 + \ldots$$

In a parallel combination the equivalent capacitance is always greater than any individual capacitance.

Example 18.36 *In the circuit shown in figure, find*
(a) the equivalent capacitance
(b) the charge stored in each capacitor.

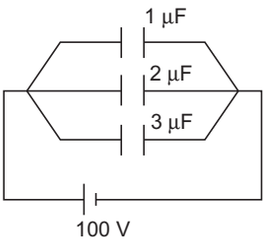


Fig. 18.86

Sol. (a) The capacitors are in parallel. Hence, the equivalent capacitance is,

$$C = C_1 + C_2 + C_3$$

or $$C = (1 + 2 + 3) = 6\,\mu F$$

(b) Total charge drawn from the battery,

$$q = CV = 6 \times 100\,\mu C = 600\,\mu C$$

This charge will be distributed in the ratio of their capacities. Hence,

$$q_1 : q_2 : q_3 = C_1 : C_2 : C_3 = 1 : 2 : 3$$

$$\therefore \quad q_1 = \left(\frac{1}{1+2+3}\right) \times 600 = 100\,\mu C$$

$$q_2 = \left(\frac{2}{1+2+3}\right) \times 600 = 200\,\mu C$$

and $$q_3 = \left(\frac{3}{1+2+3}\right) \times 600 = 300\,\mu C.$$

Alternate solution Since, the capacitors are in parallel, the PD across each of them is 100 V. Therefore, from $q = CV$, the charge stored in 1 μF capacitor is 100 μC, in 2 μF capacitor is 200 μC and that in 3 μF capacitor is 300 μC.

Two laws in capacitors Like an electric circuit having resistances and batteries, in a circuit containing capacitors the Kirchhoff's laws can be applied in the manner discussed below.

First law This is basically the law of conservation of charge. Following two points are important regarding the first law :

(i) In case of a battery, both terminals of the battery supply equal amount of charge.

(ii) In an isolated system (not connected to either of the terminals of a battery or to the earth), net charge remains constant.

e.g. in the figure shown, the positive terminal of the battery supplies a positive charge $q_1 + q_2$. Similarly, the negative terminal supplies a negative charge of magnitude $q_3 + q_4$.

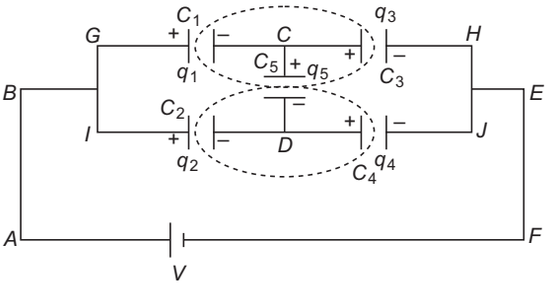


Fig. 18.87

Hence, $$q_1 + q_2 = q_3 + q_4$$

Further, the plates enclosed by the dotted lines form an isolated system, as they are neither connected to a battery terminal nor to the earth. Initially no charge was present in these plates. Hence, after charging net charge on these plates should be zero or,

$$q_3 + q_5 - q_1 = 0 \quad \text{and} \quad q_4 - q_2 - q_5 = 0$$

So, these are the three equations which can be obtained from the first law.

Second law In a capacitor, potential drops when one moves from positive plate to the negative plate by q/C and in a battery it drops by an amount equal to the emf of the battery. Applying second law in loop *ABGHEFA*, we have

$$-\frac{q_1}{C_1} - \frac{q_3}{C_3} + V = 0$$

Similarly, the second law in loop *GCDIG* gives the equation,

$$-\frac{q_1}{C_1} - \frac{q_5}{C_5} + \frac{q_2}{C_2} = 0$$

Example 18.37 *Find the charges on the three capacitors shown in figure.*

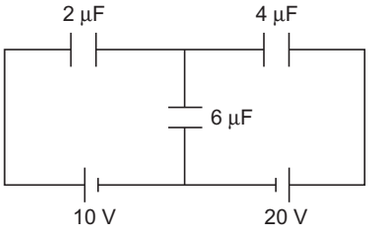


Fig. 18.88

Sol. Let the charges in three capacitors be as shown in Fig. 18.89.

Charge supplied by 10 V battery is q_1 and that from 20 V battery is q_2. Then,

$$q_1 + q_2 = q_3 \quad \ldots(i)$$

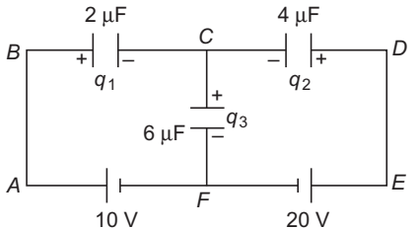


Fig. 18.89

This relation can also be obtained in a different manner. The charges on the three plates which are in contact add to zero. Because these plates taken together form an isolated system which can't receive charges from the batteries.

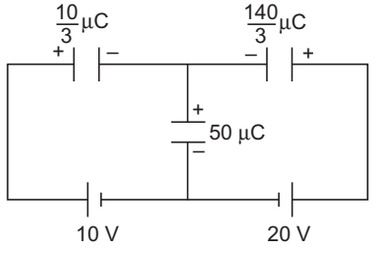


Fig. 18.90

Thus, $q_3 - q_1 - q_2 = 0$

or $q_3 = q_1 + q_2$

Applying second law in loops *BCFAB* and *CDEFC*, we have

$$-\frac{q_1}{2} - \frac{q_3}{6} + 10 = 0$$

or $q_3 + 3q_1 = 60 \quad \ldots(ii)$

and $\frac{q_2}{4} - 20 + \frac{q_3}{6} = 0$

or $3q_2 + 2q_3 = 240 \quad \ldots(iii)$

Solving the above three equations, we have

$$q_1 = \frac{10}{3}\,\mu C,\ q_2 = \frac{140}{3}\,\mu C$$

and $q_3 = 50\,\mu C$

Thus, charges on different capacitors are as shown in Fig. 18.90.

In the problem, q_1, q_2 and q_3 are already in microcoulombs.

Energy Density (u)

The potential energy of a charged conductor or a capacitor is stored in the electric field. The energy per unit volume is called the energy density (u). Energy density in a dielectric medium is given by,

$$u = \frac{1}{2}\varepsilon_0 K E^2$$

This relation shows that the energy stored per unit volume depends on E^2. If E is the electric field in a space of volume dV, then the total stored energy in an electrostatic field is given by,

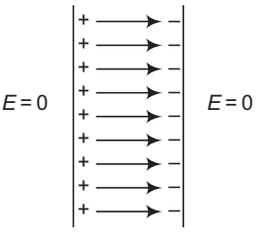
$$U = \frac{1}{2}\varepsilon_0 K \int E^2\, dV$$

and if E is uniform throughout the volume (electric field between the plates of a capacitor is uniform), then the total stored energy can be given by,

$$U = u\,(\text{total volume}) = \frac{1}{2} K \varepsilon_0 E^2 V$$

Example 18.38 *Using the concept of energy density, find the total energy stored in a*
(a) parallel plate capacitor and
(b) charged spherical conductor.

Sol. (a) Electric field is uniform between the plates of the capacitor. The magnitude of this field is,



$$E = \frac{\sigma}{\varepsilon_0}$$

Fig. 18.91

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$$

Therefore, the energy density (u) should also be constant.

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{2A^2\varepsilon_0}$$

∴ Total stored energy,

$$U = (u)(\text{total volume})$$

$$= \left(\frac{q^2}{2A^2\varepsilon_0}\right)(Ad) = \frac{q^2}{2\left(\frac{A\varepsilon_0}{d}\right)}$$

$$= \frac{q^2}{2C} \quad \text{as} \quad C = \frac{A\varepsilon_0}{d}$$

∴ $U = \frac{q^2}{2C}$

Dielectric Breakdown

When a dielectric material is subjected to a strong electric field, **dielectric breakdown** takes place and the dielectric becomes a conductor. This occurs when the electric field is so strong that electrons are ripped loose from their molecules. This maximum electric field magnitude that a material can withstand without the occurrence of breakdown is called its **dielectric strength.** The dielectric strength of dry air is about 3×10^6 V/m.

18.16 Methods of Finding Equivalent Resistance and Capacitance

- Only those methods have been discussed below, which in the opinion of author are useful for Medical Entrance Examination.

We know that in series,

$$R_{eq} = R_1 + R_2 + \ldots + R_n$$

and $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$$

and in parallel, $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

and $$C_{eq} = C_1 + C_2 + \ldots + C_n$$

Sometimes there are circuits in which resistances/capacitors are in mixed grouping. To find R_{eq} or C_{eq} for such circuits few methods are suggested here which will help you in finding R_{eq} or C_{eq}.

Method of Same Potential

Give any arbitrary potentials ($V_1, V_2, \ldots$ etc.) to all terminals of capacitors/resistors. But notice that the points connected directly by a conducting wire will have at the same potential. The capacitors/resistors having the same PD are in parallel. Make a table corresponding to the figure. Now, corresponding to this table a simplified figure can be formed and from this figure C_{eq} and R_{eq} can be calculated.

Example 18.39 *Find equivalent capacitance between points A and B shown in figure.*

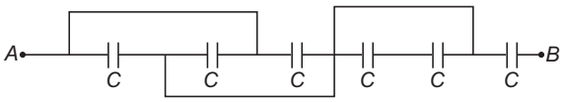


Fig. 18.92

Sol. Three capacitors have PD, $V_1 - V_2$. So, they are in parallel. Their equivalent capacitance is $3C$.

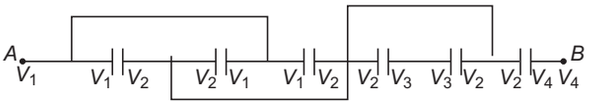


Fig. 18.93

Two capacitors have PD, $V_2 - V_3$. So, their equivalent capacitance is 2 C and lastly there is one capacitor across which PD is $V_2 - V_4$. So, let us make a table corresponding to this information.

PD	Capacitance
$V_1 - V_2$	$3C$
$V_2 - V_3$	$2C$
$V_2 - V_4$	C

Now, corresponding to this table, we make a simple figure as shown in Fig. 18.94.

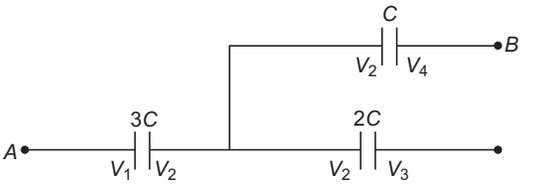


Fig. 18.94

As we had to find the equivalent capacitance between points A and B, across which PD is $V_1 - V_4$. From the simplified figure we can see that the capacitor of capacitance $2C$ is out of the circuit and points A and B are as shown. Now, $3C$ and C are in series and their equivalent capacitance is,

$$C_{eq} = \frac{(3C)(C)}{3C + C} = \frac{3}{4}C$$

EXERCISE *Find equivalent capacitance between points A and B.*

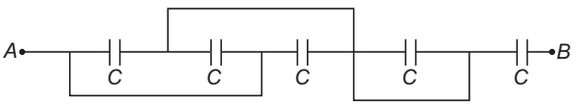


Fig. 18.95

Hint In case PD across any capacitor is zero (i.e. the plates are short circuited) then this capacitor is not to be considered.

Ans. $\frac{3}{4}C$

EXERCISE *Identical metal plates are located in air at equal distance d from one another as shown in figure. The area of each plate is A. Find the capacitance of the system between points P and Q, if plates are interconnected as shown.*

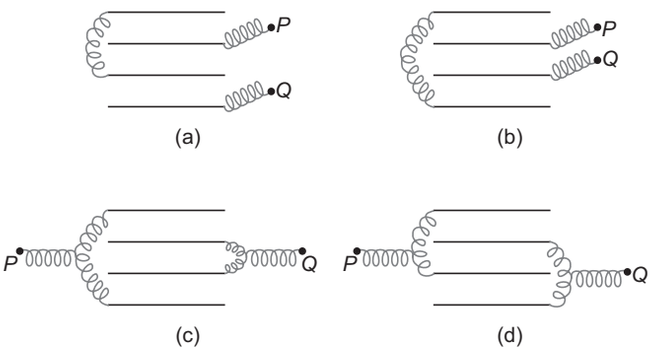


Fig. 18.96

Ans.

(a) $\frac{2}{3}\frac{\varepsilon_0 A}{d}$ (b) $\frac{3}{2}\frac{\varepsilon_0 A}{d}$

(c) $\frac{2\varepsilon_0 A}{d}$ (d) $\frac{3\varepsilon_0 A}{d}$

EXERCISE *Find equivalent resistance between points A and B.*

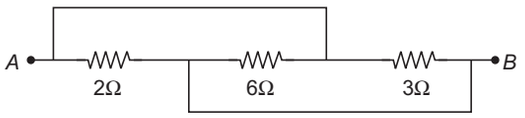


Fig. 18.97

Ans. 1 Ω

EXERCISE *Find equivalent capacitance between points A and B.*

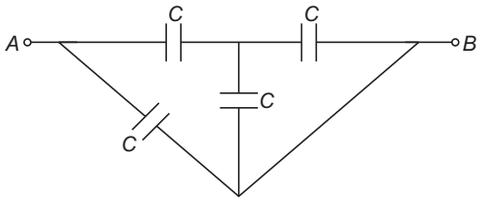


Fig. 18.98

Ans. $\frac{5}{3}C$.

Infinite Series Problems

This consists of an infinite series of identical loops. To find C_{eq} or R_{eq} of such a series first we consider by ourself a value (say x) of C_{eq} or R_{eq}. Then, we break the chain in such a manner that only one loop is left with us and in place of the remaining portion we connect a capacitor/resistor x. Then, we find the C_{eq} or R_{eq} and put it equal to x. With this we get a quadratic equation in x. By solving this equation we can find the desired value of x.

Example 18.40 *An infinite ladder network is constructed with* 1 Ω *and* 2 Ω *resistors as shown. Find the equivalent resistance between points A and B.*

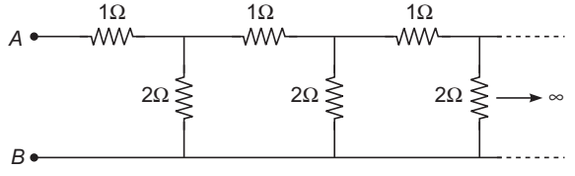


Fig. 18.99

Sol. Let the equivalent resistance between *A* and *B* is *x*. We may consider the given circuit as shown in Fig. 18.100.

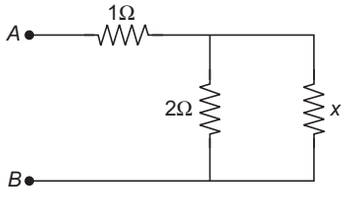


Fig. 18.100

In this diagram,

$$R_{AB} = \frac{2x}{2+x} + 1$$

or $$x = \frac{2x}{2+x} + 1 \qquad (\text{as } R_{AB} = x)$$

or $$x(2+x) = 2x + 2 + x$$

or $$x^2 - x - 2 = 0$$

$$x = \frac{1 \pm \sqrt{1+8}}{2} = -1\,\Omega \text{ and } 2\,\Omega$$

Ignoring the negative value, we have

$$R_{AB} = x = 2\ \Omega$$

- Care should be taken while breaking the chain. It should be broken from such points that identical chain repeats after breaking. For example,

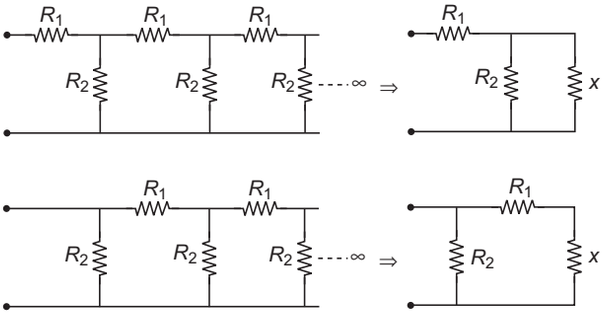


Fig. 18.101

Connection Removal Method

This method is useful when the circuit diagram is symmetric except for the fact that the input and output are reversed. That is the flow of current is a mirror image between input and output above a particular axis. In such cases some junctions are unnecessarily made. Even if we remove that junction there is no difference in the remaining circuit or current distribution. But after removing the junction, the problem becomes very simple.

The following example illustrates the theory:

Example 18.41 *Find the equivalent resistance between points A and B.*

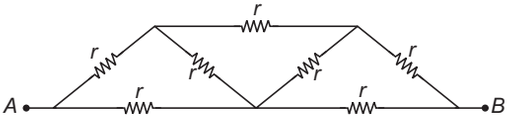


Fig. 18.102

Sol.

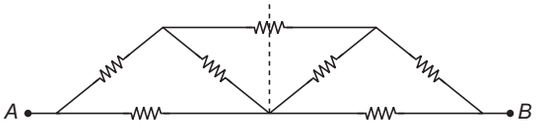


Fig. 18.103

Input and output circuits are mirror images of each other about the dotted line as shown. So, if a current i enters from A and leaves from B it will distribute as shown below.

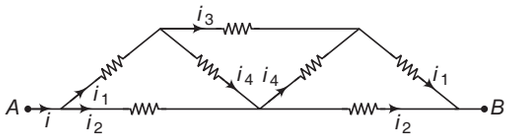


Fig. 18.104

Now, we can see that the junction where i_2 and i_4 are meeting can be removed easily and then the circuit becomes simple.

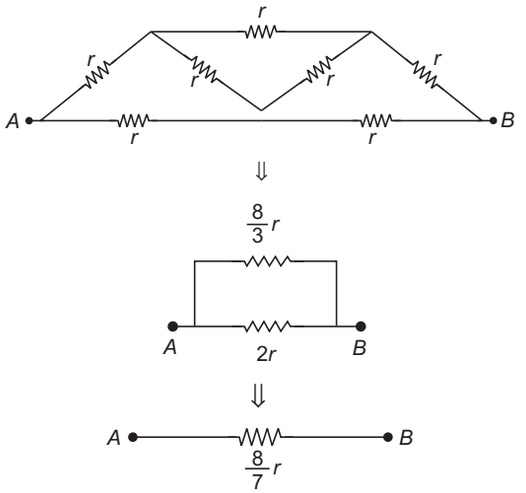


Fig. 18.105

Hence, the equivalent resistance between A and B is $\frac{8}{7}r$.

EXERCISE *Twelve resistors each of resistance r are connected as shown. Find equivalent resistance between points A and B.*

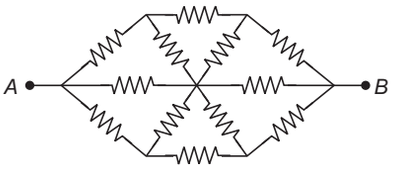


Fig. 18.106

Ans. (4/5) *r.*

Wheatstone Bridge Circuits

Wheatstone bridge in case of resistors has already been discussed in chapter 11.

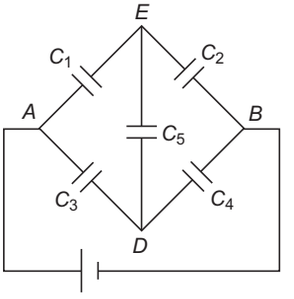


Fig. 18.107

For capacitors theory is same.

If $\frac{C_1}{C_2}=\frac{C_3}{C_4}$, bridge is said to be balanced and in that case

$$V_E = V_D$$

or $$V_E - V_D$$

or $$V_{ED} = 0$$

i.e. no charge is stored in C_5. Hence, it can be removed from the circuit.

Chapter Summary with Formulae

(i) $q = \pm ne$, where $n = 1, 2, 3$ .....

(ii) $e = 1.6 \times 10^{-19}$ C

(iii) $|\mathbf{F}| = F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$ (in vacuum)

(iv) $|\mathbf{F}| = F = \frac{1}{4\pi\varepsilon_0 K}\frac{|q_1 q_2|}{r^2}$ (in dielectric medium)

(v) $\varepsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$

= Permittivity of free space (or vacuum)

(vi) $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2C^{-2}

(vii) $\varepsilon_0 K = \varepsilon$ = Permittivity of dielectric medium

(viii) K = dielectric constant of dielectric medium

= 1 for vacuum

= 80 for water = slightly greater than 1 for air

(ix) Two like charges repel each other and unlike charges attract each other.

(x) Force on q_1 due to q_2 in vector form :

$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}(\mathbf{r}_1 - \mathbf{r}_2).$$

Here $r = |\mathbf{r}_1 - \mathbf{r}_2|$ = distance between two charges. In this formula q_1 and q_2 are to be substituted with sign.

(xi) Principle of superposition,

$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13}$. Here, $\mathbf{F}_1$ = net force on charge q_1,

$\mathbf{F}_{12}$ = force on q_1 due to q_2

$\mathbf{F}_{13}$ = force on q_1 due to q_3

▪ Electric Field

(i) Electric field strength :

$$\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0}$$

(ii) $\mathbf{F} = q\mathbf{E}$

(iii) $F = qE$ force will act on $\pm q$ charge in the direction of field if charge is positive and in opposite direction of field if charge is negative.

(iv) Magnitude of electric field due to a point charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Direction of this electric field is away from the charge if q is positive and towards the charge if q is negative.

(v) Electric field at some point due to two or more than two charges

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 \quad \text{(Principle of superposition)}$$

(vi) Electric field at some point due to a point charge in vector form is

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}(\mathbf{r}_P - \mathbf{r}_A)$$

In this formula, q is to be substituted with sign.

$\mathbf{r}_P$ = position vector of that point where electric field is asked.

$\mathbf{r}_A$ = position vector of that point where charge is kept

$\mathbf{r} = |\mathbf{r}_P - \mathbf{r}_A|$ = distance between P and A

▪ Electric Dipole

(i) Magnitude of electric dipole moment

$$p = q(2a)$$

Here, $2a$ = distance between two charges $+q$ and $-q$

(ii) Direction of p is from $-q$ towards $+q$.

(iii) On the axis of a dipole

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2 - a^2)^2} \approx \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} \quad (\text{if } r >> a)$$

This electric field is in the direction of p.

(iv) On perpendicular bisector of dipole,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2 + a^2)^{3/2}} \approx \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} \quad (\text{if } r >> a)$$

This electric field is in opposite direction of p.

(v) Linear charge density, $\lambda = \frac{\Delta q}{\Delta l}$ or $\frac{q}{l}$

Surface charge density, $\sigma = \frac{\Delta q}{\Delta S}$ or $\frac{q}{S}$

Volume charge density, $\rho = \frac{\Delta q}{\Delta V}$ or $\frac{q}{V}$

▪ Dipole in Uniform Field

(i) Electric dipole in uniform electric field

$\mathbf{F}_{net} = 0 \Rightarrow \tau = \mathbf{P} \times \mathbf{E}$ or $\tau = PE\sin\theta$

(ii) $\theta = 0^\circ$ is called stable equilibrium position. $\theta = 180^\circ$ is called unstable equilibrium position. Net force and net torque in both equilibrium positions are zero.

(iii) When the dipole is released from any other position (other than 0° or 180°), the torque acting on the dipole tends to align it towards stable equilibrium position.

(iv) In non-uniform field, dipole will experience both force and torque.

▪ Electric Flux

(i) Magnitude of area vector $|\Delta S| = \Delta S$.

(ii) Direction of area vector is perpendicular to the surface.

(iii) For a closed surface direction of area vector is outward normal.

(iv) Solid angle,

$$\Delta\Omega = \frac{\Delta S}{r^2} \text{ or } \frac{\Delta S\cos\theta}{r^2}$$

(v) Electric flux from a small area

$$\Delta\phi = \mathbf{E}\cdot\Delta\mathbf{S} = E\Delta S\cos\theta$$

(vi) Electric flux from a large surface

$$\phi = \int d\phi = \int \mathbf{E}\cdot d\mathbf{S} \text{ or } \int E\, dS\cos\theta$$

(vii) $\phi = 0$, if electric lines are tangential to the surface at all points.

(viii) $\phi = ES$, if magnitude of electric field is uniform at all points on the surface and electric lines are perpendicular at all points.

(ix) **Gauss's Law** Electric flux passing through any closed surface is equal to q_{in}/ε_0, where q_{in} is total charged enclosed by that surface.

(x) Mathematically, Gauss's law can be written as $\oint \mathbf{E}\cdot d\mathbf{S} = \frac{q_{in}}{\varepsilon_0}$

(xi) If magnitude of electric field is uniform and lines are perpendicular to every point on the closed surface (Gaussian Surface), then we can write.

$$\oint \mathbf{E} \cdot d\mathbf{S} = ES = \frac{q_{in}}{\varepsilon_0}$$

or $$E = \frac{q_{in}}{S\varepsilon_0}$$

(xii) Electric field due to thin infinitely long straight wire of uniform linear charge density λ is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

(xiii) Electric field due to infinitely long thin sheet of charge is

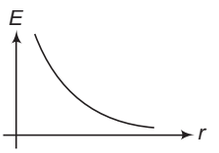
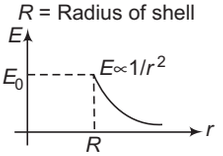
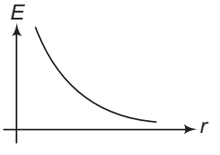
$$E = \frac{\sigma}{2\varepsilon_0} \quad (\sigma = \text{surface charge density})$$

(xiv) Electric field due to thin spherical charged shell is

$$E = 0 \qquad (r < R)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad (r \geq R)$$

(xv)

Charge Distribution	*E*-*r* Equation	*E*-*r* Graph
Point charge	$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \propto \frac{1}{r^2}$	
Thin charged spherical shell	$E_{in} = 0$; $E_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} = E_0$ (say); $E_{out} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \propto \frac{1}{r^2}$	
Infinitely long line charge	$E = \frac{\lambda}{2\pi\varepsilon_0 r} \propto \frac{1}{r}$ λ = Linear charge density	

- **Electric Potential**

(i) Absolute potential at some point P

$V_P = -W_{R\to P}$ (by electrostatic forces to unit positive test charge)

$V_P = -W_{R\to P}$ (by electrostatic force, on unit positive charge)

Further,

$V_P = +W_{R\to P}$ (by external agent, on unit positive test charge, without change in kinetic energy)

Here R is a reference point where $V = 0$.

(ii) Potential difference between two points,

$$V_Q - V_P = -W_{P\to Q}$$

(by electrostatic forces on unit positive test charge)

or $$V_Q - V_P = +W_{P\to Q}$$

(by external agent on unit positive test charge, without change in kinetic energy)

(iii) Electric potential due to a point charge,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad (q \to \text{with sign})$$

(iv) Electric potential due to a system of charges

$$V = V_1 + V_2 + \ldots = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \ldots.\right]$$

(v) Electric potential due to an electric dipole

$$V_{axis} = \pm\frac{1}{4\pi\varepsilon_0}\frac{P}{r^2 - a^2} \approx \pm\frac{1}{4\pi\varepsilon_0}\frac{P}{r^2} \qquad (\text{for } r >> a)$$

Take positive sign if point (where potential is required) lies on $+q$ side and take negative sign if point lies on $-q$ side.

$$V_{\perp \text{ bisector}} = 0$$

$$V_{(r,\,\theta)} = \frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta}{r^2} \qquad (\text{for } r >> a)$$

(vi) In equipotential surface

→ $V_A = V_B$

→ Potential difference = 0, between any two points

→ Work done is zero in moving a test charge on this surface.

→ Electric lines of forces are perpendicular to this surface.

→ Two equipotential surfaces never intersect each other.

(vii) $W_{A\to B} = -q(V_B - V_A)$ By electrostatic forces

$W_{A\to B} = q(V_B - V_A)$ By external agent without change in kinetic energy

(viii) $U_P = -W_{\infty\to P}$ By electrostatic forces

$U_P = +W_{\infty\to P}$ By external agent without change in kinetic energy

(ix) $\Delta U = -W_{el}$

$\Delta U = +W_{ext}$ Without change in kinetic energy

(x) $U_2 - U_1 = -W_{1\to 2}$ By electrostatic forces

$U_2 - U_1 = +W_{1\to 2}$

By external agent without change in kinetic energy

(xi) Potential energy of two point charges,

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \qquad (q_1 \text{ and } q_2 \to \text{with sign})$$

(xii) For potential energy of n charges make $\frac{n(n-1)}{2}$ pair of charges.

(xiii) $U = qV$ or $\Delta U = q \cdot \Delta V$

(xiv) Potential energy of a single charge in an electric field

$U = qV$

(xv) Potential energy of two point charges :

$$U = q_1V_1 + q_2V_2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

(xvi) Potential energy of dipole in uniform electric field,

$$U = -PE\cos\theta$$

(xvii) At $\theta = 0^\circ$, $\tau = 0$, $U = -PE$ = minimum.

This is called stable equilibrium position.

(xviii) At $\theta = 180^\circ$, $\tau = 0$, $U = +PE$ = minimum

This is called unstable equilibrium position.

(xix) $W_{\theta_1 \to \theta_2} = \Delta U = \mathbf{PE}(\cos\theta_1 - \cos\theta_2)$ by external agent without change in rotational kinetic energy of dipole.

(xx) When a dipole is left from a general value of $\theta (\neq 0^o$ or $180^o)$ electrostatic torque $(= \mathbf{PE}\sin\theta)$ rotates this dipole towards $\theta = 0^o$, stable equilibrium position having minimum potential energy.

(xxi) In uniform electric field,

$$V = Ed$$

(xxii) $E = -\dfrac{dV}{dr}$

(xxiii) $dV = -Edr$

(xxiv) $V_1 - V_2 = -\int_2^1 Edr$

- **Capacitance**

(i) Under electrostatic conditions, electric field just inside a conductor is zero.

(ii) Electric field just outside a conductor is σ/ε_0 and normal to the conductor.

(iii) The potential of a charged conductor remains same throughout its volume.

(iv) Potential inside a charged spherical shell $(r < R)$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} = \text{constant}$$

(v) Potential outside the charged spherical shell $(r \geq R)$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

or $$V \propto \frac{1}{r}$$

(vi) For linear isotropic dielectrics,

$$\mathbf{P} = \chi_e \mathbf{E}$$

(vii) $\sigma_i = \sigma\left(1 - \dfrac{1}{K}\right)$

(viii) Capacitance of a conductor,

$$C = \frac{q}{V}$$

(ix) Capacitance of a spherical conductor,

$$C = 4\pi\varepsilon_0 R$$

or $$C \propto R \quad \text{or} \quad \frac{C_1}{C_2} = \frac{R_1}{R_2}$$

(x) Capacitance of a capacitor,

$$C = \frac{q}{V}$$

V = potential difference between the plates

(xi) Capacitance of parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$ (vacuum between two plates)

(xii) When a dielectric slab is completely filled between the plates,

$$C = \frac{K\varepsilon_0 A}{d}$$

(xiii) When a dielectric slab is partially filled between the plates,

$$C = \frac{\varepsilon_0 A}{d - t}$$

(xiv) Potential energy stored in the electric field of a charged conductor or capacitor,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$

(xv) Energy density,

$$u = \frac{1}{2}\varepsilon_0 E^2$$

(xvi) Capacitors in series

→ Charge on capacitors is same

→ $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + ... + \dfrac{1}{C_n}$

→ Potential difference distributes in inverse ratio of capacity

(xvii) Capacitors in parallel

→ Potential difference is same

→ $C = C_1 + C_2 + ... + C_n$

→ Charge distributes in direct ratio of capacity

(xviii) Capacitance of spherical capacitor

$$C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$

(xix) $\chi_e = \varepsilon_0(K - 1)$

(xx) Dielectric strength of air is 3.6×10^6 V/m

Additional Examples

Example 1. *A metal rod held in hand and rubbed with wool does not show any sign of being charged. Explain why ?*

Sol. Human body and metal both being conductors, the excess charge of metal (produced by rubbing with wool) transfers to ground.

Example 2. *If we comb our hair on a dry day and bring the comb near small pieces of paper, the comb attracts the pieces, why?*

Sol. This is an example of frictional electricity and induction. When we comb our hair, it gets positively charged by rubbing. When the comb is brought near the pieces of paper, some of the electrons accumulate at the edge of the paper piece which is closer to the comb. At the farther end of the piece, there is deficiency of electrons and hence, positive charge appears there. Such a redistribution of charge in a material, due to presence of a nearby charged body is called induction.

The comb exerts larger attraction on the negative charges of the paper piece as compared to the repulsion on the positive charge. This is because the negative charges are closer to the comb. Hence, there is a net attraction between the comb and the paper piece.

Example 3. *In Coulomb's law, on what factors the value of electrostatic force constant k depends?*

Sol. On the system of units adopted and the nature of medium in which two charges are placed.

Example 4. *State the limitations of Coulomb's law.*

Sol.

1. It holds for point charges only.
2. It holds for stationary charges only.

Example 5. *Electrostatic force between two charges is called central force. Why ?*

Sol. The force between two electric charges always acts along the line joining the two charges. For this reason, it is called central force.

Example 6. *A positively charged glass rod attracts a suspended pith-ball. Does it imply that the pith-ball is negatively charged ?*

Sol. The pith-ball can be uncharged also. The positively charged glass rod can attract the pith-ball due to induced charges of opposite kind (towards nearer side) produced on the pith-ball.

Example 7. (*a*) *An electrostatic field line is a continuous curve. i.e. a field line cannot have sudden breaks. Why not?*

(*b*) *Explain why two field lines never cross each other at any point?*

Sol. (*a*) Tangent to electric field line at any point shows the direction of electrostatic force on positive test charge at that point. Now, sudden break of field line at some point means, suddenly force on test charge becomes zero and then again starts acting. But, in any case, force will not become suddenly zero. That's why electric field lines can be taken to be curves without any breaks.

(*b*) If two field lines cross each other at some point, then two tangents can be drawn at the point of intersection. It means, at the point of intersection, electric field has two directions, which is not possible.

Example 8. *Give two reasons, why the test charge used to measure the electric field at a point should be vanishingly small?*

Sol. (i) The source charge, of whom, we have to measure the electric field should remain stationary. If test charge is big then it will exert large force on source charge and under this source charge cannot remain stationary.

(ii) In case, test charge is not vanishingly small, then it will produce its own field and original will be disturbed.

Example 9. *Does an electric charge experience a force due to its own field ?*

Sol. No, an electric charge does not experience any force due to the electric field produced by itself.

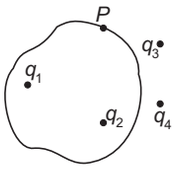
Example 10. *Suppose the net electric flux from a closed surface is zero. Does it necessarily means that electric field is equal to zero for all points on the surface.*

Sol. No. It does not mean that electric field is zero at all points. Rather it means that net charge enclosed by that closed surface is zero.

Example 11. *Gauss's law can be applied only for symmetrical charge distribution. Is this statement true or false?*

Sol. False. Gauss's law can be applied for any type of charge distributions. But it makes calculations simple where charge distribution is symmetrical type.

Example 12. *There is a point P on a closed surface as shown in figure. q_1 and q_2 are two charges lying inside the closed surface and q_3 and q_4 are lying outside it. Net electric field at point P is only due to q_1 and q_2. Is this statement true or false?*



Sol. False. Electric field at point P is the vector sum of four fields due to four charges q_1, q_2, q_3 and q_4. But electric flux of this net electric field over the closed surface depends only on the charges q_1 and q_2.

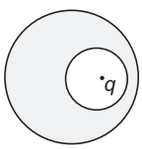
Example 13. *Electric potential of earth is taken to be zero. Why ?*

Sol. Earth is a good conductor of very large size. When some small charge is given to earth, its potential does not change. Hence, potential of earth is assumed to be zero. It is like sea level being taken as zero level for measuring all heights.

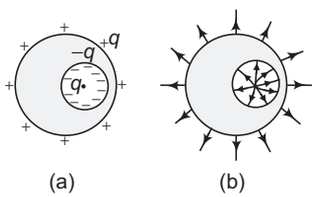
Example 14. *If electric potential at some point is zero, then does it mean that electric field is also zero at that point.*

Sol. No. This is not necessary. For example on the perpendicular bisector of an electric dipole, electric potential is zero but electric field is non zero.

Example 15. *Charge q is placed asymmetrically inside a cavity in a conducting sphere as shown in figure. Show the induced charge on the inner surface of cavity and on the outer surface of spherical conductor. Also show the electric lines of forces.*



Sol. On the inner surface of cavity induced charges $(=-q)$ will distribute asymmetrically, as shown in figure. Further, electric lines of forces do not enter inside a conducting body. Therefore distribution of induced charges $(=+q)$ on outer surface of the spherical conductor will be uniform. Electric lines of forces are also shown in figure (b) according to charge distribution.



Example 16. *A capacitor has capacitance C. Is this information sufficient to know what maximum charge the capacitor can obtain? If no what other information is needed?*

Sol. Answer is no. Only from capacitance we cannot determine the maximum charge which can be stored by the capacitor. The dielectric strength of the dielectric filled between the plates of capacitor will be required to calculate the maximum charge which can be stored by the capacitor.

Example 17. *A hollow metal sphere and a solid metal sphere of equal radii are given equal charges. Which of the two will have higher potential?*

Sol. Both will have the same potential given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

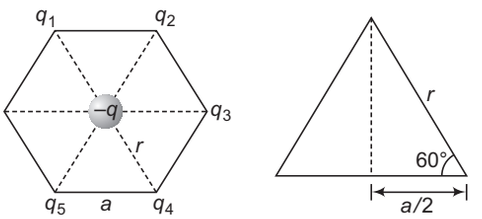
This is because in both cases, the excess charge given to the spheres will distribute on their outer surfaces.

Example 18. *As $C = \dfrac{q}{V}$, can you say that capacitance C is proportional to the charge q ?*

Sol. No capacitance C depends only upon the size, shape and the dielectric medium filled between the plates of capacitor.

Example 19. *Five point charges each of value + q are placed on five vertices of a regular hexagon of side a metre. What is the magnitude of the force on a point charge of value – q coulomb placed at the centre of the hexagon?*

Sol. $\dfrac{a/2}{r} = \cos 60° = \dfrac{1}{2}$



$\therefore$ $a = r$

$q_1 = q_2 = \ldots = q_5 = q$

Net force on $-q$ is only due to q_3 because forces due to q_1 and due to q_4 are equal and opposite so cancel each other. Similarly forces due to q_2 and q_5 also cancel each other. Hence, the net force on $-q$ is,

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(q)(q)}{r^2} \qquad \text{(towards } q_3\text{)}$$

or

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a^2}$$

Example 20. *A point charge $q_1 = 9.1\,\mu C$ is held fixed at origin. A second point charge $q_2 = -0.42\,\mu C$ and a mass 3.2×10^{-4} kg is placed on the x-axis, $0.96\,m$ from the origin. The second point charge is released at rest. What is its speed when it is $0.24\,m$ from the origin?*

Sol. From conservation of mechanical energy, we have decrease in gravitational potential energy = increase in kinetic energy.

$$\text{or} \quad \frac{1}{2}mv^2 = U_i - U_f = \frac{q_1q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_i}-\frac{1}{r_f}\right) = \frac{q_1q_2}{4\pi\varepsilon_0}\left(\frac{r_f - r_i}{r_ir_f}\right)$$

$$v = \sqrt{\frac{q_1q_2}{2\pi\varepsilon_0 m}\left(\frac{r_f - r_i}{r_ir_f}\right)}$$

$$= \sqrt{\frac{(2\times 9\times 10^9)(9.1\times 10^{-6})(-0.42\times 10^{-6})}{3.2\times 10^{-4}}\left(\frac{0.24-0.96}{(0.24)(0.96)}\right)}$$

$$= 26 \text{ m/s}$$

Example 21. *A point charge $q_1 = -5.8\,\mu C$ is held stationary at the origin. A second point charge $q_2 = +4.3\,\mu C$ moves from the point $(0.26\,m, 0, 0)$ to $(0.38\,m, 0, 0)$. How much work is done by the electric force on q_2?*

Sol. Work done by the electrostatic forces $= U_i - U_f$

$$= \frac{q_1q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_i}-\frac{1}{r_f}\right) = \frac{q_1q_2}{4\pi\varepsilon_0}\left(\frac{r_f - r_i}{r_i\,r_f}\right)$$

$$= \frac{(-5.8\times 10^{-6})(4.3\times 10^{-6})(9\times 10^9)(0.38-0.26)}{(0.38)(0.26)}$$

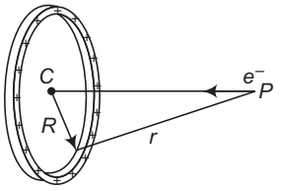
$$= -0.272 \text{ J}$$

Example 22. *A uniformly charged thin ring has radius $10.0\,cm$ and total charge $+12.0\,nC$. An electron is placed on the ring's axis a distance $25.0\,cm$ from the centre of the ring and is constrained to stay on the axis of the ring. The electron is then released from rest.*

(a) Describe the subsequent motion of the electron.

(b) Find the speed of the electron when it reaches the centre of the ring.

Sol. (a) The electron will be attracted towards the centre C of the ring. At C net force is zero, but on reaching C, electron has some kinetic energy and due to inertia it crosses C, but on the other side it is further attracted towards C. Hence, motion of electron is oscillatory about point C.



(b) As the electron approaches C, its speed (hence, kinetic energy) increases due to force of attraction towards the centre C. This increase in kinetic energy is at the cost of electrostatic potential energy. Thus,

$$\frac{1}{2}mv^2 = U_i - U_f = U_p - U_c = (-e)[V_p - V_c] \qquad ...(i)$$

Here, V is the potential due to ring.

$$V_p = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} \qquad (q = \text{charge on ring})$$

$$= \frac{(9\times 10^9)(12\times 10^{-9})}{(\sqrt{(10)^2+(25)^2})\times 10^{-2}} = 401 \text{ V}$$

$$V_c = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$

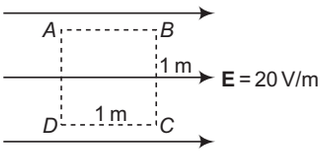
$$= \frac{(9\times 10^9)(12\times 10^{-9})}{10\times 10^{-2}} = 1080 \text{ V}$$

Substituting the proper values in Eq. (i), we have

$$\frac{1}{2}\times 9.1\times 100^{-31}\times v^2 = (-1.6\times 10^{-19})(401-1080)$$

$$\therefore \qquad v = 15.45\times 10^6 \text{ m/s}$$

Example 23. *In the uniform electric field shown in figure, find*



(a) $V_A - V_D$ *(b) $V_A - V_C$*

(c) $V_B - V_D$ *(d) $V_C - V_D$*

Sol. Using the relation

$$V = Ed \qquad (\text{in uniform } \mathbf{E})$$

where, V = PD between the two points

E = magnitude of $\mathbf{E}$ and

d = projection of line (joining two points) along $\mathbf{E}$

(a) $V_A - V_D = 0$ as $d = 0$

(b) $V_A - V_C = +(20)(1) = 20$ V

(c) $V_B - V_D = -(20)(1) = -20$ V

(d) $V_C - V_D = -(20)(1) = -20$ V

Example 24. *The electric field in a region is given by $\mathbf{E} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$. Here a and b are constants. Find the net flux passing through a square area of side l parallel to y-z plane.*

Sol. A square area of side l parallel to y-z plane in vector form can be written as,

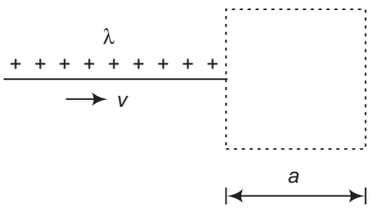
$$\mathbf{s} = l^2\hat{\mathbf{i}}$$

Given, $\mathbf{E} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$

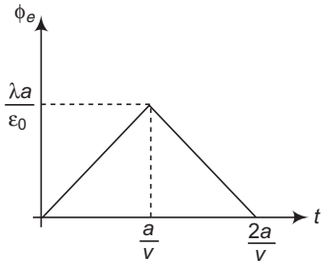
$\therefore$ Electric flux passing through the given area will be,

$$\phi_e = \mathbf{E}\cdot\mathbf{s} = (a\hat{\mathbf{i}} + b\hat{\mathbf{j}})\cdot(l^2\hat{\mathbf{i}}) = al^2$$

Example 25. *Figure shows an imaginary cube of side a. A uniformly charged rod of length a moves towards right at a constant speed v. At $t = 0$, the right end of the rod just touches the left face of the cube. Plot a graph between electric flux passing through the cube versus time.*

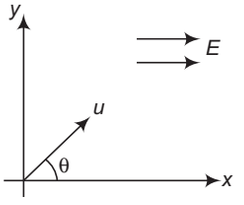


Sol. The electric flux, passing through a closed surface depends on the net charge inside the surface. Net charge in this case first increases, reaches a maximum value and finally decreases to zero. The same is the case with the electric flux. The electric flux ϕ_e *versus* time graph is as shown in figure.



Example 26. *A charged particle of mass $m = 1\,kg$ and charge $q = 2\,\mu C$ is thrown from a horizontal ground at an angle $\theta = 45°$ with speed $20\,m/s$. In space a horizontal electric field $E = 2 \times 10^7\,V/m$ exist. Find the range on horizontal ground of the projectile thrown.*

Sol. The path of the particle will be a parabola, but along *x*-axis also motion of the particle will be accelerated. Time of flight of the projectile is



$$T = \frac{2u_y}{a_y} = \frac{2u_y}{g}$$

$$= \frac{2 \times 20 \cos 45°}{10}$$

$$= 2\sqrt{2}\ \text{s}$$

Horizontal range of the particle will be,

$$R = u_x T + \frac{1}{2} a_x T^2$$

Here, $$a_x = \frac{qE}{m} = \frac{(2 \times 10^{-6})(2 \times 10^{+7})}{1} = 40\ \text{m/s}^2$$

$\therefore$ $$R = (20 \cos 45°)(2\sqrt{2}) + \frac{1}{2}(40)(2\sqrt{2})^2$$

$$= 40 + 160 = 200\ \text{m}$$

Example 27. *Two points A and B are 2 cm apart and a uniform electric field E acts along the straight line AB directed from A to B with $E = 200\,N/C$. A particle of charge $+10^{-6}\,C$ is taken from A to B along AB. Calculate*

(a) the force on the charge
(b) the potential difference $V_A - V_B$ and
(c) the work done on the charge by **E**.

Sol. (a) Electrostatic force on the charge,

$$F = qE = (10^{-6})(200) = 2 \times 10^{-4}\ \text{N}$$

(b) In uniform electric field,

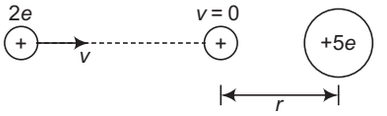
$$\text{PD}, V = Ed \quad \text{or} \quad V_A - V_B = 200 \times 2 \times 10^{-2} = 4\ \text{V}$$

(c) $$W = F \cdot s \cos\theta = (2 \times 10^{-4})(2 \times 10^{-2}) \cos 0°$$

$$= 4 \times 10^{-6}\ \text{J}$$

Example 28. *An alpha particle with kinetic energy 10 MeV is heading towards a stationary tin nucleus of atomic number 50. Calculate the distance of closest approach.*

Sol. Due to repulsion by the tin nucleus, the kinetic energy of the α-particle gradually decreses at the expense of electrostatic potential energy.



$\therefore$ Decrease in kinetic energy

= increase in potential energy

or $$\frac{1}{2}mv^2 = U_f - U_i \quad \text{or} \quad \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r} - 0$$

$\therefore$ $$r = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2e)(s_0 e)}{(\text{KE})}$$

Substituting the values,

$$r = \frac{(9 \times 10^9)(2 \times 1.6 \times 10^{-19})(1.6 \times 10^{-19} \times 50)}{10 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$= 14.4 \times 10^{-15}\ \text{m}$$

Example 29. *Three point charges of $1\,C, 2\,C$ and $3\,C$ are placed at the corners of an equilateral triangle of side $1\,m$. Calculate the work required to move these charges to the corners of a smaller equilateral triangle of side $0.5\,m$.*

Sol. Work done $= U_f - U_i$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)[q_3 q_2 + q_3 q_1 + q_2 q_1]$$

$$= 9 \times 10^9 \left(\frac{1}{0.5} - \frac{1}{1}\right)[(3)(2) + (3)(1) + (2)(1)]$$

$$= 99 \times 10^9\ \text{J}$$

NCERT Selected Questions

Q 1. What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?

Sol. $F=\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}$

$$\therefore \qquad F=9\times10^{9}\times\frac{2\times10^{-7}\times3\times10^{-7}}{(0.30)^2}=6\times10^{-3}\ \text{N}$$

Q 2. The electrostatic force on a small sphere of charge $0.4\,\mu$C due to another small sphere of charge $-0.8\,\mu$C in air is 0.2 N. (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first?

Sol. (a) Using the relation $F=\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_1q_2}{r^2}$, we get

$$r^2=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{F}$$

$$=9\times10^{9}\times\frac{0.4\times10^{-6}\times(0.8\times10^{-6})}{0.2}$$

$$=144\times10^{-4}\ \text{m}^2$$

$$r=12\times10^{-2}\ \text{m}=0.12\ \text{m}$$

(b) Electrostatic forces always appear in pairs and follows Newton's Third law of motion.

$\therefore\ |\mathbf{F}_{21}|=$ Force on q_2 due to $q_1=0.2$ N and is attractive in nature.

Q 3. Check that the ratio ke^2/Gm_em_p is dimensionless. Look up a table of physical constants and determine the value of this ratio. What does the ratio signify?

Sol. Dimensions of e^2 are $=[\text{C}^2]$

Dimensions of k are $=[\text{Nm}^2\text{C}^{-2}]=[\text{ML}^3\text{T}^{-2}\text{C}^{-2}]$

Dimensions of G are $=[\text{M}^{-1}\text{L}^3\text{T}^{-2}]$

Dimensions of m_e are $=[\text{M}]$

$\therefore$ Dimensions of $\dfrac{ke^2}{Gm_em_p}$ are

$$\frac{ke^2}{Gm_em_p}=\frac{[\text{ML}^3\text{T}^{-2}\ \text{C}^{-2}]\,[\text{C}^2]}{[\text{M}^{-1}\text{L}^3\text{T}^{-2}]\,[\text{M}]\,[\text{M}]}$$

$$=[\text{M}^0\ \text{L}^0\text{T}^0]$$

$\therefore\ \dfrac{ke^2}{Gm_em_p}$ is a dimensionless quantity.

$$\frac{ke^2}{Gm_em_p}=\frac{(9\times10^{9})\times(1.6\times10^{-19})^2}{6.67\times10^{-11}\times9.1\times10^{-31}\times1.66\times10^{-27}}$$

$$=2.29\times10^{39}$$

This factor represents the ratio of electrostatic force and the gravitational force between an electron and a proton.

Q 4. (a) Explain the meaning of the statement 'electric charge of a body is quantised'.

(b) Why can one ignore quantisation of electric charge when dealing with macroscopic, i.e. large scale charges?

Sol. (a) The meaning of the statement 'electric charge of a body is quantised' is that the charge on it is always some integral multiple of elementary charge on an electron or a proton, i.e. $q=ne$ where, n is an integer, $e=$ magnitude of the charge on an electron or proton $=1.6\times10^{-19}$ C. A fraction of the fundamental charge e has never been observed in free state.

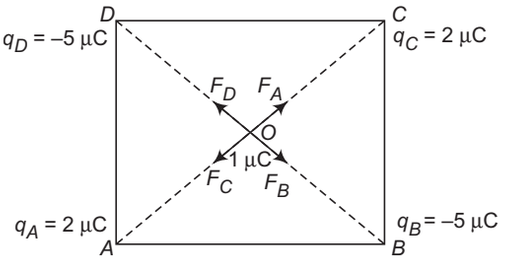
(b) In practice, the charge on a charged body at macroscopic level is very large while the charge on an electron is very small. When electrons are added to or removed from a body, the change taking place in the total charge on the body is so small that the charge seems to be varying in a continuous manner. Thus, quantisation of electric charge can be ignored at macroscopic level.

Q 5. When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain, how this observation is consistent with the law of conservation of charge?

Sol. Before rubbing, both the glass rod and silk cloth are electrically neutral. When the glass rod is rubbed with silk cloth, a few electrons get transferred from the rod to the silk cloth, thus glass rod becomes positively charged and silk cloth negatively charged. The positive charge on the glass rod is exactly equal to the negative charge on the silk cloth, so net charge on the system is constant. Thus, the appearance of charge on the glass rod and silk cloth is in accordance with the law of conservation of charge as the total charge of the isolated system is constant.

Q 6. Four point charges $q_A=2\,\mu$C, $q_B=-5\,\mu$C, $q_C=2\,\mu$C and $q_D=-5\,\mu$C are located at the corners of a square $ABCD$ of side 10 cm. What is the force on a charge of $1\,\mu$C placed at centre of the square?

Sol.



$F_A=F_C$ and $F_B=F_D$

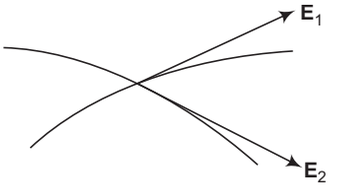
Thus, the net force on the charge of $1\,\mu$C due to the given arrangement of charges is zero.

Q 7. (a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?

(b) Explain, why two fields lines never cross each other at any point?

Sol. (a) The electrostatic line of force are continuous curves and cannot have sudden breaks because if it is so, then it will indicate the absence of electric field at the break points.

(b) The electric lines of force never cross each other because if they do so, then at the point of their intersection, we can draw two which give two directions of electric field at that point which is not possible.

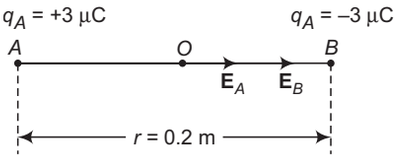


Q 8. Two point charges $q_A = 3\,\mu C$ and $q_B = -3\,\mu C$ are located 20 cm apart in vacuum.

(a) What is the electric field at the mid-point O of the line AB joining the two charges?

(b) If a negative test charge of magnitude 1.5×10^{-9} C is placed at this point, what is the force experienced by the test charge?

Sol.



$$OA = OB = \frac{r}{2} = \frac{0.2}{2} = 0.1\text{m}$$

(a) If $\mathbf{E}_A$ and $\mathbf{E}_B$ be the electric fields at point O due to q_A and q_B, respectively. Then,

$$\mathbf{E}_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A}{(OA)^2}$$

$$= 9 \times 10^9 \times \frac{3 \times 10^{-6}}{(0.1)^2}$$

$$= 2.7 \times 10^{-6}\ \text{NC}^{-1} \text{ along } OB$$

and $$\mathbf{E}_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_B}{(OB)^2}$$

$$= 9 \times 10^9 \times \frac{3 \times 10^{-6}}{(0.1)^2}$$

$$= 2.7 \times 10^{-6}\ \text{NC}^{-1} \text{ along } OB$$

If $\mathbf{E}$ be the net electric field at point O due to q_A and q_B, then

$$\mathbf{E} = \mathbf{E}_A + \mathbf{E}_B$$

$$= 2.7 \times 10^{-6} + 2.7 \times 10^{-6}$$

$$= 5.4 \times 10^{-6}\ \text{NC}^{-1} \text{ along } OB$$

(b) Force on a negative charge of magnitude 1.5×10^{-9} C is given by the formula.

$$\mathbf{F} = q\mathbf{E}$$

$$\therefore \quad \mathbf{F} = -1.5 \times 10^{-9} \times 5.4 \times 10^{-6} = -8.1 \times 10^{-3}\ \text{N}$$

Negative sign shows that F acts opposite to **E**, i.e. along OA.

Q 9. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points $A(0, 0, -15\text{ cm})$ and $B(0, 0, +15\text{ cm})$ respectively. What is the electric dipole moment of the system?

Sol. Electric dipole moment

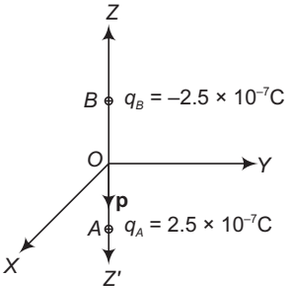
$$p = \text{either charge} \times \text{dipole length}$$

$$= q_A \times AB$$

$$= 2.5 \times 10^{-7} \times 0.30$$

$$= 7.5 \times 10^{-8}\ \text{C-m}$$

The electric dipole moment is directed from B to A, i.e. from negative charge to positive charge.



Q 10. An electric dipole with dipole moment 4×10^{-9} C-m is aligned at 30° with the direction of a uniform electric field of magnitude $5 \times 10^4\ \text{NC}^{-1}$. Calculate the magnitude of the torque acting on the dipole.

Sol. Using the formula, $\tau = pE \sin\theta$, we get

$$\tau = 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30°$$

$$= 20 \times 10^{-5} \times \frac{1}{2}$$

$$= 10^{-4}\ \text{N-m}$$

Q 11. A polythene piece rubbed with wool is found to have a negative charge of -3×10^{-7} C.

(a) Estimate the number of electrons transferred (from which to which?)

(b) Is there a transfer of mass from wool to polythene?

Sol. (a) From quantisation of charge,

We know that, $q = ne$

$$n = \frac{q}{e} = \frac{-3 \times 10^{-7}}{-1.6 \times 10^{-19}} = 1.875 \times 10^{12}$$

(b) Yes, there is a transfer of mass from wool to polythene as electrons are material particles and are transferred from wool to polythene piece.

Q 12. (a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C? The radii of A and B are negligible compared to the distance of separation.

(b) What is the force of repulsion of each sphere if charge doubles the above amount, and the distance between them is halved?

Sol. (a) $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_A q_B}{r^2} = \dfrac{9\times 10^9 \times 6.5\times 10^{-7} \times 6.5\times 10^{-7}}{(0.5)^2}$

$\approx 1.5 \times 10^{-2}$ N

(b) If charge on each sphere is doubled then

$q_A = q_B = 2 \times 6.5 \times 10^{-7}$ C

$= 13 \times 10^{-7}$ C

and $r = \dfrac{1}{2} \times 50$ cm $= 0.25$ m

$\therefore \quad F' = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q'_A\, q'_B}{r^2}$

$= \dfrac{9\times 10^9 \times 13\times 10^{-7} \times 13\times 10^{-7}}{(0.25)^2}$

≈ 0.24 N

Q 13. Suppose the spheres A and B of previous exercise have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the force of repulsion between A and B?

Sol. When 3rd sphere C having charge zero is brought in contact with 1st sphere A, then

Charge on A = Charge on $C = q'_1$ (say)

$\therefore \quad q'_1 = \dfrac{q_A + q_C}{2} = \dfrac{6.5\times 10^{-7} + 0}{2} = 3.25 \times 10^{-7}$ C

or $q'_1 = 3.25 \times 10^{-7}$ C

When 3rd sphere C having charge 3.25×10^{-7} C is brought in contact with 2nd sphere B, then charge on B say q'_2 is given by

Charge on B = Charge on C

or $q'_2 = \dfrac{q_B + q'_1}{2} = \dfrac{6.5\times 10^{-7} + 3.25\times 10^{-7}}{2}$

or $q'_2 = 4.875 \times 10^{-7}$ C

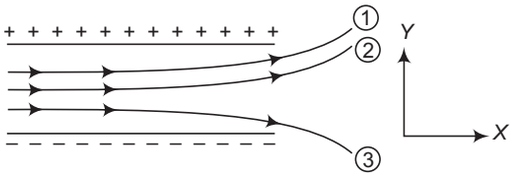
If F be the force of repulsion between spheres A and B, then according to Coulomb's law of electrostatic forces,

$F = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q'_1\, q'_2}{r^2}$

$F = \dfrac{9\times 10^9 \times 3.25\times 10^{-7} \times 4.875\times 10^{-7}}{(0.5)^2}$

$\approx 5.7 \times 10^{-3}$ N

Q 14. Figure below shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?



Sol. Charges on the two plates are shown in the figure. Since, charged particles are deflected towards the oppositely charged plates, therefore (1) and (2) are negatively charged while particle (3) is positively charged.

The deflection produced in the path of a charged particle along vertical direction is given by, $y = \dfrac{1}{2}at^2 = \dfrac{1}{2}\dfrac{eE}{m}t^2$.

So, the displacement $y \propto \left(\dfrac{e}{m}\right)$. As the charged particle 3 suffers maximum deflection along vertical, so value of y is maximum for it, hence, it has the highest charge to mass ratio.

Q 15. Consider a uniform electric field $\mathbf{E} = 3 \times 10^3\ \hat{\mathbf{i}}$ N/C.

(a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the Y-Z plane?

(b) What is the flux through the same square if the normal to its plane makes at 60° angle with the X-axis?

Sol. Here, $\mathbf{E} = 3 \times 10^3\ \hat{\mathbf{i}}$ NC^{-1}, i.e. the electric field acts along positive direction of x-axis.

Side of square = 10 cm

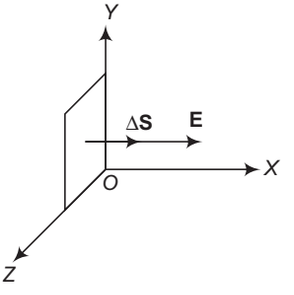
$\therefore$ Surface area, $\Delta S = (10\text{ cm})^2 = 10^{-2}$ m^2

or $\Delta\mathbf{S} = 10^{-2}\ \hat{\mathbf{i}}$ m^2

as normal to the square is along X-axis.

(a) If ϕ be the electric flux through the square, then

$\phi = \mathbf{E} \cdot \Delta\mathbf{S} = (3 \times 10^3\ \hat{\mathbf{i}}) \cdot (10^{-2}\ \hat{\mathbf{j}}) = 30$ N-m^2-C^{-1}

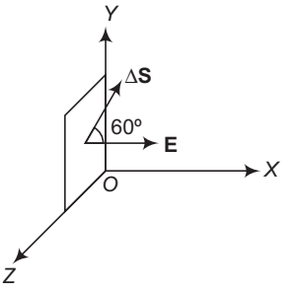


(b) Here, angle between normal to the square, i.e. area vector and the electric field is 60°.

i.e. $\theta = 60°$

$\therefore \quad \phi = \mathbf{E} \cdot \Delta\mathbf{S} = E \cdot \Delta S \cos 60°$

$= 3 \times 10^3 \times 10^{-2} \times \frac{1}{2} = 15 \text{ Nm}^2 \text{ C}^{-1}$



Q 16. What is the net flux of the uniform electric field of above example through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Sol. Zero. Net flux from any closed surface in uniform electric field = 0

Q 17. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0×10^3 Nm^2/C.

(a) What is the net charge inside the box?

(b) If the net outward flux through the surface of the box was zero, could you conclude that there were no charges inside the box? Why or why not?

Sol. (a) If the net charge inside the black box is q, then

$$\phi = \frac{q}{\varepsilon_0}$$

We get, $q = \varepsilon_0 \phi$

or $\quad q = 8.854 \times 10^{-12} \times 8 \times 10^3 \text{ C} \approx 0.071 \mu\text{C}$

(b) We cannot conclude that there is no charge inside the box because there might be equal amounts of positive and negative charges cancelling each other and thus making the resultant charge equal to zero. Thus, we can only conclude that the net charge inside the box is zero.

Q 18. A point charge $+10\,\mu\text{C}$ is a distance 5 cm directly above the centre of a square of side 10 cm as shown in figure. What is the magnitude of the electric flux through the square? (Hint think of the square as one face of a cube with edge 10 cm.)

Sol. The given square can be imagined as one of the side faces of a cube of side 0.10 m. The given charge can be imagined to be at the centre of this cube at a distance of 5 cm.

Here, $\quad q = +10\,\mu\text{C} = 10^{-5}\text{C}$

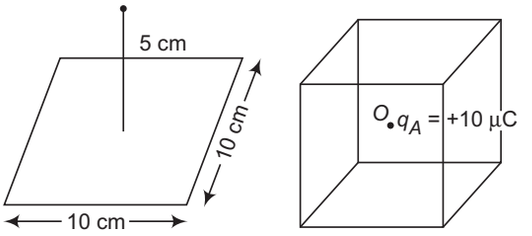
Then, according to Gauss's theorem, the total electric flux through all the 6 faces of the cube is given by

$$\phi = \frac{q}{\varepsilon_0}$$

If ϕ' be the electric flux through the given square, then

$$\phi' = \frac{1}{6}\phi = \frac{1}{6} \cdot \frac{q}{\varepsilon_0} = \frac{1}{6} \times \frac{10^{-5}}{8.854 \times 10^{-12}} \text{ NC}^{-1} \text{ m}^2$$

$$= 1.88 \times 10^5 \text{ NC}^{-1} \text{ m}^2$$



Q 19. A point charge of $2\,\mu\text{C}$ is at the centre of a cubic Gaussian surface 9 cm on edge. What is the net electric flux through the surface?

Sol. According to Gauss's theorem, the electric flux through the six faces of the cube (a closed surface) is given by

$$\phi = \frac{q}{\varepsilon_0} = \frac{2 \times 10^{-6}}{8.856 \times 10^{-12}} = 2.26 \times 10^5 \text{ N-m}^2/\text{C}$$

Q 20. A point charge causes an electric flux of -1.0×10^3 Nm^2/C to pass through a spherical Gaussian surface of 10 cm radius centred on the charge.

(a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface?

(b) What is the value of the point charge?

Sol. (a) According to Gauss's law, the electric flux through a Gaussian surface depends upon the charge enclosed inside the surface and not upon its size. Thus, the electric flux will remain unchanged, i.e. -1.0×10^3 Nm^2C^{-1}.

(b) Using the formula, $\phi = \frac{q}{\varepsilon_0}$, we get

$$q = \varepsilon_0 \phi = 8.854 \times 10^{-12} \times (-1.0 \times 10^3)$$

$$= -8.854 \times 10^{-9} \text{ C} = 8.8 \text{ nC}$$

Q 21. A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 N/C and points radially inward, what is the net charge on the sphere?

Sol. Using the formula, $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$

We get, $\quad q = 4\pi\varepsilon_0 E r^2$

$$= \frac{1}{9 \times 10^9} \times 1.5 \times 10^3 \times (0.20)^2$$

$$= 6.67 \times 10^{-9} \text{ C} = 6.67 \text{ nC}$$

Also as E acts in the inward direction, so charge on the sphere is negative.

$$q = -6.67 \times 10^{-9} \text{ C} = -6.67 \text{ nC}$$

Q 22. A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of $80\,\mu C/m^2$.

(a) Find the charge on the sphere.

(b) What is the total electric flux leaving the surface of the sphere?

Sol. (a) Using the relation, $\sigma = \dfrac{q}{4\pi R^2}$

We get, $q = 4\pi R^2 \times \sigma = 4 \times \dfrac{22}{7} \times (1.2)^2 \times 80 \times 10^{-6}$

$= 1.45 \times 10^{-3}$ C

(b) Using Gauss's theorem, we get

$$\phi = \frac{q}{\varepsilon_0} = \frac{1.45 \times 10^{-3}}{8.854 \times 10^{-12}} = 1.64 \times 10^8 \text{ Nm}^2\text{C}^{-1}$$

Q 23. An infinite line charge produces a field of 9×10^4 N/C at a distance of 2 cm. Calculate the linear charge density.

Sol. Using the relation, $E = \dfrac{1}{2\pi\varepsilon_0} \cdot \dfrac{\lambda}{r}$ or $\lambda = 2\pi\varepsilon_0 rE$

$$= \frac{1}{9 \times 10^9} \times \frac{0.02 \times 9 \times 10^4}{2} = 10^{-7} \text{ C/m}$$

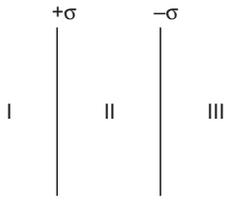
Q 24. Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-22} C/m^2. What is E

(a) in the outer region of the first plate,

(b) in the outer region of the second plate, and

(c) between the plates?

Sol.



In regions III and I, two fields of magnitude $\sigma / 2\varepsilon_0$ are substractive.

$\therefore \quad E_I = E_{III} = 0$

In region II, two fields are additive.

$\therefore \quad E_{II} = \dfrac{\sigma}{2\varepsilon_0} + \dfrac{\sigma}{2\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0}$

Substituting the values of σ and ε_0, we get

$E = 1.92 \times 10^{-8}$ N/C

Q 25. An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 NC^{-1} in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3}. Estimate the radius of the drop. ($g = 9.81$ ms^{-2}, $e = 1.60 \times 10^{-19}$ C).

Sol. If q = charge on the drop, then

$$q = ne = 12 \times 1.6 \times 10^{-19} \text{ C} = 19.2 \times 10^{-19} \text{ C}$$

If F_e be the electrostatic force on the oil drop due to electric field.

Then, $F_e = qE = 19.2 \times 10^{-19} \times 2.55 \times 10^{-4}$ N ...(i)

Also, let F_g = Force on the drop due to gravity, then

$$F_g = mg = \frac{4}{3}\pi r^3 \rho g \qquad \text{...(ii)}$$

Here, ρ = density of oil $= 1.26$ g cm^{-3}

$= 1.26 \times 10^3$ kg - m^{-3}

Putting these values in Eq. (ii), we get

$$F_g = \frac{4}{3}\pi r^3 \times 1.26 \times 10^3 \times 9.81 \qquad \text{...(iii)}$$

As the drop remains stationary,

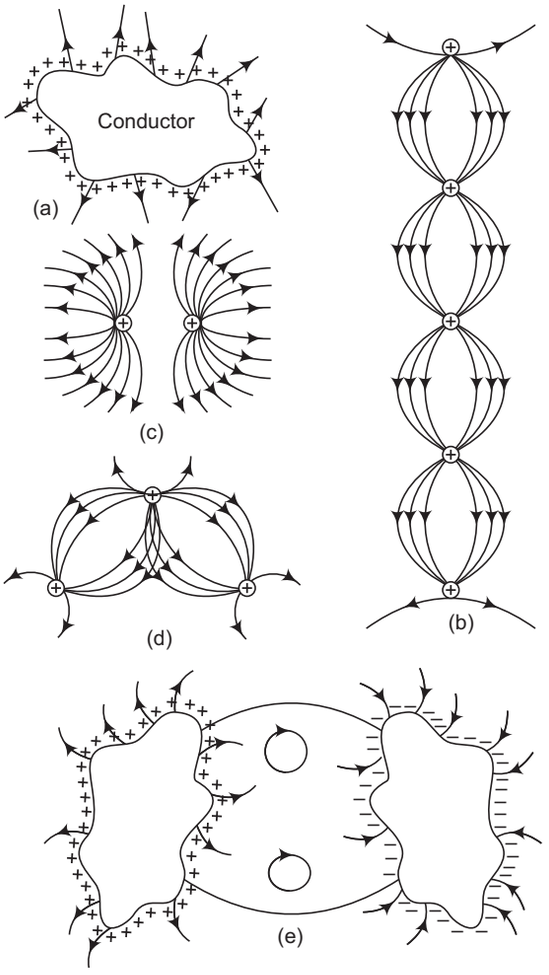
$$F_e = F_g$$

or $19.2 \times 10^{-19} \times 2.55 \times 10^4 = 4/3\ \pi r^3 \times 1.26 \times 10^3 \times 9.81$

After solving, we get

$r = 9.82 \times 10^{-4}$ mm

Q 26. Which among the curves shown in figure cannot possibly represent electrostatic field lines?



Sol. Only (c) represents electric field lines.

(a) As the electrostatic field lines start or end only normally to the surface of the conductor, so Fig. (a) cannot represent electrostatic field lines as they are not normal at all points to the surface.

(b) The electrostatic lines of force cannot start from negative charge and cannot end on positive charge, so Fig. (b) cannot represent such lines.

(c) This figure represents electrostatic lines of force due to two isolated positive charges separated by some distance.

(d) As no two electrostatic lines of force intersect each other, so this figure cannot represent such lines.

(e) This figure does not represent electrostatic lines of force because they cannot form closed loop.

Q 27. It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called up quark (denoted by u) of charge $+(2/3)\,e$, and the down quark (denoted by d) of charge $(-1/3)\,e$, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.

Sol. The charge on the proton is $+e$ and it is made of three quarks. Therefore, the possible quark composition a proton is *uud*.

$$\therefore \text{ Total charge} = \frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = \frac{4-1}{3}e = e$$

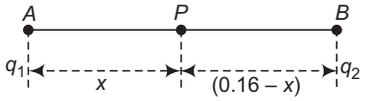
On the other hand, neutron is a neutral particle but it is also made of three quarks. For this, the possible composition of the neutron is *udd*.

$$\therefore \text{ Total charge on neutron} = +\frac{2}{3}e + \left(\frac{-e}{3}\right) + \left(\frac{-e}{3}\right) = 0$$

Q 28. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Sol. Here, $q_1 = 5\times10^{-8}$ C, $q_2 = -3\times10^{-3}$ C

Let, P be the required point, i.e. the point on the line AB at a distance x m from q_1 at which the electric potential is zero. If V_1 and V_2 be the potential at P due to q_1 and q_2 respectively, then using the formula,



$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}, \text{ we get}$$

$$V_1 = \frac{9\times10^9\times5\times10^{-8}}{x} \qquad ...(i)$$

and $$V_2 = 9\times10^9\times\frac{(-3\times10^{-8})}{(0.16-x)} \qquad ...(ii)$$

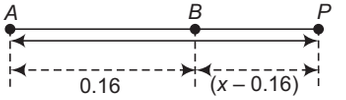
$$V_1 + V_2 = 0 \qquad \text{(given)}$$

$$9\times10^9\times\frac{5\times10^{-8}}{x} + \frac{9\times10^9\times(-3\times10^{-8})}{(0.16-x)} = 0$$

Solving the equation, we get

$$x = 0.1 \text{ m} = 10 \text{ cm from } 5\times10^{-8}\text{ C}$$

If P lies on the extended line AB, then required condition is



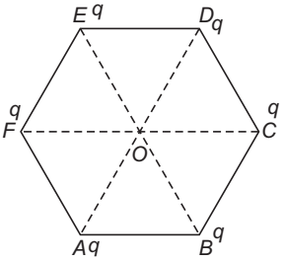
$$\frac{5}{x} - \frac{3}{(x-0.16)} = 0$$

Solving the above equation, we get

$$x = 0.4 \text{ m} = 40 \text{ cm}$$

On the side of negative charge from 5×10^{-8} C.

Q 29. A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.



Sol. $r = OA = OB = OC = OD = OE = OF = AB = 0.10$ m

Using the relation $V = \sum_{i=1}^{n}\frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$,

we get $$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} + \ldots \text{ 6 times}\right] = 6\times\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$= 6\times9\times10^9\times\frac{5\times10^{-6}}{0.1} = 2.7\times10^6\text{ V}$$

Q 30. Two charges 2 μC and −2 μC are placed at points A and B, 6 cm apart.

(a) Identify an equipotential surface of the system.

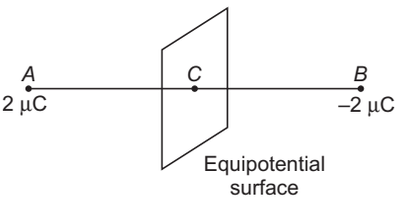
(b) What is the direction of the electric field at every point on this surface?

Sol. (a) For the given system of two charges, the equipotential surface is a plane normal to the line joining points A and B. The plane passes through the mid-point C of the line AB. The potential at C is

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{2\times10^{-6}}{0.03\text{ m}} + \frac{(-2\times10^{-6})}{0.03\text{ m}}\right] = 0$$

Thus, potential at all points lying on this plane is equal to zero, so it is an equipotential surface.

(b) We know that the electric field always acts from positively to negatively charge, thus here the electric field acts from point A (having positively charge) to point B (having negatively charge) and is normal to the equipotential surface.



Q 31. A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly on its surface. What is the electric field
(a) inside the sphere,
(b) just outside the sphere,
(c) at a point 18 cm from the centre of the sphere?

Sol. (a) Charge given to a spherical conductor resides on its outer surface. From Gauss theorem, electric field inside the spherical conductor will be zero.

(b) For a point just outside the sphere, i.e. for a point lying on the surface of the sphere, the charge may be supposed to be concentrated on the centre of the sphere.
Thus, using the relation,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}$$

we get, $$E = 9\times 10^9 \times \frac{1.6\times 10^{-7}}{(0.12)^2} = 10^5 \text{ NC}^{-1}$$

(c) $E = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{r^2}$

Here, $r = 0.18$ m

$$\therefore \quad E = 9\times 10^9 \times \frac{1.6\times 10^{-7}}{(0.18)^2} = 4.44\times 10^4 \text{ NC}^{-1}$$

Q 32. A parallel plate capacitor with air between the plates has a capacitance of 8 pF ($1\text{ pF} = 10^{-12}$ F). What will be the capacitance if the distance between the plates is reduced by half and the space between them is filled with a substance of dielectric constant 6?

Sol. $C_0 = \dfrac{\varepsilon_0 A}{d}$ …(i)

Distance between the plates with a dielectric substance between them $= \dfrac{d}{2}$

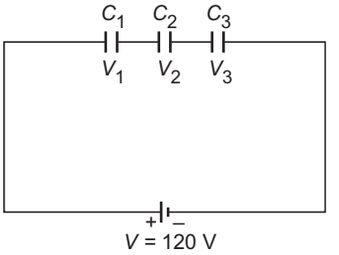
$$C = \frac{\varepsilon_0 K\cdot A}{\left(\frac{d}{2}\right)} = 2K\,\frac{\varepsilon_0 A}{d} \qquad \text{...(ii)}$$

$$\therefore \quad \frac{C}{C_0} = 2K$$

or $$C = 2KC_0 = 2\times 6\times 8\times 10^{-12}$$
$$= 96\times 10^{-12} \text{ F} = 96 \text{ pF}$$

Q 33. Three capacitors each of capacitance 9 pF are connected in series.
(a) What is the total capacitance of the combination?
(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Sol. (a) C = Capacitance of series combination



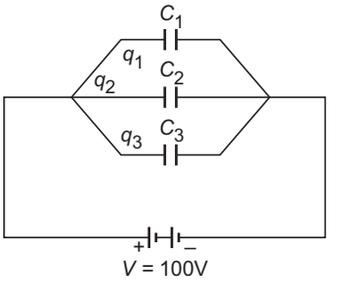
We know that the total capacitance of the series combination is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
$$= \frac{1}{9\times 10^{-12}} + \frac{1}{9\times 10^{-12}} + \frac{1}{9\times 10^{-12}}$$
$$= \frac{3}{9\times 10^{-12}} = \frac{1}{3\times 10^{-12}}$$
$$C = 3\times 10^{-12} \text{ F} = 3 \text{ pF}$$

(b) All capacitors have same capacity,

$$\therefore \quad V_1 = V_2 = V_3 = \frac{V}{3} = \frac{120}{3} = 40 \text{ V}$$

Q 34. Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.



(a) What is the total capacitance of the combination?
(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Sol. (a) C = Total capacitance of the parallel grouping.

We know that the total capacitance of the parallel combination is given by

$$C = C_1 + C_2 + C_3$$
$$= 2\times 10^{-12} + 3\times 10^{-12} + 4\times 10^{-12}$$
$$= 9\times 10^{-12} \text{ F} = 9 \text{ pF}$$

(b) Let q_1, q_2 and q_3 be the charges on the capacitors C_1, C_2 and C_3, respectively.

Also we know that in parallel combination, the potential difference across each capacitor.

$= \text{Supply voltage} = 100 \text{ V}$

$q_1 = C_1V = 2 \times 10^{-12} \times 100 = 2 \times 10^{-10}$ C

$q_2 = C_2V = 3 \times 10^{-12} \times 100 = 3 \times 10^{-10}$ C

$q_3 = C_3V = 4 \times 10^{-12} \times 100 = 4 \times 10^{-10}$ C

Q 35. In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Sol. Using the relation, $C_0 = \dfrac{\varepsilon_0 A}{d}$

We get, $C_0 = \dfrac{8.854 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$

$= 17.708 \times 10^{-12}$ F ≈ 18 pF

Now, $V_0 =$ Potential difference applied across the capacitor $= 100$ V

Using the relation, $q_0 = C_0V_0$

We get $q_0 = 17.708 \times 10^{-12} \times 100 \approx 1.8 \times 10^{-9}$ C

Q 36. Explain what would happen if in the capacitor given in above example a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

(a) while the voltage supply remained connected.

(b) after the supply was disconnected.

Sol. (a) As the mica sheet completely fills the space between the plates, thus the capacitance of the capacitor (C) is given by

$C = KC_0 = 6 \times 18 \times 10^{-12}$ F $= 108 \times 10^{-12}$ F $= 108$ pF

Thus, the capacitance of the capacitor increases by K times on inserting the mica sheet.

Potential difference across this capacitor, $V = 100$ V

$\therefore$ Charge q' on the capacitor with mica sheet as medium is given by

$q' = CV = 108 \times 10^{-12} \times 100$

$= 108 \times 10^{-8}$ C

Now, clearly $q' = KC_0V = Kq$

$= 6 \times 1.8 \times 10^{-9}$

$= 1.8 \times 10^{-8}$ C

(b) If the mica sheet is inserted after the battery is disconnected, then the charge on the capacitor with mica as medium remains same.

Q 37. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?

Sol. Using the relation, $U = \dfrac{1}{2} CV^2$

We get, $U = \dfrac{1}{2} \times 12 \times 10^{-12} \times (50)^2 = 1.5 \times 10^{-8}$ J

Q 38. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Sol. $U_1 = \dfrac{1}{2} C_1V_1^2 = \dfrac{1}{2} \times 6 \times 10^{-10} \times (200)^2 = 12 \times 10^{-6}$ J

Let V be the common potential, then

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$= \frac{6 \times 10^{-10} \times 200 + 6 \times 10^{-10} \times 0}{6 \times 10^{-10} + 6 \times 10^{-10}}$$

$$= \frac{12 \times 10^{-8}}{12 \times 10^{-10}} = 100 \text{ V}$$

If U_2 be the final electrostatic energy, then

$$U_2 = \frac{1}{2} (C_1 + C_2) V^2$$

$$= \frac{1}{2} \times (6 \times 10^{-10} + 6 \times 10^{-10}) \times (100)^2$$

$$= 6 \times 10^{-6} \text{ J}$$

$\therefore$ Loss in electrostatic energy $= U_1 - U_2$

$= 12 \times 10^{-6} - 6 \times 10^{-6} = 6 \times 10^{-6}$ J

Q 39. A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point $P(0, 0, 3\text{ cm})$ to a point $Q(0, 4\text{ cm}, 0)$, *via* a point $R(0, 6\text{ cm}, 9\text{ cm})$.

Sol. Here, $q =$ Charge at origin $O = 8$ mC $= 8 \times 10^{-3}$ C

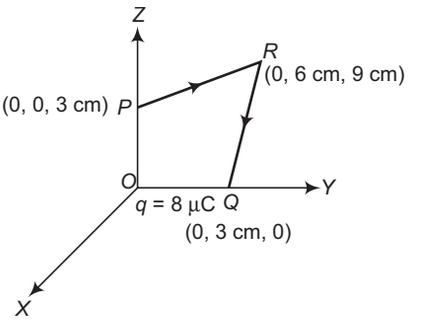
$q_0 =$ Charge to be carried from P to Q *via* R

$= -2 \times 10^{-9}$ C

$\therefore$ $r_1 = 3$ cm $= 3 \times 10^{-2}$ m

$r_2 = 4$ cm $= 4 \times 10^{-2}$ m

As electrostatic force are conservative forces, the work done in moving q_0 is independent of the path followed. Thus there is no relevance of the point R.



Let W_{PQ} be the work done in moving q_0 from P to Q, then using the relation,

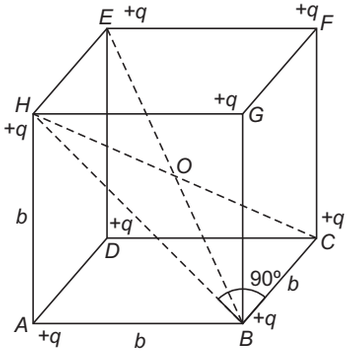
$$W_{PQ} = \frac{1}{4\pi\varepsilon_0} q_0 q \left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

We get, $W_{PQ} = 9 \times 10^9 \times (-2 \times 10^{-9}) \times 8 \times 10^{-3} \left(\frac{1}{4 \times 10^{-2}} - \frac{1}{3 \times 10^{-2}}\right)$

$= 1.2$ J

Q 40. A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Sol. Distance of each charge from $O = \frac{\sqrt{3}}{2} b$



$$V = \frac{1}{4\varepsilon_0\pi} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{OA} + \frac{q}{OB} + \frac{q}{OC} + \frac{q}{OD} - \frac{q}{OE} + \ldots + \ldots + \frac{q}{OH}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\left(\frac{b\sqrt{3}}{2}\right)} + \ldots \text{ 8 times}\right]$$

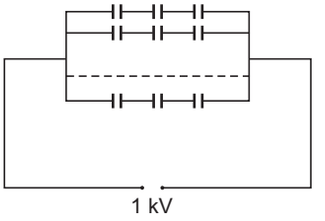
$$= \left(\frac{1}{4\pi\varepsilon_0} \times \frac{2}{b\sqrt{3}} \times q\right) \times 8$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{16q}{b\sqrt{3}} = \frac{4q}{\pi\varepsilon_0 b\sqrt{3}}$$

The electric field at O due to charges at the opposite corners such as A and F, B and E, C and H, D and G are equal in magnitude and opposite in direction. So, the net electric field at O is zero due to the symmetry of charges.

Q 41. An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference which is not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Sol. Let N be the total number of capacitors used by the technician and arrange them in m rows each row having n capacitors.



$$N = mn \quad \ldots(i)$$

C_1 = Capacitance of the each capacitor = 1 μF

C = Required capacitance of the combination = 2 μF

Maximum potential difference across each capacitor = 400 V

Potential difference across the circuit

= 1000 V = PD across each row

We know that when the capacitors are connected in series, then potential differences across them gets added.

$\therefore$ n capacitors in a row will stand a voltage = $400 \times n$

$\therefore \quad 400 \times n = 1000 \quad \text{or} \quad n = \frac{1000}{400} = 2.5$

As n has to be a whole number not less than 2.5

$\therefore \quad n = 3$

Let C' = Total capacitance of the capacitors in a row.

$\therefore \quad \frac{1}{C'} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 3$

or $\quad C' = \frac{1}{3}$ μF

Total capacitance of m such rows in parallel is given by

$$C = m \times C'$$

or $\quad m = \frac{C}{C'} = \frac{2}{\left(\frac{1}{3}\right)} = 2 \times \frac{3}{1} = 6$

$\therefore \quad N = 3 \times 6 = 18$

Thus, he must connect eighteen 1 μF capacitors in 6 parallel rows, each row containing 3 capacitors.

Q 42. What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realise from your answer why ordinary capacitors are in the range of μF or less? However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

Sol. Using the relation $C = \frac{\varepsilon_0 A}{d}$,

We get, $A = \frac{Cd}{\varepsilon_0} = \frac{2 \times 5 \times 10^{-3}}{8.854 \times 10^{-12}}$

$= 1130 \times 10^6 \text{ m}^2 = 1130 \text{ km}^2$

This area is impossible to realise as it is very large. That is why ordinary capacitors are of range much lesser than of μF.

Objective Problems

[Level 1]

Charge and Electrostatic Force

1. When 10^{14} electrons are removed from a neutral metal sphere, the charge on the sphere becomes
(a) $16\,\mu C$ (b) $-16\,\mu C$
(c) $32\,\mu C$ (d) $-32\,\mu C$

2. A conductor has 14.4×10^{-19} C positive charge. The conductor has (charge on electron $= 1.6\times10^{-19}$ C)
(a) 9 electrons in excess
(b) 27 electrons in short
(c) 27 electrons in excess
(d) 9 electrons in short

3. Charge on α-particle is
(a) 4.8×10^{-19} C (b) 1.6×10^{-19} C
(c) 3.2×10^{-19} C (d) 6.4×10^{-19} C

4. When a glass rod is rubbed with silk, it
(a) gains electrons from silk
(b) gives electrons to silk
(c) gains protons from silk
(d) gives protons to silk

5. When a body is earth connected, electrons from the earth flow into the body. This means the body is
(a) uncharged
(b) charged positively
(c) charged negatively
(d) an insulator

6. There are two charges +1 μC and +5 μC. The ratio of the forces acting on them will be
(a) 1 : 5 (b) 1 : 1
(c) 5 : 1 (d) 1 : 25

7. F_g and F_e represents gravitational and electrostatic force respectively between electrons situated at a distance 10 cm. The ratio of F_g/F_e is of the order of
(a) 10^{42} (b) 10^{-21}
(c) 10^{24} (d) 10^{-43}

8. Two similar small spheres having $+q$ and $-q$ charge are kept at a certain distance. F force acts between the two. If in the middle of two spheres, another similar small sphere having $+q$ charge is kept, then it will experience a force in magnitude and direction as
(a) zero having no direction
(b) $8F$ towards $+q$ charge
(c) $8F$ towards $-q$ charge
(d) $4F$ towards $+q$ charge

9. The force between two charges 0.06 m apart is 5 N. If each charge is moved towards the other by 0.01 m, then the force between them will become
(a) 7.20 N (b) 11.25 N (c) 22.50 N (d) 45.00 N

10. Two charged spheres separated at a distance d exert a force F on each other. If they are immersed in a liquid of dielectric constant 2, then what is the force? (if all conditions are same)
(a) $\frac{F}{2}$ (b) F (c) $2F$ (d) $4F$

11. Electric charges of $1\mu C, -1\mu C$ and $2\mu C$ are placed in air at the corners A, B and C respectively of an equilateral triangle ABC having length of each side 10 cm. The resultant force on the charge at C is
(a) 0.9 N (b) 1.8 N (c) 2.7 N (d) 3.6 N

12. Two small conducting spheres of equal radius have charges $+10\,\mu C$ and $-20\,\mu C$ respectively and placed at a distance R from each other. They experience force F_1. If they are brought in contact and separated to the same distance, they experience force F_2. The ratio of F_1 to F_2 is
(a) 1 : 8 (b) −8 : 1
(c) 1 : 2 (d) −2 : 1

13. Equal charges Q are placed at the four corners A, B, C and D of a square of length a. The magnitude of the force on the charge at B will be
(a) $\frac{3Q^2}{4\pi\varepsilon_0 a^2}$ (b) $\frac{Q^2}{4\pi\varepsilon_0 a^2}$
(c) $\left(\frac{1+2\sqrt{2}}{2}\right)\frac{Q^2}{4\pi\varepsilon_0 a^2}$ (d) $\left(2+\frac{1}{\sqrt{2}}\right)\frac{Q^2}{4\pi\varepsilon_0 a^2}$

14. Two equally charged, identical metal spheres A and B repel each other with a force F. The spheres are kept fixed with a distance r between them. A third identical, but uncharged sphere C is brought in contact with A and then placed at the mid-point of the line joining A and B. The magnitude of the net electric force on C is
(a) F (b) $F/4$
(c) $F/2$ (d) $4F$

15. Two charges $+4e$ and $+e$ are at a distance x apart. At what distance, a charge q must be placed from charge $+e$, so that it is in equilibrium?
(a) $x/2$ (b) $2x/3$
(c) $x/3$ (d) $x/4$

16. A charge Q is placed at each of the two opposite corners of a square. A charge q is placed at each of the other two corners. If the resultant force on Q is zero, then
(a) $Q = \sqrt{2}q$ (b) $Q = -\sqrt{2}q$
(c) $Q = 2\sqrt{2}q$ (d) $Q = -2\sqrt{2}q$

17. A charge q is lying at mid-point of the line joining the two similar charges Q. The system will be in equilibrium if the value of q is
(a) $Q/2$ (b) $-Q/2$
(c) $Q/4$ (d) $-Q/4$

18. Two point charges +2 C and +6 C repel each other with a force of 12 N. If a charge of – 4 C is given to each of these charges, the force now is
(a) 4 N (repulsive)
(b) 4 N (attractive)
(c) 12 N (attractive)
(d) 8 N (repulsive)

19. A charge q_1 exerts some force on a second charge q_2. If a third charge q_3 is brought near q_2, the force exerted by q_1 on q_2
(a) decreases
(b) increases
(c) remains the same
(d) increases if q_3 is of same sign as q_1 and decreases if q_3 is of opposite sign as q_1

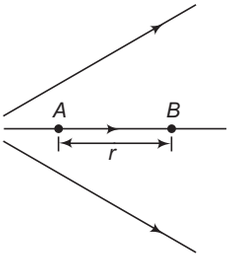
20. Three equal charges are placed on the three corners of a square. If the force between q_1 and q_2 is F_{12} and that between q_1 and q_3 is F_{13}, then the ratio of magnitudes (F_{12}/F_{13}) is
(a) 1/2 (b) 2
(c) $1/\sqrt{2}$ (d) $\sqrt{2}$

21. The centres of two identical small conducting spheres are 1 m apart. They carry charges of opposite kind and attract each other with a force F. When they are connected by a conducting thin wire they repel each other with a force $F/3$. What is the ratio of magnitude of charges carried by the spheres initially?
(a) 1 : 1 (b) 2 : 1
(c) 3 : 1 (d) 4 : 1

22. A metallic sphere having no net charge is placed near a finite metal plate carrying a positive charge. The electric force on the sphere will be
(a) towards the plate
(b) away from the plate
(c) parallel to the plate
(d) zero

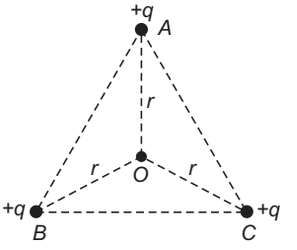
Electric Field Strength and Electric Potential

23. Figure shows the electric lines of force emerging from a charged body. If the electric field at A and B are E_A and E_B respectively and if the distance between A and B is r, then



(a) $E_A > E_B$ (b) $E_A < E_B$
(c) $E_A = \frac{E_B}{r}$ (d) $E_A = \frac{E_B}{r^2}$

24. ABC is an equilateral triangle. Charges $+q$ are placed at each corner. The electric intensity at O will be



(a) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$
(c) zero (d) $\frac{1}{4\pi\varepsilon_0}\frac{3q}{r^2}$

25. The insulation property of air breaks down at $E = 3 \times 10^6$ V/m. The maximum charge that can be given to a sphere of diameter 5 m is approximately (in coulombs)
(a) 2×10^{-2} (b) 2×10^{-3}
(c) 2×10^{-4} (d) 2×10^{-5}

26. The distance between the two charges 25 μC and 36 μC is 11 cm. At what point on the line joining the two, the intensity will be zero?
(a) At a distance of 5 cm from 25 μC
(b) At a distance of 5 cm from 36 μC
(c) At a distance of 4 cm from 25 μC
(d) At a distance of 4 cm from 36 μC

27. The electric field near a conducting surface having a uniform surface charge density σ is given by

(a) $\frac{\sigma}{\varepsilon_0}$ and is parallel to the surface

(b) $\frac{2\sigma}{\varepsilon_0}$ and is parallel to the surface

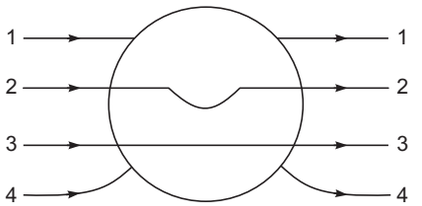
(c) $\frac{\sigma}{\varepsilon_0}$ and is normal to the surface

(d) $\frac{2\sigma}{\varepsilon_0}$ and is normal to the surface

28. The unit of electric field is not equivalent to

(a) N/C
(b) J/C
(c) V/m
(d) J/C-m

29. A metallic solid sphere is placed in a uniform electric field.The lines of force follow the path(s) shown in figure as

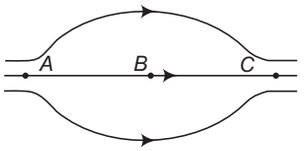


(a) 1 (b) 2 (c) 3 (d) 4

30. The unit of intensity of electric field is

(a) Newton/coulomb (b) Joule/coulomb
(c) Volt-metre (d) Newton/metre

31. The figure shows some of the electric field lines corresponding to an electric field. The figure suggests



(a) $E_A > E_B > E_C$ (b) $E_A = E_B = E_C$
(c) $E_A = E_C > E_B$ (d) $E_A = E_C < E_B$

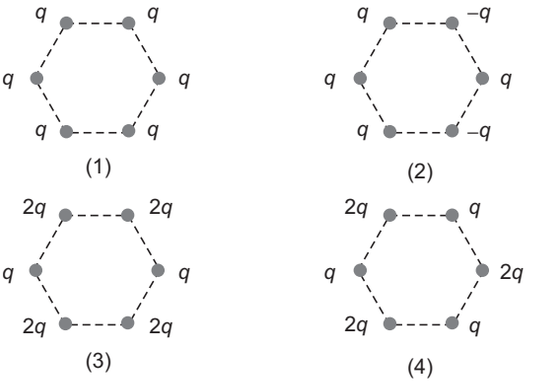
32. Two spheres of radius a and b respectively are charged and joined by a wire. The ratio of electric field of the spheres is

(a) a/b (b) b/a
(c) a^2/b^2 (d) b^2/a^2

33. A cube of side b has a charge q at each of its vertices. The electric field due to this charge distribution at the centre of this cube will be $\frac{1}{4\pi\varepsilon_0}$ times

(a) q/b^2 (b) $q/2b^2$
(c) $32q/b^2$ (d) zero

34. Figures below show regular hexagons, with charges at the vertices. In which of the following cases the electric field at the centre is not zero?

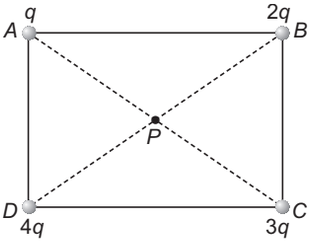


(a) 1 (b) 2 (c) 3 (d) 4

35. Electric field intensity at a point in between two parallel sheets with like charges of same surface charge densities (σ) is

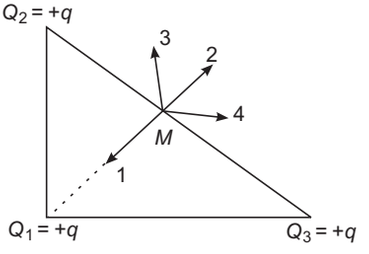
(a) $\frac{\sigma}{2\varepsilon_0}$ (b) $\frac{\sigma}{\varepsilon_0}$
(c) zero (d) $\frac{2\sigma}{\varepsilon_0}$

36. q, $2q$, $3q$ and $4q$ charges are placed at the four corners A, B, C and D of a square. The field at the centre P of the square has the direction along



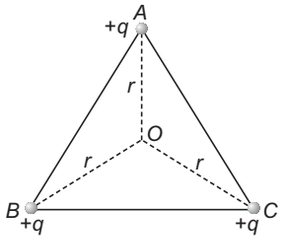
(a) AB (b) CB
(c) AC (d) BD

37. Three point charges as shown are placed at the vertices of an isosceles right angled triangle. Which of the numbered vectors coincides in direction with the electric field at the mid-point M of the hypotenuse ?



(a) 1 (b) 2
(c) 3 (d) 4

38. ABC is an equilateral triangle. Charges $+q$ are placed at each corner. The electric intensity at O, the centroid of the triangle will be



(a) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$ (b) $\frac{3}{4\pi\varepsilon_0}\cdot\frac{q}{r}$

(c) zero (d) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{3q}{r^2}$

39. Two point charges $(+Q)$ and $(-2Q)$ are fixed on the X-axis at positions a and $2a$ from origin, respectively. At what positions on the axis, the resultant electric field is zero?

(a) Only $x=\sqrt{2}a$ (b) Only $x=-\sqrt{2}a$

(c) Both $x=\pm\sqrt{2}a$ (d) Only $x=3a/2$

40. Two point charges q and $2q$ are placed some distance apart. If the electric field at the location of q be **E**, then that at the location of $2q$ will be

(a) 3**E** (b) **E**/2

(c) **E** (d) None of these

41. Infinite charges of magnitude q each are lying at $x=1, 2, 4, 8,\ldots$ metre on X-axis. The value of intensity of electric field at point $x=0$ due to these charges will be

(a) $12\times10^9\,q$N/C

(b) zero

(c) $6\times10^9\,q$N/C

(d) $4\times10^9\,q$N/C

42. Angle between equipotential surface and lines of force is

(a) zero (b) 180°

(c) 90° (d) 45°

43. If a charged spherical conductor of radius 10 cm has potential V at a point distant 5 cm from its centre, then the potential at a point distant 15 cm from the centre will be

(a) $\frac{1}{3}$ V (b) $\frac{2}{3}$ V

(c) $\frac{3}{2}$ V (d) 3 V

44. Two unlike charges of magnitude q are separated by a distance $2d$. The potential at a point mid-way between them is

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}$

(c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{d}$ (d) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{2q}{d}$

45. Two spheres A and B of radius a and b respectively, are at same electric potential. The ratio of the surface charge densities of A and B is

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$

(c) $\frac{a^2}{b^2}$ (d) $\frac{b^2}{a^2}$

46. Point charge $q_1=2\,\mu$C and $q_2=-1\,\mu$C are kept at points $x=0$ and $x=6$, respectively. Electrical potential will be zero at points

(a) $x=2$ and $x=9$ (b) $x=1$ and $x=5$

(c) $x=4$ and $x=12$ (d) $x=-2$ and $x=2$

47. Eight small drops, each of radius r and having same charge q are combined to form a big drop. The ratio between the potentials of the bigger drop and the smaller drop is

(a) 8 : 1 (b) 4 : 1 (c) 2 : 1 (d) 1 : 8

48. Eight oil drops of same size are charged to a potential of 50 V each. These oil drops are merged into one single large drop. What will be the potential of the large drop?

(a) 50 V (b) 100 V

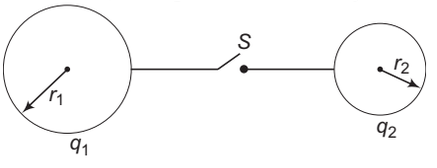
(c) 200 V (d) 400 V

49. Electric potential at a point x from the centre inside a conducting sphere of radius R and carrying charge Q is

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{x}$

(c) $\frac{1}{4\pi\varepsilon_0}xQ$ (d) zero

50. Consider a system composed of two metallic spheres of radii r_1 and r_2 connected by a thin wire and switch S as shown in the figure. Initially, S is in open position and the spheres carry charges q_1 and q_2, respectively. If the switch is closed, then potential of the system is



(a) $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_1r_2}$ (b) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2}{r_1+r_2}\right)$

(c) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1+q_2}{\sqrt{r_1r_2}}\right)$

51. Two equal positive charges are kept at points A and B. The electric potential at the points between A and B (excluding these points) is studied while moving from A to B. The potential

(a) continuously increases

(b) continuously decreases

(c) increases then decreases

(d) decreases then increases

52. A uniform electric field having a magnitude E_0 and direction along the positive X-axis exists. If the potential V is zero at $x = 0$, then its value at $X = +x$ will be
(a) $+xE_0$ (b) $-xE_0$
(c) $+x^2E_0$ (d) $-x^2E_0$

53. A charge of 5 C experiences a force of 5000 N when it is kept in a uniform electric field. What is the potential difference between two points separated by a distance of 1 cm?
(a) 10 V (b) 250 V
(c) 1000 V (d) 2500 V

54. The electric potential V is given as a function of distance x (metre) by $V = (5x^2 + 10x - 9)$ volt. The value of electric field at $x = 1$ is
(a) – 20 V/m (b) 6 V/m
(c) 11 V/m (d) –23 V/m

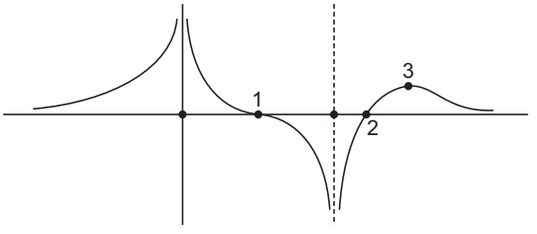
55. Two plates are at potentials –10 V and +30 V. If the separation between the plates be 2 cm. The electric field between them is
(a) 2000 V/m (b) 1000 V/m
(c) 500 V/m (d) 3000 V/m

56. Charges $2q, -q$ and $-q$ lie at the vertices of an equilateral triangle. The value of E and V at the centroid of the triangle will be
(a) $E \neq 0$ and $V \neq 0$ (b) $E = 0$ and $V = 0$
(c) $E \neq 0$ and $V = 0$ (d) $E = 0$ and $V \neq 0$

57. In a uniform electric field a charge of 3 C experiences a force of 3000 N. The potential difference between two points 1 cm apart along the electric lines of force will be
(a) 10 V (b) 100 V
(c) 30 V (d) 300 V

58. From a point charge there is a fixed point A. At that point there is an electric field of 500 V/m and potential of 3000 V. Then, the distance of A from the point charge is
(a) 6 m (b) 12 m
(c) 36 m (d) 144 m

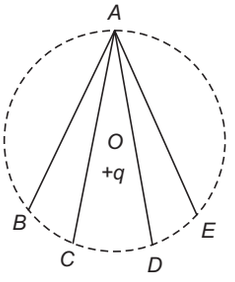
59. Two point charges are kept at a certain distance from one another. The graph represents the variation of the potential along the straight line connecting the two charges. At what point is the electric field zero?



(a) 1 (b) 2
(c) 3 (d) None of these

Potential Energy, Work done, Kinetic Energy and Speed of Charged Particle in Electric Field

60. In the electric field of a point charge q, a certain charge is carried from point A to B, C, D and E. Then, the work done



(a) is least along the path AB
(b) is least along the path AD
(c) is zero along all the paths AB, AC, AD and AE
(d) is least along AE

61. If E is the electric field intensity of an electrostatic field, then the electrostatic energy density is proportional to
(a) E (b) E^2 (c) $1/E^2$ (d) E^3

62. A particle A has charge $+q$ and a particle B has charge $+4q$ with each of them having the same mass m. When allowed to fall from rest through the same electric potential difference, the ratio of their speed $\frac{v_A}{v_B}$ will become
(a) 2 : 1 (b) 1 : 2
(c) 1 : 4 (d) 4 : 1

63. Three particles, each having a charge of 10 μC are placed at the corners of an equilateral triangle of side 10 cm. The electrostatic potential energy of the system is (given $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N-m^2/C^2)
(a) zero (b) infinite
(c) 27 J (d) 100 J

64. A mass $m = 20$ g has a charge $q = 3.0$ mC. It moves with a velocity of 20 m/s and enters a region of electric field of 80 N/C in the same direction as the velocity of the mass. The velocity of the mass after 3 s in this region is
(a) 80 m/s (b) 56 m/s
(c) 44 m/s (d) 40 m/s

65. Four identical charges +50 μC each are placed, one at each corner of a square of side 2 m. How much external energy is required to bring another charge of + 50 μC from infinity to the centre of the square?
(a) 64 J (b) 41 J
(c) 16 J (d) 10 J

66. Two equal charges q are placed at a distance of $2a$ and a third charge $-2q$ is placed at the mid-point. The potential energy of the system is

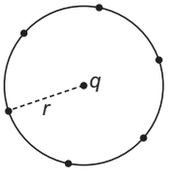
(a) $\frac{q^2}{8\pi\varepsilon_0 a}$ (b) $\frac{6q^2}{8\pi\varepsilon_0 a}$

(c) $-\frac{7q^2}{8\pi\varepsilon_0 a}$ (d) $-\frac{9q^2}{8\pi\varepsilon_0 a}$

67. An α-particle is accelerated through a potential difference of 10^6 V. Its kinetic energy will be

(a) 1 MeV (b) 2 MeV
(c) 4 MeV (d) 8 MeV

68. A charge of 5 C is given a displacement of 0.5 m. The work done in the process is 10 J. The potential difference between the two points will be



(a) 2 V (b) 0.25 V
(c) 1 V (d) 25 V

69. How much kinetic energy will be gained by an α-particle in going from a point at 70 V to another point at 50 V

(a) 40 eV (b) 40 keV
(c) 40 MeV (d) 0 eV

70. The ratio of momenta of an electron and an α-particle which are accelerated from rest by a potential difference of 100 V is

(a) 1 (b) $\sqrt{\frac{2m_e}{m_\alpha}}$

(c) $\sqrt{\frac{m_e}{m_\alpha}}$ (d) $\sqrt{\frac{m_e}{2m_\alpha}}$

71. An electron enters in higher potential region V_2 from lower potential region V_1, then its velocity

(a) will increase
(b) will change in direction but not in magnitude
(c) will not change in direction of field
(d) will not change in direction perpendicular to field

72. At a distance of 1 m from a fixed charge of 1 mC, a particle of mass 2 g and charge 1μC is held stationary. Both the charges are placed on a smooth horizontal surface. If the particle is made free to move, then its speed at a distance of 10 m from the fixed charge will be

(a) 10 ms^{-1} (b) 20 ms^{-1}
(c) 60 ms^{-1} (d) 90 ms^{-1}

73. An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_1/t_2 is nearly equal to

(a) m_e/m_p (b) $(m_p/m_e)^{1/2}$
(c) $(m_e/m_p)^{1/2}$ (d) m_p/m_e

74. A charged particle of mass m and charge q is released from rest in an electric field of constant magnitude E. The KE of the particle after time t is

(a) $\frac{Eq^2m}{2t^2}$ (b) $\frac{2E^2t^2}{mq}$ (c) $\frac{E^2q^2t^2}{2m}$ (d) $\frac{Eqm}{2t}$

75. A particle A has a charge $+q$ and particle B has a charge $+4q$ each having the same mass m. When released from rest through same potential difference the ratio of their speeds v_A/v_B is

(a) 2 : 1 (b) 1 : 2
(c) 1 : 4 (d) 4 : 1

76. A point charge q is surrounded by six identical charges at distance r shown in the figure. How much work is done by the force of electrostatic repulsion, when the point charge at the centre is removed to infinity?

(a) $6q/4\pi\varepsilon_0 r$ (b) $6q^2/4\pi\varepsilon_0 r$
(c) $36q^2/4\pi\varepsilon_0 r$ (d) Zero

77. Three charges are placed at the vertices of an equilateral triangle of side 10 cm. Assume $q_1 = 1\mu C$, $q_2 = -2\mu C$ and $q_3 = 4\mu C$. Work done in separating the charges to infinity is

(a) − 4.5 J (b) 0.54 J
(c) 45 J (d) None of these

78. Two positive charges 12μC and 8μC are 10 cm apart. The work done in bringing them 4 cm closer is

(a) 5.8 J (b) 5.8 eV
(c) 13 J (d) 13 eV

79. Three charges $Q, (+q)$ and $(+q)$ are placed at the vertices of an equilateral triangle. If the net electrostatic energy of the system is zero, then Q is equal to

(a) $(-q/2)$ (b) $-q$
(c) $+q$ (d) zero

80. Two particles of masses m and $2m$ with charges $2q$ and q are placed in a uniform electric field E and allowed to move for same time. Find the ratio of their kinetic energies.

(a) 8 : 1 (b) 4 : 1
(c) 2 : 1 (d) 16 : 1

81. When the separation between two charges is increased, the electric potential energy of the charges

(a) increases
(b) decreases
(c) remains the same
(d) may increase or decrease

82. If a positive charge is shifted from a low potential region to a high potential region, the electric potential energy

(a) increases
(b) decreases
(c) remains the same
(d) may increase or decrease

83. The work done in carrying a charge of $5\,\mu C$ from a point A to a point B in an electric field is 10 mJ. Then, potential difference $(V_B - V_A)$ is
(a) + 2 kV (b) − 2 kV
(c) + 200 kV (d) − 200 V

84. If $E = 0$, at all points of a closed surface
(a) the electric flux through the surface is zero
(b) the total charge enclosed by the surface is zero
(c) no charge resides on the surface
(d) All of the above

Electric Dipole, Electric Flux and Gauss Theorem

85. Flux coming out from a positive unit charge placed in air, is
(a) ε_0 (b) ε_0^{-1} (c) $(4\pi\varepsilon_0)^{-1}$ (d) $4\pi\varepsilon_0$

86. An electric dipole is kept in non-uniform electric field. It experiences
(a) a force and a torque
(b) a force but not a torque
(c) a torque but not a force
(d) neither a force nor a torque

87. An electric dipole of moment p is placed normal to the lines of force of electric intensity E, then the work done in deflecting it through an angle of 180° is
(a) pE (b) $+2pE$ (c) $-2pE$ (d) zero

88. Electric charges $q, q, -2q$ are placed at the corners of an equilateral triangle ABC of side l. The magnitude of electric dipole moment of the system is
(a) ql (b) $2ql$
(c) $\sqrt{3}ql$ (d) $4ql$

89. The torque acting on a dipole of moment $\mathbf{p}$ in an electric field $\mathbf{E}$ is
(a) $\mathbf{p}\cdot\mathbf{E}$ (b) $\mathbf{p}\times\mathbf{E}$
(c) zero (d) $\mathbf{E}\times\mathbf{p}$

90. Two opposite and equal charges 4×10^{-8} C when placed 2×10^{-2} cm away form a dipole. If this dipole is placed in an external electric field 4×10^{8} N/C, then value of maximum torque and the work done in rotating it through 180° will be
(a) 64×10^{-4} N-m and 64×10^{-4} J
(b) 32×10^{-4} N-m and 32×10^{-4} J
(c) 64×10^{-4} N-m and 32×10^{-4} J
(d) 32×10^{-4} N-m and 64×10^{-4} J

91. If E_a be the electric field strength of a short dipole at a point on its axial line and E_e that on the equatorial line at the same distance, then
(a) $E_e = 2E_a$ (b) $E_a = 2E_e$
(c) $E_a = E_e$ (d) None of these

92. The potential at a point due to an electric dipole will be maximum and minimum when the angles between the axis of the dipole and the line joining the point to the dipole are respectively
(a) 90° and 180° (b) 0° and 90°
(c) 90° and 0° (d) 0° and 180°

93. A molecule with a dipole moment p is placed in an electric field of strength E. Initially, the dipole is aligned parallel to the field. If the dipole is to be rotated to be anti-parallel to the field, the work required to be done by an external agency is
(a) $-2pE$ (b) $-pE$
(c) pE (d) $2pE$

94. The electric field due to an electric dipole at a distance r from its centre in axial position is E. If the dipole is rotated through an angle of 90° about its perpendicular axis, the magnitude of electric field at the same point will be
(a) E (b) $E/4$ (c) $E/2$ (d) $2E$

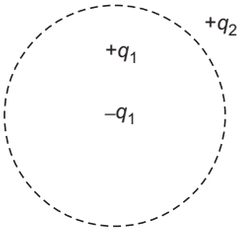
95. An electric dipole when placed in a uniform electric field E will have minimum potential energy if the dipole moment makes the following angle with E
(a) π (b) $\pi/2$
(c) zero (d) $3\pi/2$

96. Electric field at a far away distance r on the axis of a dipole is E_0. What is the electric field at a distance $2r$ on perpendicular bisector?
(a) $\frac{E_0}{16}$ (b) $-\frac{E_0}{16}$ (c) $\frac{E_0}{8}$ (d) $-\frac{E_0}{8}$

97. The SI unit of electric flux is
(a) Weber (b) Newton/coulomb
(c) Volt × metre (d) Joule/coulomb

98. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2 the electric charge inside the surface will be
(a) $(\phi_1 + \phi_2)\varepsilon_0$ (b) $(\phi_2 - \phi_1)\varepsilon_0$
(c) $(\phi_1 + \phi_2)/\varepsilon_0$ (d) $(\phi_2 - \phi_1)/\varepsilon_0$

99. Consider the charge configuration and spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface, the electric field will be due to



(a) q_2 (b) Only the positive charges
(c) all the charges (d) $+q_1$ and $-q_1$

100. A wire of linear charge density λ passes through a cuboid of length l, breadth b and height h in such a manner that flux through the cuboid is maximum. The position of wire is now changed, so that the flux through the cuboid is minimum. $l > b > h$, then the ratio of maximum flux to minimum flux will be

(a) $\frac{\sqrt{l^2 + b^2 + h^2}}{h}$ (b) $\frac{\sqrt{l^2 + b^2}}{h}$

(c) $\frac{h}{\sqrt{l^2 + b^2}}$ (d) $\frac{l}{\sqrt{l^2 + b^2 + h^2}}$

101. If the flux of the electric field through a closed surface is zero,

(a) the electric field must be zero everywhere on the surface
(b) the electric field may be zero everywhere on the surface
(c) the charge inside the surface must be zero
(d) the charge in the vicinity of the surface must be zero

102. Charge of 2 C is placed at the centre of a cube. What is the electric flux passing through one face?

(a) $\frac{1}{3\varepsilon_0}$ (b) $\left(\frac{1}{4}\right)\varepsilon_0$

(c) $\frac{2}{\varepsilon_0}$ (d) $\frac{3}{\varepsilon_0}$

103. The electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 20 V-m. The flux over a concentric sphere of radius 20 cm will be

(a) 20V-m (b) 10 V-m (c) 40 V-m (d) 5 V-m

104. For a given surface, the Gauss's law is stated as $\int \mathbf{E} \cdot d\mathbf{S} = 0$.

From this, we can conclude that

(a) E is necessarily zero on the surface
(b) E is perpendicular to the surface at every point
(c) The total flux through the surface is zero
(d) The flux is only going out of the surface

105. A cube of side a is placed in a uniform electric field $\mathbf{E} = E_0\hat{\mathbf{i}} + E_0\hat{\mathbf{j}} + E_0\hat{\mathbf{k}}$. Total electric flux passing through the cube would be

(a) E_0a^2 (b) $2E_0a^2$

(c) $6E_0a^2$ (d) None of these

106. A surface $\mathbf{E} = 10\hat{\mathbf{j}}$ is kept in an electric field $\mathbf{E} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$. How much electric flux will come out through this surface?

(a) 40 units (b) 50 units (c) 30 units (d) 20 units

Capacity and Equivalent Capacity

107. Separation between the plates of a parallel plate capacitor is d and the area of each plate is A. When a slab of material of dielectric constant K and thickness $t(t < d)$ is introduced between the plates, its capacitance becomes

(a) $\frac{\varepsilon_0 A}{d + t\left(1 - \frac{1}{K}\right)}$ (b) $\frac{\varepsilon_0 A}{d + t\left(1 + \frac{1}{K}\right)}$

(c) $\frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$ (d) $\frac{\varepsilon_0 A}{d - t\left(1 + \frac{1}{K}\right)}$

108. The distance between the circular plates of a parallel plate condenser 40 mm in diameter, in order to have same capacity as a sphere of radius 1 m is

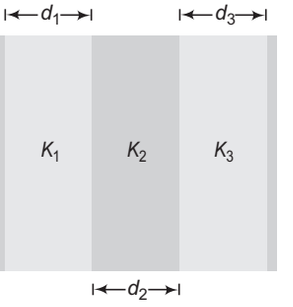
(a) 0.01 mm (b) 0.1 mm
(c) 1 mm (d) 10 mm

109. A spherical condenser has inner and outer spheres of radii a and b respectively. The space between the two is filled with air. The difference between the capacities of two condensers formed when outer sphere is earthed and when inner sphere is earthed will be

(a) zero (b) $4\pi\varepsilon_0 a$

(c) $4\pi\varepsilon_0 b$ (d) $4\pi\varepsilon_0 a\left(\frac{b}{b-a}\right)$

110. The expression for the capacity of the capacitor formed by compound dielectric placed between the plates of a parallel plate capacitor as shown in figure, will be (area of plate = A)



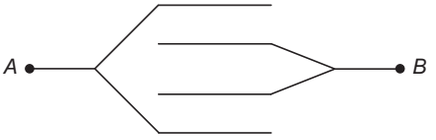
(a) $\frac{\varepsilon_0 A}{\left(\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}\right)}$ (b) $\frac{\varepsilon_0 A}{\left(\frac{d_1 + d_2 + d_3}{K_1 + K_2 + K_3}\right)}$

(c) $\frac{\varepsilon_0 A(K_1K_2K_3)}{d_1d_2d_3}$ (d) $\varepsilon_0\left(\frac{AK_1}{d_1} + \frac{AK_2}{d_2} + \frac{AK_3}{d_3}\right)$

111. If a slab of insulating material 4×10^{-3} m thick is introduced between the plates of a parallel plate capacitor, the separation between plates has to be increased by 3.5×10^{-3} m to restore the capacity to original value. The dielectric constant of the material will be

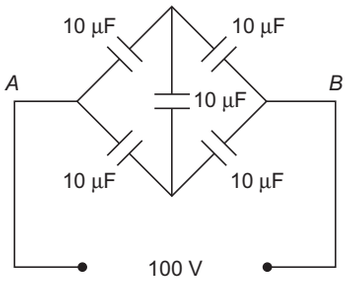
(a) 6 (b) 8 (c) 10 (d) 12

112. Four plates of equal area A are separated by equal distances d and are arranged as shown in the figure. The equivalent capacity is



(a) $\frac{2\varepsilon_0 A}{d}$ (b) $\frac{3\varepsilon_0 A}{d}$ (c) $\frac{4\varepsilon_0 A}{d}$ (d) $\frac{\varepsilon_0 A}{d}$

113. Five capacitors of 10 μF capacity each are connected to a DC potential of 100 V as shown in the below figure. The equivalent capacitance between the points A and B will be equal to

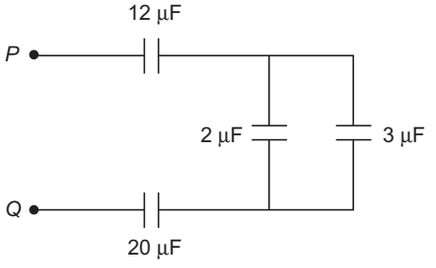


(a) 40 μF (b) 20 μF
(c) 30 μF (d) 10 μF

114. Three capacitors of capacitances 3 μF, 9 μF and 18 μF are connected once in series and another time in parallel. The ratio of equivalent capacitance in the two cases $\left(\frac{C_s}{C_p}\right)$ will be

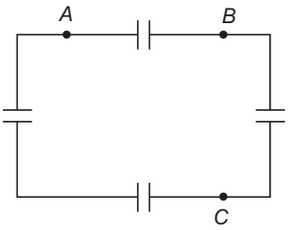
(a) 1 : 15 (b) 15 : 1 (c) 1 : 1 (d) 1 : 3

115. In the circuit diagram shown in the below figure, the resultant capacitance between points P and Q is



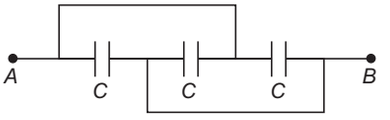
(a) 47 μF (b) 3 μF (c) 60 μF (d) 10 μF

116. Four capacitors each of capacity 3 μF are connected as shown in the below figure. The ratio of equivalent capacitance between A and B and between A and C will be



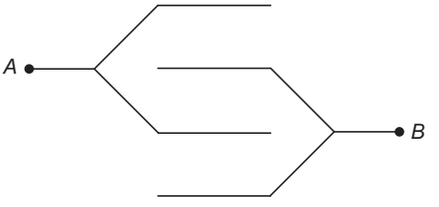
(a) 4 : 3 (b) 3 : 4
(c) 2 : 3 (d) 3 : 2

117. Three equal capacitors, each with capacitance C are connected as shown in figure. Then the equivalent capacitance between point A and B is



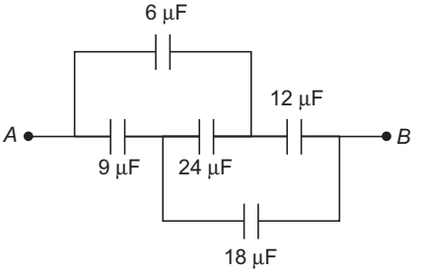
(a) C (b) $3C$
(c) $\frac{C}{3}$ (d) $\frac{3C}{2}$

118. Four plates of the same area of cross-section are joined as shown in the figure. The distance between each plate is d. The equivalent capacity across A and B will be



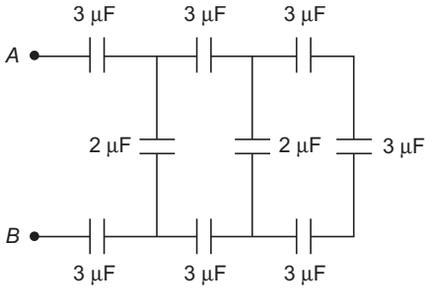
(a) $\frac{2\varepsilon_0 A}{d}$ (b) $\frac{3\varepsilon_0 A}{d}$
(c) $\frac{3\varepsilon_0 A}{2d}$ (d) $\frac{\varepsilon_0 A}{d}$

119. In the connections shown in the below figure, the equivalent capacity between points A and B will be



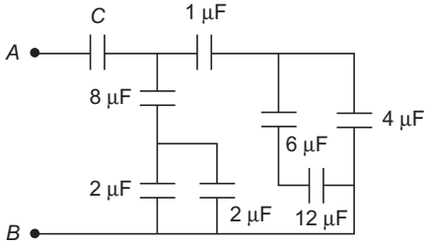
(a) 10.8 μF (b) 69 μF
(c) 15 μF (d) 10 μF

120. The resultant capacitance between A and B in the following figure is equal to



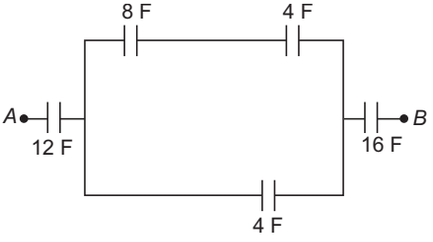
(a) 1 μF (b) 3 μF (c) 2 μF (d) 1.5 μF

121. In the following circuit, the resultant capacitance between A and B is 1 μF. The value of C is



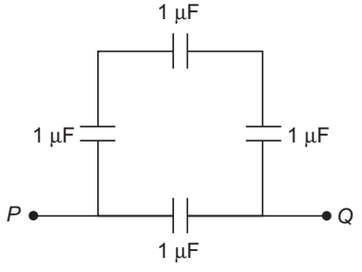
(a) $\frac{32}{11}$ μF (b) $\frac{11}{32}$ μF
(c) $\frac{23}{32}$ μF (d) $\frac{32}{23}$ μF

122. What is the equivalent capacitance between A and B in the given figure (all are in farad)?



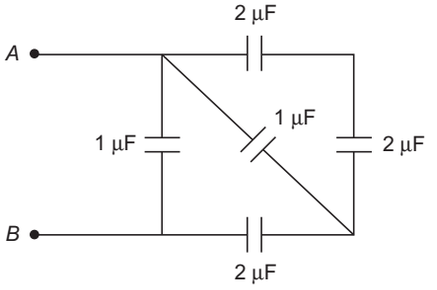
(a) $\frac{13}{18}$ F (b) $\frac{48}{13}$ F (c) $\frac{1}{31}$ F (d) $\frac{240}{71}$ F

123. Four capacitors are connected as shown. The equivalent capacitance between the points P and Q is



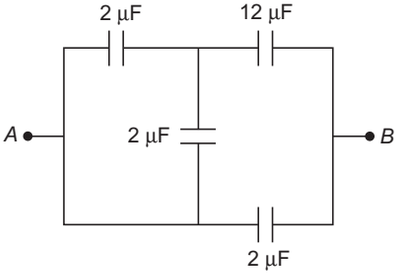
(a) 4 μF (b) $\frac{1}{4}$ μF (c) $\frac{3}{4}$ μF (d) $\frac{4}{3}$ μF

124. The total capacity of the system of capacitors shown in the below figure between the points A and B is



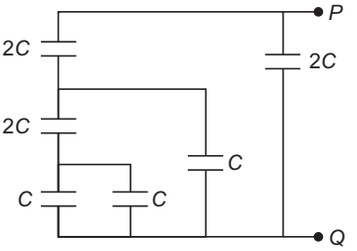
(a) 1 μF (b) 2 μF
(c) 3 μF (d) 4 μF

125. Four capacitors are connected in a circuit as shown in the figure. The effective capacitance in μF between points A and B will be



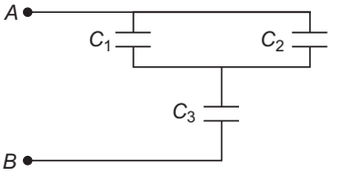
(a) $\frac{28}{9}$ (b) 4 (c) 5 (d) 18

126. The resultant capacitance of given circuit between points P and Q is



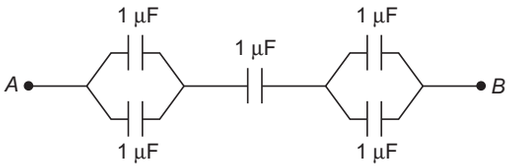
(a) $3C$ (b) $2C$ (c) C (d) $\frac{C}{3}$

127. In the given network capacitance, $C_1 = 10\,\mu F, C_2 = 5\,\mu F$ and $C_3 = 4\,\mu F$. What is the resultant capacitance between A and B (approximately)?



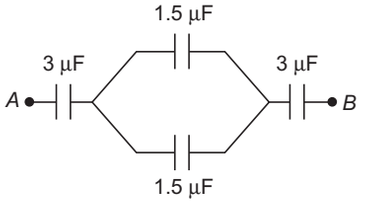
(a) 2.2 μF (b) 3.2 μF
(c) 1.2 μF (d) 4.7 μF

128. The equivalent capacitance between points A and B is



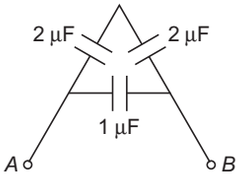
(a) 2 μF (b) 3 μF (c) 5 μF (d) 0.5 μF

129. The capacitance between the points A and B in the given circuit will be



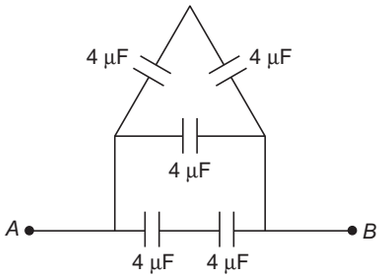
(a) 1 μF (b) 2 μF (c) 3 μF (d) 4 μF

130. What is the effective capacitance between points A and B in the given figure?



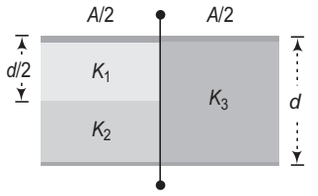
(a) 1 μF (b) 2 μF (c) 1.5 μF (d) 2.5 μF

131. Equivalent capacitance between points A and B is



(a) 8 μF (b) 6 μF (c) 26 μF (d) 10/3 μF

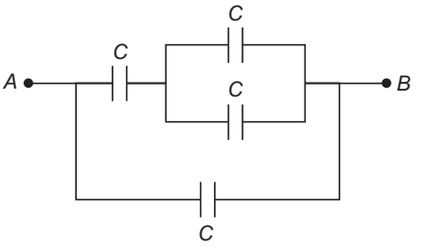
132. In the figure, a capacitor is filled with dielectrics. The resultant capacitance is



(a) $\frac{2\varepsilon_0 A}{d}\left[\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}\right]$ (b) $\frac{\varepsilon_0 A}{d}\left[\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}\right]$

(c) $\frac{2\varepsilon_0 A}{d}[K_1+K_2+K_3]$ (d) None of these

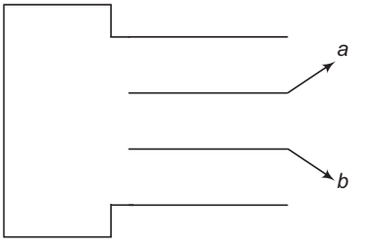
133. Four equal capacitors, each of capacity C, are arranged as shown. The effective capacitance between points A and B is



(a) $\frac{5}{8}C$ (b) $\frac{3}{5}C$

(c) $\frac{5}{3}C$ (d) C

134. Four metallic plates each with a surface area of one side A are placed at a distance d from each other. The plates are connected as shown in the circuit diagram. Then, the capacitance of the system between points a and b is



(a) $\frac{3\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$

(c) $\frac{2\varepsilon_0 A}{3d}$ (d) $\frac{3\varepsilon_0 A}{2d}$

135. There are 7 identical capacitors. The equivalent capacitance when they are connected in series is C. The equivalent capacitance when they are connected in parallel is

(a) $C/49$ (b) $C/7$

(c) $7C$ (d) $49C$

136. The capacitance of a parallel plate capacitor is 16 μF. When a glass slab is placed between the plates, the potential difference reduces to 1/8th of the original value. What is dielectric constant of glass?

(a) 4 (b) 8

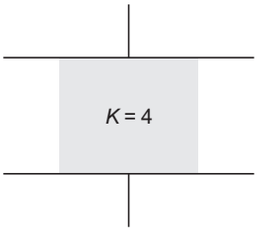
(c) 16 (d) 32

137. A parallel plate condenser with air between the plates possesses the capacity of 10^{-12} F. Now, the plates are removed apart so that the separation is twice the original value. The space between the plates is filled with a material of dielectric constant 4.0. Then, new value of the capacity is (in farad)

(a) 4×10^{-12} (b) 3×10^{-12}

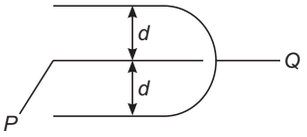
(c) 2×10^{-12} (d) 0.5×10^{-12}

138. Three condensers each of capacity C microfarad are connected in series. An exactly similar set is connected in parallel to the first one. The effective capacity of the combination is 4 μF. Then, the value of C in microfarad is



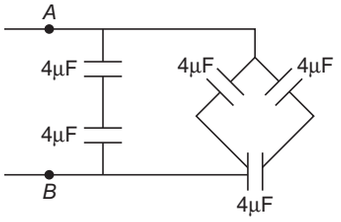
(a) 8 (b) 6 (c) 4 (d) 2

139. Three plates of common surface area A are connected as shown. The effective capacitance between points P and Q will be



(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{3\varepsilon_0 A}{d}$ (c) $\frac{3}{2}\frac{\varepsilon_0 A}{d}$ (d) $\frac{2\varepsilon_0 A}{d}$

140. The equivalent capacitance between points A and B in the circuit will be



(a) 10/3 μF (b) 4 μF
(c) 6 μF (d) 8 μF

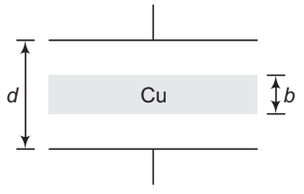
141. The capacitance of the earth, viewed as a spherical conductor of radius 6408 km is

(a) 980 μF (b) 1424 μF
(c) 712 μF (d) 356 μF

142. Eight drops of mercury of equal radii combine to form a big drop. The capacitance of the bigger drop as compared to each smaller drop is

(a) 2 times (b) 8 times (c) 4 times (d) 16 times

143. A slab of copper of thickness b is inserted in between the plates of parallel plate capacitor as shown in figure. The separation of the plate is d. If $b = d/2$, then the ratio of capacities of the capacitor after and before inserting the slab will be



(a) $\sqrt{2}:1$ (b) 2 : 1 (c) 1 : 1 (d) $1:\sqrt{2}$

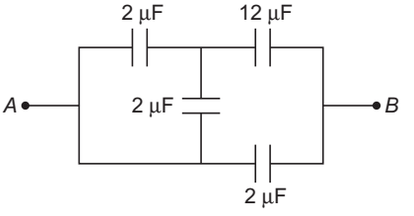
144. Consider a parallel plate capacitor of capacity 10 μF with air filled in the gap between the plates. Now, one-half of the space between the plates is filled with a dielectric of dielectric constant 4, as shown in the figure. The capacity of the capacitor changes to

(a) 25 μF (b) 20 μF (c) 40 μF (d) 5 μF

145. The capacity of a spherical condenser is 1μF. If the spacing between the two spheres is 1 mm, the radius of the outer sphere is

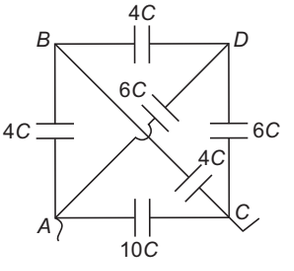
(a) 30 cm (b) 3 cm (c) 6 m (d) 3 m

146. Four capacitors are connected in a circuit as shown in the figure. The effective capacitance between points A and B will be



(a) 28/9 μF (b) 4 μF (c) 5 μF (d) 18 μF

147. The equivalent capacitance between the points A and C is given by



(a) $\frac{10}{3}C$ (b) $15C$ (c) $\frac{3}{10}C$ (d) $20C$

148. A thin metal plate P is inserted between the plates of a parallel-plate capacitor of capacitance C in such a way that its edges touch the two plates. The capacitance now becomes

(a) $C/2$ (b) $2C$ (c) zero (d) ∞

149. The capacitance of a capacitor does not depend on

(a) the medium between the plates
(b) the size of the plates
(c) the charges on the plates
(d) the separation between the plates

150. A parallel plate capacitor has plate separation d and capacitance 25 μF. If a metallic foil of thickness $\frac{2}{7}d$ is introduced between the plates, the capacitance would become

(a) 25 μF (b) 35 μF (c) $\frac{125}{7}$μF (d) $\frac{175}{2}$ μF

General Problems of Capacitors

151. In a charged capacitor, the energy resides in
(a) the positive charges
(b) both the positive and negative charges
(c) the field between the plates
(d) around the edge of the capacitor plates

152. A capacitor of capacity C has charge Q and stored energy is W. If the charge is increased to $2Q$, the stored energy will be
(a) $2W$ (b) $W/2$
(c) $4W$ (d) $W/4$

153. The capacity and the energy stored in a charged parallel plate condenser with air between its plates are respectively C_0 and W_0. If the air is replaced by glass (dielectric constant = 5) between the plates, the capacity of the plates and the energy stored in it will respectively be
(a) $5C_0, 5W_0$ (b) $5C_0, \frac{W_0}{5}$
(c) $\frac{C_0}{5}, 5W_0$ (d) $\frac{C_0}{5}, \frac{W_0}{5}$

154. A capacitor of capacity C is connected with a battery of potential V. The distance between its plates is reduced to half, assuming that the battery remains the same. Then, the new energy given by the battery will be
(a) $CV^2/4$ (b) $CV^2/2$ (c) $3CV^2/4$ (d) CV^2

155. A charged capacitor when filled with a dielectric $K = 3$ has charge Q_0, voltage V_0 and field E_0. If the dielectric is replaced with another one having $K = 9$. The new values of charge voltage and field will be respectively
(a) $3Q_0, 3V_0, 3E_0$ (b) $Q_0, 3V_0, 3E_0$
(c) $Q_0, \frac{V_0}{3}, 3E_0$ (d) $Q_0, \frac{V_0}{3}, \frac{E_0}{3}$

156. A 2 μF capacitor is charged to 100 V and then its plates are connected by a conducting wire. The heat produced is
(a) 1 J (b) 0.1 J (c) 0.01 J (d) 0.001 J

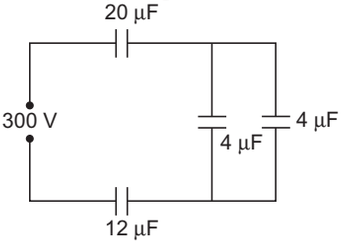
157. The force between the plates of a parallel plate capacitor of capacitance C and distance of separation of the plates d with a potential difference V between the plates, is
(a) $\frac{CV^2}{2d}$ (b) $\frac{C^2V^2}{2d^2}$
(c) $\frac{C^2V^2}{d^2}$ (d) $\frac{V^2d}{C}$

158. A parallel plate air capacitor is charged to a potential difference of V. After disconnecting the battery, distance between the plates of the capacitor is increased using an insulating handle. As a result, the potential difference between the plates
(a) decreases (b) increases
(c) becomes zero (d) does not change

159. A variable condenser is permanently connected to a 100 V battery. If the capacity is changed from 2 μF to 10 μF, then change in energy is equal to
(a) 2×10^{-2} J (b) 2.5×10^{-2} J
(c) 3.5×10^{-2} J (d) 4×10^{-2} J

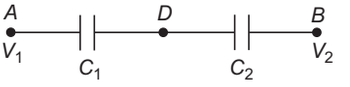
160. Two condensers of capacity 0.3 μF and 0.6 μF respectively are connected in series. The combination is connected across a potential of 6 V. The ratio of energies stored by the condensers will be
(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{4}$ (d) 4

161. In the below figure, four capacitors are shown with their respective capacities and the PD applied. The charge and the PD across the 4 μF capacitor will be



(a) 600 μC, 150 V (b) 300 μC, 75 V
(c) 800 μC, 200 V (d) 580 μC, 145 V

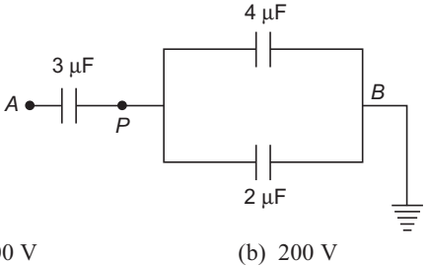
162. Two condensers C_1 and C_2 in a circuit are joined as shown in figure. The potential of point A is V_1 and that of B is V_2. The potential of point D will be



(a) $\frac{1}{2}(V_1 + V_2)$ (b) $\frac{C_2V_1 + C_1V_2}{C_1 + C_2}$
(c) $\frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ (d) $\frac{C_2V_1 - C_1V_2}{C_1 + C_2}$

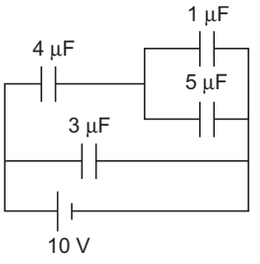
163. Three capacitors of 2 μF, 3 μF and 6 μF are joined in series and the combination is charged by means of a 24 V battery. The potential difference between the plates of the 6 μF capacitor is
(a) 4 V (b) 6 V (c) 8 V (d) 10 V

164. In the figure, a potential of + 1200 V is given to point A and point B is earthed, what is the potential at the point P?



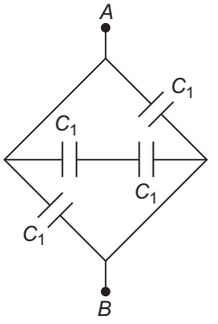
(a) 100 V (b) 200 V
(c) 400 V (d) 800 V

165. The charge on 4 μF capacitor in the given circuit is (in μC)



(a) 12 (b) 24 (c) 36 (d) 32

166. Four identical capacitors are connected as shown in diagram. When a battery of 6 V is connected between A and B, the charge stored is found to be 1.5 μC. The value of C_1 is



(a) 2.5 μF (b) 15 μF (c) 1.5 μF (d) 0.1 μF

167. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As one goes from 0 to $3d$,

(a) the magnitude of the electric field remains the same
(b) the direction of the electric field remains the same
(c) the electric potential increases continuously
(d) the electric potential increases at first, then decreases and again increases

168. A capacitor of capacity C_1, is charged by connecting it across a battery of emf V_0. The battery is then removed and the capacitor is connected in parallel with an uncharged capacitor of capacity C_2. The potential difference across this combination is

(a) $\frac{C_2}{C_1 + C_2} V_0$ (b) $\frac{C_1}{C_1 + C_2} V_0$

(c) $\frac{C_1 + C_2}{C_2} V_0$ (d) $\frac{C_1 + C_2}{C_1} V_0$

169. A 2 μF condenser is charged upto 200 V and then battery is removed. On combining this with another uncharged condenser in parallel, the potential differences between two plates are found to be 40 V. The capacity of second condenser is

(a) 2 μF (b) 4 μF
(c) 8 μF (d) 16 μF

170. A capacitor is charged by using a battery which is then disconnected. A dielectric slab is then slipped between the plates which results in

(a) reduction of charges on the plates and increase of potential difference across the plates
(b) increase in the potential difference across the plates, reduction in stored energy, but no change in the charge on the plates
(c) decrease in the potential difference across the plates, reduction in stored energy, but no change in the charge on the plates
(d) None of the above

171. Consider two conductors. One of them has a capacity of 2 units and the capacity of the other is unknown. They are charged until their potentials are 4 and 5 units, respectively. The two conductors are now connected by a wire when their common potential is found to be 4.6 units. Then, the unknown capacity has the value (in the same units as above)

(a) 6 (b) 5
(c) 4 (d) 3

172. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to

(a) nCV^2 (b) $\frac{1}{2} nCV^2$ (c) $\frac{CV^2}{n}$ (d) $\frac{1}{2n} CV^2$

173. The energy stored in a condenser is in the form of

(a) kinetic energy (b) potential energy
(c) elastic energy (d) magnetic energy

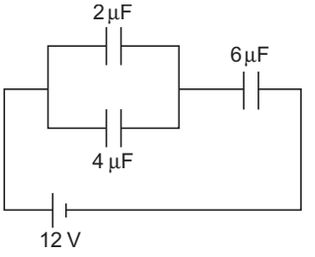
174. On increasing the plate separation of charged condenser, the energy

(a) increases (b) decreases
(c) remains unchanged (d) becomes zero

175. The electric potential difference between two parallel plates is 2000 V. If the plates are separated by 2 mm, what is the magnitude of electrostatic force on a charge of 4×10^{-6} C located mid-way between the plates?

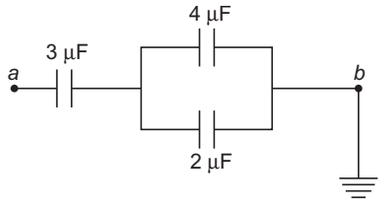
(a) 4 N (b) 6 N
(c) 8 N (d) 1.5×10^{-8} N

176. Two capacitors 2 μF and 4 μF are connected in parallel. A third capacitor of 6 μF capacity is connected in series. The combination is connected across a 12 V battery. The voltage across a 2 μF capacitor is



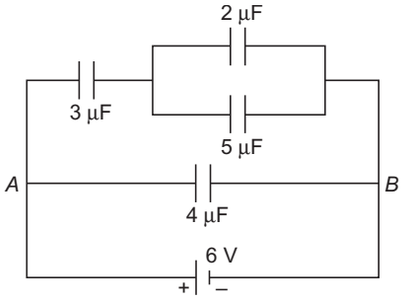
(a) 2 V (b) 6 V (c) 8 V (d) 1 V

177. In the given circuit, if point b is connected to earth and a potential of 1200 V is given to a point a, the charge on 4 μF capacitor is



(a) 800 μC
(b) 1600 μC
(c) 2400 μC
(d) 3000 μC

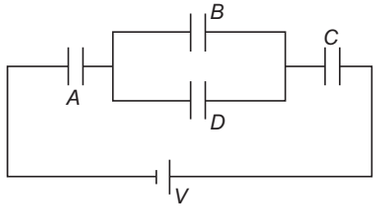
178. A circuit is shown in the given figure. Find out the charge on the condenser having capacity 5 μF.



(a) 4.5 μC
(b) 9 μC
(c) 7 μC
(d) 30 μC

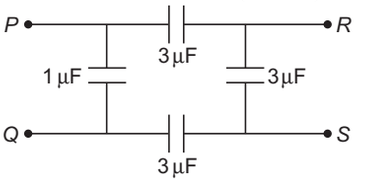
179. A sphere of capacity 2 F can be given a potential of 2 V by
(a) connecting it across a 4 V battery
(b) connecting it to the positive terminal of a 4 V battery and earthing the other terminal
(c) giving it 1 C of charge
(d) giving it 4 C of charge

180. A potential of $V = 3000$ V is applied to a combination of four initially uncharged capacitors as shown in the figure. Capacitors A, B, C and D have capacitances $C_A = 6.0\,\mu F$, $C_B = 5.2\,\mu F$, $C_C = 1.5\,\mu F$ and $C_D = 3.8\,\mu F$ respectively. If the battery is then disconnected then potential difference across capacitor B is (approximately)



(a) 3000 V
(b) zero
(c) 530 V
(d) 350 V

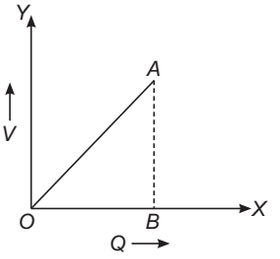
181. Four capacitors are arranged as shown. All are initially uncharged. A 30 V battery is placed across terminal PQ to charge the capacitors and is then removed. The voltage across the terminals RS is then (in volt)



(a) 10
(b) 20
(c) 30
(d) 40

182. The dimensional formula for capacitance is
(a) $[M^{-1}L^{-1}TA]$
(b) $[MLT^{-2}A^2]$
(c) $[M^{-1}L^{-2}T^4A^2]$
(d) $[M^{-1}L^{-2}T^2A^2]$

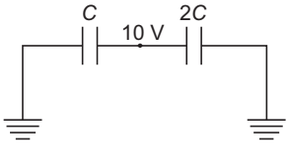
183. Charge Q on a capacitor varies with voltage V as shown in the figure, where Q is taken along the X-axis and V along the Y-axis. The area of triangle OAB represents



(a) capacitance
(b) capacitive reactance
(c) electric field between the plates
(d) energy stored in the capacitor

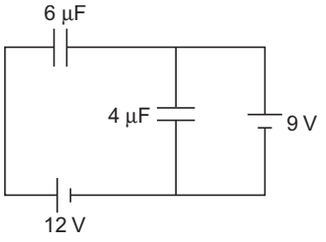
184. Two identical capacitors are joined in parallel, charged to a potential V, separated and then connected in series i.e. the positive plate of one is connected to the negative plate of the other. Then
(a) the charges on the free plates connected together are destroyed
(b) charges on the free plates are enhanced
(c) the energy stored in the system increases
(d) the potential difference between the free plates is 2 V

185. In the circuit shown in figure $C = 6\,\mu F$. The charge stored in the capacitor of capacity C is



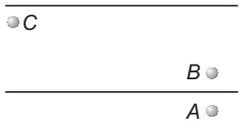
(a) zero
(b) 90 μC
(c) 40 μC
(d) 60 μC

186. In the circuit shown in figure. Charge stored in 6 μF capacitor will be



(a) 18 μC (b) 54 μC (c) 36 μC (d) 72 μC

187. For a charged parallel plate capacitor shown in the figure, the force experienced by an α - particle will be



(a) maximum at C (b) zero at A
(c) same at B and C (d) zero at C

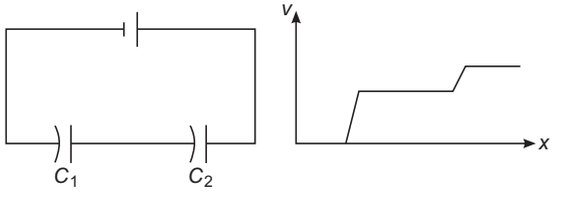
188. Two capacitors each having capacitance C and breakdown voltage V are joined in series. The capacitance and the breakdown voltage of the combination will be

(a) $2C$ and $2V$ (b) $C/2$ and $V/2$
(c) $2C$ and $V/2$ (d) $C/2$ and $2V$

189. If the capacitors in the previous question are joined in parallel, the capacitance and the breakdown voltage of the combination will be

(a) $2C$ and $2V$ (b) C and $2V$
(c) $2C$ and V (d) C and V

190. Figure shows two capacitors connected in series and joined to a battery. The graph shows the variation in potential as one moves from left to right on the branch containing the capacitors.



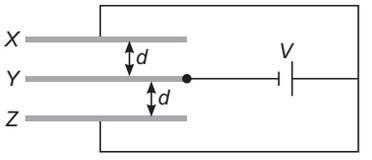
(i) (ii)

(a) $C_1 > C_2$
(b) $C_1 = C_2$
(c) $C_1 < C_2$
(d) the information is not sufficient to decide the relation between C_1 and C_2

191. Two spherical conductors each of capacity C are charged to potential V and $-V$. These are then connected by means of a fine wire. The loss of energy is

(a) zero (b) $\frac{1}{2}CV^2$ (c) CV^2 (d) $2CV^2$

192. Consider the arrangement of three plates X, Y and Z each of area A and separation d. The energy stored when the plates are fully charged is



(a) $\frac{\varepsilon_0 AV^2}{2d}$ (b) $\frac{\varepsilon_0 AV^2}{d}$
(c) $\frac{2\varepsilon_0 AV^2}{d}$ (d) $\frac{3\varepsilon_0 AV^2}{2d}$

Miscellaneous Problems

193. A soap bubble is given a negative charge, then its radius

(a) decreases
(b) increases
(c) remains unchanged
(d) Nothing can be predicted as information is insufficient

194. Dielectric constant of pure water is 81. Its permittivity will be

(a) 7.17×10^{-10} MKS units
(b) 8.86×10^{-12} MKS units
(c) 1.02×10^{13} MKS units
(d) Cannot be calculated

195. There are two metallic spheres of same radii but one is solid and the other is hollow, then

(a) solid sphere can be given more charge
(b) hollow sphere can be given more charge
(c) they can be charged equally (maximum)
(d) None of the above

196. Dielectric constant for metal is

(a) zero (b) infinite
(c) 1 (d) None of these

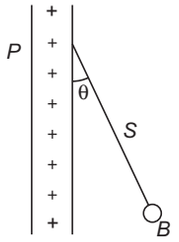
197. Two spheres A and B of radius 4 cm and 6 cm are given charges of 80 μC and 40 μC, respectively. If they are connected by a fine wire, the amount of charge flowing from one to the other is

(a) 20 μC from A to B
(b) 20 μC from B to A
(c) 32 μC from B to A
(d) 32 μC from A to B

198. In Millikan's oil drop experiment an oil drop carrying a charge Q is held stationary by a potential difference 2400 V between the plates. To keep a drop of half the radius stationary the potential difference had to be made 600 V. What is the charge on the second drop?

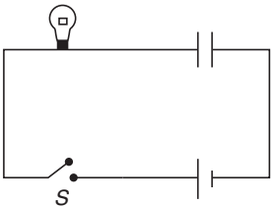
(a) $\frac{Q}{4}$ (b) $\frac{Q}{2}$
(c) Q (d) $\frac{3Q}{2}$

199. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to



(a) $\sin\theta$ (b) $\tan\theta$ (c) $\cos\theta$ (d) $\cot\theta$

200. A light bulb, a capacitor and a battery are connected together as shown here, with switch S initially open. When the switch S is closed, which one of the following is true



(a) the bulb will light up for an instant capacitor starts charging
(b) the bulb will light up when the capacitor is fully charged
(c) the bulb will not light up at all
(d) the bulb will light up and go off at regular intervals

201. The unit of electric permittivity is
(a) Volt/m^2 (b) Joule/C (c) Farad/m (d) Henry/m

202. Two concentric metallic spherical shells are given equal amount of positive charges. Then,
(a) the outer sphere is always at a higher potential
(b) the inner sphere is always at a higher potential
(c) both the spheres are at the same potential
(d) no prediction can be made about their potentials unless the actual value of charges and radii are known

203. Two conducting spheres A and B of radii 4 cm and 2 cm carry charges of 18×10^{-8} stat-coulomb and 9×10^{-8} stat-coulomb respectively of positive electricity. When they are put in electrostatic contact, the charge will
(a) not flow at all (b) flow from A to B
(c) flow from B to A (d) disappear

204. Two insulated charged spheres of radii R_1 and R_2 having charges Q_1 and Q_2 are respectively connected to each other. There is
(a) an increase in the energy of the system
(b) no change in the energy of the system
(c) always decrease in energy
(d) a decrease in energy of the system unless $Q_1R_2 = Q_2R_1$

205. At the centre of a charged ring
(a) $E = 0, V = 0$
(b) $E =$ maximum, $V = 0$
(c) $E =$ maximum, $V =$ maximum
(d) $E = 0, V =$ maximum

206. A small sphere is charged to a potential of 50 V and a big hollow sphere is charged to a potential of 100 V. Electricity will flow from the smaller sphere to the bigger one when
(a) the smaller one is placed inside the bigger one and connected by a wire
(b) bigger one placed by the side of the smaller one and connected by a wire
(c) both are correct
(d) both are wrong

207. Point charges $+4q, -q$ and $+4q$ are kept on the x-axis at points $x = 0$, $x = a$ and $x = 2a$, respectively. Then,
(a) only $-q$ is in stable equilibrium
(b) none of the charges are in equilibrium
(c) all the charges are in unstable equilibrium
(d) all the charges are in stable equilibrium

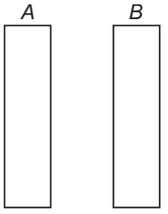
208. Two metallic spheres of radii 1 cm and 2 cm are given charges 10^{-2} C and 5×10^{-2} C, respectively. If they are connected by a connecting wire, the final charge on the smaller sphere is
(a) 1×10^{-2} C
(b) 2×10^{-2} C
(c) 3×10^{-2} C
(d) 4×10^{-2} C

209. A particle having charge q and mass m is projected with velocity $\mathbf{v} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$ in a uniform electric field $\mathbf{E} = E_0\hat{\mathbf{j}}$. Change in momentum $|\Delta\mathbf{p}|$ during any time interval t is given by
(a) $\sqrt{13}\,m$ (b) qE_0t
(c) $\frac{qE_0t}{m}$ (d) zero

210. Two positive charges are kept at $(a, 0)$ and $(-a, 0)$. A negative charge then left from a position $(0, 2a)$. The negative charge will perform
(a) oscillatory motion only
(b) simple harmonic motion
(c) will come to rest at the centre
(d) None of the above

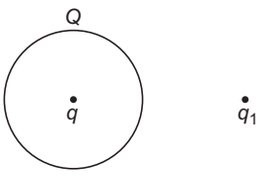
211. Three concentric conducting spherical shells carry charges as follows; $+Q$ on the inner shell, $-2Q$ on the middle shell and $-5Q$ on the outer shell. The charge on the inner surface of the outermost shell is
(a) zero (b) $+Q$
(c) $-2Q$ (d) $-3Q$

212. Two conducting plates A and B each having large surface area S (on one side) are placed parallel to each other. The plate A is given a charge q, while the plate B is neutral. Then, the electric field at a point in between the plates is



(a) $\frac{q}{2S\varepsilon_0}$ (b) $\frac{q}{S\varepsilon_0}$
(c) $\frac{2q}{S\varepsilon_0}$ (d) $\frac{3}{2}\frac{q}{S\varepsilon_0}$

213. A thin, metallic spherical shell contains a charge Q on it. A point charge q is placed at the centre of the shell and another charge q_1 is placed outside it as shown in figure. All the three charges are positive. The force on the charge at the centre is



(a) towards left (b) towards right
(c) upward (d) zero

214. Consider the situation of the previous problem. The force on the central charge due to the shell is
(a) towards left (b) towards right
(c) upward (d) zero

[Level 2]

Only One Correct Option

1. A small element l is cut from a circular ring of radius a and λ charge per unit length. The net electric field at the centre of ring is

(a) zero (b) $\frac{\lambda l}{4\pi\varepsilon_0 a^2}$
(c) infinity (d) $\frac{\lambda}{4\pi\varepsilon_0 l}$

2. A point charge is located at origin. At point (a, a), electric field is $\mathbf{E}_1$. At point $(-a, a)$, electric field is $\mathbf{E}_2$ and a point $(-a, -a)$ electric field is $\mathbf{E}_3$. Choose the correct option.
(a) $\mathbf{E}_1 \cdot \mathbf{E}_2 = 0$
(b) $|\mathbf{E}_1 \times \mathbf{E}_3| = 0$
(c) Both are correct
(d) Both are wrong

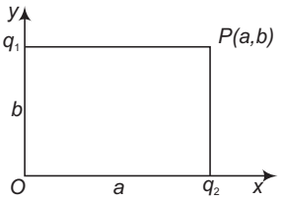
3. Two spherical conductors of radii 4 cm and 5 cm are charged to the same potential. If σ_1 and σ_2 be respective value of surface density of charge on both the conductors, then the ratio of σ_1 / σ_2 will be

(a) $\frac{16}{25}$ (b) $\frac{25}{10}$
(c) $\frac{4}{5}$ (d) $\frac{5}{4}$

4. A hollow charged metal sphere has radius r. If the potential difference between its surface and a point at a distance $3r$ from the centre is V, then electric field intensity at a distance $3r$ is

(a) $\frac{V}{2r}$ (b) $\frac{V}{3r}$
(c) $\frac{V}{6r}$ (d) $\frac{V}{4r}$

5. Two point charges $q_1 = 2\mu C$ and $q_2 = 1\mu C$ are placed at distances $b = 1$ cm and $a = 2$ cm from the origin of the y and x-axes as shown in figure. The electric field vector at point $P(a, b)$ will subtend angle θ with the x-axis given by

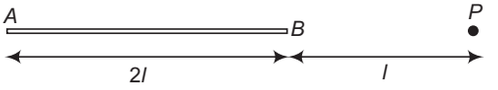


(a) $\tan\theta = 1$ (b) $\tan\theta = 2$
(c) $\tan\theta = 3$ (d) $\tan\theta = 4$

6. Two identical charges are placed at the two corners of an equilateral triangle. The potential energy of the system is U. The work done in bringing an identical charge from infinity to the third vertex is

(a) U (b) $2U$
(c) $3U$ (d) $4U$

7. Charge Q is uniformly distributed on a dielectric rod AB of length $2l$. The potential at P shown in the figure is equal to



(a) $\frac{Q}{4\pi\varepsilon_0(2l)}$
(b) $\frac{Q}{4\pi\varepsilon_0(l)} \ln 2$
(c) $\frac{Q}{4\pi\varepsilon_0(2l)} \ln 3$
(d) None of the above

8. A and B are two thin concentric hollow conductors having radii a and $2a$ and charge $2Q$ and Q respectively. Its potential of outer sphere is 5 V, then potential of inner sphere is

(a) 20 V (b) 10 V (c) $\frac{25}{3}$ V (d) $\frac{50}{3}$ V

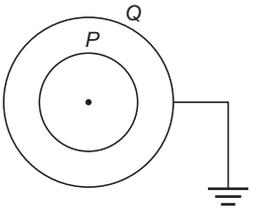
9. A spherical conductor of radius 2 m is charged to a potential of 120 V. It is now placed inside another hollow spherical conductor of radius 6 m. Calculate the potential of bigger sphere if the smaller sphere is made to touch the bigger sphere

(a) 120 V (b) 60 V
(c) 80 V (d) 40 V

10. Potential difference between two points $(V_A - V_B)$ in an electric field $\mathbf{E} = (2\hat{\mathbf{i}} - 4\hat{\mathbf{j}})\,\text{N/C}$, where $A = (0, 0)$ and $B = (3\text{ m}, 4\text{ m})$ is

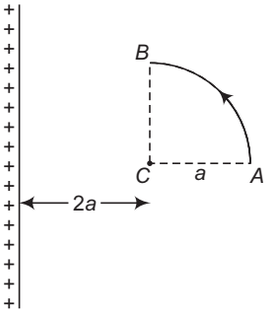
(a) 10 V (b) −10 V
(c) 16 V (d) −16 V

11. P and Q are two concentric metallic spheres. P is positively charged and Q is earthed. Then,



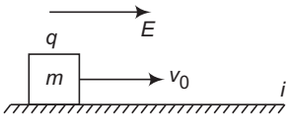
(a) charge density on Q is same as on P
(b) electric field between P and Q is uniform
(c) electric potential inside P is zero
(d) electric field outside Q is zero

12. The arc AB with the centre C and the infinitely long wire having linear charge density λ are lying in the same plane. The minimum amount of work to be done to move a point charge q_0 from point A to B through a circular path AB of radius a is equal to



(a) $\frac{q_0\lambda}{4\pi\varepsilon_0}\ln\left(\frac{2}{3}\right)$ (b) $\frac{q_0\lambda}{2\pi\varepsilon_0}\ln\left(\frac{3}{2}\right)$
(c) $\frac{q_0\lambda}{2\pi\varepsilon_0}\ln\left(\frac{2}{3}\right)$ (d) None of these

13. A charged block is projected on a rough horizontal surface with speed v_0. The value of coefficient of friction if the kinetic energy of the block remains constant is

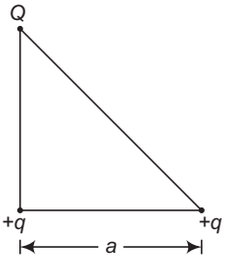


(a) $\frac{qE}{mg}$ (b) $\frac{qE}{m}$
(c) qE (d) None of these

14. There are four concentric shells A, B, C and D of radii $a, 2a, 3a$ and $4a$, respectively. Shells B and D are given charges $+q$ and $-q$, respectively. Shell C is now earthed. The potential difference $V_A - V_C$ is, where $k = \frac{1}{4\pi\varepsilon_0}$

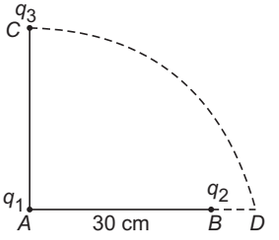
(a) $\frac{kq}{2a}$ (b) $\frac{kq}{3a}$
(c) $\frac{kq}{4a}$ (d) $\frac{kq}{6a}$

15. Three charges $Q, +q$ and $+q$ are placed at the vertices of a right angled isosceles triangle as shown in figure. The net electrostatics energy of the configuration is zero if Q is equal to



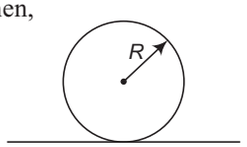
(a) $\frac{-q}{1+\sqrt{2}}$ (b) $\frac{-2q}{2+\sqrt{2}}$
(c) $-2q$ (d) $+q$

16. Two charges q_1 and q_2 are placed 30 cm apart as shown in figure. A third charge q_3 is moved along the arc of a circle of radius 40 cm from C to D. The change in the potential energy of the system is $\frac{q_3}{4\pi\varepsilon_0}k$, where k is



(a) $8q_2$ (b) $8q_1$
(c) $6q_2$ (d) $6q_1$

17. If linear charge density of a bent wire as shown in the figure is λ. Then,



(a) potential at the centre is $\frac{\lambda}{2\varepsilon_0}$

(b) electric field at the centre of the loop is $\frac{\lambda}{2\pi\varepsilon_0 R}$

(c) electric field at the centre of the loop is $\frac{\lambda}{2\pi\varepsilon_0 R} + \frac{\lambda}{2\varepsilon_0 R}$

(d) None of the above

18. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surface is

(a) V (b) $2V$
(c) $4V$ (d) $-2V$

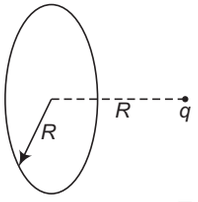
19. Three identical metallic uncharged spheres *A, B* and *C* of radius *a* are kept on the corners of an equilateral triangle of side $d(d >> a)$. A fourth sphere (radius *a*) which has charge *Q* touches *A* and is then removed to a position far away. *B* is earthed and then the earth connection is removed. *C* is then earthed. The charge on *C* is

(a) $\frac{Qa}{2d}\left(\frac{2d-a}{2d}\right)$ (b) $\frac{Qa}{2d}\left(\frac{2d-a}{d}\right)$
(c) $\frac{Qa}{2d}\left(\frac{a-d}{d}\right)$ (d) $\frac{2Qa}{d}\left(\frac{d-a}{2d}\right)$

20. A ring of radius *R* is uniformly charged. Linear charge density is λ. An imaginary sphere of radius *R* is drawn with its centre on circumference of ring. Total electric flux passing through the sphere would be

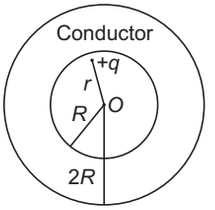
(a) $\frac{2\pi R\lambda}{\varepsilon_0}$ (b) $\frac{\pi R\lambda}{\varepsilon_0}$
(c) zero (d) None of these

21. A ring of radius *R* is placed at a distance *R* from a point charge *q* as shown in figure. Total electric flux passing through the ring would be



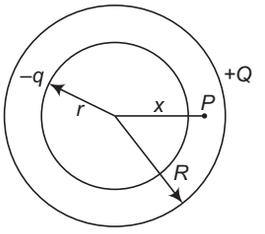
(a) $\frac{q}{4\varepsilon_0}$ (b) $\frac{\sqrt{3}q}{2\varepsilon_0}$
(c) $\frac{q}{2\varepsilon_0}$ (d) None of these

22. A point charge *q* is placed at a distance *r* from the centre *O* of an uncharged spherical shell of inner radius *R* and outer radius 2*R*. The distance is $r < R$. The electric potential at the centre of the shell will be



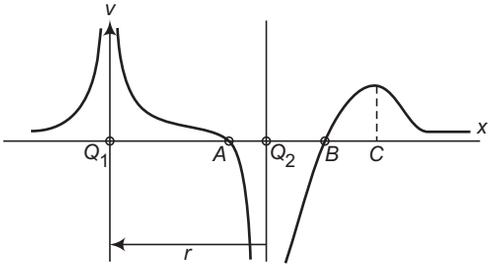
(a) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{2R}\right)$ (b) $\frac{q}{4\pi\varepsilon_0 r}$
(c) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}+\frac{1}{2R}\right)$ (d) $\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{R}\right)$

23. A hollow sphere of radius *r* is put inside another hollow sphere of radius *R*. The charges on the two are $+Q$ and $-q$ as shown in the figure. A point *P* is located at a distance *x* from the common centre such that $r < x < R$. The potential at the point *P* is



(a) $\frac{1}{4\pi\varepsilon_0}\left(\frac{Q-q}{x}\right)$ (b) $\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}-\frac{q}{r}\right)$
(c) $\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}-\frac{q}{x}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}-\frac{Q}{x}\right)$

24. The curve represents the distribution of potential along the straight line joining the two charges Q_1 and Q_2 (separated by a distance *r*), then which is of the following statements are correct?



1. $|Q_1| > |Q_2|$
2. Q_1 is positive in nature
3. *A* and *B* are equilibrium points
4. *C* is a point of unstable equilibrium

Which of the above statements are correct?

(a) 1 and 2 (b) 1, 2 and 3
(c) 1, 2 and 4 (d) 1, 2, 3 and 4

25. A charge $+Q$ is uniformly distributed over a thin ring of the radius R. The velocity of an electron at the moment when it passes through the centre O of the ring, if the electron was initially at far away on the axis of the ring is (m = mass of electron, $k = \frac{1}{4\pi\varepsilon_0}$)

(a) $\sqrt{\left(\frac{2kQe}{mR}\right)}$ (b) $\sqrt{\left(\frac{kQe}{m}\right)}$

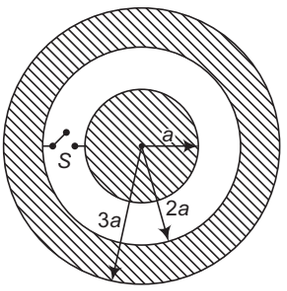
(c) $\sqrt{\left(\frac{kme}{QR}\right)}$ (d) $\sqrt{\left(\frac{kQe}{mR}\right)}$

26. Three identical charges are placed at corners of an equilateral triangle of side l. If force between any two charges is F, the work required to double the dimensions of triangle is

(a) $-3Fl$ (b) $3Fl$ (c) $\left(-\frac{3}{2}\right)Fl$ (d) $\left(\frac{3}{2}\right)Fl$

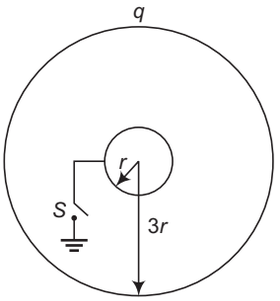
27. A solid conducting sphere of radius a having a charge q is surrounded by a concentric conducting spherical shell of inner radius $2a$ and outer radius $3a$ as shown in figure. Find the amount of heat produced when switch is closed.

$$k = \frac{1}{4\pi\varepsilon_0}$$



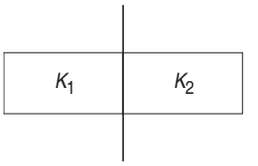
(a) $\frac{kq^2}{2a}$ (b) $\frac{kq^2}{3a}$ (c) $\frac{kq^2}{4a}$ (d) $\frac{kq^2}{6a}$

28. Figure shows two conducting thin concentric shells of radii r and $3r$. The outer shell carries charge q. Inner shell is neutral. Find the charge that will flow from inner shell to earth after the switch S is closed.



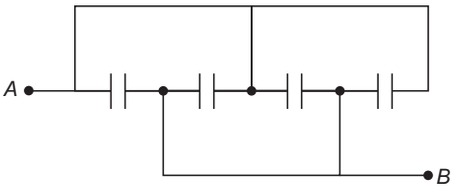
(a) $+\frac{q}{3}$ (b) $-\frac{q}{3}$ (c) $+3q$ (d) $-3q$

29. A parallel plate capacitor with air as medium between the plates has a capacitance of 10 μF. The area of capacitor is divided into two equal halves and filled with two media as shown in the figure having dielectric constant $K_1 = 2$ and $K_2 = 4$. The capacitance of the system will now be



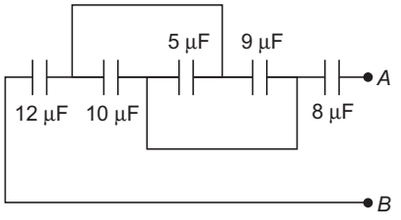
(a) 10 μF (b) 20 μF
(c) 30 μF (d) 40 μF

30. Four condensers are joined as shown in the below figure. The capacity of each is 8 μF. The equivalent capacity between the points A and B will be



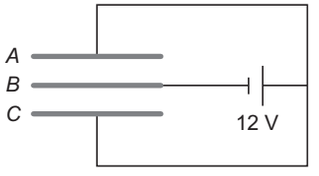
(a) 32 μF (b) 2 μF
(c) 8 μF (d) 16 μF

31. The capacities and connection of five capacitors are shown in the below figure. The potential difference between the points A and B is 60 V. Then, the equivalent capacity between A and B and the charge on 5 μF capacitance will be respectively



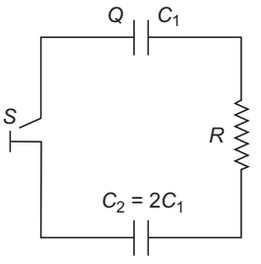
(a) 44 μF, 300 μC
(b) 16 μF, 150 μC
(c) 15 μF, 200 μC
(d) 4 μF, 50 μC

32. Three plates A, B, C each of area 50 cm^2 have separation 3 mm between A and B and 3 mm between B and C. The energy stored when the plates are fully charged is



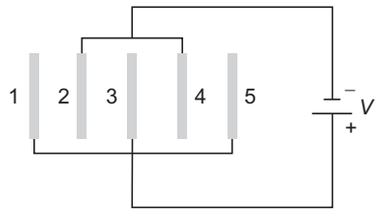
(a) 1.6×10^{-9} J (b) 2.1×10^{-9} J
(c) 5×10^{-9} J (d) 7×10^{-9} J

33. Two capacitors C_1 and $C_2 = 2C_1$ are connected in a circuit with a switch between them as shown in the figure. Initially, the switch is open and C_1 holds charge Q. The switch is closed. At steady state, the charge on each capacitor will be



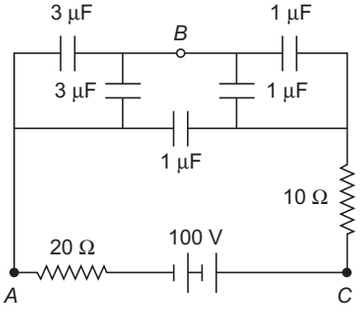
(a) $Q, 2Q$ (b) $Q/3, 2Q/3$
(c) $3Q/2, 3Q$ (d) $2Q/3, 4Q/3$

34. Five identical plates each of area A are joined as shown in the figure. The distance between the plates is d. The plates are connected to a potential difference of V volt. The charge on plates 1 and 4 will be



(a) $-\frac{\varepsilon_0 AV}{d} \cdot \frac{2\varepsilon_0 AV}{d}$ (b) $\frac{\varepsilon_0 AV}{d} \cdot \frac{2\varepsilon_0 AV}{d}$
(c) $\frac{\varepsilon_0 AV}{d} \cdot \frac{-2\varepsilon_0 AV}{d}$ (d) $\frac{-\varepsilon_0 AV}{d} \cdot \frac{-2\varepsilon_0 AV}{d}$

35. In the figure shown, what is the potential difference between the points A and B and between B and C respectively in steady state



(a) $V_{AB} = V_{BC} = 100$ V (b) $V_{AB} = 75$ V, $V_{BC} = 25$ V
(c) $V_{AB} = 25$ V, $V_{BC} = 75$ V (d) $V_{AB} = V_{BC} = 50$ V

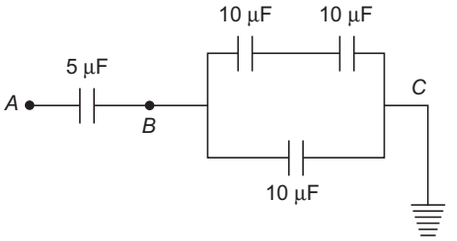
36. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is connected to another battery and is charged to potential difference $2V$. The charging batteries are now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

(a) zero (b) $\frac{25CV^2}{6}$
(c) $\frac{3CV^2}{2}$ (d) $\frac{9CV^2}{2}$

37. Condenser A has a capacity of 15 μF when it is filled with a medium of dielectric constant 15. Another condenser B has a capacity of 1 μF with air between the plates. Both are charged separately by a battery of 100 V. After charging, both are connected in parallel without the battery and the dielectric medium being removed. The common potential now is

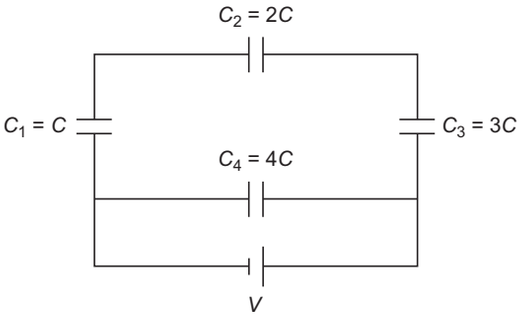
(a) 400 V (b) 800 V
(c) 1200 V (d) 1600 V

38. In the given circuit, if point C is connected to the earth and a potential of +2000 V is given to the point A, the potential at B is



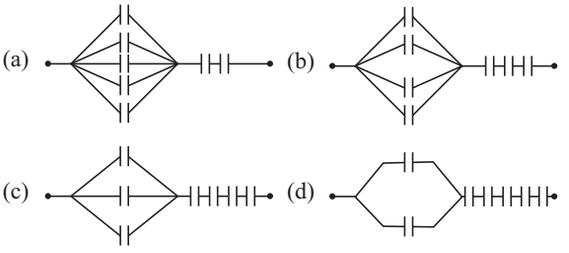
(a) 1500 V (b) 1000 V
(c) 500 V (d) 400 V

39. A network of four capacitors of capacity equal to $C_1 = C$, $C_2 = 2C$, $C_3 = 3C$ and $C_4 = 4C$ are connected to in a battery as shown in the figure. The ratio of the charges on C_2 and C_4 is

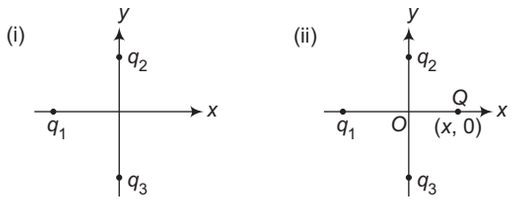


(a) $\frac{22}{3}$ (b) $\frac{3}{22}$
(c) $\frac{7}{4}$ (d) $\frac{4}{7}$

40. Seven capacitors each of capacity 2 μF are to be so connected to have a total capacity 10/11 μF. Which will be the necessary figure as shown?

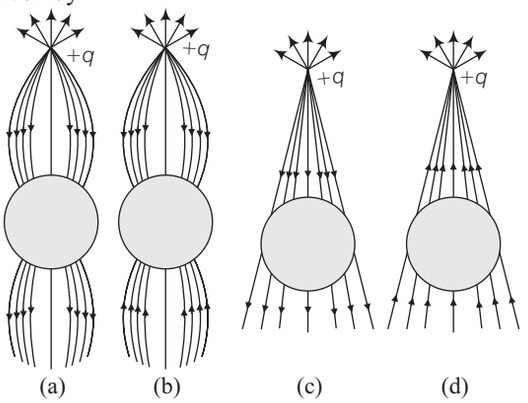


41. In figure two positive charges q_2 and q_3 fixed along the y-axis, exert a net electric force in the + x-direction on a charge q_1 fixed along the x-axis. If a positive charge Q is added at $(x, 0)$, the force on q_1

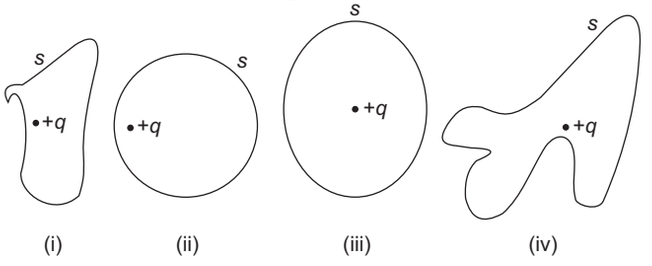


(a) shall increase along the positive x-axis
(b) shall decrease along the positive x-axis
(c) shall point along the negative x-axis
(d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3

42. A point positive charge is brought near an isolated conducting sphere (figure). The electric field is best given by

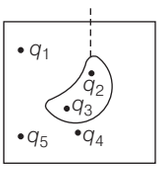


43. The electric flux through the surface



(a) in Fig. (iv) is the largest
(b) in Fig. (iii) is the least
(c) in Fig. (ii) is same as Fig. (iii) but is smaller than Fig. (iv)
(d) is the same for all the figures

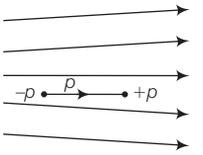
44. Five charges q_1, q_2, q_3, q_4, and q_5 are fixed at their positions as shown in figure, S is a Gaussian surface. The Gauss' law is given by $\int_S E \cdot dS = \frac{q}{\varepsilon_0}$.



Which of the following statements is correct?

(a) **E** on the LHS of the above equation will have a contribution from q_1, q_5 and q_1, q_5 and q_3 while q on the RHS will have a contribution from q_2 and q_4 only
(b) **E** on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only
(c) **E** on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_1, q_3 and q_5 only
(d) Both **E** on the LHS and q on the RHS will have contributions from q_2 and q_4 only

45. Figure shows electric field lines in which an electric dipole p is placed as shown. Which of the following statements is correct?

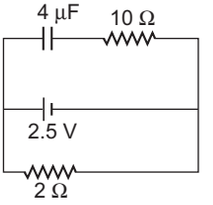


(a) The dipole will not experience any force
(b) The dipole will experience a force towards right
(c) The dipole will experience a force towards left
(d) The dipole will experience a force upwards

46. A point charge $+q$ is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is
(a) directed perpendicular to the plane and away from the plane
(b) directed perpendicular to the plane but towards the plane
(c) directed radially away from the point charge
(d) directed radially towards the point charge

47. A hemisphere is uniformely charged positively. The electric field at a point on a diameter away from the centre is directed
(a) perpendicular to the diameter
(b) parallel to the diameter
(c) at an angle tilted towards the diameter
(d) at an angle tilted away from the diameter

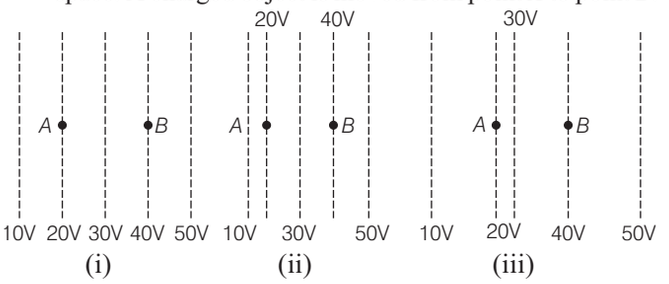
48. A capacitor of 4 μF is connected as shown in the circuit. The internal resistance of the battery is 0.5 Ω. The amount of charge on the capacitor plates will be



(a) 0 (b) 4 μ C
(c) 16 μC (d) 8 μC

49. A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge.
(a) remains a constant because the electric field is uniform
(b) increases because the charge moves along the electric field
(c) decreases because the charge moves along the electric field
(d) decreases because the charge moves opposite to the electric field

50. Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B.



(a) The work done in Fig. (i) is the greatest
(b) The work done in Fig. (ii) is least
(c) The work done is the same in Fig. (i), Fig.(ii) and Fig. (iii)
(d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in Fig. (i)

51. The electrostatic potential on the surface of a charged conducting sphere is 100 V. Two statements are made in this regard

S_1 at any point inside the sphere, electric intensity is zero.

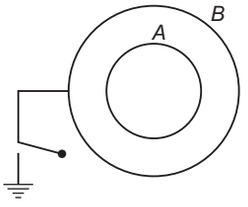
S_2 at any point inside the sphere, the electrostatic potential is 100 V.

Which of the following is a correct statement?

(a) S_1 is true but S_2 is false
(b) Both S_1 and S_2 are false
(c) S_1 is true, S_2 is also true and S_1 is the cause of S_2
(d) S_1 is true, S_2 is also true but the statements are independant

More than One Correct Options

1. Two concentric shells have radii R and $2R$ charges q_A and q_B and potentials $2\,\text{V}$ and $(3/2)\,\text{V}$, respectively. Now, shell B is earthed and let charges on them become q_A' and q_B'. Then,



(a) $q_A/q_B = 1/2$
(b) $q_A'/q_B' = 1$
(c) potential of A after earthing becomes $(3/2)$ V
(d) potential difference between A and B after earthing becomes $V/2$

2. A particle of mass 2 kg and charge 1 mC is projected vertically with a velocity $10\,\text{ms}^{-1}$. There is a uniform horizontal electric field of 10^4 N/C

(a) The horizontal range of the particle is 10 m
(b) The time of flight of the particle is 2 s
(c) The maximum height reached is 5 m
(d) The horizontal range of the particle is 5 m

3. At the distance of 5 cm and 10 cm from surface of a uniformly charged solid sphere, the potentials are 100 V and 75 V, respectively. Then,

(a) potential at its surface is 150 V
(b) the charge on the sphere is $\frac{50}{3}\times 10^{-10}$C
(c) the electric field on the surface is 1500 V/m
(d) the electric potential at its centre is 25 V

4. Three charged particles are in equilibrium under their electrostatic forces only. Then,

(a) the particles must be collinear
(b) all the charges cannot have the same magnitude
(c) all the charges cannot have the same sign
(d) the equilibrium is unstable

5. Charges Q_1 and Q_2 lie inside and outside respectively of a closed surface S. Let E be the field at any point on S and ϕ be the flux of E over S

(a) If Q_1 changes, both E and ϕ will change
(b) If Q_2 changes, E will change but ϕ will not change
(c) If $Q_1 = 0$ and $Q_2 \neq 0$, then $E \neq 0$ but $\phi = 0$
(d) If $Q_1 \neq 0$ and $Q_2 = 0$, then $E = 0$ but $\phi \neq 0$

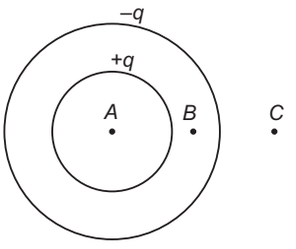
6. An electric dipole is placed at the centre of a sphere. Mark the correct options.

(a) The flux of the electric field through the sphere is zero
(b) The electric field is zero at every point of the sphere
(c) The electric field is not zero at any where on the sphere
(d) The electric field is zero on a circle on the sphere

7. Mark the correct options.

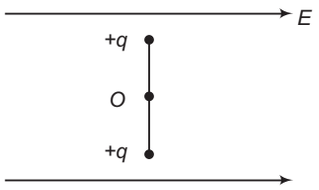
(a) Gauss's law is valid only for uniform charge distributions
(b) Gauss's law is valid only for charges placed in vacuum
(c) The electric field calculated by Gauss's law is the field due to all the charges
(d) The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface

8. Two concentric spherical shells have charges $+q$ and $-q$ as shown in figure. Choose the correct options.



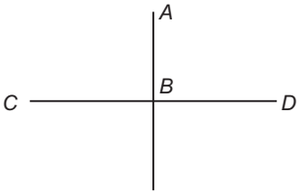
(a) At A electric field is zero, but electric potential is non-zero
(b) At B electric field and electric potential both are non-zero
(c) At C electric field is zero but electric potential is non-zero
(d) At C electric field and electric potential both are zero

9. A rod is hinged (free to rotate) at its centre O as shown in figure. Two point charges $+q$ and $+q$ are kept at its two ends. Rod is placed in uniform electric field E as shown. Space is gravity free. Choose the correct options.



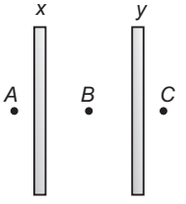
(a) Net force from the hinge on the rod is zero
(b) Net force from the hinge on the rod is leftwards
(c) Equilibrium of rod is neutral
(d) Equilibrium of rod is stable

10. Two charges $+Q$ each are fixed at points C and D. Line AB is the bisector line of CD. A third charge $+q$ is moved from A to B, then from B to C.



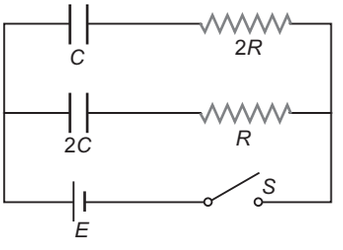
(a) From A to B electrostatic potential energy will decrease
(b) From A to B electrostatic potential energy will increase
(c) From B to C electrostatic potential energy will increase
(d) From B to C electrostatic potential energy will decrease

11. X and Y are large, parallel conducting plates close to each other. Each face has an area A. X is given a charge Q. Y is without any charge. Points A, B and C are as shown in the figure.



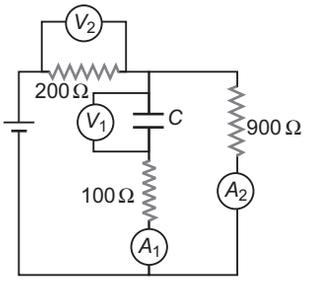
(a) The field at B is $\frac{Q}{2\varepsilon_0 A}$
(b) The field at B is $\frac{Q}{\varepsilon_0 A}$
(c) The field at A, B and C are of the same magnitude
(d) The fields at A and C are of the same magnitude, but in opposite directions

12. In the circuit shown in the figure, switch S is closed at time $t = 0$. Select the correct statements.



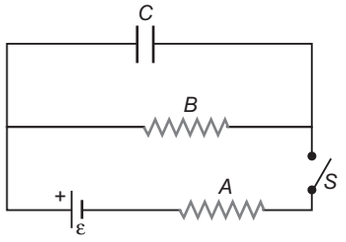
(a) Rate of increase of charge is same in both the capacitors
(b) Ratio of charge stored in capacitors C and $2C$ at any time t would be 1 : 2
(c) Time constants of both the capacitors are equal
(d) Steady state charge in capacitors C and $2C$ are in the ratio of 1 : 2

13. An electrical circuit is shown in the given figure. The resistance of each voltmeter is infinite and each ammeter is 100 Ω. The charge on the capacitor of 100 μF in steady state is 4 mC. Choose correct statements (s) regarding the given circuit.



(a) Reading of voltmeter V_2 is 16 V
(b) Reading of ammeter A_1 is zero and A_2 is 1/25 A
(c) Reading of voltmeter V_1 is 40 V
(d) EMF of the ideal cell is 66 V

14. In the circuit shown, A and B are equal resistances. When S is closed, the capacitor C charges from the cell of emf ε and reaches a steady state

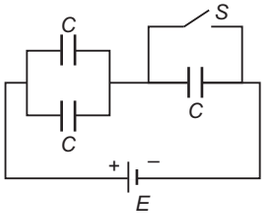


(a) During charging, more heat is produced in A then in B
(b) In steady state, heat is produced at the same rate in A and B
(c) In the steady state, energy stored in C is $1/4\ C\varepsilon^2$
(d) In the steady state energy stored in C is $1/8\ C\varepsilon^2$

15. A parallel plate capacitor is charged from a cell and then isolated from it. The separation between the plates is now increased

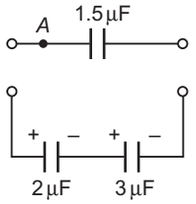
(a) The force of attraction between the plates will decrease
(b) The field in the region between the plates will not change
(c) The energy stored in the capacitor will increase
(d) The potential difference between the plates will decrease

16. In the circuit shown, each capacitor has a capacitance C. The emf of the cell is E. If the switch S is closed, then



(a) positive charge will flow out of the positive terminal of the cell
(b) positive charge will enter the positive terminal of the cell
(c) the amount of the charge flowing through the cell will be $1/3\ CE$
(d) the amount of charge flowing through the cell is $(4/3)\ CE$

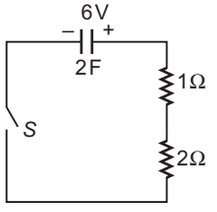
17. Two capacitors of 2 μF and 3 μF are charged to 150 V and 120 V,respectively. The plates of capacitor are connected as shown in the figure. An uncharged capacitor of capacity 1.5 μF falls to the free end of the wire. Then,



(a) charge on 1.5 μF capacitor is 180 μC
(b) charge on 2 μF capacitor is 120 μC
(c) positive charge flows through A from right to left
(d) positive charge flows through A from left to right

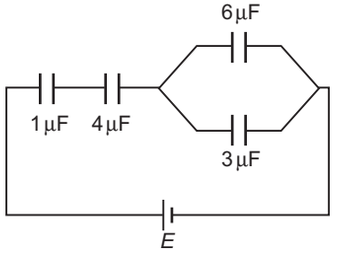
18. A parallel plate capacitor is charged and then the battery is disconnected. When the plates of the capacitor are brought closer, then
(a) energy stored in the capacitor decreases
(b) the potential difference between the plates decreases
(c) the capacitance increases
(d) the electric field between the plates decreases

19. A capacitor of 2 F (practically not possible to have a capacity of 2 F) is charged by a battery of 6 V. The battery is removed and circuit is made as shown. Switch is closed at time $t = 0$. Choose the correct options.



(a) At time $t = 0$, current in the circuit is 2 A
(b) At time $t = (6 \ln 2)$ second potential difference across capacitor is 3 V
(c) At time $t = (6 \ln 2)$ second, potential difference across 1 Ω resistance is 1 V
(d) At time $t = (6 \ln 2)$ second potential difference across 2 Ω resistance is 2 V

20. Given that potential difference across 1μF capacitor is 10 V. Then,



(a) potential difference across 4 μF capacitor is 40 V
(b) potential difference across 4 μF capacitor is 2.5 V
(c) potential difference across 3 μF capacitor is 5 V
(d) value of E is 70 V

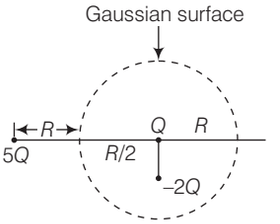
21. If $\int_S \mathbf{E} \cdot d\mathbf{S} = 0$ over a surface, then
(a) the electric field inside the surface and on it is zero
(b) the electric field inside the surface is necessarily uniform
(c) the number of flux lines entering the surface must be equal to the number of flux lines leaving it
(d) all charges must necessarily be outside the surface

22. The electric field at a point is
(a) always continuous
(b) continuous if there is no charge at that point
(c) discontinuous only if there is a negative charge at that point
(d) discontinuous if there is a charge at that point

23. If there were only one type of charge in the universe, then
(a) $\oint_S \mathbf{E} \cdot d\mathbf{S} \neq 0$ on any surface
(b) $\oint_S \mathbf{E} \cdot d\mathbf{S} = 0$ if the charge is outside the surface
(c) $\oint_S \mathbf{E} \cdot d\mathbf{S}$ could not be defined
(d) $\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$ if charges of magnitude q were inside the surface

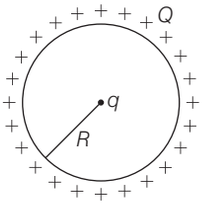
24. Consider a region inside which there are various types of charges but the total charge is zero. At points outside the region,
(a) the electric field is necessarily zero
(b) the electric field is due to the dipole moment of the charge distribution only
(c) the dominant electric field is $\propto \frac{1}{r_3}$, for large r, where r is the distance from a origin in this regions
(d) the work done to move a charged particle along a closed path, away from the region, will be zero

25. Refer to the arrangement of charges in figure and a Gaussian surface of radius R with Q at the centre. Then,



(a) total flux through the surface of the sphere is $\frac{-Q}{\varepsilon_0}$
(b) field on the surface of the sphere is $\frac{-Q}{4\pi\varepsilon_0 R^2}$
(c) flux through the surface of sphere due to $5Q$ is zero
(d) field on the surface of sphere due to $-2Q$ is same everywhere

26. A positive charge Q is uniformly distributed along a circular ring of radius R. A small test charge q is placed at the centre of the ring in figure. Then,



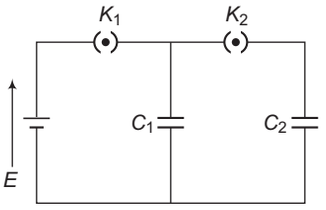
(a) if $q > 0$ and is displaced away from the centre in the plane of the ring, it will be pushed back towards the centre
(b) if $q < 0$ and is displaced away from the centre in the plane of the ring, it will never return to the centre and will continue moving till it hits the ring
(c) if $q < 0$, it will perform SHM for small displacement along the axis
(d) q at the centre of the ring is in an unstable equilibrium within the plane of the ring for $q > 0$

27. Equipotential surfaces
(a) are closer in regions of large electric fields compared to regions of lower electric fields
(b) will be more crowded near sharp edges of a conductor
(c) will be more crowded near regions of large charge densities
(d) will always be equally spaced

28. The work done to move a charge along an equipotential from A to B
(a) cannot be defined as $-\int_A^B \mathbf{E}\cdot d\mathbf{l}$
(b) must be defined as $-\int_A^B \mathbf{E}\cdot d\mathbf{l}$
(c) is zero
(d) can have a non-zero value

29. In a region of constant potential,
(a) the electric field is uniform
(b) the electric field is zero
(c) there can be no charge inside the region
(d) the electric field shall necessarily change if a charge is placed outside the region

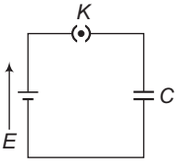
30. In the circuit shown in figure initially key K_1 is closed and key K_2 is open. Then, K_1 is opened and K_2 is closed (order is important).
[Take Q_1' and Q_2' as charges on C_1 and C_2 and V_1 and V_2 as voltage, respectively.]



Then,
(a) charge on C_1 gets redistributed such that $V_1 = V_2$
(b) charge on C_1 gets redistributed such that $Q_1' = Q_2'$
(c) charge on C_1 gets redistributed such that $C_1V_1 + C_2V_2 = C_1E$
(d) charge on C_1 gets redistributed such that $Q_1' + Q_2' = Q$

31. If a conductor has a potential $V \neq 0$ and there are no charges anywhere else outside, then
(a) there must be charges on the surface or inside itself
(b) there cannot be any charge in the body of the conductor
(c) there must be charges only on the surface
(d) there must be charges inside the surface

32. A parallel plate capacitor is connected to a battery as shown in figure. Consider two situations.



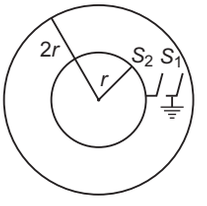
A. Key K is kept closed and plates of capacitors are moved apart using insulating handle.
B. Key K is opened and plates of capacitors are moved apart using insulating handle.

Choose the correct option(s).
(a) In A, Q remains same but C changes
(b) In B, V remains same but C changes
(c) In A, V remains same and hence Q changes
(d) In B, Q remains same and hence V changes

Comprehension Based Questions

Passage I (Q. 1 to 3)

There are two concentric spherical shell of radii r and 2r. Initially, a charge Q is given to the inner shell and both the switches are open.



1. If switch S_1 is closed and then opened, charge on the outer shell will be
(a) Q (b) $Q/2$
(c) $-Q$ (d) $-Q/2$

2. Now S_2 is closed and opened. The charge flowing through the switch S_2 in the process is
(a) Q (b) $Q/4$
(c) $Q/2$ (d) $2Q/3$

3. The two steps of the above two problems are repeated n times, the potential difference between the shells will be
(a) $\frac{1}{2^{n+1}}\left[\frac{Q}{4\pi\varepsilon_0 r}\right]$ (b) $\frac{1}{2^{n}}\left[\frac{Q}{4\pi\varepsilon_0 r}\right]$
(c) $\frac{1}{2^{n}}\left[\frac{Q}{2\pi\varepsilon_0 r}\right]$ (d) $\frac{1}{2^{n-1}}\left[\frac{Q}{2\pi\varepsilon_0 r}\right]$

Passage II (Q. 4 to 7)

A sphere of charge of radius R carries a positive charge whose volume charge density depends only on the distance r from the ball's centre as $\rho = \rho_0\left(1-\frac{r}{R}\right)$, *where* ρ_0 *is a constant. Assume* ε *as the permittivity of space.*

4. The magnitude of electric field as a function of the distance r inside the sphere is given by

(a) $E=\frac{\rho_0}{\varepsilon}\left[\frac{r}{3}-\frac{r^2}{4R}\right]$ (b) $E=\frac{\rho_0}{\varepsilon}\left[\frac{r}{4}-\frac{r^2}{3R}\right]$

(c) $E=\frac{\rho_0}{\varepsilon}\left[\frac{r}{3}+\frac{r^2}{4R}\right]$ (d) $E=\frac{\rho_0}{\varepsilon}\left[\frac{r}{4}+\frac{r^2}{3R}\right]$

5. The magnitude of the electric field as a function of the distance r outside the ball is given by

(a) $E=\frac{\rho_0 R^3}{8\varepsilon r^2}$ (b) $E=\frac{\rho_0 R^3}{12\varepsilon r^2}$

(c) $E=\frac{\rho_0 R^2}{8\varepsilon r^3}$ (d) $E=\frac{\rho_0 R^2}{12\varepsilon r^3}$

6. The value of distance r_m at which electric field intensity is maximum, is given by

(a) $r_m=\frac{R}{3}$ (b) $r_m=\frac{3R}{2}$ (c) $r_m=\frac{2R}{3}$ (d) $r_m=\frac{4R}{3}$

7. The maximum electric field intensity is

(a) $E_m=\frac{\rho_0 R}{9\varepsilon}$ (b) $E_m=\frac{\rho_0 \varepsilon}{9R}$

(c) $E_m=\frac{\rho_0 R}{3\varepsilon}$ (d) $E_m=\frac{\rho_0 R}{6\varepsilon}$

Passage III (Q. 8 to 12)

A solid metallic sphere of radius a is surrounded by a conducting spherical shell of radius b $(b>a)$. *The solid sphere is given a charge Q. A student measures the potential at the surface of the solid sphere as* V_a *and the potential at the surface of spherical shell as* V_b. *After taking these readings, he decides to put a charge of* $-4Q$ *on the shell. He then noted the readings of the potential of solid sphere and the shell and found that the potential difference is* ΔV. *He then connected the outer spherical shell to the earth by a conducting wire and found that the charge on the outer surface of the shell as* q_1.

He then decides to remove the earthing connection from the shell and earthed the inner solid sphere. Connecting the inner sphere with the earth he observes the charge on the solid sphere as q_2. *He then wanted to check what happens if the two are connected by the conducting wire. So he removed the earthing connection and connected a conducting wire between the solid sphere and the spherical shell. After the connections were made he found the charge on the outer shell as* q_3.

Answer the following questions based on the readings taken by the student at various stages.

8. Potential difference (ΔV) measured by the student between the inner solid sphere and outer shell after putting a charge $-4Q$ is

(a) V_a-3V_b (b) $3(V_a-V_b)$

(c) V_a (d) V_a-V_b

9. What is the value of q_2?

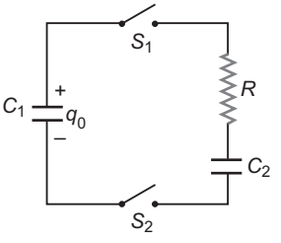
(a) Q (b) $Q\left(\frac{a}{b}\right)$ (c) $-4Q$ (d) zero

10. What is the value of q_3?

(a) $\frac{Q(a+b)}{a-b}$ (b) $\frac{Qa^2}{b}$ (c) $\frac{Q(a-b)}{b}$ (d) $-\frac{Qb}{a}$

Passage IV (Q. 11 to 12)

The capacitor C_1 *in the figure shown initially carries a charge* q_0. *When the switches* S_1 *and* S_2 *are closed, capacitor* C_1 *is connected in series to a resistor R and a second capacitor* C_2, *which is initially uncharged.*



11. Find the charge flown through wires as a function of time.

(a) $q_0e^{-t/RC}+\frac{C}{C_2}q_0$

(b) $\frac{q_0C}{C_1}\times[1-e^{-t/RC}]$

(c) $q_0\frac{C}{C_1}e^{-t/CR}$

(d) $q_0e^{-t/RC}$, where $C=\frac{C_1C_2}{C_1+C_2}$

12. Find the total heat dissipated in the circuit during the discharging process of C_1.

(a) $\frac{q_0^2}{2C_1^2}\times C$ (b) $\frac{q_0^2}{2C}$

(c) $\frac{q_0^2C_2}{2C_1^2}$ (d) $\frac{q_0^2}{2C_1C_2}$

Passage V (Q. 13 to 14)

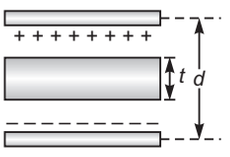


Figure shows a parallel plate capacitor with plate area A and plate separation d. A potential difference is being applied between the plates. The battery is then disconnected and a dielectric slab of dielectric constant K is placed in between the plates of the capacitor as shown.

Now, answer the following questions based on above information.

13. The electric field in the gaps between the plates and the dielectric slab will be

(a) $\frac{\varepsilon_0 AV}{d}$ (b) $\frac{V}{d}$

(c) $\frac{KV}{d}$ (d) $\frac{V}{d-t}$

14. The electric field in the dielectric slab is

(a) $\frac{V}{Kd}$ (b) $\frac{KV}{d}$

(c) $\frac{V}{d}$ (d) $\frac{KV}{t}$

Assertion and Reason

Directions (Q. Nos. 1-20) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions, you are required to choose any one of the following five responses*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

(e) If both Assertion and Reason are false.

1. **Assertion** Due to two point charges electric field and electric potential cannot be zero at some point simultaneously.

Reason Field is a vector quantity and potential is a scalar quantity.

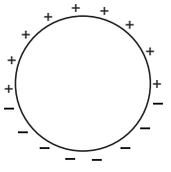
2. **Assertion** When two positive point charges move away from each other, their electrostatic potential energy decreases.

Reason Change in potential energy between two points is equal to the work done by electrostatic forces.

3. **Assertion** When two charged spheres are connected to each other by a thin conducting wire, charge flow from bigger sphere to smaller sphere if initial charges on them are same.

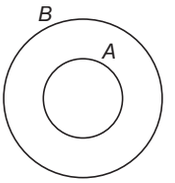
Reason Electrostatic potential energy will be lost in redistribution of charges.

4. **Assertion** Half of the ring is uniformly positively charged and other half uniformly negatively charged. Then, electric field and electric potential both are zero at centre.



Reason At the centre of uniformly charged ring, electric field is zero.

5. **Assertion** Spherical shell A in the shown figure has a charge $-q$ and shell B has charge $+q$. When these two are connected by a thin conducting wire, whole of the electrostatic potential stored between the shell is lost.



Reason Whole of the inner charge will transfer to outer shell B.

6. **Assertion** A charged sphere has a radius R. At a distance $\frac{R}{2}$ from the surface towards centre potential is V_1 and a distance $\frac{R}{2}$ from surface towards infinity potential is V_2. Then, $V_1 > V_2$.

Reason Inside a solid sphere, V-r variation is parabolic. Where r is the distance from centre.

7. **Assertion** For a non-uniformly charged thin circular ring with net charge zero, the electric field at any point on axis of the ring is zero.

Reason For a non-uniformly charged thin circular ring with net charge zero, the electric potential at each point on axis of the ring is zero.

8. **Assertion** In a region where uniform electric field exists, the net charge within volume of any size is zero.

Reason The electric flux within any closed surface in region of uniform electric field is zero.

9. **Assertion** We have a solid charged sphere. There are two points P and Q. P is a point lying between centre and surface of the sphere. Q is at finite and non-zero distance from the surface outside the sphere. There will be no situation where electric potential is equal at P and Q.

Reason Rate of change of electric field with distance from centre merely represents, magnitude of electric potential at that point.

10. **Assertion** Cyclotron is used to accelerate heavy charged particles, such as proton. It is not suitable for accelerating light charged particles such as electrons.

Reason Specific charge of proton is more as compared to electron.

11. **Assertion** Five charges $+q$ each are placed at five vertices of a regular pentagon. A sixth charge $-Q$ is placed at the centre of pentagon. Then, net electrostatic force on $-Q$ is zero.

Reason Net electrostatic potential at the centre is zero.

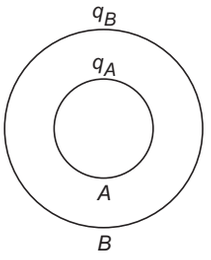
12. **Assertion** Inside a solid charged sphere E-r variation is linear.

Reason Inside that solid charged sphere V-r variation is parabolic.

13. Assertion Two charges $+Q$ each are fixed at two points. If a third charge $+q$ is placed at centre, then this charge is in stable equilibrium condition line joining the charges.

Reason Electrostatic potential energy of the system in this position is minimum.

14. Assertion Two concentric spherical shells A and B are charged by charges q_A and q_B. If q_A is negative, then $V_A - V_B$ is also negative.



Reason

$$V_A - V_B = \left(\frac{q_A}{4\pi\varepsilon_0}\right)\left(\frac{1}{R_A} - \frac{1}{r_B}\right)$$

Therefore, if q_A is negative, then $V_A - V_B$ is also negative.

15. Assertion Two identical balls are charged by q. They are suspended from a common point by two insulating threads l each. In equilibrium, the angle between the threads is 180°. (Ignore gravity)

Reason In equilibrium tension in the springs is

$$T = \frac{1}{4\pi\varepsilon_0}\frac{q \cdot q}{l^2}$$

16. Assertion With the help of Gauss theorem, we can find electric field at any point.

Reason Gauss theorem can be applied for any type of charge distribution.

17. Assertion When a capacitor is charged by a battery half of the energy supplied by the battery is stored in the capacitor and rest half is lost.

Reason If resistance in the circuit is zero, then there will be no loss of energy.

18. Assertion A point charge produces a uniform electric field.

Reason Due to a point charge electric lines of forces are parallel and equidistant.

19. Assertion When a metallic plate is partially inserted between the plates of a capacitor its capacity increases.

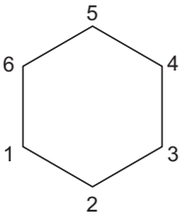
Reason If conductivity of conducting plate is more, increase in capacity will be more.

20. Assertion No work is done in moving an electric dipole transnationally in uniform electric field.

Reason Net force on electric dipole in uniform electric field is zero.

Match the Columns

1. At six vertices of a regular hexagon six point charges $+q, +q, +q, -q, -q, +q$ and $-q$ are placed. In Column I some conditions are given in which positions of two charges are interchanged. Corresponding to these conditions let potential at centre changes by x-times and magnitude of electric field changes by y-times. Then, in Column II values of x and y are given. Match the two columns.



Column I	Column II
(A) Charges at 6 and 3 are interchanged	(p) $y = 1$
(B) Charges at 2 and 4 are interchanged	(q) $y = \frac{\sqrt{3}}{2}$
(C) Charges at 2 and 5 are interchanged	(r) $x = 1$
(D) Charges at 5 and 6 are interchanged	(s) $x = \frac{\sqrt{3}}{2}$

2. Two charges $+q$ and $-q$ are placed at distance r. Match the following two columns when distance between them is changed to r'.

Column I	Column II
(A) $r' = 2r$	(p) Potential energy will become half
(B) $r' = \frac{r}{2}$	(q) Force between them will become 1/4th
(C) $r' = 4r$	(r) Potential energy will become four times
(D) $r' = \frac{r}{4}$	(s) None

3. A capacitor is connected with a battery. With battery remains connected some changes are done in capacitor/battery, which are given in Column I. Corresponding match the two columns.

Column I	Column II
(A) Distance between capacitor plates is halved	(p) Capacity of capacitor will become two times
(B) A metallic plate is half filled between the plates	(q) Charge stored in capacitor will become two times
(C) A dielectric slab of dielectric constant $K = 2$ is completely filled between the plates	(r) Energy stored in capacitor will become two times
(D) EMF of battery is doubled	(s) None

4. Match the following two columns.

Column I	Column II
(A) Electric charge	(p) $[ML^2T^{-3}A^{-1}]$
(B) Electric field strength	(q) $[MLT^{-3}A^{-1}]$
(C) Electric potential	(r) $[MT^{-3}A^{-1}]$
(D) Electric dipole	(s) None

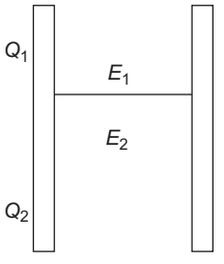
5. Four metallic plates are charged as shown in figure. Now match the following two columns.

Column I	Column II
(A) Electric field in region-I	(p) $\frac{\sigma}{\varepsilon_0}$
(B) Electric field in region-II	(q) $\frac{2\sigma}{\varepsilon_0}$
(C) Electric field in region-III	(r) $\frac{\sigma}{2\varepsilon_0}$
(D) Electric field in region-IV	(s) zero

Entrance Gallery

2014

1. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects. **[JEE Advanced]**



(a) $\frac{E_1}{E_2}=1$ (b) $\frac{E_1}{E_2}=\frac{1}{K}$

(c) $\frac{Q_1}{Q_2}=\frac{3}{K}$ (d) $\frac{C}{C_1}=\frac{2+K}{K}$

2. Assume that electric field, $\mathbf{E}=30x^2\hat{\mathbf{i}}$ exists in space. Then, the potential difference $V_A - V_O$, where, V_O is the potential at the origin and V_A the potential at $x=2$ m, is **[JEE Main]**

(a) 120 J (b) – 120 J

(c) – 80 J (d) 80 J

3. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to **[JEE Main]**

(a) 6×10^{-7} C/m^2 (b) 3×10^{-7} C/m^2

(c) 3×10^4C/m^2 (d) 6×10^4C/m^2

4. The energy stored in a capacitor of capacitance C having a charge Q under a potential V is **[Kerala CEE]**

(a) $\frac{1}{2}Q^2V$ (b) $\frac{1}{2}C^2V$ (c) $\frac{1}{2}\frac{Q^2}{V}$

(d) $\frac{1}{2}QV$ (e) $\frac{1}{2}CV$

5. Two capacitors of 10 pF and 20 pF are connected to 200 V and 100 V sources, respectively. If they are connected by the wire, what is the common potential of the capacitors? **[Karnataka CET]**

(a) 133.3 V (b) 150 V

(c) 300 V (d) 400 V

6. If the charge on the body in 1 nC, then how many electrons are present on the body? **[Karnataka CET]**

(a) 1.6×10^{-19} (b) 6.25×10^9

(c) 6.25×10^{27} (d) 6.25×10^{28}

7. A spherical conductor of radius 2 cm is uniformly charged with 3 nC. What is the electric field at a distance of 3 cm from the centre of the sphere? **[Karnataka CET]**

(a) 3×10^6 Vm^{-1} (b) 3 Vm^{-1}

(c) 3×10^4 Vm^{-1} (d) 3×10^{-4} Vm^{-1}

8. Two equal and opposite charges of masses m_1 and m_2 are accelerated in an uniform electric field through the same distance. What is the ratio of their accelerations, if their ratio of masses is $\frac{m_1}{m_2}=0.5$? **[Karnataka CET]**

(a) $\frac{a_1}{a_2}=0.5$ (b) $\frac{a_1}{a_2}=1$

(c) $\frac{a_1}{a_2}=2$ (d) $\frac{a_1}{a_2}=3$

9. What is the nature of Gaussian surface involved in Gauss law of electrostatic? **[Karnataka CET]**

(a) Scalar (b) Electrical

(c) Magnetic (d) Vector

10. Consider two concentric spherical metal shells of radii r_1 and r_2 ($r_2 > r_1$). If the outer shell has a charge q and the inner one is grounded, the charge on the inner shell is **[WB JEE]**

(a) $\frac{-r_2}{r_1}q$ (b) zero

(c) $\frac{-r_1}{r_2}q$ (d) $-q$

11. What is the electric potential at a distance of 9 cm from 3 nC? [Karnataka CET]
(a) 270 V (b) 3 V (c) 300 V (d) 30 V

12. A parallel plate capacitor is charged and then disconnected from the charging battery. If the plates are now moved farther apart by pulling at them by means of insulating handles, then [WB JEE]
(a) the energy stored in the capacitor decreases
(b) the capacitance of the capacitor increases
(c) the charge on the capacitor decreases
(d) the voltage across the capacitor increases

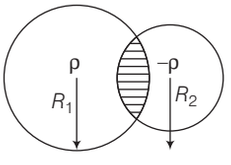
13. Three capacitors 3 μF, 6 μF and 6 μF are connected in series to a source of 120 V. The potential difference, in volt, across the 3 μF capacitor will be [WB JEE]
(a) 24 (b) 30
(c) 40 (d) 60

14. A 400 pF capacitor is charged by a 100 V supply. How much electrostatic energy is lost in the process of disconnecting from the supply and connecting another uncharged 400 pF capacitor? [J&K CET]
(a) 10^{-5} J (b) 10^{-6} J
(c) 10^{-7} J (d) 10^{-4} J

15. A 15 pF capacitor is connected to a 60 V battery. How much electrostatic energy is stored? [J&K CET]
(a) 2.70×10^{-8} J (b) 1.77×10^{-8} J
(c) 2.35×10^{-8} J (d) 1.35×10^{-8} J

2013

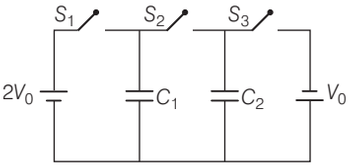
16. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in figure. At all points in the overlapping region [JEE Advanced]



(a) the electrostatic field is zero
(b) the electrostatic potential is constant
(c) the electrostatic field is constant in magnitude
(d) the electrostatic field has same direction

17. Two-conduction solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centre of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be [JEE Advanced]
(a) -4 (b) $-\frac{32}{25}$ (c) $\frac{32}{25}$ (d) 4

18. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After sometime, S_2 is released and then S_3 is pressed. After sometime, [JEE Advanced]

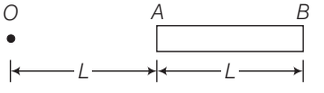


(a) the charge on the upper plate of C_1 is $2CV_0$
(b) the charge on the upper plate of C_1 is CV_0
(c) the charge on the upper plate of C_2 is 0
(d) the charge on the upper plate of C_2 is $-CV_0$

19. Two capacitors C_1 and C_2 are charged to 120 V and 200 V, respectively. It is found that by connecting them together the potential on each one can be made zero. Then, [JEE Main]
(a) $5C_1 = 3C_2$
(b) $3C_1 = 5C_2$
(c) $3C_1 \pm 5C_2 = 0$
(d) $9C_1 = 4C_2$

20. Two charges each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement $y\,(y << a)$ along the y-axis, the net force acting on the particle is proportional to [JEE Main]
(a) y (b) $-y$
(c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

21. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is [JEE Main]

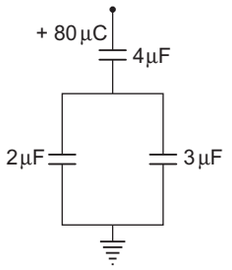


(a) $\frac{Q}{8\pi\varepsilon_0 L}$ (b) $\frac{3Q}{4\pi\varepsilon_0 L}$
(c) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$ (d) $\frac{Q \ln 2}{4\pi\varepsilon_0 L}$

22. Two capacitors of capacitance C are connected in series. If one of them is filled with dielectric substance K, what is the effective capacitance? [OJEE]
(a) $\frac{KC}{(1+K)}$ (b) $C\,(K+1)$
(c) $\frac{2KC}{1+K}$ (d) None of these

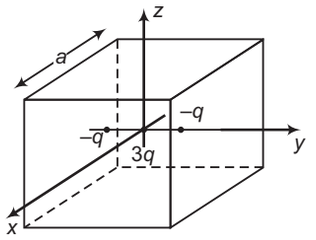
2012

23. In the given circuit, a charge of + 80 μC is given to the upper plate of the 4 μF capacitor. Then, in the steady state, the charge on the upper plate of the 3 μF capacitor is **[IIT JEE]**



(a) + 32 μC (b) + 48 μC (c) + 40 μC (d) + 80 μC

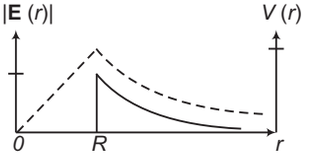
24. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $(0, a/4, 0)$, $+3q$ and $-q$ at $(0, +a/4, 0)$. Choose the correct option(s). **[IIT JEE]**



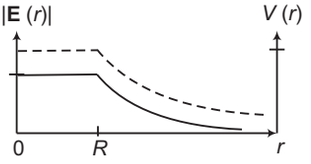
(a) The net electric flux crossing the plane $x = +a/2$ is equal to the net electric flux crossing the plane $x = -a/2$
(b) The net electric flux crossing the plane $y = +a/2$ is more than the net electric flux crossing the plane $y = -a/2$
(c) The net electric flux crossing the entire region is q/ε_0
(d) The net electric flux crossing the plane $z = +a/2$ is equal to the net electric flux crossing the plane $x = +a/2$

25. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\mathbf{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by which graph? **[IIT JEE]**

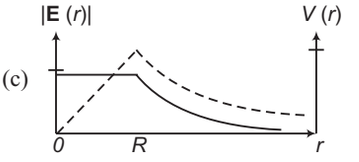
(a)



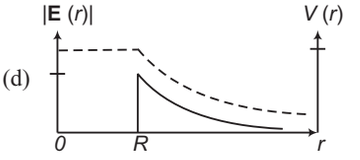
(b)



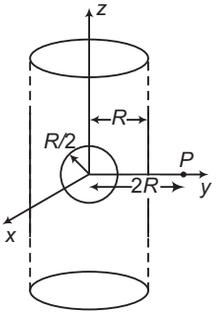
(c)



(d)

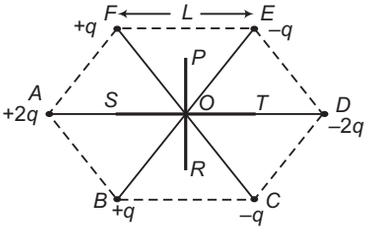


26. A infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder is given by the expression $\frac{23\rho R}{16 K \varepsilon_0}$. The value of k is **[IIT JEE]**



(a) 6 (b) 7
(c) 8 (d) 9

27. Six point charges are kept at the vertices of a regular hexagon of side L and centre O as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is (are) correct? **[IIT JEE]**



(a) The electric field at O is $6K$ along OD
(b) The potential at O is zero
(c) The potential at all points on the line PR is same
(d) The potential at all points on the line ST is same

28. This question has statement 1 and statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

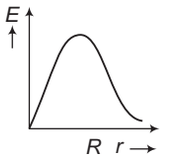
An insulating solid sphere of radius R has a uniform positive charge density ρ. As a result of this uniform charge distribution, there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero.

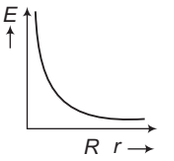
Statement 1 When a charge q is taken from the centre of the surface of the sphere its potential energy u changes by $\frac{q\rho}{3\varepsilon_0}$.

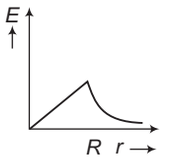
Statement 2 The electric field at a distance $r\,(r<R)$ from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$. [AIEEE]

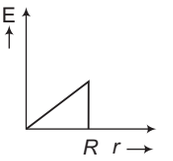
(a) Statement 1 is false, Statement 2 is true
(b) Statement 1 is true, Statement 2 is false
(c) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1

29. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as the function of distanace from the centre. The graph which would correspond to the above will be [AIEEE]

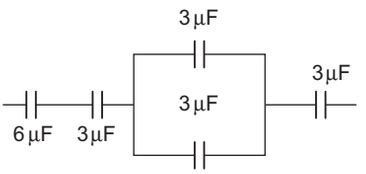
(a) 

(b) 

(c) 

(d) 

30. In this diagram, the PD between A and B is 60 V. The PD across 6 μF capacitor is [Karnataka CET]



(a) 4 V (b) 10 V
(c) 5 V (d) 20 V

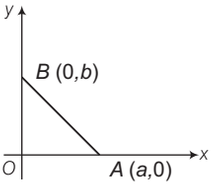
31. Charge q of mass m is placed in a uniform electric field E, then KE after time t [OJEE]

(a) $\frac{qEt}{2m}$ (b) $\frac{qE^2t^2}{2m}$ (c) $\frac{2m}{q^2E^2t^2}$ (d) $\frac{q^2E^2t^2}{2m}$

32. The two infinite parallel metal plates, contain electric charges with charge densities $+\sigma$ and $-\sigma$, respectively and they are separated by a small distance in air. If the permittivity of air is ε_0, then the magnitude of the field between the two plates with its directions will be

(a) σ/ε_0 towards the positively charged plane [WB JEE]
(b) σ/ε_0 towards the negatively charged plane
(c) $\sigma/(2\varepsilon_0)$ towards the positively charged plane
(d) 0 and towards any direction

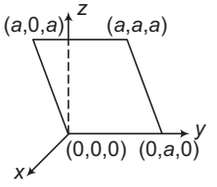
33. A charge $+q$ is placed at the origin O of X-Y axes as shown in the figure. The work done in taking a charge Q from A to B along the straight line AB is [WB JEE]



(a) $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{a-b}{ab}\right)$ (b) $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$
(c) $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{b}{a^2}-\frac{1}{b}\right)$ (d) $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{a}{b^2}-\frac{1}{b}\right)$

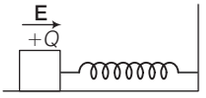
2011

34. Consider an electric field $\mathbf{E}=E_0\hat{\mathbf{x}}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is [IIT JEE]



(a) $2E_0a^2$ (b) $\sqrt{2}E_0a^2$ (c) E_0a^2 (d) $\frac{E_0a^2}{\sqrt{2}}$

35. A wooden block performs SHM on a frictionless surface with frequency ν_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field $\mathbf{E}$ is switched on as shown, then the SHM of the block will be [IIT JEE]



(a) of the same frequency and with shifted mean position
(b) of the same frequency and with the same mean position
(c) of changed frequency and with shifted mean position
(d) of changed frequency and with the same mean position

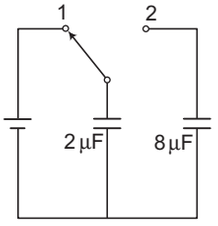
36. A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B\,(<R_A)$ are kept far apart and each are given charge $+Q$. Now, they are connected by a thin metal wire. Then, [IIT JEE]

(a) $E_A^{\text{inside}}=0$ (b) $Q_A>Q_B$
(c) $\frac{\sigma_A}{\sigma_B}=\frac{R_B}{R_A}$ (d) $E_A^{\text{on surface}}<E_B^{\text{on surface}}$

37. Which of the following statement(s) is/are correct? [IIT JEE]

(a) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss' law will still be valid
(b) The Gauss' law can be used to calculate the field distribution around an electric dipole
(c) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same
(d) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is (V_B-V_A)

38. A 2 μF capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch *s* is turned to position 2 is **[IIT JEE]**



(a) 0% (b) 20%
(c) 75% (d) 80%

39. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result charges approach each other with a velocity v. Then, as a function of distance x between them **[AIEEE]**

(a) $v \propto x^{-1}$ (b) $v \propto x^{1/2}$
(c) $v \propto x$ (d) $v \propto x^{-1/2}$

40. Two positive charges of magnitude q are placed at the ends of a side 1 of a square of side $2a$. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is **[AIEEE]**

(a) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right)$
(b) zero
(c) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1+\frac{1}{\sqrt{5}}\right)$
(d) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{2}{\sqrt{5}}\right)$

41. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$, where r is the distance from the centre a and b are constants. Then, the charge density inside the ball is **[AIEEE]**

(a) $-6a\,\varepsilon_0 r$
(b) $-24\pi\,a\,\varepsilon_0$
(c) $-6a\,\varepsilon_0$
(d) $-24\pi\,a\,\varepsilon_0 r$

42. A charged particle q is shot towards another charged particle Q, which is fixed, with a speed v. It approaches Q upto a closest distance r and then returns. If q is shot with speed $2v$, the closest distance of approach would be **[Kerala CEE]**

(a) $\frac{r}{4}$ (b) $\frac{r}{2}$
(c) $2r$ (d) r
(e) $\frac{3}{2}r$

43. Two conducting spheres of radii 3 cm and 1 cm are separated by a distance of 10 cm in free space. If the spheres are charged to same potential of 10 V each, the force of repulsion between them is **[Kerala CEE]**

(a) $\frac{1}{3}\times10^{-9}$ N (b) $\frac{2}{9}\times10^{-9}$ N
(c) $\frac{1}{9}\times10^{-9}$ N (d) $\frac{4}{3}\times10^{-9}$ N
(e) $\frac{2}{3}\times10^{-9}$ N

44. A dipole of electric dipole moment p is placed in a uniform electric field of strength E. If θ is the angle between positive directions of p and E, then the potential energy of the electric dipole is largest, when θ is **[Kerala CEE]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) zero
(e) $\frac{2}{3}\pi$

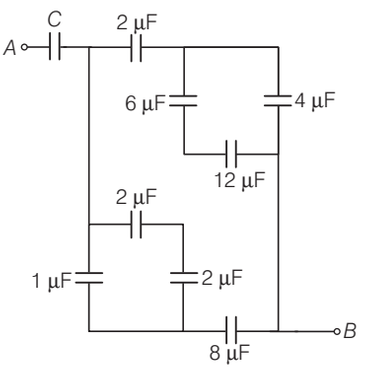
45. The electrostatic force of repulsion between two positively charged ions carrying equal charge is 3.7×10^{-9} N, when they are separated by a distance of 5 Å. How much electrons are missing from each ion? **[BITSAT]**

(a) 10 (b) 8
(c) 2 (d) 1

46. Electric charge is uniformly distributed along a long straight wire of radius 1 mm. The charge per cm length of the wire is Q coulomb. Another cylindrical surface of radius 50 cm and length 1 m symmetrically encloses the wire. The total electric flux passing through the cylindrical surface is **[Kerala CEE]**

(a) $\frac{Q}{\varepsilon_0}$ (b) $\frac{100\,Q}{\varepsilon_0}$ (c) $\frac{10\,Q}{\pi\varepsilon_0}$ (d) $\frac{100\,Q}{\pi\varepsilon_0}$
(e) $\frac{Q}{100\varepsilon_0}$

47. In the given network, the value of C, so that an equivalent capacitance between A and B is 3μF, is **[Karnataka CET]**



(a) 36 μF (b) 48 μF (c) $\frac{31}{5}$ μF (d) $\frac{1}{5}$ μF

48. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium, the value of q is **[WB JEE]**

(a) $-\frac{Q}{4}(1+2\sqrt{2})$

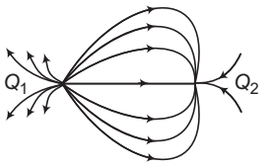
(b) $\frac{Q}{4}(1+2\sqrt{2})$

(c) $-\frac{Q}{2}(1+2\sqrt{2})$

(d) $\frac{Q}{2}(1+2\sqrt{2})$

2010

49. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that **[IIT JEE]**



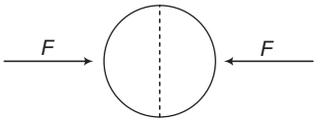
(a) $|Q_1| > |Q_2|$

(b) $|Q_1| < |Q|$

(c) at a finite distance to the left of Q_1 the electric field is zero.

(d) at a finite distance to the right of Q_2 the electric field is zero.

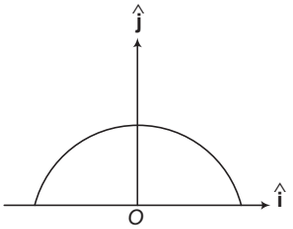
50. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemi-spherical shells, held together by pressing them with force F (see figure). F is proportional to **[IIT JEE]**



(a) $\frac{1}{\varepsilon_0}\sigma^2R^2$ (b) $\frac{1}{\varepsilon_0}\sigma^2R$

(c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ (d) $\frac{1}{\varepsilon_0}\cdot\frac{\sigma^2}{R^2}$

51. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field E at the centre O is **[AIEEE]**



(a) $\frac{q}{4\pi^2\varepsilon^0 r^2}(\hat{\mathbf{i}})$ (b) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{\mathbf{j}}$

(c) $\frac{-q}{2\pi^2\varepsilon_0 r^2}\hat{\mathbf{j}}$ (d) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{\mathbf{j}}$

52. Let there be a spherically symmetric charge distribution with charge density varying as $\psi(r)=\psi\left(\frac{5}{4}-\frac{r}{R}\right)$ upto $r=R$ and $\psi(r)=0$ for $r>R$, where r is the distance from the origin. The electric field at a distance $r(r<R)$ from the origin is given by **[AIEEE]**

(a) $\frac{4\pi\psi_0 r}{3\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (b) $\frac{\psi_0 r}{4\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$

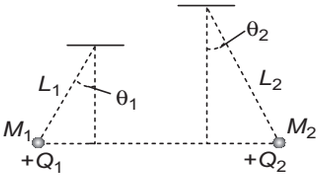
(c) $\frac{4\psi_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$ (d) $\frac{\psi_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$

53. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density $0.8\,\text{gcm}^{-3}$, the angle remains the same. If density of the material of the sphere is $1.6\ \text{gcm}^{-3}$, the dielectric constant of the liquid is **[AIEEE]**

(a) 4 (b) 3

(c) 2 (d) 1

54. Two small spheres of masses M_1 and M_2 are suspended by weightless insulating threads of lengths L_1 and L_2. The spheres carry charges Q_1 and Q_2, respectively. The spheres are suspended such that they are in level with one another and the threads are inclined to the vertical at angles of θ_1 and θ_2 as shown. Which one of the following conditions is essential, if $\theta_1=\theta_2$? **[Karnataka CET]**



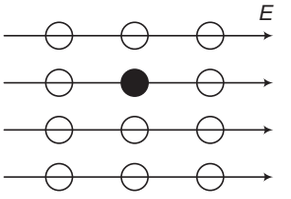
(a) $M_1 \neq M_2$, but $Q_1=Q_2$

(b) $M_1=M_2$

(c) $Q_1=Q_2$

(d) $L_1=L_2$

55. There is a uniform electric field of intensity E, which is as shown in the figure. How many labelled points have the same electric potential as the fully shaded point? **[Karnataka CET]**



(a) 2 (b) 3 (c) 8 (d) 11

56. Two identical conducting balls A and B have positive charges q_1 and q_2 respectively, but $q_1 \neq q_2$. The balls are brought together so that they touch each other and then kept in their original positions. The force between them is **[Karnataka CET]**

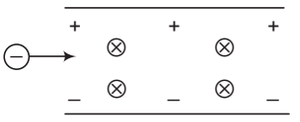
(a) less than that before the balls touched

(b) greater than that before the balls touched

(c) same as that before the balls touched

(d) zero

57. An electron enters the space between the plates of a charged capacitor as shown. The charge density on the plate is σ, electric intensity in the space between the plates is E. A uniform magnetic field B also exists in the space perpendicular to the direction of E.



The electron moves perpendicular to both **E** and **B** without any change in direction. The time taken by the electron to travel a distance l in the space is **[Karnataka CET]**

(a) $\frac{\sigma l}{\varepsilon_0 B}$ (b) $\frac{\sigma B}{\varepsilon_0 l}$

(c) $\frac{\varepsilon_0 lB}{\sigma}$ (d) $\frac{\varepsilon_0 l}{\sigma B}$

58. In a parallel plate capacitor, the capacity increases, if **[MHT CET]**

(a) area of the plate is decreased
(b) distance between the plates increases
(c) area of the plate is increased
(d) dielectric constant decrease

59. Capacity of a capacitor is 48 μF. When it is charged from 0.1 C to 0.5 C, change in the energy stored is **[MHT CET]**

(a) 2500 J
(b) 2.5×10^{-3} J
(c) 2.5×10^{6} J
(d) 2.42×10^{-2} J

60. The electric intensity outside a charged sphere of radius R at a distance $r(r>R)$ is **[MHT CET]**

(a) $\frac{\sigma R^2}{\varepsilon_0 r^2}$ (b) $\frac{\sigma r^2}{\varepsilon_0 R^2}$

(c) $\frac{\sigma r}{\varepsilon_0 R}$ (d) $\frac{\sigma R}{\varepsilon_0 r}$

Answers

Level 1

Objective Problems

1. (a)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (a)
11. (b)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (d)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (b)
21. (c)	**22.** (a)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (a)	**27.** (c)	**28.** (b)	**29.** (d)	**30.** (a)
31. (c)	**32.** (b)	**33.** (d)	**34.** (b)	**35.** (c)	**36.** (b)	**37.** (b)	**38.** (c)	**39.** (b)	**40.** (d)
41. (a)	**42.** (c)	**43.** (b)	**44.** (a)	**45.** (b)	**46.** (c)	**47.** (b)	**48.** (c)	**49.** (a)	**50.** (b)
51. (d)	**52.** (b)	**53.** (a)	**54.** (a)	**55.** (a)	**56.** (c)	**57.** (a)	**58.** (a)	**59.** (c)	**60.** (c)
61. (b)	**62.** (b)	**63.** (c)	**64.** (b)	**65.** (a)	**66.** (c)	**67.** (b)	**68.** (a)	**69.** (a)	**70.** (d)
71. (a,d)	**72.** (d)	**73.** (c)	**74.** (c)	**75.** (b)	**76.** (b)	**77.** (b)	**78.** (a)	**79.** (a)	**80.** (d)
81. (d)	**82.** (a)	**83.** (a)	**84.** (d)	**85.** (b)	**86.** (a)	**87.** (d)	**88.** (c)	**89.** (b)	**90.** (d)
91. (d)	**92.** (d)	**93.** (d)	**94.** (c)	**95.** (c)	**96.** (a)	**97.** (c)	**98.** (b)	**99.** (c)	**100.** (a)
101. (b,c)	**102.** (a)	**103.** (a)	**104.** (c)	**105.** (d)	**106.** (a)	**107.** (c)	**108.** (b)	**109.** (c)	**110.** (a)
111. (b)	**112.** (a)	**113.** (d)	**114.** (a)	**115.** (b)	**116.** (a)	**117.** (b)	**118.** (b)	**119.** (d)	**120.** (a)
121. (d)	**122.** (d)	**123.** (d)	**124.** (b)	**125.** (c)	**126.** (a)	**127.** (b)	**128.** (d)	**129.** (a)	**130.** (b)
131. (a)	**132.** (d)	**133.** (c)	**134.** (d)	**135.** (d)	**136.** (b)	**137.** (c)	**138.** (b)	**139.** (d)	**140.** (d)
141. (c)	**142.** (a)	**143.** (b)	**144.** (a)	**145.** (d)	**146.** (c)	**147.** (b)	**148.** (d)	**149.** (c)	**150.** (b)
151. (c)	**152.** (c)	**153.** (b)	**154.** (d)	**155.** (d)	**156.** (c)	**157.** (a)	**158.** (b)	**159.** (d)	**160.** (b)
161. (d)	**162.** (c)	**163.** (a)	**164.** (c)	**165.** (b)	**166.** (d)	**167.** (b,c)	**168.** (b)	**169.** (c)	**170.** (c)
171. (d)	**172.** (b)	**173.** (b)	**174.** (a)	**175.** (a)	**176.** (b)	**177.** (b)	**178.** (b)	**179.** (d)	**180.** (d)
181. (a)	**182.** (c)	**183.** (d)	**184.** (d)	**185.** (d)	**186.** (a)	**187.** (b,c)	**188.** (d)	**189.** (c)	**190.** (c)
191. (c)	**192.** (b)	**193.** (b)	**194.** (a)	**195.** (c)	**196.** (b)	**197.** (d)	**198.** (b)	**199.** (b)	**200.** (a)
201. (c)	**202.** (b)	**203.** (a)	**204.** (d)	**205.** (d)	**206.** (a)	**207.** (c)	**208.** (b)	**209.** (b)	**210.** (a)
211. (b)	**212.** (a)	**213.** (d)	**214.** (b)						

Level 2

Only One Correct Option

1. (a) **2.** (b) **3.** (d) **4.** (c) **5.** (b) **6.** (b) **7.** (c) **8.** (c) **9.** (d) **10.** (b)
11. (d) **12.** (b) **13.** (a) **14.** (d) **15.** (b) **16.** (a) **17.** (d) **18.** (a) **19.** (c) **20.** (d)
21. (d) **22.** (a) **23.** (c) **24.** (a) **25.** (a) **26.** (c) **27.** (c) **28.** (a) **29.** (c) **30.** (a)
31. (d) **32.** (b) **33.** (b) **34.** (c) **35.** (c) **36.** (c) **37.** (b) **38.** (c) **39.** (b) **40.** (a)
41. (a) **42.** (a) **43.** (d) **44.** (b) **45.** (c) **46.** (a) **47.** (a) **48.** (d) **49.** (c) **50.** (c)
51. (c)

More than One Correct Options

1. (a,d) **2.** (a,b,c) **3.** (a,c,d) **4.** (all) **5.** (a,b,c) **6.** (a,c) **7.** (c,d) **8.** (a,b,d) **9.** (b,c) **10.** (b,c)
11. (a,b,c) **12.** (b,c,d) **13.** (b,c) **14.** (a,b,d) **15.** (b,c) **16.** (a,d) **17.** (a,b,d) **18.** (a,b,c) **19.** (all) **20.** (b)
21. (c,d) **22.** (b,d) **23.** (c,d) **24.** (c,d) **25.** (a,c) **26.** (a,b,c) **27.** (a,b,c) **28.** (c) **29.** (b,c) **30.** (a,d)
31. (a,b) **32.** (c,d)

Comprehension Based Questions

1. (c) **2.** (c) **3.** (a) **4.** (a) **5.** (b) **6.** (c) **7.** (a) **8.** (d) **9.** (b) **10.** (c)
11. (b) **12.** (a) **13.** (b) **14.** (a)

Assertion and Reason

1. (b) **2.** (c) **3.** (d) **4.** (d) **5.** (a,b) **6.** (d) **7.** (d) **8.** (a) **9.** (c) **10.** (c)
11. (c) **12.** (a,b) **13.** (a,b) **14.** (a) **15.** (c) **16.** (d) **17.** (c) **18.** (e) **19.** (c) **20.** (a)

Match the Columns

1. (A→q,r; B→p,r; C→p,r; D→q,r) **2.** (A→p,q; B→s; C→s; D→r) **3.** (A→p,q,r; B→p,q,r; C→p,q,r; D→q)
4. (A→s; B→q; C→p; D→s) **5.** (A→s; B→p; C→p; D→s)

Entrance Gallery

1. (d) **2.** (c) **3.** (a) **4.** (d) **5.** (a) **6.** (b) **7.** (c) **8.** (c) **9.** (d) **10.** (c)
11. (c) **12.** (d) **13.** (d) **14.** (b) **15.** (a) **16.** (c,d) **17.** (b) **18.** (b) **19.** (c) **20.** (a)
21. (d) **22.** (a) **23.** (b) **24.** (a,c) **25.** (d) **26.** (a) **27.** (a,b,c) **28.** (a) **29.** (c) **30.** (b)
31. (d) **32.** (b) **33.** (a) **34.** (c) **35.** (d) **36.** (a,b,c,d) **37.** (c,d) **38.** (d) **39.** (d) **40.** (a)
41. (c) **42.** (a) **43.** (a) **44.** (c) **45.** (c) **46.** (b) **47.** (b) **48.** (b) **49.** (a) **50.** (a)
51. (c) **52.** (b) **53.** (c) **54.** (b) **55.** (b) **56.** (b) **57.** (c) **58.** (c) **59.** (a) **60.** (a)

Solutions

Level 1 : Objective Problems

1. $q = ne = 10^{14} \times 1.6 \times 10^{-19}$

$\therefore \quad q = 1.6 \times 10^{-5}\ \text{C} = 16\,\mu\text{C}$

2. As conductor has positive charge. So, there is a deficiency of electrons.

$\therefore$ Number of electrons $= \dfrac{14.4 \times 10^{-19}}{1.6 \times 10^{-19}} = 9$

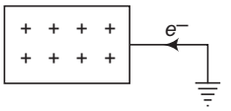
3. $q = ne$

$\therefore \quad q = +2e = 2 \times 1.6 \times 10^{-19}$

$= +3.2 \times 10^{-19}\ \text{C}$

4. When we rub glass rod with silk, excess electrons are transferred from glass to silk. So, glass rod becomes positive and silk becomes negative.

5.



When positively charged body connected to earth, electrons flow from earth to body and body becomes neutral.

6. $F_1 = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{(1 \times 10^{-6})(5 \times 10^{-6})}{r^2}$

(if distance between them is r)

also, $\quad F_2 = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{(5 \times 10^{-6})(1 \times 10^{-6})}{r^2}$

$\therefore \quad \dfrac{F_1}{F_2} = \dfrac{1}{1}$



But directions of F_1 and F_2 are different.

7. $F_g = \dfrac{G(m_e)(m_e)}{r^2}$

Also, $\quad F_e = \dfrac{1}{4\pi\varepsilon_0} \dfrac{(e)(e)}{r^2}$

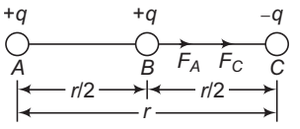
$\therefore \quad \dfrac{F_g}{F_e} = \dfrac{G(m_e)^2}{\left(\dfrac{1}{4\pi\varepsilon_0}\right)e^2}$

$= \dfrac{6.67 \times 10^{-11} \times (9.1 \times 10^{-31})^2}{9 \times 10^9 \times (1.6 \times 10^{-19})^2}$

$= 2.39 \times 10^{-43}$

So, ratio of F_g / F_e is of order 10^{-43}.

8. Situation is shown in figure.



Force between A and C

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r^2}$$

When sphere B is kept at the mid-point of line joining A and C, then net force on B is

$$F_{\text{net}} = F_A + F_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(r/2)^2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(r/2)^2}$$

$$= 8 \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r^2} = 8F$$

9. According to Coulomb's law,

$$F \propto \frac{1}{r^2} \quad \Rightarrow \frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)^2$$

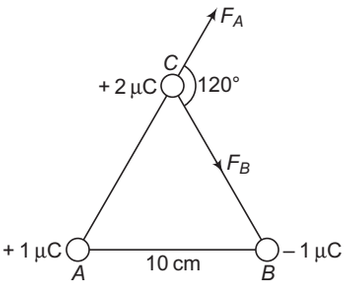
$\therefore \quad \dfrac{5}{F_2} = \left(\dfrac{0.04}{0.06}\right)^2$

$\therefore \quad F_2 = 11.25\ \text{N}$

10. Dielectric constant, $K = \dfrac{F_{\text{in medium}}}{F_{\text{in air}}}$

So, $F' = \dfrac{F}{2}$

11. F_A = Force on charge at C due to charge at A



$$= 9 \times 10^9 \times \frac{10^{-6} \times 2 \times 10^{-6}}{(10 \times 10^{-2})^2} = 1.8\ \text{N}$$

F_B = Force on point C due to charge at B

$$= 9 \times 10^9 \times \frac{10^{-6} \times 2 \times 10^{-6}}{(0.1)^2} = 1.8\ \text{N}$$

$\therefore$ Net force on C

$$F_{\text{net}} = \sqrt{(F_A)^2 + (F_B)^2 + 2F_A F_B \cos 120^\circ}$$

$$= \sqrt{(1.8)^2 + (1.8)^2 + 2(1.8)(1.8)(-1/2)} = 1.8\ \text{N}$$

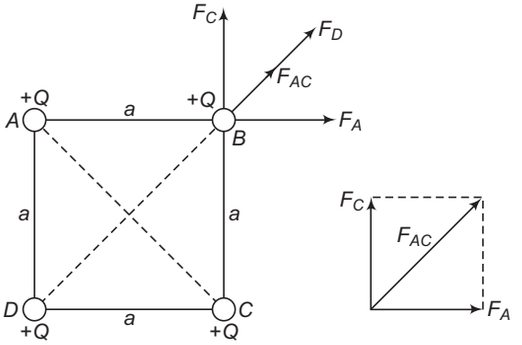
12. $q_1' = q_2' = \left(\dfrac{q_1 + q_2}{2}\right) = \left(\dfrac{10 - 20}{2}\right) = -5\,\mu\text{C}$

$\therefore \quad \dfrac{F_1}{F_2} = \dfrac{q_1 q_2}{q_1' q_2'}$

$= \dfrac{(10)(-20)}{(-5) \times (-5)}$

$= -\dfrac{8}{1}$

13.



Here, $$F_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a^2}$$

$$F_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a^2}$$

Net force on B,

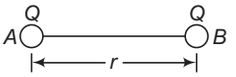
$$F_{\text{net}} = F_{AC} + F_D$$

$$= \sqrt{F_A^2 + F_C^2} + F_D$$

$$= \sqrt{\left\{\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a^2} + \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a^2}\right\}} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{(a\sqrt{2})^2}$$

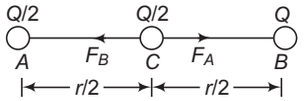
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{a^2}\left(\frac{1+2\sqrt{2}}{2}\right)$$

14. In Ist case,



$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{r^2} \qquad \ldots \text{(i)}$$

In IInd case,



When sphere C is touched to A, then equal charge $Q/2$ distributes on A and C.

$$\therefore \quad F_A = \frac{1}{4\pi\varepsilon_0}\frac{(Q/2)^2}{(r/2)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{r^2}$$

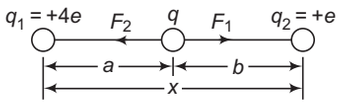
$$F_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q)(Q/2)}{(r/2)^2} = 2\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{r^2}$$

$\therefore$ Net force on C,

$$F_{\text{net}} = F_B - F_A$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{r^2} = F$$

15. For equilibrium of charge q, forces on chagre q due to charges q_1 and q_2 should be equal.



$$\therefore \quad F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(4e)(q)}{a^2}$$

and $$F_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(e)(q)}{b^2}$$

Also, $$x = a + b$$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{(4e)(q)}{a^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(e)q}{(x-a)^2}$$

or $$\frac{4}{a^2} = \frac{1}{(x-a)^2}$$

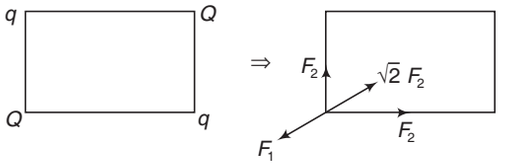
or $$\frac{2}{a} = \frac{1}{x-a}$$

or $$2x - 2a = a$$

$$\therefore \quad a = \frac{2x}{3}$$

$$\Rightarrow \quad b = x/3$$

16. Q and q should be of opposite sign.



$$\sqrt{2}F_2 = F_1$$

$$\therefore \quad \sqrt{2}\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q(q)}{a^2}\right] = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\cdot Q}{(\sqrt{2}a)^2},$$

$$Q = -2\sqrt{2}q$$

17. Net force on charge Q should be zero. Q and q should be unlike in nature.

$$\therefore \quad \frac{K\cdot Qq}{r^2} = \frac{KQ\cdot Q}{(2r)^2} \quad \text{or} \quad q = \frac{-Q}{4}$$

18. $12 = K\,(2)\,(6), \quad F = K\,(2)\,(2)$

From these two equations,

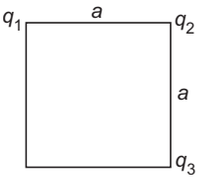
$$F = 4\text{ N} \qquad \text{(attractive)}$$

20.

$$F_{12} = \frac{K(q^2)}{a^2}$$

$$F_{13} = \frac{K(q^2)}{(\sqrt{2}a)^2}$$

$$\therefore \quad \frac{F_{12}}{F_{13}} = \frac{2}{1}$$



21. $q_1 q_2 = F$ …(i)

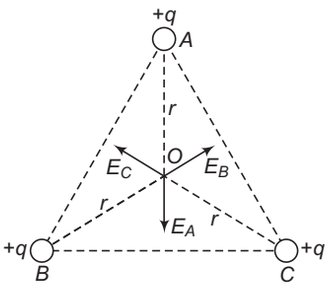
$$\left(\frac{q_1 - q_2}{2}\right)\left(\frac{q_1 - q_2}{2}\right) = \frac{F}{3} \qquad \ldots\text{(ii)}$$

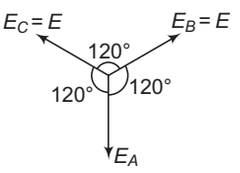
Solving these equations, we get

$$\frac{q_1}{q_2} = \frac{3}{1}$$

23. Electric lines are closer at A.

24. The resultant of E_B and E_C is





$$= \sqrt{E^2 + E^2 + 2E \cdot E\cos 120^\circ}$$

$$= \sqrt{2E^2 - 2E^2 \cdot \left(-\frac{1}{2}\right)} = E$$



Now, situation is shown in figure.

Here, E_A and E_{BC} are equal and opposite so, they cancel out. So, resultant electric field due to E_A, E_B and E_C is zero.

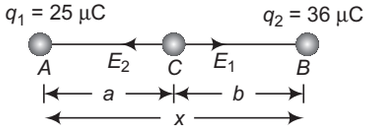
25. Electric field on the surface of a conducting sphere is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

$$\therefore \quad q = Er^2 \cdot 4\pi\varepsilon_0$$

$$= \frac{3\times10^6 \times (2.5)^2}{9\times10^9} = 2.0831\times10^{-3}\ \text{C}$$

26. Let electric field is zero at point O in the figure.



$$\therefore \quad E_1 = E_2$$

$$\therefore \quad E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{a^2}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{b^2}$$

Also, $x = a + b$

or $11 = a + b$

$\therefore$ $b = 11 - a$

Now, $$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{a^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{(11-a)^2}$$

$$\therefore \quad \frac{q_1}{q_2} = \frac{a^2}{(11-a)^2}$$

or $$\sqrt{\frac{q_1}{q_2}} = \frac{a}{11-a}$$

or $$\sqrt{\frac{25}{36}} = \frac{a}{11-a}$$

or $$\frac{5}{6} = \frac{a}{11-a}$$

or $6a = 55 - 5a$

$\therefore$ $a = 5\,\text{cm}$

So, intensity will be zero at a distance of 5 cm from 25 μC.

28. Unit of E in SI system $\left(E = \frac{F}{q_0}\right)$ = newton/coulomb

As, $$E = -\frac{dV}{dr}$$

So, unit of E is also volt/metre

Also, $q = CV$

$\therefore$ $q = CEd$ $(\because V = Ed)$

$$E = \frac{q}{Cd} = \frac{qV}{qd} = \frac{K}{qd}$$

$$= \frac{\text{Joule}}{\text{Coulomb - metre}}$$

while J/C is unit of electric potential.

29. Electric line of force are perpendicular to the surface of a conductor. Inside the sphere no lines are present.

30. Electric field = force per unit charge = $\frac{F}{q_0}$

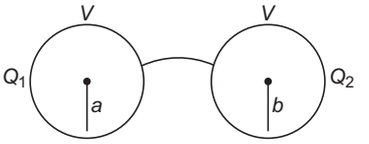
This gives unit of electric field as newton/coulomb.

31. At point A and C, electric field lines are dense and equally spaced, so $E_A = E_C$

While at B, they are far apart.

$\therefore$ $E_A = E_C > E_B$

32. When spheres are joined by wire, then they have same potential.



$$\therefore \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1}{a} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_2}{b}$$

$$\therefore \quad \frac{Q_1}{Q_2} = \frac{a}{b}$$

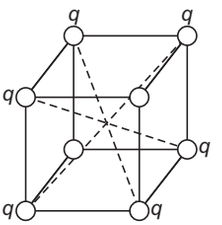
Also, electric field due to two spheres,

$$\frac{E_1}{E_2} = \frac{\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1}{a^2}}{\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_2}{b^2}} = \frac{Q_1}{Q_2} \times \frac{b^2}{a^2}$$

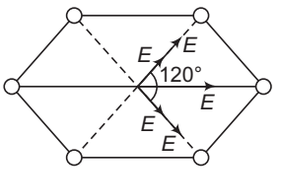
But, $$\frac{Q_1}{Q_2} = \frac{a}{b}$$

$\therefore$ $$\frac{E_1}{E_2} = \frac{a}{b} \times \frac{b^2}{a^2} = \frac{b}{a}$$

33. Due to symmetry of charges, electric field intensity is equal and opposite due to charges. So, they cancel out. Hence, net electric field due to charge distribution at centre of cube is zero.



34. In Fig. (1), (3) and (4) net electric field is zero, because electric field at a point due to positive charge acts away from the charge and due to negative charge it acts towards the charge for Fig. (2) net electric field is not zero.

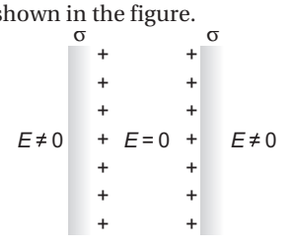


Here, net electric field in Fig. (2) is

$$= \sqrt{(2E)^2 + (2E)^2 + (2E)(2E) \cdot 2\cos 120^\circ}$$

$$= 2E$$

35. Situation is shown in the figure.



Electric field between sheets

$$= \frac{2}{2\varepsilon_0}(\sigma - \sigma) = 0$$

36. $\mathbf{E}_q + \mathbf{E}_{3q}$ is along PA

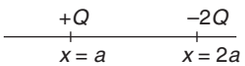
$\mathbf{E}_{2q} + \mathbf{E}_{4q}$ is along PB

$\therefore$ $\mathbf{E}_{net}$ is along CB.

37. Electric field due to Q_2 and Q_3 cancel each other.

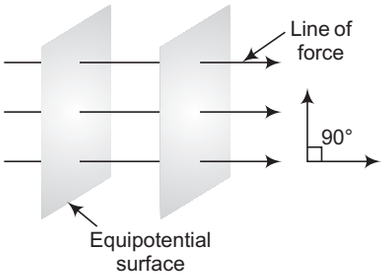
38. Three vectors of equal magnitude at 120° with the next.

39. Electric field can be zero only at a point, where $x < a$.



40. It will be $-\frac{\mathbf{E}}{2}$.

42. Angle between equipotential surface and line of force is 90°



43. For charged spherical conductor. Potential inside the sphere is same as that on its surface

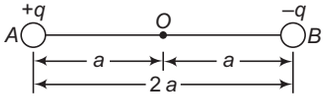
$$V_{in} = V_{surface} = \frac{q}{10}\text{stat-volt}$$

$$V_{out} = \frac{q}{15}\text{stat-volt}$$

$\therefore$ $\frac{V_{out}}{V_{in}} = \frac{2}{3}$ $\therefore$ $V_{out} = \frac{2}{3}$

$$V_{in} = \frac{2}{3}\text{V}$$

44. Potential at O due to charge at A,



$\therefore$ $V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a}$

Potential at O due to charge at B,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{a}$$

Potential at mid-point O,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{a} = 0$$

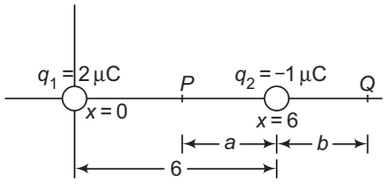
45. As electric potential of spheres are same, i.e.

$$V_A = V_B$$

$\therefore$ $\frac{\sigma_A \cdot a}{\varepsilon_0} = \frac{\sigma_B b}{\varepsilon_0}$

or $\frac{\sigma_A}{\sigma_B} = \frac{b}{a}$

46. Let potential will be zero at two points P and Q, then



$$\frac{1}{4\pi\varepsilon_0}\times\left[\frac{2\times10^{-6}}{(6-a)} + \frac{(-1\times10^{-6})}{a}\right] = 0$$

$\therefore$ $a = 2$

So, distance of P from origin,

$$x = 6 - 2 = 4$$

At external point Q,

$$\frac{1}{4\pi\varepsilon_0}\times\left[\frac{2\times10^{-6}}{(6+b)} + \frac{(-1\times10^{-6})}{b}\right] = 0$$

$\therefore$ $b = 6$

So, distance of Q from origin, $x = 6 + 6 = 12$

47. Let radius of big drop is R and radius of small drops is r,

$\therefore$ Volume of big drop $= 8 \times$ volume of small drop

or $\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$

or $R = 2r$

Potential of big drop,

$V = \frac{Q}{C} = \frac{8q}{(8)^{1/3}C}$ (q is charge on small drop)

$\therefore$ $V = (8)^{2/3}V$

$\therefore$ $\frac{V_{big}}{V_{small}} = (8)^{2/3} = \frac{4}{1}$

48. $8 \times$ Volume of small drop = Volume of big drop

$8\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$, $R = 2r$, $Q = 8q$

Further, $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} = 50\text{ V}$

and $V' = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{8q}{2r}$

$= 4 \times 50 = 200\text{ V}$

49. Inside a sphere, potential remains uniform.

50. $V = \frac{\text{Net charge}}{\text{Net capacity}} = \frac{q_1 + q_2}{4\pi\varepsilon_0(r_1 + r_2)}$

52. $E_0 = -\frac{dV}{dx}$

or $dV = -E_0\,dx$

Integrating on the both sides,

$$\int dV = -\int E_0\,dx$$

$$V_x = -xE_0$$

53. Force on charged particle in electric field E is given by

$$F = qE = q\left(\frac{V}{d}\right)$$

$$\therefore \quad 5000 = \frac{5 \times V}{10^{-2}}$$

$$\therefore \quad V = 10\,\text{V}$$

54. Electric field is negative of potential gradient, i.e.

$$E = -\frac{dV}{dx} = -\frac{d}{dx}(5x^2 + 10x - 9)$$

$$= -10x - 10$$

$$\therefore \quad (E)_{x=1} = -10 \times 1 - 10$$

$$= -20\,\text{V/m}$$

55. Electric field is given by

$$E = -\frac{\Delta V}{\Delta r} = \frac{30 - (-10)}{2 \times 10^{-2}}$$

$$= 2000\,\text{V/m}$$

57. $F = qE = q\left(\frac{V}{d}\right)$

$$\therefore \quad V = \frac{F \cdot d}{q} = \frac{(3000)\,(10^{-2})}{3} = 10\,\text{V}$$

58. $E = \frac{V}{r}$

$$\therefore \quad r = \frac{V}{E} = \frac{3000}{500} = 6\,\text{m}$$

60. A, B, C, D and E lie on equipotential surface as on sphere at surface potential is same. So,

$$W_{AB} = W_{AC} = W_{AD} = W_{AE} = q(V_f - V_i) = \text{zero}$$

61. Electrostatic energy density is energy stored per unit volume

$$u_e = \frac{\text{Electric potential energy}}{\text{Volume}}$$

$$= \frac{1}{2}\varepsilon_0 E^2$$

$$\therefore \quad u_e \propto E^2$$

62. When charge particle enters in a potential field, then

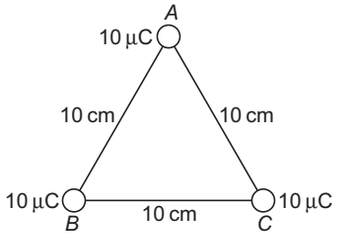
$$\frac{1}{2}mv^2 = qV$$

$$\therefore \quad v = \sqrt{\left(\frac{2qV}{m}\right)}$$

$$v \propto \sqrt{q}$$

$$\therefore \quad \frac{v_A}{v_B} = \sqrt{\frac{q}{4q}} = \frac{1}{2}$$

63.



$$U_{AB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(10)(10) \times 10^{-12}}{10 \times 10^{-2}}$$

$$U_{BC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(10)(10) \times 10^{-12}}{10 \times 10^{-2}}$$

$$U_{AC} = \frac{1}{4\pi\varepsilon_0} \frac{(10)(10) \times 10^{-12}}{10 \times 10^{-2}}$$

$$\therefore \quad U_{\text{Total}} = U_{AB} + U_{BC} + U_{CA}$$

$$= \frac{3}{4\pi\varepsilon_0}\left[\frac{100 \times 10^{-12} \times 100}{10}\right]$$

$$= 27\,\text{J}$$

64. When charge particle enters in a uniform electric field, then force on charged particle is

$$F = qE$$

Also, $F = ma$

$$\therefore \quad ma = qE$$

or $$a = \frac{qE}{m} = \frac{3 \times 10^{-3} \times 80}{20 \times 10^{-3}}$$

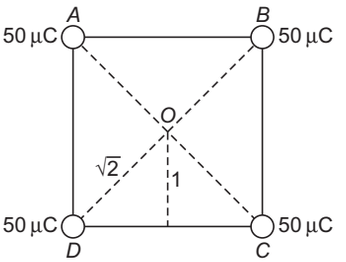
$$= 12\ \text{m/s}^2$$

So, from equations of motion

$$v = u + at = 20 + 12 \times 3$$

$$= 56\ \text{m/s}$$

65.



Now, $$V_A = V_B = V_C = V_D$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{(50 \times 10^{-6})}{(\sqrt{2})}$$

$\therefore$ Potential at the centre of square

$$V_o = 4 \times \left[\frac{9 \times 10^9 \times 50 \times 10^{-6}}{\sqrt{2}}\right]$$

$$= 90\sqrt{2} \times 10^4\ \text{V}$$

Work done in bringing a charge ($q = 50\,\mu\text{C}$) from ∞ to centre of the square is

$$\therefore \quad W = qV_0$$

$$= 50 \times 10^{-6} \times 90\sqrt{2} \times 10^4$$

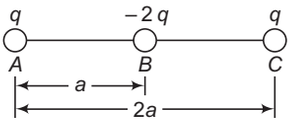
$$= 64\,\text{J}$$

66.

$$U_{AB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(q)(-2q)}{a}$$

$$U_{BC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(-2q)(q)}{a}$$

$$U_{CA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(q)(q)}{2a}$$



$$\therefore \quad U_{\text{system}} = \frac{1}{4\pi\varepsilon_0} \cdot \left[\frac{-2q^2}{a} - \frac{2q^2}{a} + \frac{q^2}{2a}\right]$$

$$U_{\text{system}} = \frac{-7q^2}{8\pi\varepsilon_0 a}$$

67. When α-particle is accelerated through a potential difference V, then kinetic energy of α-particle

$$K = qV$$

$$= (2e)V \text{ J} = 2\times1.6\times10^{-19}\times10^{6} \text{ J}$$

$$= \frac{2\times1.6\times10^{-19}\times10^{6}}{1.6\times10^{-19}} \text{eV}$$

$$= 2 \text{ MeV}$$

68. $\because$ Potential difference $= \dfrac{\text{Work done}}{\text{Charge}}$

$\therefore \quad W = qV$

$20 = 5\times V$

$\therefore \quad V = 2\text{V}$

69. Kinetic energy gained by α-particle



$$\text{KE} = q\cdot\Delta V = q(V_1 - V_2)$$

$$= 2e(V_1 - V_2)$$

$$= 2\times1.6\times10^{-19}\,(70-50)$$

$$= 40 \text{ eV}$$

70. $\therefore \quad p = \sqrt{2mK}$

Also, $K = qV$

$\therefore \quad p = \sqrt{2mqV}$

$\therefore \quad \dfrac{p_e}{p_\alpha} = \sqrt{\dfrac{m_e q_e}{m_\alpha q_\alpha}} = \sqrt{\dfrac{m_e}{2m_\alpha}} \qquad (\because q_\alpha = 2q_e)$

71. A negative charge when moves from higher potential to lower potential its velocity increases.

72. $\dfrac{1}{2}mv^2 = \dfrac{q_1 q_2}{4\pi\varepsilon_0}\left(\dfrac{1}{r_i} - \dfrac{1}{r_f}\right)$

$$\frac{1}{2}\times2\times10^{-3}\times v^2 = (10^{-9})\,(9\times10^{9})\,(0.9)$$

$$v = \sqrt{8.1\times10^{3}} \text{ m/s}$$

$$= 90 \text{ m/s}$$

73. From $s = \dfrac{1}{2}at^2 = \dfrac{1}{2}\dfrac{F}{m}\cdot t^2$

$$t = \sqrt{\frac{2\text{ms}}{F}}$$

$t \propto \sqrt{m} \quad \therefore \quad \dfrac{t_1}{t_2} = \sqrt{\dfrac{m_e}{m_p}}$

74. $\text{KE} = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m(at)^2 = \dfrac{1}{2}m\left(\dfrac{qE}{m}\right)^2 t^2$

$$\text{KE} = \frac{q^2E^2t^2}{2m}$$

75. $\text{KE} = \dfrac{q^2E^2t^2}{2m}$

76. $W = U_i - U_f = 6\left[\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q\cdot q}{r}\right] - 0 = \dfrac{6q^2}{4\pi\varepsilon_0 r}$

77. $W = U_f - U_i$

Here, $U_f = 0$

$\therefore \quad W = -U_i$

$$= -\frac{(9\times10^{9})}{0.1}\,[(1)(-2) + (1)(4) + (-2)(4)]\times10^{-12}$$

$$= 0.54 \text{ J}$$

78. $W = U_f - U_i$

$$= (9\times10^{9})\,(10^{-12})\,(12)\,(8)\left[\frac{1}{0.06} - \frac{1}{0.1}\right]$$

$$= 5.8 \text{ J}$$

79. $2Qq + q^2 = 0$

So, either $q = 0$ or $Q = -\dfrac{q}{2}$

80. $v = at = \left(\dfrac{qE}{m}\right)t$

$$v \propto \frac{q}{m}$$

or $\quad \text{KE} \propto \dfrac{q^2}{m^2}$

$\therefore \quad \dfrac{K_1}{K_2} = \left(\dfrac{2}{1/2}\right)^2 = 16$

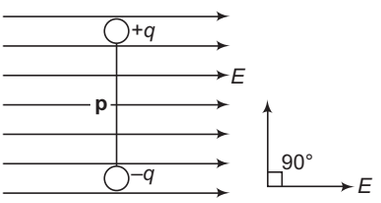
83. $W_{A\to B} = U_B - U_A = q\,(V_B - V_A)$

$$V_B - V_A = \frac{W_{A\to B}}{q} = \frac{10\times10^{3}}{5\times10^{-6}} = 2\text{kV}$$

87. Here, $\theta_1 = 90°$

$\theta_2 = 90° + 180°$

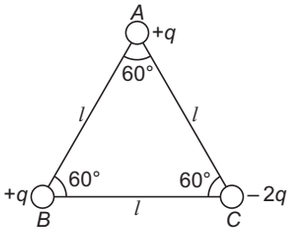
$= 270°$



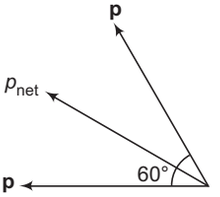
$\therefore \quad \text{Work done} = \displaystyle\int_{\theta_1=90}^{\theta_2=270} pE\sin\theta\, d\theta$

$$= [-pE\cos\theta]_{90}^{270} = 0$$

88.



The direction of dipole moment is shown in figure.



Net electric dipole moment

$$p_{\text{net}} = \sqrt{p^2 + p^2 + 2pp\cos60°}$$

$$= \sqrt{3}p$$

$$= \sqrt{3}\,ql \qquad (\because p = ql)$$

90. Maximum torque is given by

$$\tau_{max} = pE \qquad (\because \sin\theta = 1)$$
$$= (ql)E$$
$$= (4\times10^{-8}\times2\times10^{-4})\times4\times10^{8}$$
$$= 32\times10^{-4}\ \text{N-m}$$

If $\theta = 180°$, then

$$W = pE(1-\cos180°)$$
$$= pE\,[1-(-1)]$$
$$W = 2pE = 2\times32\times10^{-4}$$
$$= 64\times10^{-4}\ \text{J}$$

91. On equatorial line electric field is given by

$$E_{equal} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}$$

On axial line,

$$E_{axial} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3}$$

$\therefore$ $E_{axial} = 2E_{equal}$

But since their directions are opposite. Hence,

$$E_a = -2E_e$$

92. Potential at a point due to electric dipole

$$V = \frac{p\cos\theta}{r^2}$$

If $\theta = 0°$, then V = max

If $\theta = 180°$, then V = min

93. Work done in rotating the dipole

$$W = pE(\cos\theta_1 - \cos\theta_2)$$
$$= pE(\cos0° - \cos180°)$$
$$= pE\,[1-(-1)] = 2pE$$

94. If dipole is rotated through an angle of 90° about its perpendicular axis, then given point comes on equatorial line. So, field becomes half of previous value, i.e. $E/2$.

95. $U = -pE\cos\theta$, U is minimum at $\theta = 0°$.

96. $E_0 = \frac{2KP}{r^3}$ (along **P**)

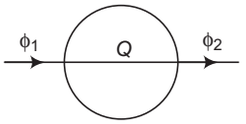
$$E_\perp = \frac{KP}{(2r)^3} \qquad \text{(opposite to } \mathbf{P}\text{)}$$
$$E_\perp = \frac{KP}{8r^3}$$
$$E_\perp = \frac{E_0}{16}$$

97. Electric flux, $\phi = EA$

$\therefore$ SI unit of electric flux

$$= \frac{\text{Newton}}{\text{Coulomb}}\times\text{metre}^2$$
$$= \frac{(\text{Newton-metre})\ \text{metre}}{\text{Coulomb}}$$
$$= \frac{\text{Joule}\times\text{metre}}{\text{Coulomb}} = \text{Volt-metre}$$

98. From Gauss's law,



$$\text{Net flux} = \frac{\text{Total charge enclosed}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\times Q$$

$\therefore$ $Q = \varepsilon_0(\phi_2 - \phi_1)$

100. $\frac{\phi_{max}}{\phi_{min}} = \frac{q_{max}}{q_{min}}$

102. Flux from one face $= \frac{1}{6}$ (Total flux)

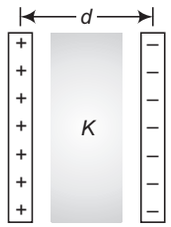
$$= \frac{1}{6}\left(\frac{q}{\varepsilon_0}\right) = \frac{1}{3\varepsilon_0}$$

103. Same flux will pass through two concentric spherical shells if charge is kept at centre.

105. Net electric flux from a closed surface in uniform electric field is always zero.

106. $\phi = \mathbf{E}\cdot\mathbf{S} = 40$ unit

107. Potential difference between plates A and B



V = PD in air + PD in medium

$$V = \frac{\sigma}{\varepsilon_0}(d-t) + \frac{\sigma}{K\varepsilon_0}t$$

$\therefore$ $$V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{K}\right]$$
$$= \frac{Q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right) \qquad \left(\because \sigma = \frac{Q}{A}\right)$$

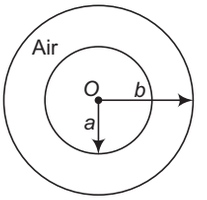
$\therefore$ $$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right)}$$
$$= \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$
$$= \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

108. According to question,
Capacity of spherical condenser = Capacity of parallel plate capacitor

$\therefore$ $$4\pi\varepsilon_0 r = \frac{\varepsilon_0 A}{d}$$

$\therefore$ $$d = \frac{A}{4\pi r} = \frac{\pi R^2}{4\pi r} = \frac{\pi(20\times10^{-3})^2}{4\pi\times1}$$
$$= 0.1\ \text{mm}$$

109. Capacity of spherical condenser when outer sphere is earthed



$$C_1 = 4\pi\varepsilon_0\cdot\frac{ab}{b-a}$$

Capacity of spherical condenser when inner sphere is earthed

$$C_2 = 4\pi\varepsilon_0 b + \frac{4\pi\varepsilon_0 ab}{b-a}$$

$\therefore$ Difference in their capacity $= C_2 - C_1 = 4\pi\varepsilon_0 b$

110. Three capacitors are in series their, resultant capacity is given by

$$\frac{1}{C_s} = \frac{1}{\left(\frac{\varepsilon_0 K_1 A}{d_1}\right)} + \frac{1}{\left(\frac{\varepsilon_0 K_2 A}{d_2}\right)} + \frac{1}{\left(\frac{\varepsilon_0 K_3 A}{d_3}\right)}$$

or $$\frac{1}{C_s} = \frac{d_1}{\varepsilon_0 K_1 A} + \frac{d_2}{\varepsilon_0 K_2 A} + \frac{d_3}{\varepsilon_0 K_3 A}$$

$$\frac{1}{C_s} = \frac{1}{\varepsilon_0 A}\left(\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}\right)$$

$\therefore$ $$C_s = \frac{\varepsilon_0 A}{\left(\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}\right)}$$

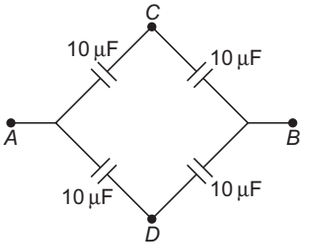
111. $$\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{(d + \Delta d) - t + \frac{t}{K}}$$

or $$K = \frac{t}{t - \Delta d}$$

112. The given figure is equivalent to two identical capacitors in parallel combination

$\therefore$ $$C = \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{d} = \frac{2\varepsilon_0 A}{d}$$

113. Given circuit is balanced Wheatstone bridge circuit.



There is no current in branch *CD*.

So, equivalent capacitance between *AB*,

$$C_{AB} = 5 + 5$$
$$= 10\ \mu F$$

114. When capacitors are connected in series, then

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
$$= \frac{1}{3} + \frac{1}{9} + \frac{1}{18}$$
$$\frac{1}{C_s} = \frac{1}{2}$$

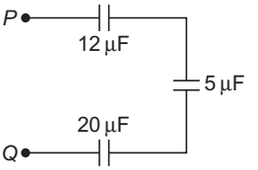
$\Rightarrow$ $$C_s = 2\ \mu F$$

When capacitors are joined in parallel, then

$$C_p = 3 + 9 + 18$$
$$= 30\ \mu F$$

$\therefore$ $$\frac{C_s}{C_p} = \frac{2}{30} = \frac{1}{15}$$

115. In circuit, condenser of capacity 2 μF and 3 μF are in parallel. Their resultant capacity is 5 μF.

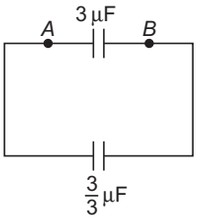


Now, capacitor 12 μF, 5 μF and 20 μF are in series. So, their resultant capacity

$$\frac{1}{C} = \frac{1}{5} + \frac{1}{20} + \frac{1}{12} = \frac{1}{3}$$

$\therefore$ $$C = 3\mu F$$

116. Capacitance between *A* and *B*

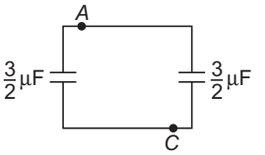


$$C_{AB} = 3 + 1$$
$$= 4\mu F$$

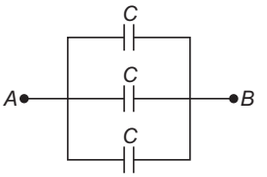
Capacitance between *A* and *C*

$$C_{AC} = \frac{3}{2} + \frac{3}{2} = 3\mu F$$

$\therefore$ $$\frac{C_{AB}}{C_{AC}} = \frac{4}{3}$$



117. Three capacitors are in parallel. So, their equivalent capacity

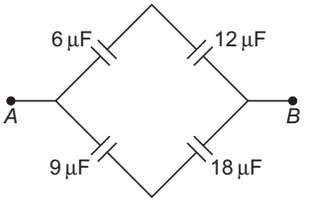


$$C_p = C + C + C = 3C$$

118. Given plates are equivalent to 3 identical capacitors in parallel combination. Hence, equivalent capacitance

$$C_p = C + C + C = 3C = 3\frac{\varepsilon_0 A}{d}$$

119. Given circuit is balanced Wheatstone bridge circuit.

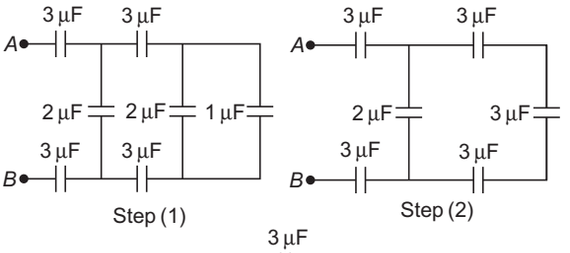


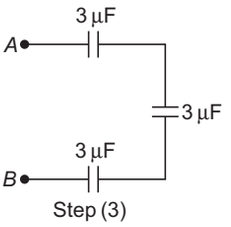
Now, capacitor of capacity 6μF,12μF are in series and 9μF,18μF are also in series.

$\therefore$ Equivalent capacitance between *A* and *B* is

$$C_{AB} = 4 + 6$$
$$= 10\ \mu F$$

120. The given circuit can be reduced in following manner

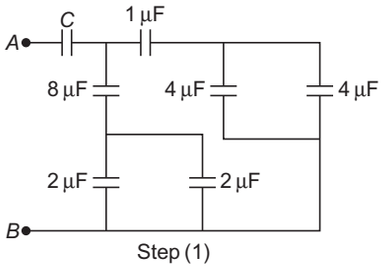


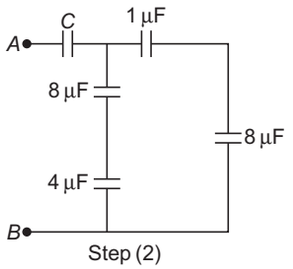


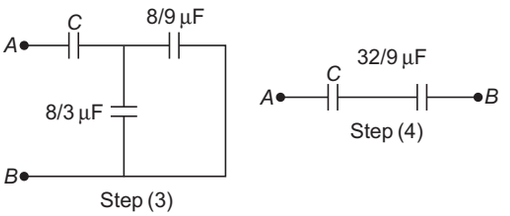
$\therefore$ Resultant capacity between A and B

i.e. $C_{AB} = 1\,\mu F$

121. Given circuit can be reduced in following manner :



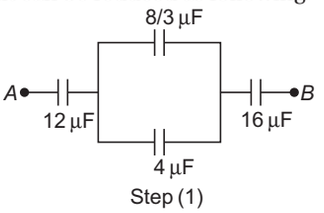


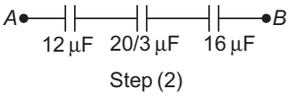


So, equivalent capacitance between A and B

$$C_{eq.} = 1 = \frac{\frac{32}{9} \times C}{\frac{32}{9} + C} \quad \therefore \quad C = \frac{32}{23}\ \mu F$$

122. Given circuit can be reduced as following.



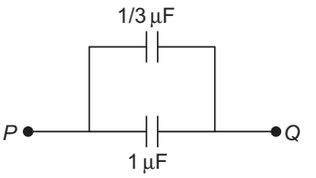


Hence, equivalent capacitance between A and B

$$\frac{1}{C_{AB}} = \frac{1}{12} + \frac{1}{20/3} + \frac{1}{16}$$

$\therefore$ $$C_{AB} = \frac{240}{71}\ F$$

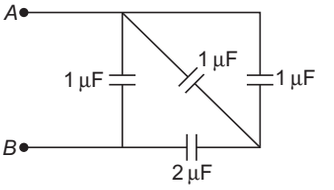
123. Given circuit can be simplified as shown,

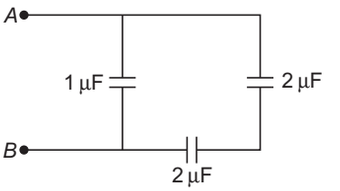


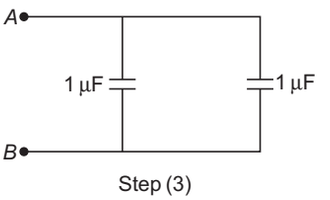
$\therefore$ $$C_{PQ} = \frac{1}{3} + 1$$

$$= \frac{4}{3}\ \mu F$$

124. Given circuit can be simplified as follows





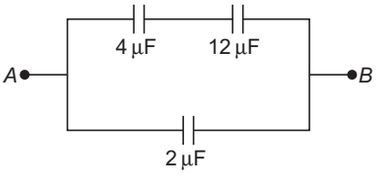


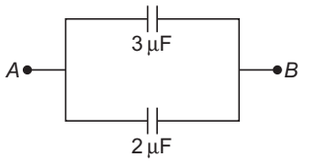
So, equivalent capacitance between A and B,

$$C_{AB} = 1 + 1$$

$$= 2\,\mu F$$

125. Given circuit can be expressed as



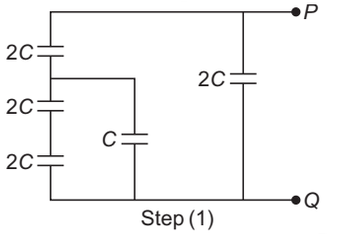


Step (3)

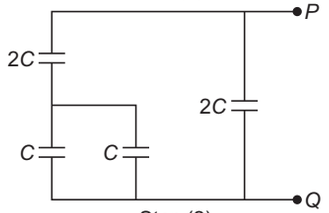
So, net capacitance between AB

$$C_{AB} = 3 + 2 = 5\,\mu F$$

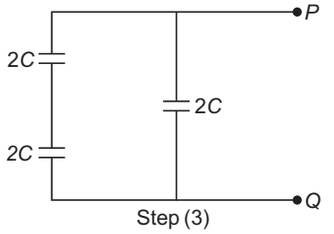
126. The given circuit can be simplified in the following way.



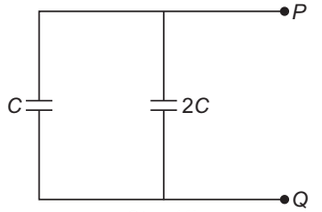
Step (1)



Step (2)



Step (3)



Step (4)

So, capacity between P and Q

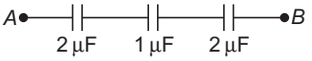
$$C_{PQ} = 2C + C = 3C$$

127. Capacitors C_1 and C_2 are in parallel and they are in series with C_3, then equivalent capacity between A and B

$$C = \frac{C_P \times C_3}{C_P + C_3} = \frac{15 \times 4}{15 + 4}$$

$$= \frac{60}{19} = 3.2\,\mu F$$

128. The situation can be simplified as follows

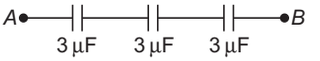


$\therefore$ Equivalent capacity between A and B

$$C_{AB} = \frac{2}{1 + 2 + 1} = \frac{2}{4}$$

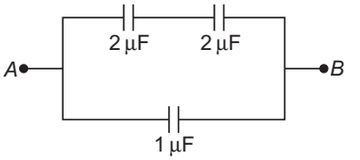
$\therefore$ $$C_{AB} = 0.5\,\mu F$$

129. The circuit can be redrawn as

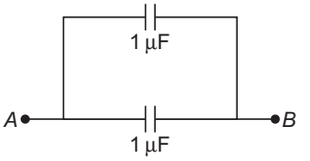


So, $$C_{AB} = \frac{3}{3} = 1\,\mu F$$

130. Given circuit can be redrawn as follows



Step (1)

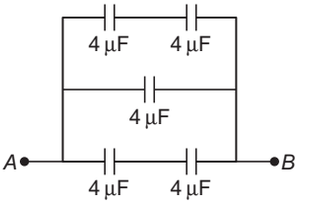


Step (2)

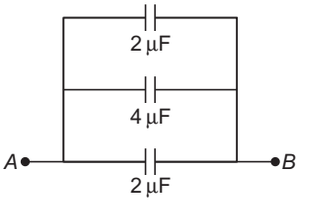
So, equivalent capacitance between A and B

$$C_{AB} = 1 + 1$$
$$= 2\,\mu F$$

131. Given circuit can be redrawn as



Step (1)



Step (2)

So, equivalent capacitance between A and B,

$$C_{AB} = 2 + 4 + 2$$
$$= 8\,\mu F$$

132. Capacitors C_1 and C_2 are in series with C_3 in parallel with them.

Now, $$C_1 = \frac{K_1 \varepsilon_0 (A/2)}{(d/2)} = \frac{K_1 \varepsilon_0 A}{d}$$

$$C_2 = \frac{K_2 \varepsilon_0 (A/2)}{(d/2)} = \frac{K_2 \varepsilon_0 A}{d}$$

and $$C_3 = \frac{K_3 \varepsilon_0 A}{2d}$$

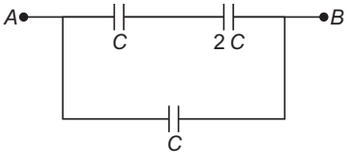
$$C_{\text{equivalent}} = C_3 + \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{K_3\varepsilon_0 A}{2d} + \frac{\left(\frac{K_1\varepsilon_0 A}{d}\right)\left(\frac{K_2\varepsilon_0 A}{d}\right)}{\frac{K_1\varepsilon_0 A}{d} + \frac{K_2\varepsilon_0 A}{d}}$$

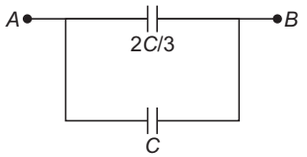
$$= \frac{\varepsilon_0 A}{d}\left(\frac{K_3}{2} + \frac{K_1 K_2}{K_1 + K_2}\right)$$

So, none option is correct.

133. Given circuit can be redrawn as follows.



Step (1)



Step (2)

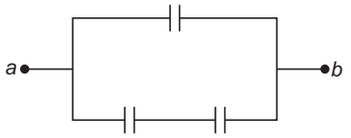
Equivalent capacitance between A and B

$$C_{AB} = \frac{2C}{3} + C$$

$$= \frac{5C}{3}$$

134. In the given figure, the metallic plates form a combination of two capacitors in series with one capacitor in parallel.

Let capacity of each capacitor is C, then equivalent capacitance between a and b



$$C_{ab} = \frac{C}{2} + C = \frac{3C}{2}$$

$\because$ $$C = \frac{\varepsilon_0 A}{d}$$

$\therefore$ $$C_{ab} = \frac{3}{2}\frac{\varepsilon_0 A}{d}$$

135. $C_s = \dfrac{C_0}{7}$

and $$C_P = 7C_0$$

$\therefore$ $$C_P = 49C_s$$

Here, C_0 = capacity of one capacitor.

136. $V = \dfrac{q}{C}$ or $V \propto \dfrac{1}{C}$

V has reduced to $\frac{1}{8}$th its original value. Therefore, C has increased 8 times.

137. $C = \dfrac{K\varepsilon_0 A}{d}$ or $C \propto K$

$\therefore$ $$C' = (10^{-12})\left(\frac{4.0}{2}\right)$$

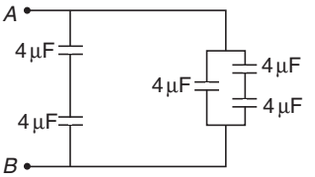
$$= 2 \times 10^{-12}\ \text{F}$$

138. Here, $\dfrac{2C}{3} = 4\ \mu\text{F} \Rightarrow \therefore C = 6\ \mu\text{F}$

139. By such an arrangement two capacitors in parallel are formed.

$\therefore$ $$C_{\text{net}} = 2C = 2\left(\frac{\varepsilon_0 A}{d}\right)$$

140. The simple circuit is shown as below



141. $C = (4\pi\varepsilon_0 R)$

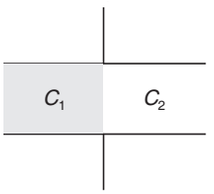
142. $C \propto R$

$8 \times$ Volume of small drop = Volume of big drop

$$8\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3 \Rightarrow \therefore R = 2r$$

143. $\dfrac{C'}{C} = \dfrac{\varepsilon_0 A/d - t}{\varepsilon_0 A/d} = \dfrac{d}{d-t} = \dfrac{d}{d/2} = 2:1$

144. $C = C_1 + C_2 = (4)(5) + 5 = 25\ \mu\text{F}$



145. $C = (4\pi\varepsilon_0)\dfrac{ab}{b-a}$

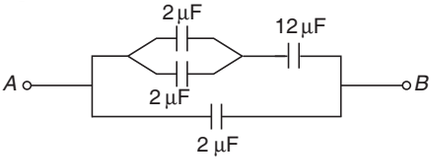
Assuming $b \approx a$

$$C = (4\pi\varepsilon_0)\frac{b^2}{b-a}$$

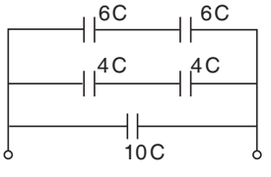
$$b = \sqrt{\frac{C(b-a)}{4\pi\varepsilon_0}}$$

$$= \sqrt{(10^{-6})(10^{-3})(9\times10^9)} = 3\ \text{m}$$

146. The simple circuit is as under,



146. The given combination is a balanced Wheatstone bridge in parallel with 10 C.



150. $\dfrac{C'}{C} = \dfrac{\varepsilon_0 A/d - t}{\varepsilon_0 A/d}$

$$= \frac{d}{d-t} = \frac{d}{(5/7)d} = \frac{7}{5}$$

$\therefore$ $$C' = \frac{7}{5} \times 25\,\mu\text{F} = 35\,\mu\text{F}$$

152. Energy stored, $W = \frac{Q^2}{2C}$

$\therefore \quad W \propto Q^2$ (If C is a constant)

or $\frac{W}{W'} = \frac{Q^2}{(2Q)^2} = \frac{Q^2}{4Q^2}$

$\therefore \quad W' = 4W$

153. On introducing dielectric K in a parallel plate capacitor, its capacity becomes

$C' = KC_0$

$\therefore \quad C' = 5C_0$

Also, energy stored $W_0 = \frac{q^2}{2C_0}$

$\therefore \quad W' = \frac{q^2}{2C'}$

$= \frac{q^2}{2 \times 5C_0}$

$\therefore \quad \frac{W_0}{W'} = \frac{5}{1}$

$\therefore \quad W' = \frac{W_0}{5}$

154. $C \propto \frac{1}{d}$

$\therefore \quad C' = 2C$

$\therefore$ Extra charge flow

$q = (2CV - CV)$

$= CV$

$\therefore$ Work done, $W = qV$

$= (CV)V$

$= CV^2$

155. When charging battery is disconnected, than charge remains constant V becomes $\frac{1}{K}$ times and E becomes $\frac{1}{K}$ times.

So, new value of charge is Q_0

New value of $V = \frac{V_0 \times 3}{9} = \frac{V_0}{3}$

New value of $E = \frac{E_0 \times 3}{9} = \frac{E_0}{3}$

156. Heat produced in capacitor

= Energy of charged capacitor

$= \frac{1}{2}CV^2$

$= \frac{1}{2} \times (2 \times 10^{-6}) \times (100)^2$

$= 0.01$ J

157. Force between the plates of a parallel plate capacitor is given by

$$|F| = \frac{\sigma^2 A}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A} = \frac{CV^2}{2d}$$

158. Potential on parallel plate capacitor

$$V = \frac{Q}{C}$$

Also, capacity of parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 KA}{d}$$

$\therefore \quad V = \frac{Qd}{\varepsilon_0 KA}$

$\Rightarrow \quad V \propto d$

So, on increasing the distance between plates of capacitor, the potential difference between plates also increases.

159. Change in energy of condenser

$= \Delta U = U_2 - U_1$

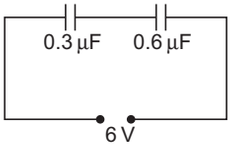
$= \frac{1}{2}C_2V^2 - \frac{1}{2}C_1V^2$

$= \frac{V^2}{2}(C_2 - C_1)$

$= \frac{(100)^2}{2}(10 \times 10^{-6} - 2 \times 10^{-6})$

$= 4 \times 10^{-2}$ J

160. In series, charge remains same on both capacitors,



$\therefore \quad U = \frac{Q^2}{2C}$

or $U \propto \frac{1}{C}$

$\therefore \quad \frac{U_1}{U_2} = \frac{C_2}{C_1} = \frac{0.6}{0.3}$

$\therefore \quad U_1 : U_2 = 2:1$

161. Here, capacitors of capacity 4 μF, each are in parallel, their equivalent capacity is 8 μF. Now, there is a combination of three capacitors in series of capacity 20 μF, 8 μF and 12 μF so, their resultant capacity

$\frac{1}{C} = \frac{1}{20} + \frac{1}{8} + \frac{1}{12}$

$\therefore \quad C = \frac{120}{31}$ μF

Total charge, $Q = CV$

$= \frac{120}{31} \times 300$

$= 1161$ μC

$\because$ Charge through 4 μF capacitor $= \frac{1161}{2}$

$= 580$ μC

and potential difference across 4 μF condenser

$= \frac{580}{4} = 145$ V

162. As capacitors C_1 and C_2 are in series, then there should be equal charge on them, i.e.

Charge on C_1 = Charge on C_2

$\therefore \quad C_1(V_A - V_D) = C_2(V_D - V_B)$

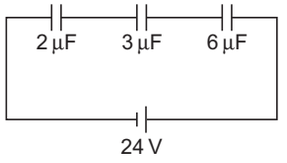
or $C_1(V_1 - V_D) = C_2(V_D - V_2)$

or $C_1V_1 - C_1V_D = C_2V_D - C_2V_2$

or $V_D(C_1 + C_2) = C_1V_1 + C_2V_2$

$\therefore \quad V_D = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

163. Equivalent capacitance of circuit,

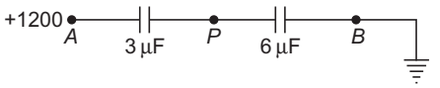


$$C_{eq} = \frac{6}{3+2+1} = 1\,\mu F$$

Total charge, $Q = 1 \times 24 = 24\,\mu C$

Now, potential difference across 6 μF capacitor $= \frac{24}{6} = 4$ V.

164. Given circuit can be reduced as



Let potential at P is V_P and potential at B is V_B.

As capacitors 3 μF and 6 μF are in series, they have same charge

$\therefore$ Charge on 3 μF = Charge on 6 μF

$\therefore$ $C_1V_1 = C_2V_2$

or $3(1200 - V_P) = 6(V_P - V_B)$

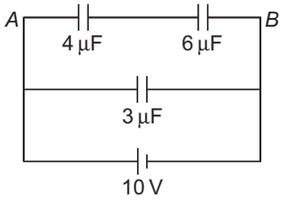
As B point is attached to earth.

So, $V_B = 0$

So, $1200 - V_P = 2V_P$

$\therefore$ $V_P = 400$ V

165. The circuit can be redrawn as



Here, 4 μF and 6 μF are in series. So, charge is same on both.

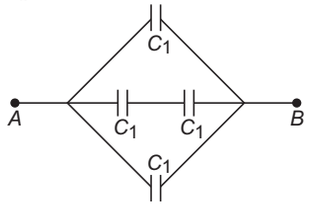
Now, equivalent capacity between A and B

$$C_{AB} = \frac{6 \times 4}{6+4} = 2.4\ \mu F$$

So, charge on 4 μF capacitor

$$Q = C_{AB} \times 10$$
$$= 2.4 \times 10$$
$$= 24\,\mu C$$

166. Equivalent capacitance between A and B



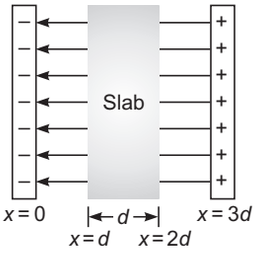
$$C_{AB} = C_1 + \frac{C_1}{2} + C_1 = \frac{5}{2}C_1$$

As charge, $Q = CV$

So, $1.5\,\mu C = \frac{5}{2}C_1 \times 6 \Rightarrow C_1 = \frac{1.5}{15} \times 10^{-6}$

$$= 0.1 \times 10^{-6}\ F = 0.1\,\mu F$$

167. After the introduction of dielectric slab, direction of electric field remains perpendicular to plate and is directed from positive to negative plate.



$$\text{Electric field in air} = \frac{\sigma}{\varepsilon_0}$$

and $$\text{electric field in dielectric} = \frac{\sigma}{K\varepsilon_0}$$

Positive plate is at higher potential and negative plate is at lower potential. So, electric potential increases continuosly as we move from $x = 0$ to $x = 3d$.

168. $V = \frac{q_{net}}{C_{net}} = \frac{C_1V_0}{C_1 + C_2}$

169. $V = \frac{q_{net}}{C_{net}}$ or $40 = \frac{(2)(200)}{2+C}$

$\therefore$ $C = 8\,\mu F$

170. q remains same, while C increases.

171. $q_i = q_f$

$$C_1V_1 + C_2V_2 = (C_1 + C_2)V$$

$\therefore$ $(2)(4) + (C)5 = (2 + C)4.6$

$$0.4C = 1.2$$

$$C = 3 \text{ units}$$

172. $U = \frac{1}{2}(nC)V^2$

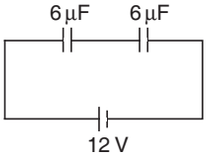
174. $U = \frac{1}{2}\frac{q^2}{V}$

On increasing the plate reparation value of C' decreases, hence U will increases.

175. $F_e = qE = q\left(\frac{V}{d}\right)$

$$= (4 \times 10^{-6})\left(\frac{2000}{2 \times 10^{-3}}\right) = 4\ N$$

176. The simple circuit will be as under



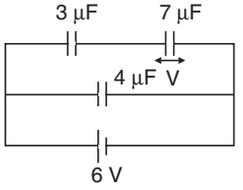
The potential drop 12 V will be equally distribute.

177. PD across 4 μF will be 400 V.



$\therefore$ $q = 1600\ \mu C$

178. In parallel potential difference is same and in series it distributes in inverse ratio of capacity.



$$V = 6\left(\frac{3}{3+7}\right)$$

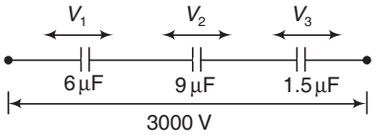
$$= 1.8\,\text{V}$$

$\therefore$ Charge stored in 5 μF capacitor will be

$$q = CV = (5\,\mu\text{F})(1.8\,\text{V})$$

$$= 9\,\mu\text{C}$$

180. In series potential difference distributes in inverse ratio of capacities. Hence,



$$V_1 : V_2 : V_3 = \frac{1}{6} : \frac{1}{9} : \frac{1}{1.5} = 1.5 : 1 : 6$$

$$V_2 = (3000)\left(\frac{1}{1+1.5+6}\right) = 352.9\,\text{V}$$

181. In three capacitors of 3 μF each, 30 V will be equally distributed.

183. Area $= \frac{1}{2}QV$ = energy stored in the capacitor.

185. Potential difference across C is 10 V.

$\therefore$ $q = CV = 60\,\mu\text{C}$

186. Potential difference across 4 μF capacitor is 9 V and potential difference across 6 μF capacitor will be (12 – 9) V = 3 V.

$\therefore$ $q = CV = 18\,\mu\text{C}$

191. Both the conductors carry equal and opposite charges. So, after connecting by a wire, there will be no charge in any conductor. Hence, all the stored energy will be destroyed.

$$\text{Loss of energy} = 2\left(\frac{1}{2}CV^2\right) = CV^2$$

192. $U = \frac{1}{2}(2C)V^2 = CV^2 = \left(\frac{\varepsilon_0 A}{d}\right)V^2$

193. On the surface of bubble, same charge will distribute, so there is repulsion of charges so radius of bubble increases.

194. Dielectric constant,

$$K = \frac{\text{permittivity of medium}}{\text{permittivity of free space}}$$

$$K = \frac{\varepsilon}{\varepsilon_0}$$

$\therefore$ $\varepsilon = K\varepsilon_0$

$$= 81 \times 8.86 \times 10^{-12}$$

$$= 7.16 \times 10^{-10}\ \text{MKS units}$$

195. Both the spheres can be charged equally. Because for metallic sphere either solid or hollow, the charge will inside the surface of the sphere. Since, both spheres have same surface area, so they can hold equal maximum charge.

196. Permittivity of metals is very high comparable to permittivity of free space. So, dielectric constant for metal is infinite.

197. Net charge distributes in direct ratio of capacity (or radius in case of spherical conductor)

$\therefore$ $q'_A = q_{\text{net}}\left[\frac{4}{4+6}\right] = 120\left[\frac{4}{4+6}\right]$

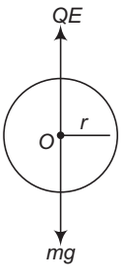
$= 48\,\mu\text{C}$

So, $\Delta q = 80 - 48 = 32\,\mu\text{C}$

198. Let radius of drop is r. According to Millikan, for balance of drop

$$QE = mg$$

or $$Q\frac{V}{d} = \left[\left(\frac{4}{3}\pi r^3\right)\rho\right]g$$



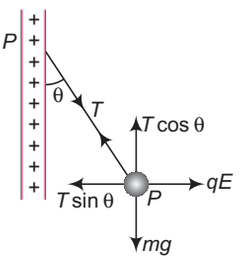
where, V is potential difference and ρ is density of drop

$\therefore$ $\frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2}\right)^3 \times \frac{V_2}{V_1}$

$\therefore$ $\frac{Q}{Q_2} = \left(\frac{r}{r/2}\right)^3 \times \frac{600}{2400} = 2$

$\therefore$ $Q_2 = Q/2$

199. At point P for equilibrium,



$$T\sin\theta = qE \quad \text{and} \quad T\cos\theta = mg$$

Dividing, we get

$$\tan\theta = \frac{qE}{mg}$$

$\because$ $E = \frac{\sigma}{2\varepsilon_0}$

$\therefore$ $\tan\theta = \frac{q}{mg}\left(\frac{\sigma}{2\varepsilon_0}\right)$

$\therefore$ $\sigma \propto \tan\theta$

201. When key is closed, then capacitor becomes short-circuited and bulb will light up. But when capacitor becomes fully charged, it acts as open circuit and this time bulb will not glow.

201. Capacity of parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d}$$

$$\therefore \quad \varepsilon_0 = \frac{Cd}{A}$$

$$\text{Unit} = \frac{\text{Farad} \times \text{metre}}{\text{metre}^2}$$

$$= \frac{\text{Farad}}{\text{metre}}$$

202. $\text{PD} = V_i - V_0 = \dfrac{q_i}{4\pi\varepsilon_0}\left(\dfrac{1}{r_i} - \dfrac{1}{r_0}\right)$

If q_i is positive, $V_i - V_0 =$ positive or $V_i > V_0$.

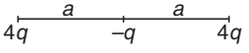
203. $\left(\dfrac{q}{R}\right)_1 = \left(\dfrac{q}{R}\right)_2$

$$\therefore \quad V_1 = V_2$$

204. In redistribution of charge there is always loss of energy, unless their potentials are same or $Q_1 R_2 = Q_2 R_1$.

206. Principle of generator.

207. At the present position, all the charges are in equilibrium. But when they displaced slightly from their present position, they do not return back. So, they are in unstable equilibrium position.

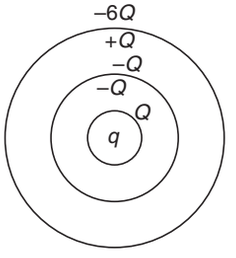


208. Net charge distributes in direct ratio of their capacities.

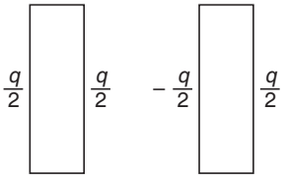
209. If force is constant,

$$|\Delta \text{P}| = |\text{F}|t = qE_0 t$$

211. Charge distribution is as given below



212. The distribution of charge is as under



$$E = \frac{\sigma}{\varepsilon_0} = \frac{(q/2)/S}{\varepsilon_0}$$

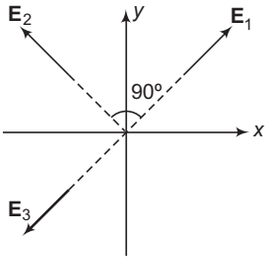
$$= \frac{q}{2S\varepsilon_0}$$

214. Net force on q is zero. Due to q_1 it is towards left. Hence, due to shell it should be towards right.

Level 2 : Only One Correct Option

1. If ring is complete, net field at centre is zero. If small portion is cut, field opposite to this is not cancelled out.

2. $\mathbf{E}_1 \perp \mathbf{E}_2 \therefore \mathbf{E}_1 \cdot \mathbf{E}_2 = 0$



$$\therefore \quad \mathbf{E}_1 \downarrow \uparrow \mathbf{E}_3$$

$$\therefore \quad |\mathbf{E}_1 \times \mathbf{E}_3| = 0$$

3. $V \propto \dfrac{q}{R} \propto \dfrac{\sigma R^2}{R}$ or $V \propto \sigma R$

$$V_1 = V_2$$

$$\therefore \quad \sigma_1 R_1 = \sigma_2 R_2 \quad \Rightarrow \quad \frac{\sigma_1}{\sigma_2} = \frac{R_1}{R_2} = \frac{5}{4}$$

4. $V = \dfrac{q}{4\pi\varepsilon_0 r} - \dfrac{q}{4\pi\varepsilon_0 3r} = \dfrac{q}{6\pi\varepsilon_0 r}$

Electric field intensity at a distance $3r$ is

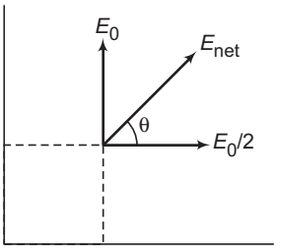
$$E = \frac{q}{4\pi\varepsilon_0 (3r)^2} = \frac{q}{4\pi\varepsilon_0 \cdot 9r^2}$$

Thus, $\dfrac{E}{V} = \dfrac{q(6\pi\varepsilon_0 r)}{4\pi\varepsilon_0 9r^2 (q)} = \dfrac{1}{6r}$

$$\Rightarrow \quad E = \frac{V}{6r}$$

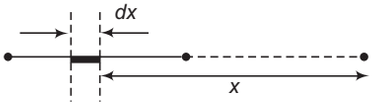
5. $E \propto \dfrac{q}{r^2}$

$$\tan\theta = \frac{E_0}{E_0/2} = 2$$



6. $W = \Delta U = U_f - U_i = (3U) - U = 2U$

7. $dQ = \left(\dfrac{Q}{2l}\right) \cdot dx$



$$dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{d\theta}{x} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{2l} \cdot \frac{dx}{x}$$

$$\therefore \quad V = \int_{x=l}^{x=3l} dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{2l} \ln 3$$

8. $\frac{k(2Q)}{2a}+\frac{kQ}{2a}=5\,\text{V}$

or $\frac{k(3Q)}{2a}=5$

$\therefore \quad \frac{kQ}{a}=\frac{10}{3}$

Now, potential of inner sphere

$$V_m=\frac{k2Q}{a}+\frac{kQ}{2a}=\frac{5}{2}\frac{kQ}{a}=\frac{5}{2}\left(\frac{10}{3}\right)$$
$$=\frac{25}{3}\,\text{V}$$

9. We know that, $V=\frac{kq}{2}$

$$120=\frac{kq}{2} \quad \therefore \quad kq=240\,\text{units}$$

When it is made to touch the bigger sphere, whole charge will transfer to bigger sphere.

$\therefore \quad V=\frac{kq}{6}$

$$=\frac{240}{6}=40\,\text{V}$$

10. $V_A-V_B=\int_A^B(2\hat{\mathbf{i}}-4\hat{\mathbf{j}})\cdot(dx\hat{\mathbf{i}}+dy\hat{\mathbf{j}})$

$$=[2x-4y]_{(0,\,0)}^{(3m,\,4m)}=-10\,\text{V}$$

11. $q_Q=-q_P$

From Gauss law, we can prove that net electric field outside Q is zero.

12. $W_{AB}=U_B-U_A=q_0(V_B-V_A)$

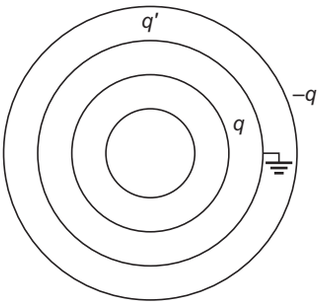
$$V_B-V_A=-\int_A^B E\,dr$$
$$=-\int_{3a}^{2a}\frac{\lambda}{2\pi\varepsilon_0 r}\,dr=\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{3}{2}\right)$$

$\therefore \quad W_{A\to B}=\frac{q_0\lambda}{2\pi\varepsilon_0}\ln\left(\frac{3}{2}\right)$

13. $qE=\mu mg$

$\therefore \quad \mu=\frac{qE}{mg}$

14. $V_C=0$



$\therefore \quad \frac{k(q+q')}{3a}-\frac{kq}{4a}=0$

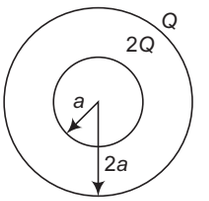
$\therefore \quad q'=-\frac{q}{4}$

Now $\quad V_A-V_C=\frac{kq}{2a}-\frac{kq/4}{3a}-\frac{kq}{4a}-0$

$$=\frac{kq}{6a}$$

15. $\frac{kQq}{a}+\frac{kq^2}{a}+\frac{kQq}{\sqrt{2}\,a}=0$

Solving, we get $\quad Q=\frac{-2q}{2+\sqrt{2}}$



16. $\Delta U=U_f-U_i$

$$=\left[\frac{kq_1q_2}{0.3}+\frac{kq_1q_3}{0.4}+\frac{kq_2q_3}{0.1}\right]$$
$$-\left[\frac{kq_1q_2}{0.3}+\frac{kq_1q_3}{0.4}+\frac{kq_2q_3}{0.5}\right]$$
$$=8kq_2q_3$$

17. Electric field at centre due to circular portion is zero.

18. Potential difference only depends upon the inner charge.

19. $q_A=\frac{Q}{2}$

When B is earthed

$$V_B=0$$

$\therefore \quad \frac{kq_B}{a}+\frac{k(Q/2)}{d}=0$

$\therefore \quad q_B=\frac{-Qa}{2d}$

When C is earthed

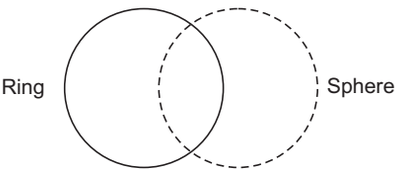
$$V_C=0$$

$\therefore \quad \frac{kq_C}{a}+\frac{k(Q/2)}{d}-\frac{k(Qa/2d)}{d}=0$

$\therefore \quad q_C=\frac{Qa^2}{2d^2}-\frac{Qa}{2d}$

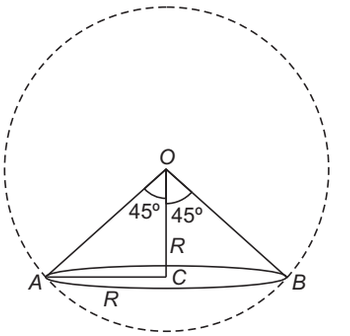
$$a_C=\frac{Qa}{2d}\left(\frac{a-d}{d}\right)$$

20. $q_{\text{in}}<\pi R\lambda$

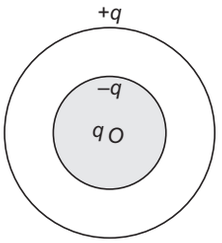


$\therefore \quad \phi<\frac{\pi R\lambda}{\varepsilon_0}$

21. $\phi=\frac{2\pi R^2(1-\cos 45^\circ)}{4\pi R^2}\cdot\frac{q}{\varepsilon_0}$



22. The induced charges will be as under.



Potential at point O (the centre),

$$V_O = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} - \frac{q}{R} + \frac{q}{2R}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{2R}\right]$$

23. Electric potential, $V = \frac{kQ}{R} - \frac{kq}{x} = k\left[\frac{Q}{R} - \frac{q}{x}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R} - \frac{q}{x}\right]$

24. At unstable equilibrium position potential energy is maximum. But

$$U = QV$$

Therefore, C is stable or unstable, it will depend on the sign of Q, placed at C.

25. $k_i + U_i = k_f + U_f$

$$0 + 0 = \frac{1}{2}mv^2 + (-e)\frac{kQ}{R}$$

$$\therefore \quad v = \sqrt{\frac{2kQe}{mR}}$$

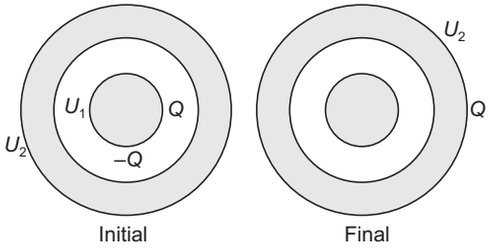
26. $F = \frac{k \cdot q^2}{l^2}$ or $\frac{kq^2}{l} = F \cdot l$

Now, $W = U_f - U_i = \frac{3k \cdot q^2}{2l} - \frac{3kq^2}{l}$

$$= -\frac{3kq^2}{2l} = -\frac{3}{2}Fl$$

27. Heat produced $= U_i - U_f$

$$= (U_1 + U_2) - U_2 = U_1$$



$$= \frac{q^2}{2C} = \frac{q^2}{2\left[4\pi\varepsilon_0\left\{\frac{a \times 2a}{2a - a}\right\}\right]} = \frac{kq^2}{4a}$$

28. $q_{\text{in}} = 0$

$$\therefore \quad \frac{kq'}{r} + \frac{kq}{3r} = 0 \quad (q' \rightarrow \text{on inner shell})$$

or $$q' = -\frac{q}{3}$$

$\therefore$ $+\frac{q}{3}$ charge will flow from inner shell to outer shell.

29. In figure, there is a combination of two capacitors in parallel

$$\therefore \quad C_p = C_1 + C_2$$

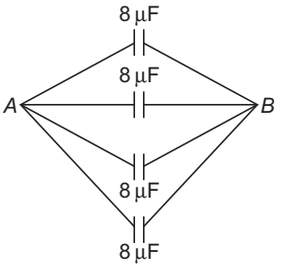
$$= \frac{K_1\varepsilon_0(A/2)}{d} + \frac{K_2\varepsilon_0(A/2)}{d}$$

$$= \frac{2\varepsilon_0(A/2)}{d} + \frac{4\varepsilon_0(A/2)}{d}$$

$$= 2 \times \frac{10}{2} + 4 \times \frac{10}{2}$$

$$= 30\ \mu\text{F}$$

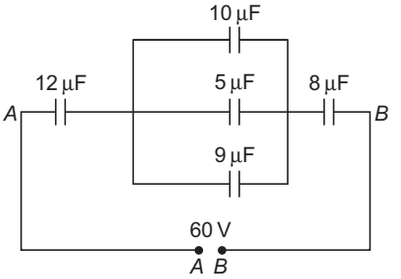
30. Here, circuit can be redrawn as



Equivalent capacitance of their capacitors

$$C_{\text{eq}} = 8 + 8 + 8 + 8 = 32\,\mu\text{F}$$

31. Given circuit can be redrawn as follows.



Equivalent capacitance of the circuit

$$C_{AB} = \frac{24}{2 + 1 + 3} = 4\ \mu\text{F}$$

Total charge given by battery

$$q = C_{AB} \cdot V = 4 \times 60 = 240\ \mu\text{C}$$

Charge on 5μF capacitor

$$q_2 = \left(\frac{5}{10 + 5 + 9}\right) \times 240 = 50\ \mu\text{C}$$

32. Here, circuit is equivalent to two capacitors in parallel

$$\therefore \quad C_{\text{eq}} = C_1 + C_2$$

$$= \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{d} = \frac{2\varepsilon_0 A}{d}$$

$$\therefore \quad \text{Energy stored} = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}\left(\frac{2\varepsilon_0 A}{d}\right)V^2$$

$$= \frac{8.86 \times 10^{-12} \times 50 \times 10^{-4} \times 12 \times 12}{3 \times 10^{-3}}$$

$$= 2.1 \times 10^{-9}\ \text{J}$$

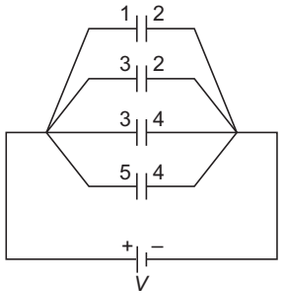
33. Applying charge distribution law on C_1 and C_2 at steady state. Charge on C_1

$$Q_1 = \left(\frac{C_1}{C_1 + 2C_1}\right) \times Q = \frac{Q}{3}$$

Charge on C_2

$$Q_2 = \left(\frac{2C_1}{C_1 + 2C_1}\right) \times Q = \frac{2}{3}Q$$

34. Given circuit can be redrawn as shown



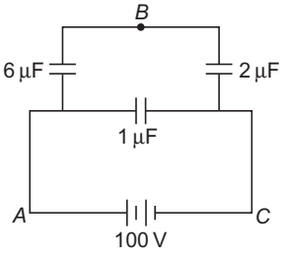
Capacity of each capacitor is $C = \frac{\varepsilon_0 A}{d}$.

So, magnitude of charge on each capacitor = Magnitude of charge on each plate $= \frac{\varepsilon_0 A}{d} V$

As plate 1 is connected with +ve terminal of battery, so charge on $= + \frac{\varepsilon_0 A}{d} \cdot V$

Plate 4 comes twice and it is connected with negative terminal of battery. So, charge on plate $4 = -\frac{2\varepsilon_0 AV}{d}$

35. The given circuit can be reduced as follows. (Resistance does not matter in considering equivalent capacitance).



$$C_{eq} = \frac{6 \times 2}{6+2} + 1 = \frac{5}{2}\ \mu F$$

Total charge, $Q = CV$

$$= \frac{5}{2} \times 100 = 250\ \mu C$$

So, charge in 6 μF branch $= CV$

$$= \left(\frac{6 \times 2}{6+2}\right) \times 100$$

$$= 150\ \mu C$$

$\therefore$ $V_{AB} = \frac{150}{6} = 25\ V$

and $V_{BC} = 100 - V_{AB} = 75\ V$

36. Charge on first capacitor $= C(-V)$

Charge on second capacitor $= (2C)(2V)$

$$= 4CV$$

$\therefore$ Total charge on both capacitors,

$$= 4CV - CV = 3CV$$

$\therefore$ Common potential on them $= \frac{3CV}{3C} = V$

$\therefore$ Energy $= \frac{1}{2}(3C)V^2$

$$= \frac{3}{2} CV^2$$

37. $q_1 = C_1 V = (15) \times 100 = 1500\ \mu C$

$q_2 = C_2 V = (1) \times 100 = 100\ \mu C$

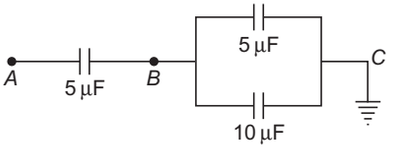
$\therefore$ $q_{net} = q_1 + q_2 = 1600\ \mu C$

When dielectric is removed

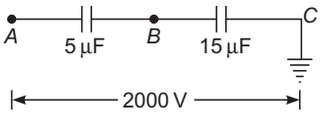
$$C'_1 = \frac{C_1}{K} = \frac{15}{15} = 1\ \mu F$$

Now $V = \frac{q}{C'_1 + C_2} = \frac{1600}{1+1} = 800\ V$

38. Given circuit can be redrawn as



Step (1)



Step (2)

Potential defference between A and B

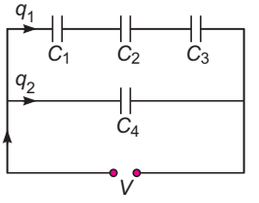
i.e. $V_A - V_B = \left(\frac{15}{5+15}\right) \times 2000$

$\therefore$ $V_A - V_B = 1500\ V$

$\therefore$ $2000 - V_B = 1500\ V$

$\therefore$ $V_B = 500\ V$

39. Given circuit can be reduced to

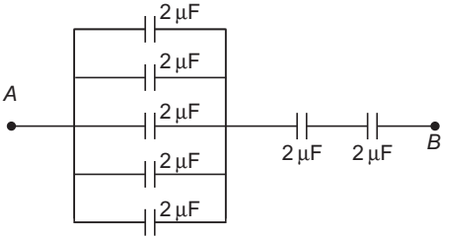


For series combination of C_1, C_2, C_3 resultant capacity

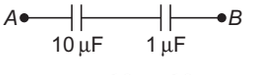
$$C_s = \frac{6C}{6C + 3C + 2C} \quad \text{or} \quad C_s = \frac{6C}{11}$$

Now, $\frac{q_1}{q_2} = \frac{C_s}{C_4} = \frac{6C/11}{4C} = \frac{3}{22}$

40. From concept of series and parallel combination, we can easily find that in option (a) the resultant capacity is $\frac{10}{11}\ \mu F$.

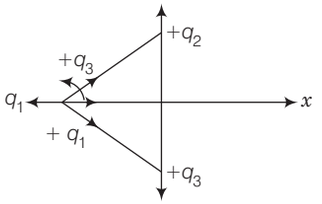


The circuit can be redrawn as



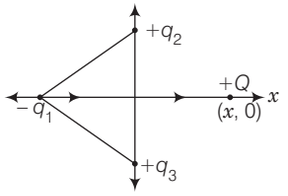
$\therefore$ $C_{eq} = \frac{10}{10+1} = \frac{10}{11}\ \mu F$

41. The net force on q_1 by q_2 and q_3 is along the + x-direction, so nature of force between q_1, q_2 and q_1, q_3 is attractive. This can be represent by the figure given below



The attractive force between these charges states that q_1 is a negative charge (since, q_2 and q_3 are positive).

Thus, nature of force between q_1 and newly introduced charge Q (positive) is attractive and net force on q_1 by q_2, q_3 and Q are along the same direction as given in the diagram below

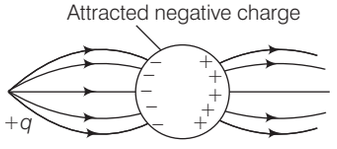


The figure given above clearly shows that the force on q_1 shall increase along the positive x-axis due to the positive charge Q.

42. When a positive point charge is brought near an isolated conducting sphere without touching the sphere, then the free electrons in the sphere are attracted towards the positive charge. This leaves an excess of positive charge on the rear (right) surface of sphere.

Both kinds of charges are bound in the metal sphere and cannot escape. They, therefore, reside on the surface.

Thus, the left surface of sphere has an excess of negative charge and the right surface of sphere has an excess of positive charge as given in the figure below



An electric field lines start from positive charge and ends at negative charge (in this case from point positive charge to negative charge created inside the sphere).

Also, electric field line emerges from a positive charge, in case of single charge and ends at infinity.

Here, all these conditions are fulfilled in Fig. (a).

43. Gauss' law of electrostatics state that the total of the electric flux out of a closed surface is equal to the charge enclosed decided by the permittivity, i.e. Q electric $= \frac{Q}{\varepsilon_0}$.

Thus, electric flux through a surface doesn't depend on the shape, size or area of a surface but it depends on the number of charges enclosed by the surface.

So, here in this question, all the figures same electric flux as all of them has single positive charge.

44. According to Gauss' law, the term q on the right side of the equation $\int_s \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$ includes the sum of all charges enclosed by the surface. The charges may be located anywhere inside the surface, if the surface is so chosen that there are some charges inside and some outside, the electric field on the left side of equation is due to all the charges, both inside and outside S.

So, E on LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only.

45. The space between the electric field lines is increasing, here from left to right and its characteristics states that, strength of electric field decreases with the increase in the space between electric field lines. As a result force on charges also decreases from left to right.

Thus, the force on charge $-q$ is greater than force on charge $+q$ in turn dipole will experience a force towards left.

46. When a point positive charge brought near an isolated conducting plane, some negative charge developes on the surface of the plane towards the charge and an equal positive charge developes on opposite side of the plane. This process is called charging by induction.

47. When the point is situated at a point on diameter away from the centre of hemisphere charged uniformly positively, the electric field is perpendicular to the diameter. The component of electric intensity parallel to the diameter cancel out.

48. Current flows through 2Ω resistance from left to right, is given by

$$I = \frac{V}{R+r} = \frac{2.5\,\text{V}}{2+0.5} = 1\,\text{A}$$

The potential difference across 2Ω resistance

$$V = IR = 1 \times 2 = 2\,\text{V}$$

Since, capacitor is in parallel with 2Ω resistance, so it also has 2V potential difference across it.

The charge on capacitor

$$q = CV = (2\,\mu\text{F}) \times 2\text{V} = 8\,\mu\text{C}$$

49. The direction of electric field is always perpendicular to one equipotential surface maintained at high electrostatic potential to other equipotential surface maintained at low electrostatic potential.

The positively charged particle experiences electrostatic force along the direction of electric field *i.e.*, from high electrostatic potential to low electrostatic potential. Thus, the work is done by the electric field on the positive charge, hence electrostatic potential energy of the positive charge decreases.

50. The work done by a electrostatic force is given by $W_{12} = q(V_2 - V_1)$. Here initial and final potentials are same in all three cases and same charge is moved, so work done is same in all three cases.

51. In this problem, the electric field intensity E and electric potential V are related as

$$E = -\frac{dV}{dr}$$

Electric field intensity $E = 0$ suggest that $\frac{dV}{dr} = 0$

This imply that V = constant.

Thus, $E = 0$ inside the charged conducting sphere causes , the same electrostatic potential 100V at any point inside the sphere.

More than One Correct Options

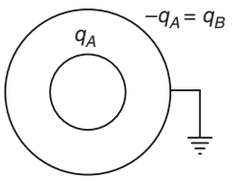
1. ***(a)*** $V_A = 2\text{V} = \dfrac{Kq_A}{R} + \dfrac{Kq_B}{2R}$

$$V_B = \frac{3}{2}\text{V} = \frac{Kq_A}{2R} + \frac{Kq_B}{2R}$$

Solving these two equations, we get

$$\frac{q_A}{q_B} = \frac{1}{2}$$

(b)



$$\frac{q_A'}{q_B'} = \frac{q_A}{-q_A} = -1$$

(c) & ***(d)*** Potential difference between A and B will remain unchanged as by earthing B, charge on will not change.

$\therefore \quad V_A' - V_B' = V_A - V_B = 2V - \dfrac{3}{2}V = \dfrac{V}{2}$

$\therefore \quad V_A' = \dfrac{V}{2}$ as $V_B' = 0$

2. $T = \dfrac{2u_y}{g} = \dfrac{2\times 10}{10} = 2\text{ s}$

$$H = \frac{u_y^2}{2g} = \frac{(10)^2}{20} = 5\text{ m}$$

$$R = \frac{1}{2}a_x T^2 = \frac{1}{2}\left(\frac{qE}{m}\right)T^2$$

$$= \frac{1}{2}\left(\frac{10^{-3}\times 10^4}{2}\right)(2)^2$$

$$= 10\text{ m}$$

3. $100 = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{(R+0.05)}$ …(i)

$75 = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{(R+0.1)}$ …(ii)

Solving these equations, we get $q = \dfrac{5}{3}\times 10^{-9}\text{ C}$ and $R = 0.1\text{ m}$

(a) $V = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{R} = \dfrac{(9\times 10^9)\left(\frac{5}{3}\times 10^{-9}\right)}{0.1}$

$= 150\text{ V}$

(c) $E = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q}{R^2} = \dfrac{V}{R} = \dfrac{150}{0.1}$

$= 1500\text{ V/m}$

(d) $V_{\text{centre}} = 1.5\,V_{\text{surface}}$

4. See the hint of Q. No. 6 of subjective questions.

5. Electric field at any point depends on both charges Q_1 and Q_2. But electric flux passing from any closed surface depends on the charged enclosed by that closed surface only.

6. Flux from any closed surface $= \dfrac{q_{\text{in}}}{\varepsilon_0}$, $q_{\text{in}} = 0$, due to a dipole.

7. Solution is not required.

8. $E = \dfrac{K\,q_{\text{in}}}{r^2}$ $\left(K = \dfrac{1}{4\pi\varepsilon_0}\right)$

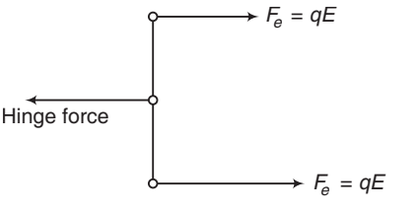
$\therefore \quad E_A = E_C = 0$

but $E_B \neq 0$

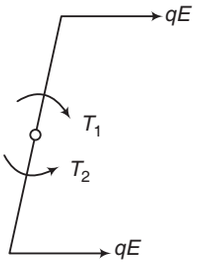
$V = \dfrac{Kq}{R}$ $(r \le R)$

$V = \dfrac{Kq}{r}$ $(r \ge R)$

9.



Higher force $= 2qE$ (towards left)

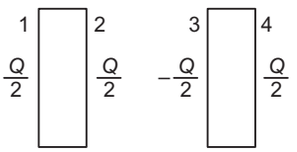


If we displace the rod, $\tau_1 = \tau_2$ or $\tau_{\text{net}} = 0$ in displaced position too. Hence, equilibrium is neutral.

10. Along the line AB, charge q is at unstable equilibrium position at B (When displaced from B along AB, net force on it is away from B, whereas force at B is zero). Hence potential energy at B is maximum.

Along CD equilibrium of q is stable. Hence, potential energy at B is minimum along CD.

11.



$$q_1 = q_4 = \frac{q_{\text{total}}}{2} = \frac{Q}{2}$$

$$q_2 = Q - q_1 = \frac{Q}{2}$$

$$q_3 = -q_2 = \frac{-Q}{2}$$

$E_A = E_1 + E_4$ (towards left)

$$= \frac{Q}{4A\varepsilon_0} + \frac{Q}{4A\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

$E_C = -E_A$

$= \dfrac{Q}{2A\varepsilon_0}$ (towards right)

$E_B = E_2 + E_3$ (towards right)

$$= \frac{Q}{4A\varepsilon_0} + \frac{Q}{4A\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

12. In steady state,

$$q_C = EC \quad \text{and} \quad q_{2C} = 2EC$$

$\tau_C = 2CR$ of both circuits.

At time t,

$$q_C = EC\,(1 - e^{t/\tau_C})$$

$$q_{2C} = 2EC\,(1 - e^{-t/\tau_C})$$

$$\therefore \quad \frac{q_C}{q_{2C}} = \frac{1}{2}$$

13. In steady state, the current through capacitor wire is zero. Current flows through 200 Ω, 900 Ω and A_2.

$$V_C = \frac{q}{C} = \frac{4\times10^{-3}}{100\times10^{-6}} = 40\text{ V}$$

This is also potential drop across 900 Ω resistance and 100 Ω ammeter A_2 (Total resistance = 1000 Ω). Now, this 1000 Ω and 200 Ω are in series. Therefore,

$$V_2 = V_{200\,\Omega} = \frac{V_{1000\,\Omega}}{5} = \frac{40}{5} = 8\text{ V}$$

$$\text{EMF} = V_{1000\,\Omega} + V_{200\,\Omega} = 48\text{ V}$$

$$i = \frac{\text{EMF}}{\text{Net resistance}} = \frac{48}{1200} = \frac{1}{25}\text{A}$$

14. Current through A is the main current passing through the battery. So, this current is more than the current passing through B. Hence, during charging more heat is produced in A.

In steady state,

$$i_C = 0 \quad \text{and} \quad i_A = i_B$$

Hence, heat is produced at the same rate in A and B.

Further in steady state,

$$V_C = V_B = \frac{\varepsilon}{2}$$

$$\therefore \quad U = \frac{1}{2}CV_C^2 = \frac{1}{8}C\varepsilon^2$$

15. $F = qE = (q)\left(\frac{\sigma}{2\varepsilon_0}\right) = \frac{q^2}{2\varepsilon_0 A}$

q remains unchanged. Hence, F remains unchanged.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$$

q remains unchanged. Hence, E also remains unchanged.

$$U = \frac{q^2}{2C}$$

or $$U \propto \frac{1}{C}$$

C will decrease. Hence U will increase.

$$V = Ed \quad \text{or} \quad V \propto d$$

d is increasing. Hence V will increase.

16. $C_i = \frac{(C)(2C)}{C + 2C} = \frac{2}{3}C$

$$q_i = C_i E = \frac{2}{3}EC$$

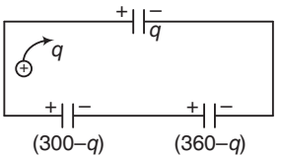
$$C_f = 2C$$

$$\therefore \quad q_f = 2EC$$

$$\Delta q = q_f - q_i$$

$$= \frac{4}{3}CE$$

17. Let $(+q)$ μC charge flows in the closed loop in clockwise direction. Then final charges on different capacitors are as shown in figure.



Now, applying Kirchhoff's loop law,

$$\frac{360 - q}{3} + \frac{300 - q}{2} = \frac{q}{1.5}$$

Solving the above equation, we get

$$q = 180\,\mu\text{C}$$

18. If the battery is disconnected, then q = constant

$$C = \frac{\varepsilon_0 A}{d} \quad \text{or} \quad C \propto \frac{1}{d}$$

d is decreased. Hence, C will increase.

$$U = \frac{1}{2}\frac{q^2}{C} \quad \text{or} \quad U \propto \frac{1}{C}$$

C in increasing. Hence, U will decrease.

$$V = \frac{q}{C} \quad \text{or} \quad V \propto \frac{1}{C}$$

C is increasing. Hence V will decrease.

19. ***(a)*** At $t = 0$, emf of the circuit = PD across the capacitor = 6 V.

$$\therefore \quad i = \frac{6}{1+2} = 2\text{A}$$

Half-life of the circuit

$$= (\ln 2)\,\tau_C\ (\ln 2)\,CR = (6 \ln 2)\text{ s}$$

In half-life time all values get halved. For example,

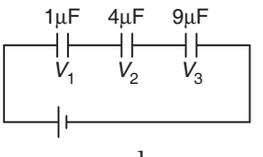
$$V_C = \frac{6}{2} = 3\text{V}$$

$$i = \frac{2}{1} = 1\text{A}$$

$$\therefore \quad V_{1\,\Omega} = iR = 1\text{V}$$

$$V_{2\Omega} = iR = 2\text{V}$$

20.



In series $V \propto \frac{1}{C}$ (as q = constant)

$$\therefore \quad \frac{V_2}{V_1} = \frac{1}{4}$$

or $$V_2 = \frac{V_1}{4}$$

$$= \frac{10}{4} = 2.5\text{ V}$$

$$\frac{V_3}{V_1} = \frac{1}{9}$$

$$\therefore \quad V_3 = \frac{V_1}{9} = \frac{10}{9}\text{ V}$$

Now $$E = \left(10 + 2.5 + \frac{10}{9}\right)\text{V} = \frac{325}{9}\text{ V}$$

21. $\oint_S \mathbf{E}\cdot d\mathbf{S}=0$ represents electric flux over the closed surface.

In general, $\oint_S \mathbf{E}\cdot d\mathbf{S}$ means the algebraic sum of number of flux lines entering the surface and number of flux lines leaving the surface.

When $\oint_S \mathbf{E}\cdot d\mathbf{S}=0$, it means that the number of flux lines entering the surface must be equal to the number of flux lines leaving it.

Now, from Gauss' law, we know that $\oint_S \mathbf{E}\cdot d\mathbf{S}=\frac{q}{\varepsilon_0}$ where q is charge enclosed by the surface. When $\int_S \mathbf{E}\cdot d\mathbf{S}=0$, $q=0$, i.e. net charge enclosed by the surface must be zero.

Therefore, all other charges must necessarily be outside the surface. This is because charges outside because of the fact that charges outside the surface do not contribute to the electric flux.

22. The electric field due to a charge Q at a point in space may be defined as the force that a unit positive charge would experience if placed at that point. Thus, electric field due to the charge Q will be continuous, if there is no charge at that point. It will be discontinuous if there is a charge at that point.

23. Gauss' law states that $\oint_S \mathbf{E}\cdot d\mathbf{S}=\frac{q}{\varepsilon_0}$, where q is the charge enclosed by the surface. If the charge is outside the surface, then charge enclosed by the surface is $q=0$ and thus, $\oint_S \mathbf{E}\cdot d\mathbf{S}=0$. Here, electric flux doesn't depend on the type or nature of charge.

24. When there are various types of charges in a region, but the total charge is zero, the region can be supposed to contain a number of electric dipoles.

Therefore, at points outside the region (may be anywhere w.r.t. electric dipoles), the dominant electric field $\propto \frac{1}{r_3}$ for large r.

Further, as electric field is conservative, work done to move a charged particle along a closed path, away from the region will be zero.

25. Gauss' law states that total electric flux of an enclosed surface is given by $\frac{q}{\varepsilon_0}$, where q is the charge enclosed by the surface. Thus, from figure,

Total charge inside the surface is $=Q-2Q=-Q$

$\therefore$ Total flux through the surface of the sphere $=\frac{-Q}{\varepsilon_0}$

Now, considering charge $5Q$. Charge $5Q$ lies outside the surface, thus it makes no contribution to electric flux through the given surface.

26. The positive charge Q is uniformly distributed at the outer surface of the enclosed sphere. Thus, electric field inside the sphere is zero.

So, the effect of electric field on charge q due to the positive charge Q is zero.

Now, the only governing factor is the attractive and repulsive forces between charges (Q and q) there are two cases arise.

***Case* I** When charge $q>0$, i.e. q is a positive charge, they creates a repulsive force between charge q and Q.

The repulsive forces of charge Q from all around the charge q will push it towards the centre if it is displaced from the centre of the ring.

Case **II** When charge $q<0$, i.e. q is a negative charge, then there is an attractive force between charge Q and q.

If q is shifted from the centre, then the positive charges nearer to this charge will attract it towards itself and charge q will never return to the centre.

27. The electric field intensity E is inversely proportional to the separation between equipotential surfaces. So, equipotential surfaces are closer in regions of large electric fields.

Since, the electric field intensities is large near sharp edges of charged conductor and near regions of large charge densities. Therefore, equipotential surfaces are closer at such places.

28. Work done in displacing a charge particle is given by $W_{12}=q(V_2-V_1)$ and the line integral of electrical field from point 1 to 2 gives potential difference $V_2-V_1=-\int_1^2 E.\mathrm{dI}$ For equipotential surface, $V_2-V_1=0$ and $W=0$.

29. The electric field intensity E and electric potential V are related as $E=0$ and for $V=$ constant, $\frac{dV}{dr}=0$

This imply that electric field intensity $E=0$.

30. The charge stored by capacitor C_1 gets redistributed between C_1 and C_2 till their potentials become same, i.e. $V_2=V_1$. By law of conservation of charge, the charge stored in capacitor C_1 when key K_1 is closed and key K_2 is open is equal to sum of charges on capacitors C_1 and C_2 when K_1 is opened and K_2 is closed, i.e.,

$$Q'_1+Q'_2=Q$$

31. The charge resides on the outer surface of a closed charged conductor.

32. ***Case* A** When key K is kept closed and plates of capacitors are moved apart using insulating handle, the separation between two plates increases which in turn decreases its capacitance $\left(C=\frac{K\varepsilon_0 A}{d}\right)$ and hence, the charge stored decreases as $Q=CV$ (potential continue to be the same as capacitor is still connected with cell).

***Case* B** When key K is opened and plates of capacitors are moved apart using insulating handle, charge stored by disconnected charged capacitor remains conserved and with the decreases of capacitance, potential difference V increases as $V=Q/C$.

Comprehension Based Questions

1. $V_{\text{outer}}=0$

$\therefore \quad \frac{KQ}{2r}+\frac{KQ_1}{2r}=0$

$\therefore \quad Q_1=-Q=$ charge on outer shell

2. $V_{\text{inner}}=0$

$\therefore \quad \frac{KQ_2}{r}+\frac{KQ_1}{2r}=0$

$\therefore \quad Q_2=-\frac{Q_1}{2}=\frac{Q}{2}=$ charge on inner shell

Charge flown through S_2 = initial charge on inner shell final charge on it

$$= Q - Q_2 = \frac{Q}{2}$$

3. After two steps charge on inner shell remains $\frac{Q}{2}$ or half.

So, after n-times

$$q_{\text{in}} = \frac{Q}{(2)^n}$$

Now, according to the principle of generator, potential difference depends on the inner charge only.

$$\therefore \quad \text{PD} = \frac{q_{\text{in}}}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{2r}\right]$$

$$= \frac{1}{2^{n+1}}\left[\frac{Q}{4\pi\varepsilon_0 r}\right]$$

4. According to Gauss theorem,

$$E = \left(\frac{1}{4\pi\varepsilon}\right)\left(\frac{q_{\text{in}}}{r^2}\right) \quad \ldots(\text{i})$$

For $r \le R$,

$$q_{\text{in}} = \int_0^r (4\pi r^2)\cdot dr \cdot \rho$$

$$= \int_0^r (4\pi r^2)(\rho_0)\left(1 - \frac{r}{R}\right)dr$$

$$= 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^4}{4R}\right)$$

Substituting in Eq. (i), we get

$$E = \frac{\rho_0}{\varepsilon}\left[\frac{r}{3} - \frac{r^2}{4R}\right]$$

5. For outside the ball,

$$E = \frac{1}{4\pi\varepsilon}\frac{q_{\text{total}}}{r^2} \quad \ldots(\text{i})$$

where, $q_{\text{total}} = \int_0^R (4\pi r^2)(\rho_0)\left(1 - \frac{r}{R}\right)dr$

Substituting this value in Eq. (i), we get

$$E = \frac{\rho_0 R^3}{12\,\varepsilon r^2}$$

6. For outside the ball, electric field will continuously decrease.

Hence, it will be maximum somewhere inside the ball. For maximum value,

$$\frac{dE}{dr} = 0$$

$$\therefore \quad \frac{d}{dr}\left[\frac{\rho_0}{\varepsilon}\left(\frac{r}{3} - \frac{r^2}{4R}\right)\right] = 0$$

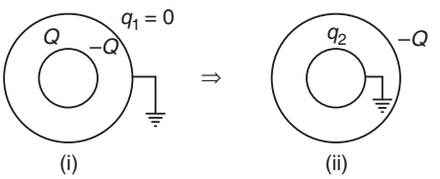
Solving, we get $r = \frac{2R}{3}$

7. Submitting $r = \frac{2R}{3}$ in the same expression of electric field, we get its maximum value.

8. Potential difference in such situation depends on inner charge only. So, potential difference will remain unchanged. Hence,

$$\Delta V = V_a - V_b$$

9.



$V_{\text{inner}} = 0$ when, solid sphere is earthed

$$\because \quad \frac{Kq_2}{a} - \frac{KQ}{b} = 0$$

$$\because \quad q_2 = Q\left(\frac{a}{b}\right)$$

10. Whole inner charge transfers to shell.

$\therefore$ Total charge on shell $= q_2 - Q$

$$= Q\left(\frac{a}{b} - 1\right)$$

11. Finally, the capacitors are in parallel and total charge ($= q_0$) distributes between them in direct ratio of capacity.

$$\therefore \quad q_{C_2} = \left(\frac{C_2}{C_1 + C_2}\right)q_0 \rightarrow \text{in steady state.}$$

But this charge increases exponentially.

Hence, charge on C_2 at any time t is

$$q_{C_2} = \left(\frac{C_2\, q_0}{C_1 + C_2}\right)(1 - e^{-t/\tau_C})$$

Initially, C_2 is uncharged so, whatever is the charge on C_2, it is charge flown through switches.

12. Common potential in steady state when they finally come in parallel is

$$V = \frac{\text{Total charge}}{\text{Total capacity}}$$

$$= \frac{q_0}{C_1 + C_2}$$

Total heat dissipated $= U_i - U_f$

$$= \frac{q_0^2}{2C_1} - \frac{1}{2}(C_1 + C_2)\left(\frac{q_0}{C_1 + C_2}\right)^2$$

$$= \left(\frac{q_0^2}{2C_1}\right)\left(\frac{C_1 C_2}{C_1 + C_2}\right)$$

13. $E_{\text{air}} = E_0 = \frac{V}{d}$

14. $E_{\text{dielectric}} = \frac{E_0}{K} = \frac{V}{Kd}$

Assertion and Reason

2. $\Delta U = -$ work done by electrostatic field.

3. Positive charge flows from higher potential to lower potential.

4. For given ring $V = 0$ at centre but electric field is downwards.

5. Charge on both will become zero.

$$\therefore \quad U_f = 0$$

Hence, U_i is completely lost.

6. If sphere is positively charged $V_1 > V_2$ and if it is negatively charged $V_1 < V_2$.

7. For non-uniformly charged ring with $Q_{net} = 0$

$$V_x = 0 \quad \text{but } E_X \neq 0$$

8. $\phi_{net} = \dfrac{q_{in}}{\varepsilon_0}$

$$\phi_{net} = 0$$

$$\therefore \quad q_{in} = 0$$

9. Rate of change of electric field with distance from centre at some point, represents magnitude of electric potential at that point, that is wrong. Because $E = -\dfrac{dV}{dr}$. Hence, the correct is $E =$ – (rate of change of electric potential at that point). Further, electric potential inside the sphere varies from $1.5\dfrac{Kq}{R}$ to $\dfrac{Kq}{R}$ inside the sphere, whereas outside the sphere it varies from $\dfrac{Kq}{R}$ to zero. Hence, $V_P \neq V_Q$ on surface.

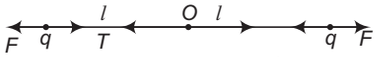
10. Specific charge $= \dfrac{e}{m}$

11. Five forces of equal magnitude are acting on –Q. When they are added as per polygon law their vector sum is zero.

12. $E = -\dfrac{dV}{dr}$

Therefore, if V is parabolic, E will be linear.

15.



$$T = \frac{1}{4\pi\varepsilon_0}\frac{q \cdot q}{(2l)^2}$$

16. Not required.

17. Energy supplied by battery $= qV$

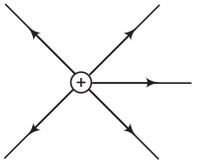
$$= (CV)\,V = CV^2$$

$$\text{Energy stored} = \frac{1}{2}CV^2$$

$$\therefore \quad \text{Energy lost} = CV^2 - \frac{1}{2}CV^2$$

$$= \frac{1}{2}CV^2$$

18. Lines are not parallel and equidistant. Therefore, field is non-uniform.



19. Increase does not depend on the conductivity of conducting plate.

Match the Columns

1. Under all conditions potential at centre will remain unchanged.

2. $U \propto \dfrac{1}{r}$ and $F \propto \dfrac{1}{r^2}$

3. $q = CV, U - \dfrac{1}{2}CV^2,$

$C = \dfrac{k\varepsilon_0 A}{d}$ or $\dfrac{\varepsilon_0 A}{d - t}$

Entrance Gallery

1. For the combination of parallel plate capacitor, $C = C_1 + C_2$

But $\quad C_1 = \dfrac{K\varepsilon_0 A/3}{d}$ and $C_2 = \dfrac{\varepsilon_0 2A/3}{d}$

$$\therefore \quad C = \frac{(K+2)\varepsilon_0 A}{3d} \quad \Rightarrow \quad \frac{C}{C_1} = \frac{K+2}{K}$$

Also, $E_1 = E_2 = \dfrac{V}{d}$, where V is potential difference between the plates.

2. As we know that potential difference $V_A - V_0$ is

$$dV = -Edx \quad \Rightarrow \quad \int_{V_0}^{V_A} dV = -\int_0^2 30x^2\,dx$$

$$V_A - V_0 = -30 \times \left[\frac{x^3}{3}\right]_0^2 = -10 \times [2^3 - (0)^3] = -10 \times 8 = -80\,\text{J}$$

3. When free space between parallel plates of capacitor, the electric field, $E = \dfrac{\sigma}{\varepsilon_0}$

When dielectric is introduced between parallel plates of capacitor, $E' = \dfrac{\sigma}{K\varepsilon_0}$.

Electric field inside dielectric

$$\frac{\sigma}{K\varepsilon_0} = 3 \times 10^4$$

where, K = dielectric constant of medium = 2.2

and $\quad \varepsilon_0 =$ permittivity of free space $= 8.85 \times 109^{-12}\ \text{C}^2/\text{N - m}^2$.

$$\Rightarrow \quad \sigma = 2.2 \times 8.55 \times 10^{-12} \times 3 \times 10^4$$

$$= 6.6 \times 8.85 \times 10^{-8} = 5.643 \times 10^{-7}$$

$$= 6 \times 10^{-7}\,\text{C/m}^2$$

4. By energy stored in a charged conductor.

Suppose, a conductor of capacity C is charged to a potential V and Q the charge on conductor at an instant.

The potential of the conductor,

$$V = \frac{Q}{C}$$

Now, work done in bringing a small charge dQ at this potential is given by

$$dW = V\,dQ = \frac{Q}{C}\,dQ$$

$\therefore$ Total work done in charging it from 0 to Q is

$$W = \int_0^Q dW = \int_0^Q \frac{Q}{C}\,dQ$$

$$= \frac{1}{2}\frac{Q^2}{C}$$

The work is stored as the potential energy,

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV \qquad (\because Q = VC)$$

5. Given, $C_1 = 10\,\text{pF} = 10 \times 10^{-12}\,\text{F}$,

$C_2 = 20\,\text{pF} = 20 \times 10^{-12}\,\text{F}$,

$V_1 = 200\,\text{V}$ and $V_2 = 100\,\text{V}$

C_1 = Capacitance of Ist capacitor

C_2 = Capacitance of IInd capacitor

V_1 = Voltage across Ist capacitor

V_2 = Voltage across IInd capacitor

We know that, $V_1 = \frac{q_1}{C_1}$ and $V_2 = \frac{q_2}{C_2}$

$\Rightarrow \quad q_1 = V_1 C_1 \quad$ and $\quad q_2 = V_2 C_2$

So, common potential of capacitors

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{V_1 C_1 + V_2 C_2}{C_1 + C_2}$$

$$= \frac{200 \times 10 \times 10^{-12} + 100 \times 20 \times 10^{-12}}{10 \times 10^{-12} + 20 \times 10^{-12}}$$

$$= \frac{200 \times 10 + 100 \times 20}{10 + 20}$$

$$= \frac{2000 + 2000}{30} = \frac{4000}{30} = 133.3 \text{ V}$$

6. Given, $q = 1 \text{ nC} = 1 \times 1 \times 10^{-9} \text{ C} \quad (\because 1 \text{ nC} = 1 \times 10^{-9} \text{ C})$

$e = 1.6 \times 10^{-19} \text{ C} \quad \Rightarrow \quad n = ?$

From the property of quantisation charge,

$$q = ne$$

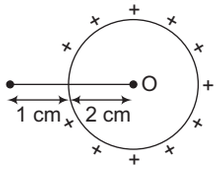
$$n = \frac{q}{e} = \frac{1 \times 10^{-9}}{1.6 \times 10^{-19}}$$

$$= 0.625 \times 10^{-9} \times 10^{19}$$

$$= 0.625 \times 10^{10} = 6.25 \times 10^{9}$$

7. Given,

Distance from the centre of sphere



$$r = (1 + 2) \text{ cm} = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$$

Charge on sphere, $q = 3 \text{ nC} = 3 \times 10^{-9} \text{ C}$

We know that $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$

$$= 9 \times 10^{9} \times \frac{3 \times 10^{-9}}{(3 \times 10^{-2})^2}$$

$$= \frac{9 \times 3}{9 \times 10^{-4}} = 3 \times 10^{4} \text{ V/m}$$

8. Given, $\frac{m_1}{m_2} = 0.5 \quad \Rightarrow \quad F_1 = F_2 = F$

$\because$ (force experienced by equal charge in electric field)

We know that, $F_1 = q_1 E$

$\Rightarrow \qquad F_2 = q_2 E \qquad$ (as $q_1 = q_2$)

Also, $\qquad F_1 = m_1 a_1 \qquad \ldots$(i)

$\qquad F_2 = m_2 a_2 \qquad \ldots$(ii)

From Eqs. (i) and (ii), we get

$$\frac{a_1}{a_2} = \frac{F_1 / m_1}{F_2 / m_2} \quad \Rightarrow \quad \frac{a_1}{a_2} = \frac{F \cdot m_2}{F \cdot m_1}$$

$$\frac{a_1}{a_2} = \frac{m_2}{m_1} = \frac{1}{0.5} = \frac{10}{5} = 2 \qquad \left\{\because \frac{m_1}{m_2} = 0.5\right\}$$

9. The Gauss's law in electrostatics gives a relation between electric flux through any closed hypothetical surface (called a Gaussian surface) and the charge enclosed by the surface.

So, the nature is area vector.

10. Let, the charge on the inner shell be q.

The total charge will be zero.

So, $\qquad \frac{Kq'}{r_1} + \frac{Kq}{r_2} = 0 \qquad (\because r_2 > r_1)$

$\Rightarrow \qquad q' = -\frac{Kq/r_2}{K/r_1} = -\left(\frac{r_1}{r_2}\right) q \quad$ or $\quad -\frac{r_1}{r_2} \cdot q$

11. Given, $q = 3 \text{ nC} = 3 \times 10^{-9} \text{ C}$ and $r = 9 \text{ cm} = 9 \times 10^{-2} \text{ m}$

We know that potential due to point charge is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} = 9 \times 10^{9} \times \frac{3 \times 10^{-9}}{9 \times 10^{-2}}$$

$$V = 3 \times 10^{2} = 300 \text{ V} \qquad \left(\because \frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9} \text{ N-m}^2/\text{C}^2\right)$$

12. We know that, The parallel plate capacitor, $C = \frac{\varepsilon_0 \cdot A}{d}$

When d (distance between two parallel plate) increase, then C will be decreases.

$\because \quad Q = CV$, where, $Q =$ constant

$\therefore \quad$ The voltage across the capacitor increases.

13. The combination of three charges in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{6} + \frac{1}{6}$$

$$\Rightarrow \qquad C = \frac{6}{4} = 1.5 \text{ } \mu\text{F}$$

The charge of this circuit, $q = CV = 1.5 \times 20 = 180 \text{ } \mu\text{C}$

The potential difference across the 3 μF

$$q = CV$$

$$\Rightarrow \qquad V = \frac{q}{C} = \frac{180}{3} = 60 \text{ V}$$

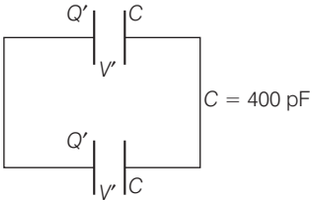
14. Given, capacitor $C = 400 \times 10^{-12} \text{ F}$ and $V = 100 \text{ V}$

Initially, energy stored $= \frac{1}{2} CV^2 = \frac{1}{2} [400 \times 10^{-12} \times (10^2)^2]$

$$= \frac{1}{2} \times 4 \times 10^{-10} \times 10^{4} = 2 \times 10^{-6} \text{ J}$$

$\Rightarrow \qquad$ Charge, $Q = CV = 4 \times 10^{-10} \times 100 = 4 \times 10^{-8} \text{ C}$

By again connecting the charged capacitor with a uncharged ideal capacitor.



Now, the charge Q will divide into two capacitors.

As V' and C are same for both, Q' will be same for both.

So, $\qquad Q' = \frac{Q}{2} = 2 \times 10^{-8} \text{ C}$

So, energy stored in both the capacitors $= \left(\frac{(Q')^2}{2C}\right) = \frac{(Q')^2}{2C}$

$$= \frac{(2 \times 10^{-8})^2}{8 \times 10^{-10}} = 10^{-6} \text{ J}$$

$$= 0.5 \times 10^{-6} \text{ J}$$

So, energy lost $= 2 \times 10^{-6} - 0.5 \times 10^{-6} = 1.5 \times 10^{-6} \text{ J} \approx 10^{-6} \text{ J}$

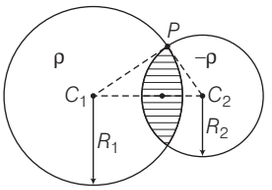
$\approx 1.5 \times 10^{-6} \text{ J} = 10^{-6} \text{ J}$

15. Electrostatic energy, $U=\frac{1}{2}CV^2=\frac{1}{2}(15\times10^{-12})\times(60)^2$

$$=\frac{1}{2}\times15\times36\times10^{-12}\times10^2$$

$$=270\times10^{-10}=2.70\times10^{-8}\ \text{J}$$

16.

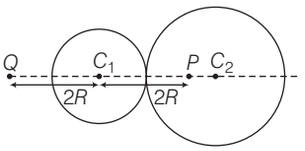


For electrostatic field, $\mathbf{E}_p=\mathbf{E}_1+\mathbf{E}_2=\frac{\rho}{3\varepsilon_0}\mathbf{C}_1\mathbf{P}+\frac{(\rho)}{3\varepsilon_0}\mathbf{C}_1\mathbf{P}$

$$=\frac{\rho}{3\varepsilon_0}(\mathbf{C}_1\mathbf{P}+\mathbf{C}\mathbf{P}_2)=\frac{\rho}{3\varepsilon_0}\mathbf{C}_1\mathbf{C}_2$$

Since, electric field is non-zero, so it is not equipotential.

17. At point P,



If resultant electric field is zero, then

$$\frac{KQ_1}{4R^2}=\frac{KQ_2}{8R^3}R,\quad \frac{\rho_1}{\rho_2}=4$$

At point Q,

If resultant electric field is zero, then

$$\frac{KQ_1}{4R^2}+\frac{KQ_2}{25R^2}=0$$

$$\frac{\rho_1}{\rho_2}=-\frac{32}{25}\qquad(\rho_1\text{ must be negative})$$

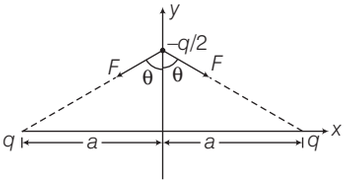
18. After pressing S_1, charge on upper plate of C_1 is $+2CV_0$. After pressing S_2, this charge equally distributes in two capacitors. Therefore, charge on upper plates of both capacitors will be $+CV_0$.

When S_2 is released and S_3 is pressed, charge on upper plate of C_1 remains unchanged $(=CV_0)$ but charge on upper plate of C_2 is according to new battery $(=CV_0)$.

19. Polarity should be mentioned in the question. Potential on each of them can be zero if, $q_{net}=0$.

$q_1\pm q_2=0\ \Rightarrow\ 120C_1\pm200C_2=0\ \Rightarrow\ 3C_1\pm5C_2=0$

20.

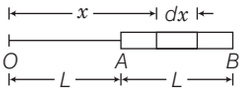


$F_{net}=2F\cos\theta$

$$F_{net}=\frac{2kq\left(\frac{q}{2}\right)}{(\sqrt{y^2+a^2})^2}\cdot\frac{y}{\sqrt{y^2+a^2}}$$

$$F_{net}=\frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}}\ \Rightarrow\ \frac{kq^2y}{a^3}\propto y$$

21.



$$V=\int_L^{2L}\frac{kdQ}{x}=\int_L^{2L}\frac{k\left(\frac{Q}{L}\right)dx}{x}$$

$$=\frac{Q}{4\pi\varepsilon_0L}\int_L^{2L}\left(\frac{1}{x}\right)dx=\frac{Q}{4\pi\varepsilon_0L}[\log_e x]_L^{2L}$$

$$=\frac{Q}{4\pi\varepsilon_0L}[\log_e 2L-\log_e L]=\frac{Q}{4\pi\varepsilon_0L}\ln 2$$

22. When plates of capacitor are separated by a dielectric medium of dielectric constant K, its capacity

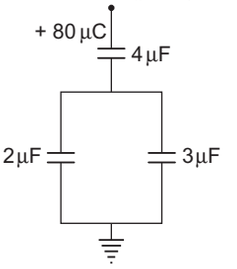
$$C_m=\frac{K\varepsilon_0A}{d}=KC_0$$

$\therefore\quad C_m=KC\qquad(\text{here, }C_0=C)$

Now, two capacitors of capacities KC and C are in series, that effective capacitance

$$\frac{1}{C'}=\frac{1}{KC}+\frac{1}{C}\ \Rightarrow\ \frac{1}{C'}=\frac{1+K}{KC}\ \Rightarrow\ C'=\frac{KC}{K+1}$$

23. Between 3 μF and 2 μF (in parallel) total charge of 80 μC will distribute in direct ratio of capacity.



$\therefore\quad \frac{q_3}{q_2}=\frac{3}{2}\ \Rightarrow\ q_3=\left(\frac{3}{3+2}\right)(80)=48\,\mu\text{C}$

24. Option (a) is correct due to symmetry.

Option (b) is wrong again due to symmetry.

Option (c) is correct because as per Gauss' theorem, net electric flux passing through any closed surface $=\frac{q_{in}}{\varepsilon_0}$.

Here, $q_{in}=3q-q-q=q$

$\therefore\quad$ Net electric flux $=\frac{q}{\varepsilon_0}$

Option (d) is wrong, because there is no symmetry in two given planes.

25. **For inside point** $(r\le R)$, $E=0$

$$V=\text{constant}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$

For outside point $(r\ge R)$,

$$E=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\quad\text{or}\quad E\propto\frac{1}{r^2}$$

and $$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\quad\text{or}\quad V\propto\frac{1}{r}$$

On the surface $(r=R)$

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}\ \Rightarrow\ E=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2}=\frac{\sigma}{\varepsilon_0}$$

where, $\sigma = \dfrac{q}{4\pi R^2}$ = surface charge density corresponding to above equations, the correct graphs are shown in option (d).

26. Volume of cylinder per unit length ($l = 1$) is

$$V = \pi R^2 l = \pi R^2$$

$\therefore$ Charge per unit length, λ = Volume per unit length $\times$ Volume charge density

$$= \pi R^2 \rho$$

Now at P, electric field, $E_R = E_T - E_C$

where, R = remaining portion

T = total portion and C = cavity

$$\therefore \quad E_R = \frac{\lambda}{2\pi\varepsilon_0(2R)} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)^2}$$

Q = Charge on sphere $= \dfrac{4}{3}\pi\left(\dfrac{R}{2}\right)^3 \rho = \dfrac{\pi R^3 \rho}{6}$

Substituting the values, we have

$$E_R = \frac{(\pi R^2\rho)}{4\pi\varepsilon_0 R} - \frac{1}{4\pi\varepsilon_0}\cdot\frac{(\pi R^3\rho/6)}{4R^2}$$

$$= \frac{\rho R}{4\varepsilon_0} - \frac{1}{4\varepsilon_0}\cdot\frac{\rho R}{24} = \frac{1}{4\varepsilon_0}\left[\rho R - \frac{\rho R}{24}\right]$$

$$E_R = \frac{23\rho R}{96\,\varepsilon_0}$$

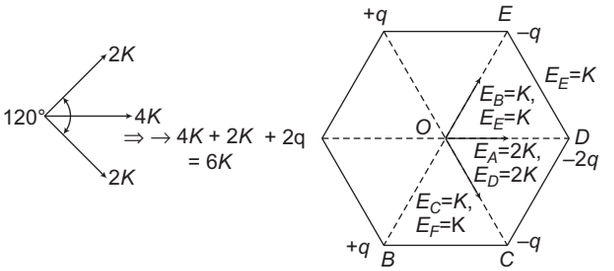
But $$E_R = \frac{23\rho R}{16\,K\,\varepsilon_0} \qquad \text{(given)}$$

So, $$\frac{23\rho R}{16K\,\varepsilon_0} = \frac{23\rho R}{(16)(6)\,\varepsilon_0}$$

Comparing the both sides, we get

$$K = 6$$

27.



(a) Resultant of $2K$ and $2K$ (at 120°) is also $2K$ towards $4K$.

Therefore, net electric field is $6K$.

(b) $V_0 = \dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{q_A}{L} + \dfrac{q_B}{L} + \dfrac{q_C}{L} + \dfrac{q_D}{L} + \dfrac{q_E}{L} + \dfrac{q_F}{L}\right]$

$$= \frac{1}{4\pi\varepsilon_0 L}(q_A + \ldots + q_F) = 0$$

Because $q_A + q_B + q_C + q_D + q_E + q_F = 0$

(c) Only line PR, potential is same (= 0).

28. Statement 1 is dimensionally wrong while from Gauss' law,

$$E(4\pi r^2) = \frac{\rho\cdot\dfrac{4}{3}\pi r^3}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho r}{3\varepsilon_0}$$

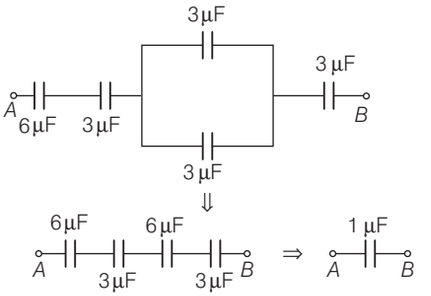
This gives statement 2 is correct.

29. Electric field inside the uniformly charged sphere varies linearly, $E = \dfrac{KQ}{R^3}\cdot r, (r \le R)$, while outside the sphere, it varies as inverse square of distance, $E = \dfrac{KQ}{r^2}; (r \ge R)$

which is correctly represented in option (c).

30. Let us first calculate equivalent capacitance between A and B.

Hence, $C_e = 1\ \mu F$



$\therefore$ Charge on the combination, $q = CV = 1 \times 60 = 60\ \mu C$

As 6 μF capacitor is in series with all other capacitors, hence charge on 6 μF capacitance is also 60 μC.

$\therefore$ Potential difference across 6 μF capacitor

$$V_1 = \frac{q}{C_1} = \frac{60}{6} = 10\ V$$

31. When charge q is placed in uniform electric field E, then its acceleration,

$$a = \frac{qE}{m}$$

So, its motion will be uniformly accelerated motion and its velocity after time t is given by

$$v = at = \frac{qE}{m}t$$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qEt}{m}\right)^2 = \frac{q^2E^2t^2}{2m}$$

32. Electric field due to the two infinite parallel metal plates are given by

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$E = \dfrac{\sigma}{\varepsilon_0}$ towards negatively charged plate.

33. We know that $W = q\Delta V$ and $V = \dfrac{Q}{4\pi\varepsilon_0 r}$

$$W = q\left[\frac{Q}{4\pi\varepsilon_0 b} - \frac{Q}{4\pi\varepsilon_0 a}\right]$$

$$= \frac{Qq}{4\pi\varepsilon_0}\left[\frac{a-b}{ab}\right]$$

34. Electric flux, $\phi = \mathbf{E}\cdot\mathbf{S}$ or $\phi = ES\cos\theta$

Here, θ is the angle between E and S. In this question, $\theta = 45°$, because $\mathbf{S}$ is perpendicular to the surface.

$$E = E_0$$

$$S = (\sqrt{2}a)(a) = \sqrt{2}a^2$$

$$\therefore \quad \phi = (E_0)(\sqrt{2}a^2)\cos 45° = E_0 a^2$$

35. Frequency or time period of SHM depends on variable forces. It does not depend on constant external force. Constant external force can only change the mean position. For example, in the given question, mean position is at natural length of spring in the absence of electric field. Whereas in the presence of electric field, mean position will be obtained after a compression of x_0. Where, x_0 is given by

$$kx_0 = QE \quad \Rightarrow \quad x_0 = \frac{QE}{k}$$

36. Inside a conducting shell, electric field is always zero. Therefore, option (a) is correct. When the two are connected, their potentials become the same

$$\therefore \qquad V_A = V_B \quad \text{or} \quad \frac{Q_A}{R_A} = \frac{Q_B}{R_B} \left(V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \right)$$

Since, $\qquad R_A > R_B \quad \Rightarrow \quad Q_A > Q_B$

Potential is also equal to,

$$V = \frac{\sigma R}{\varepsilon_0} \quad \Rightarrow \quad V_A = V_B$$

$$\therefore \qquad \sigma_A R_A = \sigma_B R_B \quad \Rightarrow \quad \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A} \quad \text{or} \quad \sigma_A < \sigma_B$$

Electric field on surface, $E = \dfrac{\sigma}{\varepsilon_0}$ or $E \propto \sigma$

Since, $\qquad \sigma_A < \sigma_B \quad \Rightarrow \quad E_A < E_B$

$\therefore$ Correct options are (a), (b), (c) and (d).

37. If charges are of opposite signs, then the two fields are along the same direction. So, they cannot be zero.

Hence, the charges should be of same sign.

Therefore, option (c) is correct.

Further, work done by external force = change in potential energy

$$\therefore \qquad W_{A \to B} = q(\Delta V) = (+1)(V_B - V_A) = V_B - V_A$$

Therefore, option (d) is also correct.

$\therefore$ Correct options are (c) and (d).

38. This charge will remain constant after switch is shifted from position 1 to position 2.

i.e. $\qquad q_i = C_i V = 2V = q \qquad$ (say)

$$U_i = \frac{1}{2}\frac{q^2}{C_i} = \frac{q^2}{2\times 2} = \frac{q^2}{4}$$

$$U_f = \frac{1}{2}\frac{q^2}{C_f} = \frac{q^2}{2\times 10} = \frac{q^2}{20}$$

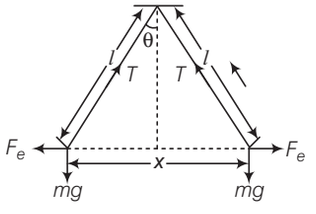
$\therefore$ Energy dissipated $\left(u_i - u_f = \dfrac{q^2}{5} \right)$ is 80% of the initial stored energy $\left(= \dfrac{q^2}{4} \right)$

39. At any instant,

$$T\cos\theta = mg \qquad \ldots(\text{i})$$

$$T\sin\theta = F_e \qquad \ldots(\text{ii})$$

$$= \frac{kq^2}{x^2}$$



From Eqs. (i) and (ii), we have

$$\frac{kq^2}{x^2} = mg\tan\theta$$

$$q^2 = \frac{mg}{k}\frac{x}{2l}x^2 \qquad \left(\tan\theta \approx \frac{x}{2l}\right)$$

$$\Rightarrow \qquad q^2 = \frac{mg}{2kl}x^3 \qquad \ldots(\text{iii})$$

$$\Rightarrow \qquad 2q\frac{dq}{dt} = \frac{3mg}{2kl}x^2\frac{dx}{dt}$$

$$\Rightarrow \qquad 2\left(\frac{mg}{2kl}x^3\right)^{1/2}\frac{dq}{dt} = \frac{3mg}{2kl}x^2 v \qquad \left[\because q = \left(\frac{mg}{2kl}x^3\right)^{1/2}\right]$$

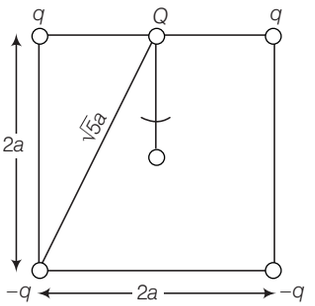
$$\Rightarrow \qquad v x^{1/2} = \text{constant}$$

$$\Rightarrow \qquad v \propto x^{-1/2}$$

40. Potential Energy, $U_i = \dfrac{2kqQ}{a} + \dfrac{2k(-q)Q}{\sqrt{5}a}$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQ}{a}\left(1 - \frac{1}{\sqrt{5}}\right)$$

$$U_f = 0$$



By conservation of energy,

Gain in KE = Loss in PE

$$K = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQ}{a}\left(1 - \frac{1}{\sqrt{5}}\right)$$

41. Electric field, $E = -\dfrac{d\phi}{dt} = -2ar$

By Gauss' theorem,

$$E(4\pi r^2) = \frac{q}{\varepsilon_0} \quad \Rightarrow \quad q = -8\pi\varepsilon_0 a r^3$$

$$\rho = \frac{dq}{dV} = \frac{dq}{dr}\times\frac{dr}{dV}$$

$$= (-24\pi\varepsilon_0 a r^2)\times\frac{1}{4\pi r^2}$$

$$\Rightarrow \qquad \rho = -6\varepsilon_0 a$$

42. $\text{KE}_1 = \dfrac{1}{2}mv^2 = \dfrac{kQq}{r_1}$ ...(i)

$$\text{KE}_2 = \frac{1}{2}m(4v^2) = \frac{kQq}{r_2} \qquad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\frac{r_1}{r_2} = 4$$

$$\Rightarrow \qquad r_2 = \frac{r}{4} \qquad (\because r_1 = r)$$

43. We know that, $V = \dfrac{Kq}{r}$

$$\frac{Kq_1}{3\times10^{-2}} = 10V, \frac{Kq_2}{1\times10^{-2}} = 10V$$

$$Kq_1 = 30\times10^{-2}\text{ C} \quad \Rightarrow \quad Kq_2 = 10\times10^{-2}\text{ C}$$

$$F = \frac{Kq_1q_2}{r^2} = \frac{(30\times10^{-2})\times(10\times10^{-2})}{K\times10^{-2}}$$

$$F = \frac{1}{3}\times10^{-9}\text{ N}$$

44. Dipole energy, $U = -pE\cos\theta$

when, U is maximum

$$\cos\theta = -1 \quad \Rightarrow \quad \theta = \pi$$

45. Here, $F = 3.7\times10^{-9}$ N

Let, $q_1 = q_2 = q \quad \Rightarrow \quad r = 5\text{ Å} = 5\times10^{-10}$ m

$\because$ $$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}$$

$\Rightarrow$ $$3.7\times10^{-9} = 9\times10^9\times\frac{q\times q}{(5\times10^{-10})^2}$$

$$q^2 = \frac{3.7\times10^{-9}\times25\times10^{-20}}{9\times10^9}$$

$$= 10.28\times10^{-38}$$

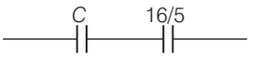
or $$q = 3.2\times10^{-19}\text{ C}$$

Now, $$q = ne$$

$\therefore$ $$n = \frac{q}{e} = \frac{3.2\times10^{-19}}{1.6\times10^{-19}} = 2$$

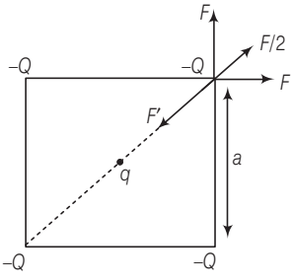
46. Electric flux, $\phi = \dfrac{1}{\varepsilon_0}q = \dfrac{100Q}{\varepsilon_0}$

47. The equivalent capacitance between A and B,



$$3 = \frac{\frac{16}{5}C}{\frac{16}{5}+C} \quad \text{or} \quad C = 48\,\mu\text{F}$$

48. Force, $F' = \sqrt{2}F + \dfrac{F}{2} = F\left(\sqrt{2}+\dfrac{1}{2}\right)$



$$\frac{qQ}{\left(\frac{\sqrt{2}a}{2}\right)^2} = \frac{Q^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

$$q = \frac{Q}{2}\left(\frac{2\sqrt{2}+1}{2}\right)$$

$$= \frac{Q}{4}(2\sqrt{2}+1)$$

49. From the behaviour of electric lines, we can say that Q_1 is positive and Q_2 is negative. Further, $|Q_1| > |Q_2|$.

At some finite distance to the right of Q_2, electric field will be zero. Because, electric field due to Q_1 is towards right (away from Q_1) and due to Q_2 is towards left (towards Q_2). But since magnitude of Q_1 is more, the two fields may cancel each other because distance of that point from Q_1 will also be more.

$\therefore$ The correct options are (a) and (d).

50. Electrical force per unit area $= \dfrac{1}{2}\varepsilon_0 E^2 = \dfrac{1}{2}\varepsilon_0\left(\dfrac{\sigma}{\varepsilon_0}\right)^2 = \dfrac{\sigma^2}{2\varepsilon_0}$

Projected area $= \pi R^2$

$\therefore$ Net electrical force $= \left(\dfrac{\sigma^2}{2\varepsilon_0}\right)(\pi R^2)$

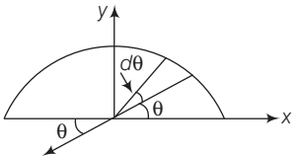
In equilibrium, this force should be equal to the applied force.

$\therefore$ $$F = \frac{\pi\sigma^2R^2}{2\varepsilon_0} \quad \text{or} \quad F \propto \frac{\sigma^2R^2}{\varepsilon_0}$$

51. Linear charge density, $\lambda = \left(\dfrac{q}{\pi r}\right)$

$$E = \int dE\sin\theta(-\hat{\mathbf{j}})$$

$$= \int\frac{K\cdot dq}{r^2}\sin\theta(-\hat{\mathbf{j}})$$



$$E = \frac{K}{r^2}\int\frac{qr}{\pi r}\,d\theta\sin\theta(-\hat{\mathbf{j}})$$

$$= \frac{K}{r^2}\cdot\frac{q}{\pi}\int_0^{\pi}\sin\theta(-\hat{\mathbf{j}}) \quad \Rightarrow \quad E = \frac{q}{2\pi^2\varepsilon_0r^2}(-\hat{\mathbf{j}})$$

52. Apply shell theorem, the total charge upto distance r can be calculated as followed

$$dq = 4\pi r^2\cdot dr\cdot\rho$$

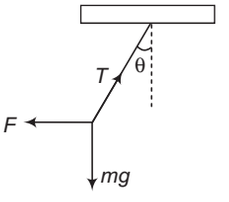
$$= 4\pi r^2\cdot dr\cdot\rho_0\left[\frac{5}{4}-\frac{r}{R}\right] = 4\pi\rho_0\left[\frac{5}{4}r^2\,dr - \frac{r^3}{R}\,dr\right]$$

$$\int dq = q = 4\pi\rho_0\int_0^r\left(\frac{5}{4}r^2\,dr - \frac{r^3}{R}\,dr\right) = 4\pi\rho_0\left[\frac{5}{4}\frac{r^3}{3} - \frac{1}{R}\frac{r^4}{4}\right]$$

$$E = \frac{kq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\cdot4\pi\rho_0\left[\frac{5}{4}\left(\frac{r^3}{3}\right) - \frac{r^4}{4R}\right] = \frac{\rho_0r}{4\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$$

53. From FBD of sphere, using Lami's theorem,

$$\frac{F}{mg} = \tan\theta \qquad \ldots(\text{i})$$



When suspended in liquid, as θ remains same,

$$\therefore \quad \frac{F'}{mg\left(1-\frac{\rho}{d}\right)} = \tan\theta \qquad \ldots(ii)$$

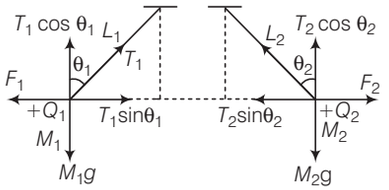
Using Eqs. (i) and (ii), we get

$$\frac{F}{mg} = \frac{F'}{mg\left(1-\frac{\rho}{d}\right)}$$

where, $F' = \frac{F}{k}$

$$\therefore \quad \frac{F}{mg} = \frac{F}{mgK\left(1-\frac{\rho}{d}\right)} \quad \Rightarrow \quad K = \frac{1}{1-\frac{\rho}{d}} = \frac{1}{\left(1-\frac{0.8}{1.6}\right)} = 2$$

54. For sphere 1, in equilibrium



$$T_1 \cos\theta_1 = M_1 g$$

and $T_1 \sin\theta_1 = F_1$

$$\therefore \quad \tan\theta_1 = \frac{F_1}{M_1 g}$$

Similarly for sphere 2, $\tan\theta_2 = \frac{F_2}{M_2 g}$

F is same on both the charges, θ will be same only if their masses M are equal.

55. From the given figure, the labelled point have the same electric potential as the fully shaded point is three.

56. Original charges on spheres A and B be q_1 and q_2 respectively.

Distance between the two spheres $= r$

Since, both the spheres are of same size, they will possess equal charges on being brought in contact.

$$\therefore \quad q_1' = \frac{q_1 + q_2}{2}$$

Similarly, $q_2' = \frac{q_1 + q_2}{2}$

Therefore, new force of repulsion between spheres A and B is

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{\left[\frac{q_1+q_2}{2}\right]\left[\frac{q_1+q_2}{2}\right]}{r^2} = \frac{\left[\frac{q_1+q_2}{2}\right]^2}{4\pi\varepsilon_0 r^2}$$

As, $\left[\frac{q_1+q_2}{2}\right]^2 > q_1 q_2$

$$\therefore \quad F' > F$$

57. Here, magnetic force = electrostatic force

$$qvB = qE$$

$$\Rightarrow \quad v = \frac{E}{B} = \frac{\sigma}{\varepsilon_0 B}$$

The time taken by an electron to travel a distance l in that space, then

$$t = \frac{l}{v} = \frac{l}{\frac{\sigma}{\varepsilon_0 B}} = \frac{\varepsilon_0\, lB}{\sigma}$$

58. In a parallel plate capacitor, the capacity of capacitor,

$$C = \frac{k\,\varepsilon_0 A}{d}$$

$$\therefore \quad C \propto A$$

So, the capacity of capacitor increase if area of the plate is increased.

59. Change in energy, $\Delta U = \frac{1}{2}\left[\frac{q_1^2 - q_2^2}{C}\right]$

$$= \frac{1}{2}\left[\frac{(0.5)^2 - (0.1)^2}{48\times10^{-6}}\right]$$

$$= \frac{1}{2}\left[\frac{0.25 - 0.01}{48\times10^{-6}}\right]$$

$$= \frac{1}{2}\left[\frac{0.24}{48\times10^{-6}}\right]$$

$$= \frac{1}{2}\left[\frac{10^4}{2}\right] = 2500\,\text{J}$$

60. The electric intensity outside a charged sphere,

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

19

Current Electricity

19.1 Introduction

An electrical circuit consists of some active and passive elements. The active elements such as a battery or a cell supply, electric energy to the circuit. On the contrary, passive elements consume or store the electric energy. The basic passive elements are resistor, capacitor and inductor.

A resistor opposes the flow of current through it and if some current is passed by maintaining a potential difference across it, some energy is dissipated in the form of heat. A capacitor is a device which stores energy in the form of electric potential energy. It opposes the variations in voltage. An inductor opposes the variations in current. It does not oppose the steady current through it. Fundamentally, electric circuits are a means for conveying energy from one place to another. As charged particles move within a circuit, electric potential energy is transferred from a source (such as a battery or a cell) to a device in which that energy is either stored or converted to another form, into sound in a stereo system or into heat and light in a toaster or light bulb. Electric circuits are useful, because they allow energy to be transported without any moving parts (other than the moving charged particles themselves).

In this chapter, we will study the basic properties of electric currents and batteries and how they cause current and energy transfer in a circuit. In this analysis we will use the concepts of current, potential difference, resistance and electromotive force.

19.2 Electric Current

At room temperature, the free electrons in a conductor move randomly with speeds on the order of 10^5 m/s. Since, the motion of the electrons is random, there is no net charge flow in any direction. For any imaginary plane passing through the conductor, the number of electrons crossing the plane in one direction is equal to the number crossing it in the other direction.

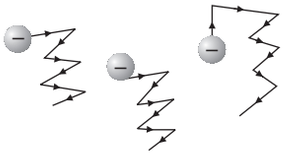


Fig. 19.1 Random motion of free electrons in a conductor

Chapter Snapshot

- Introduction
- Electric Current
- Resistance and Ohm's Law
- Temperature dependence of Resistivity and Resistance
- The Battery and Electromotive Force
- Direct Current Circuits, Kirchhoff's Laws
- Heating Effects of Current
- Grouping of Cells
- Electrical Measuring Instruments
- Charging of a Capacitor in *C-R* Circuit
- Chemical Effects of Current
- Thermo Electricity
- Primary and Secondary Cells

When a constant potential difference V is applied between the ends of the conductor as shown in Fig. 19.2, an electric field **E** is produced inside the conductor. The conduction electrons within the conductor are then subjected to a force $-e\mathbf{E}$ and move overall in the direction of increasing potential. However, this force does not cause the electrons to move faster and faster. Instead, a conduction electron accelerates through a very small distance (about 5×10^{-8} m) and then collides with one of the atoms of the conductor.

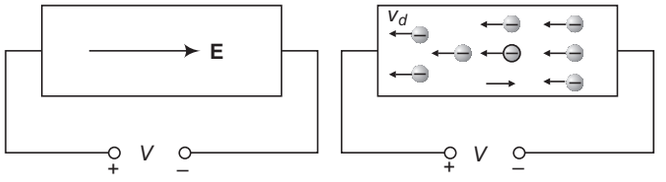


Fig. 19.2

Each collision transfers some of the electron's kinetic energy to the atoms, resulting in an increase in the vibrational energy (and therefore in the temperature) of the atoms. Because of the electric and collision forces a conduction electron moves slowly along the conductor or we can say it acquires a **drift velocity** $\mathbf{v}_d$ in the direction opposite to **E** in addition to its random motion.

This chaotic motion of the electrons can be understood by a simple model. Let us imagine that the conduction electrons form an electron gas whose particles are moving randomly, while the gas as a whole is moving slowly down the conductor with a drift velocity v_d in the direction opposite to **E**.

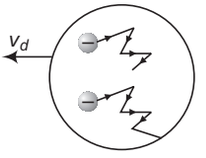


Fig. 19.3

It's interesting to note that the magnitude of the drift velocity is on the order of 10^{-4} m/s, or about 10^9 times smaller than the average speed of the electrons between collisions. When averaged over time, the random motion of an electron due to its collisions does not result in any net displacement. Hence, we can ignore this random motion and treat the electrons as though they were all moving down the conductor at the drift velocity v_d.

The net movement of charge through a conductor is represented by **electric current** (or just current) I. Current is defined quantitatively in terms of the rate at which net charge passes through a cross-sectional area of the conductor.

Thus, $$I = i = \frac{dq}{dt}$$

We can have following two concepts of current, as in the case of velocity, instantaneous current and average current.

$$\text{Instantaneous current} = \frac{dq}{dt}$$

$$= \text{ current at any point of time}$$

$$\text{and average current} = \frac{q}{t}$$

Hence, forth unless otherwise referred to current would signify instantaneous current. By convention, the direction of the current is assumed to be that in which positive charge moves. In the SI system, the unit of current is ampere (A).

$$1\text{ A} = 1\text{ C/s}$$

Household currents are of the order of few amperes.

Current, Drift Velocity and Current Density

We can express current in terms of the drift velocity v_d of the moving charges (may be free electrons in case of conducting wire or positive and negative ions in case of electrolytes). The relation is as follows,

$$i = nqAv_d$$

where, n = moving charged particles per unit volume,

A = area of cross-section.

The current per unit cross-section area is called the current density J. Thus,

$$J = \frac{i}{A} = nqv_d$$

The current i and current density J don't depend on the sign of the charge and so, in the above expressions for i and J, we replace the charge q by its modulus $|q|$.

$\therefore$ $$i = n|q|Av_d$$

and $$J = n|q|v_d$$

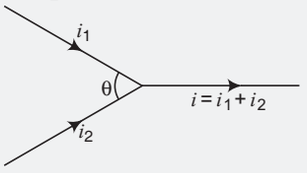
We can also define a vector current density **J** that includes the direction of the drift velocity.

$$\mathbf{J} = n|q|\mathbf{v}_d$$

Extra Knowledge Points

- The current is the same for all cross-sections of a conductor of non-uniform cross-section. Similar to the water flow, charge flows faster, where the conductor is smaller in cross- section and slower where the conductor is larger in cross-section, so that charge rate remains unchanged.
- Though conventionally a direction is associated with current (opposite to the motion of electrons), it is not a vector as the direction merely represents the sense of charge flow and not a true direction.

Further, current does not obey the law of parallelogram of vectors, i.e. if two currents i_1 and i_2 reaches a point we always have $i = i_1 + i_2$, whatever be the angle between i_1 and i_2.



- According to its magnitude and direction current is usually divided into two types:
  - (i) **Direct Current (DC)** If the magnitude and direction of current does not vary with time, it is said to be direct current (DC). Cell, battery or DC dynamo are its sources.
  - (ii) **Alternating Current (AC)** If a current is periodic (with constant amplitude) and has half cycle positive and half negative, it is said to be alternating current (AC). AC dynamo is the source of it.
- If a charge q revolves in a circle with frequency f, the equivalent current,

$$i = q f$$

Example 19.1 *A wire carries a current of* $2.0\,A$. *What is the charge that has flowed through its cross-section in* $1.0\,s$. *How many electrons does this correspond to?*

Sol. $i = \frac{q}{t} \Rightarrow \therefore\ q = it = (2.0\text{ A})\,(1.0\text{ s})$

$$= 2.0\text{ C}$$

$$q = ne$$

$$\therefore \quad n = \frac{q}{e} = \frac{2.0}{1.6 \times 10^{-19}}$$

$$= 1.25 \times 10^{19}$$

Example 19.2 *The current in a wire varies with time according to the relation*

$$i = (3.0\,A) + \left(2.0\,\frac{A}{s}\right)t$$

(a) How many coulombs of charge pass a cross-section of the wire in the time interval between $t = 0$ *and* $t = 4.0\,s$ *?*

(b) What constant current would transport the same charge in the same time interval?

Sol. (a) $i = \frac{dq}{dt}$

$$\therefore \quad \int_0^q dq = \int_0^4 i\,dt$$

$$\therefore \quad q = \int_0^4 (3 + 2t)\,dt$$

$$= [3t + t^2]_0^4 = [12 + 16] = 28\text{ C}$$

(b) $i = \frac{q}{t} = \frac{28}{4} = 7\text{ A}$

19.3 Resistance and Ohm's Law

The current density **J** in a conductor depends on the electric field **E** and on the properties of the material. In general, this dependence is quite complex. But for some materials, especially metals, at a given temperature, **J** is nearly directly proportional to **E** and the ratio of the magnitudes E and J is constant. This ratio is called the **resistivity (ρ)** and this relationship is called the **Ohm's law.**

Note Point

- Ohm's law, like the ideal gas equation and Hooke's law is an idealised model that describes the behaviour of some materials quite well but is not a general description of all matters. In the following discussion we will assume that Ohm's law is valid, even though there are many situations in which it is not.

Thus, resistivity $\rho = \frac{E}{J}$

SI units of resistivity are Ω-m (ohm-metre). A perfect conductor would have zero resistivity and a perfect insulator would have an infinite resistivity. Metals and alloys have the smallest resistivities and are the best conductors.

The reciprocal of resistivity is **conductivity** (σ).

Thus, $\sigma = \frac{1}{\rho}$

Its units are $(\Omega\text{m})^{-1}$. Good electrical conductors, such as metals, are usually also good conductors of heat. In a metal the free electrons that carry charge in electrical conduction also provide the principal mechanism for heat conduction so we can expect a correlation between electrical and thermal conductivity. Semiconductors have resistivities between those of metals and those of insulators.

A material that obeys Ohm's law reasonably well is called an **ohmic conductor** or a **linear conductor**. For such materials at a given temperature, ρ is a constant and does not depend on the value of E. Materials which show substantial departures from Ohm's law are called **non-ohmic** or **non-linear**. In these materials, J depends on E in a more complicated manner.

Resistance

For ohmic conductors, $\mathbf{E} \propto \mathbf{J}$ or $\mathbf{E} = \rho\,\mathbf{J}$

Here, resistivity (ρ) is constant and independent of the magnitude of the electric field **E** often, however, we are more interested in the total current in a conductor than in **J** and more interested in the potential difference between the ends of the conductor than in **E**. This is because current and potential difference are much easier to measure than are **J** and **E**.

Suppose a conducting wire has a uniform cross-sectional area A and length l as shown in Fig. 19.4. Let V be the potential difference between the ends of the wire. The direction of current (i) and electric field **(E)** inside the conductor is from higher potential to the lower potential. If the magnitudes of the current density **J** and the electric field **E** are uniform throughout the conductor, the total current i is given by $i = JA$ and the potential difference V between the ends is $V = El$.

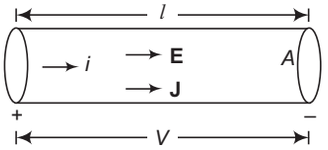


Fig. 19.4

The ratio of V and i is therefore,

$$\frac{V}{i} = \frac{El}{JA} = \rho\frac{l}{A}$$

where, $\rho\frac{l}{A}$ is constant for ohmic materials. This is called the **resistance *R***.

Thus, $$R = \frac{V}{i}$$

The resistance R of a particular conductor is related to the resistivity ρ of its material by,

$$R = \rho\frac{l}{A}$$

The equation, $V = iR$

is often called **Ohm's law,** but it is important to understand that the real content of Ohm's law is the direct proportionality (for some materials) of V to i or of J to E.

Note Point

- The equation $R = \frac{V}{i}$ defines resistance R for any conductor, whether or not it obeys Ohm's law, but only when R is constant we can correctly call this relationship Ohm's law. Thus, for Ohmic conductors V-i graph is a straight line passing through origin. The slope of this line is equal to the resistance of the conductor.

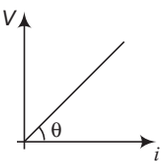


Fig. 19.5

$$\therefore \quad R = \frac{V}{i} = \tan\theta$$

Reciprocal of resistance is called conductance (G), i.e.

$$G = \frac{1}{R} = \frac{i}{V}$$

SI unit of G is ohm^{-1} which is called mho.

Example 19.3 *Two copper wires of the same length have got different diameters,*
(a) which wire has greater resistance?
(b) which wire has greater specific resistance?

Sol. (a) For a given wire,

$$R = \rho\frac{l}{A}$$

i.e. $$R \propto \frac{1}{A}$$

So, the thinner wire will have greater resistance.

(b) Specific resistance (ρ) is a material property. It does not depend on l or A.

So, both the wires will have same specific resistance.

Example 19.4 *A wire has a resistance R. What will be its resistance, if it is stretched to double its length?*

Sol. Let V be the volume of wire, then

$$V = Al \quad \Rightarrow \quad A = \frac{V}{l}$$

Substituting this in $R = \rho\frac{l}{A}$, we have

$$R = \rho\frac{l^2}{V}$$

So, for given volume and material (i.e. V and ρ are constant)

$$R \propto l^2$$

When l is doubled, resistance will become four times or the new resistance will be $4R$.

Example 19.5 *The dimensions of a conductor of specific resistance* ρ *are shown below.*

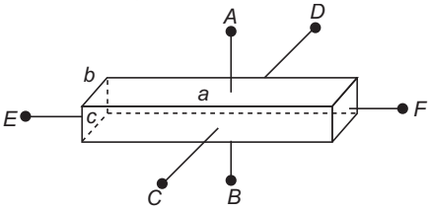


Fig. 19.6

Find the resistance of the conductor across AB, CD and EF.

Sol. $R = \rho\frac{l}{A}$

Resistance across *AB, CD* and *EF* in tabular form is shown below.

Table 19.1

	l	A	R
AB	c	$a \times b$	$\rho\frac{c}{ab}$
CD	b	$a \times c$	$\rho\frac{b}{ac}$
EF	a	$b \times c$	$\rho\frac{a}{bc}$

19.4 Temperature dependence of Resistivity and Resistance

The resistivity of a metallic conductor nearly increases with increasing temperature as shown in Fig. 19.7. This is because, with the increase in temperature the ions of the conductor vibrate with greater amplitude and the collision between electrons and ions become more frequent. Over a small temperature range (upto 100°C), the resistivity of a metal can be represented approximately by the equation.

$$\rho(T) = \rho_0 [1 + \alpha (T - T_0)] \quad \ldots(i)$$

where, ρ_0 is the resistivity at a reference temperature T_0 (often taken as 0°C or 20°C) and $\rho(T)$ is the resistivity at temperature T, which may be higher or lower than T_0. The factor α is called the **temperature coefficient of resistivity.**

The resistance of a given conductor depends on its length and area of cross-section besides the resistivity. As temperature changes, the length and area also change. But these changes are quite small and the factor l/A may be treated as constant.

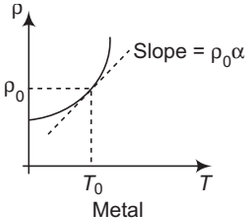


Fig. 19.7

Then, $R \propto \rho$

and hence, $R(T) = R_0 [1 + \alpha (T - T_0)] \quad \ldots(ii)$

In this equation $R(T)$ is the resistance at temperature T and R_0 is the resistance at temperature T_0, often taken to be 0°C or 20°C. The temperature coefficient of resistance α is the same constant that appears in Eq. (i), if the dimensions l and A in equation $R = \rho \frac{l}{A}$ do not change with temperature.

Thermistor

The temperature coefficient of resistivity is negative for semiconductors. This means that the resistivity decreases as we raise the temperature of such a material.

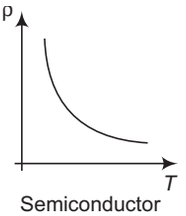


Fig. 19.8

The magnitude of the temperature coefficient of resistivity is often quite large for a semiconducting material. This fact is used to construct thermometers to detect small changes in temperatures. Such a device is called a thermistor. The variation of resistivity of a semiconductor with temperature is shown in Fig. 19.8. A typical thermistor can easily measure a change in temperature of the order of 10^{-3} °C.

Superconductors

Superconductivity was first discovered in 1911 by the Dutch physicist Heike Kamerlingh Onnes.

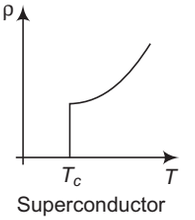


Fig. 19.9

There are certain materials, including several metallic alloys and oxides for which as the temperature decreases, the resistivity first decreases smoothly, like that of any metal. But then at a certain critical temperature T_c a phase transition occurs, and the resistivity suddenly drops to zero as shown in Fig. 19.9.

Once, a current has been established in a superconducting ring, it continues indefinitely without the presence of any driving field. Possible applications of superconductors are ultrafast computer switches and transmission of electric power through superconducting power lines. However, the requirement of low temperature is posing difficulty. For instance the critical temperature for mercury is 4.2 K. Scientists are putting great effort to construct compounds and alloys which would be superconducting at room temperature (300 K). Superconductivity at around 125 K has already been achieved.

Example 19.6 *The resistance of a thin silver wire is* 1.0 Ω *at* 20°C. *The wire is placed in a liquid bath and its resistance rises to* 1.2 Ω. *What is the temperature of the bath?* α *for silver is* 3.8×10^{-3} *per* °C.

Sol. $R(T) = R_0 [1 + \alpha (T - T_0)]$

where, $R(T) = 1.2\ \Omega$, $R_0 = 1.0\ \Omega$,

$\alpha = 3.8 \times 10^{-3}$ per °C and $T_0 = 20$°C

Substituting the values, we have

$1.2 = 1.0 [1 + 3.8 \times 10^{-3} (T - 20)]$

or $3.8 \times 10^{-3} (T - 20) = 0.2$

Solving this, we get $T = 72.6$°C

19.5 The Battery and the Electromotive Force

Before studying the electromotive force (emf) of a cell let us take an example of a pump which is more easy to understand.

Suppose, we want to recycle water between a overhead tank and a ground water tank. Water flows from overhead tank to ground water tank by itself (by gravity). No external agent is required for this purpose. But to raise the water from ground water tank to overhead tank a pump is required or some external work has to be done. In an electric circuit a battery or a cell plays the same role as the pump played in the above example.

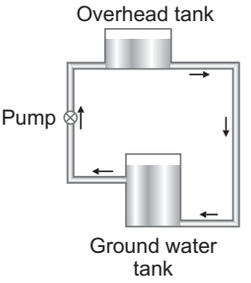


Fig. 19.10

Suppose a resistance (R) is connected across the terminals of a battery. A potential difference is developed across its ends. Current (or positive charge) flows from higher potential to lower potential across the resistance by itself. But inside the battery work has to be done to bring the positive charge from lower potential to higher potential. The influence that makes current flow from lower to higher potential (inside the battery) is called **electromotive force** (abbreviated **emf**). If W work is done by the battery in taking a charge q from negative terminal to positive terminal, then work done by the battery per unit charge is called emf (E) of the battery.

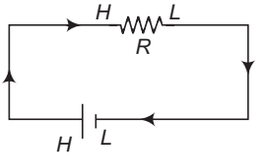


Fig. 19.11

Thus, $$E = \frac{W}{q}$$

The name electromotive force is misleading in the sense that emf is not a force it is work done per unit charge. The SI unit of emf is J/C or V (1 V = 1 J/C).

Example 19.7 *Why cannot we use a capacitor as a battery? What is the difference between a battery and a capacitor?*

Sol. A capacitor cannot maintain constant potential difference across a circuit. After the capacitor discharges completely no current is obtained. A capacitor discharges in a very short interval of time. On the other hand, a battery can maintain a constant potential difference for a long period of time. A battery also gets discharged, but discharging may take years also.

19.6 Direct Current Circuits, Kirchhoff's Laws

Current in a simple circuit can be found by the relation,

$$i = \frac{\text{Net emf}}{\text{Net resistance}} = \frac{E_{\text{net}}}{R_{\text{net}}}$$

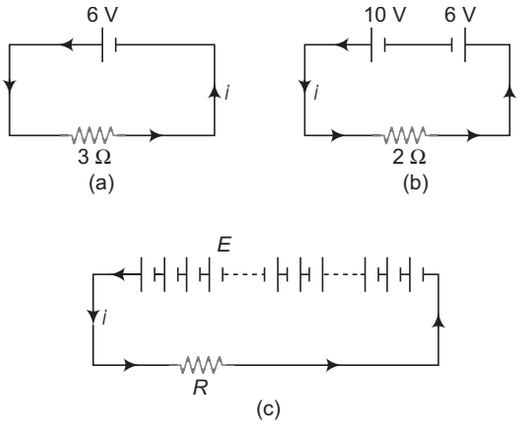


Fig. 19.12

For example

In Fig. (a) Net emf is 6 V and net resistance is 3 Ω. Therefore,

$$i = \frac{6}{3} = 2 \text{ A}$$

In Fig. (b) Net emf = (10 – 6) V = 4 V

and net resistance = 2 Ω

Therefore, $$i = \frac{4}{2} = 2 \text{ A}$$

In Fig. (c) We have n cells each of emf E. Of these polarity of m cells (where, $n > 2m$) is reversed. Then, net emf in the circuit is $(n - 2m)\,E$ and resistance of the circuit is R. Therefore,

$$i = \frac{(n - 2m)\,E}{R}$$

Resistors in Series and in Parallel

In series

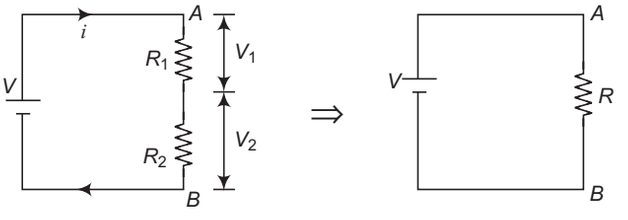


Fig. 19.13

Figure represents a circuit consisting of a source of emf and two resistors connected in series. We are interested in finding the resistance R of the network lying between A and B. i.e. what single **equivalent resistor *R*** would have the same resistance as the two resistors linked together.

Because there is only one path for electric current to follow, i must have the same value everywhere in the circuit. The potential difference between A and B is V. This potential difference must some how be divided into two parts V_1 and V_2 as shown.

Subject to the condition

$$V = V_1 + V_2 = iR_1 + iR_2$$

or $$V = i(R_1 + R_2) \quad \ldots(i)$$

Let R be the equivalent resistance between A and B, then

$$V = iR \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$R = R_1 + R_2 \quad \text{(for resistors in series)}$$

This result can be readily extended to a network consisting of n resistors in series.

$$\therefore \quad R = R_1 + R_2 + \ldots + R_n$$

In parallel

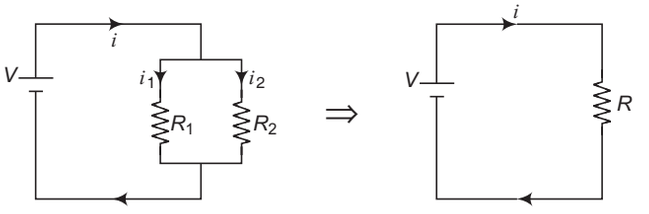


Fig. 19.14

In Fig. 19.14, the two resistors are connected in parallel. The voltage drop across each resistor is equal to the source voltage V. The current i, however, divides into two branches, which carry currents i_1 and i_2.

$$i = i_1 + i_2 \quad \ldots(iii)$$

If R be the equivalent resistance, then

$$i = \frac{V}{R}, \quad i_1 = \frac{V}{R_1}$$

and $$i_2 = \frac{V}{R_2}$$

Substituting in Eq. (iii), we get

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{(for resistors in parallel)}$$

This result can also be extended to a network consisting of n resistors in parallel. The result is,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

Example 19.8 *Compute the equivalent resistance of the network shown in figure and find the current i drawn from the battery.*

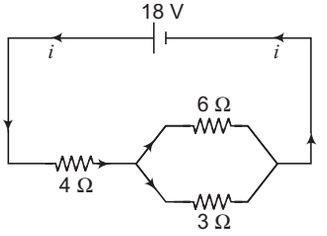


Fig. 19.15

Sol. The 6 Ω and 3 Ω resistances are in parallel. Their equivalent resistance is

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{3} \quad \text{or} \quad R = 2\ \Omega$$

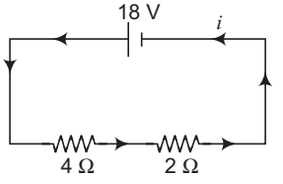


Fig. 19.16

Now, this 2 Ω and 4 Ω resistances are in series and their equivalent resistance is, 4 + 2 = 6 Ω.

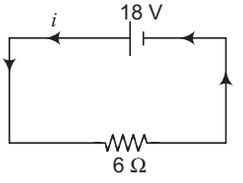


Fig. 19.17

Therefore, equivalent resistance of the network = 6 Ω.

Current drawn from the battery is,

$$i = \frac{\text{net emf}}{\text{net resistance}} = \frac{18}{6} = 3\ \text{A}$$

Kirchhoff's Laws

Many electric circuits cannot be reduced to simple series-parallel combinations. For example, two circuits that cannot be so broken down are shown in Fig. 19.18.

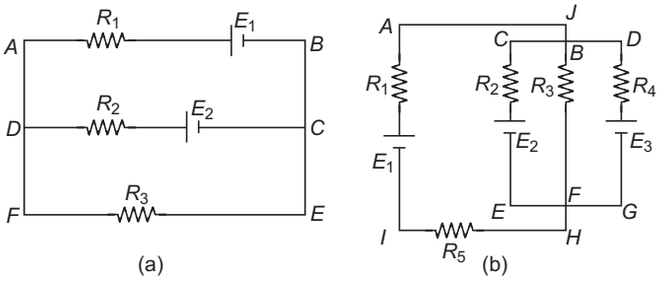


Fig. 19.18

However, it is always possible to analyse such circuits by applying two rules, devised by Kirchhoff in 1845 and 1846 when he was still a student.

First here are two terms that we will use often.

Junction

A junction in a circuit is a point, where three or more conductors meet. Junctions are also called nodes or branch points.

For example, in Fig. (a) points *D* and *C* are junctions. Similarly in Fig. (b) points *B* and *F* are junctions.

Loop

A loop is any closed conducting path. For example, in Fig. (a) *ABCDA*, *DCEFD* and *ABEFA* are loops. Similarly, in Fig. (b), *CBFEC*, *BDGFB* are loops.

Kirchhoff's rules consist of the following two statements.

Kirchhoff's Junction Rule

The algebraic sum of the currents into any junction is zero.

i.e. $$\sum_{\text{junction}} i = 0$$

This law can also be written as, the sum of all the currents directed towards a point in a circuit is equal to the sum of all the currents directed away from that point.

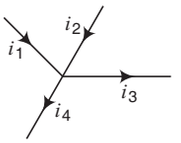


Fig. 19.19

Thus, in figure $i_1 + i_2 = i_3 + i_4$

The junction rule is based on **conservation of electric charge.** No charge can accumulate at a junction, so the total charge entering the junction per unit time must equal to charge leaving per unit time. Charge per unit time is current so, if we consider the currents entering to be positive and those leaving to be negative, the algebraic sum of currents into a junction must be zero.

Kirchhoff's Loop Rule

The algebraic sum of the potential differences in any loop including those associated emf's and those of resistive elements, must equal zero.

i.e. $$\sum_{\text{closed loop}} \Delta V = 0$$

Kirchhoff's second rule is based on the fact that the **electrostatic field is conservative** in nature. This result states that there is no net change in electric potential around a closed path. Kirchhoff's second rule applies only for circuits in which an electric potential is defined at each point. This criterion may not be satisfied, if changing electromagnetic fields are present.

In applying the loop rule, we need sign conventions. First assume a direction for the current in each branch of the circuit. Then, starting at any point in the circuit, we imagine, travelling around a loop, adding emf's and *iR* terms as we come to them.

When we travel through a source in the direction from negative to positive, the emf is considered to be positive, when we travel from positive to negative, the emf is considered to be negative.

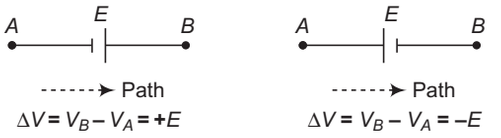


Fig. 19.20

When we travel through a resistor in the same direction as the assumed current, the *iR* term is negative, because the current goes in the direction of decreasing potential. When we travel through a resistor in the direction opposite to the assumed current, the *iR* term is positive, because this represents a rise of potential.

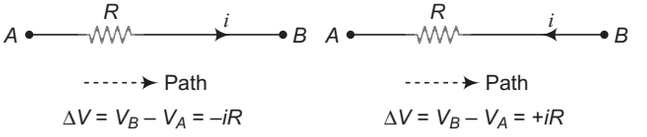


Fig. 19.21

Note Point

- It is advised to write *H* (for higher potential) and *L* (for lower potential) across all the batteries and resistances of the loop under consideration while using the loop law. Then write – while moving from *H* to *L* and + for *L* to *H*. Across a battery write *H* on positive terminal and *L* on negative terminal. Across a resistance keep in mind the fact that current always flows from higher potential (*H*) to lower potential (*L*). For example, in the loop shown in figure we have marked *H* and *L* across all batteries and resistances. Now, let us apply the second law in the loop *ADCBA*. The equation will be

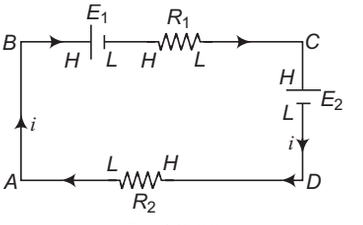


Fig. 19.22

$$iR_2 + E_2 + iR_1 + E_1 = 0$$

Example 19.9 *Find currents in different branches of the electric circuit shown in figure.*

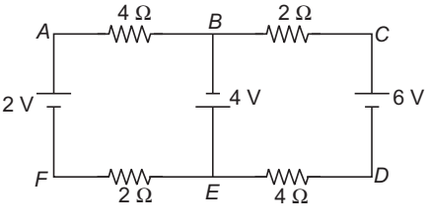


Fig. 19.23

How To Proceed *In this problem there are three wires EFAB, BE and BCDE. Therefore, we have three unknowns i_1, i_2 and i_3. So, we require three equations. One equation will be obtained by applying Kirchhoff's junction law (either at B or at E) and the remaining two equations, we get from the second law (loop law). We can make three loops ABEFA, ACDFA and BCDEB. But we have to choose any two of them. Further, we can choose any arbitrary directions of i_1, i_2 and i_3.*

Sol. Applying Kirchhoff's first law (junction law) at junction *B*,

$$i_1 = i_2 + i_3 \quad \ldots(i)$$

Applying Kirchhoff's second law in loop 1 (*ABEFA*),

$$-4i_1 + 4 - 2i_1 + 2 = 0 \quad \ldots(ii)$$

Applying Kirchhoff's second law in loop 2 (*BCDEB*),

$$-2i_3 - 6 - 4i_3 - 4 = 0 \quad \ldots(iii)$$

Solving Eqs. (i), (ii) and (iii), we get

$$i_1 = 1\,\text{A}$$

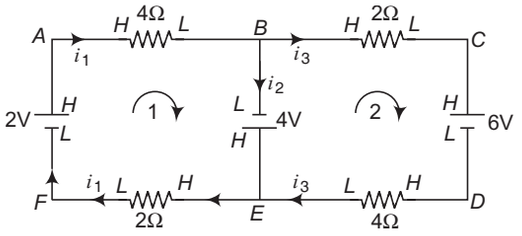


Fig. 19.24

$$i_2 = \frac{8}{3}\,\text{A} \quad \Rightarrow \quad i_3 = -\frac{5}{3}\,\text{A}$$

Here, negative sign of i_3 implies that current i_3 is in opposite direction of what we have assumed.

Example 19.10 *In example number* 19.9*, find the potential difference between points F and C.*

How To Proceed *To find the potential difference between any two points of a circuit you have to reach from one point to the other via any path of the circuit. It is advisable to choose a path in which we come across the least number of resistors preferably a path which has no resistance.*

Sol. Let us reach from *F* to *C via A* and *B*,

$$V_F + 2 - 4i_1 - 2i_3 = V_C$$

$$\therefore \quad V_F - V_C = 4i_1 + 2i_3 - 2$$

Substituting, $i_1 = 1\,\text{A}$ and $i_3 = -\frac{5}{3}\,\text{A}$, we get

$$V_F - V_C = -\frac{4}{3}\,\text{V}$$

Here, negative sign implies that $V_F < V_C$.

Internal Resistance (r) and Potential Difference (V) across the Terminals of a Battery

The potential difference across a real source in a circuit is not equal to the emf of the cell. The reason is that charge moving through the electrolyte of the cell encounters resistance. We call this the **internal resistance** of the source, denoted by *r*. If this resistance behaves according to Ohm's law *r* is constant and independent of the current *i*. As the current moves through *r*, it experiences an associated drops in potential equal to *ir*. Thus, when a current is drawn through a source, the potential difference between the terminals of the source is

$$V = E - ir$$

This can also be shown as below.

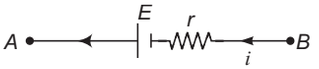


Fig. 19.25

$$V_A - E + ir = V_B$$

or $$V_A - V_B = E - ir$$

Following three special cases are possible

(i) If the current flows in opposite direction (as in case of charging of a battery), then $V = E + ir$.

(ii) $V = E$, if the current through the cell is zero.

(iii) $V = 0$, if the cell is short circuited.

This is because current in the circuit.

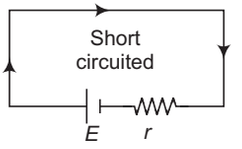


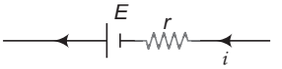
Fig. 19.26

$$i = \frac{E}{r} \quad \text{or} \quad E = ir$$

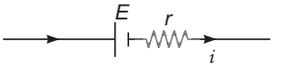
$$\therefore \quad E - ir = 0 \quad \text{or} \quad V = 0$$

Thus, we can summarise it as follows

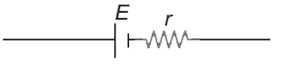
$$V = E - ir \quad \text{or} \quad V < E$$



$$V = E + ir \quad \text{or} \quad V > E$$



$$V = E \quad \text{if,} \quad i = 0$$



$$V = 0, \quad \text{if short circuited}$$

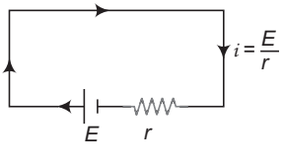
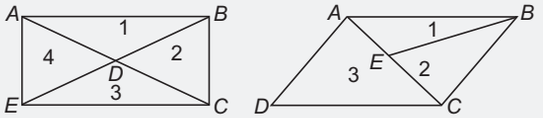


Fig. 19.27

Extra Knowledge Points

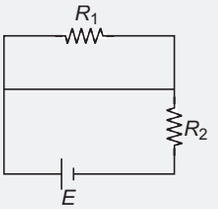
- **In Fig. (a)** There are eight wires and hence, will have eight currents or eight unknowns. The eight wires are *AB*, *BC*, *CE*, *EA*, *AD*, *BD*, *CD* and *ED*. Number of loops are four. Therefore, from the second law we can make only four equations. Total number of junctions are five (*A, B, C, D* and *E*). But by using the first law, we can make only four equations (one less). So, the total number of equations are eight.



(a) (b)

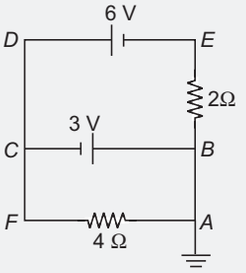
In Fig. (b) Number of wires are six (*AB, BC, CDA, BE, AE* and *CE*). Number of loops are three so, three equations will be obtained from the second law. Number of junctions are four (*A, B, C* and *E*) so, we can make only three (one less) equations from the first law. But total number of equations are again six.

- **Short circuiting** Two points in an electric circuit directly connected by a conducting wire are called short circuited. Under such condition both points are at same potential



For example, resistance R_1 in the adjoining circuit is short circuited, i.e. potential difference across it is zero. Hence, no current will flow through R_1 and the current through R_2 is therefore, E/R_2.

- **Earthing** If some point of a circuit is earthed then its potential is taken to be zero.



For example, in the adjoining figure,

$$V_A = V_B = 0$$
$$V_F = V_C = V_D = -3\ \text{V}$$
$$V_E = -9\ \text{V}$$
$$\therefore V_B - V_E = 9\ \text{V}$$

or Current through 2 Ω resistance is

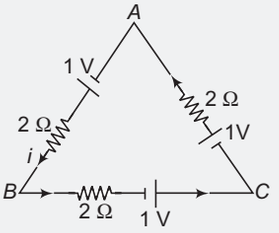
$$\frac{V_B - V_E}{2} \quad \text{or} \quad \frac{9}{2}\ \text{A} \qquad \text{(from } B \text{ to } E\text{)}$$

Similarly, $V_A - V_F = 3\ \text{V}$

and the current through 4 Ω resistance is

$$\frac{V_A - V_F}{4} \quad \text{or} \quad \frac{3}{4}\ \text{A} \qquad \text{(from } A \text{ to } F\text{)}$$

- For a current flow through a resistance there must be a potential difference across it but between any two points of a circuit the potential difference may be zero.



For example, in the circuit,net emf = 3 V and net resistance = 6 Ω

∴ Current in the circuit,

$$i = \frac{3}{6} = \frac{1}{2}\ \text{A}$$

$$V_A - V_B : V_A + 1 - 2 \times \frac{1}{2} = V_B \quad \text{or} \quad V_A - V_B = 0$$

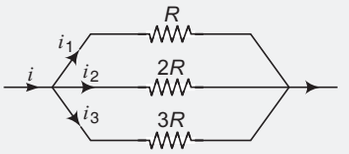
or by symmetry, we can say that

$$V_A = V_B = V_C$$

So, the potential difference across any two vertices of the triangle is zero, while the current in the circuit is non-zero.

- **Distribution of current in parallel connections**

When more than one resistances are connected in parallel, the potential difference across them is equal and the current is distributed among them in inverse ratio of their resistance, as



$$i = \frac{V}{R} \quad \text{or} \quad i \propto \frac{1}{R} \qquad \text{(for same value of } V\text{)}$$

e.g. in the figure,

$$i_1 : i_2 : i_3 = \frac{1}{R} : \frac{1}{2R} : \frac{1}{3R} = 6 : 3 : 2$$

$$\therefore \quad i_1 = \left(\frac{6}{6+3+2}\right) i = \frac{6}{11}\, i$$

$$i_2 = \left(\frac{3}{6+3+2}\right) i = \frac{3}{11}\, i$$

and $$i_3 = \left(\frac{2}{6+3+2}\right) i = \frac{2}{11}\, i$$

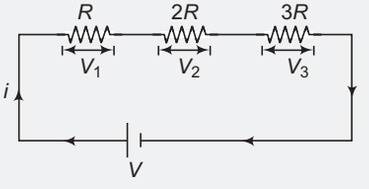
- **Distribution of potential in series connections** When more than one resistances are connected in series, the current through them is same and the potential is distributed in the ratio of their resistance, as

$$V = iR$$

or $$V \propto R \qquad \text{(for same value of } i\text{.)}$$

For e.g. in the figure,

$$V_1 : V_2 : V_3 = R : 2R : 3R = 1 : 2 : 3$$



$$V_1 = \left(\frac{1}{1+2+3}\right) V = \frac{V}{6} \Rightarrow V_2 = \left(\frac{2}{1+2+3}\right) V = \frac{V}{3}$$

and $$V_3 = \left(\frac{3}{1+2+3}\right) V = \frac{V}{2}$$

Example 19.11 *In the circuit shown in figure,* $E_1 = 10\ \text{V}$, $E_2 = 4\ \text{V}$, $r_1 = r_2 = 1\ \Omega$ *and* $R = 2\ \Omega$. *Find the potential difference across battery* 1 *and battery* 2.

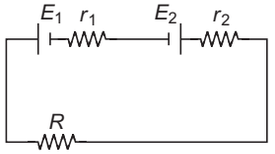


Fig. 19.28

Sol. Net emf of the circuit = $E_1 - E_2 = 6$ V

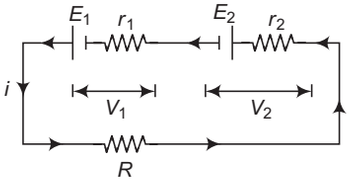


Fig.19.29

Total resistance of the circuit = $R + r_1 + r_2 = 4\ \Omega$

$\therefore$ Current in the circuit, $i = \dfrac{\text{Net emf}}{\text{Total resistance}} = \dfrac{6}{4} = 1.5$ A

Now, $V_1 = E_1 - i r_1 = 10 - (1.5)\ (1) = 8.5$ V

and $V_2 = E_2 + i r_2 = 4 + (1.5)\ (1) = 5.5$ V

19.7 Heating Effects of Current

An electric current through a resistor increases its thermal energy. Also, there are other situations in which an electric current can produce or absorb thermal energy.

Power Supplied or Power Absorbed by a Battery

When charges are transported across a source of emf, their potential energy changes. If a net charge Δq moves through a potential difference E in a time Δt, the change in electric potential energy of the charge is $E\ \Delta q$. Thus, the source of emf does work,

$$\therefore \qquad \Delta W = E\ \Delta q$$

Dividing both sides by Δt, then taking the limit as $\Delta t \to 0$, we find,

$$\frac{dW}{dt} = E\,\frac{dq}{dt}$$

By definition, $\dfrac{dq}{dt} = i$, the current through the battery and $\dfrac{dW}{dt} = P$, the power output of (or input to) the battery.

Hence,

$$P = Ei$$

The quantity P represents the rate at which energy is transferred from a discharging battery or to a charging battery.

Energy is transferred from the source at a rate Ei,

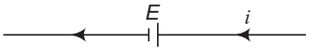


Fig. 19.30

Energy is transferred to the source at a rate Ei,

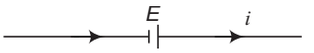


Fig.19.31

Power Dissipated across a Resistance

Now let's consider the power dissipated in a conducting element. Suppose it has a resistance R and the potential difference between its ends is V. In moving through the element from higher to lower potential, a positive charge Δq loses energy $\Delta U = V\ \Delta q$. This electric energy is absorbed by the conductor through collisions between its atomic lattice and the charge carriers, causing its temperature to rise. This effect is commonly called **Joule heating.** Since, power is the rate at which energy is transferred, we have,

$$P = \frac{\Delta U}{\Delta t} = V \cdot \frac{\Delta q}{\Delta t} = V i \quad \Rightarrow \quad \therefore \quad P = V i$$

which with the help of Ohm's law can also be written in the forms,

$$P = i^2 R \quad \text{or} \quad P = \frac{V^2}{R}$$

Power is always dissipated in a resistance. With this rate the heat produced in the resistor in time t is,

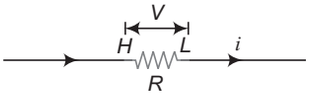


Fig. 19.32

$$H = Pt \quad \text{or} \quad H = Vit = i^2 Rt = \frac{V^2}{R}\,t$$

Joule heating occurs whenever a current passes through an element that has resistance. To prevent the overheating of delicate electronic components, many electric devices like video cassette recorders, televisions and computer monitors have fans in their chassis to allow some of the heat produced to escape.

Note Point

- $P = Vi$, can also be derived in a different manner.

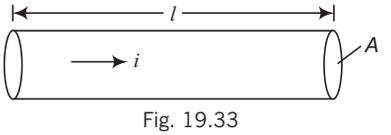


Fig. 19.33

Suppose a potential difference V is applied across a resistor of length l and area A. The work done per unit time on an electron drifting with velocity v_d is $\mathbf{F} \cdot \mathbf{v}_d = eEv_d$. If n is the number of electrons per unit volume, then

$$P = (nAl)\ (eEv_d) = (nAl)\ (e) \left(\frac{V}{l}\right)\left(\frac{i}{neA}\right) = Vi$$

Example 19.12 *In the circuit shown in figure, find*

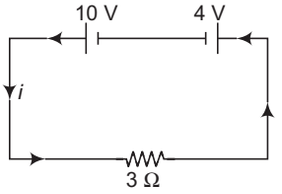


Fig. 19.34

(a) the power supplied by 10 *V battery,*
(b) the power consumed by 4 *V battery and*
(c) the power dissipated in 3 Ω *resistance.*

Sol. Net emf of the circuit = (10 – 4) V = 6 V

Total resistance of the circuit = 3 Ω

∴ Current in the circuit, $i = \frac{\text{net emf}}{\text{total resistance}} = \frac{6}{3} = 2$ A

(a) Power supplied by 10 V battery = Ei = (10) (2)
= 20 W

(b) Power consumed by 4 V battery = Ei = (4) (2)
= 8 W

(c) Power consumed by 3 Ω resistance = $i^2R = (2)^2(3)$
= 12 W

Note Point

- Here, we can see that total power supplied by 10 V battery (i.e. 20 W) = power consumed by 4 V battery and 3 Ω resistance. Which proves that conservation of energy holds good in electric circuits also.

Example 19.13 *In the circuit shown in figure, find the heat developed across each resistance in* 2 *s.*

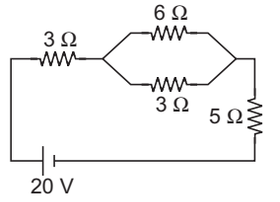


Fig. 19.35

Sol. The 6 Ω and 3 Ω resistances are in parallel. So, their combined resistance is

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2} \quad \text{or} \quad R = 2\ \Omega$$

The equivalent simple circuit can be drawn as shown in figure.

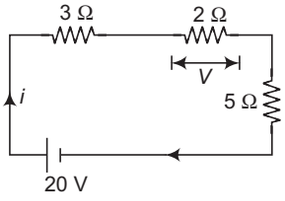


Fig. 19.36

Current in the circuit,

$$i = \frac{\text{net emf}}{\text{total resistance}} = \frac{20}{3+2+5} = 2\ \text{A}$$

$$V = iR = (2)(2) = 4\ \text{V}$$

i.e. potential difference across 6 Ω and 3 Ω resistances are 4 V. Now,

$$H_{3\,\Omega} = i^2Rt = (2)^2\,(3)\,(2) = 24\ \text{J}$$

$$H_{6\,\Omega} = \frac{V^2}{R}t = \frac{(4)^2}{6}(2) = \frac{16}{3}\ \text{J}$$

$$H_{3\,\Omega} = \frac{V^2}{R}t = \frac{(4)^2\,(2)}{3} = \frac{32}{3}\ \text{J}$$

and $$H_{5\,\Omega} = i^2Rt = (2)^2\,(5)\,(2) = 40\ \text{J}$$

19.8 Grouping of Cells

Cells are usually grouped in following three ways.

Series Grouping

Suppose n cells each of emf E and internal resistance r are connected in series as shown in figure. Then,

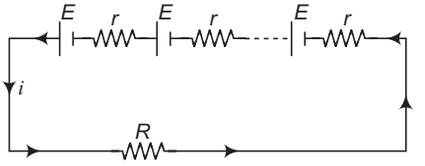


Fig. 19.37

$$\text{Net emf} = nE$$

$$\text{Total resistance} = nr + R$$

∴ Current in the circuit,

$$i = \frac{\text{net emf}}{\text{total resistance}}$$

or $$i = \frac{nE}{nr + R}$$

Note Point

- If polarity of m cells is reversed, then equivalent emf

$$= (n - 2m)\,E$$

while total resistance is still $nr + R$

∴ $$i = \frac{(n - 2m)\,E}{nr + R}$$

Parallel Grouping

Here three cases are possible.

Case 1

When E and r of each cell has same value and positive terminals of all cells are connected at one junction while negative at the other.

In this situation, the net emf is E. The net internal resistance is $\frac{r}{n}$ as n resistances each of r are in parallel. Net external resistance is R. Therefore, total resistance is $\left(\frac{r}{n}+R\right)$ and so, the current in the circuit will be

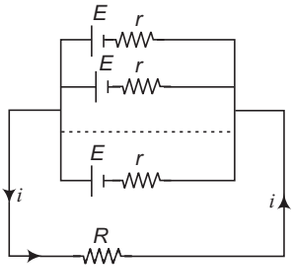


Fig.19.38

$$i=\frac{\text{Net emf}}{\text{Total resistance}} \quad \text{or} \quad i=\frac{E}{R+\frac{r}{n}}$$

Note Point

- A comparison of series and parallel grouping reveals that to get maximum current, cells must be connected in series, if effective internal resistance is lesser than external and in parallel, if effective internal resistance is greater than external.

Case 2

If E and r of each cell are different but still the positive terminals of all cells are connected at one junction while negative at the other.

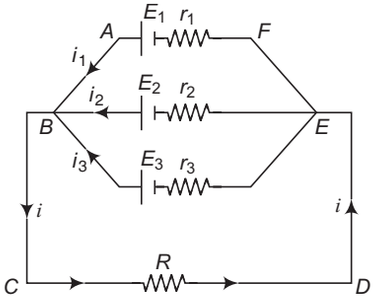


Fig. 19.39

Applying Kirchhoff's second law in loop *ABCDEFA*,

$$E_1 - iR - i_1 r_1 = 0$$

or $$i_1 = -i\frac{R}{r_1}+\frac{E_1}{r_1} \qquad \ldots(i)$$

Similarly, we can write

$$i_2 = -i\frac{R}{r_2}+\frac{E_2}{r_2} \qquad \ldots(ii)$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

Adding all above equations, we have

$$(i_1 + i_2 + \ldots + i_n) = -iR\,\Sigma\left(\frac{1}{r}\right)+\Sigma\left(\frac{E}{r}\right)$$

But $i_1 + i_2 + \ldots + i_n = i$

$$\therefore \qquad i = -iR\,\Sigma\left(\frac{1}{r}\right)+\Sigma\left(\frac{E}{r}\right)$$

$$\therefore \qquad i=\frac{\Sigma\,(E/r)}{1+R\,\Sigma\,(1/r)}=\frac{E_{\text{eq}}}{R_{\text{eq}}}$$

where, $E_{\text{eq}}=\frac{\Sigma\,(E/r)}{\Sigma\,(1/r)}$ and $R_{\text{eq}}=R+\frac{1}{\Sigma\,(1/r)}$

From the above expression, we can see that,

$$i=\frac{E}{R+\frac{r}{n}}$$

if n cells of same emf E and internal resistance r are connected in parallel. This is because

$$\Sigma\,(E/r) = nE/r \quad \text{and} \quad \Sigma\,(1/r) = n/r$$

$$\therefore \qquad i=\frac{nE/r}{1+\frac{nR}{r}}$$

Multiplying the numerator and denominator by r/n, we have

$$i=\frac{E}{R+\frac{r}{n}}$$

Exercise In parallel grouping (Case 2) prove that, $E_{\text{eq}} = E$, if $E_1 = E_2 = \ldots = E$ and $r_1 = r_2 = \ldots = r$

Case 3

This is the most general case of parallel grouping in which E and r of different cells are different and the positive terminals of few cells are connected to the negative terminals of the others as shown in figure.

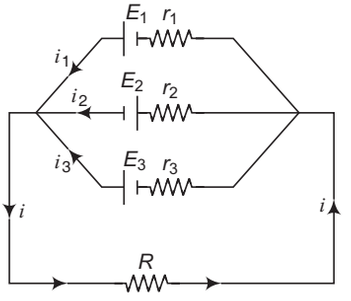


Fig. 19.40

Kirchhoff's second law in different loops gives the following equations,

$$E_1 - iR - i_1 r_1 = 0$$

or $$i_1 = \frac{E_1}{r_1}-\frac{iR}{r_1} \qquad \ldots(i)$$

$$-E_2 - iR - i_2 r_2 = 0$$

or $$i_2 = -\frac{E_2}{r_2} - \frac{iR}{r_2} \quad \ldots(ii)$$

Similarly, $$i_3 = \frac{E_3}{r_3} - \frac{iR}{r_3} \quad \ldots(iii)$$

Adding Eqs. (i), (ii) and (iii), we get

$$i_1 + i_2 + i_3$$

$$= (E_1/r_1) - (E_2/r_2) + (E_3/r_3) - iR\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

or $$i[1 + R(1/r_1 + 1/r_2 + 1/r_3)] = (E_1/r_1) - (E_2/r_2) + (E_3/r_3)$$

$$\therefore \quad i = \frac{(E_1/r_1) - (E_2/r_2) + (E_3/r_3)}{1 + R(1/r_1 + 1/r_2 + 1/r_3)}$$

Mixed Grouping

The situation is shown in figure.

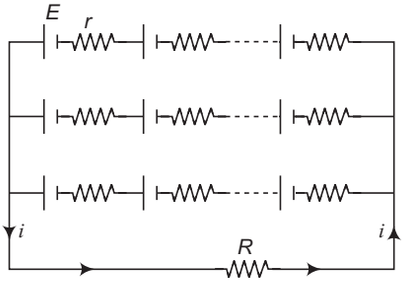


Fig.19.41

There are n identical cells in a row and number of rows are m. Emf of each cell is E and internal resistance is r. Treating each row as a single cell of emf nE and internal resistance nr, we have

$$\text{net emf} = nE$$

$$\text{Total internal resistance} = \frac{nr}{m}$$

$$\text{Total external resistance} = R$$

$\therefore$ Current through the external resistance R is

$$i = \frac{nE}{R + \frac{nr}{m}}$$

This expression after some rearrangements can also be written as,

$$i = \frac{mnE}{(\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mnrR}}$$

From this expression we see that i is maximum when,

$$\sqrt{mR} = \sqrt{nr} \quad \text{or} \quad R = \frac{nr}{m}$$

or total external resistance = total internal resistance.

Thus, we can say that the current and hence power transferred to the load is maximum when load resistance is equal to internal resistance. This is known as maximum power transfer theorem.

Example 19.14 *Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.*

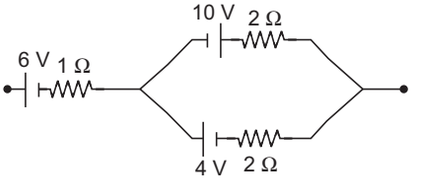


Fig.19.42

Sol. The given combination consists of two batteries in parallel and resultant of these two in series with the third one.

For parallel combination we can apply,

$$E_{eq} = \frac{\frac{E_1}{r_1} - \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{\frac{10}{2} - \frac{4}{2}}{\frac{1}{2} + \frac{1}{2}} = 3\text{ V}$$

Further,

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{2} + \frac{1}{2} = 1$$

$\therefore$ $$r_{eq} = 1\ \Omega$$

Now this is in series with the third one, i.e.

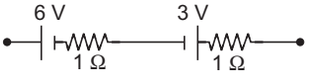


Fig. 19.43

The equivalent emf of these two is (6 – 3) V or 3 V and the internal resistance will be, (1 + 1) or 2 Ω.

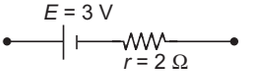


Fig. 19.44

19.9 Electrical Measuring Instruments

So, far we have studied about current, resistance, potential difference and emf. Now, in this article we will study how these are measured. The basic measuring instrument is galvanometer, whose pointer shows a deflection when current passes through it. A galvanometer can easily be converted into an ammeter for measuring current, into a voltmeter for measuring potential difference. For accurate measurement of potential difference or emf a potentiometer is more preferred. Resistances are accurately measured by using post office box or meter bridge which are based on the principle of Wheatstone bridge. All these are discussed here one by one in brief.

Galvanometer

Many common devices including car instrument panels, battery chargers measure potential difference, current or resistance using d'Arsonval Galvanometer. It consists of a pivoted coil placed in the magnetic field of a permanent magnet. Attached to the coil is a spring. In the equilibrium position, with no current in the coil, the pointer is at zero and spring is relaxed. When there is a current in the coil, the magnetic field exerts a torque on the coil that is proportional to current. As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement. Thus, the angular deflection of the coil and pointer is directly proportional to the coil current and the device can be calibrated to measure current.

The maximum deflection, typically 90° to 120° is called full scale deflection. The essential electrical characteristics of the galvanometer are the current i_g required for full scale deflection (of the order of 10 μA to 10 mA) and the resistance G of the coil (of the order of 10 to 1000 Ω).

The galvanometer deflection is proportional to the current in the coil. If the coil obeys Ohm's law, the current is proportional to potential difference. The corresponding potential difference for full scale deflection is

$$V = i_g G$$

Ammeter

A current measuring instrument is called an ammeter. A galvanometer can be converted into an ammeter by connecting a small resistance S (called shunt) in parallel with it.

Suppose, we want to convert a galvanometer with full scale current i_g and coil resistance G into an ammeter with full scale reading i. To determine the shunt resistance S needed, note that at full scale deflection, the total current through the parallel combination is i, the current through the galvanometer is i_g and the current through the shunt is $i - i_g$. The potential difference V_{ab} $(= V_a - V_b)$ is the same for both paths, so

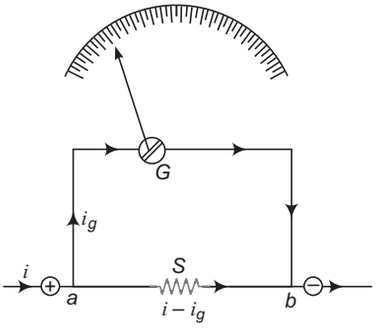


Fig. 19.45

$$i_g G = (i - i_g)\, S \quad \Rightarrow \quad \therefore \quad S = \left(\frac{i_g}{i - i_g}\right) G$$

Note Point

- The reading of an ammeter is always lesser than actual current in the circuit.

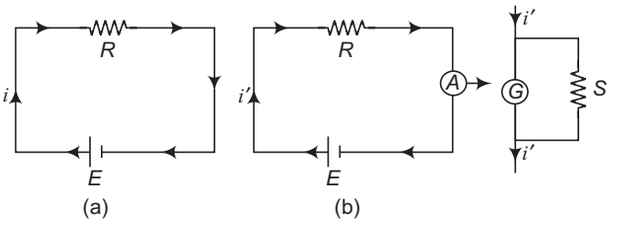


Fig. 19.46

For example, in Fig. (a),

actual current through R is, $i = \dfrac{E}{R}$ …(i)

While the current after connecting an ammeter of resistance, $A\left(= \dfrac{GS}{G+S}\right)$ in series with R is

$$i' = \frac{E}{R + A} \qquad \text{…(ii)}$$

From Eqs. (i) and (ii), we see that $i' < i$ and $i' = i$ when $A = 0$.

i.e. resistance of an ideal ammeter should be zero.

Percentage error in measuring a current through an ammeter is

$$\left(\frac{i - i'}{i}\right) \times 100 = \left(\frac{1/R - 1/(R + A)}{1/R}\right) \times 100$$

or $$\%\ \text{error} = \left(\frac{A}{R + A}\right) \times 100$$

Resistance of ammeter, $A = \dfrac{GS}{G+S}$.

Example 19.15 *What shunt resistance is required to make the* $1.00\,mA$, $20\,\Omega$ *galvanometer into an ammeter with a range of* 0 *to* $50.0\,mA$*?*

Sol. Here, $i_g = 1.00\ \text{mA} = 10^{-3}$ A, $G = 20\ \Omega$,

$$i = 50.0 \times 10^{-3}\ \text{A}$$

Substituting in, $$S = \left(\frac{i_g}{i - i_g}\right) G$$

$$= \frac{(10^{-3})\,(20)}{(50.0 \times 10^{-3}) - (10^{-3})}$$

$$= 0.408\,\Omega$$

Note Point

- The resistance of ammeter is given by

$$\frac{1}{A} = \frac{1}{G} + \frac{1}{S} = \frac{1}{20} + \frac{1}{0.408} \quad \text{or} \quad A = 0.4\ \text{W}$$

The shunt resistance is so small in comparison to the galvanometer resistance that the ammeter resistance is very nearly equal to the shunt resistance. This shunt resistance gives us a low resistance ammeter with the desired range of 0 to 50.0 mA. At full scale deflection i =50.0mA the current through the galvanometer is 1.0 mA while the current through the shunt is 49.0 mA. If the current i is less than 50.0 mA, the coil current and the deflection are proportionally less, but the ammeter resistance is still 0.4 Ω.

Voltmeter

A voltage measuring device is called a voltmeter. It measures the potential difference between two points.

A galvanometer can be converted into a voltmeter by connecting a high resistance (R) in series with it. The whole assembly called the voltmeter is connected in parallel between the points where potential difference has to be measured.

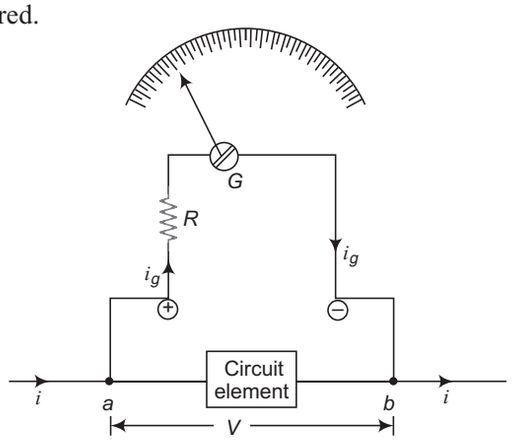


Fig. 19.47

For a voltmeter with full scale reading V, we need a series resistor R such that

$$V = i_g\ (G + R) \text{ or } R = \frac{V}{i_g} - G$$

Note Points

- The reading of a voltmeter is always lesser than true value.

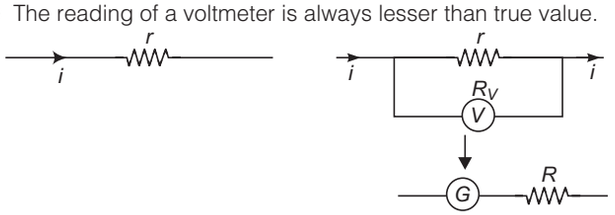


Fig. 19.48

For example, if a current i is passing through a resistance r, the actual value is

$$V = ir \qquad \ldots(i)$$

Now if a voltmeter of resistance $R_V\ (= G + R)$ is connected across the resistance r, the new value will be

$$V' = \frac{i \times (rR_V)}{r + R_V} \quad \text{or} \quad V' = \frac{ir}{1 + \frac{r}{R_V}} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we can see that,

$$V' < V \text{ and } V' = V \text{ if } R_V = \infty$$

Thus, resistance of an ideal voltmeter should be infinite.

- Percentage error in measuring the potential difference by a voltmeter is

$$\left(\frac{V - V'}{V}\right) \times 100 = \left(\frac{1}{1 + \frac{r}{R_V}}\right) \times 100 \text{ or \% error} = \left(\frac{1}{1 + \frac{r}{R_V}}\right) \times 100$$

- Resistance of voltmeter, $R_V = R + G$

Example 19.16 *How can we make a galvanometer with $G = 20\ \Omega$ and $i_g = 1.0$ mA into a voltmeter with a maximum range of 10 V?*

Sol. Using $R = \frac{V}{i_g} - G$

We have, $R = \frac{10}{10^{-3}} - 20$

$= 9980\ \Omega$

Thus, a resistance of 9980 Ω is to be connected in series with the galvanometer to convert it into the voltmeter of desired range.

- At full scale deflection current through the galvanometer, the voltage drop across the galvanometer

$$V_g = i_g G = 20 \times 10^{-3}\ \text{V} = 0.02\ \text{V}$$

and the voltage drop across the series resistance R is,

$$V = i_g R = 9980 \times 10^3\ \text{V} = 9.98\ \text{V}$$

or we can say that most of the voltage appears across the series resistor.

Potentiometer

The potentiometer is an instrument that can be used to measure the emf or the internal resistance of an unknown source. It also has a number of other useful applications.

Principle of Potentiometer

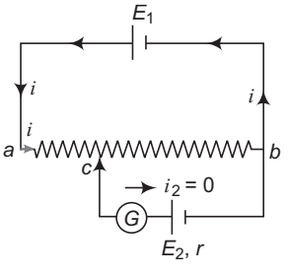


Fig. 19.49

The principle of potentiometer is schematically shown in above figure. A resistance wire ab of total resistance R_{ab} is permanently connected to the terminals of a source of known emf E_1. A sliding contact c is connected through the galvanometer G to a second source whose emf E_2 is to be measured. As contact c is moved along the potentiometer wire, the resistance R_{cb} between points c and b varies. If the resistance wire is uniform R_{cb} is proportional to the length of the wire between c and b. To determine the value of E_2, contact c is moved until a position is found at which the galvanometer shows no deflection. This corresponds to zero current passing through E_2. With $i_2 = 0$, Kirchhoff's second law gives,

$$E_2 = iR_{cb}$$

With $i_2 = 0$, the current i produced by the emf E_1 has the same value no matter what the value of emf E_2. A potentiometer has following applications.

To find emf of an Unknown Battery

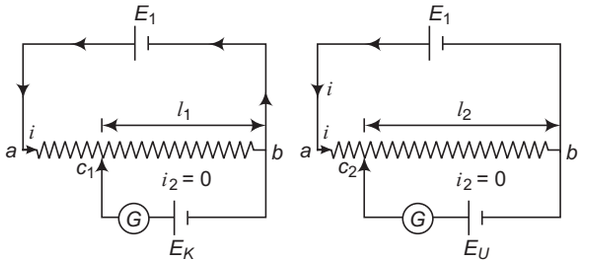


Fig. 19.50

We calibrate the device by replacing E_2 by a source of known emf E_K and then by unknown emf E_U. Let the null points are obtained at lengths l_1 and l_2. Then,

$$E_K = i(\rho l_1)$$

and $$E_U = i(\rho l_2)$$

where, ρ = resistance of wire *ab* per unit length.

$$\therefore \quad \frac{E_K}{E_U} = \frac{l_1}{l_2} \quad \text{or} \quad E_U = \left(\frac{l_2}{l_1}\right) E_K$$

So, by measuring the lengths l_1 and l_2, we can find the emf of an unknown battery.

To find the Internal Resistance of an Unknown Battery

To find the internal resistance of an unknown battery let us derive a formula.

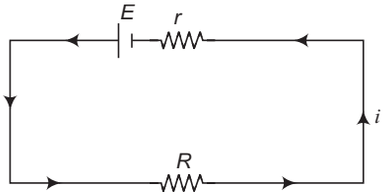


Fig. 19.51

In the circuit shown in figure,

$$i = \frac{E}{R + r} \qquad \ldots(i)$$

and $V =$ potential difference across the terminals of the battery

or $$V = E - ir = iR \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we can prove that

$$r = R\left(\frac{E}{V} - 1\right)$$

Thus, if a battery of emf E and internal resistance r is connected across a resistance R and the potential difference across its terminals comes out to be V, then the internal resistance of the battery is given by the above formula. Now let us apply it in a potentiometer for finding the internal resistance of the unknown battery.

The circuit shown in Fig. 19.52 is similar to the previous one.

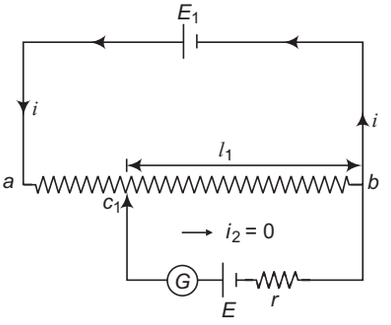


Fig. 19.52

Hence, $$E = i\rho l_1 \qquad \ldots(i)$$

Now, a known resistance R is connected across the terminals of the unknown battery as shown in Fig. 19.53.

This time $V_{cb} \neq E$, but $V_{cb} = V$

where, V = potential difference across the terminals of the unknown battery. Hence,

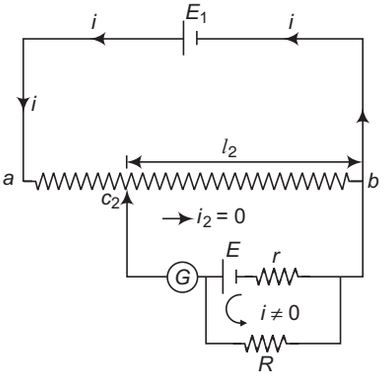


Fig. 19.53

$$V = i\rho l_2 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{E}{V} = \frac{l_1}{l_2}$$

Substituting in $r = R\left(\frac{E}{V} - 1\right)$, we get

$$r = R\left(\frac{l_1}{l_2} - 1\right)$$

So, by putting R, l_1 and l_2 we can determine the internal resistance r of unknown battery.

Example 19.17 *A potentiometer wire of length* 100 *cm has a resistance of* 10 Ω. *It is connected in series with a resistance and a cell of emf* 2 *V and of negligible internal resistance. A source of emf* 10 *mV is balanced against a length of* 40 *cm of the potentiometer wire. What is the value of external resistance?*

Sol. From the theory of potentiometer,

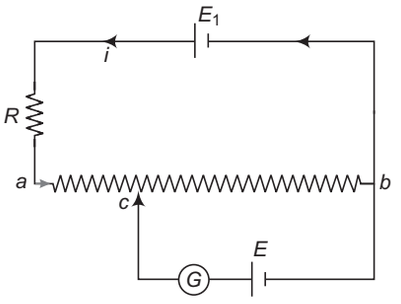


Fig. 19.54

$V_{cb} = E$, if no current is drawn from the battery

or $$\left(\frac{E_1}{R + R_{ab}}\right) R_{cb} = E$$

Here, $E_1 = 2$ V, $R_{ab} = 10\ \Omega$

$$R_{cb} = \left(\frac{40}{100}\right) \times 10 = 4\ \Omega$$

and $E = 10 \times 10^{-3}$ V

Substituting in above, we get

$R = 790\ \Omega$

Principle of Wheatstone's Bridge

The scientist Wheatstone designed a circuit to find unknown resistance. Such a circuit is popularly known as Wheatstone's bridge. This is an arrangement of four resistances which can be used to measure one of them in terms of the rest.

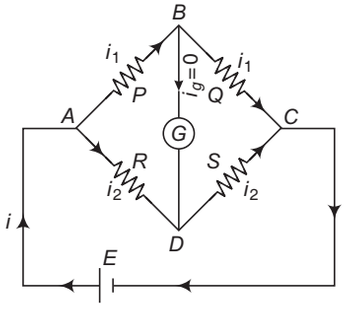


Fig. 19.55

The figure shows the circuit designed by him. The bridge is said to be balanced when deflection in galvanometer is zero, i.e. $i_g = 0$, and hence,

$$V_B = V_D$$

Under this condition,

$$V_A - V_B = V_A - V_D \quad \text{or} \quad i_1 P = i_2 R$$

or $$\frac{i_1}{i_2} = \frac{R}{P} \qquad \ldots(i)$$

Similarly,

$$V_B - V_C = V_D - V_C \quad \text{or} \quad i_1 Q = i_2 S$$

or $$\frac{i_1}{i_2} = \frac{S}{Q} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{R}{P} = \frac{S}{Q} \quad \text{or} \quad \frac{P}{Q} = \frac{R}{S}$$

So, this is a condition for which a Wheatstone bridge is balanced.

To measure the resistance of an unknown resistor, it is connected as one of the four resistors in the bridge. One of the other three should be a variable resistor. Let us suppose P is the unknown resistance and Q is the variable resistance. The value of Q is so adjusted so that deflection through the galvanometer is zero. In this case the bridge is balanced and

$$P = \left(\frac{R}{S}\right) \cdot Q$$

Knowing R, S and Q, the value of P is calculated. Following two points are important regarding a Wheatstone's bridge.

(i) In Wheatstone bridge, cell and galvanometer arms are interchangeable.

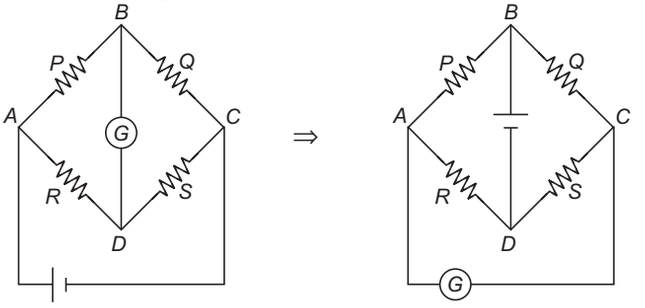


Fig. 19.56

In both the cases, condition of balanced bridge is

$$\frac{P}{Q} = \frac{R}{S}$$

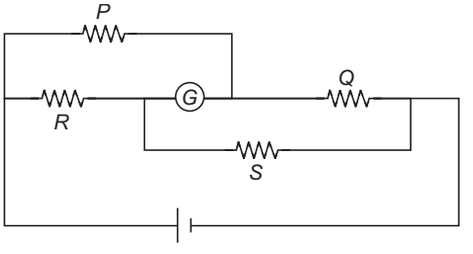
(ii) If bridge is not balanced current will flow from D to B if,

$$PS > RQ$$

Exercise Try and prove the statements of both the points yourself.

Different forms of Wheatstone's Bridge

Following are given few circuits which are basically Wheatstone's bridge circuits.



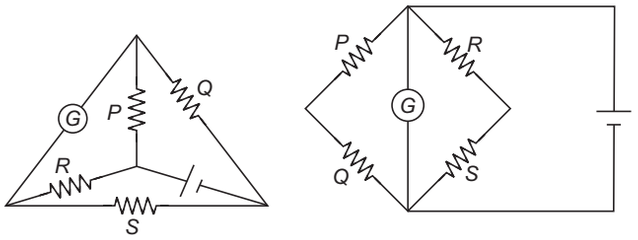
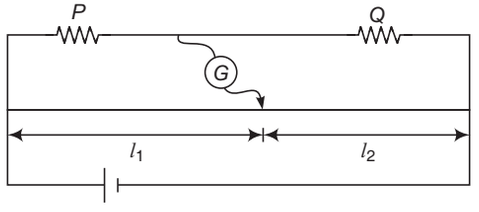


Fig. 19.57

Most simple practical application of this bridge is the **meter bridge.**

It consists of a straight and uniform wire of manganin or constantan stretched along a meter scale. The balance is obtained on the wire, if lengths of two parts are l_1 and l_2, then the ratio of two resistances



Meter bridge

Fig. 19.58

$$\frac{P}{Q} = \frac{l_1}{l_2}$$

Example 19.18 *The potentiometer wire AB is* 100 *cm long. When AC* = 40 *cm, no deflection occurs in the galvanometer. Find R.*

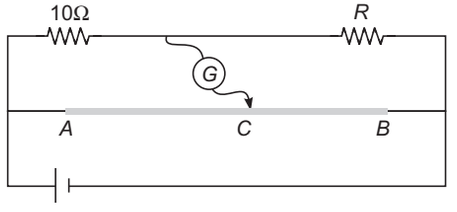


Fig. 19.59

Sol. $\frac{10}{R} = \frac{AC}{CB}$

$$\therefore \quad R = 10\left(\frac{CB}{AC}\right)$$

$$= (10)\left(\frac{100 - 40}{40}\right)$$

$$= 10\left(\frac{60}{40}\right)$$

$$= 15\ \Omega$$

Post Office Box

Initially this appratus was used for measuring the resistance of the telephone or the telegraph wires, or for finding faults in these wires.

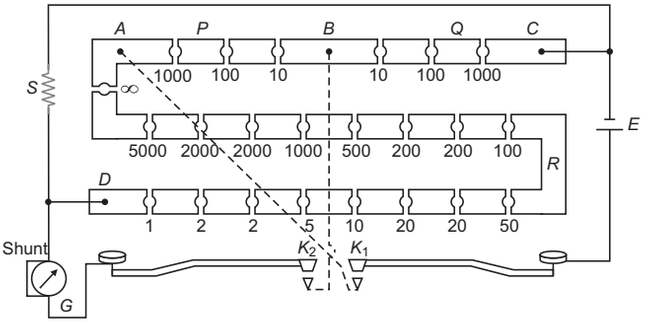


Fig. 19.60

The post office box is a resistance box in which two arms *AB* and *BC* are connected in series. Each of these arms contains resistances 10, 100 and 1000 Ω. In the third known arm *AD*, there are resistances from 1Ω to 5000 Ω arranged in a U-shape. The unknown resistance *S* to be determined is connected in the arm *CD*. The galvanometer *G* is connected between *B* and *D* through the key K_2 and the cell is connected between *A* and *C* through the key K_1.

In balanced condition, galvanometer shows zero deflection.

From Wheatstone's principle,

$$\frac{P}{Q} = \frac{R}{S}$$

∴ The unknown resistance, $S = \frac{RQ}{P}$.

By putting the values of *P*, *Q* and *R*, we can find *S*.

Chapter Summary with Formulae

▪ Formulae for Current

(i) Instantaneous current, i_{in} or $i = \frac{dq}{dt}$

(ii) Average current, $i_{av} = \frac{q}{t}$ or $\frac{\Delta q}{\Delta t}$

(iii) Direction of current is the direction of motion of positive charge or opposite to the direction of motion of negative charge.

(iv) $i_{net} = i_{+q} + i_{-q}$

(v) Order of thermal (or random) motion of free electrons is 10^5 m/s.

(vi) Order of drift motion of free electrons is 10^{-4} m/s.

(vii) Drift velocity, $v_d = \frac{eE\tau}{m}$

(viii) Current, $i = neAv_d$

(ix) Current density $j = \frac{I}{A} = nev_d$

(x) Mobility, $\mu = \frac{v_d}{E} = \frac{e\tau}{m}$

(xi) **Ohm's law for certain materials**,

$$V \propto i \quad \text{or} \quad \frac{V}{i} = \text{constant} = R$$

V-i graph is a straight line passing through origin. Slope of this graph is *R*.

(xii) Electrical conductivity, $\sigma = \frac{ne^2\tau}{m}$

(xiii) $j = \sigma E$ is another form of Ohm's law

(xiv) $j = \left(\frac{ne^2\tau}{m}\right)E$

(xv) Resistivity, $\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$

(xvi) $R = \rho\frac{l}{A}$

(xvii) $\rho_T = \rho_0\,[1 + \alpha\,(T - T_0)]$

(xviii) $R_T = R_0\,[1 + \alpha\,(T - T_0)]$

(xix) $\alpha = \frac{1}{\rho}\left(\frac{d\rho}{dT}\right)$

▪ Formulae in Vector Form

(xx) $i = \mathbf{J} \cdot \Delta\mathbf{A}$

(xxi) $\mathbf{E} = \rho\mathbf{j}$ or $\mathbf{j} = \sigma\mathbf{E}$

(xxii) $\mathbf{j} = -ne\,\mathbf{v}_d$ i.e. $\mathbf{j}$ and $\mathbf{v}_d$ are anti-parallel

(xxiii) $\mu = \frac{|\mathbf{v}_d|}{|\mathbf{E}|} = \frac{v_d}{E}$

▪ Formulae for Electrical Circuits

(i) Emf of a cell, $E = \frac{W}{q}$

(ii) Current in a simple circuit (single wire, single current),

$$i = \frac{\text{Net emf}}{\text{Net resistance}}$$

(iii) Current in a complex circuit (two or more than two wires) can be obtained by Kirchhoff's two laws.

(iv) Kirchhoff's junction rule

$$\sum_{\text{junction}} i = 0$$

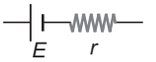
This is law of conservation of charge.

(v) Kirchhoff's loop rule,

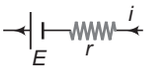
$$\sum_{\text{closed loop}} = \Delta V = 0$$

This is law of conservation of energy.

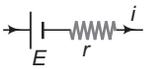
(vi) Potential difference across the terminals of a battery



$V = E$, if $i = 0$



$V = E - ir$ during discharging



$V = E + ir$ during charging

(vii) **Resistors in series**

(a) $R = R_1 + R_2$

(b) Current is same

(c) Potential difference distributes in direct ratio of resistance,

$$V = R \quad \text{or} \quad \frac{V_1}{V_2} = \frac{R_1}{R_2}$$

(viii) **Resistors in parallel**

(a) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

(b) $R = \frac{R_1R_2}{R_1 + R_2}$

(c) Equivalent value of resistance is less than the minimum value of individual resistors.

(d) *R* and *R* has a net value of $\frac{R}{2}$.

(e) *R*, *R* and *R* has a net value of $\frac{R}{3}$ and so on.

(f) Current distributes in inverse ratio of resistance. More resistance, less current.

$$i \propto \frac{1}{R} \quad \text{or} \quad \frac{i_1}{i_2} = \frac{R_2}{R_1}$$

(ix) **Cells in series**

(a) If polarities are additive, then

net emf = nE

total internal resistance = nr

total external resistance = R

∴ Current in the circuit, $i = \frac{nE}{nr + R}$

(b) If polarity of *m* cells is reversed, then

$$i = \frac{(n - 2m)\,E}{nr + R} \qquad (n > 2m)$$

(x) **Identical cells in parallel**

net emf = E

total internal resistance = r/n

total external resistance = R

∴ Current through external resistance,

$$i = \frac{E}{r/n + R}$$

(xi) **Unidentical cells in parallel**

$$\text{net emf} = \frac{\Sigma (E/r)}{\Sigma (1/r)}$$

$$\text{total internal resistance} = \frac{1}{\Sigma (1/r)}$$

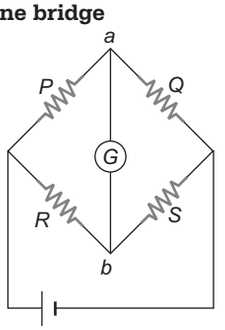
total external resistance $= R$

$\therefore$ Current through external resistance, $i = \dfrac{\Sigma (E/r)/\Sigma(1/r)}{R + (1/\Sigma 1/r)}$

In this case, if polarity of one cell (say of 2nd cell) is reversed than,

$$E_{eq} = \frac{E_1/r_1 - E_2/r_2 + E_3/r_3}{1/r_1 + 1/r_2 + 1/r_3}.$$

(xii) **Wheatstone bridge**



If $\dfrac{P}{Q} = \dfrac{R}{S}$, then bridge is said to be balanced.

In balanced condition,

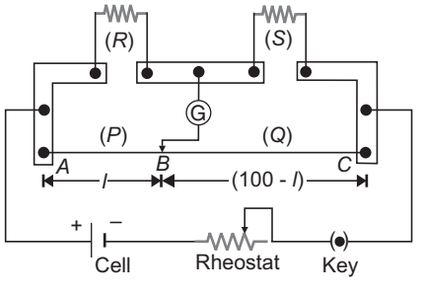
$$V_a = V_b$$

or $V_{ab} = 0$

or $i_G = 0$

(xiii) **Meter bridge**

This is based on balanced Wheatstone bridge.



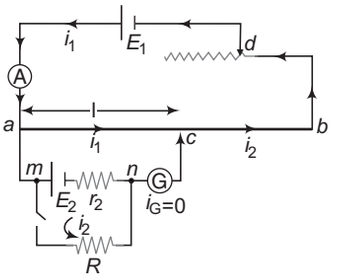
$$\frac{P}{Q} = \frac{R}{S}$$

$\therefore$ $$\frac{l}{100 - l} = \frac{R}{S}$$

or $$R = \left(\frac{l}{100 - l}\right) S$$

Most accurate results are obtained when null point (when $i_G = 0$) is obtained near the centre of wire AC. Or the ratio $\dfrac{P}{Q}$ or $\dfrac{R}{S}$ is kept 1 : 1.

(xiv) **Potentiometer**



(a) $V_a = V_m$ and $V_c = V_n$

or $V_a - V_c = V_m - V_n$ or $V_{ac} = V_{mn}$

or $i_1 \lambda l = E_2 - i_2 r_2$

i_2 may or may not be zero.

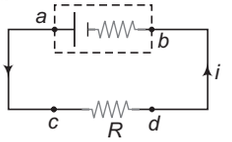
(b) λ = resistance per unit length of potentiometer wire ab.

(c) $i_1 \lambda = \phi$ = potential gradient in wire ab

= potential drop per unit length.

(d) $\dfrac{E_2}{E_2'} = \dfrac{l_1}{l_2}$

(e) $V_{ab} = V_{cd} = V = E - ir$



where, $i = \dfrac{E}{R + r}$, $r = R\left(\dfrac{E}{V} - 1\right)$

(f) In potentiometer, $r = R\left(\dfrac{l_1}{l_2} - 1\right)$

(xv) **Thermal heat**

$$H = i^2 Rt = Vit = \frac{V^2}{R} t$$

Here, V and i are the values across that resistance in which thermal heat is asked.

(xvi) **Thermal power**

$$P = i^2 R = Vi = \frac{V^2}{R}$$

(xvii) 1 kWh = 3.6×10^6 J

(xv) Conversion of galvanometer into ammeter

(a) Shunt, $S = \left(\dfrac{i_g}{i - i_g}\right) G$

(b) Resistance of an ideal ammeter = 0

(xvi) Conversion of galvanometer into voltmeter

(a) Resistance required, $R = \dfrac{V}{i_g} - G$

(b) Resistance of an ideal voltmeter should be infinite.

(c) Voltage sensitivity = $\dfrac{\phi}{V} = \left(\dfrac{NAB}{k}\right)\left(\dfrac{1}{R}\right)$

Additional Examples

Example 1. *The electron drift arises due to the force experienced by electrons in the electric field inside the conductor. But force should cause acceleration. Why then do the electrons acquire a steady average drift speed?*

Sol. Free electrons do not accelerate continuously. They collide with fixed ions or atoms. They loses their speed during collision. After collision they further accelerate. So, the process of acceleration and collision take place simultaneously. On the average therefore, electrons acquire only a drift speed.

Example 2. *If the electron drift speed is so small, and the electron's charge is small, how can we still obtain large amounts of current in a conductor?*

Sol. Free electron density or the number of electrons per unit volume is very high ($\approx 10^{29}\,\text{m}^{-3}$) and current is given by

$$i = neAv_d$$

where, e and v_d are small but n is large.

Example 3. *When electrons drift in a metal from lower to higher potential, does it mean that all the free electrons of the metal are moving in the same direction?*

Sol. No, all the free electrons are not moving in the same direction. The drift velocity is superposed over the large random velocities of electrons.

Example 4. *Are the paths of electrons straight lines between successive collisions (with the positive ions of the metal) in the*

(i) absence of electric field,

(ii) presence of electric field?

Sol. In the absence of electric field, when there is only random motion of electrons, path is straight line. In the presence of electric field when the drift motion is superposed over the random motion, paths are in general curved.

Example 5. *A steady current flows in a metallic conductor of non-uniform cross-section. Which of these quantities is constant along the conductor : current, current density, electric field, drift speed?*

Sol. Only current. Rest all are inversely proportional to area of cross-section.

Example 6. *Is Ohm's law universally applicable for all conducting elements? If not give examples of elements which do not obey Ohm's law?*

Sol. No. Examples of non-ohmic elements are vacuum diode and semiconductor diode.

Example 7. Choose the correct alternative.

(a) Alloys of metals usually have (greater/less) resistivity than that of their constituent metals.

(b) Alloys usually have much (lower/higher) temperature coefficients of resistance than pure metals.

(c) The resistivity of the alloy manganin is (nearly independent of/increases rapidly) with increase of temperature.

(d) The resistivity of a typical insulator (e.g. Amber) is greater than that of a metal by a factor of the order of $(10^{22}/10^{3})$

Sol. (a) Greater, (b) Lower, (c) Nearly independent of, (d) 10^{22}

Example 8. *A battery of emf E and internal resistance r is connected with an external resistance R. Draw*

(a) the current versus external resistance graph (*i*-*R graph*)

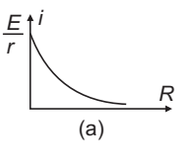
(b) the current versus potential difference across the terminals of the battery graph (*i* - *V graph*).

Sol. (a) $i = \dfrac{E}{r+R}$

At $R = 0$, $i = \dfrac{E}{r}$ = maximum value

At $R \to \infty$, $i \to 0$

Therefore, i *versus* R graph is as shown in Fig. (a) :



(a)

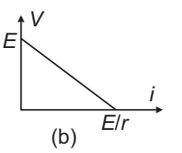
(b) $3V = E - ir$

i.e. V *versus* i graph is a straight line of intercept $+E$ and slope $-r$.

At $i = 0$, $V = E$ and at $i = \dfrac{E}{r}$

(when, $R = 0$, short circuiting) $V = 0$.

Corresponding V - i graph is as shown in Fig. (b) :



(b)

Example 9. *When a motor car is started then its head light becomes slightly dim. Explain why ?*

Sol. When the motor car is started, the starter draws heavy current from the car battery. Hence, due to an increase in the potential drop ir within the battery actual available potential difference ($V = E - ir$) for head light is less. As a result the head light becomes dim.

Example 10. *Distinguish between kilowatt and kilowatt-hour.*

Sol. Kilowatt is the unit of power, whereas kilowatt hour is the unit of energy.

1 kilowatt = 1000 W = 1000 J/s

1 kilowatt-hour = 3.6×10^{6} J

Example 11. *What is/are the material (s) used in (i) electric heater and (ii) fuse wire?*

Sol. (i) Nichrome is used for making an electric heater.
(ii) Fuse wire is made up of an alloy of tin and lead.

Example 12. *We have two bulbs one of* $60W$ *and other of* $100W$.
(a) Which bulb has more resistance ?
(b) Which bulb will consume more power, if they are connected in parallel?
(c) Which bulb will consume more power, if they are connected in series?

Sol. (a) Resistance of a bulb is given by,

$$R = \frac{V^2}{P}$$

where, P and V are the written values over the bulb. In both bulbs voltage V will be remain same. Hence,

$$R = \frac{1}{P}$$

$$\therefore \quad R_{60\text{W}} > R_{100\text{W}}$$

(b) In parallel V remains same. Hence, for comparison of two powers, we should write

$$P = \frac{V^2}{R} \quad \text{or} \quad P \propto \frac{1}{R}$$

Since, $R_{100\text{W}} < R_{60\text{W}}$, therefore 100W bulb will consume more power.

(c) In series current i is same. Hence, for comparison of two powers, we should write,

$P = i^2 R$ or $P \propto R$

Since, $R_{60\text{W}} > R_{100\text{W}}$, therefore 60W bulb will consume more power.

Example 13. *What is potential gradient? State its SI units.*

Sol. Potential difference per unit length of potentiometer wire is called potential gradient.

Potential gradient $= i\lambda$

Here, i = current passing through potentiometer wire and

λ = resistance per unit length of potentiometer wire.

SI unit of potential gradient is V/m.

Example 14. *Which one of the two, an ammeter or a milliammeter, has a higher resistance and why?*

Sol. Milliammeter has a higher resistance, because its shunt is of higher resistance than that of an ammeter.

Example 15. *What would happen, if we connect an ammeter in parallel and a voltmeter in series in a circuit?*

Sol. An ammeter (very low resistance) in parallel will draw almost the entire current of the circuit and may be damaged. A voltmeter (very high resistance) in series will reduce the current to almost zero.

Example 16. *In the Bohr model of hydrogen atom, the electron is pictured to rotate in a circular orbit of radius* $5\times10^{-11}\,m$, *at a speed* $2.2\times10^{6}\,m/s$. *What is the current associated with electron motion?*

Sol. Frequency of revolution of electron,

$$f = \frac{v}{2\pi r} = \frac{2.2\times10^{6}}{2\pi\,(5\times10^{-11})}$$

$$= 7.0\times10^{15}\ \text{Hz}$$

Current associated, $i = q\,f$

$$= (1.6\times10^{-19})\,(7.0\times10^{15})\ \text{A}$$

$$= 11.2\times10^{-4}\ \text{A}$$

$$= 1.12\ \text{mA}$$

Example 17. *Copper has one conduction electron per atom. Its density is* $8.89\,g/cm^3$ *and its atomic mass is* $63.54\,g/mol$. *If a copper wire of diameter* $1.0\,mm$ *carries a current of* $2.0\,A$, *what is the drift speed of the electrons in the wire?*

Sol. As we know, $i = neAv_d$

$$\therefore \quad v_d = \frac{i}{neA}$$

Here, n = number of conduction electrons per unit volume.

Mass of unit volume is ρ.

$$\text{Number of atoms in unit volume} = \left(\frac{\text{mass}}{M}\right)(N) = \frac{\rho N}{M}$$

As, there is one conduction electron per atom.

Hence, $$n = \frac{\rho N}{M}$$

$$\therefore \quad v_d = \frac{i}{\left(\frac{\rho N}{M}\right)e\,(\pi r^2)} \quad \text{or} \quad v_d = \frac{iM}{\rho Ne\pi r^2}$$

Substituting the values, we have

$$v_d = \frac{(2.0)\,(63.54\times10^{-3})}{(8.89\times10^{3})\,(6.02\times10^{23})\,(1.6\times10^{-19})\,(3.14)\,(0.5\times10^{-3})^2}$$

$$= 1.9\times10^{-4}\ \text{m/s}$$

Example 18. *In household wiring, copper wire* $2.05\,mm$ *in diameter is often used. Find the resistance of a* $35.0\,m$ *long wire. Specific resistance of copper is* $1.72\times10^{-8}\ \Omega\text{-}m$.

Sol. Resistance of the wire,

$$R = \frac{\rho l}{A}$$

Substituting the values, we have

$$R = \frac{(1.72\times10^{-8})\,(35.0)}{\frac{\pi}{4}(2.05\times10^{-3})^2} = 0.18\,\Omega$$

Example 19. *An aluminium wire carrying a current has diameter* 0.84 *mm, the electric field in the wire is* $0.49\,V/m$. *What is,*

(a) the current carried by the wire?

(b) the potential difference between two points in the wire 12.0 *m apart?*

(c) the resistance of a 12.0 *m length of this wire? Specific resistance of aluminium is* $2.75\times10^{-8}\ \Omega$-*m.*

Sol. (a) Current carried by the wire,

$$i=\frac{V}{R} \qquad (V = \text{potential difference})$$

$$=\frac{El}{\rho\frac{l}{A}} \qquad (\rho = \text{specific resistance})$$

$$=\frac{EA}{\rho}$$

Substituting the values, we have

$$i=\frac{(0.49)\frac{\pi}{4}(0.84\times10^{-3})^2}{2.75\times10^{-8}}$$

$$=9.9\text{ A}$$

(b) Potential difference, $V = El$

$$=(0.49)\,(12)$$

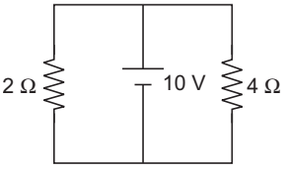
$$=5.9\text{ V}$$

- Potential difference can also be obtained by $V = iR$, after calculating a resistance of 12 m long wire and multiplying it with the current flowing through the wire.

(c) $$R=\rho\frac{l}{A}=\frac{(2.75\times10^{-8})\,(12.0)}{\frac{\pi}{4}(0.84\times10^{-3})^2}$$

$$=0.6\ \Omega$$

Example 20. *Find the current through* $2\ \Omega$ *and* $4\ \Omega$ *resistance.*



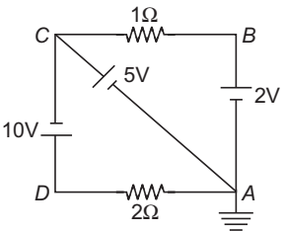
Sol. Potential difference across both the resistances is 10 V. Hence,

$$i_{2\,\Omega}=\frac{10}{2}=5\text{ A}$$

and

$$i_{4\,\Omega}=\frac{10}{4}=2.5\text{ A}$$

Example 21. *In the circuit shown in figure, find the potentials of A, B, C and D and the current through* $1\ \Omega$ *and* $2\ \Omega$ *resistance.*



Sol. Point A is earthed. Therefore, potential of point A is zero.

There is a battery connected between points A and B with positive terminal connected to point B. Therefore, potential of point B is 2 V.

On the similar logic, potential of point C is 5 V. Between points A and D, there are two batteries (additive in nature) of 5 V and 10 V each. Therefore, net potential at point D will be 15 V. Thus,

$$V_A=0,\ V_B=2\text{ V},\ V_C=5\text{ V},\ \text{and}\ V_D=15\text{ V}$$

Current through $2\ \Omega$ resistance,

$$i_2=\frac{V_{DA}}{2}=\frac{V_D-V_A}{2}$$

$$=\frac{15-0}{2}=7.5\text{ A} \qquad (\text{from } D \text{ to } A)$$

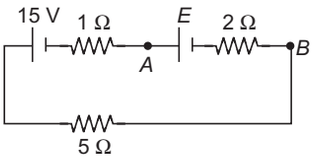
Similarly, current through $1\ \Omega$ resistance,

$$i_1=\frac{V_{CB}}{1}$$

$$=\frac{V_C-V_B}{1}$$

$$=\frac{5-2}{1}=3\text{ A} \qquad (\text{from } C \text{ to } B)$$

Example 22. *For what value of E, the potential of A is equal to the potential of B?*



Sol. Both the batteries are additive in nature. Hence, current in the circuit will be,

$$i=\frac{E+15}{1+2+5}$$

$$=\frac{E+15}{8}\text{ A}$$

Potential of A is given equal to the potential at B, i.e.

$$V_{AB}=0$$

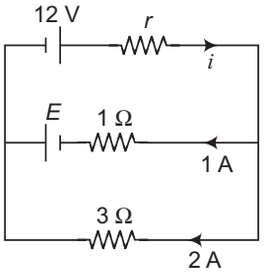
or $$E-ir=0 \qquad (\text{here, } r=2\ \Omega)$$

$$\therefore \qquad E-\left(\frac{E+15}{8}\right)(2)=0$$

Solving this equation, we get

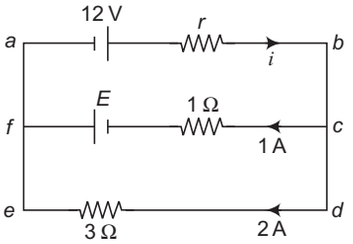
$$E=5\text{ V}$$

Example 23. *In the circuit shown in figure, a* 12 *V power supply with unknown internal resistance r is connected to a battery with unknown emf E and internal resistance* 1 Ω *and to a resistance of* 3 Ω *carrying a current of* 2 *A. The current through the rechargeable battery is* 1 *A in the direction shown in figure. Find the unknown current i, internal resistance r and the emf E.*



Sol. Applying Kirchhoff 's junction law at *c*, we have

$$i = (1 + 2)\ \text{A} = 3\text{A}$$



From Kirchhoff's loop law in *eabde*, we have, $+12 - ir - 3 \times 2 = 0$

$$\therefore \quad r = \frac{12 - 6}{i} = \frac{6}{3} = 2\ \Omega$$

Further, applying second law in loop *efcde*, we have,

$$-E + 1 \times 1 - 3 \times 2 = 0$$

$$\therefore \quad E = -5\ \text{V}$$

Example 24. *The full scale deflection current of a galvanometer of resistance* 1 Ω *is* 5 *mA. How will you convert it into a voltmeter of range* 5 *V?*

Sol. From the relation, $V = i_g\ (G + R)$, we have

$$R = \frac{V}{i_g} - G = \left(\frac{5}{5 \times 10^{-3}}\right) - 1 = 999\ \Omega$$

i.e. a resistance of 999 Ω should be connected in series with the galvanometer to convert it into a voltmeter of desired range.

Example 25. *A micrometer has a resistance of* 100 Ω *and full scale deflection current of* 50 μ*A. How can it be made to work as an ammeter of range* 5 *mA*?

Sol. From the relation,

$$i_g G = (i - i_g)S$$

We have,

$$S = \left(\frac{i - i_g}{i_g}\right) G$$

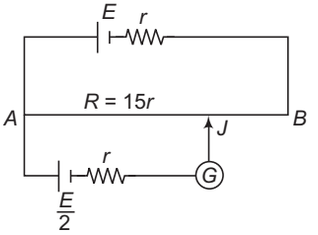
$$= \left(\frac{5 \times 10^{-3} - 50 \times 10^{-6}}{50 \times 10^{-6}}\right) \times 100$$

$$= 1\ \Omega$$

i.e. a resistance of 1 Ω should be connected in parallel with galvanometer to convert it into the ammeter of desired range.

Example 26. *The potentiometer wire AB is* 600 *cm long.*

(a) At what distance from A should the jockey J touch the wire to get zero deflection in the galvanometer?



(b) If the jockey touches the wire at a distance 560 *cm from A, what will be the current through the galvanometer?*

Sol. (a) In case of zero deflection in galvanometer,

$$V_{AJ} = \frac{E}{2} \ \Rightarrow \ \therefore \ iR_{AJ} = \frac{E}{2}$$

or $$\left(\frac{E}{15r + r}\right)\left(\frac{15r}{600}\right) AJ = \frac{E}{2}$$

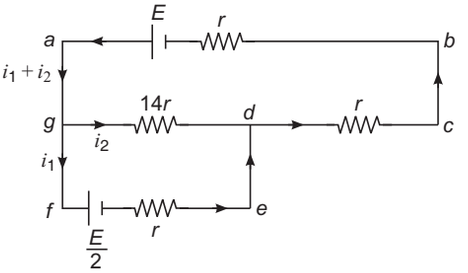
Solving this equation, we get

$$AJ = 320\ \text{cm}$$

(b) R_{AJ} in this case will be,

$$R_{AJ} = \frac{15r}{600} \times 560 = 14r$$

The circuit can be drawn as under,



Applying second law in loop *abcdga*, we have

$$-E + (i_1 + i_2)r + (i_1 + i_2)r + 14i_2 r = 0 \qquad \text{...(i)}$$

Further applying second law in loop *gdefg*, we have

$$-14i_2 r + i_1 r + \frac{E}{2} = 0 \qquad \text{...(ii)}$$

Solving these two equations, we get

$$i_1 = \frac{3E}{22r}$$

NCERT Selected Questions

Q 1. The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω, what is the maximum current that can be drawn from the battery?

Sol. The maximum current is drawn from the battery, when the external resistance in the circuit is zero, i.e. $R = 0$.

Thus, $$I_{max} = \frac{E}{r} = \frac{12}{0.4}$$
$$= 30 \text{ A}$$

Q 2. A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?

Sol. Using the relation, $I = \frac{E}{R + r}$, we get

$$R = \frac{E}{I} - r$$
$$= \frac{10}{0.5} - 3$$
$$R = 17\ \Omega$$

Using $V = IR$, we get
$$V = 0.5 \times 17$$
$$= 8.5 \text{ V}$$

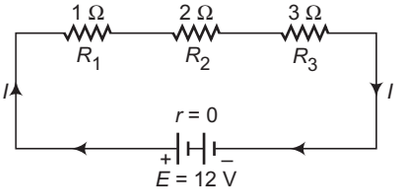
Q 3. (a) Three resistors 1 Ω, 2 Ω and 3 Ω are combined in series. What is the total resistance of the combination?

(b) If the combination is connected to a battery of emf 12V and negligible internal resistance, obtain the potential drop across each resistor.

Sol. (a) In series, the total resistance is given by,

$$R_s = R_1 + R_2 + R_3$$
$$= 1 + 2 + 3$$
$$= 6\ \Omega$$

(b)

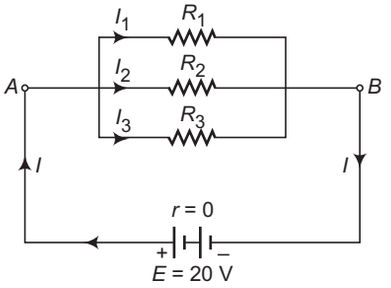


$$I = \frac{E}{R_s} = \frac{12}{6} = 2 \text{ A}$$

$\therefore$ $V_1 =$ Potential drop across $R_1 = IR_1 = 2 \times 1 = 2$ V
$V_2 =$ Potential drop across $R_2 = IR_2 = 2 \times 2 = 4$ V
$V_3 =$ Potential drop across $R_3 = IR_3 = 2 \times 3 = 6$ V

Q 4. (a) Three resistors 2 Ω, 4 Ω and 5 Ω are combined in parallel. What is the total resistance of the combination?

(b) If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor and the total current drawn from the battery.



Sol. (a) If R_p be the resistance of the parallel combination.

Then, $$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$$
$$= \frac{10 + 5 + 4}{20}$$
$$= \frac{19}{20}$$

$\therefore$ $$R_p = \frac{20}{19}\ \Omega$$

(b) Let I = total current in the circuit and I_1, I_2, I_3 be the currents through R_1, R_2 and R_3 respectively,

Then, $$I = \frac{E}{R_p} = \frac{20}{(20/19)} = 19 \text{ A}$$

$\therefore$ $$I_1 = \frac{E}{R_1} = \frac{20}{2} = 10 \text{ A}$$

$$I_2 = \frac{E}{R_2} = \frac{20}{4} = 5 \text{ A}$$

$$I_3 = \frac{E}{R_3} = \frac{20}{5} = 4 \text{ A}$$

Q 5. At room temperature (27.0°C) the resistance of a heating element is 100 Ω. What is the temperature of the element, if the resistance is found to be 117 Ω, given that the temperature coefficient of the material of the resistor is $1.70 \times 10^{-4}\ °C^{-1}$.

Sol. Using the relation,

$$\alpha = \frac{R_t - R_0}{R_0\, \Delta t}, \text{we get}$$

$$\alpha = \frac{R_t - R_{27}}{R_{27} \times (t - 27)}$$

or $$t - 27 = \frac{R_t - R_{27}}{R_{27} \times \alpha}$$

$$= \frac{117 - 100}{100 \times 1.7 \times 10^{-4}}$$

$$= 1000$$

or $$t = 27 + 1000$$

$$= 1027°\text{C}$$

Q 6. A negligibly small current is passed through a wire of length 15m and uniform cross-section 6.0×10^{-7} m^2 and its resistance is measured to be 5.0 Ω. What is the resistivity of the material at the temperature of the experiment?

Sol. Using the relation, $R = \frac{\rho l}{A}$, we get

$$\rho = \frac{RA}{l} = \frac{5.0 \times 6.0 \times 10^{-7}}{15}$$

$$= 2 \times 10^{-7}\ \Omega\text{-m}$$

Q 7. A silver wire has a resistance of 2.1 Ω at 27.5°C and a resistance of 2.7 Ω at 100°C. Determine the temperature coefficient of resistivity of silver.

Sol. Using the relation,

$$\alpha = \frac{R_t - R_0}{R_0\, \Delta t}, \text{we get}$$

$$\alpha = \frac{R_{100} - R_{27.5}}{R_{27.5} \times \Delta t}$$

$$= \frac{2.7 - 2.1}{2.1 \times 72.5}$$

$$= 3.94 \times 10^{-3}\ °\text{C}^{-1} \approx 4 \times 10^{-3}\ °\text{C}^{-1}$$

Q 8. A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element, if the room temperature is 27.0°C? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is $1.70 \times 10^{-4}\ °\text{C}^{-1}$.

Sol. If R_1 and R_2 be the resistance of wire at θ_1 and θ_2 respectively, then

$$R_1 = \frac{V}{I_1} = \frac{230}{3.2} = 71.875\ \Omega$$

and $$R_2 = \frac{V}{I_2} = \frac{230}{2.8} = 82.143\ \Omega$$

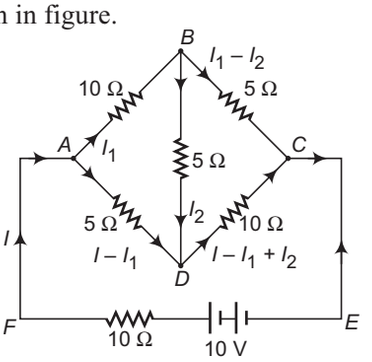
Using the relation, $\alpha = \frac{R_2 - R_1}{R_1\, \Delta\theta}$, we get

$$\Delta\theta = \frac{R_2 - R_1}{R_1 \times \alpha} = \frac{82.143 - 71.875}{71.875 \times 1.7 \times 10^{-4}}$$

or $$\theta_2 - 27 = 840.35$$

$\therefore$ $$\theta_2 = 840.35 + 27 = 867.35°\text{C}$$

Q 9. Determine the current in each branch of network shown in figure.



Sol. Let I be the total current in the circuit.

I_1 = current flowing through AB

$\therefore$ $I - I_1$ = current flowing through AD

I_2 = current flowing through BD

$\therefore$ $I_1 - I_2$ = current flowing through BC

and $I - I_1 + I_2$ = current flowing through DC

Applying loop law in loop *ABDA,* we get

$$10I_1 + 5I_2 - 5(I - I_1) = 0$$

or $$3I_1 + I_2 - I = 0 \quad \text{...(i)}$$

Again applying loop law in loop *BCDB,* we get

$$5(I_1 - I_2) - 10(I - I_1 + I_2) - 5I_2 = 0$$

or $$15I_1 - 20I_2 - 10I = 0$$

or $$3I_1 - 4I_2 - 2I = 0 \quad \text{...(ii)}$$

Applying loop law in loop *ABCEFA,* we get

$$10I + 10I_1 + 5(I_1 - I_2) = 10$$

or $$3I_1 - I_2 + 2I = 2 \quad \text{...(iii)}$$

Eq. (ii) + Eq. (iii) gives, we get

$$6I_1 - 5I_2 = 2 \quad \text{...(iv)}$$

Solving the above equations, we get

$$I_1 = \frac{4}{17}\ \text{A}$$

$$I_2 = -\frac{2}{17}\ \text{A} \quad \text{and} \quad I = \frac{10}{17}\ \text{A}$$

$\therefore$Current in *AD,*

$$I - I_1 = \frac{10}{17} - \frac{4}{17}$$

$$= \frac{6}{17}\ \text{A}$$

Current in *BC,*

$$I_1 - I_2 = \frac{4}{17} - \left(\frac{-2}{17}\right)$$

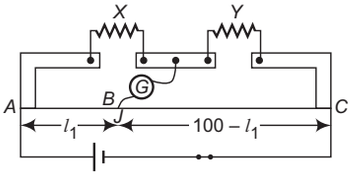
$$= \frac{6}{17}\ \text{A}$$

Current in *DC,*

$$I - I_1 + I_2 = \frac{10}{17} - \frac{4}{17} + \left(-\frac{2}{17}\right)$$

$$= \frac{4}{17}\ \text{A}$$

Q 10. (a) In a meter bridge figures, the balance point is found to be at 39.5 cm from the end A, when the resistor Y is of 12.5 Ω. Determine the resistance of X. Why are the connections between resistors in a Wheatstone or meter bridge made of thick copper strips?



(b) Determine the balance point of the bridge above if X and Y are interchanged.

Sol. (a) $\frac{X}{Y} = \frac{l_1}{100 - l_1}$ or $X = \frac{l_1}{100 - l_1} \times Y$

$$= \frac{39.5}{100 - 39.5} \times 12.5 = 8.16\ \Omega$$

Thick copper strips are used as they have negligible resistance.

(b) When X and Y are interchanged, then

$$\frac{Y}{X} = \frac{l}{100 - l}$$

$$\therefore \quad X = \left(\frac{100 - l}{l}\right) Y$$

$$\therefore \quad 8.16 = \frac{100 - l}{l} \times 12.5$$

or $\quad l = 60.5$ cm

Q 11. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell?

Sol. Using the relation,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}, \text{we get}$$

$$E_2 = \frac{l_2}{l_1} \times E_1$$

$$= \frac{63}{35} \times 1.25$$

$$= 2.25\ \text{V}$$

Q 12. Answer the following questions.

(a) A steady current flows in a metallic conductor of non-uniform cross-section. Which of these quantities is a constant along the conductor, current, current density, electric field, drift speed?

(b) Is Ohm's law universally applicable for all conducting elements? If not, give examples of elements which do not obey Ohm's law.

Sol. (a) Current density, electric field and drift speed all are inversely proportional to the area of cross-section of the conductor. Only current remains constant, when it flows through a conductor of non-uniform area of cross-section.

(b) No, Ohm's law is not universally applicable for all conducting elements. It is valid only for metallic conductors and that also when the physical conditions like temperature, stress, etc of the conductor remains the same.

The examples of elements which do not obey Ohm's law are vacuum diode semiconductor, diodes, thermistors, thyristor, SCR, etc.

Q 13. Choose the correct alternative.

(a) Alloys of metals usually have (greater/less) resistivity than that of their constituent metals.

(b) Alloys usually have much (lower/higher) temperature coefficients of resistance than pure metals.

(c) The resistivity of the alloy manganin is nearly independent of/increases rapidly with increase of temperature.

(d) The resistivity of a typical insulator (e.g. amber) is greater than that of a metal by a factor of the order of $(10^{22} / 10^{23})$.

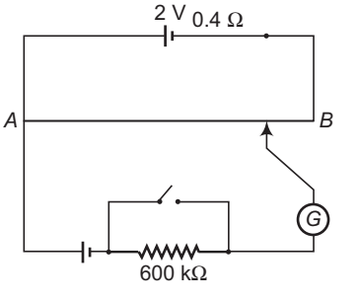
Sol. (a) greater

(b) lower

(c) nearly independent of

(d) 10^{22}

Q 14. Figure shows a potentiometer with a cell of 2.0 V and internal resistance 0.40 Ω maintaining a potential drop across the resistor wire AB.



A standard cell which maintains a constant emf of 1.02 V (for very moderate currents upto a few mA) gives a balance point of 67.3 cm length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of 600 kΩ is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown emf and the balance point found similarly, turns out to be at 82.3 cm length of the wire.

(a) What is the value of unknown emf ?

(b) What purpose does the high resistance of 600 kΩ have?

(c) Is the balance point affects by this high resistance?

(d) Is the balance point affected by the internal resistance of the driver cell?

(e) Would the method work in the above situation, if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V?

(f) Would the circuit work well for determining an extremely small emf, say of the order of few mV (such as the typical emf of a thermocouple)? If not, how will you modify the circuit?

Sol. (a) As, $\frac{E_1}{E_2} = \frac{l_1}{l_2}$ or $E_2 = E_1 \times \frac{l_2}{l_1}$

$\therefore \quad E_2 = 1.02 \times \frac{82.3}{67.3} = 1.247 \text{ V} \approx 1.25 \text{ V}$

(b) Here, $R = 600\ \text{k}\Omega$ is connected in series with the cell. The purpose of using this high resistance is to allow very small current through the galvanometer when the movable contact is far from the balance point.

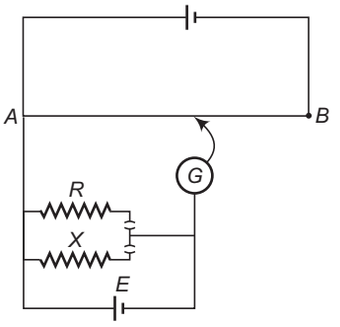
(c) No, the balance point is not affected by the presence of this high resistance.

(d) No, the balance point is not affected by the internal resistance of the driver cell.

(e) No, the method will not work as the balance point will not be obtained on the potentiometer wire, if the emf of the driver cell is less than the emf of the standard cell.

(f) No, the circuit will not work for measuring extremely small emf of the order of millivolt, because the balance point will be just close to the end A and the percentage error in measurement will be very large. The circuit is modified by putting a suitable resistance R in series with the wire AB so that, potential drop across AB is only slightly greater than the emf to be measured. Then, the balance point will be at larger length of the wire and the percentage error will be much smaller.

***Q* 15.** Figure shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor $R = 10.0\ \Omega$ is found to be 58.3 cm, while that with the unknown resistance X is 68.5 cm. Determine the value of X. What might you do if you failed to find a balance point with the given cell of emf E?



Sol. $\frac{E_2}{E_1} = \frac{l_2}{l_1}$

Also we know that,

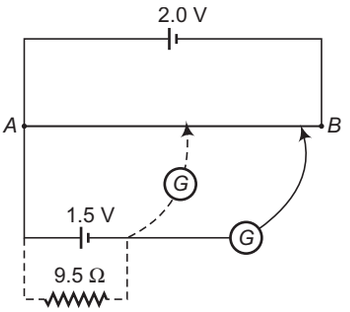
$$\frac{R_2}{R_1} = \frac{E_2}{E_1} = \frac{l_2}{l_1} \quad \text{or} \quad \frac{X}{R} = \frac{l_2}{l_1}$$

$$\therefore \quad X = \left(\frac{l_2}{l_1}\right) R = \frac{68.5}{58.3} \times 10$$

$$= 11.75\ \Omega$$

In case we do not get balance point on potentiometer wire, emf of driver cell is less than the emf to be measured. Then we need driver cell of higher emf. When we do not get balance point, it also means that the potential drop across R or X is greater than the potential drop across wire AB. A suitable series resistor can be put in the external circuit to reduce the current in the outside circuit and hence potential drop across R or X.

***Q* 16.** Figure shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell.



Sol. $r = R\left(\frac{l_1}{l_2} - 1\right)$ or $r = \left(\frac{l_1 - l_2}{l_2}\right) R$

$$= \frac{76.3 - 64.8}{64.8} \times 9.5 = 1.68\ \Omega \approx 1.7\ \Omega$$

***Q* 17.** A galvanometer coil has a resistance of 12 Ω and the meter shows full scale deflection for a current of 3 mA. How will you convert the meter into a voltmeter of range 0 to 18 V?

Sol. Using the relation,

$$V = I_g(R + G), \text{we get}$$

$$R = \frac{V}{I_g} - G = \frac{18}{3 \times 10^{-3}} - 12$$

$$= 6000 - 12 = 5988\ \Omega$$

***Q* 18.** A galvanometer coil has a resistance of 15 Ω and the meter shows full scale deflection for a current of 4 mA. How will you convert the meter into an ammeter of range 0 to 6 A?

Sol. Using the relation,

$$I_g G = (I - I_g)\, S \quad \text{or} \quad S = \frac{I_g G}{I - I_g}$$

$$= \frac{4 \times 10^{-3} \times 15}{6 - 0.004} = \frac{60 \times 10^{-3}}{5.996}$$

$$= 10.01 \times 10^{-3}\ \Omega = 10.01\ \text{m}\Omega$$

Objective Problems

[Level 1]

Electric Current, Drift Velocity and Resistance

1. When the current i is flowing through a conductor, the drift velocity is v. If $2i$ current is flowed through the same metal but having double the area of cross-section, then the drift velocity will be
(a) $v/4$ (b) $v/2$ (c) v (d) $4v$

2. When current flows through a conductor, then the order of drift velocity of electrons will be
(a) 10^{10} m/s (b) 10^{-2} cm/s
(c) 10^{4} cm/s (d) 10^{-1} cm/s

3. If a 0.1% length is increased due to stretching, then percentage increase in its resistance will be
(a) 0.2% (b) 2% (c) 1% (d) 0.1%

4. The specific resistance of manganin is $50 \times 10^{-8}\ \Omega \times \text{m}$. The resistance of a cube of length 50 cm across two sides will be
(a) $10^{-6}\ \Omega$ (b) $2.5 \times 10^{-5}\ \Omega$
(c) $10^{-8}\ \Omega$ (d) $5 \times 10^{-4}\ \Omega$

5. The specific resistance of a wire is ρ, its volume is 3 m^3 and its resistance is 3 Ω, then its length will be
(a) $\sqrt{\frac{1}{\rho}}$ (b) $\frac{3}{\sqrt{\rho}}$
(c) $\frac{1}{\rho}\sqrt{3}$ (d) $\rho\sqrt{\frac{1}{3}}$

6. In hydrogen atom, the electron makes 6.6×10^{15} revolutions per second around the nucleus in an orbit of radius 0.5×10^{-10} m. It is equivalent to a current nearly
(a) 1 A (b) 1 mA
(c) 1 μA (d) 1.6×10^{-19} A

7. The positive temperature coefficient of resistance is for
(a) carbon (b) germanium
(c) copper (d) an electrolyte

8. If n, e, τ and m respectively represent the electron density, relaxation time and mass of the electron, then the resistance of a wire of length l and area of cross-section A will be
(a) $\frac{ml}{ne^2\tau A}$ (b) $\frac{m\tau^2 A}{ne^2 l}$
(c) $\frac{ne^2\tau A}{2ml}$ (d) $\frac{ne^2 A}{2m\tau l}$

9. The resistance of a wire is 10 Ω. Its length is increased by 10% by stretching. The new resistance will now be
(a) 12 Ω (b) 1.2 Ω
(c) 13 Ω (d) 11 Ω

10. The reciprocal of resistance is
(a) conductance (b) resistivity
(c) voltage (d) None of these

11. Conductivity increases in the order of
(a) Al, Ag, Cu (b) Al, Cu, Ag
(c) Cu, Al, Ag (d) Ag, Cu, Al

12. The charge of an electron is 1.6×10^{-19} C. How many electrons strike the screen of a cathode ray tube each second, when the beam current is 16 mA?
(a) 10^{17} (b) 10^{19}
(c) 10^{-19} (d) 10^{-17}

13. Calculate the amount of charge flowing in 2 min in a wire of resistance 10 Ω, when a potential difference of 20 V is applied between its ends
(a) 120 C (b) 240 C
(c) 20 C (d) 4 C

14. A strip of copper and another of germanium are cooled from room temperature to 80 K. The resistance of
(a) each of these increases
(b) each of these decreases
(c) copper strip decreases and that of germanium increases
(d) copper strip increases and that of germanium decreases

15. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter, the change in the resistance of the wire will be
(a) 300% (b) 200%
(c) 100% (d) 50%

16. The following four wires are made of the same material and are at the same temperature. Which one of them has highest electrical resistance?
(a) Length = 50 cm, diameter = 0.5 mm
(b) Length = 100 cm, diameter = 1 mm
(c) Length = 200 cm, diameter = 2 mm
(d) Length = 300 cm, diameter = 3 mm

17. The electric field E, current density J and conductivity σ of a conductor are related as
(a) $\sigma = E/J$
(b) $\sigma = J/E$
(c) $\sigma = JE$
(d) $\sigma = 1/E$

18. Two wires of equal diameters, of resistivities ρ_1 and ρ_2 and lengths l_1 and l_2, respectively, are joined in series. The equivalent resistivity of the combination is

(a) $\frac{\rho_1 l_1 + \rho_2 l_2}{l_1 + l_2}$ (b) $\frac{\rho_1 l_2 + \rho_2 l_1}{l_1 - l_2}$

(b) $\frac{\rho_1 l_2 - \rho_2 l_1}{l_1 + l_2}$ (d) $\frac{\rho_1 l_1 - \rho_2 l_2}{l_1 + l_2}$

19. Two wires of the same dimensions but resistivities ρ_1 and ρ_2 are connected in series. The equivalent resistivity of the combination is

(a) $\rho_1 + \rho_2$ (b) $\frac{\rho_1 + \rho_2}{2}$

(c) $\sqrt{\rho_1 \rho_2}$ (d) $2(\rho_1 + \rho_2)$

20. A steady current is flowing in a cylindrical conductor. Then, which one of the following statements is true?

(a) Electric field within the conductor is zero
(b) Electric field exists only on the outer surface of the conductor
(c) Electric field exists within the conductor
(d) None of the above

21. The electron with charge ($q = 1.6 \times 10^{-19}$ C) moves in an orbit of radius 5×10^{-11} m with a speed of 2.2×10^{6} ms^{-1}, around an atom. The equivalent current is

(a) 1.12×10^{-6} A (b) 1.12×10^{-3} A

(c) 1.12×10^{-9} A (d) 1.12 A

22. In an inert gas discharge tube 2.9×10^{18} positive ions (e^+) move to the right through a cross-section of the tube each second, while 1.2×10^{18} negative ions (e^-) move to the left in the same time. The magnitude of current is

(a) 2.384 A (b) 2.384×10^{-3} A

(c) 2.384×10^{-6} A (d) None of these

23. A conductor carries a current of 0.2 A. In 30 show many electrons will flow through the cross-section of the conductor ($q = 1.6 \times 10^{-19}$ C)?

(a) 0.375×10^{19} (b) 375×10^{19}

(c) 3.75×10^{19} (d) 37.5×10^{19}

24. A wire of radius r has resistance R. If it is stretched to a wire of radius $r/2$, then the resistance of the wire becomes

(a) $2R$ (b) $4R$

(c) $16R$ (d) zero

25. Ohm-metre is the unit of

(a) resistance (b) conductance

(c) resistivity (d) voltmeter

26. The product of resistivity and conductivity of a cylindrical conductor depends on

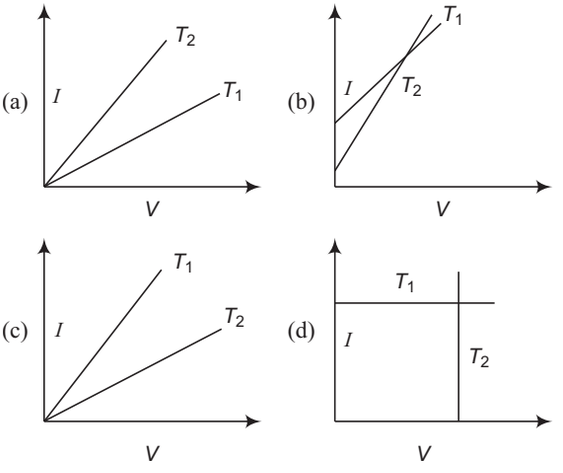
(a) temperature
(b) material
(c) area of cross-section
(d) None of the above

27. The potential difference across a wire of 10^{-3} cm^2 cross-sectional area and 50 cm length is 2 V, when a current of 0.25 A exists in the wire. The conductivity of the material is

(a) 2.5×10^{6} mho/m (b) 6.25×10^{5} mho/m

(c) 2.5×10^{9} mho/m (d) 6.25×10^{10} mho/m

28. The current-voltage graphs for a given metallic wire at two different temperatures T_1 and T_2 ($T_2 > T_1$) would be



29. For which of the following dependences of drift velocity v_d on electric field E, is Ohm's law obeyed?

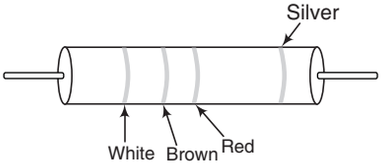
(a) $v_d \propto E$ (b) $v_d \propto E^2$

(c) $v_d \propto \sqrt{E}$ (d) v_d = constant

30. The temperature coefficient of resistance of a wire is 0.00125 per °C. At 300 K its resistance is 1 Ω. The resistance of the wire will be 2 Ω at

(a) 1154 K
(b) 1100 K
(c) 1400 K
(d) 1127 K

31. In the figure, a carbon resistor has bands of different colours on its body as mentioned in the figure. The value of the resistance is



(a) 2.2 kΩ (b) 3.3 kΩ

(c) 5.6 kΩ (d) 9.1 kΩ

32. Carbon resistors, used in electronic circuits are marked for their value of resistance and tolerance by a colour code. A given resistor has colour scheme brown, red, green and gold. Its value in ohm is

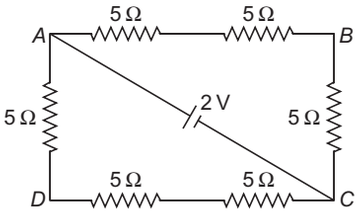
(a) $5.2 \times 10^{6} \pm 10\%$ (b) $2.4 \times 10^{5} \pm 5\%$

(c) $1.2 \times 10^{4} \pm 10\%$ (d) $1.2 \times 10^{6} \pm 5\%$

33. A metallic resistor is connected across a battery. If the number of collisions of the free electrons with the lattice is somehow decreased in the resistor (for example, by cooling it), the current will
(a) increase
(b) decrease
(c) remain constant
(d) become zero

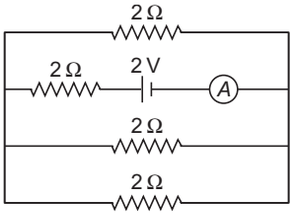
Kirchhoff's Laws and Potential Difference between two Points

34. The potential difference between points A and B of adjoining figure is



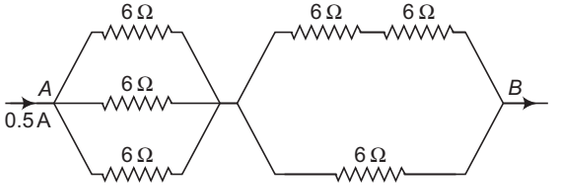
(a) $\frac{2}{3}$ V (b) $\frac{8}{9}$ V
(c) $\frac{4}{3}$ V (d) 2 V

35. The reading of the ammeter as per figure shown is



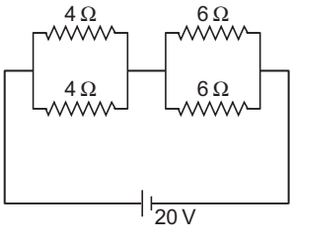
(a) $\frac{1}{8}$ A (b) $\frac{3}{4}$ A
(c) $\frac{1}{2}$ A (d) 2A

36. Resistance of 6 Ω each are connected in the manner shown in figure. With the current 0.5 A, the potential difference $V_A - V_B$ is



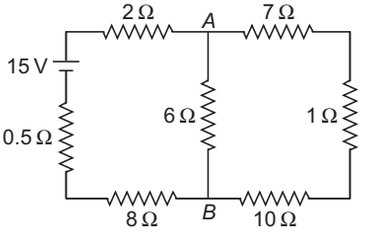
(a) 3.6 V (b) 6.0 V
(c) 3.0 V (d) 7.2 V

37. Four resistances are connected in circuit in the given figure. The electric current flowing through 4 Ω and 6 Ω resistance is respectively



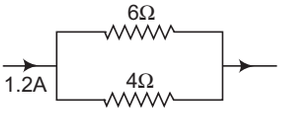
(a) 2 A and 4 A (b) 1 A and 2 A
(c) 1 A and 1 A (d) 2 A and 2 A

38. The current from the battery in circuit diagram shown is

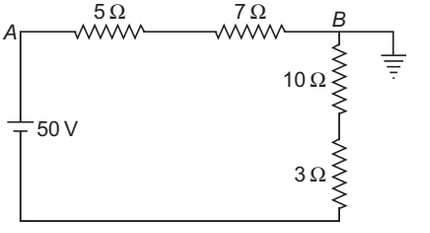


(a) 1 A (b) 2 A
(c) 1.5 A (d) 3 A

39. In the figure given below, the current passing through 6 Ω resistor is
(a) 0.40 A
(b) 0.48 A
(c) 0.72 A
(d) 0.80 A

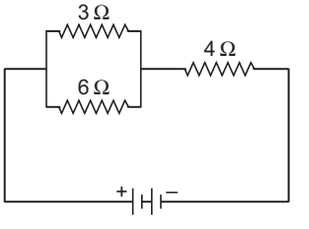


40. In the circuit shown, the point B is earthed. The potential at the point A is



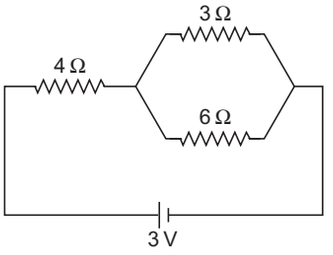
(a) 14 V (b) 24 V
(c) 26 V (d) 50 V

41. In the figure, current through the 3 Ω resistor is 0.8 A, then potential drop through 4 Ω resistor is



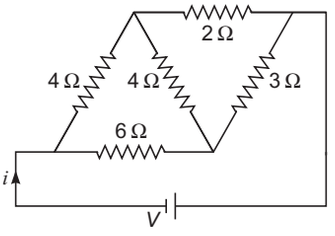
(a) 9.6 V
(b) 2.6 V
(c) 4.8 V
(d) 1.2 V

42. The potential drop across the 3 Ω resistor is



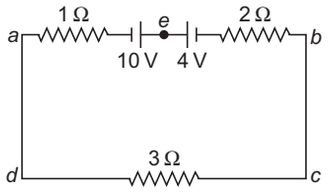
(a) 1 V (b) 1.5 V
(c) 2 V (d) 3 V

43. For the network shown in the figure, the value of the current i is



(a) $\frac{9V}{35}$ (b) $\frac{5V}{18}$
(c) $\frac{5V}{9}$ (d) $\frac{18V}{5}$

44. The magnitude and direction of the current in the circuit shown will be

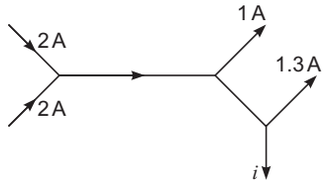


(a) $\frac{7}{3}$ A from a to e
(b) $\frac{7}{3}$ A from b to e
(c) 1A from b to e
(d) 1A from a to e

45. Kirchhoff's first law i.e. $\Sigma i = 0$ at a junction is based on the law of conservation of
(a) charge
(b) energy
(c) momentum
(d) angular momentum

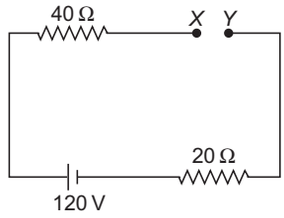
46. Kirchhoff's second law is based on the law of conservation of
(a) charge
(b) energy
(c) momentum
(d) sum of mass and energy

47. The figure shows currents in a part of electric circuit. The current i is



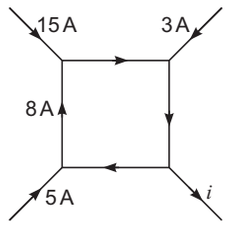
(a) 1.7 A (b) 3.7 A (c) 1.3 A (d) 1 A

48. In the circuit shown, potential difference between X and Y will be



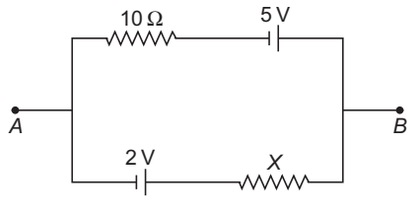
(a) zero (b) 20 V (c) 60 V (d) 120 V

49. The figure shows a network of currents. The current i will be



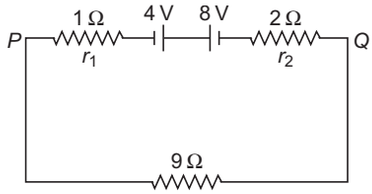
(a) 3 A (b) 13 A (c) 23 A (d) –3 A

50. If $V_{AB} = 4$ V in the given figure, then resistance X will be



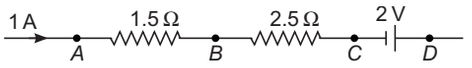
(a) 5 Ω (b) 10 Ω (c) 15 Ω (d) 20 Ω

51. Two batteries of emf 4 V and 8 V with internal resistances 1Ω and 2Ω are connected in a circuit with a resistance of 9Ω as shown in figure. The current and potential difference between the points P and Q are



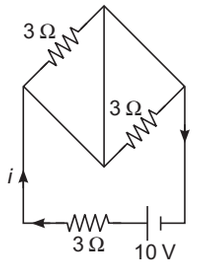
(a) $\frac{1}{3}$ A and 3 V (b) $\frac{1}{6}$ A and 4 V
(c) $\frac{1}{9}$ A and 9 V (d) $\frac{1}{2}$ A and 12 V

52. In the circuit element given here, if the potential at point $B, V_B = 0$, then the potentials of A and D are given as



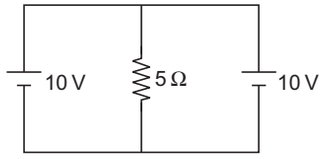
(a) $V_A = -1.5$ V, $V_D = +2$ V
(b) $V_A = -1.5$ V, $V_D = +0.5$ V
(c) $V_A = +1.5$ V, $V_D = +0.5$ V
(d) $V_A = +1.5$ V, $V_D = -0.5$ V

53. Current i as shown in the circuit will be



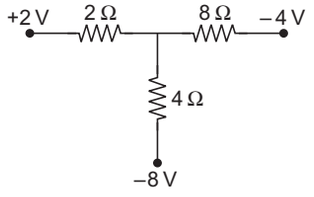
(a) 10 A (b) $\frac{10}{3}$ A
(c) zero (d) infinite

54. Current through the 5 Ω resistor is



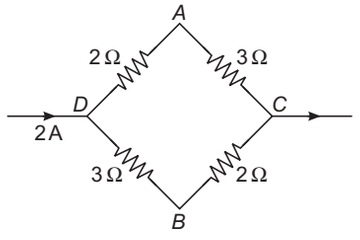
(a) 2A (b) 4A (c) zero (d) 1A

55. Three resistances are connected in the form of a T-shape as shown in the figure. Then, the current in the 4 Ω resistor is



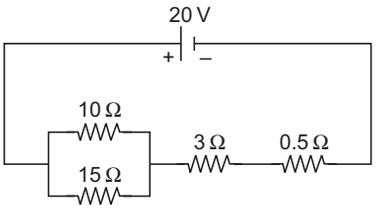
(a) 0.93 A (b) 1.42 A
(c) 2.5 A (d) 1.57 A

56. A current of 2 A flows in resistances as shown. The potential difference $V_A - V_B$ will be



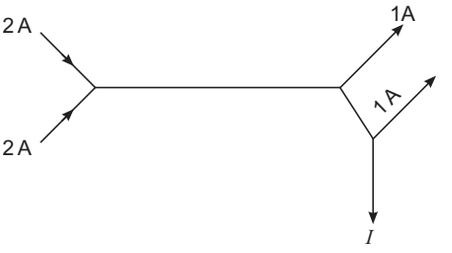
(a) + 4 V (b) + 2V
(c) + 1V (d) – 1V

57. In the circuit, the current flowing through 10Ω resistance is



(a) 12 A (b) 1.2 A
(c) 0.8 A (d) 0.4 A

58. The figure given shows current in a part of electric circuit. Then, current I is

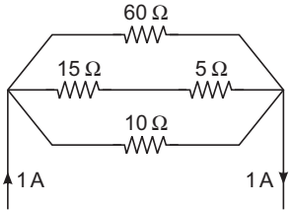


(a) 2.5 A (b) 2.7 A
(c) 2 A (d) 3 A

59. A cell of negligible internal resistance and emf 1.2 V is connected in series with three resistors of resistances 1 Ω, 2 Ω and 3 Ω respectively. What is the potential difference across 2 Ω resistor?

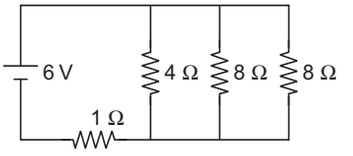
(a) 0.2 V (b) 0.4 V
(c) 0.6 V (d) 0.8 V

60. The value of current in the 60 Ω resistance of the circuit shown is



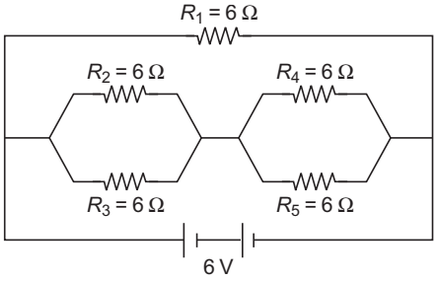
(a) 1/6 A (b) 1/3 A
(c) 0.1 A (d) 0.5 A

61. In the given figure, what is the current (in amp) through the 1 Ω resistor?



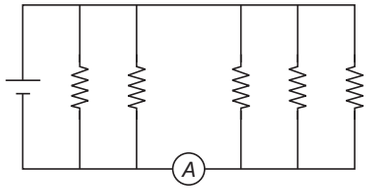
(a) 1
(b) 2
(c) 8
(d) 9

62. Consider an electric circuit given below. What is the potential difference across the resistor R_4? (Assume that internal resistance of battery is zero)



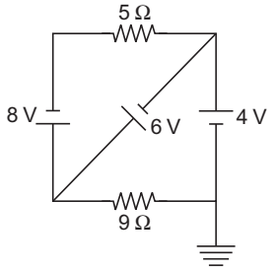
(a) 6 V (b) 4.5 V
(c) 3 V (d) 1.5 V

63. Five identical resistors, each of value 1100 Ω are connected to a 220 V battery as shown in figure. The reading of the ideal ammeter A is



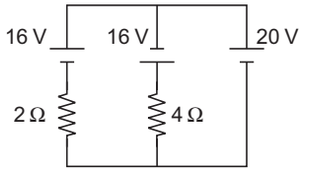
(a) 1/5 A (b) 2/5 A
(c) 3/5 A (d) 4/5 A

64. The current through the 5 Ω resistor is



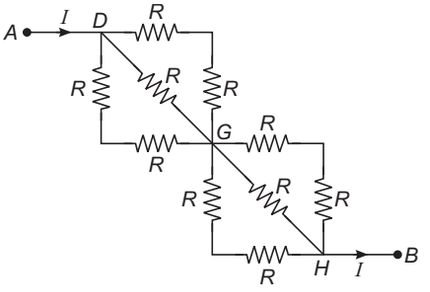
(a) 10 A
(b) 1 A
(c) 2.5 A
(d) 0.4 A

65. In the given figure, the current through the 20 V battery is



(a) 11 A
(b) 12 A
(c) 7 A
(d) 14 A

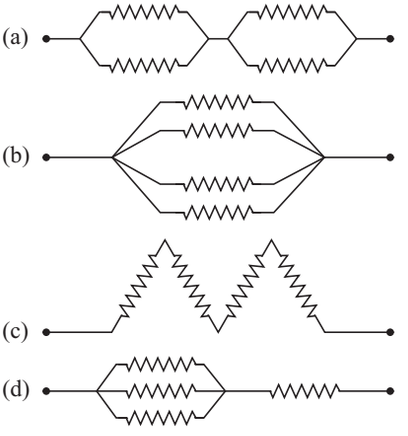
66. In the circuit shown each resistance is $R = 2\ \Omega$. If a DC source of 10 V is connected between A and B, then the current I is



(a) 2.5 A (b) 10 A (c) 1.25 A (d) 5 A

Thermal Heat, Power and Bulbs

67. Which arrangement of four identical resistances should be used to draw maximum energy from a cell of voltage V?



68. One kilowatt hour is equal to
(a) 36×10^5 J (b) 36×10^3 J
(c) 10^3 J (d) 10^5 J

69. If R_1 and R_2 are respectively, the filament resistances of a 200 W bulb and 100 W bulb designed to operate on the same voltage, then
(a) R_1 is two times R_2 (b) R_2 is two times R_1
(c) R_2 is four times R_1 (d) R_1 is four times R_2

70. The electric bulb have tungsten filaments of same length. If one of them gives 60 W and other 100 W, then
(a) 100 W bulb has thicker filament
(b) 60 W bulb has thicker filament
(c) both filaments are of same thickness
(d) it is not possible to get different wattage unless the lengths are different

71. 3 identical bulbs are connected in series and these together dissipate a power P. If now the bulbs are connected in parallel, then the power dissipated will be
(a) $\frac{P}{3}$ (b) $3P$ (c) $9P$ (d) $\frac{P}{9}$

72. Two bulbs of wattage 40 W and 100 W rated at 220 V are connected in series across a 440 V. What will happen?
(a) 40 W bulb will fuse
(b) 100 W bulb will fuse
(c) Both bulbs will fuse
(d) Nothing will happen

73. A and B are two bulbs connected in parallel. A is glowing brighter than B, then the relation between R_A and R_B is
(a) $R_A = R_B$ (b) $R_B > R_A$
(c) $R_A > R_B$ (d) None of these

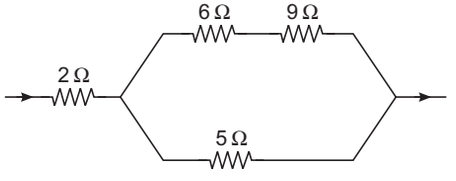
74. A resistor of resistance R is connected to an ideal battery. If the value of R is decreased, the power dissipated in the resistor will
(a) increase (b) decrease
(c) remain unchanged (d) data insufficient

75. Two resistors R and $2R$ are connected in series in an electric circuit. The thermal energy developed in R and $2R$ are in the ratio
(a) 1 : 2 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

76. Two resistances R and $2R$ are connected in parallel in an electric circuit. The thermal energy developed in R and $2R$ are in the ratio
(a) 1 : 2 (b) 2 : 1
(c) 1 : 4 (d) 4 : 1

77. Two electric bulbs one if 200 V-40W and other 200 V-100 W are connected in series to a 200 V line, then the potential drop across
(a) the two bulbs is the same
(b) Both the bulbs is 200 V
(c) 40 W bulb is greater than the potential drop across 100 W bulb
(d) 100 W bulb is greater than the potential drop across 40 W bulb

78. In the circuit shown in figure the 5 Ω resistor develops a heat of 10 cal/s due to the current flowing through it. The heat developed per second in the 2 Ω resistor is



(a) 4.1 cal (b) 7.1 cal
(c) 9.2 cal (d) 10.2 cal

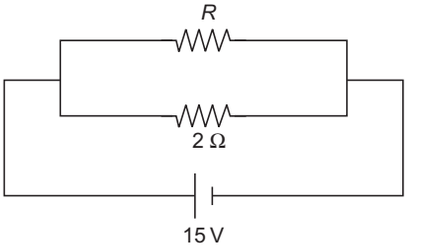
79. Two wires A and B of same material and mass, have their lengths in the ratio 1 : 2. On connecting them, one at a time to the same source of potential, the rate of heat dissipation in B is found to be 2.5 W. The rate of heat dissipation in A would be
(a) 2.5 W (b) 5.0 W
(c) 8 W (d) 10 W

80. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled, if
(a) both the length and radius of the wire are halved
(b) both the length and radius of the wire are doubled
(c) the radius of the wire is doubled
(d) the length of the wire is doubled

81. A cell of emf E and internal resistance r drives a current I through an external resistance R. Then,
(a) the cell supplies (EI) power
(b) heat is produced in R at the rate EI
(c) heat is produced in R at the rate $\frac{EIR}{R+r}$
(d) Both (a) and (c)

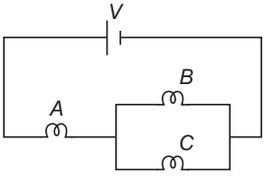
82. A wire when connected to 220 V mains supply has power dissipation P_1. Now, the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P_2. Then, $P_2 : P_1$ is
(a) 1 (b) 4 (c) 2 (d) 3

83. If in the circuit, power dissipation is 150 W, then R is



(a) 2 Ω (b) 6 Ω
(c) 5 Ω (d) 4 Ω

84. In the circuit shown, three identical light bulbs are connected to a battery. The brightness of the bulb



(a) $A < B = C$ (b) $A > B = C$
(c) $A = B = C$ (d) $A < B < C$

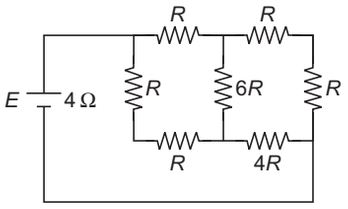
85. The resistor of resistance R is connected to 25 V supply and heat produced in it is 25 J/s. The value of R is
(a) 225 Ω (b) 1 Ω
(c) 25 Ω (d) 50 Ω

86. Some electric bulbs are connected in series across a 220 V supply in a room. If one bulb is fused, then remaining bulbs are connected again in series (after removing the fused bulb) across the same supply. The illumination in the room will
(a) increase (b) decrease
(c) remain the same (d) not continuous

87. If a wire of resistance 20 Ω is covered with ice (at 0°C) and a voltage of 210 V is applied across the wire, then the rate of melting of ice is
(a) 0.85 g/s (b) 1.92 g/s
(c) 6.59 g/s (d) None of these

88. An electric kettle has two heating elements. One brings it to boil 10 min and the other in 15 min. If two heating filaments are connected in parallel, the water in kettle will boil in
(a) 6 min (b) 8 min (c) 25 min (d) 5 min

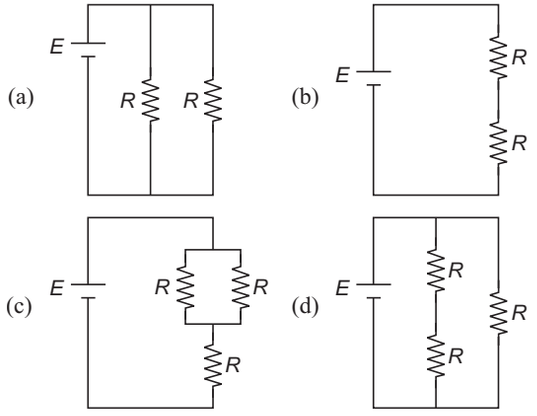
89. A battery of internal resistance 4 Ω is connected to the network of resistances as shown. In order to give the maximum power to the network, the value of R in Ω should be



(a) 4/9 (b) 8/9 (c) 2 (d) 18

90. Just as electricity is supplied at 220 V for domestic use in India, it is supplied at 110 V in USA. If a resistance of 60 W bulb for use in India is R, that of 60 W bulb for use in USA will be
(a) $R/4$ (b) $R/2$ (c) R (d) $2R$

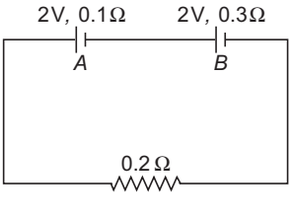
91. Consider four circuits shown in the figure below. In which circuit power dissipated is greatest? (Neglect the internal resistance of the power supply)



Combination of Cells and Potential Difference Across Battery

92. The terminal potential difference of a cell is greater than its emf when it is
(a) being discharged
(b) open circuit
(c) being charged
(d) being either charged or discharged

93. The internal resistances of two cells shown are 0.1 Ω and 0.3 Ω. If $R = 0.2\,\Omega$, the potential difference across the cell



(a) B will be zero
(b) A will be zero
(c) A and B will be 2 V
(d) A will be > 2 V and B will be < 2 V

94. How much work is required to carry a 6 μC charge from the negative terminal to the positive terminal of a 9 V battery?
(a) 54×10^{-3} J
(b) 54×10^{-6} J
(c) 54×10^{-9} J
(d) 54×10^{-12} J

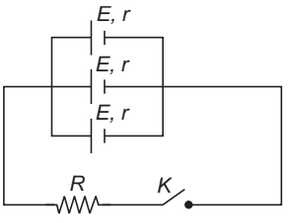
95. The n rows each containing m cells in series are joined in parallel. Maximum current is taken from this combination across an external resistance of 3Ω resistance. If the total number of cells used are 24 and internal resistance of each cell is 0.5 Ω, then
(a) $m = 8, n = 3$
(b) $m = 6, n = 4$
(c) $m = 12, n = 2$
(d) $m = 2, n = 12$

96. If E is the emf of a cell of internal resistance r and external resistance R, then potential difference across R is given as
(a) $V = E/(R + r)$
(b) $V = E$
(c) $V = E/(1 + r/R)$
(d) $V = E/(1 + R/r)$

97. Two non-ideal batteries are connected in series. Consider the following statements.
(A) The equivalent emf is larger than either of the two emf's.
(B) The equivalent internal resistance is smaller than either of the two internal resistances.
(a) Each of A and B is correct
(b) A is correct but B is wrong
(c) B is correct but A is wrong
(d) Each of A and B is wrong

98. A cell which has an emf 1.5 V is connected in series with an external resistance of 10 Ω. If the potential difference across the cell is 1.25 V, the internal resistance of the cell is (in Ω)
(a) 2
(b) 0.25
(c) 1.5
(d) 0.3

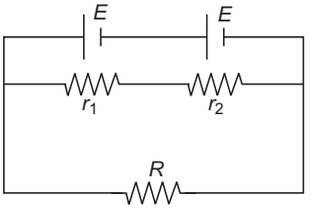
99. When n cells are joined in parallel as shown, the strength of the current i is given by



(a) $\frac{nE}{R+nr}$ (b) $\frac{E}{R+(r/n)}$

(c) $\frac{E}{r+Rn}$ (d) None of these

100. Two cells of the same emf E but different internal resistances r_1 and r_2 are connected in series with an external resistance R as shown in the figure. The terminal potential difference across, the second cell is found to be zero. The external resistance R must then be



(a) r_1r_2 (b) $r_1 - r_2$

(c) $r_2 - r_1$ (d) $r_1 + r_2$

101. A cell supplies a current i_1 through a resistance R_1 and a current i_2 through a resistance R_2. The internal resistance of this cell is

(a) $R_2 - R_1$

(b) $\frac{i_1R_2 - i_2R_1}{i_1 - i_2}$

(c) $\frac{i_2R_2 - i_1R_1}{i_1 - i_2}$

(d) $\left(\frac{i_1 + i_2}{i_1 - i_2}\right)\sqrt{R_1R_2}$

102. There are n cells, each of emf E and internal resistance r, connected in series with an external resistance R. One of the cells is wrongly connected, so, that it sends current in the opposite direction. The current flowing in the circuit is

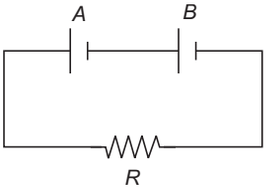
(a) $\frac{(n-1)E}{(n+1)\,r+2}$

(b) $\frac{(n-1)E}{nr+R}$

(c) $\frac{(n-2)E}{nr+R}$

(d) $\frac{(n-2)E}{(n-2)r+R}$

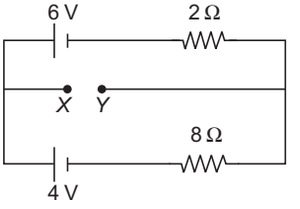
103. Two batteries A and B each of emf 2 V are connected in series to an external resistance $R = 1\,\Omega$. If the internal resistance of battery A is 1.9 Ω and that of B is 0.9 Ω, what is the potential difference between the terminals of battery A?



(a) 2 V (b) 3.8 V

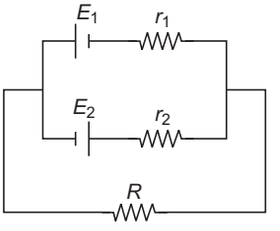
(c) Zero (d) None of these

104. Two batteries in opposition are joined as shown. The internal resistance of 6 V battery is 2 Ω and 4 V battery is 8 Ω. The potential difference between the points X and Y is



(a) 2 V (b) zero (c) 5.6 V (d) 3 V

105. The current in the resistance R will be zero, if



(a) $E_1r_1 = E_2r_2$ (b) $\frac{E_1}{r_1} = \frac{E_2}{r_2}$

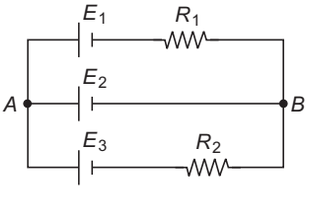
(c) $(E_1 + E_2)\,r_1 = E_1r_2$ (d) $(E_1 - E_2)\,r_1 = E_2r_1$

106. The least number of cells each of emf 2.0 V and internal resistance 0.5 Ω to produce a current of 2 A in an external resistance 4.0 Ω will be

(a) 12 cell (b) 10 cell

(c) 8 cell (d) 14 cell

107. In the circuit shown here $E_1 = E_2 = E_3 = 2\,\text{V}$ and $R_1 = R_2 = 4\Omega$. The current flowing between points A and B through battery E_2 is

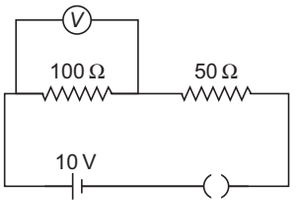


(a) zero (b) 2 A from A to B

(c) 2 A from B to A (d) None of these

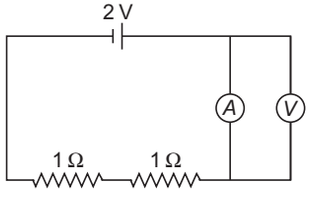
Electrical Instruments

108. In the given circuit, the voltmeter records 5 V. The resistance of the voltmeter in Ω is



(a) 200 (b) 100
(c) 10 (d) 50

109. In the circuit shown, A and V are ideal ammeter and voltmeter respectively. Reading of the voltmeter will be



(a) 2 V (b) 1 V
(c) 0.5 V (d) zero

110. A cell of internal resistance 1.5 Ω and of emf 1.5 V balances 500 cm on a potentiometer wire. If a wire of 15 Ω is connected between the balance point and the cell, then the balance length will be

(a) zero (b) = 500 cm
(c) > 500 cm (d) < 500 cm

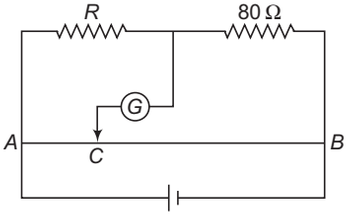
111. In order to pass 10% of main current through a moving coil galvanometer of 99 Ω, the resistance of the required shunt is

(a) 9.9 Ω (b) 10 Ω
(c) 11 Ω (d) 9 Ω

112. A potentiometer is used for the comparison of emf of two cells E_1 and E_2. For cell E_1, the no deflection point is obtained at 20 cm and for E_2, the no deflection point is obtained at 30 cm. The ratio of their emf's will be

(a) 2/3 (b) 3/2
(c) 1 (d) 2

113. AB is a wire of uniform resistance. The galvanometer G shows no current, when the length $AC = 20$ cm and $CB = 80$ cm. The resistance R is equal to

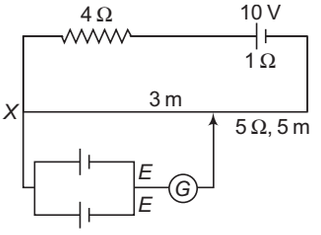


(a) 320 Ω (b) 8 Ω
(c) 20 Ω (d) 40 Ω

114. In an experiment to measure the internal resistance of a cell by potentiometer, it is found that the balance point is at a length of 2 m, when the cell is shunted by a 5 Ω resistance; and is at a length of 3 m, when the cell is shunted by a 10 Ω resistance. The internal resistance of the cell is, then

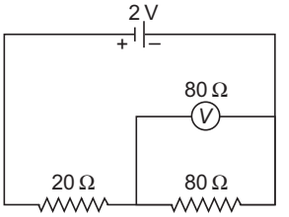
(a) 1.5 Ω (b) 10 Ω
(c) 15 Ω (d) 1 Ω

115. A resistance of 4 Ω and a wire of length 5 m and resistance 5 Ω are joined in series and connected to a cell of emf 10 V and internal resistance 1 Ω. A parallel combination of two identical cells is balanced across 3 m of the wire. The emf E of each cell is



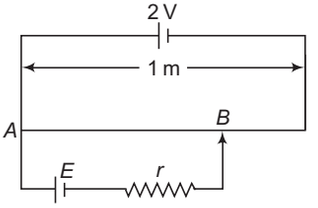
(a) 1.5 V (b) 3.0 V
(c) 0.67 V (d) 1.33 V

116. In the adjoining circuit, the emf of the cell is 2 V and the internal resistance is negligible. The resistance of the voltmeter is 80 Ω. The reading of the voltmeter will be



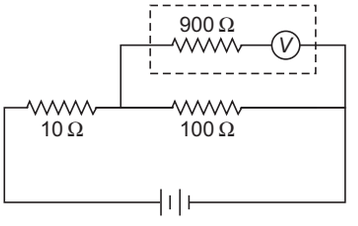
(a) 0.80 V (b) 1.60 V (c) 1.33 V (d) 2.00 V

117. In the given figure, battery E is balanced on 55 cm length of potentiometer wire but when a resistance of 10 Ω is connected in parallel with the battery, then it balances on 50 cm length of the potentiometer wire. Then, internal resistance r of the battery is



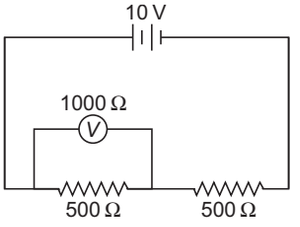
(a) 1 Ω (b) 3 Ω
(c) 10 Ω (d) 5 Ω

118. The potential difference across the 100 Ω resistance in the circuit is measured by a voltmeter of 900 Ω resistance. The percentage error made in reading the potential difference is



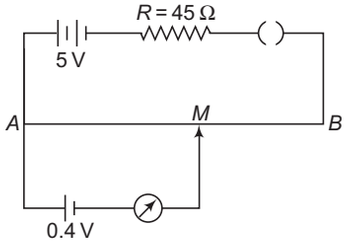
(a) $\frac{10}{9}$ (b) 0.1 (c) 1.0 (d) 10.0

119. What is the reading of voltmeter in the figure?



(a) 3 V (b) 2 V
(c) 5 V (d) 4 V

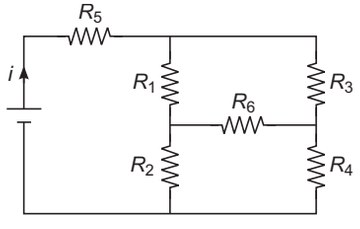
120. In given figure, the potentiometer wire AB has a resistance of 5 Ω and length 10 m. The balancing length AM for the emf of 0.4 V is



(a) 0.4 m (b) 4 m
(c) 0.8 m (d) 8 m

121. A galvanometer of resistance 18 Ω shows a deflection of 30 division. When this galvanometer is shunted with a 12 Ω resistance, the deflection shall fall to
(a) zero
(b) 10 division
(c) 12 division
(d) 18 division

122. In the given circuit, it is observed that the current I is independent of the value of the resistance R_6. Then, the resistance values must satisfy.



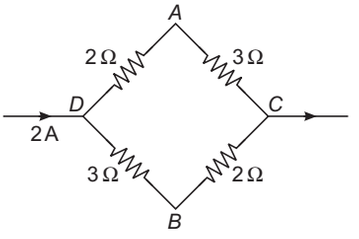
(a) $R_1 R_2 R_3 = R_3 R_4 R_5$
(b) $\frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$
(c) $R_1 R_4 = R_2 R_3$
(d) $R_1 R_3 = R_2 R_4 = R_5 R_6$

123. A voltmeter having an internal resistance of 20 kΩ, when connected in series with a large resistance R across a 110 V line reads 5 V. The magnitude of the resistance R is
(a) 210 k Ω (b) 310 k Ω
(c) 420 k Ω (d) 440 k Ω

124. An ammeter gives full deflection when a current of 2 A flows through it. The resistance of ammeter is 12 Ω. If the same ammeter is to be used for measuring a maximum current of 5 A, then the ammeter must be connected with a resistance of
(a) 8 Ω in series
(b) 18 Ω in series
(c) 8 Ω in parallel
(d) 18 Ω in parallel

125. A galvanometer of 25 Ω and having full scale deflection for a current of 10 mA is changed into voltmeter of range 100 V by connecting a resistance R in series with the galvanometer. The resistance R in ohm is
(a) 10,000 (b) 975
(c) 10,025 (d) 9,975

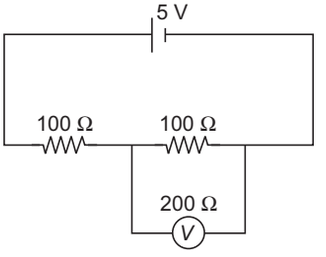
126. In the given circuit, current in 2 Ω resistor is



(a) 1.4 A (b) 1.2 A
(c) 0.4 A (d) 1.0 A

127. An ammeter and a voltmeter are joined in series to a cell. Their readings are A and V respectively. If a resistance is now joined in parallel with the voltmeter
(a) Both A and V will decrease
(b) Both A and V will increase
(c) A will decrease, V will increase
(d) A will increase V will decrease

128. In the circuit shown in the figure, the voltmeter reading is

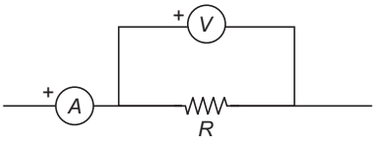


(a) 2 V (b) 2.5 V
(c) 3 V (d) 5 V

129. Two galvanometers A and B require 3 mA and 6 mA respectively to produce the same deflection of 10 division. Then,
(a) A is more sensitive than B
(b) B is more sensitive than A
(c) A and B are equally sensitive
(d) sensitiveness of B is twice that of A

130. In a potentiometer experiment two cells of emf E_1 and E_2 are used in series and the balancing length is found to be 58 cm of the wire. If the polarity of E_2 is reversed, then the balancing length becomes 29 cm. The ratio E_1/E_2 of the emf's of the two cells is
(a) 1 : 1 (b) 2 : 1 (c) 3 : 1 (d) 4 : 1

131. An ammeter A, a voltmeter V and a resistance R are connected as shown in the figure. If the voltmeter reading is 1.6 V and the ammeter reading is 0.4 A, then R is



(a) equal to 4 Ω
(b) greater than 4 Ω
(c) less than 4 Ω
(d) between 3 Ω and 4 Ω

132. To send 10% of the main current through a moving coil galvanometer of resistance 99 Ω, the shunt required is
(a) 9.9 Ω (b) 10 Ω (c) 11 Ω (d) 9 Ω

133. A microammeter has a resistance of 100 Ω and a full scale range of 50 μA. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combinations
(a) 50 V range with 10 kΩ resistance in series
(b) 10 V range with 200 kΩ resistance in series
(c) 10 mA range with 1Ω resistance in parallel
(d) None of the above

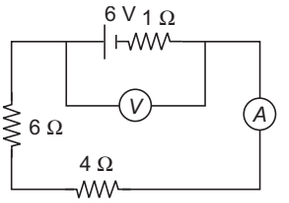
134. A 36 Ω galvanometer is shunted by a resistance of 4 Ω. The percentage of total current passing through the instrument is
(a) 34.9% (b) 10% (c) 6.2% (d) 3.14%

135. When an additional resistance of 1980 Ω is connected in series with a voltmeter, the scale division reads 100 times larger value. Resistance of the voltmeter is
(a) 10 Ω (b) 20 Ω (c) 30 Ω (d) 40 Ω

136. By mistake, a voltmeter is connected in series and an ammeter in parallel. When the circuit is switched on
(a) both ammeter and voltmeter will be damaged
(b) neither the ammeter nor the voltmeter will be damaged
(c) only the ammeter will be damaged
(d) only the voltmeter will be damaged

137. A potentiometer having the potential gradient of 2 mV/cm is used to measure, the difference of potential across a resistance of 10 Ω in some circuit. If a length of 50 cm of the potentiometer wire is required to get the null point, the current passing through the 10 Ω resistor is (in mA)
(a) 1 (b) 2
(c) 5 (d) 10

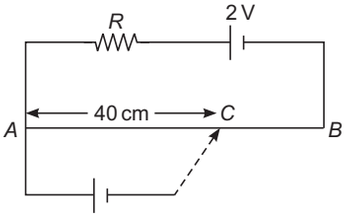
138. In the circuit shown here, the readings of the ammeter and voltmeter are



(a) 6 A, 60 V
(b) 0.6 A, 6 V
(c) 6 A, 6 V
(d) 6/11 A, 60/11 V

139. For a cell of emf 2 V, a balance is obtained for 50 cm of the potentiometer wire. If the cell is shunted by a 2 Ω resistor, the balance is obtained across 40 cm of the wire, then the internal resistance of the cell is
(a) 0.25 Ω (b) 0.50 Ω
(c) 0.80 Ω (d) 1.00 Ω

140. AB is a potentiometer wire of length 100 cm and its resistance is 10 Ω. It is connected in series with a resistance $R = 40$ Ω and a battery of emf 2 V and negligible internal resistance. If a source of unknown emf E is balanced by 40 cm length of the potentiometer wire, the value of E is



(a) 0.8 V (b) 1.6 V
(c) 0.08 V (d) 0.16 V

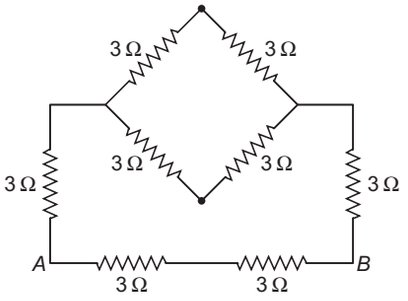
Equivalent Resistance

141. A piece of wire of resistance 4 Ω is bent through 180° at its mid-point and the two halves are twisted together, then the resistance is
(a) 8 Ω (b) 1 Ω
(c) 2 Ω (d) 5 Ω

142. A wire of resistance R is divided in 10 equal parts. These parts are connected in parallel, the equivalent resistance of such connection will be
(a) 0.01 R (b) 0.1 R
(c) 10 R (d) 100 R

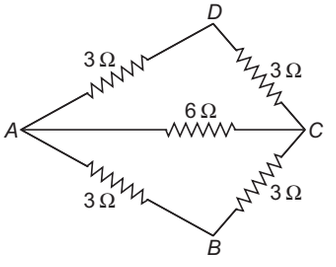
143. Three resistors each of 2 Ω are connected together in a triangular shape. The resistance between any two vertices will be
(a) 4/3 Ω (b) 3/4 Ω
(c) 3 Ω (d) 6 Ω

144. Equivalent resistance between A and B will be



(a) 2 Ω (b) 18 Ω
(c) 6 Ω (d) 3.6 Ω

145. The effective resistance between the points A and B in the figure is

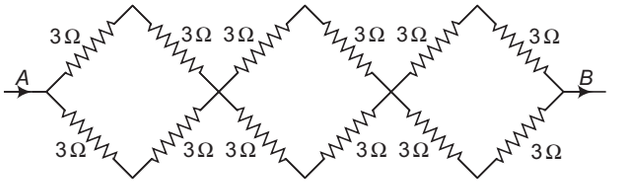


(a) 5 Ω (b) 2 Ω (c) 3 Ω (d) 4 Ω

146. Two resistances are joined in parallel whose resultant is $\frac{6}{8}\Omega$. One of the resistance wire is broken and the effective resistance becomes 2 Ω. Then, the resistance in ohm of the wire that got broken was

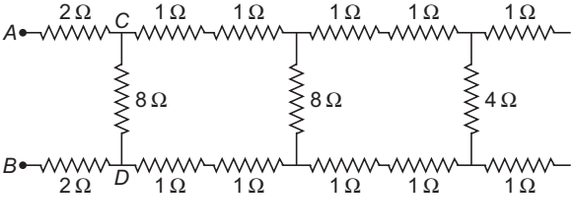
(a) 3/5 (b) 2
(c) 6/5 (d) 3

147. In the network of resistors shown in the figure, the equivalent resistance between A and B is



(a) 54 Ω (b) 18 Ω
(c) 36 Ω (d) 9 Ω

148. In the figure shown, the total resistance between A and B is



(a) 12 Ω (b) 4 Ω
(c) 6 Ω (d) 8 Ω

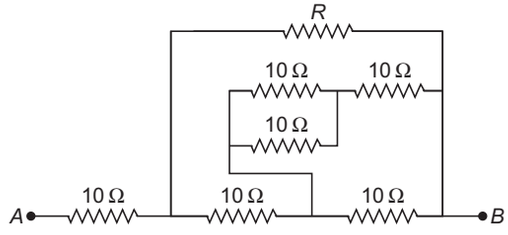
149. A student has 10 resistors of resistance r each. The minimum resistance made by him from given resistors is

(a) $10r$ (b) $\frac{r}{10}$ (c) $\frac{r}{100}$ (d) $\frac{r}{5}$

150. A wire has resistance 12 Ω. It is bent in the form of a circle. The effective resistance between the two points on any diameter is equal to

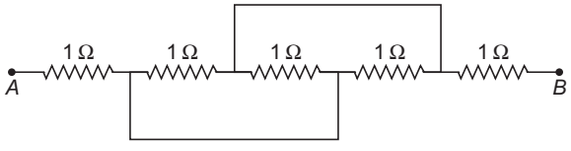
(a) 12 Ω (b) 6 Ω (c) 3 Ω (d) 24 Ω

151. For what value of R, the net resistance of the circuit will be 18 Ω?



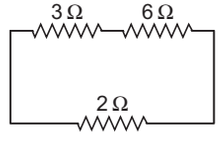
(a) 8 Ω (b) 10 Ω (c) 16 Ω (d) 24 Ω

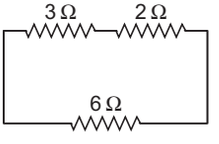
152. Equivalent resistance between the points A and B is (in Ω)

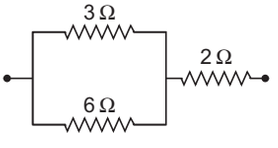


(a) $\frac{1}{5}$ (b) $\frac{5}{4}$ (c) $\frac{7}{3}$ (d) $\frac{7}{2}$

153. If you are provided three resistance 2 Ω, 3 Ω and 6 Ω. How will you connect them so, as to obtain the equivalent resistance of 4 Ω ?

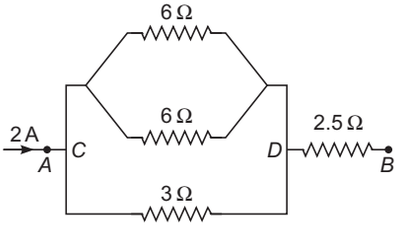
(a)


(b)


(c)


(d) None of these

154. The equivalent resistance and potential difference between A and B for the circuit is respectively,

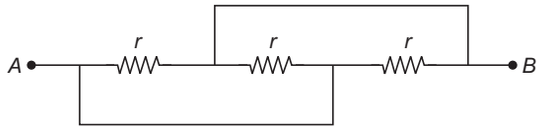


(a) 4 Ω, 8 V (b) 8 Ω, 16 V (c) 2 Ω, 4 V (d) 16 Ω, 8 V

155. A wire has resistance of 9 Ω it is broken into three equal pieces and they are now connected in parallel. The net resistance will become
(a) 3 Ω (b) 1 Ω
(c) 9 Ω (d) 27 Ω

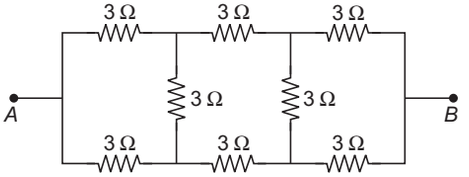
156. Three resistances each of 4 Ω are connected in the form of an equilateral triangle, the effective resistance between two corners is
(a) 8 Ω (b) 12 Ω
(c) $\frac{3}{8}$ Ω (d) $\frac{8}{3}$ Ω

157. Three equal resistors each equal to r are connected as shown in the figure. The equivalent resistance between points A and B is



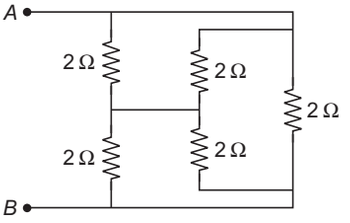
(a) r (b) $3r$
(c) $r/3$ (d) $2/3r$

158. Calculate the equivalent resistance between A and B.



(a) 9/2 Ω (b) 3 Ω
(c) 6 Ω (d) 5/3 Ω

159. Find the equivalent resistance across AB.

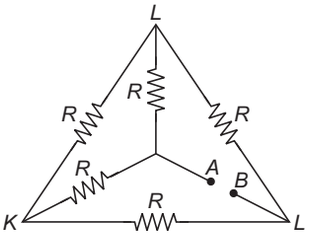


(a) 1 Ω (b) 2 Ω
(c) 3 Ω (d) 4 Ω

160. A cube is made from 12 identical wires. Current enters one corner of the cube and it leaves the opposite corner. If the resistance of each wire is r, then equivalent resistance will be
(a) $6r/5$ (b) $5r/6$
(c) $5r/12$ (d) $12r/5$

161. For two resistance wires joined in parallel, the resultant resistance is 6/5 Ω. When one of the resistance wire breaks, the effective resistance becomes 2 Ω. The resistance of the broken wire is
(a) 3/5 Ω (b) 2 Ω
(c) 6/5 Ω (d) 3 Ω

162. Each of the resistance in the net work shown in the figure is equal to R. The resistance between the terminals A and B is

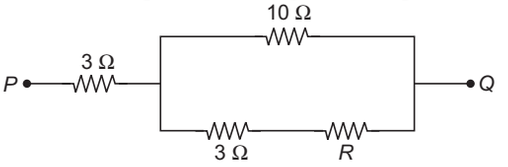


(a) R (b) $5R$ (c) $3R$ (d) $6R$

163. If an ammeter is joined in parallel in a circuit, it can be damaged due to excess
(a) resistance (b) current
(c) voltage (d) None of these

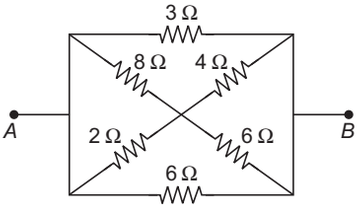
164. A wire of resistance 9Ω is broken in two parts. The length ratio being 1 : 2. The two pieces are connected in parallel. The net resistance will be
(a) 2 Ω (b) 3 Ω (c) 4 Ω (d) 6 Ω

165. In the circuit shown here, what is the value of the unknown resistor R, so, that the total resistance of the circuit between points P and Q is also equal to R?



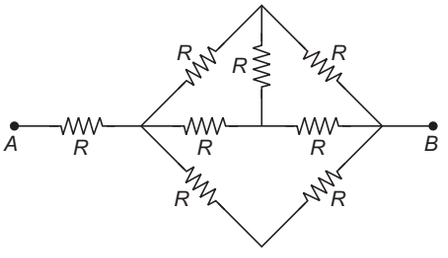
(a) 3 Ω (b) $\sqrt{39}$ Ω (c) $\sqrt{69}$ Ω (d) 10 Ω

166. In the net work shown, the equivalent resistance between A and B is



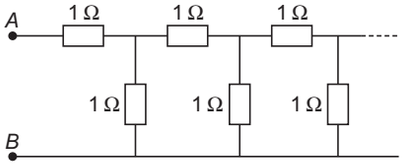
(a) $\frac{4}{3}$ Ω (b) $\frac{3}{4}$ Ω (c) $\frac{24}{17}$ Ω (d) $\frac{17}{24}$ Ω

167. In the given net work of resistances, the effective resistance between A and B is



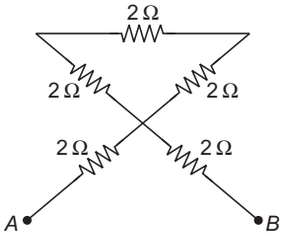
(a) $\frac{5}{3}R$ (b) $\frac{8}{3}R$ (c) $5R$ (d) $8R$

168. The equivalent resistance between points A and B, of an infinite net work of resistances, each of 1 Ω, connected as shown, is



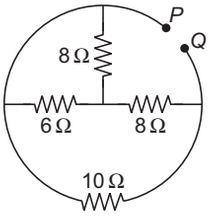
(a) infinite (b) 2 Ω
(c) $\frac{1+\sqrt{5}}{2}$ Ω (d) zero

169. For the circuit shown in the figure, the equivalent resistance between A and B is



(a) 10 Ω (b) 8 Ω
(c) 6 Ω (d) 4 Ω

170. The equivalent resistance between P and Q in the figure is approximately.



(a) 6 Ω (b) 5 Ω
(c) 7.5 Ω (d) 20 Ω

Miscellaneous Problems

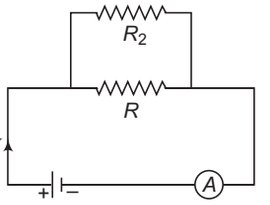
171. If an electric current is passed through a nerve of a man, then man
(a) begins to laugh (b) begins to weep
(c) is excited (d) becomes insensitive to pain

172. If potential $V = 100 \pm 0.5$ V and current $I = 10 \pm 0.2$ A are given to us. Then, what will be the value of resistance?
(a) $10 \pm 0.7\ \Omega$ (b) $5 \pm 2\ \Omega$
(c) $0.1 \pm 0.2\ \Omega$ (d) None of these

173. Which of the following is vector quantity?
(a) Current density (b) Current
(c) Wattless current (d) Power

174. Given three equal resistors, how many different combination of all the three resistors can be made?
(a) Six (b) Five
(c) Four (d) Three

175. If a resistance R_2 is connected in parallel with the resistance R in the circuit shown, then possible value of current through R and the possible value of R_2 will be



(a) $\frac{I}{3}, R$ (b) $I, 2R$ (c) $\frac{I}{3}, 2R$ (d) $\frac{I}{2}, R$

176. Two wires of same metal have the same length but their cross-sections are in the ratio 3 : 1. They are joined in series. The resistance of the thicker wire is 10 Ω. The total resistance of the combination will be
(a) 40 Ω (b) $\frac{40}{3}$ Ω (c) $\frac{5}{2}$ Ω (d) 100 Ω

177. Three unequal resistors in parallel are equivalent to a resistance 1 Ω. If two of them are in the ratio 1 : 2 and if no resistance value is fractional, the largest of the three resistances in ohms is
(a) 4 (b) 6 (c) 8 (d) 12

178. A battery has emf 4 V and internal resistance r. When this battery is connected to an external resistance of 2 Ω, a current of 1 A, flows in the circuit. How much current will flow, if the terminals of the battery are connected directly?
(a) 1 A (b) 2 A (c) 4 A (d) Infinite

179. What is immaterial for an electric fuse wire?
(a) Its specific resistance (b) Its radius
(c) Its length (d) Current flowing through it

180. The heat generated through 2 Ω and 8 Ω resistances separately, when a condenser of 200 μF capacity charged to 200 V is discharged one by one, will be
(a) 4 J and 16 J respectively
(b) 16 J and 4 J respectively
(c) 4 J and 8 J respectively
(d) 4 J and 4 J respectively

181. Electric resistance is defined as the ratio of voltage to current. Its dimensional formulae is given by
(a) $[M^2LT^{-3}A^{-2}]$ (b) $[ML^2T^{-3}A^{-2}]$
(c) $[ML^{-3}T^{-2}A^{-1}]$ (d) $[M^{-2}L^{-2}T^2A^{-1}]$

182. Identify the material commonly used for making coil of a resistance box
(a) molybdenum (b) manganese
(c) manganin (d) magnesium

183. Which has not the same unit as others?
(a) W-s (b) kWh (c) eV (d) J-s

184. The charge flowing in a conductor varies with time as $Q = at - bt^2$, then the current
(a) reaches a maximum and then decreases
(b) falls to zero after $t = \frac{a}{2b}$
(c) changes at a rate of $(-2b)$
(d) Both (b) and (c)

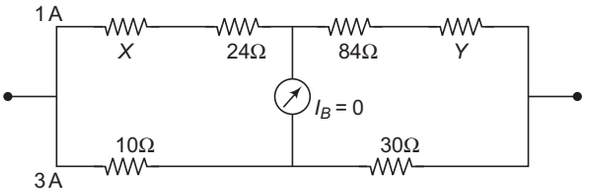
185. A composite resistance of 50 Ω which can carry a current of 4 A is to be made from resistances each of resistance 100 Ω which can carry a current of 1 A. The minimum number of resistances to be used is
(a) 4 (b) 8 (c) 12 (d) 16

186. A source of emf $E = 15$ V and having negligible internal resistance is connected to a variable resistance, so that the current in the circuit increases with time as $i = 1.2t + 3$. Then, the total charge that will flow in first five second will be
(a) 10 C (b) 20 C (c) 30 C (d) 40 C

187. A wire of length 100 cm is connected to a cell of emf 2 V and negligible internal resistance. The resistance of the wire is 3 Ω. The additional resistance required to produce a potential drop of 1 millivolt per cm is
(a) 60 Ω (b) 47 Ω (c) 57 Ω (d) 35 Ω

188. Two uniform wires A and B are of same metal and have equal masses. The radius of wire A is twice that of wire B. The total resistance of A and B when connected in parallel is
(a) 4 Ω when the resistance of wire A is 4.25 Ω
(b) 5 Ω when the resistance of wire A is 4 Ω
(c) 4 Ω when the resistance of wire B is 4.25 Ω
(d) 5 Ω when the resistance of wire B is 4 Ω

189. In the given circuit, the resistances are given in ohm. The current through the 10 Ω resistance is 3 A while that through the resistance X is 1 A. No current passes through the galvanometer. The values of the unknown resistances X and Y are respectively (in ohm).

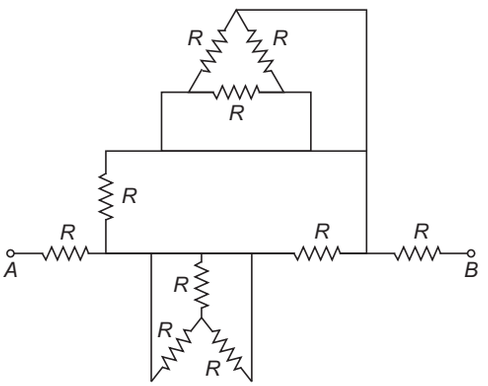


(a) 14 and 54 (b) 12 and 6
(c) 6 and 12 (d) 6 and 6

[Level 2]

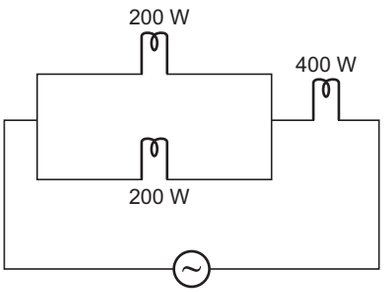
Only One Correct Option

1. Equivalent resistance between points A and B is



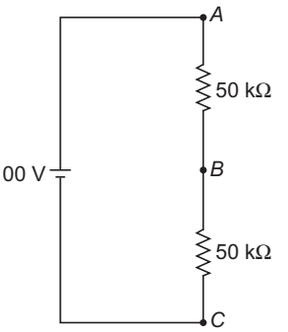
(a) $3R$
(b) $4R$
(c) $\frac{5R}{2}$
(d) $\frac{7R}{2}$

2. Three electric bulbs of 200 W and 400 W are shown in figure. The resultant power of the combination, if rated voltage is applied across the combination is



(a) 800 W (b) 400 W
(c) 200 W (d) 600 W

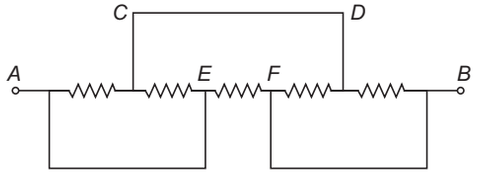
3. In the adjacent shown circuit, a voltmeter of internal resistance R, when connected across B and C reads $\frac{100}{3}$ V. Neglecting the internal resistance of the battery, the value of R is



(a) 100 kΩ
(b) 75 kΩ
(c) 50 kΩ
(d) 25 kΩ

4. A cell in secondary circuit gives null deflection for 2.5 m length of potentiometer having 10 m length of wire. If the length of the potentiometer wire is increased by 1 m without changing the cell in the primary, the position of the null point now is
(a) 2.9 m (b) 3 m (c) 2.75 m (d) 2.5 m

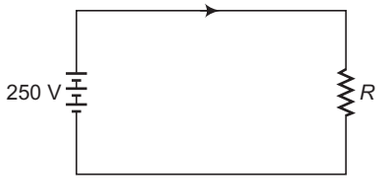
5. If $V_A - V_B = V_0$ and the value of each resistance is R, then



(a) net resistance between AB is $\frac{A}{2}$
(b) net resistance between AB is $\frac{3R}{5}$
(c) current through CD is $\frac{V_0}{R}$
(d) current through EF is $\frac{2V_0}{3R}$

6. Two non-ideal unidentical batteries are connected in parallel with positive terminals of both connected together. Consider the following statements.
(A) The equivalent emf is smaller than either of the two emfs.
(B) The equivalent internal resistance is smaller than either of the two internal resistances.
(a) Both A and B are correct
(b) A is correct but B is wrong
(c) B is correct but A is wrong
(d) Both A and B are wrong

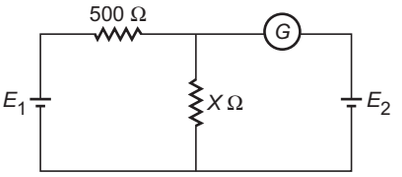
7. In the circuit shown in figure, the resistance R has a value that depends on the current. Specifically R is 20 Ω, when i is zero and the amount of increase is resistance is numerically equal to one-half of the current.



What is the value of current I in circuit?
(a) 15 A (b) 10 A (c) 20 A (d) 5 A

8. An electric current of 16 A exists in a metal wire of cross-section 10^{-6} m^2 and length 1 m. Assuming one free electrons per atom. The drift speed of the free electrons in the wire will be
(density of metal = 5×10^3 kg/m^3, atomic weight = 60)
(a) 5×10^{-3} m/s (b) 2×10^{-3} m/s
(c) 4×10^{-3} m/s (d) 7.5×10^{-3} m/s

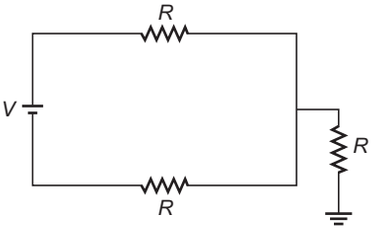
9. In the adjoining circuit, the battery E_1 has an emf of 12 V and zero internal resistance. While the battery E_2 has an emf of 2 V.



If the galvanometer G reads zero, then the value of the resistance X in ohms is
(a) 250 (b) 100 (c) 50 (d) 200

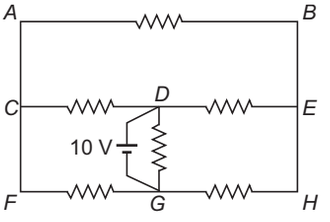
10. Two wires of same material and lengths L and $2L$ have areas of cross-section A and $2A$, respectively. These are joined in series and connected across a battery. The values of the electric field and the drift velocity in the two wires will be respectively, in the ratio of
(a) 2 : 1, 2 : 1 (b) 2 : 1, 1 : 1
(c) 1 : 1, 1 : 2 (d) 1 : 1, 2 : 1

11. The current drawn from the battery shown in the figure is



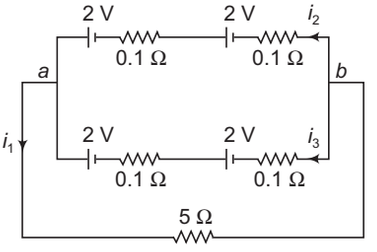
(a) $\frac{V}{R}$ (b) $\frac{V}{2R}$ (c) $\frac{2V}{R}$ (d) $\frac{3V}{2R}$

12. All resistances shown in circuit are 2 Ω each. The current in the resistance between D and E is



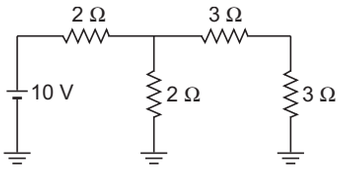
(a) 5 A (b) 2.5 A (c) 1 A (d) 7.5 A

13. In the circuit shown in figure, the values of i_1, i_2 and i_3 are



(a) 0.784 A, 0.392 A, 0.392 A
(b) 0.468 A, 0.234 A, 0.234 A
(c) 0.396 A, 0.198 A, 0.198 A
(d) None of the above

14. In the circuit shown, the current in 3 Ω resistance is

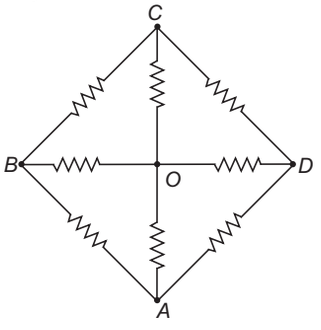


(a) 1 A (b) $\frac{1}{7}$ A (c) $\frac{5}{7}$ A (d) $\frac{15}{7}$ A

15. Under what conditions current passing through the resistance R can be increased by short circuiting the battery of emf E_2. The internal resistances of the two batteries are r_1 and r_2, respectively.

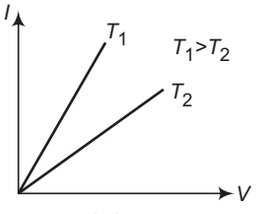
(a) $E_2 r_1 > E_1(R + r_2)$ (b) $E_1 r_2 < E_2(R + r_1)$
(c) $E_2 r_2 < E_1(R + r_2)$ (d) $E_1 r_2 > E_2(R + r_1)$

16. In the arrangement shown, the magnitude of each resistance is 1 Ω. The equivalent resistance between O and A given by



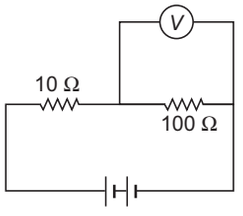
(a) $\frac{14}{13}\Omega$ (b) $\frac{7}{15}\Omega$ (c) $\frac{2}{3}\Omega$ (d) $\frac{5}{6}\Omega$

17. The figure shows, the variation of V with 1 at temperatures T_1 and T_2. $(T_1 - T_2)$ is proportional to



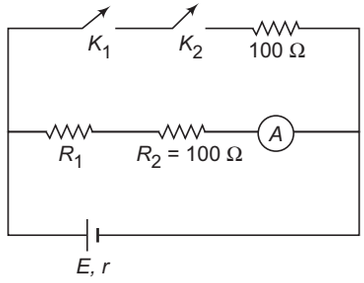
(a) tan 2α (b) $\sec^2\left(\frac{\alpha}{2}\right)$ (c) sin α (d) cos 2α

18. The potential difference across 100 Ω resistance in the circuit is measured by a voltmeter of 900 Ω resistance. The percentage error in reading the potential difference is



(a) $\frac{10}{9}$ (b) 0.1 (c) 1.0 (d) 10.0

19. In the circuit shown in the figure, when key K_1 is closed, the ammeter reads I_0 whether K_2 is open or closed.

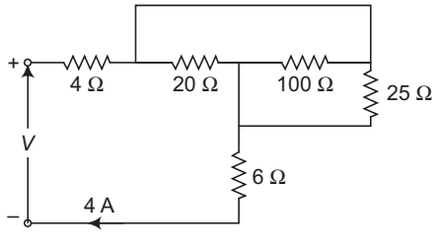


But when K_1 is open, the ammeter reads $I_0/2$, when $I_0/2$ when K_2 is closed.

Assuming that ammeter resistance is much less than R_2, the values of r and R_1 in ohms are

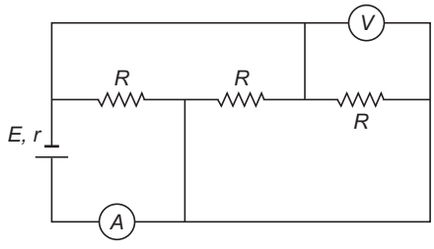
(a) 100, 50 (b) 50, 100
(c) 0, 100 (d) 0, 50

20. In the circuit shown in the figure, V must be



(a) 50 V (b) 80 V
(c) 100 V (d) 1290 V

21. In the circuit shown in the figure, ammeter and voltmeter are ideal. If $E = 4$V, $R = 9\,\Omega$ and $r = 1\,\Omega$, then readings of ammeter and voltmeter are

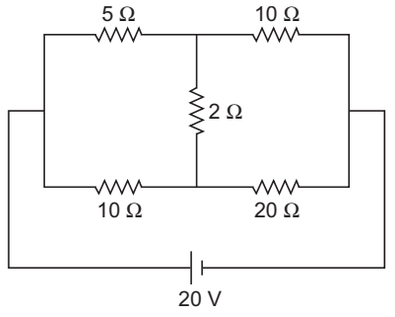


(a) 1 A, 3 V (b) 2 A, 3 V
(c) 3 A, 4 V (d) 4 A, 4 V

22. Two wires A and B made of same material and having their lengths in the ratio 6 : 1 are connected in series. The potential difference across the wires are 3 V and 2 V respectively. If r_A and r_B are the radii of A and B respectively, then $\frac{r_B}{r_A}$ is

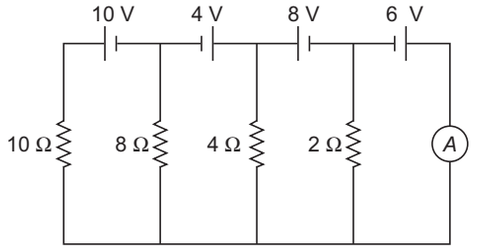
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

23. In the given circuit diagram, potential difference across 2 Ω resistance is



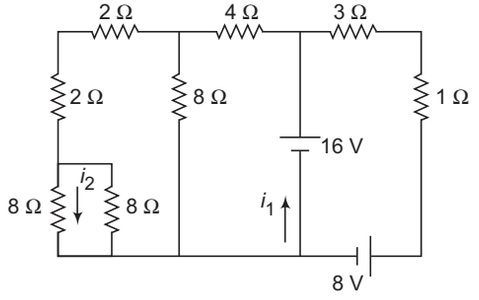
(a) 4 V (b) 6 V
(c) 10 V (d) None of these

24. Find the reading of the ideal ammeter connected in the given circuit. Assume that the cells have negligible internal resistance.



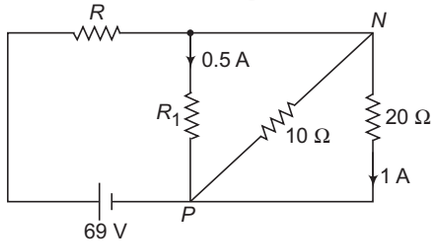
(a) 0.8 A (b) 0.25 A
(c) 1.95 A (d) 1.0 A

25. In the circuit shown in figure, the ratio of currents i_1 / i_2 is



(a) 2 : 4 (b) 8 : 1
(c) 6 : 1 (d) 4 : 2

26. In the circuit shown in the figure.



(a) The current through *NP* is 0.5 A
(b) The value of $R_1 = 40\ \Omega$
(c) The value of $R = 14\ \Omega$
(d) The potential difference across, $R = 49$ V

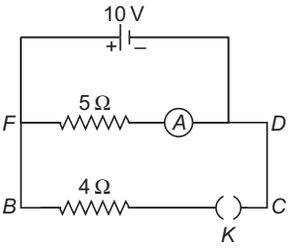
27. Ratio of power developed across R, $2R$ and $6R$ will be

(a) 2 : 4 : 3 (b) 1 : 2 : 6
(c) 2 : 4 : 6 (d) 1 : 2 : 3

28. If the resistance R is removed from the circuit, then

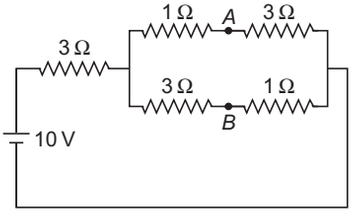
(a) power consumed by $2R$ will increase
(b) power consumed by $6R$ will decrease
(c) Both (a) and (b) are correct
(d) Both (a) and (b) wrong

29. In the given figure, when key K is opened, the reading of the ammeter A will be



(a) 50 A (b) 2 A (c) 0.5 A (d) $\frac{10}{9}$ A

30. A battery of emf 10 V is connected to a group of resistances as shown in figure. The potential difference $V_A - V_B$ between the points A and B is

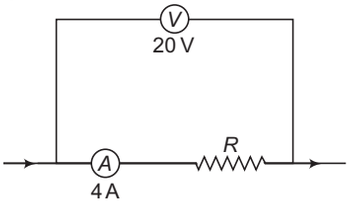


(a) –2 V (b) 2 V (c) 5 V (d) $\frac{20}{11}$ V

31. A cell of constant emf first connected to a resistance R_1 and then connected to a resistance R_2. If power delivered in both cases is same then the internal resistance of the cell is

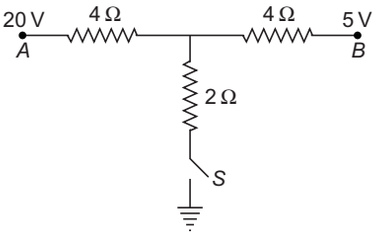
(a) $\sqrt{R_1 R_2}$ (b) $\sqrt{\frac{R_1}{R_2}}$ (c) $\frac{R_1 - R_2}{2}$ (d) $\frac{R_1 + R_2}{2}$

32. In the diagram shown, the reading of voltmeter is 20 V and that of ammeter is 4 A. The value of R should be (consider given ammeter and voltmeter are not ideal)



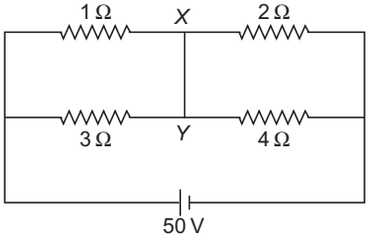
(a) greater than 5 Ω
(b) less than 5 Ω
(c) greater or less than 5 Ω depends on the material of R
(d) None of the above

33. As the switch S is closed in the circuit shown in figure, current passing through it is



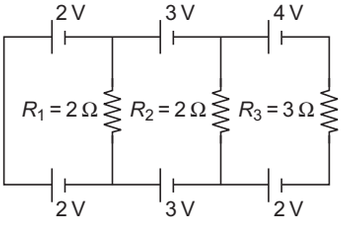
(a) 4.5 A (b) 6.0 A
(c) 3.0 A (d) zero

34. Current through wire XY of circuit shown is



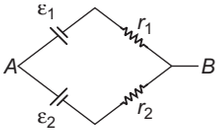
(a) 1 A (b) 4 A
(c) 2 A (d) 3 A

35. The current in resistance R_3 in the given circuit is



(a) 1 A (b) 2/3 A (c) 0.25 A (d) 0.50 A

36. Two batteries of emf ε_1 and $\varepsilon_2 (\varepsilon_2 > \varepsilon_1)$ and internal resistances r_1 and r_2 respectively are connected in parallel as shown in figure.



(a) Two equivalent emf ε_{eq} of the two cells is between ε_1 and ε_2, i.e. $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$
(b) The equivalent emf ε_{eq} is smaller than ε_1
(c) The ε_{eq} is given by $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ always
(d) ε_{eq} is independent of internal resistances r_1 and r_2

37. A resistance R is to be measured using a meter bridge, student chooses the standard resistance S to be 100 Ω. He finds the null point at $l_1 = 2.9$ cm. He is told to attempt to improve the accuracy.
Which of the following is a useful way?
(a) He should measure l_1 more accurately
(b) He should change S to 1000Ω and repeat the experiment
(c) He should change S to 3Ω and repeat the experiment
(d) He should given up hope of a more accurate measurement with a meter bridge

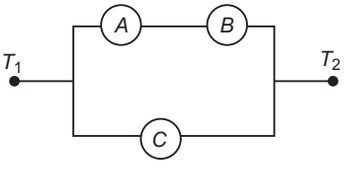
38. Two cells of emf"s approximately 5 V and 10 V are to be accurately compared using a potentiometer of length 400 cm.
(a) The battery that runs the potentiometer should have voltage of 8V
(b) The battery of potentiometer can have a voltage of 15 V and R adjusted so that the potential drop across the wire slightly exceeds 10 V
(c) The first portion of 50 cm of wire itself should have a potential drop of 10 V
(d) Potentiometer is usually used for comparing resistances and not voltages

More than One Correct Options

1. Two heaters designed for the same voltage V have different power ratings. When connected individually across a source of voltage V, they produce H amount of heat each in time t_1 and t_2 respectively. When used together across the same source, they produce H amount of heat in time t.
(a) If they are in series, then $t = t_1 + t_2$
(b) If they are in series, then $t = 2(t_1 + t_2)$
(c) If they are in parallel, then $t = \dfrac{t_1 t_2}{(t_1 + t_2)}$
(d) If they are in parallel, then $t = \dfrac{t_1 t_2}{2(t_1 + t_2)}$

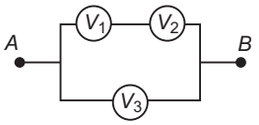
2. Two cells of emf $E_1 = 6$ V and $E_2 = 5$ V are joined in parallel with same polarity on same side, without any external load. If their internal resistances are $r_1 = 2\,\Omega$ and $r_2 = 3\,\Omega$ respectively, then
(a) terminal potential difference across any cell is less than 5 V
(b) terminal potential difference across any cell is 5.6 V
(c) current through the cells is 0.2 A
(d) current through the cells is zero if $E_1 = E_2$

3. Three ammeters A, B and C of resistances R_A, R_B and R_C respectively are joined as shown in the figure. When some potential difference is applied across the terminals T_1 and T_2 their readings are I_A, I_B and I_C respectively. Then,



(a) $I_A = I_B$
(b) $I_A R_A + I_B R_B = I_C R_C$
(c) $\dfrac{I_A}{I_C} = \dfrac{R_C}{R_A}$
(d) $\dfrac{I_B}{I_C} = \dfrac{R_C}{R_A + R_B}$

4. Three voltmeters all having different resistances are joined as shown in the figure. When some potential difference is applied across A and B, their readings are V_1, V_2 and V_3. Then

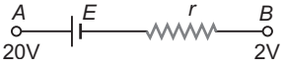


(a) $V_1 = V_2$ (b) $V_1 \neq V_2$
(c) $V_1 + V_2 = V_3$ (d) $V_1 + V_2 > V_3$

5. Two conductors made of the same material have lengths L and $2L$ but have equal resistances. The two are connected in series in a circuit in which current is flowing. Which of the following is/are correct ?

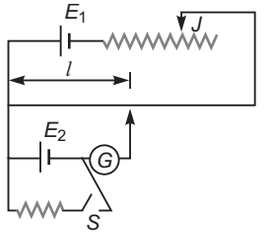
(a) The potential difference across the two conductors is the same
(b) The drift speed is larger in the conductor of length L
(c) The electric field in the first conductor is twice that in the second
(d) The electric field in the second conductor is twice that in the first

6. In the figure shown



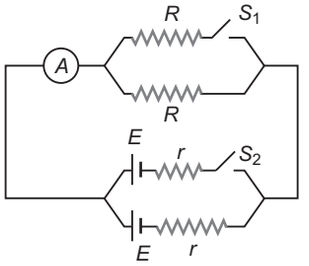
(a) current will flow from A to B
(b) current may flow A to B
(c) current may flow from B to A
(d) the direction of current will depend on E

7. In the potentiometer experiment shown in the figure, the null point length is l. Choose the correct options given below.



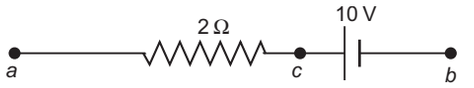
(a) If jockey J is shifted towards right, l will increase
(b) If value of E_1 is increased, l is decreased
(c) If value of E_2 is increased, l is increased
(d) If switch S is closed, l will decrease

8. In the circuit shown in the figure, reading of ammeter will



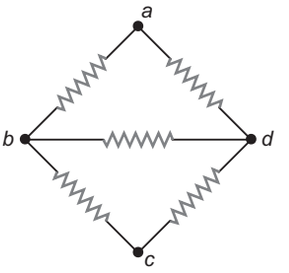
(a) increase if S_1 is closed (b) decrease if S_1 is closed
(c) increase if S_2 is closed (d) decrease if S_2 is closed

9. In the circuit shown in the figure it is given that $V_b - V_a = 2$ V. Choose the correct options.



(a) Current in the wire is 6 A
(b) Direction of current is from a to b
(c) $V_a - V_c = 12$ V
(d) $V_c - V_a = 12$ V

10. Each resistance of the network shown in figure is r. Net resistance between



(a) a and b is $\frac{7}{3}r$
(b) a and c is r
(c) b and d is r
(d) b and d is $\frac{r}{2}$

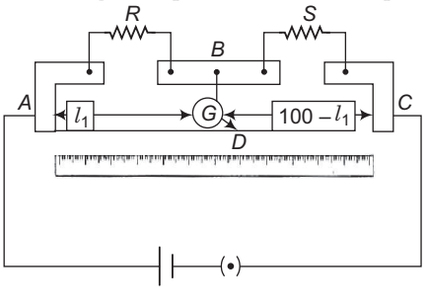
11. Kirchhoff's junction rule is a reflection of

(a) conservation of current density vector
(b) conservation of charge
(c) the fact that the momentum with which a charged particle approaches a junction is unchanged (as a vector) as the charged particle leaves the junction
(d) the fact that there is no accumulation of charges at a junction

12. Temperature dependence of resistivity $\rho(T)$ of semiconductors, insulators and metals is significantly based on the following factors

(a) number of charge carriers can change with temperature T
(b) time interval between two successive collisions can depend on T
(c) length of material can be a function of T
(d) mass of carriers is a function of T

13. In a meter bridge, the point D is a neutral point (figure).



(a) The meter bridge can have no other neutral. A point for this set of resistances
(b) When the jockey contacts a point on meter wire left of D, current flows to B from the wire
(c) When the jockey contacts a point on the meter wire to the right of D, current flows from B to the wire through galvanometer
(d) When R is increased, the neutral point shifts to left

Comprehension Based Questions

Passage

The length of a potentiometer wire is 600 *cm and it carries a current of* 40 *mA. For a cell of emf* 2 *V and internal resistance* 10 Ω*, the null point is found to be at* 500 *cm. On connecting a voltmeter across the cell, the balancing length is decreased by* 10 *cm*.

1. The voltmeter reading will be
(a) 1.96 V (b) 1.8 V
(c) 1.64 V (d) 0.96 V

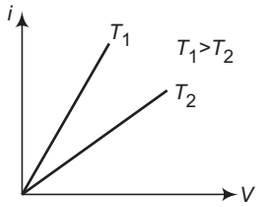
2. The resistance of the voltmeter is
(a) 500 Ω (b) 290 Ω (c) 490 Ω (d) 20 Ω

Assertion and Reason

Directions (Q. Nos. 1-19) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion
(c) If Assertion is true but Reason is false
(d) If Assertion is false but Reason is true
(e) If both Assertion and Reason are false

1. **Assertion** Current *versus* potential difference (i-V) graph for a conductor at two different temperatures T_1 and T_2 is shown in figure. Hence, $T_1 > T_2$.



Reason Resistance of a conductor increases with rise in temperature.

2. **Assertion** If a current flows through a wire of non-uniform cross-section, potential difference per unit length of wire is same throughout the length of wire.

Reason Current through the wire is same at all cross-sections.

3. **Assertion** In case of potentiometer experiment, if emf's of known and unknown both batteries are made two times, null point length will remain unchanged.

Reason Null point length does not depend on emf of either of the two batteries.

4. **Assertion** A resistance wire is broken into four pieces and all are connected in parallel. Then, net resistance becomes 1/16 times.

Reason In parallel net resistance is less than the smallest value of individual resistance.

5. **Assertion** Potential difference across the terminals of a battery is always less than the emf of the battery.

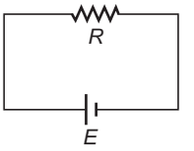
Reason During discharging of a battery potential difference across to terminals of a battery is

$$V = E - ir \quad \Rightarrow \quad \therefore \quad V < E$$

6. **Assertion** In potentiometer experiment, null point cannot be obtained, if emf of unknown battery is more than the emf of known battery.

Reason By increasing the emf of known battery, null point length increases.

7. **Assertion** In the circuit shown in figure, battery is ideal. If a resistance R_0 is connected in parallel with R, power across R will increase.

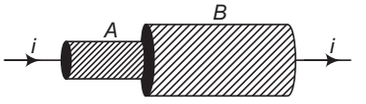


Reason Current drawn from the battery will increase.

8. **Assertion** Two identical bulbs when connected across a battery, produce a total power P. When they are connected across the same battery in series total power consumed will be $\frac{P}{4}$.

Reason In parallel, $P = P_1 + P_2$ and in series $P = \frac{P_1 P_2}{P_1 + P_2}$.

9. **Assertion** Two resistance wires shown in figure are of same material. They have equal length. More heat is generated in wire A.



Reason In series, $H \propto R$ and resistance of wire A is more.

10. **Assertion** Inside a conductor, electrons have no motion in the absence of some potential difference across it.

Reason In the absence of potential difference no electrostatic force will act on the electrons inside the conductor.

11. **Assertion** If a current is flowing through a conducting wire of non-uniform cross-section, then drift speed and resistance both will increase at a section, where cross-sectional area is less.

Reason Current density at such sections is more, where cross-sectional area is less.

12. **Assertion** In our houses, when we start switching on different light buttons, main current goes on increasing.

Reason Different connections in houses are in parallel. When we start switching on different light buttons, net resistance of the circuit decreases. Therefore, main current increases.

13. Assertion Current between two points in an electrical circuit always flows from higher potential to lower potential.

Reason During charging of a battery current inside the battery flows from negative terminal to positive terminal.

14. Assertion Resistance of an ammeter is less than the resistance of a milliammeter.

Reason Value of shunt required in case of ammeter is more than a milliammeter.

15. Assertion Kilovolt-ampere (kV-A) and kilowatt-hour have the same dimensions.

Reason Both are the units of energy.

16. Assertion If by mistake, a voltmeter is connected in series, then circuit will burn.

Reason Current will drastically decrease in the circuit.

17. Assertion $V = iR$ is Ohm's law.

Reason V-i graph is always a straight line passing through origin.

18. Assertion Net power supplied by a non-ideal battery is $Ei = i^2 r$.

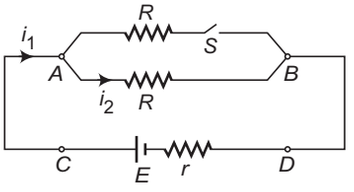
Reason Power consumed by internal resistance of a battery is $i^2 r$.

19. Assertion Two 60 W bulbs are first connected in series and then in parallel with same battery. Total power produced in second case will become four times. Rated voltage is same for two bulbs.

Reason In series total power produced will be 30 W and in parallel 120 W.

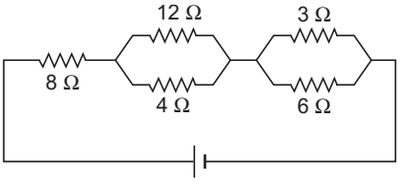
Match the Columns

1. In the circuit diagram shown in figure, match the following two columns when switch S is closed.



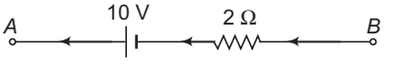
Column I	Column II
(A) Current i_1	(p) will increase
(B) Current i_2	(q) will decrease
(C) Potential difference across A and B	(r) will remain same
(D) Potential difference ac-ross C and D	(s) may increase or decrease. It will depend on value of r

2. In the circuit diagram shown in figure, potential difference across 3 Ω resistance is 20 V. Then, match the following two columns.



Column I	Column II
(A) Potential difference across 6 Ω resistance	(p) 30 V
(B) Potential difference across 4 Ω resistance	(q) 40 V
(C) Potential difference across 12 Ω resistance	(r) 20 V
(D) Potential difference across 8 Ω resistance	(s) 80 V

3. In the part of circuit shown in the figure, match the following two columns for V_{AB}.

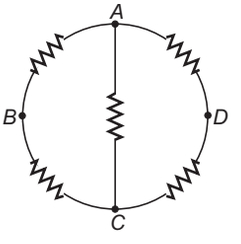


Column I	Column II
(A) If $i = 5$ A	(p) $V_{AB} = 0$ V
(B) If $i = -5$ A	(q) $V_{AB} = 10$ V
(C) If $i = 0$	(r) $V_{AB} = 20$ V
(D) If points A and B are connected by a conducting wire	(s) $V_{AB} = -10$ V

4. Rated power of a bulb at V voltage is P. Now same voltage V is applied in all conditions mentioned in Column I. Match this Column I with Column II in which actual total power consumed is given.

Column I	Column II
(A) Two bulbs are connected in parallel	(p) P
(B) Two bulbs are connected in series	(q) $2P$
(C) Two bulbs are connected in parallel and one bulb in series with this combination	(r) $\frac{P}{2}$
(D) A group of two-two bulbs in parallel are mutually connected in series	(s) None

5. In the circuit diagram shown in the figure, each resistance is R. Match the following two columns for equivalent resistance.



	Column I	Column II
(A)	Between points *A* and *B*	(p) R
(B)	Between points *A* and *C*	(q) $\frac{R}{2}$
(C)	Between points *B* and *D*	(r) None

Entrance Gallery

2014

1. The equivalent resistance of two resistors connected in series is 6Ω and their parallel equivalent resistance is $4/3\Omega$. What are the values of resistance? **[Karnataka CET]**

(a) 4 Ω, 6 Ω (b) 8 Ω, 1 Ω
(c) 4 Ω, 2 Ω (d) 6 Ω, 2 Ω

2. Two resistors of resistances 2Ω and 6Ω are connected in parallel. This combination is then connected to a battery of emf 2V and internal resistance 0.5Ω . What is the current flowing through the battery? **[Karnataka CET]**

(a) 4 A (b) 4/3 A
(c) 4/17 A (d) 1 A

3. In a potentiometer of a cell of emf 1.25 V gives balancing length of 30 cm.If the cell is replaced by another cell, balancing length is found to be 40 cm. What is the emf of second cell? **[Karnataka CET]**

(a) $\simeq$ 1.57 V (b) $\simeq$ 1.67 V
(c) $\simeq$ 1.47 V (d) $\simeq$ 1.37 V

4. What is the value of shunt resistance required to convert a galvanometer of resistance 100Ω into an ammeter of range 1 A? Given, full scale deflection of the galvanometer is 5 mA. **[Karnataka CET]**

(a) $\frac{5}{9.95}\Omega$ (b) $\frac{9.95}{5}\Omega$
(c) 0.5Ω (d) 0.05 Ω

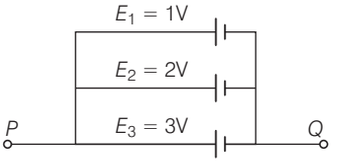
5. A galvanometer having internal resistance 10Ω requires 0.01 A for a full scale deflection. To convert this galvanometer to a voltmeter of full scale deflection at 120 V, we need to connect a resistance of **[WB JEE]**

(a) 11990 Ω in series
(b) 11990 Ω in parallel
(c) 12010 Ω in series
(d) 12010 Ω in parallel

6. Four cells, each of emf E and internal resistance r are connected in series across an external resistance R. By mistake one of the cells is connected in reverse. Then, the current in the external circuit is **[WB JEE]**

(a) $\frac{2E}{4r+R}$ (b) $\frac{3E}{4r+R}$
(c) $\frac{3E}{3r+R}$ (d) $\frac{2E}{3r+R}$

7. A circuit consists of three batteries of emf $E_1 = 1$V, $E_2 = 2$ V and $E_3 = 3$ V and internal resistances 1Ω, 2Ω and 1Ω, respectively which are connected in parallel as shown in the figure. The potential difference between points P and Q is **[WB JEE]**



(a) 1.0 V (b) 2.0 V
(c) 2.2 V (d) 3.0 V

8. A silver wire has a resistance of 1.6Ω at 25.5 ° C and a resistance of 2.5Ω at 100° C, then temperature coefficient of resistivity of silver is **[J&K CET]**

(a) $5.55\times 10^{-3}{}^{\circ}C^{-1}$ (b) $7.55\times 10^{-3}{}^{\circ}C^{-1}$
(c) $11.75\times 10^{-2}{}^{\circ}C^{-1}$ (d) $15.5\times 10^{-3}{}^{\circ}C^{-1}$

9. In a circuit, 3 resistors of resistance 1.2 Ω, 2Ω, 3Ω are connected in parallel. The value of equivalent resistance is **[J&K CET]**

(a) less than 1.2Ω (b) greater than 1.2Ω
(c) between 1.2Ω and 2Ω (d) between 2Ω and 3Ω

10. The neutral point on potentiometer's scale for two cells of emf 2.1 and E volt is observed at distances 40 cm and 56 cm respectively. The value of E is **[J & K CET]**

(a) 2.10 V (b) 2.94 V
(c) 1.50 V (d) 1.20 V

2013

Passage

A thermal power plant produces an electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends.

The drawback of the direct transmission is the large energy dissipation. In the method of using transformers, the dissipation is much smaller. In this method, a step-up transformer is used to plant side so that the current is reduced to a smaller value. At the consumers end, a step-down transformer is used to supply power to the consumers at the specified lower voltage.

It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the current and voltages mentioned are rms values.

11. If the direct transmission method with a cable of resistance $0.4\,\Omega\ \text{km}^{-1}$ is used, the power dissipation (in %) during transmission is **[JEE Advanced]**
(a) 20 (b) 30 (c) 40 (d) 50

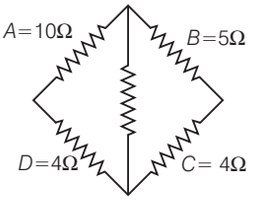
12. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is **[JEE Advanced]**
(a) 200 : 1 (b) 150 : 1
(c) 100 : 1 (d) 50 : 1

13. This equation has Statement I and Statement II of the four choices given after the statements, choose the one that best describes the two statements.

Statement I Higher the range, greater is the resistance of ammeter.

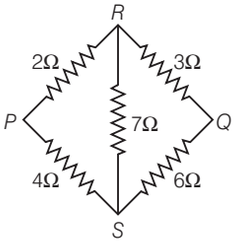
Statement II To increase the range of ammeter, additional shunt needs to be used across it. **[JEE Main]**
(a) Statement I is true, Statement II is true and Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true, but Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

14. In a typical Wheatstone network, the resistances in cyclic order are $A = 10\,\Omega$, $B = 5\,\Omega$, $C = 4\,\Omega$ and $D = 4\,\Omega$ for the bridge to be balanced **[Karnataka CET]**



(a) $10\,\Omega$ should be connected in parallel with A
(b) $10\,\Omega$ should be connected in series with A
(c) $5\,\Omega$ should be connected in series with B
(d) $5\,\Omega$ should be connected in parallel with B

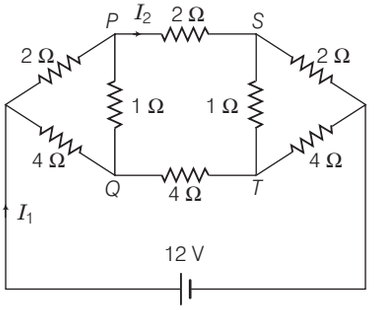
15. Five resistance are connected as shown in the figure. The equivalent resistance between P and Q will be **[Karnataka CET]**



(a) $\frac{10}{3}\Omega$
(b) $\frac{20}{3}\Omega$
(c) $\frac{16}{2}\Omega$
(d) None of the above

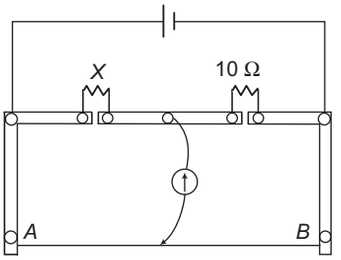
2012

16. For the resistance network shown in the figure, choose the correct option(s). **[IIT JEE]**



(a) The current through PQ is zero
(b) $I_1 = 3\,\text{A}$
(c) The potential as S is less than that at Q
(d) $I_2 = 2\text{A}$

17. A meter bridge is set up as shown in figure, to determine an unknown resistance X using a standard 10 Ω resistor. The galvanometer shows null point when tapping key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of X is **[IIT JEE]**



(a) 10.2 Ω (b) 10.6 Ω
(c) 10.8 Ω (d) 11.1 Ω

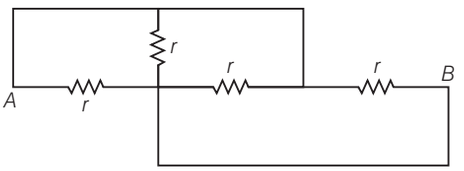
18. The current in the primary circuit of a potentiometer is 0.2 A. The specific resistance and cross-section of the potentiometer wire are 4×10^{-7} Ω-m and 8×10^{-7} m^2 respectively. The potential gradient will be equal to **[AIEEE]**
(a) 0.2 V/m (b) 1 V/m (c) 0.3 V/m (d) 0.1 V/m

19. In a potentiometer experiment, when three cells A, B and C are connected in series the balancing length is found to be 740 cm. If A and B are connected in series balancing length is 440 cm and for B and C connected in series that is 540 cm. Then, the ratio of emf's E_A, E_B and E_C are respectively (in volts) **[Kerala CEE]**
(a) 1, 1.2 and 1.5
(b) 1, 2 and 3
(c) 1.5, 2 and 3
(d) 1.5, 2.5 and 3.5
(e) 1.2, 1.5 and 3.5

20. The deflection in a moving coil galvanometer is reduced to half when it is shunted with a 40 Ω coil. The resistance of the galvanometer is **[Karnataka CET]**

(a) 15 Ω (b) 20 Ω
(c) 40 Ω (d) 80 Ω

21. The effective resistance between points A and B is **[OJEE]**



(a) $4r$ (b) $r/4$
(c) $r/2$ (d) $3r/4$

22. An electric bulb marked as 50 W-200 V is connected across a 100 V supply. The present power of the bulb is **[WB JEE]**

(a) 37.5 W (b) 25 W
(c) 12.5 W (d) 10 W

23. Heat is produced at a rate given by H in a resistor when it is connected across a supply of voltage V. If now the resistance of the resistor is doubled and the supply voltage is made $V/3$, then the rate of production of heat in the resistor will be **[WB JEE]**

(a) $H/18$
(b) $H/9$
(c) $6H$
(d) $18H$

2011

24. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistances R_{100}, R_{60} and R_{40}, respectively. The relation between these resistances is **[IIT JEE]**

(a) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$
(b) $R_{100} = R_{40} + R_{60}$
(c) $R_{100} > R_{60} > R_{40}$
(d) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

25. When two identical batteries of internal resistance 1 Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25 J_2$, then the value of R in Ω is **[IIT JEE]**

(a) 4
(b) 6
(c) 4.8
(d) 5.16

26. A resistor R and 2 μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed ($\log_{10} 2.5 = 0.4$) **[AIEEE]**

(a) 1.7×10^5 Ω (b) 2.7×10^6 Ω
(c) 3.3×10^7 Ω (d) 1.3×10^4 Ω

27. If a wire is stretched to make it 0.1% longer, its resistance will **[AIEEE]**

(a) increase by 0.2% (b) decrease by 0.2%
(c) decrease by 0.05% (d) increase by 0.05%

28. If 400 Ω of resistance is made by adding four 100 Ω resistance of tolerance 5%, then the tolerance of the combination is **[AIEEE]**

(a) 20% (b) 5% (c) 10% (d) 15%

29. Combination of two identical capacitors, a resistor R and a DC voltage source of voltage 6 V is used in an experiment on C-R circuit. It is found that for a parallel combination of the capacitor the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 s. For series combination the time needed for reducing the voltage of the fully charged series combination by half is **[AIEEE]**

(a) 200 s (b) 10 s
(c) 5 s (d) 2.5 s

30. A heater of 220 V heats a volume of water in 5 min. The same heater when connected to 110 V heats the same volume of water in (min) **[Kerala CEE]**

(a) 5 (b) 20 (c) 10
(d) 2.5 (e) 1.25

31. If 2 A of current is passed through $CuSO_4$ solution for 32 s, then the number of copper ions deposited at the cathode will be **[Kerala CEE]**

(a) 4×10^{20} (b) 2×10^{20}
(c) 4×10^{19} (d) 2×10^{19}
(e) 1.6×10^{19}

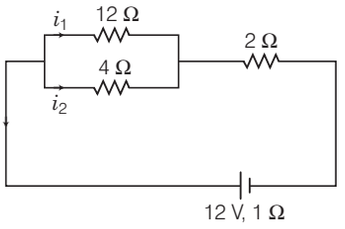
32. The resistance of a 10 m long wire is 10 Ω. It's length is increased by 25% by stretching the wire uniformly. Then, the resistance of the wire will be **[Kerala CEE]**

(a) 12.5 Ω (b) 14.5 Ω
(c) 15.6 Ω (d) 16.6 Ω
(e) 18.6 Ω

33. Find the true statement. **[Kerala CEE]**

(a) Ohm's law is applicable to all conductors of electricity
(b) In an electrolyte solution, the electric current is mainly due to the movement of electrons
(c) The resistance of an incandescent lamp is lesser when the lamp is switched on
(d) Specific resistance of a wire depends upon its dimension
(e) The resistance of carbon decreases with the increase of temperature

34. In the circuit shown, the currents i_1 and i_2 are **[Karnataka CET]**



(a) $i_1 = 3$ A, $i_2 = 1$ A
(b) $i_1 = 1$ A, $i_2 = 3$ A
(c) $i_1 = 0.5$ A, $i_2 = 1.5$ A
(d) $i_1 = 1.5$ A, $i_2 = 0.5$ A

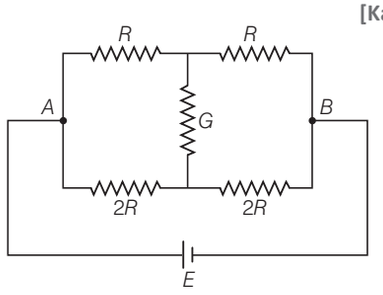
35. Consider the following statements regarding the network shown in the figure.

I. The equivalent resistance of the network between points A and B is independent of value of G.

II. The equivalent resistance of the network between points A and B is $\frac{4}{3}R$.

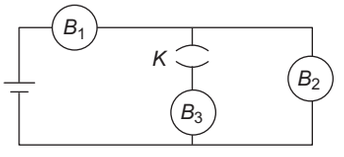
III. The current through G is zero.

Which of the above statements is/are true? **[Karnataka CET]**



(a) I, II and III (b) II and III
(c) Only II (d) Only I

36. B_1, B_2 and B_3 are the three identical bulbs connected to a battery of steady emf with key K closed. What happens to the brightness of the bulbs B_1 and B_2, when the key is opened? **[Karnataka CET]**

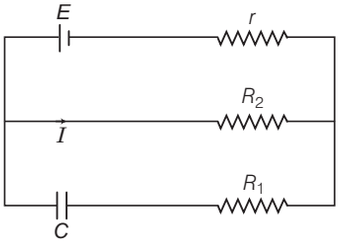


(a) Brightness of the bulb B_1 increases and that of B_2 decreases
(b) Brightness of the bulbs B_1 and B_2 increase
(c) Brightness of the bulb B_1 decreases and B_2 increases
(d) Brightness of the bulbs B_1 and B_2 decrease

37. An electric heater rated 220 V and 550 W is connected to AC mains. The current drawn by it is **[Karnataka CET]**

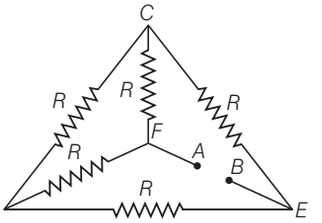
(a) 0.8 A (b) 2.5 A
(c) 0.4 A (d) 1.25 A

38. The charge on the capacitor of capacitance C shown in the figure below will be **[WB JEE]**



(a) CE (b) $\frac{CER_1}{R_1 + r}$
(c) $\frac{CER_2}{R_2 + r}$ (d) $\frac{CER_1}{R_2 + r}$

39. Five equal resistance, each of resistance R are connected as shown in figure below. A battery of V volt is connected between A and B. The current flowing in FC will be **[WB JEE]**



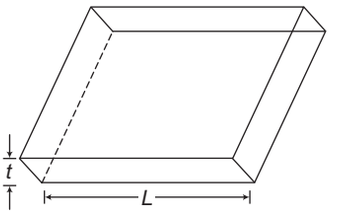
(a) $\frac{3V}{R}$ (b) $\frac{V}{R}$
(c) $\frac{V}{2R}$ (d) $\frac{2V}{R}$

40. Two cells with the same emf E and different internal resistances r_1 and r_2 are connected in series to an external resistance R. The value of R so, that the potential difference across the first cell be zero is **[WB JEE]**

(a) $\sqrt{r_1 r_2}$ (b) $r_1 + r_2$
(c) $r_1 - r_2$ (d) $\frac{r_1 + r_2}{2}$

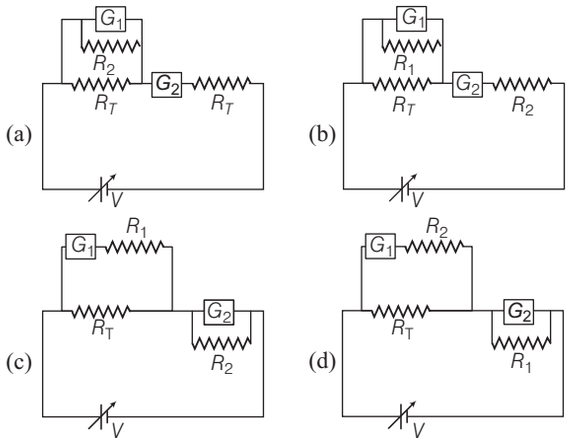
2010

41. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is **[IIT JEE]**



(a) directly proportional to L (b) directly proportional to t
(c) independent of L (d) independent of t

42. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2 and a variable voltage source V. The correct circuit to carry out the experiment is **[IIT JEE]**



43. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly **[AIEEE]**

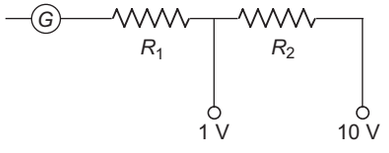
(a) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$

(b) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$

(c) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

(d) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$

44. The resistance of a galvanometer is 50 Ω and it shows full scale deflection for a current of 1 mA. To convert it into a voltmeter to measure 1 V and as well as 10 V (refer circuit diagram) the resistance R_1 and R_2 respectively are **[Kerala CEE]**



(a) 950 Ω and 9150 Ω
(b) 900 Ω and 9950 Ω
(c) 900 Ω and 9900 Ω
(d) 950 Ω and 9000 Ω
(e) 950 Ω and 9950 Ω

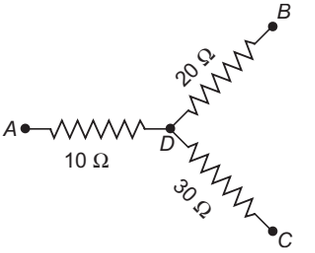
45. A potentiometer wire of length 10 m and resistance 20 Ω is connected in series with a 15 V battery and an external resistance 40 Ω. A secondary cell of emf E in the secondary circuit is balanced by 240 cm long potentiometer wire. The emf E of the cell is **[Kerala CEE]**

(a) 2.4 V
(b) 1.2 V
(c) 2.0 V
(d) 3 V
(e) 6 V

46. A battery of emf E has an internal resistance r. A variable resistance R is connected to the terminals of the battery. A current i is drawn from the battery. V is the terminal potential difference. If R alone is gradually reduced to zero, which of the following best describes i and V? **[Karnataka CET]**

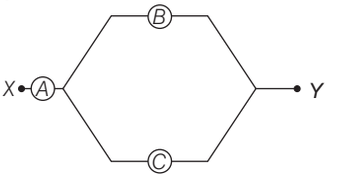
(a) i approaches zero, V approaches E
(b) i approaches $\frac{E}{r}$, V approaches zero
(c) i approaches $\frac{E}{r}$, V approaches E
(d) i approaches infinity, V approaches E

47. In the circuit given here, the points A, B and C are 70 V, 0V, 10 V respectively. Then, **[Karnataka CET]**



(a) the point D will be at a potential of 60 V
(b) the point D will be at a potential of 20 V
(c) currents in the paths AD, DB and DC are in the ratio of 1 : 2 : 3
(d) currents in the paths AD, DB and DC are in the ratio of 3 : 2 : 1

48. Three voltmeters A, B and C having resistances $R, 1.5R$ and $3R$ respectively are used in a circuit as shown in the figure. When a potential difference is applied between X and Y, the readings of the voltmeters are V_1, V_2 and V_3 respectively. Then, **[Karnataka CET]**



(a) $V_1 = V_2 = V_3$
(b) $V_1 < V_2 = V_3$
(c) $V_1 > V_2 > V_3$
(d) $V_1 > V_2 = V_3$

49. A 36 Ω galvanometer is shunted by resistance of 4 Ω. The percentage of the total current, which passes through the galvanometer is **[Karnataka CET]**

(a) 8%
(b) 9%
(c) 10%
(d) 91%

50. The accurate measurement of emf can be obtained using **[Karnataka CET]**

(a) multimeter
(b) voltmeter
(c) voltameter
(d) potentiometer

51. A galvanometer of resistance 240 Ω allows only 4% of the main current after connecting a shunt resistance. The value of the shunt resistance is [Karnataka CET]
(a) 10 Ω (b) 20 Ω (c) 8 Ω (d) 5 Ω

52. For measurement of potential difference, potentiometer is preferred in comparison to voltmeter because [MHT CET]
(a) potentiometer is more sensitive than voltmeter
(b) the resistance of potentiometer is less than voltmeter
(c) potentiometer is cheaper than voltmeter
(d) potentiometer does not take current from the circuit

53. When a resistance of 100 Ω is connected in series with a galvanometer of resistance R, its range is V. To double its range, a resistance of 1000 Ω is connected in series. Find the value of R. [MHT CET]
(a) 700 Ω (b) 800 Ω (c) 900 Ω (d) 100 Ω

54. A 2V battery, a 990 Ω resistor and a potentiometer of 2 m length, all are connected in series of the resistance of potentiometer wire is 10 Ω, then the potential gradient of the potentiometer wire is [MHT CET]
(a) 0.05 Vm^{-1}
(b) 0.5 Vm^{-1}
(c) 0.01 Vm^{-1}
(d) 0.1 Vm^{-1}

55. For a thermocouple, the inversion temperature is 600°C and the neutral temperature is 320°C. Find the temperature of the cold junction? [MHT CET]
(a) 40°C
(b) 20°C
(c) 80°C
(d) 60°C

Answers

Level 1

Objective Problems

1. (c)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (a)
11. (b)	**12.** (a)	**13.** (b)	**14.** (d)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (b)	**20.** (c)
21. (b)	**22.** (d)	**23.** (c)	**24.** (c)	**25.** (c)	**26.** (d)	**27.** (b)	**28.** (c)	**29.** (a)	**30.** (d)
31. (d)	**32.** (d)	**33.** (a)	**34.** (c)	**35.** (b)	**36.** (c)	**37.** (d)	**38.** (a)	**39.** (b)	**40.** (b)
41. (c)	**42.** (a)	**43.** (b)	**44.** (d)	**45.** (a)	**46.** (b)	**47.** (a)	**48.** (d)	**49.** (c)	**50.** (d)
51. (a)	**52.** (d)	**53.** (b)	**54.** (a)	**55.** (d)	**56.** (c)	**57.** (b)	**58.** (c)	**59.** (b)	**60.** (c)
61. (b)	**62.** (c)	**63.** (c)	**64.** (d)	**65.** (a)	**66.** (d)	**67.** (b)	**68.** (a)	**69.** (b)	**70.** (a)
71. (c)	**72.** (a)	**73.** (b)	**74.** (a)	**75.** (a)	**76.** (b)	**77.** (c)	**78.** (b)	**79.** (b)	**80.** (b)
81. (d)	**82.** (b)	**83.** (b)	**84.** (b)	**85.** (c)	**86.** (a)	**87.** (c)	**88.** (a)	**89.** (c)	**90.** (a)
91. (a)	**92.** (c)	**93.** (a)	**94.** (b)	**95.** (c)	**96.** (c)	**97.** (b)	**98.** (a)	**99.** (b)	**100.** (c)
101. (c)	**102.** (c)	**103.** (c)	**104.** (c)	**105.** (b)	**106.** (c)	**107.** (a)	**108.** (b)	**109.** (d)	**110.** (b)
111. (c)	**112.** (a)	**113.** (c)	**114.** (b)	**115.** (b)	**116.** (c)	**117.** (a)	**118.** (c)	**119.** (d)	**120.** (d)
121. (c)	**122.** (c)	**123.** (c)	**124.** (c)	**125.** (d)	**126.** (d)	**127.** (d)	**128.** (a)	**129.** (a)	**130.** (c)
131. (b)	**132.** (c)	**133.** (b)	**134.** (b)	**135.** (b)	**136.** (b)	**137.** (d)	**138.** (d)	**139.** (b)	**140.** (d)
141. (b)	**142.** (a)	**143.** (a)	**144.** (d)	**145.** (b)	**146.** (c)	**147.** (d)	**148.** (d)	**149.** (b)	**150.** (c)
151. (c)	**152.** (c)	**153.** (c)	**154.** (a)	**155.** (b)	**156.** (d)	**157.** (c)	**158.** (a)	**159.** (a)	**160.** (b)
161. (d)	**162.** (a)	**163.** (a)	**164.** (a)	**165.** (c)	**166.** (a)	**167.** (a)	**168.** (c)	**169.** (d)	**170.** (b)
171. (c)	**172.** (d)	**173.** (a)	**174.** (c)	**175.** (d)	**176.** (a)	**177.** (b)	**178.** (b)	**179.** (c)	**180.** (d)
181. (b)	**182.** (c)	**183.** (d)	**184.** (d)	**185.** (b)	**186.** (c)	**187.** (c)	**188.** (a)	**189.** (d)	

Level 2

Only One Correct Options

1. (c)	**2.** (c)	**3.** (d)	**4.** (c)	**5.** (a,c)	**6.** (c)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (b)
11. (b)	**12.** (b)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (b)	**17.** (b)	**18.** (c)	**19.** (d)	**20.** (b)
21. (a)	**22.** (b)	**23.** (d)	**24.** (c)	**25.** (b)	**26.** (b,c,d)	**27.** (a)	**28.** (a)	**29.** (b)	**30.** (b)
31. (a)	**32.** (b)	**33.** (a)	**34.** (c)	**35.** (b)	**36.** (a)	**37.** (c)	**38.** (b)		

More than One Correct Options

1. (a,c)	**2.** (b,c,d)	**3.** (a,b,d)	**4.** (b,c)	**5.** (a,b,c)	**6.** (b,c,d)	**7.** (all)	**8.** (a,c)	**9.** (a,d)	**10.** (b,d)
11. (b,d)	**12.** (a,b)	**13.** (a,c)							

Comprehension Based Questions

1. (a) **2.** (c)

Assertion and Reason

1. (d)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (e)
11. (b)	**12.** (a)	**13.** (e)	**14.** (c)	**15.** (e)	**16.** (d)	**17.** (e)	**18.** (a)	**19.** (c)	

Match the Columns

1. (A→p; B→q; C→q; D→q) **2.** (A→r; B→p; C→p; D→s) **3.** (A→p; B→r; C→q; D→p)
4. (A→q; B→r; C→s; D→p) **5.** (A→r; B→q; C→p)

Entrance Gallery

1. (c)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (b)
11. (b)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (c)
21. (b)	**22.** (c)	**23.** (a)	**24.** (d)	**25.** (a)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (d)	**30.** (b)
31. (b)	**32.** (c)	**33.** (e)	**34.** (c)	**35.** (b)	**36.** (c)	**37.** (b)	**38.** (c)	**39.** (c)	**40.** (c)
41. (c)	**42.** (c)	**43.** (d)	**44.** (d)	**45.** (b)	**46.** (b)	**47.** (d)	**48.** (a)	**49.** (c)	**50.** (d)
51. (a)	**52.** (d)	**53.** (c)	**54.** (c)	**55.** (a)					

Solutions

Level 1 : Objective Problems

1. $i = neAv_d$ or $v_d = \dfrac{i}{neA}$ or $v_d \propto \dfrac{i}{A}$.

2. The order of drift velocity is 10^{-4} m/s $= 10^{-2}$ cm/ s

3. $R = \rho \dfrac{l}{A}$

For given wire, volume $V = Al$ is constant.

or $A = V/l$

$\therefore$ $R = \dfrac{\rho l^2}{V}$

$\therefore$ $R \propto l^2$

or % increases in $R = 0.2$ (% increase in l).

4. For cube surface area is $(50 \times 10^{-2})^2$.

$\therefore$ Resistance, $R = \rho \dfrac{l}{A}$

$= 50 \times 10^{-8} \times \dfrac{50 \times 10^{-2}}{(50 \times 10^{-2})^2}$

$= 10^{-6}\,\Omega$

5. Volume of wire $= Al$

Given, $3 = Al$

$\therefore$ $A = \dfrac{3}{l}$

Now, resistance, $R = \rho \dfrac{l}{A}$

$3 = \dfrac{\rho l}{(3/l)} = \dfrac{\rho l^2}{3}$

$\therefore$ $l^2 = \dfrac{9}{\rho}$

$\therefore$ $l = \dfrac{3}{\sqrt{\rho}}$

6. Charge, $q = it$

$\therefore$ $i = \dfrac{q}{t} = qf$

$= 1.6 \times 10^{-19} \times 6.6 \times 10^{15}$

≈ 1 mA

7. Metallic conductors have positive temperature coefficient of resistance.

8. Resistance, $R = \rho \dfrac{l}{A}$ and resistivity, $\rho = \dfrac{m}{ne^2\tau}$

$\therefore$ $R = \dfrac{ml}{ne^2 \tau A}$

9. If the wire increases in length, then

$R \propto l^2$

$\dfrac{R'}{R} = \left(\dfrac{l'}{l}\right) = \left(\dfrac{1.1l}{l}\right)^2 = 1.21$

$\therefore$ $R' = 1.21\, R$

$= 1.21 \times 10 = 12.1\,\Omega$

10. Reciprocal of resistance is conductance $G = \dfrac{1}{R}$.

11. Specific resistance of silver, copper and aluminium are $1.6 \times 10^{-8}\,\Omega$-m, $1.7 \times 10^{-8}\,\Omega$-m and $2.7 \times 10^{8}\,\Omega$-m respectively.

12. Current, $i = \dfrac{q}{t}$

or $i = \dfrac{ne}{t}$

$\Rightarrow$ $16 \times 10^{-3} = \dfrac{n \times 1.6 \times 10^{-19}}{1}$

$\therefore$ $n = 10^{17}$

13. $i = \dfrac{V}{R} = \dfrac{q}{t}$

$\therefore$ $q = \dfrac{Vt}{R} = \dfrac{20 \times 2 \times 60}{10}$

$= 240\,\text{C}$

14. For semiconductors, resistance decrease with increase in temperature while for conductors, resistance increase with increase of temperature.

15. Resistance of wire, $R \propto l^2$

$\therefore$ $\dfrac{R_1}{R_2} = \left(\dfrac{l_1}{l_2}\right)^2 = \left(\dfrac{l}{2l}\right)^2$

$\therefore$ $R_2 = 4\, R_1$

$\therefore$ Change in resistance,

$\dfrac{\Delta R}{R} \times 100 = \dfrac{R_2 - R_1}{R_1} \times 100$

$= \dfrac{4\, R_1 - R_1}{R_1} \times 100$

$= 300\%$

16. Resistance, $R = \rho \cdot \dfrac{l}{A} = \rho \cdot \dfrac{l}{\pi r^2}$

For same material,

$R \propto \dfrac{l}{r^2}$

For highest resistance, $\dfrac{l}{r^2}$ should be maximum, it's maximum for option (a).

17. Resistance, $R = \rho \cdot \dfrac{l}{A}$

$\therefore$ $\rho = \dfrac{RA}{l}$

But $R = \dfrac{V}{i}$

$\therefore$ $\rho = \dfrac{VA}{il} = \dfrac{V/l}{i/A}$

But $\dfrac{V}{l} = E$ and $\dfrac{i}{A} = J$

$\therefore$ $\rho = \dfrac{E}{J}$

or $E = \rho \cdot J$

or $E = \frac{1}{\sigma} J$

where, $\sigma = \frac{1}{\rho}$

Thus, $\sigma = J/E$

18. $R_1 = \rho_1 \frac{l_1}{A}$

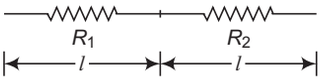
$R_2 = \rho_2 \frac{l_2}{A}$

In series, $R_{eq} = R_1 + R_2$

$\therefore \quad \rho_{eq} \frac{(l_1 + l_2)}{A} = \rho_1 \frac{l_1}{A} + \rho_2 \frac{l_2}{A}$

$\therefore \quad \rho_{eq} = \frac{A(\rho_1 l_1 + \rho_2 l_2)}{A(l_1 + l_2)} = \frac{\rho_1 l_1 + \rho_2 l_2}{l_1 + l_2}$

19.



Let resistance of two wires be R_1 and R_2. So, their equivalent resistance

$R_{eq} = R_1 + R_2$

$\rho_{eq} \frac{2l}{A} = \rho_1 \frac{l}{A} + \rho_2 \frac{l}{A}$

$\therefore \quad \rho_{eq} = \frac{\rho_1 + \rho_2}{2}$

21. $i = qf = q\left(\frac{v}{2\pi r}\right)$

$= \frac{1.6 \times 10^{-19} \ (2.2 \times 10^6)}{(2\pi)\,(5 \times 10^{-11})}$

$= 1.12 \times 10^{-3}\,\text{A}$

22. $i = (2.9 \times 1.2) + 10^{18} \times 1.6 \times 10^{-19}\ \text{A} = 0.656\,\text{A}$

23. $i = \frac{ne}{t}$

$\therefore \quad n = \frac{it}{e} = \frac{0.2 \times 30}{1.6 \times 10^{-19}} = 3.75 \times 10^{19}$

24. Radius has become half. Therefore, are will becomes $\frac{1}{4}$th and length will become four times.

Now, $R \propto \frac{l}{A}$

26. $\rho \times \sigma = 1$ which is a constant.

27. $V = iR = i\left(\frac{l}{\sigma A}\right)$

$\therefore \quad \sigma = \frac{il}{VA} = \frac{(0.25)\ (0.5)}{(2)\ (10^{-7})} = 6.25 \times 10^5\ \text{mho/m}$

28. R = slope of V - I graph (V on y-axis and I on x-axis) and R increases with increase in temperature.

30. $1 = R_0\,(1 + 27\alpha)$

$2 = R_0\,(1 + \alpha t)$

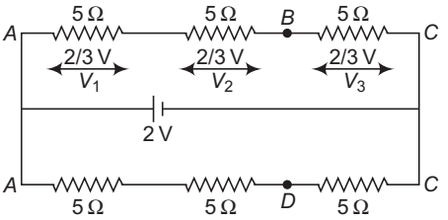
$\therefore \quad 1 + \alpha t = 2\,(1 + 27\alpha)$

or $t = \frac{1 + 54\alpha}{\alpha}$

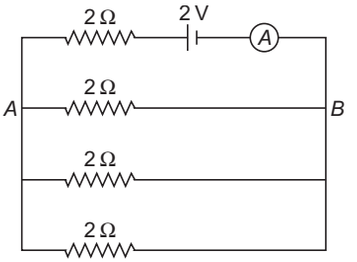
$= 54 + \frac{1}{\alpha} = 854°\text{C}$

or $T = 1127\,\text{K}$

34. Given circuit can be redrawn as follows.



35. The circuit can be redrawn as follows.



Resistance across AB,

$R_{AB} = \frac{2}{1+1+1} = \frac{2}{3}\Omega$

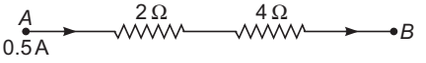
Total resistance of circuit,

$R_T = 2 + \frac{2}{3} = \frac{8}{3}\,\Omega$

Current through ammeter,

$i = \frac{2}{(8/3)} = \frac{6}{8} = \frac{3}{4}\,\text{A}$

36. The given circuit can be reduced to



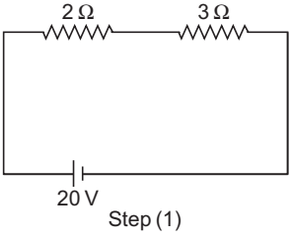
Equivalent resistance between A and B,

$R_{AB} = 2 + 4 = 6\Omega$

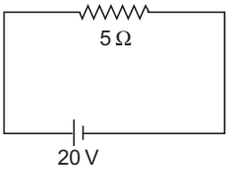
$\therefore \quad V_A - V_B = (6)\,(0.5)$

$= 3\,\text{V}$

37. Circuit can be reduced as follows.



Step (1)



Step (2)

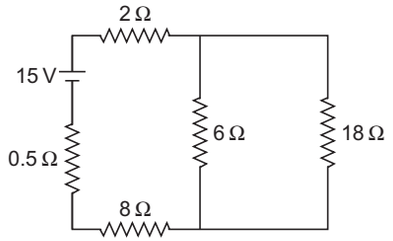
So, equivalent resistance of circuit,

$R_{eq} = 5\,\Omega$

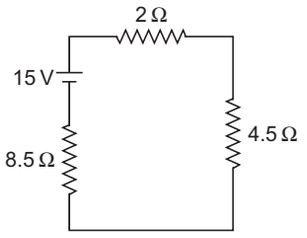
$\therefore$ Current in the circuit $= \frac{20}{5} = 4\ \text{A}$

As in parallel, current is divided according to resistance, so, current flowing through each resistance = 2A.

38. The circuit can be reduced as follows.



Step (1)



Step (2)

So, equivalent resistance across battery,

$$R_{eq} = 8.5 + 2 + 4.5 = 15\Omega$$

Hence, current from the battery

$$i = \frac{15}{15} = 1\text{A}$$

39. Potential difference across the circuit $= i\left(\frac{R_1 R_2}{R_1 + R_2}\right)$

$$= 1.2 \times \frac{6 \times 4}{6 + 4} = 2.88\text{V}$$

So, current through 6Ω resistance $= \frac{2.88}{6} = 0.48\text{A}$

Alternative Solution

From current division law,

$$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) i = 1.2\left(\frac{4}{6+4}\right) = 1.2\left(\frac{4}{10}\right)$$

$$= 0.48\text{A}$$

40. As B connected to the earth, so, potential of B is $V_B = 0$.

Now, current in the given circuit,

$$i = \frac{50}{5 + 7 + 10 + 3} = 2\text{A}$$

Potential difference between A and B is

$$V_A - V_B = 2 \times 12 \quad \text{or} \quad V_A - 0 = 24$$

$$V_A = 24\text{V}$$

41. Current divides according to resistance, so current in 6Ω resistance is $\frac{0.8}{2} = 0.4\text{A}$

So, total current in circuit is $0.8 + 0.4 = 1.2\text{A}$

$\therefore$ Potential drop across $4\Omega = 1.2 \times 4 = 4.8\text{V}$

42. Equivalent resistance of circuit

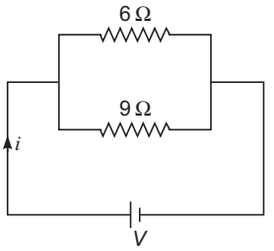
$$R = 4 + \frac{3+6}{3+6} = 6\ \Omega$$

and current through battery

$$i = \frac{3}{6} = 0.5\ \text{A}$$

43. Given circuit is balanced Wheatstone bridge and no current flows, through 4Ω resistance connected across diagonal.

Now circuit reduces to



So, equivalent resistance of circuit

$$R_{eq} = \frac{18}{5}\Omega$$

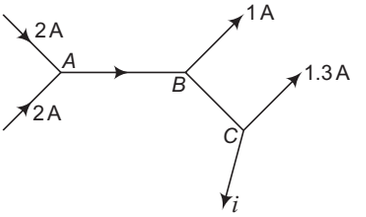
$\therefore$ Current, $i = \frac{V}{R} = \frac{V}{18/5} = \frac{5V}{18}\text{A}$

44. $i = \frac{\text{Net emf}}{\text{Net resistance}} = \frac{10 - 4}{1 + 2 + 3} = 1\text{A}$

45. Kirchhoff's first law, i.e. $\Sigma i = 0$ is based on law of conservation of charge, because at any junction, the incoming charge = outgoing charge.

46. Kirchhoff's second law is based on the law of conservation of energy as if sum of potential changes around a closed loop is not zero, unlimited energy could be gained by repeatedly carrying a charge around a loop.

47.



Applying Kirchhoff's first law to junctions A, B, C.

At A, $i_{AB} = 2 + 2 = 4\text{A}$

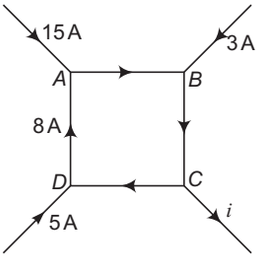
At B, $i_{BC} = i_{AB} - 1 = 3\text{A}$

At C, $i = i_{BC} - 1.3$

$= 3 - 1.3 = 1.7\text{ A}$

48. It is an open circuit and for open circuit, terminal potential difference = emf of cell, i.e. $V = E$

49.



Applying Kirchhoff's first law at junction A, B, C, D.

At A, $i_{AB} = 23\text{A}$

At B, $i_{BC} = 23 + 3 = 26\text{ A}$

At D, $i_{CD} = 8\text{A} - 5\text{A} = 3\text{A}$

At C, $i_{CD} + i = i_{BC}$

or $3 + i = 26$

$\therefore$ $i = 23\text{ A}$

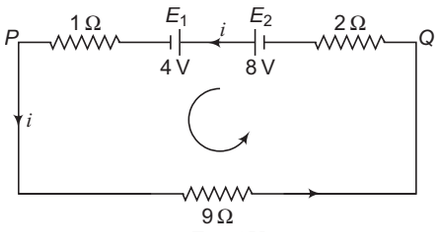
50. Potential difference between A and B is given by

$$V_A - V_B = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

$$\therefore \quad 4 = \frac{5X + 2\times 10}{X + 10}$$

$$\therefore \quad X = 20\,\Omega$$

51.



$$E_1 = 4\,\text{V}$$

and $$E_2 = 8\,\text{V}$$

So, $E_2 > E_1$ current flows from Q to P.

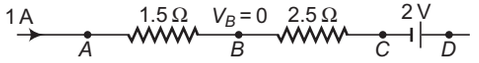
$$\therefore \quad i = \frac{1}{3}\text{A}$$

$\therefore$ Potential difference across,

$$PQ = \frac{1}{3}\times 9$$

$$= 3\text{V}$$

52.



Potential difference between A and B

$$V_A - V_B = 1\times 1.5$$

or $$V_A - 0 = 1.5$$

$$\therefore \quad V_A = 1.5\,\text{V}$$

Potential difference between B and C

$$V_B - V_C = 1\times 2.5$$

$$= 2.5\,\text{V}$$

$$\therefore \quad 0 - V_C = 2.5\,\text{V}$$

$$\therefore \quad V_C = -2.5\,\text{V}$$

So, potential difference between C and D

$$V_C - V_D = -2\,\text{V}$$

or $$-2.5 - V_D = -2$$

or $$V_D = -0.5\,\text{V}$$

53. Two resistances are short circuited.

54. PD = 10 V

56. Equal current (of 1A) will be distributed in the two branches.

$$V_D - V_A = 2\times 1 = 2\,\text{V}$$

and $$V_D - V_B = 3\times 1 = 3\,\text{V}$$

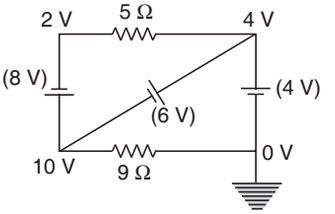
$$\therefore \quad V_A - V_B = +1\,\text{V}$$

58. Apply Kirchhoff's junction law at different junctions.

60. Current distributes in inverse ratio of resistance.

63. Reading of ammeter, $i = 3\left(\frac{220}{1100}\right) = \frac{3}{5}\text{A}$

64. The potentials of different points are as shown in figure,



Current through 5Ω resistance

$$= \frac{\text{PD}}{\text{Resistance}}$$

$$= \frac{(4-2)}{5} = 0.4\,\text{A}$$

65. Potential difference across 4Ω resistance $(20+16)\,V$ current through 4Ω resistance

$$= 36\,V = \frac{36}{4} = 9\text{A} \qquad \text{(from top to bottom)}$$

Similarly, current through 2Ω resistance $= \frac{20-46}{2} = 2\text{A}$

Therefore, total current through 20 V battery will be 11 A.

68. 1 kWh $= 1000\times 60\times 60 = 36\times 10^5$ W-s $= 36\times 10^5$ J

69. Power, $P = \frac{V^2}{R}$

$$\therefore \quad P \propto \frac{1}{R}$$

So, $$\frac{P_1}{P_2} = \frac{R_2}{R_1}$$

70. Power, $P = \frac{V^2}{R}$

$$\therefore \quad P \propto \frac{1}{R}$$

Also, resistance of wire $\propto \frac{1}{(\text{radius of filament})^2}$

$\therefore\ P \propto (\text{radius of filament})^2$

So, 100 W bulb has thicker filament.

71. When bulbs are in series,

$$P = \frac{V^2}{3R} \qquad \ldots\text{(i)}$$

When bulbs are connected in parallel,

$$P' = \frac{V^2}{(R/3)} = \frac{3V^2}{R}$$

$$= 3\times 3P \qquad \text{[from Eq. (i)]}$$

$$= 9P$$

72. $\frac{R_{40}}{R_{100}} = \frac{100}{40} = \frac{5}{2}$

In series, $$V_{40} = (440)\left(\frac{5}{7}\right) = 314.3\,\text{V}$$

and $$V_{100} = (440)\left(\frac{2}{7}\right) = 125.7\,\text{V}$$

Therefore, 40 W bulb will fuse.

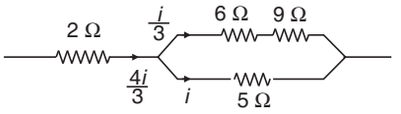
73. In parallel V is same, $P = \frac{V^2}{R}$ or $P \propto \frac{1}{R}$

75. In series i is same, $H = i^2 Rt$. Therefore, $H \propto R$.

76. In parallel V is same, $H=\frac{V^2}{R}t$. Therefore, $H\propto\frac{1}{R}$.

77. $R_{40}>R_{100}$. In series potential difference distributes in direct ratio of resistance.

78. Current will be distributed in the resistors as shown in figure.



Now, $10\,(\text{cal/s})=i^2(5)$ …(i)

$$P=\left(\frac{4i}{3}\right)^2(2)$$ …(ii)

From these two equations, we get

$$P=7.1\ \text{cal/s}$$

80. V is constant, $H=\frac{V^2}{R}t$ or $H\propto\frac{1}{R}$.

81. $P_R=i^2R=(i.i)R=\left(\frac{E}{R+r}\right)(i)(R)$

82. $R'=\frac{R}{4}, P=\frac{V^2}{R} \quad\Rightarrow\quad \therefore \quad P'=4P$

83. $$150=\frac{(15)^2}{\left(\frac{2R}{2+R}\right)}$$

$$\therefore \quad \frac{2+R}{2R}=\frac{2}{3}$$

$$\therefore \quad R=6\Omega$$

85. $R=\frac{V^2}{P}$

86. $P=\frac{V^2}{R}$, R_{net} will decrease. Therefore, P will increase.

87. $\left(\frac{dm}{dt}\right)=\frac{(V^2/R)}{L}=\frac{(210)^2/20}{80\times4.18}=6.59$ g/s.

88. $10=\frac{Q}{P_1}$

$$\therefore \quad P_1=\frac{Q}{10}$$

$$15=\frac{Q}{P_2}$$

$$\therefore \quad P_2=\frac{Q}{15}$$

$$t=\frac{Q}{P_1+P_2}=\frac{Q}{\frac{Q}{10}+\frac{Q}{15}}=6\ \text{min}$$

89. The given circuit forms a balanced Wheatstone bridge.

$$R_{\text{net}}=2R$$

Now, R_{net} = internal resistance

or $$2R=4\Omega$$

$$R=2\Omega$$

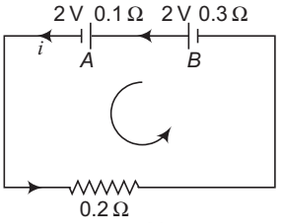
90. $R=\frac{V^2}{P}$ or $R\propto V^2$

91. $P=\frac{E^2}{R_{\text{net}}}$

92. During the charging of battery terminal potential difference is always greater than emf of circuit.

$$V=E+ir$$

93.



$$l=\frac{20}{3}\text{A}$$

Hence, potential difference across A

$$=2-0.1\times\frac{20}{3}=\frac{4}{3}\text{V}$$

Potential difference across, $B=2-0.3\times\frac{20}{3}=0$

94. $W=qV=6\times10^{-6}\times9=54\times10^{-6}$ J

95. There are n rows each containing m cells,

$\therefore$ Total cells $=m\times n=24$ …(i)

For maximum current in the circuit, external resistance should be equal to net internal resistance.

$$R=\frac{mr}{n}$$

$$\Rightarrow \quad 3=\frac{m}{n}(0.5)$$

$$\therefore \quad m=6n$$

From Eqs. (i) and (ii), we get

$$m=12, n=2$$

96. $V=E-ir=E-\left(\frac{E}{R+r}\right)r$

$$=E\left(\frac{R}{R+r}\right)$$

$$=\frac{E}{1+r/R}$$

98. $r=R\left(\frac{E}{V}-1\right)=(10)\left(\frac{1.5}{1.25}-1\right)=2\Omega$

100. $0=E-ir_2=E-\left(\frac{2E}{r_1+r_2+R}\right)\cdot r_2$

$$\therefore \quad R=r_2-r_1$$

101. $$i_1=\frac{E}{r+R_1}$$

$$i_2=\frac{E}{r+R_2}$$

From these two equations,we get, $r=\frac{i_2R_2-i_1R_1}{i_1-i_2}$

103. $V_A=E_A-ir_A=2-\left(\frac{4}{1.9+0.9+1}\right)(1.9)=0$

105. $E_{\text{net}}=\frac{E_1/r_1-E_2/r_2}{1/r_1+1/r_2}$

Current through R will be zero if $E_{\text{net}}=0$

or $$\frac{E_1}{r_1}=\frac{E_2}{r_2}$$

107. Current through R_1 and R_2 is zero (potential difference = 0). Therefore, current in wire AB is also zero.

108. Potential difference across 100Ω resistance should be 5V as voltmeter and 100 Ω resistance are in parallel. It means equivalent resistance of voltmeter and 100 Ω should be 50 Ω.

So, resistance of voltmeter must be 100 Ω.

109. Resistance in parallel with voltmeter is zero.

110. The position of balance of point in potentiometer is fixed, on introducing or wire of resistance 15 Ω, there is no effect on position of balance point, it will remain on same position.

111. Galvanometer current is given by,

$$i_g = i\left(\frac{S}{S+G}\right)$$

$\therefore$ Shunt resistance, $S = \dfrac{i_g G}{(i - i_g)}$

$\therefore$ $$S = \frac{10 \times 99}{(100-10)} = 11\,\Omega$$

112. In potentiometer, the ratio of emf's is equal to ratio of no deflection lengths.

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} = \frac{2}{3}$$

113. The galvanometer shows no current it means this is a balanced Wheatstone bridge. So,

$$\frac{R}{80} = \frac{AC}{BC} = \frac{20}{80}$$

$\therefore$ $$R = 20\,\Omega$$

114. $$r = R_1\left(\frac{E}{V_1} - 1\right) \quad \ldots(i)$$

$$r = R_2\left(\frac{E}{V_2} - 1\right) \quad \ldots(ii)$$

Solving the Eqs. (i) and (ii),we get

$$r = 10\,\Omega$$

115. For potentiometer, emf

$$E \propto l \quad \text{on} \quad E = kl$$

where, k is a constant.

Also, $$E = \frac{V}{l} = \frac{iR}{L} \times l$$

$\therefore$ $$E = \frac{E'}{(R_1 + R_2 + r)} \times \frac{R}{L} \times l$$

$\therefore$ $$E = \frac{10}{5+4+1} \times \frac{5}{5} \times 3 = 3\,\text{V}$$

116. Total resistance of given circuit

$$= \frac{80}{1+1} + 20 = 40 + 20 = 60\,\Omega$$

$\therefore$ Main current, $i = \dfrac{2}{60} = \dfrac{1}{30}$ A

Now, in parallel, there are two resistance of 80Ω each. (one of voltmeter and other 80Ω resistance) So, current is equally distributed in 80Ω resistance and voltmeter, So, $\dfrac{1}{60}$ A.

Current flows through each.

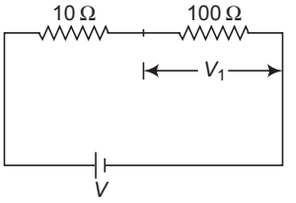
$\therefore$ Potential difference across 80Ω resistance

$$= \frac{1}{60} \times 80 = 1.33\,\text{V}$$

117. $$r = \left(\frac{l_1 - l_2}{l_2}\right) \times R$$

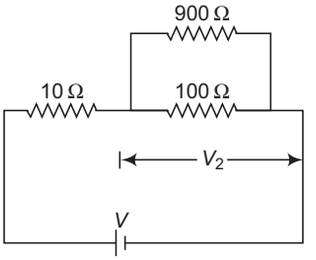
$\therefore$ $$r = \left(\frac{55+50}{50}\right) \times 10 = 1\Omega$$

118.



Before connecting the voltmeter let the potential difference across 100 Ω is V_1.

$\therefore$ $$V_1 = \left(\frac{100}{100+10}\right) \times V = \frac{10}{11} V$$



After connecting the voltmeter across 100Ω.

Equivalent resistance,

$$\frac{100 \times 900}{100 + 900} = 90\,\Omega$$

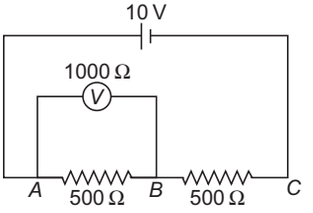
Let this time potential difference is V_2 .

$\therefore$ $$V_2 = \left(\frac{90}{90+10}\right) V = \frac{9}{10} V$$

Magnitude of % error $= \dfrac{\frac{10}{11}V - \frac{9}{10}V}{\frac{10}{11}V} \times 100$

$$= 1.0$$

119.



Equivalent resistance of circuit,

$$R_{eq} = 500 + \frac{1000}{3}$$

$$= \frac{2500}{3}$$

$\therefore$ Current drawn from the cell,

$$i = \frac{10}{2500/3} = \frac{3}{250}\,\text{A}$$

Reading of voltmeter $= \dfrac{3}{250} \times \dfrac{1000}{3}$

$$= 4\,\text{V}$$

120. Emf of cell,

$$E = Xl = \frac{V}{l} = \frac{iR}{L} \times l$$

$$\therefore \quad E = \frac{e}{(R_1 + R_2 + r)} \times \frac{R}{L} \times l$$

$$0.4 = \frac{5}{(5 + 45 + 0)} \times \frac{5}{10} \times l$$

$$\therefore \quad l = 8\,\text{m}$$

121. New deflection $\propto$ current passing through the galvanometer.

122. $\frac{R_3}{R_4} = \frac{R_1}{R_2}$ (balanced Wheatstone bridge)

or $R_1 R_4 = R_2 R_3$

123. In series current distributes in direct ratio of resistance.

$$\frac{5}{105} = \frac{20}{R}$$

$$\therefore \quad R = 420\,\text{k}\Omega$$

125. $100 = (10 \times 10^{-3})(25 + R)$

$\therefore \quad R = 9975\Omega$

127. When some resistance is connected in parallel, the effective resistance get decreased.

131. If voltmeter is ideal, then R should be $\frac{1.6}{0.4} = 4\Omega$. If it is non-ideal R should be greater than 4Ω.

132.

$$\frac{90}{10} = \frac{99}{S}$$

$$\therefore \quad S = 11\Omega$$

133. (a) $50 \neq (50 \times 10^{-6})(100 + 10^4)$. Therefore, (a) is wrong.

(b) $10 \approx (50 \times 10^{-6})(100 + 200 \times 10^3)$. Therefore, (b) is correct.

(c) $\frac{50 \times 10^{-6}}{10 \times 10^{-3} - 50 \times 10^{-6}} \neq \frac{1}{100}$. Therefore, (c) is wrong.

134. Percentage $= \frac{\left(\frac{4}{40}\right) i}{i} \times 100 = 10\%$

135. $(i_g R_v)(100) = i_g (R_v + 1980)$

$\therefore \quad R = 20\Omega$

137. $(2 \times 10^{-3})(50) = 10 \times i$

$$i = 10 \times 10^{-3}\,\text{A} = 10\,\text{mA}$$

140. $E = iR_{AC} = \left(\frac{2}{10 + 40}\right)(4) = 0.16\,\text{V}$

141. For twisted wire, there are two halves each of resistance 2Ω in parallel.

So, $R_{\text{eq}} = \frac{2}{2} = 1\Omega$

142. When wire is divided is 10 equal parts, then each part will have a resistance $= R/10 = r$

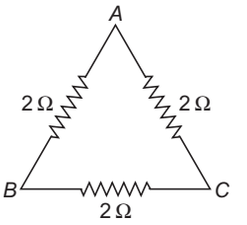
Let equivalent resistance be r_R, then

$$\frac{1}{r_R} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \ldots 10\,\text{times}$$

$$\therefore \quad \frac{1}{r_R} = \frac{10}{r} = \frac{10}{(R/10)} = \frac{100}{R}$$

$$\therefore \quad r_R = \frac{R}{100} = 0.01\,R$$

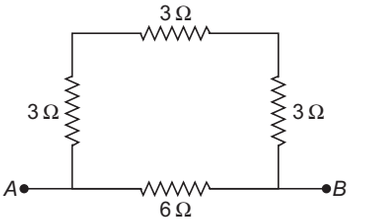
143. Equivalent resistance between B and C.



$$R_{BC} = \frac{(2 + 2) \times 2}{2 + 2 + 2}$$

$$= \frac{8}{6} = \frac{4}{3}\,\Omega$$

144. The given circuit can be reduced to



So, equivalent resistance between A and B.

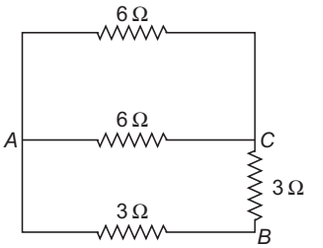
$$R_{AB} = \frac{9 \times 6}{9 + 6}$$

$$= \frac{9 \times 6}{15}$$

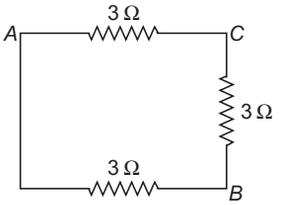
$$= \frac{18}{5}$$

$$R_{AB} = 3.6\Omega$$

145. Given circuit can be reduced to



So, equivalent resistance between points A and B is equal to



$$R_{\text{eq}} = \frac{6 \times 3}{6 + 3} = 2\Omega$$

146. $\frac{R_1 R_2}{R_1 + R_2} = \frac{6}{8}$ …(i)

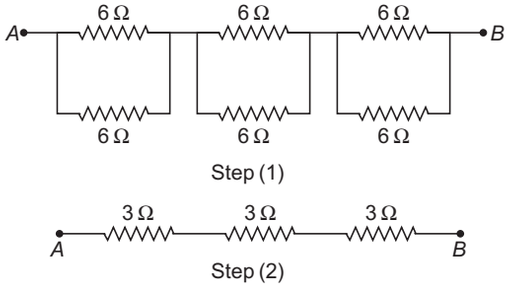
When one resistance in broken (say R_2) is broken, then

$$R_1 = 2\Omega \qquad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$R_2 = \frac{6}{5}\,\Omega$$

147. The network can be redrawn as follows.

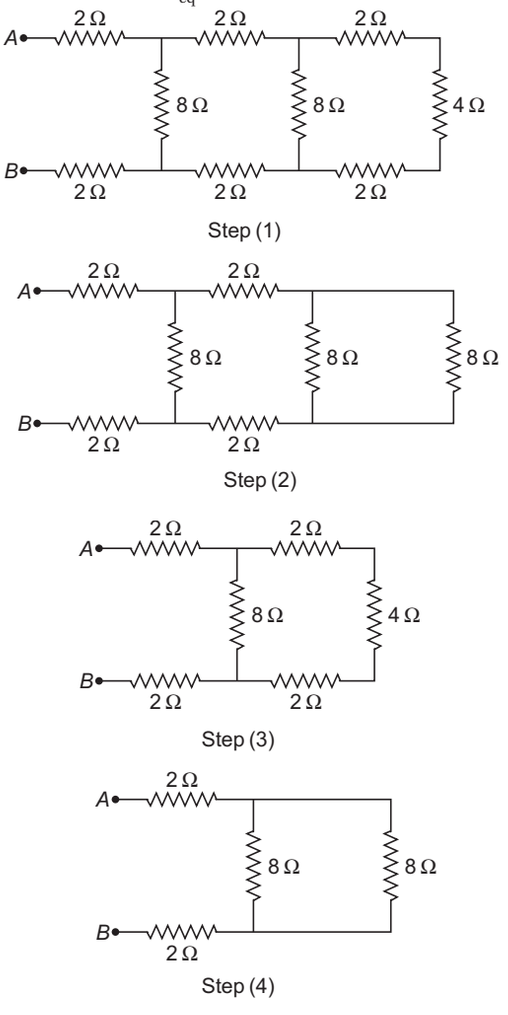


Now, there are three resistances in series.

So, equivalent resistance

$$R_{eq} = 3 + 3 + 3 = 9\,\Omega$$

148.



Now, equivalent resistance between A and B is

$$R_{eq} = 2 + 4 + 2 = 8\,\Omega$$

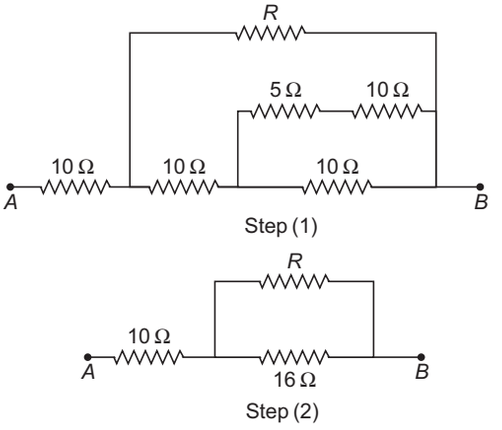
149. The minimum resistance can be achieved, when we connect all resistances in parallel.

So, equivalent resistance of combination $= \frac{r}{10}$.

150. When wire is bent in the form of a circle, then it is equivalent to two resistance in parallel

$$R_{eq} = \frac{6}{2} = 3\,\Omega$$

151. Circuit can be reduced as follows.



Resistances $16\,\Omega$ and $R\,\Omega$ are in parallel.

$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{16} = \frac{16 + R}{16R}$$

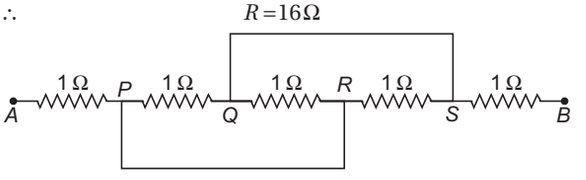
$$\therefore \quad R_p = \frac{16R}{16 + R}$$

Now, resistances R_p and $10\,\Omega$ are in series and their equivalent resistance is $18\,\Omega$ (given).

$$\therefore \quad 18 = R_p = 10 \quad \text{or} \quad 18 = \frac{16R}{16 + R} + 10$$

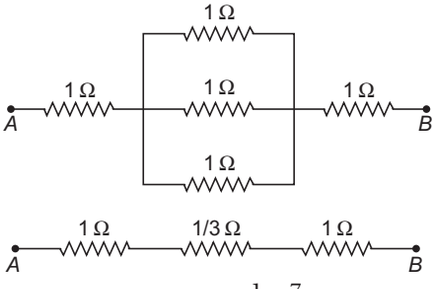
$$\therefore \quad R = 16\,\Omega$$

152.



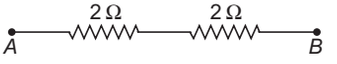
In circuit, resistance between PQ, QR and RS are in parallel.

Now, circuit reduces to



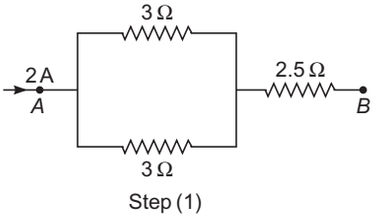
$$\therefore \quad R_{AB} = 1 + 1 + \frac{1}{3} = \frac{7}{3}\,\Omega$$

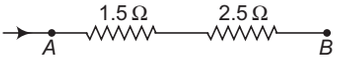
153. In circuit (C), the circuit is reduced to



Equivalent resistance between A and B is $4\,\Omega$.

154. Circuit can be reduced to





Step (2)

∴ Equivalent resistance between A and B,

$$R_{eq} = 1.5 + 2.5 = 4\ \Omega$$

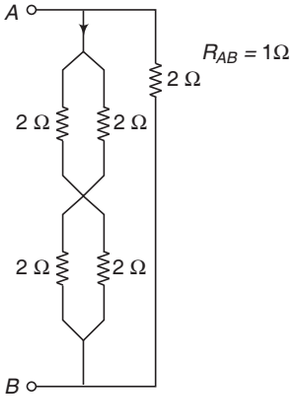
Potential difference between A and B,

$$V_A - V_B = iR = 2 \times 4.0 = 8\ \text{V}$$

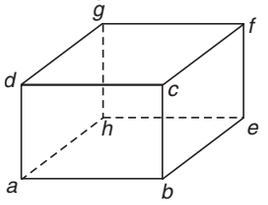
155. Three resistances of 3Ω each in the parallel.

157. All three resistances are in parallel.

159. The simple circuit is as shown below,



160. Although this question is not going to be asked in any competitive examination, but students can directly remember the following results.



$$R_{ab} = \frac{7r}{12},\ R_{ac} = \frac{9r}{12} \text{ or } \frac{3r}{4}$$

and $$R_{af} = \frac{10r}{12} \quad \text{or} \quad \frac{5r}{6}$$

161. $\dfrac{2R}{2+R} = \dfrac{6}{5} \quad \Rightarrow \quad \therefore \quad R = 3\Omega$

162. The given circuit forms a balanced Wheatstone bridge between points A and B.

163. Resistance of an ammeter is low.

164. $R \propto l$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{1}{2}$$

$$\therefore \quad R_1 = \frac{1}{3} \times 9 = 3\Omega$$

$$R_2 = \frac{2}{3} \times 9 = 6\Omega$$

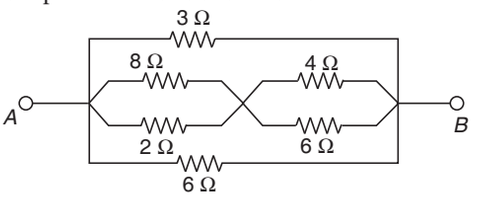
$$R_{net} = \frac{R_1 R_2}{R_1 + R_2} = 2\Omega$$

165. $R = 3 + \dfrac{10(3+R)}{13+R}$

Solving this equation, we get

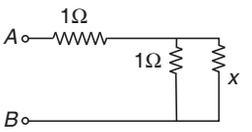
$$R = \sqrt{69}\ \Omega$$

166. The simple circuit is as shown below.



167. The given circuit consists of a balanced Wheatstone bridge.

168. Let $R_{AB} = x$. Then,



$$R_{AB} = 1 + \frac{x}{1+x}$$

or $$x = 1 + \frac{x}{1+x}$$

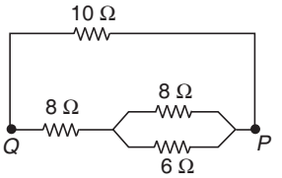
$$\therefore \quad x + x^2 = 1 + x + x$$

or $$x^2 - x - 1 = 0$$

$$x = \frac{1 + \sqrt{1+4}}{2}$$

$$= \frac{1+\sqrt{5}}{2}$$

170. The simple circuit is as shown below.



171. When current is passed through nerve of a man, he is excited, because his body is very sensitive to small current.

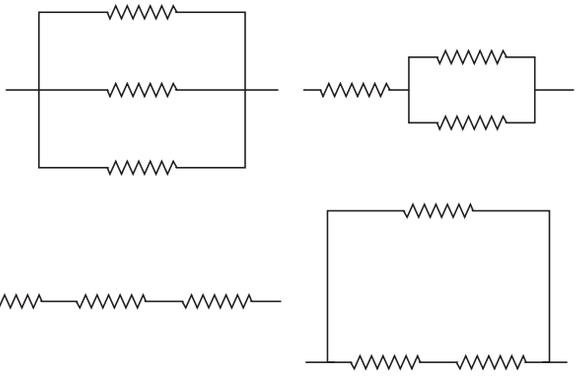
172. $R = \dfrac{V}{i} = \dfrac{100}{10} = 10\Omega$

$$\Delta R = \left(\frac{\Delta V}{V} + \frac{\Delta i}{i}\right) R = \left(\frac{0.5}{100} + \frac{0.2}{10}\right) \times 10$$

$$= 0.25\ \Omega$$

173. Although current is denoted by arrow but it is a scalar quantity and power is also scalar quantity, while current density (J) is a vector quantity.

174. There are four combinations of three resistors, they are as follows.



175. In parallel current distributes in inverse ratio of resistance.

176. Resistance, $R = \rho \cdot \frac{l}{A}$

For same material and same length,

$$\frac{R_2}{R_1} = \frac{A_1}{A_2} = \frac{3}{1}$$

$$\therefore \quad R_2 = 3R_1$$

Resistance of thick wire, $R_1 = 10\,\Omega$ (given)

$\therefore$ Resistance of thin wire.

$$R_2 = 3 \times 10 = 30\,\Omega$$

Total resistance of series combination $= 10 + 30 = 40\,\Omega$

177. Given, $\frac{R_1}{R_2} = \frac{1}{2}$

Let, third resistance is R.

So, $$\frac{1}{a} + \frac{1}{2a} + \frac{1}{R} = 1$$

$$\therefore \quad a = \frac{3R}{2(R-1)}$$

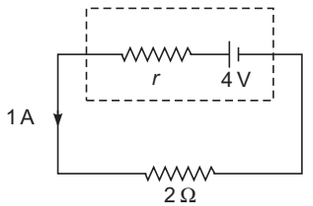
As resistance is not fractional (given),

$$\therefore \quad \frac{R}{R-1} = 2$$

So, $$R = 2$$

$$\therefore \quad a = 3 \text{ and } 2a = 6$$

178.



From, figure

$$i = \frac{E}{R + r}$$

$$\therefore \quad 1 = \frac{4}{2 + r}$$

$$\therefore \quad r = 2\Omega$$

When terminals are connected directly there is short circuit and current flows during short circuit,

$$i_{SC} = \frac{E}{r} = \frac{4}{2} = 2\text{A}$$

180. During and discharging heat generated is same, because the capacitor has the same stored energy initially.

$$= \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times (200 \times 10^{-6}) \times (200)^2$$

$$= 4\text{J}$$

181. $H = i^2\, Rt$

$$\therefore \quad R = \frac{H}{i^2 t} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{A}^2\text{T}]}$$

$$= [\text{ML}^2\text{T}^{-3}\text{A}^{-2}]$$

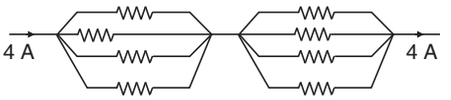
183. Rest all three are units of energy.

184. $i = \frac{dQ}{dt} = a - 2bt$

$$i = 0 \text{ at } t = \frac{a}{2b}$$

$$\frac{di}{dt} = -2b$$

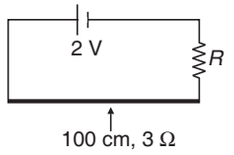
185.



186. $q = \int_0^5 i\,dt = \int_0^5 (1.2t + 3)\,dt = 30\,\text{C}$

187. Total potential drop across the given wire $= (1 \times 10^{-3})(10^2) = 0.1\,\text{V}$.

Therefore, potential difference across R should be 1.9 V



Now, $$\frac{1.9}{0.1} = \frac{R}{3}$$

or $$R = 57\,\Omega$$

188. $r_A = 2r_B$

$$\therefore \quad A_A = 4A_B$$

or $$l_A = \frac{l_B}{4}$$

$$\therefore \quad R_B = 16R_A$$

$$R_{\text{net}} = \left(\frac{16}{17}\right) R_A = \frac{R_B}{17}$$

189. $\frac{24 + x}{84 + y} = \frac{10}{30} = \frac{1}{3}$ …(i)

$$\frac{3}{1} = \frac{24 + 84 + x + y}{10 + 30}$$

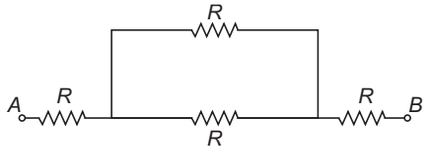
$$= \frac{108 + x + y}{40} \quad \text{…(ii)}$$

Solving this equation, we get

$$x = y = 6\Omega$$

Level 2 : Only One Correct Option

1. Topmost and bottommost figures are short circuited. Simple circuit is shown below.



2. Let resistance of 400 W is R. Then, the resistance of 200 W will be $2R$.

From $$P = \frac{V^2}{R}$$

$$R = \frac{V^2}{400}$$

Total resistance of the shown circuit will be $2R$.

$$\therefore \quad P_{\text{net}} = \frac{V^2}{R_{\text{net}}} = \frac{V^2}{2R}$$

$$= \frac{V^2}{(V^2/200)}$$

$$= 200\,\text{W}$$

3. $\dfrac{V_{AB}}{V_{BC}} = \dfrac{R_{AB}}{R_{BC}}$

$$\therefore \quad \frac{(200/3)}{(100/3)} = \frac{50\,\text{k}\Omega}{R_{BC}}$$

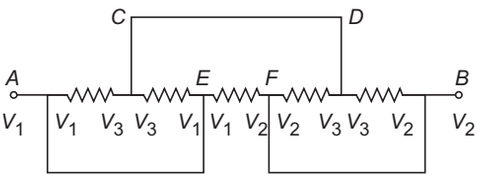
$$\therefore \quad R_{BC} = 25\,\text{k}\Omega$$

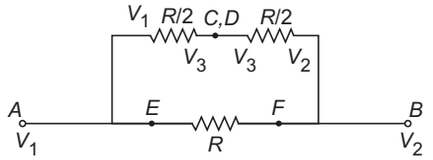
4. $\dfrac{10}{2.5} = \dfrac{11}{l}$

$$\therefore \quad l = 2.75\,\text{m}$$

5. $R_{AB} = \dfrac{R}{2}$

$$I_{CD} = \frac{V_0}{R}, I_{EF} = \frac{V_0}{R}$$





6. $E_{\text{eq}} = \dfrac{E_1/r_1 + E_2/r_2}{1/r_1 + 1/r_2} = \dfrac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = E_1\left[\dfrac{r_2 + r_1(E_2/E_1)}{r_1 + r_2}\right]$

Therefore, E_{eq} can be greater than, less than or equal to E_1 depending upon the condition whether $E_2 > E_1$, $E_2 = E_1$ or $E_2 < E_1$.

7. $R = \left(20 + \dfrac{i}{2}\right)$

Now, $$i = \frac{250}{R} = \frac{250}{20 + (i/2)}$$

Solving, we get

$$i = 10\,\text{A}$$

8. $$v_d = \frac{i}{\eta e A}$$

$$\eta = \left(\frac{5\times10^3}{60}\right)\times 6.02\times10^{26}$$

$$\therefore \quad v_d = \frac{16\times 60}{5\times10^3\times6.02\times10^{26}\times1.6\times10^{-19}\times10^{-6}}$$

$$= 2\times10^{-3}\ \text{m/s}$$

9. $I_\Omega = 0$

$$\therefore \quad V_X = E_2 = 2\,\text{V}$$

$$\frac{V_{500\,\Omega}}{V_X} = \frac{(12-2)}{2} = \frac{500}{X}$$

$$\therefore \quad X = 100\,\Omega$$

10. $\dfrac{R_1}{R_2} = \dfrac{L/A}{2L/2A} = \dfrac{1}{1}$

$\therefore$ PD will be same.

$$\frac{E_1}{E_2} = \frac{V/L}{V/2L}$$

$$= \frac{2}{1}$$

Further, $$v_d = \frac{i}{neA}$$

or $$v_d \propto \frac{1}{A}$$

$$\frac{(v_d)_1}{(v_d)_2} = \frac{A_2}{A_1}$$

$$= \frac{2}{1}$$

11. No current will flow through the grounded wire.

12. Due to balanced Wheatstone bridge resistance between A and B can be removed.

$$I_{DE} = \frac{10\,\text{V}}{R_{DE} + R_{HG}}$$

$$= \frac{10}{2+2}$$

$$= 2.5\,\text{A}$$

13. Batteries are in parallel.

Net emf = 4 V

Total internal resistance = 0.1 Ω

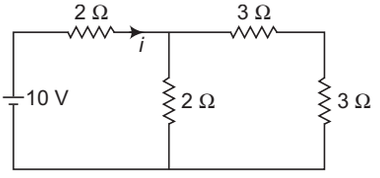
Total external resistance = 5 Ω

$$\therefore \quad i_1 = \frac{4}{5+0.1} = 0.784\,\text{A}$$

By symmetry,

$$i_2 = i_3 = \frac{i_1}{2} = 0.392\,\text{A}$$

14. $R_{\text{net}} = 2 + \dfrac{6\times2}{6+2} = \dfrac{7}{2}\,\Omega$



$$\therefore \quad i = \frac{10}{(7/2)} = \frac{20}{7}\,\text{A}$$

$$i_{3\,\Omega} = \left(\frac{2}{2+6}\right)i$$

$$= \frac{1}{4}\cdot i = \frac{5}{7}\,\text{A}$$

15. $$i_1 = \frac{E_1 + E_2}{r_1 + r_2 + R}$$

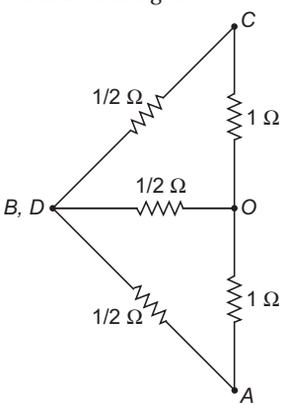
$$i_2 = \frac{E_1}{R + r_1}$$

$$i_2 > i_1$$

$$\therefore \quad \frac{E_1}{R+r_1} > \frac{E_1 + E_2}{r_1 + r_2 + R}$$

or $$E_1 r_2 > E_2(R + r_1)$$

16. B and D are symmetrically located with respect to points O and A both respect to points O and A both. Hence, the figure can be folded as shown in figure.



17. $R_{T_1} = R_{T_2}\,[1 + \alpha\,(T_1 - T_2)]$

$$\therefore \qquad T_1 - T_2 \propto \frac{R_{T_1} - R_{T_2}}{R_{T_2}}$$

$$T_1 - T_2 \propto \frac{\tan\alpha - \tan\frac{\alpha}{2}}{\tan\frac{\alpha}{2}}$$

$$T_1 - T_2 \propto \frac{\left(\frac{2\tan\alpha/2}{1-\tan^2\frac{\alpha}{2}}\right) - \tan\frac{\alpha}{2}}{\tan\frac{\alpha}{2}}$$

$$T_1 - T_2 \propto \sec^2\left(\frac{\alpha}{2}\right)$$

18. $V_{\text{actual}} = \left(\frac{100}{110}\right)E = V_1 = \frac{10}{11}E$

Equivalent resistance of 100 Ω and 900 Ω is 90 Ω.

$$\therefore \qquad V_{\text{measured}} = \left(\frac{90}{90+10}\right)E = \frac{9}{10}E = V_2$$

$$\%\ \text{error} = \frac{V_1 - V_2}{V_1} \times 100$$

$$= \frac{\left(\frac{10}{11}E\right) - \left(\frac{9}{10}\right)E}{\left(\frac{10}{11}\right)E} = 1\%$$

19. When K_1 is closed, R_1 is short circuited.

When K_2 is open,

$$i_0 = \frac{E}{r + R_2} = \frac{E}{r + 100} \qquad \ldots(\text{i})$$

When K_2 is open,

$$i_0 = \frac{1}{2}\left[\frac{E}{r + 50}\right] \qquad \ldots(\text{ii})$$

From these two equations, we get $r = 0$

When K_1 is open and K_2 is closed,

$$\frac{l_0}{2} = \frac{E}{2(R_1 + 50)} \qquad \ldots(\text{iii})$$

From Eqs. (i) and (iii), we have $R_1 = 50\,\Omega$

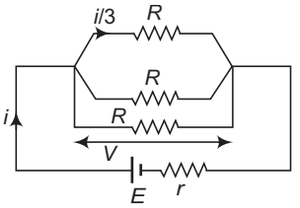
20. In the circuit we can see that 20 Ω, 100 Ω and 25 Ω are in parallel.

Net resistance of circuit

$$= 4 + 6 + \frac{1}{\frac{1}{20} + \frac{1}{100} + \frac{1}{25}} = 20\,\Omega$$

$$\therefore \qquad V = iR = 80\,\text{V}$$

21. The given three resistors are in parallel.



$$\therefore \qquad i = \frac{E}{r + R/3} = \frac{4}{1+3} = 1\,\text{A}$$

$$V = \frac{i}{3}R \quad \text{or} \quad E - ir = \left(\frac{1}{3}\right)(9) = 3\,\text{V}$$

22. In series potential difference ∝ resistance.

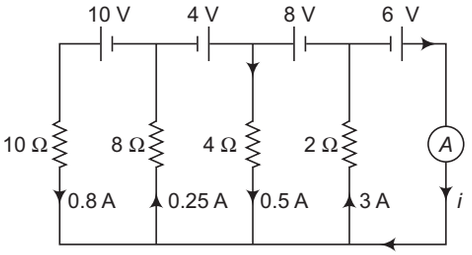
$$\frac{R_A}{R_B} = \frac{3}{2} \quad \text{or} \quad \frac{i_A}{i_B} \times \left(\frac{r_B}{r_A}\right)^2 = \frac{3}{2} \quad \text{or} \quad \frac{6}{1} \times \left(\frac{r_B}{r_A}\right)^2 = \frac{3}{2}$$

$$\therefore \qquad \frac{r_B}{r_A} = \frac{1}{2}$$

23. Balanced Wheatstone bridge.

$$\therefore \qquad V_{2\,\Omega} = 0$$

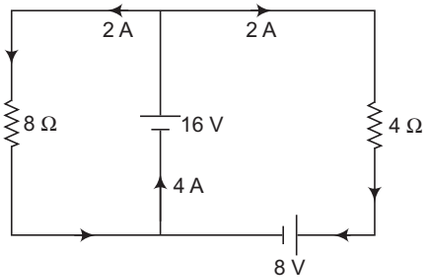
24. By finding potential difference across any resistance we can find current through each resistance directly.



$$\therefore \qquad i = 3 + 0.25 - 0.5 - 0.8 = 1.95\,\text{A}$$

25. $i_1 = 4\,\text{A}$

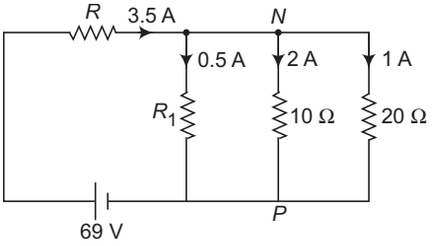
Further dividing 2 A across different resistors, we can find that



$$i_2 = \frac{2}{4} = \frac{1}{2}\,\text{A}$$

Hence, $$\frac{i_1}{i_2} = \frac{8}{1}$$

26. In parallel current distributes in inverse ratio of resistance.



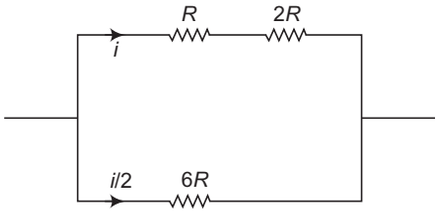
$$R_1 = 40\,\Omega$$

Further, $V_{NP} = 20\,\text{V}$

$\therefore$ $V_R = 69 - 20 = 49\,\text{V}$

$$R = \frac{49}{3.5} = 14\,\Omega$$

27. $P_R = i^2R, P_{2R} = i^2(2R), \quad P_{6R} = \left(\frac{i}{2}\right)^2 (6R)$

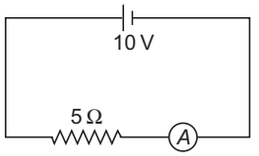


$\therefore$ $P_R : P_{2R} : P_{6R} = 1 : 2 : 1.5 = 2 : 4 : 3$

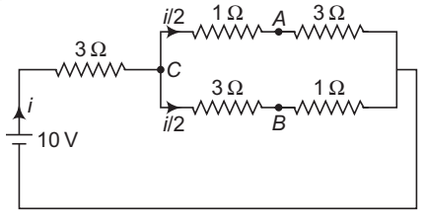
28. More current will flow through $2R$ while current through $6R$ will remain unchanged.

29. When key K is opened, then no current will flow across 4Ω resistance. Now, circuit can be reduced as follows.

$$i = \frac{10}{5} = 2\,\text{A}$$



30. Resistance between upper branch and lower branch in parallel part is same, so, equal amount of current flows through them. Let main current is i.



$\therefore$ $$i = \frac{V}{R_{\text{eq}}}$$

Equivalent resistance of circuit,

$$R_{\text{eq}} = 3 + 2 = 5\,\Omega \quad \Rightarrow \quad \therefore \quad i = \frac{10}{5} = 2\text{A}$$

So, current in each branch $= 1\text{A}$

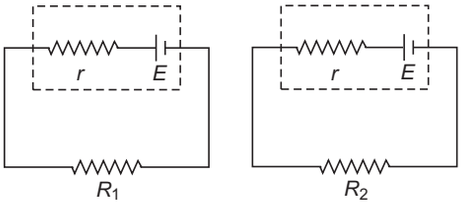
Now, $V_C - V_A = 1 \times 1 = 1\,\text{V}$ …(i)

Also $V_C - V_B = 1 \times 3 = 3\text{V}$ …(ii)

Solving Eqs. (i) and (ii), we have

$$V_A - V_B = 3 - 1 = 2\text{V}$$

31.



Power delivered in both cases is equal, then

$$i_1^2 R_1 = i_2^2 R_2$$

$\therefore$ $$\left(\frac{E}{R_1 + r}\right)^2 R_1 = \left(\frac{E}{R_2 + r}\right)^2 R_2$$

or $R_2^2 R_1 + R_1 r^2 + 2R_2 r = R_1^2 R_2 + R_2 r^2 + 2R_1 R_2 r$

or $(R_1 - R_2)\, r^2 = (R_1 - R_2)\, R_1 R_2$

$\therefore$ $r^2 = R_1 R_2 \Rightarrow r = \sqrt{R_1 R_2}$

32. Suppose resistance of ammeter is R', then equivalent resistance from ammeter side

$$R_{\text{eq}} = R' + R$$

$\therefore$ Current, $4 = \frac{20}{R' + R}$

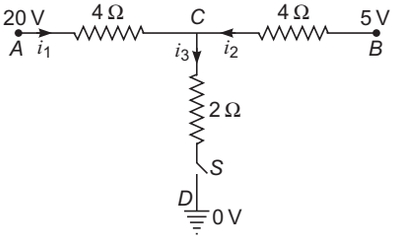
$\therefore$ $R + R' = 5$

$\therefore$ $R = 5 - R'$

So, R is less than 5Ω.

33. Let at junction C, potential is V.

Applying Kirchhoff's junction law at C,



$$i_1 + i_2 = i_3$$

or $$\frac{V_A - V_C}{2} + \frac{V_B - V_C}{4} = \frac{V_C - V_D}{2}$$

or $$\frac{20 - V}{2} + \frac{5 - V}{4} = \frac{V - 0}{2}$$

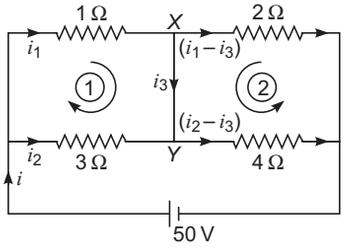
or $5V = 45$

$\therefore$ $V = 9\,\text{V}$

$\therefore$ Current through switch S,

$$i_3 = \frac{9}{2} = 4.5\,\text{A}$$

34.



Let current through XY is i_3.

Applying Kirchhoff's law to loops (1) and (2),

$$i_1 + 0 \times i_3 - 3i_2 = 0$$

$\therefore \quad i_1 = 3i_2 \quad \ldots(i)$

and $-2(i_1 - i_3) + 4(i_2 + i_3) = 0$

So, $2i - 4i_2 = 6i_3 \quad \ldots(ii)$

Also, $50 = 1i_1 - 2(i_1 - i_3)$

$50 = i_1 - 2(i_1 - i_3)$

$\therefore \quad 3i_1 - 2i_3 = 50 \quad \ldots(iii)$

From Eqs. (i), (ii) and (iii), we get

$i_3 = 2\text{A}$

35. Current through R_1 and R_2 comes out to be zero (potential difference = 0)

$\therefore$ Current through, $R_3 = \dfrac{\text{Net emf}}{\text{Total resistance}}$

$= \dfrac{(4+3+2)-(2+3+2)}{3} = \dfrac{2}{3}\text{A}$

36. The equivalent emf of this combination is given by

$$\varepsilon_{eq} = \frac{\varepsilon_2 r_1 + \varepsilon_1 r_2}{r_1 + r_2}$$

This suggest that the equivalent emf ε_{eq} of the two cells is given by

$$\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$$

37. The percentage error in R can be minimised by adjusting the balance point near the middle of the bridge, i.e. when l_1 is close to 50 cm. This requires a suitable choice of S.

Since, $\dfrac{R}{S} = \dfrac{R\,l_1}{R(100 - l_1)} = \dfrac{l_1}{100 - l_1}$

Since here, $R : S :: 2.9 : 97.1$ imply that the S is nearly 33 times to that of R. In orded to make this ratio 1:1, it is necessary to reduce the value of S nearly $\dfrac{1}{33}$ times i.e. nearly 3Ω.

38. In a potentiometer experiment, the emf of a cell can be measured, if the potential drop along the potentiometer wire is more than the emf of the cell to be determined. Here, values of emfs of two cells are given as 5V and 10V, therefore, the potential drop along the potentiometer wire must be more than 10V.

More than One Correct Options

1. $H = \dfrac{V^2}{R_1} t_1 \quad \Rightarrow \quad \therefore \quad R_1 = \dfrac{V^2 t_1}{H}$

Similarly, $R_2 = \dfrac{V^2 t_2}{H}$

In series, $H = \left(\dfrac{V^2}{R_1 + R_2}\right) t$

$t = \dfrac{H(R_1 + R_2)}{V^2}$

Substituting the values of R_1 and R_2, we get

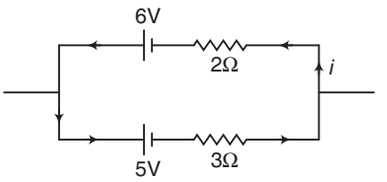
$\therefore \quad t = t_1 + t_2$

In parallel, $H = \dfrac{V^2}{R_{net}} t = V^2 t \left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$

$= V^2 t \left(\dfrac{H}{V^2 t_1} + \dfrac{H}{V^2 t_2}\right)$

Solving we get, $t = \dfrac{t_1 t_2}{t_1 + t_2}$

2.



$i = \dfrac{6-5}{2+3} = 0.2\text{ A}$

$V_1 = E_1 - ir_1 = 6 - 0.2 \times 2$

$= 5.6\text{ V}$

3. ***(a)*** In series current is same.

$\therefore \quad I_A = I_B$

(b) $V_A + V_B = V_C \quad \Rightarrow \quad \therefore \quad I_A R_A + I_B R_B = I_C R_C$

(d) In parallel current distributes in inverse ratio of resistance.

$\therefore \quad \dfrac{I_B}{I_C} = \dfrac{I_A}{I_C} = \dfrac{R_C}{R_A + R_B}$

4. Same as above.

5. ***(a)*** $V = iR$

In series i is same. Hence, V is also same, as R is given same.

(b) $R = \dfrac{\rho l}{A}$

R is same. Hence, A should be smaller in first wire. Secondly, $v_d = \dfrac{i}{ne\,A}$ or $v_d \propto \dfrac{1}{A}$

A of first wire is less. Hence, its drift velocity should be more.

(c) $E = \dfrac{V}{l}$ or $E \propto \dfrac{1}{l}$ ($V \to$ same)

6. Not required.

7. If switch S is open, $i_1 \lambda l = E_2$

where, i_1 = current in upper circuit and λ is resistance per unit length of potentiometer wire.

$\therefore$ Null point length, $l = \dfrac{E_2}{i_1 \lambda}$.

(a) If jockey is shifted towards right, resistance in upper circuit will increase. So, current i_1 will decrease. Hence, l will increase.

(b) If E_1 is increased, i_1 will also increase. So, l will decrease.

(c) $l \propto E_2$

(d) If switch is closed, then null point will be obtained corresponding to

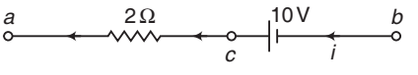
$V_2 = E_2 - i_2 r_2$

which is less than E_2. Hence, null point length will decrease.

8. By closing S_1, net external resistance will decrease. So, main current will increase.

By closing S_2, net emf will remain unchanged but net internal resistance will decrease. Hence, main current will increase.

9.



$V_b + 10 - 2i = V_a$

$V_b - V_a = 2i - 10 = 2\text{ V}$

$\therefore \quad i = 6\text{A}$

Now, $V_c - V_a = 2 \times 6 = 12\text{ V}$

10. Between a and c, balanced Wheatstone bridge is formed. Across all other points simple series and parallel grouping of resistors.

11. Kirchhoff's junction rule is also known as Kirchhoff's current law which states that the algebraic sum of the currents flowing towards any point in an electric network is zero. *i.e.,* charges are conserved in an electric network.

So, Kirchhoff's junction rule is the reflection of conservation of charge

12. The resistivity of a metallic conductor is given by,

$$e = \frac{m}{ne^2\tau}$$

where, n is number of charge carriers per unit volume which can change with temperature T and τ is time interval between two successive collisions which decreases with the increase of temperature.

13. At neutral point, potential at B and neutral point are same. When jockey is placed at to the right of D, the potential drop across AD is more than potential drop across AB, which brings the potential of point D less than that of B, hence current flows from B to D.

Comprehension Based Questions

Sol. 1 and **2.** $$r = R\left(\frac{l_1}{l_2} - 1\right)$$

$$\therefore \qquad 10 = R\left(\frac{500}{490} - 1\right)$$

Solving this equation, we get

$$R = 490\ \Omega$$

Further $$r = R\left(\frac{E}{V} - 1\right)$$

or $$10 = 490\left(\frac{2}{V} - 1\right)$$

Solving, we get $V = 1.96\ \text{V}$

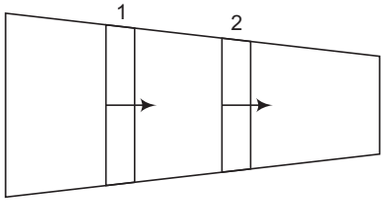
Assertion and Reason

1. $R = \frac{V}{I} \Rightarrow \therefore \quad R_2 > R_1$

Resistance of a conductor increases with increase in temperature. Hence,

$$T_2 > T_1.$$

2. R_1 and R_2 are resistances per unit length.



$$i_1 = i_2$$

$$V_1 = i_1 R_1$$

and $$V_2 = i_2 R_2$$

$$R_2 > R_1 \qquad \left(\text{as } R \propto \frac{1}{A}\right)$$

Hence, $V_2 > V_1$

3. $i\lambda l = E_U$

$$\therefore \qquad l = \frac{E_U}{i\lambda}$$

E_U and i both will become two times. Therefore, l will remain unchanged.

5. During charging of battery,

$$V = E + iR$$

$$\therefore \qquad V > E$$

7. In both cases potential difference across R is E.

$$\therefore \qquad P = \frac{E^2}{R}$$

In second case net resistance will decrease. Therefore, main current will increase.

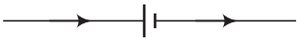
9. $R = \rho\frac{l}{A}$ or $R \propto \frac{1}{A}$

Area of cross-section of first wire is less. Hence, its resistance is more.

10. Only drift motion is absent in the absence of potential difference across it. Further, there will be electrostatic force on electrons from other electrons, nucleus, etc.

11. v_d, R and $J \propto \frac{1}{A}$.

13. Current through a resistance wire flows from higher potential to lower potential.



Further, during charging of a battery current flows in the direction shown in above figure.

14. In case of ammeter more current should pass through the shunt. Therefore, shunt resistance should be less or overall resistance of ammeter should be less.

15. k-VA is the unit of power. While kW-h is the unit of energy.

16. Resistance of voltmeter is high. If it is connected in series, then current will decrease but circuit will not burn.

17. $\frac{V}{i}$ = constant = R for ohmic circuits is Ohm's law. Further, only for ohmic circuits V-i graph is a straight line passing through origin.

19. Actual power developed will depend on the actual voltage applied.

Match the Columns

1. With increase in parallel connections net resistance will decrease. Therefore, main current i_1 will increase.

$$V_{AB} = V_{CD} = E - i_1 r$$

With increase in i_1, V_{AB} or V_{CD} will decrease.

Further, $i_2 = \frac{V_{AB}}{R}$

V_{AB} is decreasing. Therefore i_2 will also decrease.

2. The simple circuit is shown in figure.

3. During discharging $V = E - ir$

During charging $V = E + ir$

When $i = 0, V = E$ and $V = 0$ if short circuited.

4. $P = \dfrac{V^2}{R}$ (R = resistance of one bulb)

Now apply, $P_{\text{actual}} = \dfrac{V^2}{R_{\text{net}}}$

Entrance Gallery

1. Let the value of resistance be R_1 and R_2 respectively.

When R_1 and R_2 resistances are in series,

So, $R_1 + R_2 = 6\Omega$ …(i)

(according to the question)

When R_1 and R_2 resistances are in parallel,

So, $\dfrac{R_1R_2}{R_1 + R_2} = \dfrac{4}{3}\Omega$ …(ii)

From the Eq. (i), we get

$$\frac{R_1R_2}{6} = \frac{4}{3} \Rightarrow R_1R_2 = 4 \times 2$$

$$R_1R_2 = 8\Omega \quad \text{…(iii)}$$

We know that,

$$R_1 - R_2 = \sqrt{(R_1 + R_2)^2 - 4R_1R_2} = \sqrt{36 - 4 \times 8}$$

$$R_1 - R_2 = \sqrt{4}$$

$$R_1 - R_2 = 2\Omega \quad \text{…(iv)}$$

From the Eqs. (i) and (iv), we get

$$R_1 = 4\Omega \text{ and } R_2 = 2\Omega$$

2. Given,

$$R_1 = 2\Omega, R_2 = 6\Omega$$

$$E = 2\text{V}, r = 0.5\Omega, I = ?$$

R_1, R_2 are in parallel series.

So, $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

$$\Rightarrow \frac{1}{R} = \frac{1}{2} + \frac{1}{6} \Rightarrow \frac{1}{R} = \frac{3+1}{6}$$

$$R = \frac{6}{4}\Omega = 1.5\ \Omega$$

Then, the current in the circuit

$$I = \frac{E}{r + R} = \frac{2}{0.5 + 1.5} = \frac{2}{2.0} = 1\text{ A}$$

(as internal resistance, r is in series with other resistant)

3. As, per given question,

First balancing length, $l_1 = 30$ cm

Second balancing length, $l_2 = 40$ cm

$$E_1 = 1.25\text{ V}$$

$$E_2 = ?$$

So, according to the principle of potentiometer

$$E_1 = Kl_1 \quad \text{…(i)}$$

$$E_2 = Kl_2 \quad \text{…(ii)}$$

$$\frac{E_1}{E_2} = \frac{Kl_1}{Kl_2} \Rightarrow \frac{1.25}{E_2} = \frac{30}{40}$$

$$\Rightarrow E_2 = \frac{1.25 \times 40}{30} \Rightarrow E_2 = \frac{5}{3}$$

$$E_2 = 1.666\text{ V} \simeq 1.67\text{ V}$$

4. By conversion of galvanometer into ammeter

$$S = \left(\frac{I_g}{I - I_g}\right)G$$

$\begin{pmatrix} I_g = \text{Current through galvanometer} \\ S = \text{Shunt resistance} \\ G = \text{Galvanometer resistance} \end{pmatrix}$

where, $I_g = 5 \times 10^{-3}$ A

$$I = 1\text{A}$$

$$G = 100\Omega$$

$$S = \left(\frac{5 \times 10^{-3}}{1 - 5 \times 10^{-3}}\right) \times 100 = \frac{5 \times 10^{-3} \times 100 \times 10^3}{(1000 - 5)}$$

$$= \frac{500}{995} = \frac{5}{9.95}\ \Omega$$

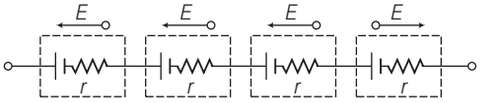
5. Resistance, $R = \dfrac{V}{i_g} - G = \dfrac{120}{0.01} - 10$

$$R = 12000 - 10$$

$$R = 11990\,\Omega$$

To convert a galvanometer into a voltmeter, high resistance should connect in series.

6. Total emf of the cell = $3E - E = 2E$



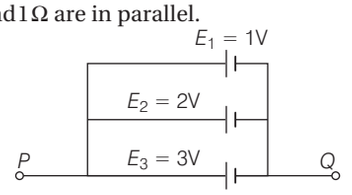
Total internal resistance = $4r$

$\therefore$ Total resistance of the circuit = $4r + R$

So, the current in the external circuit $\left(\because i = \dfrac{V}{R}\right)$

$\therefore$ $i = \dfrac{2E}{4r + R}$

7. $1\Omega, 2\Omega$ and 1Ω are in parallel.



So, the required internal resistance,

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \Rightarrow \frac{1}{r} = \frac{1}{1} + \frac{1}{2} + \frac{1}{1}$$

$$\frac{1}{r} = \frac{2+1+2}{2} \Rightarrow r = \frac{2}{5}\Omega$$

The potential difference between points P and Q

$$E_{\text{diff}} = \frac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2} + \dfrac{E_3}{r_3}}{1/r} = \frac{\dfrac{1}{1} + \dfrac{2}{2} + \dfrac{3}{1}}{5/2}$$

$$= \frac{\dfrac{2+2+6}{2}}{5/2} = \frac{10/2}{5/2} = \frac{5}{5} \times 2 = 2\text{V}$$

8. Temperature dependence of resistance is given as,

$$R_2 = R_1[1 + \alpha(T_2 - T_1)]$$

$$\frac{(R_2 - R_1)}{(T_2 - T_1)} \times \frac{1}{R_1} = \alpha$$

$$\frac{(2.5-1.6)}{(100-25.5)}\times\frac{1}{1.6}=\alpha$$

$$\frac{0.9}{74.5}\times\frac{1}{1.6}=\alpha$$

$$\Rightarrow \qquad \alpha = 7.55\times 10^{-3}\ ^\circ C^{-1}$$

9. For parallel circuit,

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} \quad\Rightarrow\quad \frac{1}{R_{eq}}=\frac{1}{1.2}+\frac{1}{2}+\frac{1}{3}$$

$$\frac{1}{R_{eq}}=\frac{6+3.6+2.4}{1.2\times 2\times 3} \quad\Rightarrow\quad R_{eq}=\frac{7.2}{12}=0.6\,\Omega$$

It is less than $1.2\,\Omega$.

10. For equating emf of two cells,

$$\frac{e_1}{e_2}=\frac{l_1}{l_2} \quad\Rightarrow\quad \frac{2.1}{E}=\frac{40}{56}$$

$$E=\frac{2.1\times 56}{40}=2.94\ V$$

11. Power dissipation, $P=Vi \quad\Rightarrow\quad i=\frac{P}{V}=\frac{600\times 10^3}{4000}=150\ A$

Total resistance of cable,

$$R=0.4\times 20=8\,\Omega$$

$\therefore$ Power loss in cable $=i^2R=(150)^2\ (8)=180000\ W=180\ kW$

This loss is 30% of 600 kW.

12. During step-up transformer, $\frac{N_p}{N_s}=\frac{V_p}{V_s}$

or $$\frac{1}{10}=\frac{4000}{V_s} \quad\text{or}\quad V_s=40000\ V$$

In step-down transformer,

$$\frac{N_p}{N_s}=\frac{V_p}{V_s}=\frac{40000}{200}=\frac{200}{1}$$

13. To increase the range of an ammeter we need to add more shunt accrues the ammeter, but in doing so the resistance of the ammeter decreases.

14. Resistance in upper arms, $A=10\,\Omega, B=5\,\Omega$

Resistance in lower arm, $C=4\,\Omega, D=4\,\Omega$

The condition required to form a Wheatstone bridge is given by

$$\frac{A}{B}=\frac{C}{D}$$

which does not satisfy this case.

If a $10\,\Omega$ resistance is connected in parallel with A [as per option (a).]

The relation for a new resistance is given by

$$\frac{1}{A'}=\frac{1}{10}+\frac{1}{10}=\frac{2}{10}=\frac{1}{5}$$

$$A'=5\,\Omega$$

Now, the network satisfies the condition of Wheatstone bridge

$$\frac{P'}{5}=\frac{R'}{5} \quad\text{or}\quad \frac{5}{5}=\frac{4}{4}$$

15. The given figure, represented balanced.

Wheatstone bridge since,

$$\frac{A}{B}=\frac{2\,\Omega}{3\,\Omega}=1.5\ \Omega \text{ and } \frac{C}{D}=\frac{6\,\Omega}{4\,\Omega}=1.5\ \Omega$$

Hence, the potential between R and S same, therefore resistance in the arm RS of $7\,\Omega$ resistance is in effective.

The resistance in the upper arm are connected in series combination.

Hence, the equivalent resistance of the upper arm is given by

$$R_V=2+3=5\,\Omega$$

Equivalent resistance in lower arm,

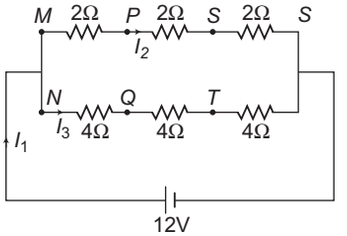
$$R_L=4+6=10\,\Omega$$

Equivalent resistance P and Q.

$$\frac{1}{R_{eq}}=\frac{1}{R_v}+\frac{1}{R_L}=\frac{1}{5}+\frac{1}{10}=\frac{2+1}{10}=3/10$$

$$\Rightarrow \qquad R_{eq}=\frac{10}{3}\,\Omega$$

16. Due to symmetry on upper side and lower side, points P and Q are at same potentials. Similarly, points S and T are at same potentials. Therefore, the simple circuit can be drawn as shown below :



$$I_2=\frac{12}{2+2+2}=2\,A$$

$$I_3=\frac{12}{4+4+4}=1\,A$$

$\therefore$ $$I_1=I_2+I_3=3\,A$$

$$I_{PQ}=0, \text{ because } V_P=V_Q$$

Potential drop (from left to right) across each resistance is $4\,\Omega$.

$$V_{MS}=2\times 4=8\,V$$

$$V_{NQ}=1\times 4=4\,V$$

or $$V_S<V_Q$$

17. Using the concept of balanced Wheatstone bridge, we have

$$\frac{P}{Q}=\frac{R}{S}$$

$$\frac{x}{(52+1)}=\frac{10}{(48+2)}$$

$$x=\frac{10\times 53}{50}=10.6\,\Omega$$

18. Potential gradient of a potentiometer,

$$K=\frac{I\rho}{A}=\frac{0.2\times 4\times 10^{-7}}{8\times 10^{-7}}$$

$$=0.1\ V/m$$

19. As, from potentiometer application measurement of emf of cell.

$$\varepsilon\propto l\Rightarrow\varepsilon=Rl \qquad \ldots(i)$$

***Case* I** When A, B and C cell are connected in series,

$$E_A+E_B+E_C=R(740) \qquad \ldots(ii)$$

***Case* II** When B and C cell are connected in series,

$$E_B+E_C=R(540)$$

***Case* III** When A and B cell are connected in series,

$$E_A+E_B=R(440) \qquad \ldots(iii)$$

From Eqs. (i), (ii) and (iii)

$$E_A=200k$$

$\Rightarrow \quad E_B = 240k$

$\Rightarrow \quad E_C = 300k$

i.e. $E_A : E_B : E_C = 1:1\cdot 2:1\cdot 5$

Hence, option (a) is correct.

20. Let the resistance of galvanometer be R_G.

As, deflection of moving coil galvanometer is directly proportional to the current through the coil.

$$\phi \propto I_g \qquad \ldots(i)$$

As, deflections is reduced to half after shunting of 40 Ω resistance with the galvanometer.

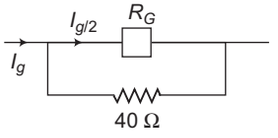
$$\phi' = \phi / 2.$$

$\Rightarrow \quad \phi' \propto I_g' \qquad \ldots(ii)$

From Eq. (i) and Eq. (ii), we get

$$\frac{\phi'}{\phi} = \frac{I_g'}{I_g} = \frac{\phi/2}{\phi} = \frac{1}{2}$$

$\Rightarrow \quad I_g' = I_g/2$

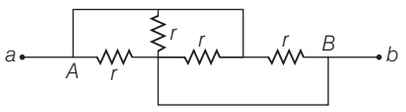


From current dividing role, current through galvanometer is,

$$\frac{I_g}{2} = \frac{40}{R_g + 40} \times I_g$$

$$R_g = 40\,\Omega$$

21.



Four resistances are in parallel,

$$\frac{1}{R} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r}$$

$$R = \frac{r}{4}$$

22. We know that,

$$R = \frac{V^2}{P}$$

Given, $V = 200\,\text{V}, P = 40\,\text{W}$

and $V' = 100\,\text{V}$

Hence, $R = \frac{V^2}{P} = \frac{200\times 200}{50}$

Power of the bulb,

$$P' = \frac{V'^2}{R} = \frac{100\times 100\times 50}{200\times 200}$$

$$= 12.5\,\text{W}$$

23. We know $H = \frac{V^2}{R}$

Here in second condition, V' becomes $\frac{V}{3}$ and R' becomes $2R$.

So, heat in resistor is given by

$$H' = \frac{\left(\frac{V}{3}\right)^2}{2R} = \frac{V^2}{18R}$$

$\Rightarrow \quad H' = \frac{H}{18}$

24. We know that resistance is given by

$$R = \frac{V^2}{P} \quad \text{or} \quad R \propto \frac{1}{P}$$

$\therefore \quad \frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

25. In series, $i = \frac{2E}{2+R}$

$\therefore \quad J_1 = i^2 R = \left(\frac{2E}{2+R}\right)^2 R \qquad \ldots(i)$

In parallel, $i = \frac{E}{0.5+R}$

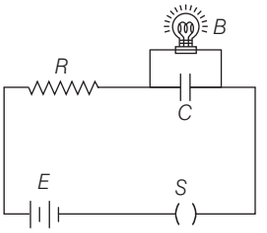
$$J_2 = i^2 R = \left(\frac{E}{0.5+R}\right)^2 R \qquad \ldots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{J_1}{J_2} = 2.25 = \frac{4(0.5+R)^2}{(2+R)^2}$$

$\Rightarrow \quad R = 4\,\Omega$

26. Neon bulb is filled with gas, so the resistance is infinite, hence no current flows through it.



Now, $V_C = E(1 - e^{-t/RC})$

$\Rightarrow \quad 120 = 200(1 - e^{-t/RC})$

$\Rightarrow \quad e^{-t/RC} = \frac{2}{5}$

$\Rightarrow \quad t = RC \ln 2.5$

$\Rightarrow \quad R = \frac{t}{C \ln 2.5}$

$$= \frac{5}{2.303\times 2\times 10^{-6}\ \log\ 2.5}$$

$$= 2.7\times 10^6\ \Omega$$

27. Resistance of a conductor is given as,

$$R = \frac{\rho l}{A} = \frac{\rho l^2}{V}$$

$\therefore \quad \frac{\Delta R}{R} = 2\frac{\Delta l}{l} = \pm 0.2\%$

28. Resistance of combination, $R_e = 4R$

$$\frac{\Delta R_e}{R_e} = \frac{\Delta R}{R}$$

$$= \frac{5\times 100}{100} = 5\%$$

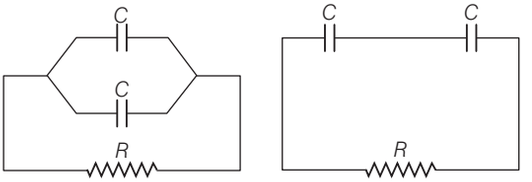
29. If C_e be the effective capacitance, then

$$V_C = \frac{1}{2}V_0 \quad \Rightarrow \quad \frac{q}{C_e} = \frac{q_0}{2C_e}$$

$\Rightarrow \quad q_0(1 - e^{-t/RC_e}) = \frac{q_0}{2}$

$\Rightarrow \quad t = RC_e \ln 2$

For parallel grouping,



$$C_e = \frac{2C}{2}$$

$\therefore$ $t_2 = 2RC \ln 2$

For series grouping,

$$C_e = \frac{C}{2} \Rightarrow t_1 = \frac{RC}{2} \ln 2$$

$$\frac{t_2}{t_1} = \frac{1}{4} \Rightarrow t_2 = 2.5 \text{ s}$$

30. Heat produced by the heater, $H = \frac{V^2}{R} \times t$

For 220 V heater heat produced, $H_1 = \frac{(220)^2}{R} \times 5$

For 110 V heater heat produced,

$$H_2 = \frac{(110)^2}{R} \times t$$

Now, $H_1 = H_2$

$$\Rightarrow \quad \frac{110 \times 110}{R} \times t = \frac{220 \times 220 \times 5}{R}$$

$$t = 20 \text{ min}$$

31. Change is given by

$$q = it = n \times 2e$$

$$n = \frac{it}{2e}$$

$$= \frac{2 \times 32}{2 \times 1.6 \times 10^{-19}}$$

$$= 2 \times 10^{20}$$

32. Resistance of the wire is given as, $R = \frac{\rho l}{A} = \frac{\rho l^2}{Al} = \frac{\rho l^2}{V}$

where V = volume of wire,

length c, $l_2 = l_1 + \frac{25}{100} l_1 = 1.25\, l_1$

$$\frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right)^2 = \frac{(1.25 l_1^2)}{l_1^2} \qquad (\text{as, } R \propto l^2)$$

$\therefore$ $R_2 = (1.25)^2 \times 10 = 15.6\, \Omega.$

33. The temperature coefficient of the carbon is negative so, the resistance of carbon decreases with the increase of temperature.

34. Resistance of circuit,

$$R = \frac{12 \times 4}{12 + 4} + 2 = 5\, \Omega$$

$$I = \frac{E}{R + r} = \frac{12}{6} = 2 \text{ A}$$

$$I_1 + I_2 = 2 \text{ A}$$

$$I_1 = \left(\frac{4 \times 2}{12 + 4} = \frac{4 \times 2}{16} = 0.5 \text{A} \right)$$

$$I_1 = 0.5 \text{ A} \quad \text{and} \quad I_2 = 1.5 \text{ A}$$

35. The circuit given is a balanced Wheat stone bridge. Hence, current through G is zero. The effective resistance is given as

$$R_{\text{eff}} = \frac{(P + Q)(R + S)}{(P + Q + R + S)} = \frac{4}{3} R$$

36. When key K is opened, bulb B_2 will not draw any current from the source, so that terminal voltage of source increases. Hence, power consumed by bulb increases, so light of the bulb becomes more. The brightness of bulb B_1 decreases.

37. Current, $i = \frac{P}{V} = \frac{550}{220} = 2.5 \text{ A}$

38. As, $I = \frac{E}{R_2 + r}$

(since, finally no current flows through the capacitor)

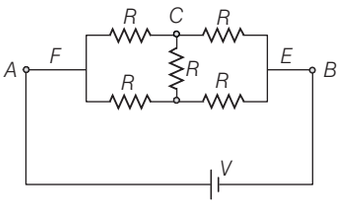
$\therefore$ Potential difference across R_2,

$$V = IR_2 = \frac{ER_2}{R_2 + r}$$

$\therefore$ Charge on the capacitor,

$$Q = CV = \frac{CER_2}{R_2 + r}$$

39. The circuit can be arranged as,

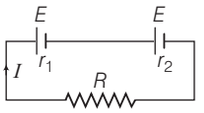


Circuit shows balanced Wheatstone bridge.

As, $I = \frac{V}{R}$

$\therefore$ Current in $FC = \frac{I}{2} = \frac{V}{2R}$

40. Current, $I = \frac{2E}{R + r_1 + r_2}$



Potential difference across first cell

$$V = E - Ir_1 = 0$$

$$E - \frac{2Er_1}{R + r_1 + r_2} = 0$$

$$\left[\frac{R + r_1 + r_2 - 2r_1}{R + r_1 + r_2} \right] = 0$$

$$\Rightarrow \quad R + r_2 - r_1 = 0 \Rightarrow R = r_1 - r_2$$

41. Resistance of a conductor is given as, $R = \frac{\rho (L)}{A} = \frac{\rho L}{tL} = \frac{\rho}{t}$

i.e. R is independent of L.

42. We will require a voltmeter, an ammeter, a test resistor and a variable battery to verify Ohm's law.

Voltmeter which is made by connecting a high resistance in series with a galvanometer is connected in parallel with the test resistor.

Further, an ammeter which is formed by connecting a low resistance in parallel with galvanometer is required to measure, the current through test resistor when connected in series.

43. Let R_0 be the initial resistance of both conductors.

$\therefore$ At temperature θ their resistances will be,

$$R_1 = R_0(1+\alpha_1\theta)$$

and $$R_2 = R_0(1+\alpha_2\theta)$$

For series combination, $R_s = R_1 + R_2$

$$R_s(1+\alpha_s\theta) = R_0(1+\alpha_1\theta) + R_0(1+\alpha_2\theta)$$

where, $$R_s = R_0 + R_0 = 2R_0$$

$\therefore$ $$2R_0(1+\alpha_s\theta) = 2R_0 + R_0\theta(\alpha_1+\alpha_2)$$

or $$\alpha_s = \frac{\alpha_1+\alpha_2}{2}$$

For parallel combination,

$$R_p = \frac{R_1R_2}{R_1+R_2}$$

$$R_p(1+\alpha_p\theta) = \frac{R_0(1+\alpha_1\theta)R_0(1+\alpha_2\theta)}{R_0(1+\alpha_1\theta)+R_0(1+\alpha_2\theta)}$$

At, $$R_p = \frac{R_0R_0}{R_0+R_0} = \frac{R_0}{2}$$

$\therefore$ $$\frac{R_0}{2}(1+\alpha_p\theta) = \frac{R_0^2(1+\alpha_1\theta+\alpha_2\theta+\alpha_1\alpha_2\theta^2)}{R_0(2+\alpha_1\theta+\alpha_2\theta)}$$

As, α_1 and α_2 are small quantities.

$\therefore \alpha_1\alpha_2$ is negligible.

or $$\alpha_p = \frac{\alpha_1+\alpha_2}{2+(\alpha_1+\alpha_2)\theta}$$

$$= \frac{\alpha_1+\alpha_2}{2}\left[1-\left(\frac{\alpha_1+\alpha_2}{2}\right)\theta\right]$$

as $(\alpha_1+\alpha_2)^2$ is negligible

$\therefore$ $$\alpha_p = \frac{\alpha_1+\alpha_2}{2}$$

44. Given, $V = 1\text{ V}$, $I_g = 1\text{ mA} = 1\times10^{-3}\text{ A}$, $R_1 = 950\,\Omega$

Resistance of a galvanometer, $R_g = 50\,\Omega$

$$R_s = \frac{V}{I_g} - R_g$$

$$= \frac{1}{10^{-3}} - 50$$

$$R_s' = \frac{10}{10^{-3}} - 50 = 9950\,\Omega$$

or $$R_s = R_s' - R_1 = 9950 - 950$$

$$= 9000\,\Omega$$

45. Total resistance, $R = 20 + 40 = 60\,\Omega$

Given, $V = 15\text{ V}$

Current $$I = \frac{V}{R} = \frac{15}{60} = 0.25\text{ A}$$

Potential gradient $= \frac{V}{l} = \frac{20\times0.25}{10} = 0.5\text{ Vm}^{-1}$

Potential difference across 240 cm, $E = 0.5\times2.4 = 1.2\text{ V}$

46. Current, $i = \frac{E}{R+r}$, when R decreases to 0,

$\therefore$ $$i = \frac{E}{r}$$

Similarly, potential difference $V = iR$, when R decreases to 0, $V = 0$.

47. Applying Kirchhoff's law at point D, we get

$$I_1 = I_2 + I_3$$

$$\frac{V_A - V_D}{10} = \frac{V_D - 0}{20} + \frac{V_D - V_C}{30}$$

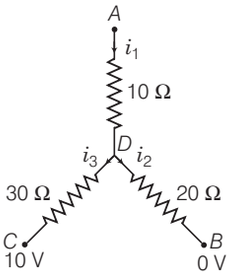
or $$70 - V_D = \frac{V_D}{2} + \frac{V_D - 10}{3}$$

$\Rightarrow$ $$V_D = 40\text{ V}$$

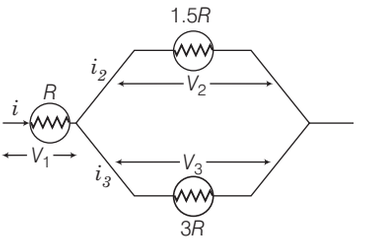
$\Rightarrow$ $$i_1 = \frac{70-40}{10} = 3\text{ A}$$

$\Rightarrow$ $$i_2 = \frac{40-0}{20} = 2\text{ A}$$

and $$i_3 = \frac{40-10}{30} = 1\text{ A}$$



48.



Given, $V_2 = V_3$

i.e. $$i_2 \times 1.5R = 3R \times i_3$$

$\Rightarrow$ $$i_2 + i_3 = i$$

$\Rightarrow$ $$i_2 = \frac{2i}{3} \quad \text{and} \quad i_3 = \frac{i}{3}$$

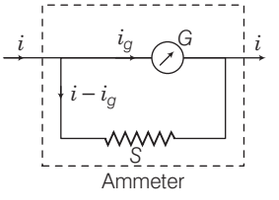
Now, $$V_1 = iR$$

$\Rightarrow$ $$V_2 = \frac{2i}{3}\times1.5R = iR$$

$$V_3 = \frac{i}{3}\times3R = iR$$

$\Rightarrow$ $$V_1 = V_2 = V_3$$

49. Let G be resistance of galvanometer and i_g, the current which on passing through the galvanometer produces full scale deflection. If i is the maximum current, since, G and S are in parallel.



$$i_g \times G = (i - i_g)\times S$$

$\Rightarrow$ $$\frac{i_g}{i} = \frac{S}{S+G}$$

Given, $G = 36\,\Omega,\ S = 4\,\Omega$

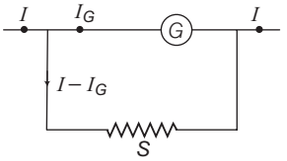
$\therefore \quad \frac{i_g}{i} = \frac{4}{36+4} = \frac{4}{40}$

$\Rightarrow \quad i_g = \frac{i}{10}$

$\therefore \quad 100 \times \frac{i_g}{i} = \frac{1}{10} \times 100 \quad \Rightarrow \quad \frac{i_g}{i}\,\% = 10\%$

50. In a potentiometer there is no current drawn from the cell whose emf is to be measured whereas a voltmeter always draws some current from the cell. Hence, the emf of a cell can be measured accurately using a potentiometer.

51. Given, galvanometer resistance, $G = 240\,\Omega$



Shunt resistance, $S = ?$

$$I_G = \frac{4}{100} I$$

From figure voltage through the circuit.

$$(I - I_G)\, S = I_G G$$

or $$\left(I - \frac{4I}{100}\right) S = \frac{4I}{100} \times 240$$

or $$S = \frac{4 \times 240}{96} = 10\,\Omega$$

52. Potentiometer works on null deflection method. In balance condition no current flows in secondary circuit.

53. When a resistance of $100\,\Omega$ is connected in series

current, $$i = \frac{V}{100 + R} \quad \ldots(\text{i})$$

When a resistance of $1000\,\Omega$ is connected in series, the its range double

current, $$i = \frac{2V}{1100 + R} \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\frac{V}{100 + R} = \frac{2V}{1100 + R}$$

$\Rightarrow \quad R = 900\,\Omega$

54. Potential gradient, $x = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L}$

$\Rightarrow \quad x = \frac{2}{(990 + 10)} \times \frac{10}{2}$

$= 0.01\ \text{Vm}^{-1}$

55. Neutral temperature, $T_n = \frac{T_c + T_f}{2}$

$$320^\circ = \frac{T_c + 600^\circ}{2}$$

$\Rightarrow \quad 640^\circ = T_c + 600^\circ$

$\Rightarrow \quad T_c = 40^\circ\text{C}$

20

Magnetic Effects of Current

20.1 Introduction

The fascinating attractive properties of magnets have been known since ancient times. The word magnet comes from ancient Greek place name Magnesia (the modern town Manisa in Western Turkey), where the natural magnets called lodestones were found.

The fundamental nature of magnetism is the interaction of moving electric charges. Unlike electric forces which act on electric charges whether they are moving or not, magnetic forces act only on moving charges and current carrying wires.

We will describe magnetic forces using the concept of a field. A magnetic field is established by a permanent magnet by an electric current or by other moving charges. This magnetic field in turn, exerts forces on other moving charges and current carrying conductors. In this chapter, first we study the magnetic forces and torques exerted on moving charges and currents by magnetic fields, then we will see how to calculate the magnetic fields produced by currents and moving charges.

20.2 Magnetic Force on a Moving Charge (F_m)

An unknown electric field can be determined by magnitude and direction of the force on a test charge q_0 at rest. To explore an unknown magnetic field (denoted by **B**), we must measure the magnitude and direction of the force on a moving test charge.

The magnetic force ($\mathbf{F}_m$) on a charge q moving with velocity **v** in a magnetic field **B** is given, both in magnitude and direction, by

$$\mathbf{F}_m = q\ (\mathbf{v} \times \mathbf{B}) \qquad \ldots(i)$$

Following points are worthnoting regarding the above expression :

(i) The magnitude of $\mathbf{F}_m$ is, $F_m = Bqv \sin\theta$
where, θ is the angle between **v** and **B**.

(ii) $\mathbf{F}_m$ is zero when,
(a) $B = 0$, i.e. no magnetic field is present.
(b) $q = 0$, i.e. particle is neutral.
(c) $v = 0$, i.e. charged particle is at rest or
(d) $\theta = 0°$ or $180°$, i.e. $\mathbf{v} \uparrow\uparrow \mathbf{B}$ or $\mathbf{v} \uparrow\downarrow \mathbf{B}$

Chapter Snapshot

- Introduction
- Magnetic Force on a Moving Charge ($\mathbf{F}_m$)
- Path of a Charged Particle in Uniform Magnetic Field
- Magnetic Force on a Current Carrying Conductor
- Magnetic Dipole
- Magnetic Dipole in Uniform Magnetic Field
- Biot-Savart Law
- Applications of Biot-Savart Law
- Ampere's Circuital Law
- Force between Parallel Current Carrying Wires
- Cyclotron

(iii) $\mathbf{F}_m$ is maximum at $\theta = 90°$ and this maximum value is Bqv.

(iv) The units of $\mathbf{B}$ must be the same as the units of F/qv. Therefore, the SI unit of $\mathbf{B}$ is equivalent to $\frac{\text{N-s}}{\text{C-m}}$. This unit is called the **tesla** (abbreviated as T), in honour of nikola tesla, the prominent Serbian-American scientist and inventor. Thus,

$$1 \text{ tesla} = 1\text{T} = \frac{1\text{N-s}}{\text{C-m}} = \frac{1\text{N}}{\text{A-m}}$$

The CGS unit of $\mathbf{B}$, the **gauss** ($1\text{G} = 10^{-4}$ T) is also in common use.

(v) In equation number (i) q is to be substituted with sign. If q is positive magnetic force is along $\mathbf{v} \times \mathbf{B}$ and if q is negative magnetic force is in a direction opposite to $\mathbf{v} \times \mathbf{B}$.

(vi) **Direction of $\mathbf{F}_m$** From the property of cross product, we can infer that $\mathbf{F}_m$ is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$ or it is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$. The exact direction of $\mathbf{F}_m$ can be given by any of the following methods :

(a) Direction of $\mathbf{F}_m$ = (sign of q) (direction of $\mathbf{v} \times \mathbf{B}$) or as we stated earlier also,

$\mathbf{F}_m \uparrow\uparrow \mathbf{v} \times \mathbf{B}$ if q is positive and

$\mathbf{F}_m \uparrow\downarrow \mathbf{v} \times \mathbf{B}$ if q is negative.

(b) **Fleming's left hand rule** According to this rule, the forefinger, the central finger and the thumb of the left hand are stretched in such a way that they are mutually perpendicular to each other. If the central finger shows the direction of velocity of positive charge ($\mathbf{v}_{+q}$) and forefinger shows the direction of magnetic field ($\mathbf{B}$), then the thumb will give the direction of magnetic force ($\mathbf{F}_m$). If instead of positive charge we have the negative charge, then $\mathbf{F}_m$ is in opposite direction.

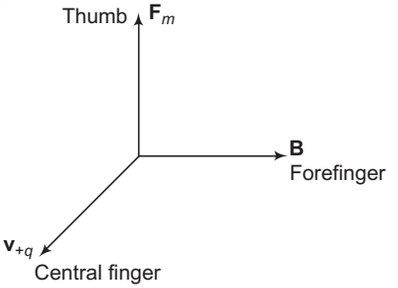


Fig. 20.1

(c) **Right hand rule** Wrap the fingers of your right hand around the line perpendicular to the plane of $\mathbf{v}$ and $\mathbf{B}$ as shown in figure, so that they curl around with the sense of rotation from $\mathbf{v}$ to $\mathbf{B}$ through the smaller angle between them. Your thumb, then points in the direction of the force $\mathbf{F}_m$ on a positive charge. (Alternatively, the direction of the force $\mathbf{F}_m$ on a positive charge is the direction in which a right hand thread screw would advance, if turned the same way).

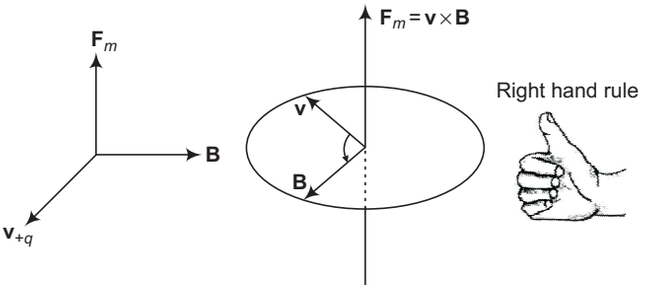


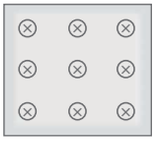
Fig. 20.2

(vii) $\mathbf{F}_m \perp \mathbf{v}$ or $\mathbf{F} \perp \frac{d\mathbf{s}}{dt}$. Therefore, $\mathbf{F}_m \perp d\mathbf{s}$ or the work done by the magnetic force in a static magnetic field is zero.

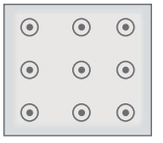
$$W_{\mathbf{F}_m} = 0$$

So, from work energy theorem KE and hence the speed of the charged particle remains constant in magnetic field. The magnetic force can change the direction only. It cannot increase or decrease the speed or kinetic energy of the particle.

By convention the direction of magnetic field $\mathbf{B}$ perpendicular to the paper going inwards is shown by ⊗ and the direction perpendicular to the paper coming out is shown by ⊙



Direction of $\mathbf{B}$ perpendicular to paper inwards



Direction of $\mathbf{B}$ perpendicular to paper outwards

Fig. 20.3

Example 20.1 *A charged particle is projected in a magnetic field,*

$$\mathbf{B} = (3\,\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \times 10^{-2}\ \text{T}$$

The acceleration of the particle is found to be,

$$\mathbf{a} = (x\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\ \text{m/s}^2$$

Find the value of x.

Sol. As we have read, $F_m \perp B$

i.e. the acceleration, $a \perp B$

or $a \cdot B = 0$

or $(x\hat{i} + 2\hat{j}) \cdot (3\hat{i} + 4\hat{j}) \times 10^{-2} = 0$

or $(3x + 8) \times 10^{-2} = 0$

$\therefore \quad x = -\frac{8}{3} \text{m/s}^2$

Example 20.2 *When a proton has a velocity* $\mathbf{v} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times 10^6$ m/s *it experiences a force* $\mathbf{F} = -(1.28 \times 10^{-13}\ \hat{\mathbf{k}})$ N. *When its velocity is along the z-axis, it experiences a force along the x-axis. What is the magnetic field?*

How to proceed In the second part of the question, it is given that magnetic force is along *x*-axis when velocity is along *z*-axis. Hence, magnetic field should be along negative *y*-direction. As in case of positive charge (here proton).

$$\mathbf{F}_m \uparrow\uparrow \mathbf{v} \times \mathbf{B}$$

So, let $\mathbf{B} = -B_0\hat{\mathbf{j}}$, where B_0 = positive constant

Now applying $\mathbf{F}_m = q\,(\mathbf{v} \times \mathbf{B})$, we can find the value of B_0 from the first part of the question.

Sol. Substituting proper values in,

$$F_m = q\,(v \times B)$$

We have, $-(1.28 \times 10^{-13}\ \hat{k}) = (1.6 \times 10^{-19})\,[(2\hat{i} + 3\hat{j}) \times (-B_0\hat{j})] \times 10^6$

$\therefore \quad 1.28 = 1.6 \times 2 \times B_0$

or $B_0 = \frac{1.28}{3.2} = 0.4$

Therefore, the magnetic field is,

$$B = (-0.4\hat{j})\ T$$

20.3 Path of a Charged Particle in Uniform Magnetic Field

The path of a charged particle in uniform magnetic field depends on the angle θ (the angle between $d\mathbf{v}$ and $\mathbf{B}$). Depending on the different values of θ, following three cases are possible.

Case 1 When θ is 0° or 180°

As we have seen in Art. 20.2, $\mathbf{F}_m = 0$, when θ is either 0° or 180°. Hence, path of the charged particle is a straight line (undeviated) when it enters parallel or anti-parallel to magnetic field.

Case 2 When θ = 90°

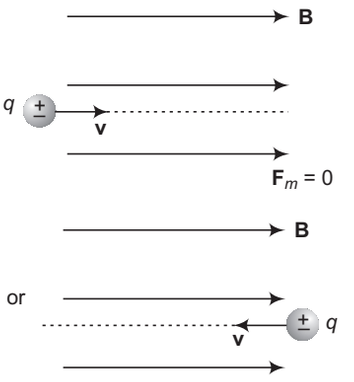


Fig. 20.4

When $\theta = 90°$, the magnetic force is $F_m = Bqv \sin 90° = Bqv$. This magnetic force is perpendicular to the velocity at every instant. Hence, path is a **circle**. The necessary centripetal force is provided by the magnetic force. Hence, if *r* be the radius of the circle, then

$$\frac{mv^2}{r} = Bqv \quad \text{or} \quad r = \frac{mv}{Bq}$$

This expression of *r* can be written in following different ways

$$r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2Km}}{Bq} = \frac{\sqrt{2qVm}}{Bq}$$

where, p = momentum of particle

$$K = \text{KE of particle} = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2Km}$$

We also know that, if the charged particle is accelerated by a potential difference of *V* volts, it acquires a KE given by,

$$K = qV$$

Further, time period of the circular path will be

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(\frac{mv}{Bq}\right)}{v}$$

$$= \frac{2\pi m}{Bq} \quad \text{or} \quad T = \frac{2\pi m}{Bq}$$

or the angular speed (ω) of the particle is,

$$\omega = \frac{2\pi}{T} = \frac{Bq}{m}$$

$\therefore \quad \omega = \frac{Bq}{m}$

Frequency of rotation is,

$$f = \frac{1}{T} \quad \text{or} \quad f = \frac{Bq}{2\pi m}$$

Following points are worthnoting regarding a circular path :

(i) The plane of the circle is perpendicular to magnetic field. If the magnetic field is along z-direction, the circular path is in x-y plane. The speed of the particle does not change in magnetic field.

Hence, if v_0 be the speed of the particle, then velocity of particle at any instant of time will be,

$$\mathbf{v} = v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}}$$

where, $$v_x^2 + v_y^2 = v_0^2$$

(ii) T, f and ω are independent of v while the radius is directly proportional to v.

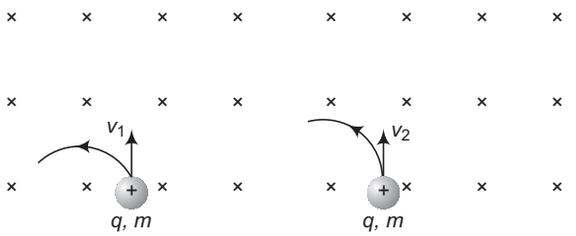


Fig. 20.5

Hence, if two charged particles of equal mass and charge enter in a magnetic field **B** with different speeds v_1 and v_2 $(> v_1)$ at right angles, then

$$T_1 = T_2 \quad \text{but} \quad r_2 > r_1$$

as shown in figure.

- Charge per unit mass $\frac{q}{m}$ is known as **specific charge**. It is sometimes denoted by α. So, in terms of α, the above formulae can be written as,

$$r = \frac{v}{B\alpha}, \quad T = \frac{2\pi}{B\alpha}, \quad f = \frac{B\alpha}{2\pi} \quad \text{and} \quad \omega = B\alpha$$

Case 3 When θ is other than $0°, 180°$ or $90°$

In this case, velocity can be resolved in two components, one along **B** and another perpendicular to **B**. Let the two components be $v_{||}$ and $v_\perp$. Then,

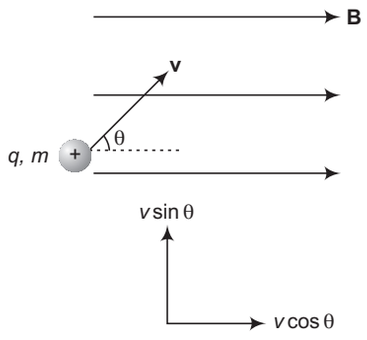


Fig. 20.6

$$v_{||} = v\cos\theta \quad \text{and} \quad v_\perp = v\sin\theta$$

The component perpendicular to field $(v_\perp)$ gives a circular path and the component parallel to field $(v_{||})$ gives a straight line path. The resultant path is a helix as shown in figure.

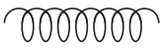


Fig. 20.7

The radius of this helical path is,

$$r = \frac{mv_\perp}{Bq} = \frac{mv\sin\theta}{Bq}$$

Time period and frequency do not depend on velocity and so they are given by,

$$T = \frac{2\pi m}{Bq} \quad \text{and} \quad f = \frac{Bq}{2\pi m}$$

There is one more term associated with a helical path, that is pitch (p) of the helical path. Pitch is defined as the distance travelled along magnetic field in one complete cycle. i.e.

$$p = v_{||} T$$

or $$p = (v\cos\theta)\frac{2\pi m}{Bq}$$

$\therefore$ $$p = \frac{2\pi m v\cos\theta}{Bq}$$

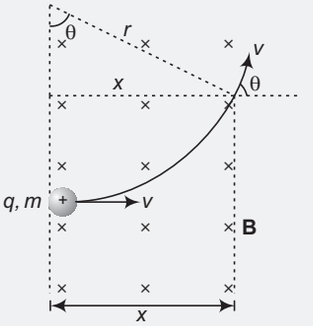
Extra Knowledge Points

- **Deviation of a charged particle in magnetic field**

Suppose a charged particle (q, m) enters a uniform magnetic field B at right angles with speed v as shown in figure. The magnetic field extends upto a length x. The path of the particle is a circle of radius r, where

$$r = \frac{mv}{Bq}$$

The speed of the particle in magnetic field does not change. But it gets deviated in the magnetic field. The deviation θ can be found in two ways.

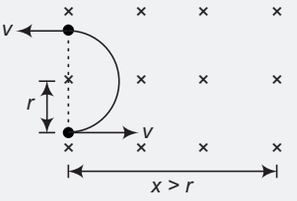


(i) After time t, deviation will be,

$$\theta = \omega t = \left(\frac{Bq}{m}\right) t$$

as, $$\omega = \frac{Bq}{m}$$

(ii) In terms of the length of the magnetic field (i.e. when the particle leaves the magnetic field) the deviation will be,



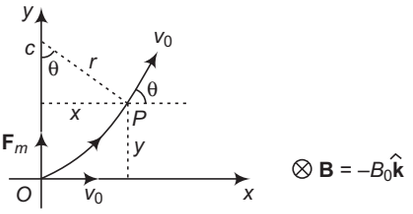
$$\theta = \sin^{-1}\left(\frac{x}{r}\right)$$

But since, $\sin\theta \nmid 1$, this relation can be used only when $x \le r$.

For $x > r$, the deviation will be 180°.

Example 20.3 *A particle of specific charge* α *enters a uniform magnetic field* $\mathbf{B} = -B_0\hat{\mathbf{k}}$ *with velocity* $\mathbf{v} = v_0\hat{\mathbf{i}}$ *from the origin. Find the time dependence of velocity and position of the particle.*

How to proceed *In such type of problems, first of all see the angle between* **v** *and* **B**. *Because it only decides the path of the particle. Here, the angle is* 90°. *Therefore, the path is a circle. If it is a circle, see the plane of the circle (perpendicular to the magnetic field). Here, the plane is x-y. Then, see the sense of the rotation.*



OC = *CP* = radius of circle

Fig. 20.8

Here, it will be anti-clockwise as shown in figure, because at origin the magnetic force is along positive y-direction (which can be seen from Fleming's left hand rule). Find the deviation and radius of the particle,

$$\theta = \omega t = B_0\alpha t \quad \text{and} \quad r = \frac{v_0}{B_0\alpha}$$

Now, according to the figure, find $\mathbf{v}(t)$ *and* $\mathbf{r}(t)$.

Sol. Velocity of the particle at any time t is,

$$\mathbf{v}(t) = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = v_0\cos\theta\hat{\mathbf{i}} + v_0\sin\theta\hat{\mathbf{j}}$$

or $$\mathbf{v}(t) = v_0\cos(B_0\alpha t)\hat{\mathbf{i}} + v_0\sin(B_0\alpha t)\hat{\mathbf{j}}$$

Position of particle at time t is,

$$\mathbf{r}(t) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = r\sin\theta\hat{\mathbf{i}} + (r - r\cos\theta)\hat{\mathbf{j}}$$

Substituting the values of r and θ, we have

$$\mathbf{r}(t) = \frac{v_0}{B_0\alpha}[\sin(B_0\alpha t)\hat{\mathbf{i}} + \{1 - \cos(B_0\alpha t)\}\hat{\mathbf{j}}]$$

Example 20.4 *A charged particle* (q, m) *enters a uniform magnetic field* **B** *at angle* α *as shown in figure with speed* v_0. *Find,*

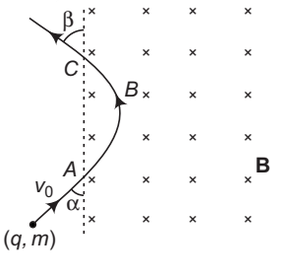


Fig. 20.9

(a) the angle β *at which it leaves the magnetic field,*
(b) time spent by the particle in magnetic field and
(c) the distance AC.

Sol. (a) Here, velocity of the particle is in the plane of paper while the magnetic field is perpendicular to the paper inwards, i.e. angle between v and B is 90°. So, the path is a circle. The radius of the circle is, $r = \frac{mv_0}{Bq}$.

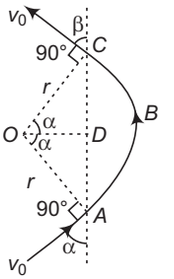


Fig. 20.10

O is the centre of the circle. In ΔAOC,

$$\angle OCD = \angle OAD$$

or $$90° - \beta = 90° - \alpha$$

$\therefore$ $$\beta = \alpha$$

(b) $$\angle COD = \angle DOA = \alpha \quad (\text{as } \angle OCD = \angle OAD = 90° - \alpha)$$

$\therefore$ $$\angle AOC = 2\alpha$$

or $$\text{length } ABC = r(2\alpha) = \frac{2mv_0}{Bq}.\alpha$$

$\therefore$ $$t_{ABC} = \frac{ABC}{v_0} = \frac{2m\alpha}{Bq}$$

Alternate Method

$$t_{ABC} = \left(\frac{T}{2\pi}\right)(2\alpha)$$

$$= \left(\frac{\alpha}{\pi}\right).T$$

$$= \left(\frac{\alpha}{\pi}\right)\left(\frac{2\pi m}{Bq}\right)$$

$$= \frac{2\alpha m}{Bq}$$

(c) Distance, $$AC = 2(AD) = 2(r\sin\alpha)$$

$$= \frac{2mv_0}{Bq}\sin\alpha$$

20.4 Magnetic Force on a Current Carrying Conductor

A charged particle in motion experiences a magnetic force in a magnetic field. Similarly, a current carrying wire also experiences a force when placed in a magnetic field. This follows from the fact that the current is a collection of many charged particles in motion. Hence, the resultant force exerted by the field on the wire is the vector sum of the individual forces exerted on all the charged particles making up the current. The force exerted on the particles is transmitted to the wire when the particles collide with the atoms making up the wire.

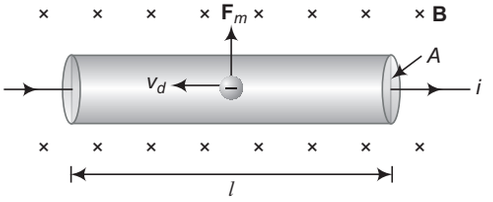


Fig. 20.11

Suppose a conducting wire carrying a current i, is placed in a magnetic field **B**. The length of the wire is l and area of cross-section is A. The free electrons drift with a speed v_d opposite to the direction of current. The magnetic force exerted on the electron is

$$d\mathbf{F}_m = -e(\mathbf{v}_d \times \mathbf{B})$$

If n be the number of free electrons per unit volume of the wire, then total number of electrons in volume Al of the wire are, nAl. Therefore, total force on the wire is,

$$\mathbf{F}_m = -e(nAl)(\mathbf{v}_d \times \mathbf{B})$$

If we denote the length l along the direction of the current by **l**, then the above equation becomes

$$\mathbf{F}_m = i(\mathbf{l} \times \mathbf{B}) \quad \ldots(i)$$

where, $neAv_d = i$

Following points are worthnoting regarding the above expression :

(i) Magnitude of $\mathbf{F}_m$ is, $F_m = ilB\sin\theta$, here θ is the angle between **l** and **B**. F_m is zero for $\theta = 0°$ or $180°$ and maximum for $\theta = 90°$.

(ii) Here, **l** is a vector that points in the direction of the current i and has a magnitude equal to the length.

(iii) The above expression applies only to a straight segment of wire in a uniform magnetic field.

(iv) For the magnetic force on an arbitrarily shaped wire segment, let us consider the magnetic force exerted on a small segment of vector length dl.

$$d\mathbf{F}_m = i(d\mathbf{l} \times \mathbf{B}) \quad \ldots(ii)$$

To calculate the total force $\mathbf{F}_m$ acting on the wire shown in figure, we integrate Eq. (ii) over the length of the wire.

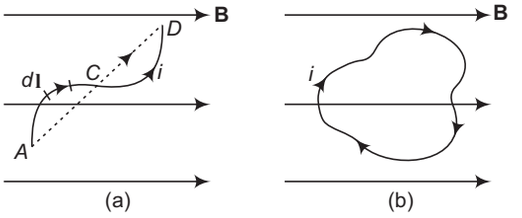


Fig. 20.12

$$\mathbf{F}_m = i\int_A^D (\mathbf{l} \times \mathbf{B}) \quad \ldots(iii)$$

Now, let us consider two special cases involving Eq. (iii). In both cases, the magnetic field is taken to be constant in magnitude and direction.

***Case* I** A curved wire ACD as shown in Fig. (a) carries a current i and is located in a uniform magnetic field **B**. Because the field is uniform, we can take **B** outside the integral in Eq. (iii) and we obtain

$$\mathbf{F}_m = i\left(\int_A^D d\mathbf{l}\right) \times \mathbf{B} \quad \ldots(iv)$$

But the quantity $\int_A^D d\mathbf{l}$ represents the vector sum of all length elements from A to D. From the polygon law of vector addition the sum equals the vector **l** directed from A to D. Thus,

$$\mathbf{F}_m = i(\mathbf{l} \times \mathbf{B})$$

or we can write, $\mathbf{F}_{ACD} = \mathbf{F}_{AD} = i(\mathbf{AD} \times \mathbf{B})$ in uniform field.

***Case* II** An arbitrarily shaped closed loop carrying a current i is placed in a uniform magnetic field as shown in Fig. (b). We can again express the force acting on the loop in the form of Eq. (iv), but this time we must take the vector sum of the length elements $d\mathbf{l}$ over the entire loop,

$$\mathbf{F}_m = i\left(\oint d\mathbf{l}\right) \times \mathbf{B}$$

Because the set of length elements forms a closed polygon, the vector sum must be zero.

$$\therefore \quad \mathbf{F}_m = 0$$

Thus, the net magnetic force acting on any closed current loop in a uniform magnetic field is zero.

(v) The direction of $\mathbf{F}_m$ can be given by Fleming's left hand rule as discussed in Art. 20.2. According to this rule, the forefinger, the central finger and the thumb of the left hand are stretched in such a way that they are mutually perpendicular to each other. If the central finger shows the direction of current (or **l**) and forefinger shows the direction of magnetic field (**B**), then the thumb will give the direction of magnetic force ($\mathbf{F}_m$).

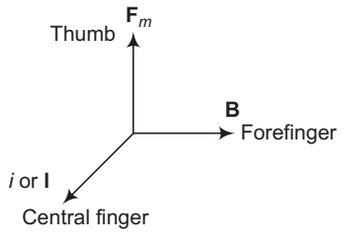


Fig. 20.13

Example 20.5 *In the figure shown a semicircular wire loop is placed in a uniform magnetic field* $B = 1.0$ T. *The plane of the loop is perpendicular to the magnetic field. Current* $i = 2$A *flows in the loop in the directions shown. Find the magnitude of the magnetic force in both the cases (a) and (b). The radius of the loop is* $1.0\,m$.

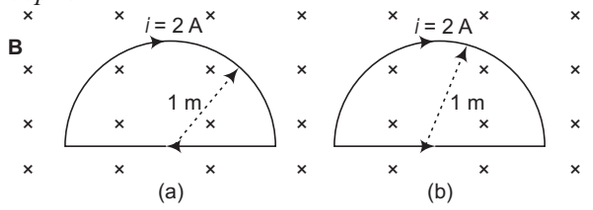


Fig. 20.14

Sol. **Refer Fig. (a)** It forms a closed loop and the current completes the loop. Therefore, net force on the loop in uniform field should be zero.

Refer Fig. (b) In this case although it forms a closed loop, but current does not complete the loop. Hence, net force is not zero.

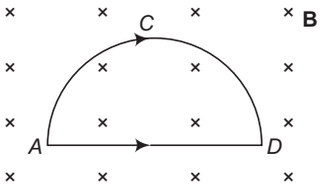


Fig. 20.15

$$F_{ACD} = F_{AD}$$

$$\therefore \quad F_{loop} = F_{ACD} + F_{AD} = 2F_{AD}$$

$$\therefore \quad |F_{loop}| = 2\,|F_{AD}|$$

$$= 2ilB\sin\theta \qquad (l = 2r = 2.0\text{ m})$$

$$= (2)\,(2)\,(2)\,(1)\sin 90°$$

$$= 8\text{ N}$$

20.5 Magnetic Dipole

Every current carrying loop is a magnetic dipole. It has two poles: South (S) and North (N). This is similar to a bar magnet. Magnetic field lines emanate from the North pole and after forming a closed path terminate on South pole. Each magnetic dipole has some magnetic moment (**M**). The magnitude of **M** is,

$$|\mathbf{M}| = NiA$$

Here, N = number of turns in the loop,

i = current in the loop and

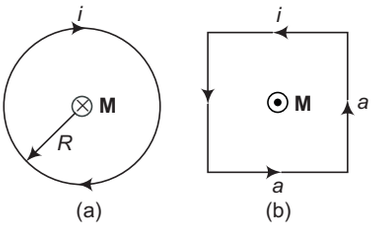
A = area of cross-section of the loop.

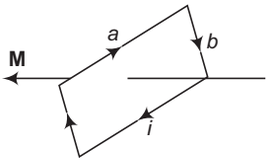
For the direction of **M** any one of the following methods can be used :

(i) As in case of an electric dipole, the dipole moment **p** has a direction from negative charge to positive charge. In the similar manner direction of **M** is from South to North pole. The South and North poles can be identified by the sense of current. The side from where the current seems to be clockwise becomes South pole and the opposite side from where it seems anti-clockwise becomes North pole.



Fig. 20.16





(c)

Fig. 20.17

Now, let us find the direction and magnitude of **M** in the three loops shown in Fig. 20.17.

Refer Fig. (a) In this case, current appears to be clockwise from outside the paper, so this side becomes the South pole. From the back of the paper it seems anti-clockwise. Hence, this side becomes the North pole. As the magnetic moment is from South to North pole it is directed perpendicular to paper inwards. Also

$$|\mathbf{M}| = NiA = \pi R^2 i$$

Refer Fig. (b) Here opposite in the case. South pole is into the paper and North pole is outside the paper. Therefore, magnetic moment is perpendicular to paper in outward direction. The magnitude of **M** is,

$$|\mathbf{M}| = a^2 i$$

Refer Fig. (c) In this case, South pole is on the right side of the loop and North pole on the left side. Hence, **M** is directed from right to left. The magnitude of magnetic moment is

$$|\mathbf{M}| = abi$$

(ii) Vector **M** is along the normal to the plane of the loop. The orientation (up or down along the normal) is given by the right hand rule. Wrap your fingers of the right hand around the perimeter of the loop in the direction of current as shown in figure. Then extend your thumb, so that it is perpendicular to the plane of the loop. The thumb points in the direction of **M**.

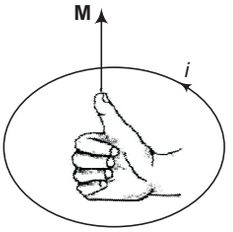


Fig. 20.18

Extra Knowledge Points

- In addition to the method discussed above for calculating **M** here are two more methods for calculating **M**.



Method 1. This method is useful for calculating **M** for a rectangular or square loop.

The magnetic moment (**M**) of the rectangular loop shown in figure is

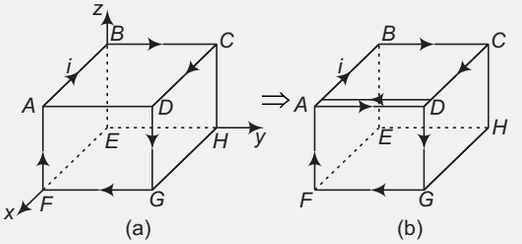
$$\mathbf{M} = i\,(\mathbf{AB} \times \mathbf{BC}) = i\,(\mathbf{BC} \times \mathbf{CD})$$
$$= i\,(\mathbf{CD} \times \mathbf{DA}) = i\,(\mathbf{DA} \times \mathbf{AB})$$

Here the cross product of any two consecutive sides (taken in order) gives the area as well as the correct direction of **M** also.

- If coordinates of vertices are known. Then, vector of any side can be written in terms of co-ordinates, e.g.

$$\mathbf{AB} = (x_B - x_A)\,\hat{\mathbf{i}} + (y_B - y_A)\,\hat{\mathbf{j}} + (z_B - z_A)\,\hat{\mathbf{k}}$$

Method 2 Sometimes a current carrying loop does not lie in a single plane. But by assuming two equal and opposite currents in one branch (which obviously makes no change in the given circuit) two (or more) closed loops are completed in different planes. Now the net magnetic moment of the given loop is the vector sum of individual loops.



For example, in Fig. (a), six sides of a cube of side *l* carry a current *i* in the directions shown. By assuming two equal and opposite currents in wire *AD*, two loops in two different planes (*xy* and *yz*) are completed.

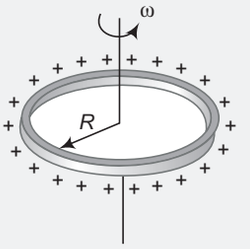
$$\mathbf{M}_{ABCDA} = -il^2\,\hat{\mathbf{k}}$$

$$\mathbf{M}_{ADGFA} = -il^2\hat{\mathbf{i}}$$

$$\therefore \quad \mathbf{M}_{net} = -il^2\,(\hat{\mathbf{i}} + \hat{\mathbf{k}})$$

- Sometimes a non-conducting charged body is rotated with some angular speed. In this case, the ratio of magnetic moment and angular momentum is constant which is equal to $q/2m$, where q is the charge and m the mass of the body.

e.g. In case of a ring, of mass m, radius R and charge q distributed on its circumference.



Angular momentum,

$$L = I\omega = (mR^2)(\omega) \qquad \ldots(i)$$

Magnetic moment,

$$M = i\,A = (q\,f)(\pi R^2)$$

here, f = frequency $= \dfrac{\omega}{2\pi}$

$$\therefore \quad M = (q)\left(\frac{\omega}{2\pi}\right)(\pi R^2)$$

$$= q\,\frac{\omega R^2}{2} \qquad \ldots(ii)$$

From Eqs. (i) and (ii),

$$\frac{M}{L} = \frac{q}{2m}$$

Although this expression is derived for simple case of a ring, it holds good for other bodies also. For example, for a disc or a sphere.

Example 20.6 *A square loop OABCO of side l carries a current i. It is placed as shown in figure. Find the magnetic moment of the loop.*

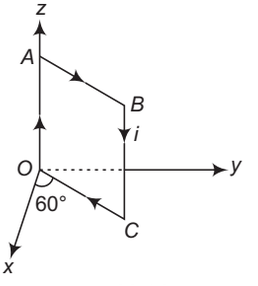


Fig. 20.19

Sol. Magnetic moment of the loop can be written as,

$$\mathrm{M} = i\,(\mathrm{BC} \times \mathrm{CO})$$

Here, $\mathrm{BC} = -l\hat{\mathrm{k}}$

$$\mathrm{CO} = -l\cos 60^\circ\,\hat{\mathrm{i}} - l\sin 60^\circ\,\hat{\mathrm{j}}$$

$$= -\frac{l}{2}\hat{\mathrm{i}} - \frac{\sqrt{3}l}{2}\hat{\mathrm{j}}$$

$$\therefore \quad \mathrm{M} = i\left[(-l\hat{\mathrm{k}}) \times \left(-\frac{l}{2}\hat{\mathrm{i}} - \frac{\sqrt{3}l}{2}\hat{\mathrm{j}}\right)\right]$$

or $$\mathrm{M} = \frac{il^2}{2}(\hat{\mathrm{j}} - \sqrt{3}\,\hat{\mathrm{i}})$$

Example 20.7 *Find the magnitude of magnetic moment of the current carrying loop ABCDEFA. Each side of the loop is* 10 *cm long and current in the loop is* $i = 2.0$ *A.*

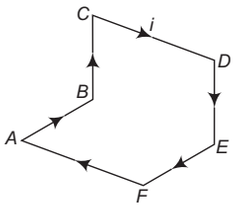


Fig. 20.20

Sol. By assuming two equal and opposite currents in *BE*, two current carrying loops (*ABEFA* and *BCDEB*) are formed. Their magnetic moments are equal in magnitude but perpendicular to each other. Hence,

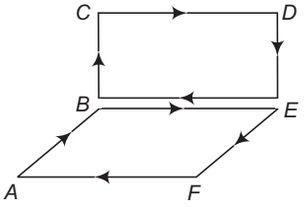


Fig. 20.21

$$M_{\text{net}} = \sqrt{M^2 + M^2} = \sqrt{2}M$$

where $M = iA = (2.0)\,(0.1)\,(0.1)$

$$= 0.02\ \text{A-m}^2$$

$$\therefore \quad M_{\text{net}} = (\sqrt{2})(0.02)\ \text{A-m}^2$$

$$= 0.028\ \text{A-m}^2$$

20.6 Magnetic Dipole in Uniform Magnetic Field

Let us consider a rectangular $(a \times b)$ current carrying loop *OACDO* placed in *x*-*y* plane. A uniform magnetic field,

$$\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

exists in space. We are interested in finding the net force and torque in the loop.

Force Net force on the loop is,

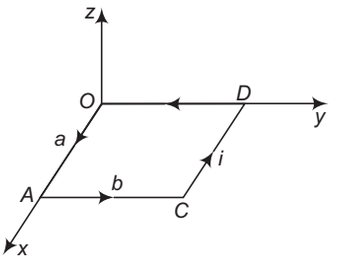


Fig. 20.22

$$\mathbf{F} = \mathbf{F}_{OA} + \mathbf{F}_{AC} + \mathbf{F}_{CD} + \mathbf{F}_{DO}$$
$$= i[(\mathbf{OA} \times \mathbf{B}) + (\mathbf{AC} \times \mathbf{B}) + (\mathbf{CD} \times \mathbf{B}) + (\mathbf{DO} \times \mathbf{B})]$$
$$= i[(\mathbf{OA} + \mathbf{AC} + \mathbf{CD} + \mathbf{DO}) \times \mathbf{B}]$$
$$= \text{null vector}$$

or $|\mathbf{F}| = 0$, as $\mathbf{OA} + \mathbf{AC} + \mathbf{CD} + \mathbf{DO}$ forms a null vector.

Torque Using, $\mathbf{F} = i\,(\mathbf{l} \times \mathbf{B})$, we have

$$\mathbf{F}_{OA} = i\,(\mathbf{OA} \times \mathbf{B}) = i[(a\hat{\mathbf{i}}) \times (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}})]$$
$$= ia\,[B_y\hat{\mathbf{k}} - B_z\hat{\mathbf{j}}]$$

$$\mathbf{F}_{AC} = i\,(\mathbf{AC} \times \mathbf{B}) = i[(b\hat{\mathbf{j}}) \times (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}})]$$
$$= ib\,[-B_x\hat{\mathbf{k}} + B_z\hat{\mathbf{i}}]$$

$$\mathbf{F}_{CD} = i\,(\mathbf{CD} \times \mathbf{B}) = i[(-a\hat{\mathbf{i}}) \times (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}})]$$
$$= ia\,[-B_y\hat{\mathbf{k}} + B_z\hat{\mathbf{j}}]$$

$$\mathbf{F}_{DO} = i\,(\mathbf{DO} \times \mathbf{B}) = i[(-b\hat{\mathbf{j}}) \times (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}})]$$
$$= ib\,[B_x\hat{\mathbf{k}} - B_z\hat{\mathbf{i}}]$$

All these forces are acting at the centre of the wires. For example, $\mathbf{F}_{OA}$ will act at the centre of *OA*. When the forces are in equilibrium net torque about any point remains the same. Let us take the torque about *O*.

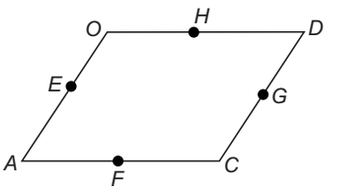


Fig. 20.23

E, *F*, *G* and *H* are the mid-points of *OA*, *AC*, *CD* and *DO* respectively. Using $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, we have

$$\boldsymbol{\tau}_0 = (\mathbf{OE} \times \mathbf{F}_{AO}) + (\mathbf{OF} \times \mathbf{F}_{AC}) + (\mathbf{OG} \times \mathbf{F}_{CD}) + (\mathbf{OH} \times \mathbf{F}_{DO})$$
$$= \left[\left(\frac{a}{2}\hat{\mathbf{i}}\right) \times \{ia\,(B_y\hat{\mathbf{k}} - B_z\hat{\mathbf{j}})\}\right] + \left[\left(a\hat{\mathbf{i}} + \frac{b}{2}\hat{\mathbf{j}}\right) \times \{ib\,(-B_x\hat{\mathbf{k}} + B_z\hat{\mathbf{i}})\}\right]$$

$$+\left[\left(\frac{a}{2}\hat{\mathbf{i}}+b\,\hat{\mathbf{j}}\right)\times\{ia\,(-B_y\hat{\mathbf{k}}+B_z\,\hat{\mathbf{j}})\}\right]+\left[\left(\frac{b}{2}\hat{\mathbf{j}}\right)\times ib\,(B_x\hat{\mathbf{k}}-B_z\,\hat{\mathbf{i}})\right]$$

$$=iab\,B_x\,\hat{\mathbf{j}}-iabB_y\,\hat{\mathbf{i}}$$

This can also be written as, $\tau_0=(iab\hat{\mathbf{k}})\times(B_x\hat{\mathbf{i}}+B_y\hat{\mathbf{i}}+B_z\hat{\mathbf{k}})$

here, $iab\hat{\mathbf{k}}=$ magnetic moment of the dipole $\mathbf{M}$ and $B_x\hat{\mathbf{i}}+B_y\hat{\mathbf{j}}+B_z\hat{\mathbf{k}}=\mathbf{B}$

$\therefore$ $\tau=\mathbf{M}\times\mathbf{B}$

- That although this formula has been derived for a rectangular loop, it comes out to be true for any shape of loop. Following points are worth noting regarding the torque acting on the loop in uniform magnetic field.

(i) Magnitude of τ is $MB\sin\theta$ or $NiAB\sin\theta$. Here, θ is the angle between $\mathbf{M}$ and $\mathbf{B}$. Torque is zero when $\theta=0°$ or $180°$ and it is maximum at $\theta=90°$.

(ii) If the loop is free to rotate in a magnetic field the axis of rotation becomes an axis parallel to τ passing through the centre of mass of the loop.

The above equation for the torque is very similar to that of an electric dipole in an electric field. The similarity between electric and magnetic dipoles extends even further as illustrated in the table below.

Table 20.1

S. No.	Field of similarity	Electric dipole	Magnetic dipole
1.	Magnitude	$\|\mathrm{p}\|=q\,(2d)$	$\|\mathrm{M}\|=NiA$
2.	Direction	from $-q$ to $+q$	from S to N
3.	Net force in uniform field	zero	zero
4.	Torque	$\tau=\mathrm{p}\times\mathrm{E}$	$\tau=\mathrm{M}\times\mathrm{B}$
5.	Potential energy	$U=-\mathrm{p}\cdot\mathrm{E}$	$U=-\mathrm{M}\cdot\mathrm{B}$
6.	Work done in rotating the dipole	$W_{\theta_1-\theta_2}=pE\,(\cos\theta_1\cos\theta_2)$	$W_{\theta_1-\theta_2}=MB\,(\cos\theta_1\cos\theta_2)$
7.	Field along axis	$\mathrm{E}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{2\mathrm{p}}{r^3}$	$\mathrm{B}=\frac{\mu_0}{4\pi}\cdot\frac{2\mathrm{M}}{r^3}$
8.	Field perpendicular to axis	$\mathrm{E}=-\frac{1}{4\pi\varepsilon_0}\cdot\frac{\mathrm{p}}{r^3}$	$\mathrm{B}=-\frac{\mu_0}{4\pi}\cdot\frac{\mathrm{M}}{r^3}$

Notice that the expressions for the magnetic dipole can be obtained from the expressions for the electric dipole by replacing $\mathbf{p}$ by $\mathbf{M}$ and ε_0 by $\frac{1}{\mu_0}$.

Here, μ_0 is called the permeability of free space. It is related with ε_0 and speed of light c as,

$$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

and it has the value,

$$\mu_0=4\pi\times10^{-7}\ \mathrm{T\text{-}m/A}$$

Dimensions of $\frac{1}{\sqrt{\varepsilon_0\mu_0}}$ are that of speed or $[\mathrm{LT}^{-1}]$.

Hence, $\left[\frac{1}{\sqrt{\varepsilon_0\mu_0}}\right]=[\mathrm{LT}^{-1}]$

Example 20.8 *A circular loop of radius $R=20\,\mathrm{cm}$ is placed in a uniform magnetic field $\mathbf{B}=2\,\mathrm{T}$ in x-y plane as shown in figure. The loop carries a current $i=1.0\,A$ in the direction shown in figure. Find the magnitude of torque acting on the loop.*

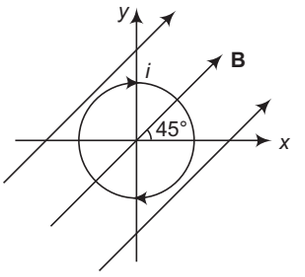


Fig. 20.24

Sol. Magnitude of torque is given by,

$$|\tau|=MB\sin\theta$$

where, $M=NiA=(1)\,(1.0)\,(\pi)\,(0.2)^2$

$$=(0.04\,\pi)\ \mathrm{A\text{-}m^2}$$

$$B=2\ \mathrm{T}$$

and $\theta=$ angle between M and B $=90°$

$\therefore$ $|\tau|=(0.04\,\pi)\,(2)\sin90°$

$$=0.25\ \mathrm{N\text{-}m}$$

- Where, the angle 45° is given just to confuse the students. **M** is along negative *z*-direction (perpendicular to paper inwards) while **B** is in *x*-*y* plane. So, the angle between **M** and **B** is 90° not 45°. So, far as only magnitude of τ is concerned there is no use of giving 45°. But when the direction of torque is desired then this angle is used. Because in that case you will write,

$$\mathbf{B} = 2\cos 45°\,\hat{\mathbf{i}} + 2\sin 45°\,\hat{\mathbf{j}} = \sqrt{2}\,(\hat{\mathbf{i}} + \hat{\mathbf{j}})\ \text{T}$$

and $\mathbf{M} = -(0.04\,\pi)\,\hat{\mathbf{k}}\ \text{A-m}^2$

$\therefore\quad \tau = \mathbf{M} \times \mathbf{B} = (0.04\,\sqrt{2}\pi)\ (-\hat{\mathbf{j}} + \hat{\mathbf{i}})$

or $\tau = 0.18\ (\hat{\mathbf{i}} - \hat{\mathbf{j}})$

20.7 Biot-Savart Law

In the preceding articles, we discussed the magnetic force exerted on a charged particle and current carrying conductor in a magnetic field. To complete the description of the magnetic interaction, this and the next article deals with the origin of the magnetic field. As in electrostatics, there are two methods of calculating the electric field at some point. One is Coulomb's law which gives the electric field due to a point charge and the another is Gauss's law which is useful in calculating the electric field of a highly symmetric configuration of charge. Similarly, in magnetics, there are basically two methods of calculating magnetic field at some point.

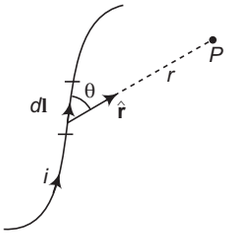


Fig. 20.25

One is **Biot-Savart law** which gives the magnetic field due to an infinitesimally small current carrying wire at some point and the another is **Ampere's law**, which is useful in calculating the magnetic field of a highly symmetric configuration carrying a steady current.

We begin by showing how to use the law of Biot and Savart to calculate the magnetic field produced at some points in space by a small current element. Using this formalism and the principle of superposition, we then calculate the total magnetic field due to various current distributions.

From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the following experimental observations for the magnetic field $d\mathbf{B}$ at a point P associated with a length element $d\mathbf{l}$ of a wire carrying a steady current i.

(i) The vector $d\mathbf{B}$ is perpendicular to both $d\mathbf{l}$ (which points in the direction of the current) and the unit vector $\hat{\mathbf{r}}$ directed from $d\mathbf{l}$ to P.

(ii) The magnitude of $d\mathbf{B}$ is inversely proportional to r^2, where r is the distance from $d\mathbf{l}$ to P.

(iii) The magnitude of $d\mathbf{B}$ is proportional to the current and to the magnitude dl of the length element $d\mathbf{l}$.

(iv) The magnitude of $d\mathbf{B}$ is proportional to $\sin\theta$, where θ is the angle between $d\mathbf{l}$ and $\hat{\mathbf{r}}$. These observations are summarized in mathematical formula known today as **Biot-Savart law.**

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i\,(d\mathbf{l} \times \hat{\mathbf{r}})}{r^2} \qquad \ldots(\text{i})$$

where, $\dfrac{\mu_0}{4\pi} = 10^{-7}\,\dfrac{\text{T - m}}{\text{A}}$

It is important to note that $d\mathbf{B}$ in Eq. (i) is the field created by the current in only a small length element $d\mathbf{l}$ of the conductor. To find the total magnetic field **B** created at some point by a current of finite size, we must sum up contributions from all current elements that make up the current. i.e. we must evaluate **B** by integrating Eq. (i).

$$\mathbf{B} = \frac{\mu_0 i}{4\pi}\int \frac{d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$$

where the integral is taken over the entire current distribution. This expression must be handled with special care because the integrand is a cross product and therefore, a vector quantity.

Following points are worth noting regarding the Biot-Savart law :

(i) Magnitude of $d\mathbf{B}$ is given by,

$$|d\mathbf{B}| = \frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2}$$

$|d\mathbf{B}|$ is zero at $\theta = 0°$ or 180° and maximum at $\theta = 90°$.

(ii) For the direction of $d\mathbf{B}$ either of the following methods can be employed.

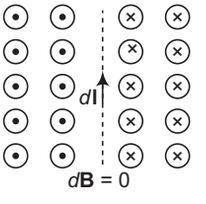


Fig. 20.26

(a) $d\mathbf{B} \uparrow\uparrow d\mathbf{l} \times \hat{\mathbf{r}}$. So, $d\mathbf{B}$ is along $d\mathbf{l} \times \hat{\mathbf{r}}$.

(b) If $d\mathbf{l}$ is in the plane of paper. $d\mathbf{B} = 0$ at all points lying on the straight line passing through $d\mathbf{l}$. The magnetic field to the right of this line is in $\otimes$ direction and to the left of this line is in $\odot$ direction.

20.8 Applications of Biot-Savart Law

Let us now consider few applications of Biot-Savart law.

Magnetic Field Surrounding a Thin, Straight Conductor

According to Biot-Savart law,

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{i\, d\mathbf{l} \times \hat{\mathbf{r}}}{r^2} \qquad \ldots(i)$$

As here every element of the wire contributes to **B** in the same direction (which is here ⊗).

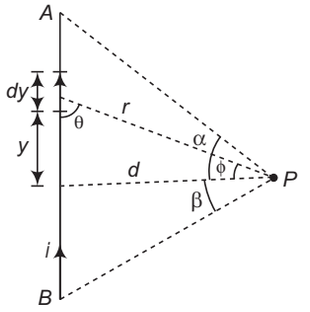


Fig. 20.27

Eq. (i) for this case becomes,

$$B = \frac{\mu_0}{4\pi}\int \frac{idl\sin\theta}{r^2} = \frac{\mu_0 i}{4\pi}\int \frac{dy\sin\theta}{r^2}$$

$$y = d\tan\phi \quad \text{or} \quad dy = d\sec^2\phi.d\phi$$

$$r = d\sec\phi \quad \text{and} \quad \theta = 90° - \phi$$

$$\therefore \quad B = \frac{\mu_0 i}{4\pi d}\int_{-\beta}^{\alpha}\cos\phi\cdot d\phi$$

$$\text{or} \quad B = \frac{\mu_0}{4\pi}\frac{i}{d}(\sin\alpha + \sin\beta)$$

Note down the following points regarding the above equation :

(i) For an infinitely long straight wire, $\alpha = \beta = 90°$

$$\therefore \quad \sin\alpha + \sin\beta = 2 \quad \text{or} \quad B = \frac{\mu_0}{2\pi}\frac{i}{d}$$

(ii) The direction of magnetic field at a point P due to a long straight wire can be found by the right hand thumb rule. If we stretch the thumb of the right hand along the current and curl our fingers to pass through P, the direction of the fingers at P gives the direction of magnetic field there.

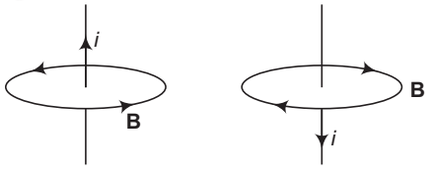


Fig. 20.28

(iii) $B \propto \frac{1}{d}$, i.e. B-d graph for an infinitely long straight wire is a rectangular hyperbola as shown in figure.

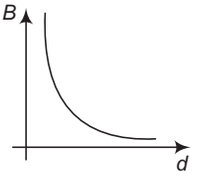


Fig. 20.29

Magnetic Field on the Axis of a Circular Coil (Derivation is Optional)

Suppose a current carrying circular loop has a radius R. Current in the loop is i. We want to find the magnetic field at a point P on the axis of the loop a distance z from the centre.

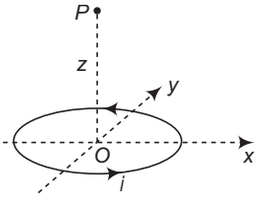


Fig. 20.30

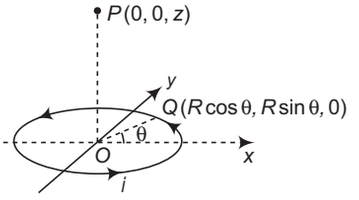


Fig. 20.31

We can take the loop in x-y plane with its centre at origin and point P on the z-axis.

Let us take a small current element at angle θ as shown.

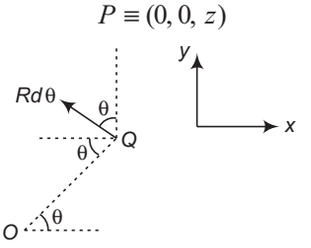


Fig. 20.32

$$Q \equiv (R\cos\theta,\ R\sin\theta,\ 0)$$

$$d\mathbf{l} = -(Rd\theta)\sin\theta\hat{\mathbf{i}} + (Rd\theta)\cos\theta\,\hat{\mathbf{j}}$$

$$\hat{\mathbf{r}} = \text{unit vector along } QP$$

$$= \frac{(-R\cos\theta\hat{\mathbf{i}} - R\sin\theta\hat{\mathbf{j}} + z\hat{\mathbf{k}})}{r}$$

where, $\quad r = \text{distance } QP = \sqrt{R^2 + z^2}$

Now, magnetic field at point P, due to current element $d\mathbf{l}$ at Q is,

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i}{r^2}(d\mathbf{l} \times \hat{\mathbf{r}})$$

$$= \frac{\mu_0}{4\pi}\frac{i}{r^3}[(-R\sin\theta\, d\theta\hat{\mathbf{i}} + R\cos\theta\, d\theta\hat{\mathbf{j}}) \times (-R\cos\theta\hat{\mathbf{i}} - R\sin\theta\hat{\mathbf{j}} + z\hat{\mathbf{k}})]$$

or $$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i}{r^3}[(zR\cos\theta\, d\theta)\hat{\mathbf{i}} + (zR\sin\theta\, d\theta)\hat{\mathbf{j}} + (R^2 d\theta)\hat{\mathbf{k}}]$$

$$= dB_x\hat{\mathbf{i}} + dB_y\hat{\mathbf{j}} + dB_z\hat{\mathbf{k}}$$

where, $$dB_x = \frac{\mu_0}{4\pi}\frac{i}{r^3}(zR\cos\theta d\theta)$$

$$dB_y = \frac{\mu_0}{4\pi}\frac{i}{r^3}(zR\sin\theta d\theta)$$

and $$dB_z = \frac{\mu_0}{4\pi}\frac{i}{r^3}(R^2 d\theta)$$

Integrating these differentials from $\theta = 0°$ to $\theta = 2\pi$ for the complete loop, we get

$$B_x = \frac{\mu_0}{4\pi}\frac{ziR}{r^3}\int_0^{2\pi}\cos\theta\, d\theta = 0$$

$$B_y = \frac{\mu_0}{4\pi}\frac{ziR}{r^3}\int_0^{2\pi}\sin\theta\, d\theta = 0$$

and $$B_z = \frac{\mu_0}{4\pi}\frac{iR^2}{r^3}\int_0^{2\pi}d\theta = \frac{\mu_0}{2}\frac{iR^2}{r^3}$$

Substituting, $r = (R^2 + z^2)^{1/2}$, we get

$$B_P = B_z = \frac{\mu_0 iR^2}{2(R^2 + z^2)^{3/2}}$$

For N number of loops,

$$B = \frac{\mu_0 NiR^2}{2(R^2 + z^2)^{3/2}}$$

Note down the following points regarding a circular current carrying loop.

(i) At the centre of the loop, $z = 0$

and $$B\ (\text{centre}) = \frac{\mu_0 Ni}{2R}$$

(ii) For $z >> R$, $z^2 + R^2 \approx z^2$

$$\therefore \quad B = \frac{\mu_0 NiR^2}{2z^3} = \left(\frac{\mu_0}{4\pi}\right)\frac{(2Ni\pi R^2)}{z^3} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{2M}{z^3}\right)$$

where, M = magnetic moment of the loop

$= NiA = Ni\pi R^2$.

This result was expected as the magnetic field on the axis of a dipole is $\frac{\mu_0}{4\pi}\frac{2M}{r^3}$.

(iii) Direction of magnetic field on the axis of a circular loop can be obtained using the right hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field.

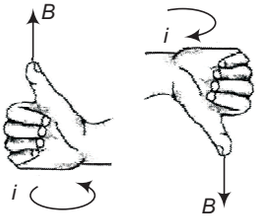


Fig. 20.33

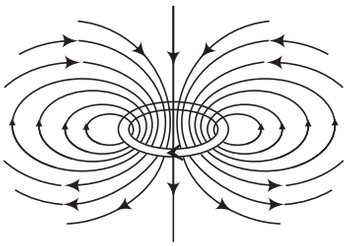


Fig. 20.34

(iv) The magnetic field at a point not on the axis is mathematically difficult to calculate. We show qualitatively in figure the magnetic field lines due to a circular current which will give some idea of the field.

(v) Magnetic field is maximum at the centre and decreases as we move away from the centre (on the axis of the loop). The B-z graph is some what like shown in figure.

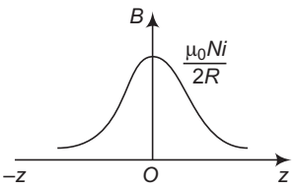


Fig. 20.35

(vi) Magnetic field due to an arc of a circle at the centre is,

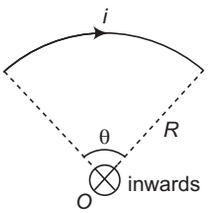


Fig. 20.36

$$B = \left(\frac{\theta}{2\pi}\right)\frac{\mu_0 i}{2R} = \frac{\mu_0}{4\pi}\left(\frac{i}{R}\right)\theta \text{ or } B = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{R}\right)\theta$$

where, θ is to be substituted in radians.

Field along the Axis of a Solenoid
(Derivation is Optional)

The name solenoid was first given by ampere to a wire wound in a closely spaced spiral over a hollow cylindrical non-conducting core. If n is the number of turns per unit length, each carries a current i uniformly wound round a cylinder of radius R, the number of turns in length dx are ndx. Thus, the magnetic field at the axial point O due to this element dx is,

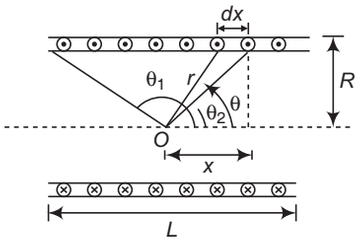


Fig. 20.37

$$dB = \frac{\mu_0}{2}\frac{(indx)R^2}{(R^2+x^2)^{3/2}}$$

Its direction is along the axis of the solenoid. From the geometry, we know

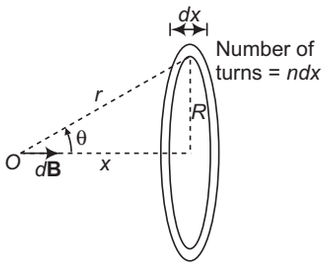


Fig. 20.38

$$x = R\cot\theta$$

$$dx = -R\,\text{cosec}^2\,\theta\cdot d\theta$$

$$\therefore \quad dB = -\frac{1}{2}\mu_0 ni\sin\theta\cdot d\theta$$

Total field B due to the entire solenoid is,

$$B = \frac{1}{2}\mu_0 ni\int_{\theta_1}^{\theta_2}(-\sin\theta)\,d\theta$$

$$\therefore \quad B = \frac{\mu_0 ni}{2}(\cos\theta_2 - \cos\theta_1)$$

If the solenoid is very long $(L >> R)$ and the point O is chosen at the middle, i.e. if $\theta_1 = 180°$ and $\theta_2 = 0°$, then, we get

$$B\,(\text{centre}) = \mu_0 ni \qquad (\text{for } L >> R)$$

At the end of the solenoid,

$$\theta_2 = 0°, \quad \theta_1 = 90°$$

and we get,

$$B\ (\text{end}) = \frac{1}{2}\mu_0 ni \qquad (\text{for } L >> R)$$

Thus, the field at the end of a solenoid is just one-half at the centre. The field lines are as shown in Fig. 20.39.

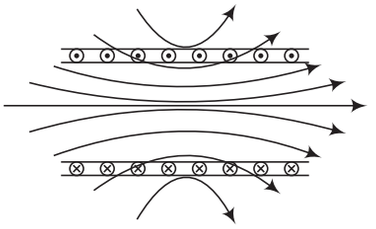
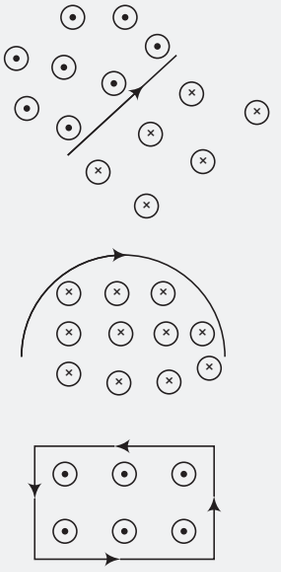


Fig. 20.39

Extra Knowledge Points

- For the direction of **B** when the current carrying wire lies in the plane of paper, the simplest approach is as discussed in Art. 20.7. To the right of the wire magnetic field is ⊗ and to the left of wire it is



Example 20.9 *In a high tension wire electric current runs from East to West. Find the direction of magnetic field at points above and below the wire.*

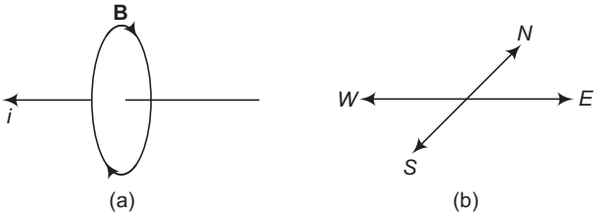


Fig. 20.40

Sol. When the current flows from East to West, magnetic field lines are circular round it as shown in Fig. (a). And so, the magnetic field above the wire is towards North and below the wire towards South.

Example 20.10 *A rectangular polygon of n sides is formed by bending a wire of total length $2\pi R$ which carries a current i.*
Find the magnetic field at the centre of the polygon.

Sol. One side of the polygon is

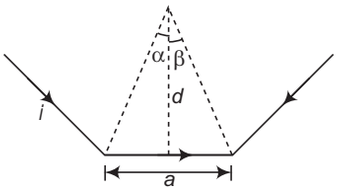


Fig. 20.41

$$a = \frac{2\pi R}{n}$$

$$\alpha = \beta = \frac{\left(\frac{2\pi}{n}\right)}{2} = \frac{\pi}{n}$$

$$\frac{d}{(a/2)} = \cot\alpha$$

$$\therefore \quad d = \left(\frac{a}{2}\right)\cot\alpha = \left(\frac{\pi R}{n}\right)\cot\left(\frac{\pi}{n}\right)$$

All sides of the polygon produce the magnetic field at the centre in same direction (here ⊙). Hence, net magnetic field,

$$B = (n)\text{ (magnetic field due to one side)}$$

$$= n\left\{\frac{\mu_0}{4\pi}\frac{i}{d}(\sin\alpha + \sin\beta)\right\}$$

or $$B = n\left(\frac{\mu_0}{4\pi}\right)\left(\frac{in}{\pi R}\right)\left(\tan\frac{\pi}{n}\right)\left(2\sin\frac{\pi}{n}\right)$$

or $$B = \left(\frac{\mu_0 i}{2R}\right)\frac{\left\{\sin\left(\frac{\pi}{n}\right)\Big/\frac{\pi}{n}\right\}^2}{\cos(\pi/n)}$$

20.9 Ampere's Circuital Law

The line integral $\oint \mathbf{B}\cdot d\mathbf{l}$ of the resultant magnetic field along a closed, plane curve is equal to μ_0 times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant. Thus,

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0\,(i_{\text{net}}) \qquad \ldots(\text{i})$$

This is known as **Ampere's circuital law**.

Eq. (i) in simplified form can be written as,

$$Bl = \mu_0\,(i_{\text{net}}) \qquad \ldots(\text{ii})$$

But this equation can be used only under following conditions.

(i) At every point of the closed path $\mathbf{B}\,||\,d\mathbf{l}$.

(ii) Magnetic field has the same magnitude B at all places on the closed path.

If this is not the case, then Eq. (i) is written as,

$$B_1 dl_1\cos\theta_1 + B_2 dl_2\cos\theta_2 + \ldots = \mu_0\,(i_{\text{net}})$$

where, θ_1 is the angle between $\mathbf{B}_1$ and $d\mathbf{l}_1$, θ_2 the angle between $\mathbf{B}_2$ and $d\mathbf{l}_2$ and so on. Besides, the Biot-Savart law, Ampere's law gives another method to calculate the magnetic field due to a given current distribution. Ampere's law may be derived from the Biot-Savart law and Biot-Savart law may be derived from the Ampere's law. However, Ampere's law is more useful under certain symmetrical conditions. To illustrate the theory now let us take few applications of Ampere's circuital law.

Magnetic Field Created by a Long Current Carrying Wire

A long straight wire of radius R carries a steady current i that is uniformly distributed through the cross-section of the wire.

For finding the behaviour of magnetic field due to this wire, let us divide the whole region into two parts. One is $r \geq R$ and the another is $r < R$. Here, r is the distance from the centre of the wire.

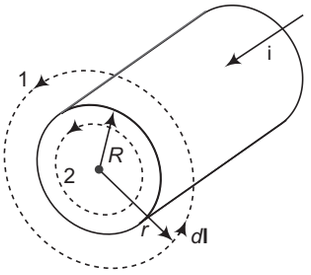


Fig. 20.42

For $r \geq R$ We should get the same result as was obtained by applying Biot-Savart law. Let us choose for our path of integration circle 1. From symmetry $\mathbf{B}$ must be constant in magnitude and parallel to $d\mathbf{l}$ at every point on this circle. Because the total current passing through the plane of the circle is i. Ampere's law gives,

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0\,(i_{\text{net}})$$

or $$Bl = \mu_0(i) \qquad \text{(simplified form)}$$

or $$B\,(2\pi r) = \mu_0 i$$

$$\therefore \quad B = \frac{\mu_0}{2\pi}\frac{i}{r} \qquad (\text{for } r \geq R) \quad \ldots(\text{iii})$$

For $r < R$ Here the current i' passing through the plane of circle 2 is less than the total current i. Because the current is uniform over the cross-section of the wire, the fraction of the current enclosed by circle 2 must equal the ratio of the area πr^2 enclosed by circle 2 to the cross-sectional area πR^2 of the wire.

$$\frac{i'}{i} = \frac{\pi r^2}{\pi R^2} \quad \Rightarrow \quad \therefore \quad i' = \left(\frac{r^2}{R^2}\right) i$$

Following the same procedure as for circle 1, we apply Ampere's law to circle 2.

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \, (i_{\text{net}})$$

$$Bl = \mu_0 \, (i') \qquad \text{(simplified form)}$$

$$\therefore \qquad B \, (2\pi r) = \mu_0 \left(\frac{r^2}{R^2} \right) i$$

$$\therefore \qquad B = \left(\frac{\mu_0 i}{2\pi R^2} \right) r \ (\text{For } r < R) \qquad \ldots(\text{iv})$$

This result is similar in form to the expression for the electric field inside a uniformly charged sphere. The magnitude of the magnetic field *versus r* for this configuration is plotted in figure. Note that inside the wire $B \to 0$ as $r \to 0$. Note also that Eqs. (iii) and (iv) give the same value of the magnetic field at $r = R$, demonstrating that the magnetic field is continuous at the surface of the wire.

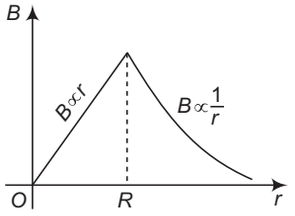


Fig. 20.43

Magnetic Field of a Solenoid

A solenoid is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire, which we shall call the interior of the solenoid, when the solenoid carries a current. When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns (as done in Art. 20.8). If the turns are closely spaced and the solenoid is of infinite length, the magnetic field lines are as shown in Fig. 20.44.

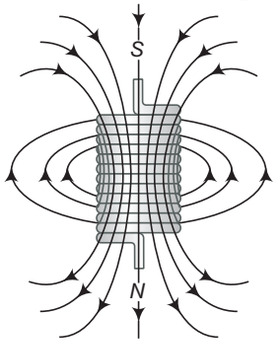


Fig. 20.44

One end of the solenoid behaves like the North pole (N) and the opposite end behaves like the South pole (S). As the length of the solenoid increases, the interior field becomes more uniform and the exterior field becomes weaker. An ideal solenoid is approached when the turns are closely spaced and the length is much greater than the radius of the turns.

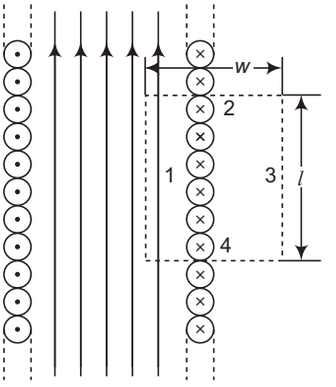


Fig. 20.45

In this case, the external field is zero, and the interior field is uniform over a great volume.

We can use Ampere's law to obtain an expression for the interior magnetic field in an ideal solenoid. Fig. 20.45 shows a longitudinal cross-section of part of such a solenoid carrying a current *i*. Because the solenoid is ideal, **B** in the interior space is uniform and parallel to the axis, and **B** in the exterior space is zero.

Consider the rectangular path of length *l* and width *w* shown in figure. We can apply Ampere's law to this path by evaluating the line integral $\mathbf{B} \cdot d\mathbf{l}$ over each side of the rectangle.

$$\int (\mathbf{B} \cdot d\mathbf{l})_{\text{side 3}} = 0 \quad \text{as} \quad B = 0$$

$$\int (\mathbf{B} \cdot d\mathbf{l})_{\text{side 2 and 4}} = 0$$

as $\mathbf{B} \perp d\mathbf{l}$ or $B = 0$ along these paths

$\int (\mathbf{B} \cdot d\mathbf{l})_{\text{side 1}} = Bl$ as **B** is uniform and parallel to $d\mathbf{l}$.

The integral over the closed rectangular path is therefore,

$$\oint \mathbf{B} \cdot \, d\mathbf{l} = Bl$$

The right side of Ampere's law involves the total current passing through the area bounded by the path of integration.

In this case,

i_{net} = (number of turns inside the area)

(current through each turn)

$= (nl)\,(i)$ (n = number of turns per unit length)

Using Ampere's law,

$$\oint \mathbf{B} \cdot \, d\mathbf{l} = \mu_0 \, (i_{\text{net}})$$

or $$Bl = (\mu_0)\,(nli)$$

or $$B = \mu_0 ni \qquad \ldots(\text{v})$$

This result is same as obtained in Art. 20.8. Eq. (v) is valid only for points near the centre (that is far from the ends) of a very long solenoid. The field near each end is half the value given by Eq. (v).

Example 20.11 *A device called a toroid (figure) is often used to create an almost uniform magnetic field in some enclosed area. The device consists of a conducting wire wrapped around a ring (a torus) made of a non-conducting material. For a toroid having N closely spaced turns of wire, calculate the magnetic field in the region occupied by the torus, a distance r from the centre.*

Sol. To calculate this field, we must evaluate $\oint \mathbf{B} \cdot d\mathbf{l}$ over the circle of radius *r*. By symmetry we see that the magnitude of the field is constant on this circle and tangent to it.

So, $$\oint \mathbf{B} \cdot d\mathbf{l} = Bl = B(2\pi r)$$

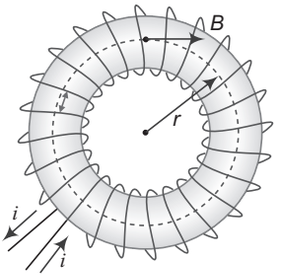


Fig. 20.46

Furthermore, the circular closed path surrounds *N* loops of wire, each of which carries a current *i.* Therefore, right side of Eq. (i) is $\mu_0 Ni$ in this case.

$$\therefore \quad \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (i_{net})$$

or $$B(2\pi r) = \mu_0 Ni$$

or $$B = \frac{\mu_0 Ni}{2\pi r}$$

This result shows that $B \propto \frac{1}{r}$ and hence is non-uniform in the region occupied by torus. However, if *r* is very large compared with the cross-sectional radius of the torus, then the field is approximately uniform inside the torus. In that case,

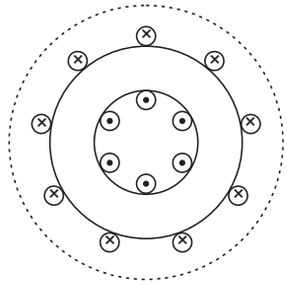


Fig. 20.47

$\frac{N}{2\pi r} = n =$ number of turns per unit length of torus

$$\therefore \quad B = \mu_0 ni$$

For an ideal toroid, in which turns are closely spaced, the external magnetic field is zero. This is because the net current passing through any circular path lying outside the toroid is zero. Therefore, from Ampere's law we find that $B = 0$, in the regions exterior to the torus.

20.10 Force between Parallel Current Carrying Wires

Consider two long wires 1 and 2 kept parallel to each other at a distance *r* and carrying currents i_1 and i_2 respectively in the same direction.

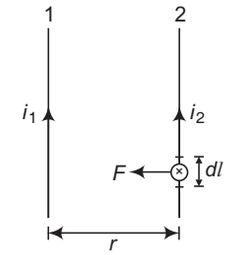


Fig. 20.48

Magnetic field on wire 2 due to current in 1 is,

$$B = \frac{\mu_0}{2\pi} \cdot \frac{i_1}{r} \qquad [\text{in } \otimes \text{ direction}]$$

Magnetic force on a small element *dl* of wire 2 due to this magnetic field is,

$$d\mathbf{F} = i_2 (d\mathbf{l} \times \mathbf{B})$$

Magnitude of this force is,

$$dF = i_2 [(dl)(B) \sin 90°]$$

$$= i_2 (dl) \left(\frac{\mu_0}{2\pi} \frac{i_1}{r} \right)$$

$$= \frac{\mu_0}{2\pi} \cdot \frac{i_1 i_2}{r} \cdot dl$$

Direction of this force is along $d\mathbf{l} \times \mathbf{B}$ or towards the wire 1.

The force per unit length of wire 2 due to wire 1 is,

$$\frac{dF}{dl} = \frac{\mu_0}{2\pi} \frac{i_1 i_2}{r}$$

The same force acts on wire 1 due to wire 2. The wires attract each other, if currents in the wires are flowing in the same direction and they repel each other, if the currents are in opposite directions.

Example 20.12 *A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of* 20 A, *is in equilibrium at a height of* 0.01 m *over another parallel long wire CD which is fixed in a horizontal plane and carries a steady current of* 30 A, *as shown in figure. Show that when AB is slightly depressed, it executes simple harmonic motion. Find the period of oscillations.*



Fig. 20.49

Sol. Let m be the mass per unit length of wire AB. At a height x above the wire CD, magnetic force per unit length on wire AB will be given by

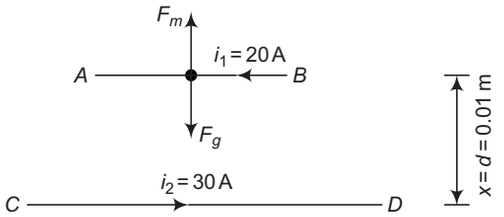


Fig. 20.50

$$F_m = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{x} \quad \text{(upwards)} \qquad \ldots(i)$$

Weight per unit length of wire AB is

$$F_g = mg \quad \text{(downwards)}$$

where, m = mass per unit length of wire AB

At $x = d$, wire is in equilibrium, i.e.

$$F_m = F_g$$

or $$\frac{\mu_0}{2\pi}\cdot\frac{i_1 i_2}{d} = mg$$

or $$\frac{\mu_0}{2\pi}\frac{i_1 i_2}{d^2} = \frac{mg}{d} \qquad \ldots(ii)$$

When AB is depressed, x decreases therefore, F_m will increase, while F_g remains the same. Let AB is displaced by dx downwards. Differentiating Eq. (i) w.r.t. x, we get

$$dF_m = -\frac{\mu_0}{2\pi}\cdot\frac{i_1 i_2}{x^2}dx \qquad \ldots(iii)$$

i.e. restoring force, $F = dF_m$

which is proportional to $-dx$.

Hence, the motion of wire is simple harmonic.

From Eqs. (ii) and (iii), we can write

$$dF_m = -\left(\frac{mg}{d}\right)dx \qquad (x = d)$$

$\therefore$ Acceleration of wire $a = -\left(\frac{g}{d}\right)dx$

Hence, period of oscillation

$$T = 2\pi\sqrt{\left|\frac{dx}{a}\right|} = 2\pi\sqrt{\left|\frac{\text{Displacement}}{\text{Acceleration}}\right|}$$

or $$T = 2\pi\sqrt{\frac{d}{g}} = 2\pi\sqrt{\frac{0.01}{9.8}}$$

or $$T = 0.2 \text{ s}$$

20.11 Cyclotron

In 1932, Lawrence developed a machine named cyclotron, for the acceleration of charged particles, such as protons or deuterons. These particles (ions), are caused to move in circular orbits by magnetic field and are accelerated by the electric field.

In its simplest form, it consists of two flat semicircular metal boxes, called dees because of their shape. These hollow chambers have their diametric edges parallel and slightly separated from each other. A radiofrequency alternating potential of the order of megacycles per second is applied between the dees, which act as electrodes. The vessel is placed between the poles of a strong electromagnet which provides a magnetic field perpendicular to the plane of the dees. A filament situated at the centre of the chamber is heated and a small potential difference is applied between the filament and the metal box to increase the energy of emitted electrons. These electrons ionise some of gas atoms and thus, produce positive ions. Another way of producing ions is the use of ion source.

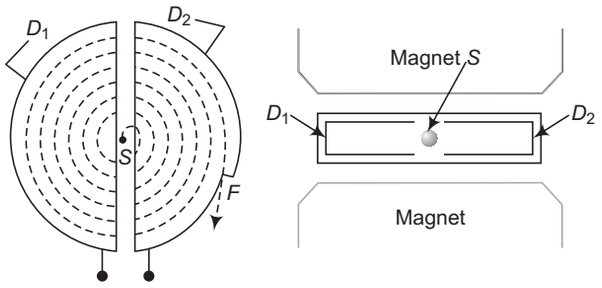


Fig. 20.51

Suppose that at any particular instant the alternating potential is in the direction which makes D_1 positive and D_2 negative. A positive ion of mass M, charge q and an initial velocity v starting from the source S will be attracted by the dee D_2. Due to magnetic field B it will move in a circular path of radius r inside the dee D_2, where

$$r = Mv/Bq$$

In the interior of the dee, the speed of the ion remains constant. After it has traversed half a cycle the ion comes to the edge of D_2. If in the mean time, the potential difference between D_1 and D_2 has changed direction, so that D_2 is now positive and D_1 negative, the positive ion will receive an additional acceleration, while going across the gap between the dees and then travel in a circular path of larger radius inside D_1 under the influence of magnetic field. After traversing a half cycle in D_2 it will reach the edge of D_1 and receive an additional acceleration between the gaps because in the meantime the direction of potential difference between the dees has changed. The ion will continue travelling in a semi-circle of increasing radii, the direction of

PD changes everytime the ion goes from D_1 to D_2 and from D_2 to D_1. The time taken by the charged particle to traverse the semicircular path in the dee is given by

$$t = \pi r / v = \pi M / Bq \qquad \ldots(i)$$

This relation indicates that time t is independent of the velocity of the particle and of the radius. For any given value of M/q, it is determined by the magnetic field intensity. By adjusting the magnetic field intensity, the time can be made the same as that required to change the potentials. On the other hand, the oscillator frequency can also be adjusted to the nature of a given ion and to the strength of the magnetic field. The frequency of the oscillations required to keep the ion in phase is given by the relation

$$f = 1/T = 1/2t = Bq/2\pi M \qquad \ldots(ii)$$

If the oscillation frequency is adjusted to keep the charged ion always in phase, each time the ion crosses the gap it receives an additional energy and at the same time it describes a flat sprial of increasing radius. Eventually, the ion reaches the periphery of the dee, where it can be brought out of the chamber by means of a deflecting plate charged to a high negative potential. This attractive force draws the ion out of its spiral path and thus, can be used easily. If R is the radius of the dee, kinetic energy of the ion emerging from the cyclotron is thus, given by

$$\begin{aligned} E &= \frac{1}{2} M\,(BqR/M)^2 \\ &= B^2R^2q^2/2M \\ &= 2\pi^2R^2f^2M \qquad \ldots(iii) \end{aligned}$$

This relation indicates that the maximum energy attained by the ion is limited by the radius R, magnetic field B or the frequency of the alternating potential f. It is independent of the alternating voltage. It can be explained by the fact that when the voltage is low the ion makes a large number of turns before reaching the periphery, but when the voltage is high the number of turns is small. The total energy remains same in both the cases provided B and R are unchanged.

Chapter Summary With Formulae

(i) Lorentz force on a charged particle

$$\mathbf{F} = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$

(ii) Magnetic force on a charged particle

(a) $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

(b) $F_m = Bqv\sin\theta$

(c) $F_m = 0$, when $B = 0$ or $q = 0$ or $v = 0$ or $\theta = 0^\circ$ or $\theta = 180^\circ$

(d) Direction of $\mathbf{F}_m$ is given either by Fleming's left hand rule or right hand rule.

(iii) Path of charged particle in uniform magnetic field

(a) depends on θ, the angle between $\mathbf{v}$ and $\mathbf{B}$

(b) If $\theta = 0^\circ$ or 180°, path is straight line

(c) If $\theta = 90^\circ$, path is a circle of

Radius, $$r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2Km}}{Bq} = \frac{\sqrt{2qvm}}{Bq}$$

Time period, $$T = \frac{2\pi m}{Bq}$$

Frequency, $$f = \frac{1}{T} = \frac{Bq}{2\pi m}$$ and,

Angular velocity, $$\omega = \frac{2\pi}{T} = 2\pi f = \frac{Bq}{m}$$

(d) If $\theta \neq 0^\circ$, 180° or 90°, then path is helix of

Radius, $$r = \frac{mv\sin\theta}{Bq}$$

Time period, $$T = \frac{2\pi m}{Bq}$$

Frequency, $$f = \frac{Bq}{2\pi m}$$ and

Pitch , $$p = (v\cos\theta)\,T = \frac{2\pi m}{Bq}(v\cos\theta)$$

(iv) Magnetic force on a current carrying conductor

(a) $\mathbf{F}_m = i(\mathbf{l} \times \mathbf{B})$

(b) $F_m = ilB\sin\theta$

(c) Direction of $\mathbf{F}_m$ is given by Fleming's left hand rule or right hand rule

(d) Total magnetic force on a current carrying loop in uniform magnetic field is zero.

(v) Work done by a magnetic force is always zero. Therefore a magnetic force cannot increase or decrease speed or kinetic energy of a particle.

(vi) Velocity selector

(a) Path of a charged particle in uniform electric and magnetic fields is zero if

$$\mathbf{F}_{\text{net}} = 0 \quad \text{or} \quad \mathbf{F}_e + \mathbf{F}_m = 0 \quad \text{or} \quad q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})] = 0 \quad \text{or} \quad \mathbf{E} = \mathbf{B} \times \mathbf{v}$$

(b) If $\mathbf{E}$, $\mathbf{v}$ and $\mathbf{B}$ are mutually perpendicular, then

$$\mathbf{v} = \mathbf{E/B}$$

(vii) Cyclotron

(a) Frequency of charged particles inside dees = Cyclotron's oscillator frequency or $f = \frac{Bq}{2\pi m}$

(b) After every half rotation speed and radius of circular path increase but frequency remains unchanged.

(c) If R is the radius of dee and v_f the final speed of charged particle, then

$$R = \frac{mv_f}{Bq} \quad \text{or} \quad v_f = \frac{BqR}{m}$$

$\therefore$ Final kinetic energy,

$$K_f = \frac{1}{2}mv_f^2 = \frac{B^2q^2R^2}{2m}$$

(i) **Biot-Savart Law**

(a) $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{i\,(d\mathbf{l} \times \mathbf{r})}{r^3}$

(b) $dB = \frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2}$

(c) Direction of $d\mathbf{B}$ can be given by right hand screw law.

(ii) $\frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$ = speed of light in vacuum.

(iii) $\frac{\mu_0}{4\pi} = 10^{-7}$ T - M/A.

(iv) Magnetic field on the axis of a circular current carrying loop

(a) $B = \frac{\mu_0 NiR^2}{2(R^2 + x^2)^{3/2}}$

(b) At centre, $B = \frac{\mu_0 Ni}{2R}$

(c) At centre due to arc of a circle, $B = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 Ni}{2R}\right)$

(v) Line integral of magnetic field over a path

(a) $\int \mathbf{B}\cdot d\mathbf{l}$

(b) $\int \mathbf{B}\cdot d\mathbf{l} = 0$, if magnetic lines are perpendicular to the path at all points.

(c) $\int \mathbf{B}\cdot d\mathbf{l} = Bl$, if magnetic lines are tangential at all points and magnitude of magnetic field is uniform.

(vi) Ampere's circuital Law

(a) $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0(i_{\text{net}})$

(b) Line integral of magnetic field over a closed path = μ_0 times the net current crossing the area bounded by that closed path

(c) Simplified version of Ampere's circuital law is

$$Bl = \mu_0\,(i_{\text{net}})$$

This simplified version is applicable in that closed path in which magnetic lines are tangential at all points and magnitude of magnetic field is uniform.

(vii) Magnetic field due to an infinitely long straight wire

$$B = \frac{\mu_0}{2\pi}\frac{i}{r}$$

(viii) Magnetic field inside a long solenoid is almost uniform and given by $B = \mu_0 ni$

where, n = number of turns per unit length.

(ix) Magnetic field outside a toroid is zero, but inside it, almost uniform (in magnitude) and is given by

$$B = \mu_0 ni$$

(x) Force between two parallel current carrying wires

(a) Force per unit length = $\frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$

(b) Wires attract each other if currents are in the same direction and they repel each other if they are in opposite directions.

(xi) **Magnetic dipole**

(a) Every current carrying loop is a magnetic dipole.

(b) Magnitude of magnetic dipole moment is given by, $M = NiA$

(c) Direction of $\mathbf{M}$ is given by right hand rule.

(xii) Magnetic dipole in uniform magnetic field

(a) $\mathbf{F}_{\text{net}} = 0$

(b) $\tau = \mathbf{M} \times \mathbf{B}$

(c) $\tau = MB\sin\theta$

(d) $U = -\mathbf{M}\cdot\mathbf{B} = -MB\cos\theta$

(e) $W_{\theta_1 \to \theta_2} = MB\,(\cos\theta_1 - \cos\theta_2)$

(f) At $\theta = 0^\circ$, $F = 0$, $\tau = 0$, $U = -MB$ or minimum. Therefore, stable equilibrium.

(g) At $\theta = 180^\circ$, $F = 0$, $\tau = 0$, $U = +MB$ or maximum. Therefore, unstable equilibrium.

where, θ is the angle between $\mathbf{M}$ and $\mathbf{B}$.

Additional Examples

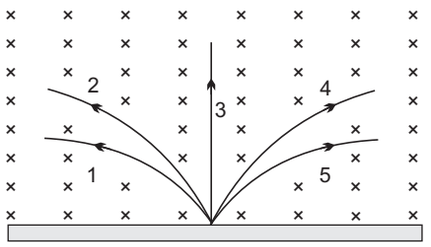
Example 1. *Both the electric and magnetic fields can deflect a charged particle. What is the difference between these deflections?*

Sol. In electric field deflection takes place with change in kinetic energy or speed. But in magnetic field deflection takes place without change in speed or kinetic energy.

Example 2. *In magnetic field speed of charged particle remains constant. Can we say that acceleration of a charged particle in a magnetic field is always zero?*

Sol. No, we cannot say that acceleration is always zero. Speed always remains constant but velocity may change. And acceleration is rate of change of velocity (not rate of change of speed). Therefore, acceleration may be non-zero also.

Example 3. *Five particles, a neutron, a proton, an* α*-particle, an electron and a* Cl^- *ion enter a uniform magnetic field perpendicularly with same speeds. Their paths are shown in figure. Identify the particles according to their path.*



Sol. Particle-3 is a neutral particle or neutron because its path is undeviated. Or magnetic force on this particle is zero. Rest four particles enter perpendicularly. So their paths are circles. From Fleming's left hand rule, we can see that magnetic force on positive particles is towards left and on negative particles towards right. Therefore, 1 and 2 are positively charged and 4 and 5 are negatively charged.

Radius of circular path is given by,

$$r = \frac{mv}{Bq}$$

or $r \propto \frac{m}{q}$ (as B and v are same for all particles)

$$\left(\frac{m}{q}\right)_{\text{proton}} < \left(\frac{m}{q}\right)_{\alpha\text{-particle}}$$

$$\therefore \quad r_P < r_{\alpha-\text{particle}}$$

So, particle-1 is proton and particle-2 is α-particle.

Similarly,

$$\left(\frac{m}{q}\right)_{e^-} < \left(\frac{m}{q}\right)_{Cl^-}$$

Therefore particle-5 is an electron and particle-4 is Cl^-.

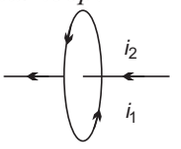
Example 4. *Describe the motion of a charged particle in a cyclotron if the frequency of the cyclotron's oscillator is doubled.*

Sol. Particle will accelerate and decelerate alternatively. So, the radius of path in the Dees will remain unchanged.

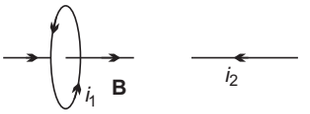
Example 5. *Can we have a single North pole or a single South pole?*

Sol. No, in magnetics, a monopole does not exist. In electrostatics monopoles, $(+q \text{ or } -q)$ can exist separately.

Example 6. *A circular loop carries a current* i_1*. A straight wire carrying a current* i_2 *is placed along its axis. What is the direction of magnetic force on straight wire due to the circular loop?*



Sol. Force on i_2 due to i_1 is zero. This is because direction of $\mathbf{B}$ due to current i_1 is anti-parallel to i_2. So using the expression,



$$F = ilB\sin\theta$$

We see that $F = 0$ at $\theta = 180°$.

Example 7. *A magnetic field of* $(4.0\times10^{-3}\ \hat{\mathbf{k}})$ T *exerts a force* $(4.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}})\times10^{-10}$ N *on a particle having a charge* 10^{-9} C *and moving in the x-y plane. Find the velocity of the particle.*

Sol. Given, $\mathbf{B} = (4\times10^{-3}\ \hat{\mathbf{k}})$ T,

$$q = 10^{-9}\ \text{C}$$

Magnetic force $\mathbf{F}_m = (4.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}})\times10^{-10}$ N

Let velocity of the particle in x-y plane be,

$$\mathbf{v} = v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}}$$

Then from the relation,

$$\mathbf{F}_m = q\,(\mathbf{v}\times\mathbf{B})$$

We have,

$$(4.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}})\times10^{-10} = 10^{-9}\,[(v_x\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}})\times(4\times10^{-3}\ \hat{\mathbf{k}})]$$

$$= (4v_y\times10^{-12}\ \hat{\mathbf{i}} - 4v_x\times10^{-12}\ \hat{\mathbf{j}})$$

Comparing the coefficients of $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ we have,

$$4\times10^{-10} = 4v_y\times10^{-12}$$

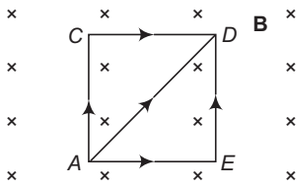
$$\therefore \quad v_y = 10^2\ \text{m/s} = 100\ \text{m/s}$$

and $3.0\times 10^{-10} = -4v_x \times 10^{-12}$

$\therefore \quad v_x = -75 \text{ m/s}$

$\therefore \quad \mathbf{v} = -75\hat{\mathbf{i}} + 100\hat{\mathbf{j}}$

Example 8. *A square of side* 2.0 m *is placed in a uniform magnetic field* $\mathbf{B} = 2.0$ T *in a direction perpendicular to the plane of the square inwards. Equal current* $i = 3.0$ A *is flowing in the directions shown in figure. Find the magnitude of magnetic force on the loop.*



Sol. Force on wire *ACD* = Force on *AD* = Force on *AED*

$\therefore$ Net force on the loop $= 3(\mathbf{F}_{AD})$

or $F_{\text{net}} = 3(i)(AD)(B)$

$= (3)\ (3.0)\ (2\sqrt{2})\ (2.0)$ N

$= 36\sqrt{2}$ N

Direction of this force is towards *EC*.

Example 9. *A charged particle carrying charge* $q = 1\mu$C *moves in uniform magnetic field with velocity* $v_1 = 10^6$ m/s *at angle* $45°$ *with x-axis in the x-y plane and experiences a force* $F_1 = 5\sqrt{2}$ mN *along the negative z-axis. When the same particle moves with velocity* $v_2 = 10^6$ m/s *along the z-axis it experiences a force* F_2 *in y-direction. Find*

(a) magnitude and direction of the magnetic field,

(b) the magnitude of the force F_2.

Sol. F_2 is in y-direction when velocity is along z-axis. Therefore, magnetic field should be along x-axis.

So let, $\mathbf{B} = B_0\hat{\mathbf{i}}$

(a) Given, $\mathbf{v}_1 = \dfrac{10^6}{\sqrt{2}}\hat{\mathbf{i}} + \dfrac{10^6}{\sqrt{2}}\hat{\mathbf{j}}$

and $\mathbf{F}_1 = -5\sqrt{2}\times 10^3\ \hat{\mathbf{k}}$

From the equation, $\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$

We have,

$$(-5\sqrt{2}\times 10^{-3})\,\hat{\mathbf{k}} = (10^{-6})\left[\left(\frac{10^6}{\sqrt{2}}\hat{\mathbf{i}} + \frac{10^6}{\sqrt{2}}\hat{\mathbf{j}}\right)\times(B_0\hat{\mathbf{i}})\right]$$

$$= -\frac{B_0}{\sqrt{2}}\hat{\mathbf{k}}$$

$\therefore \quad \dfrac{B_0}{\sqrt{2}} = 5\sqrt{2}\times 10^{-3}$

or $B_0 = 10^{-2}$ T

Therefore, the magnetic field is,

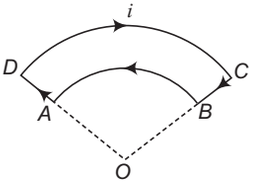
$\mathbf{B} = (10^{-2}\ \hat{\mathbf{i}})$ T

(b) $F_2 = B_0 q v_2 \sin 90°$

As the angle between $\mathbf{B}$ and $\mathbf{v}$ in this case is $90°$.

$\therefore \quad F_2 = (10^{-2})(10^{-6})(10^6) = 10^{-2}$ N

Example 10. *Figure shows a current loop having two circular arcs joined by two radial lines. Find the magnetic field B at the centre O.*



Sol. Magnetic field at point *O*, due to wires *CB* and *AD* will be zero.

Magnetic field due to wire *BA* will be,

$$B_1 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2a}\right)$$

Direction of field $\mathbf{B}_1$ is coming out of the plane of the figure.

Similarly, field at *O* due to arc *DC* will be,

$$B_2 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2b}\right)$$

Direction of $\mathbf{B}_2$ is going into the plane of the figure. The resultant field at *O* is,

$$B = B_1 - B_2 = \frac{\mu_0\, i\,\theta\,(b-a)}{4\pi\, ab}$$

Coming out of the plane.

Example 11. *The magnetic field B due to a current carrying circular loop of radius* 12 cm *at its centre is* 0.5×10^{-4} T. *Find the magnetic field due to this loop at a point on the axis at a distance of* 5.0 cm *from the centre.*

Sol. Magnetic field at the centre of a circular loop is,

$$B_1 = \frac{\mu_0\, i}{2R}$$

and that at an axial point,

$$B_2 = \frac{\mu_0\, iR^2}{2(R^2 + x^2)^{3/2}}$$

Thus, $\dfrac{B_2}{B_1} = \dfrac{R^3}{(R^2 + x^2)^{3/2}}$

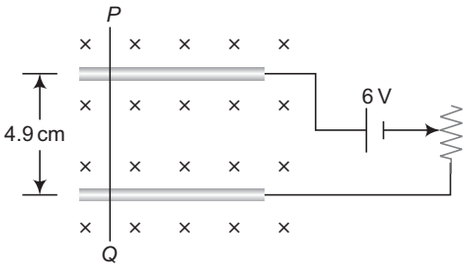
or $B_2 = B_1\left[\dfrac{R^3}{(R^2 + x^2)^{3/2}}\right]$

Substituting the values, we have

$$B_2 = (0.5\times 10^{-4})\left[\frac{(12)^3}{(144 + 25)^{3/2}}\right]$$

$$= 3.9\times 10^{-5} \text{ T}$$

Example 12. *A wire PQ of mass* 10 g *is at rest on two parallel metal rails. The separation between the rails is* 4.9 cm. *A magnetic field of* 0.80 T *is applied perpendicular to the plane of the rails, directed downwards. The resistance of the circuit is slowly decreased. When the resistance decreases to below* 20 Ω, *the wire PQ begins to slide on the rails. Calculate the coefficient of friction between the wire and the rails.*



Sol. Wire PQ begins to slide when magnetic force is just equal to the force of friction, i.e.

$$\mu mg = il\,B \sin\theta \qquad (\theta = 90°)$$

Here, $$i = \frac{E}{R} = \frac{6}{20} = 0.3 \text{ A} \Rightarrow \therefore \mu = \frac{il\,B}{mg}$$

$$= \frac{(0.3)\,(4.9\times 10^{-2})\,(0.8)}{(10\times 10^{-3})\,(9.8)}$$

$$= 0.12$$

Example 13. *What is the smallest value of B that can be set up at the equator to permit a proton of speed* 10^7 m/s *to circulate around the earth?* ($R = 6.4\times 10^6$ m, $m_p = 1.67\times 10^{-27}$ kg).

Sol. From the relation,

$$r = \frac{mv}{Bq}$$

We have, $$B = \frac{mv}{qr}$$

Substituting the values, we have

$$B = \frac{(1.67\times 10^{-27})\,(10^7)}{(1.6\times 10^{-19})\,(6.4\times 10^6)}$$

$$= 1.6\times 10^{-8} \text{ T}$$

Example 14. *Deuterons in a cyclotron describes a circle of radius* 32.0 cm. *Just before emerging from the deuterons. The frequency of the applied alternating voltage is* 10 MHz. *Find,*

(a) the magnetic flux density (i.e. *the magnetic field*).

(b) the energy and speed of the deuterons upon emergence.

Sol. (a) Frequency of the applied emf = Cyclotron frequency

or $$f = \frac{Bq}{2\pi m}$$

$$\therefore \quad B = \frac{2\pi mf}{q}$$

$$= \frac{(2)\,(3.14)\,(2\times 1.67\times 10^{-27})\,(10\times 10^6)}{1.6\times 10^{-19}}$$

$$= 1.30 \text{ T}$$

(b) The speed of deuterons on the emergence from the cyclotron,

$$v = \frac{BqR}{m} = 2\pi fR$$

$$= (2)\,(3.14)\,(10\times 10^6)\,(32\times 10^{-2})$$

$$= 2.01\times 10^7 \text{ m/s}$$

$$\therefore \text{ Energy of deuterons} = \frac{1}{2}mv^2$$

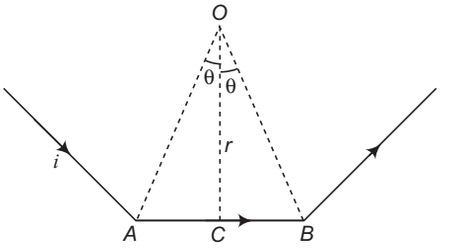
$$= \frac{1}{2}\times (2\times 1.67\times 10^{-27})\,(2.01\times 10^7)^2 \text{ J}$$

$$= 4.22 \text{ MeV}$$

✔ 1 MeV = 1.6×10^{-13} J

Example 15. *A wire shaped to a regular hexagon of side* 2 cm *carries a current of* 2 A. *Find the magnetic field at the centre of the hexagon.*

Sol.



$$\theta = 30°$$

$$\frac{BC}{OC} = \tan\theta \qquad (BC = 1 \text{ cm})$$

$$\therefore \quad \frac{1}{r} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\therefore \quad r = \sqrt{3} \text{ cm}$$

Net magnetic field at O = 6 times the magnetic field due to one side.

$$\therefore \quad B = 6\left[\frac{\mu_0}{2\pi}\frac{i}{r}(\sin\theta + \sin\theta)\right]$$

$$= \frac{6\,(2\times 10^{-7})\,(2)}{\sqrt{3}\times 10^{-2}}\left(\frac{1}{2}+\frac{1}{2}\right)$$

$$= 1.38\times 10^{-4} \text{ T}$$

Example 16. *In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a path of radius* 5×10^{-11} m *at a frequency of* 6.8×10^{15} Hz.

(a) What value of magnetic field is set up at the centre of the orbit?

(b) What is the equivalent magnetic dipole moment?

Sol. (a) An electron moving around the nucleus is equivalent to a current,

$$i = q\,f$$

Magnetic field at the centre,

$$B = \frac{\mu_0 i}{2R} = \frac{\mu_0 q\,f}{2R}$$

Substituting the values, we have

$$B = \frac{(4\pi\times10^{-7})(1.6\times10^{-19})(6.8\times10^{15})}{2\times5.1\times10^{-11}}$$

$$= 13.4 \text{ T}$$

(b) The current carrying circular loop is equivalent to a magnetic dipole, with magnetic dipole moment,

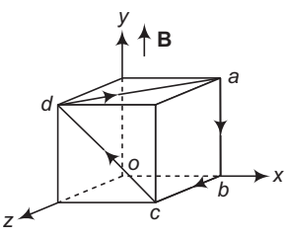
$$M = NiA = (Nq\,f\,\pi R^2)$$

Substituting the values, we have

$$M = (1)(1.6\times10^{-19})(6.8\times10^{15})(3.14)(5.1\times10^{-11})^2$$

$$= 8.9\times10^{-24} \text{ A-m}^2$$

Example 17. *In figure, the cube is* 40*cm on each side. Four straight segments of wire ab, bc, cd and da form a closed loop that carries a current* $i = 5.0$ A *in the direction shown. A uniform magnetic field of magnitude* $B = 0.02$ T *is in the positive y-direction. Determine the magnitude and direction of the magnetic force on each segment and the torque on the loop.*



Sol. Net force on a current carrying loop is zero. Therefore, torque can be calculated about any point. We will here calculate the torque of all the forces about origin *O*. Magnetic force on all the wire will act at their mid-point. So, in $\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F}$, $\mathbf{r}$ is the position vector of mid points of the wire. Force and torque in different wires in tabular form are given. Given $\mathbf{B} = (0.2\,\hat{\mathbf{j}})$ T and $i = 5.0$ A.

Name of the wire	l	$F_m = \hat{i}\,(l\times B)$	Position vector of centre point r	Torque, $\tau = r\times F_m$
ab	$(-0.4\,\hat{j})$ m	Zero	Not required	Zero
bc	$(0.4\hat{k})$ m	$(-0.4\,\hat{i})$ N	$(0.4\,\hat{i} + 0.2\,\hat{k})$ m	$(-0.08\,\hat{j})$ N-m
cd	$(-0.4\,\hat{i} + 0.4\,\hat{j})$ m	$(-0.4\hat{k})$ N	$(0.2\,\hat{i} + 0.2\,\hat{j} + 0.4\,\hat{k})$ m	$(0.08\,\hat{j} - 0.08\,\hat{i})$ N-m
da	$(0.4\,\hat{i} - 0.4\hat{k})$ m	$(0.4\,\hat{k} + 0.4\,\hat{i})$ N	$(0.2\,\hat{i} + 0.4\,\hat{j} + 0.2\,\hat{k})$ m	$(0.16\,\hat{i} - 0.16\,\hat{k})$ N-m

From this table we can see that,

$$\mathbf{F}_{\text{net}} = 0 - 0.4\,\hat{\mathbf{i}} - 0.4\,\hat{\mathbf{k}} + 0.4\,\hat{\mathbf{k}} + 0.4\,\hat{\mathbf{i}}$$

$$= \text{zero}$$

As was expected.

The torque on the loop is,

$$\boldsymbol{\tau}_{\text{net}} = 0 - 0.08\,\hat{\mathbf{j}} + 0.08\,\hat{\mathbf{j}} - 0.08\,\hat{\mathbf{i}} + 0.16\,\hat{\mathbf{i}} - 0.16\,\hat{\mathbf{k}}$$

$$= (0.08\,\hat{\mathbf{i}} - 0.16\,\hat{\mathbf{k}}) \text{ N-m}$$

NCERT Selected Questions

Q 1. A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field B at the centre of the coil?

Sol. $$B = \frac{\mu_0 Ni}{2R}$$

$$B = 10^{-7} \times \frac{2 \times 3.14 \times 100 \times 0.40}{8 \times 10^{-2}}$$

$$= 3.14 \times 10^{-4} \text{ T}$$

Q 2. A long straight wire carries a current of 35 A. What is the magnitude of the field B at a point 20 cm from the wire?

Sol. $$B = \frac{\mu_0}{2\pi} \cdot \frac{i}{r}$$

$$\therefore \quad B = 10^{-7} \times \frac{2 \times 35}{20 \times 10^{-2}}$$

$$= 3.5 \times 10^{-5} \text{ T}$$

Q 3. A long straight wire in the horizontal plane carries a current of 50 A in North to South direction. Find the magnitude and direction of B at a point 2.5 m East of the wire.

Sol. $$B = \frac{\mu_0}{2\pi} \cdot \frac{i}{r}$$

$$= 10^{-7} \times \frac{2 \times 50}{2.5}$$

$$= 0.4 \times 10^{-5} \text{ T} = 4 \times 10^{-6} \text{ T}$$

Direction of B Applying right hand thumb rule, we find that the direction of **B** at the observation point is in the vertically upward direction.

Q 4. A horizontal overhead power line carries a current of 90 A in East to West direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?

Sol. Using the relation,

$$B = \frac{\mu_0}{2\pi} \frac{i}{r}; \text{ we get}$$

$$B = 10^{-7} \times \frac{2 \times 90}{1.5} = 1.2 \times 10^{-5} \text{ T}$$

Direction of B According to right hand thumb rule, the direction of the magnetic field is towards the South at the point of observation.

Q 5. What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30° with the direction of a uniform magnetic field of 0.15 T?

Sol. $\therefore \quad F = Bil \sin\theta$

$$F = 0.15 \times 8 \times \sin 30° \text{ (Nm}^{-1}\text{)}$$

$$= 0.15 \times 8 \times \frac{1}{2}$$

$$= 0.15 \times 4$$

$$= 0.60 \text{ Nm}^{-1}$$

Q 6. A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?

Sol. $$F = Bil \sin\theta$$

$$\therefore \quad F = Bil \sin 90°$$

$$= 0.27 \times 10 \times 3 \times 10^{-2}$$

$$= 8.1 \times 10^{-2} \text{ N}$$

Q 7. Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.

Sol. If F' be the force per unit length on wire A, then using

$$F' = \frac{\mu_0}{2\pi} \frac{i_1 i_2}{r}, \text{ we get}$$

$$F' = 10^{-7} \times \frac{2 \times 8 \times 5}{4 \times 10^{-2}} \text{ Nm}^{-1}$$

$$= 20 \times 10^{-5} \text{ Nm}^{-1}$$

If F be the force on a section of length 10 cm of wire A, then

$$F = F' \times l$$

$$= 20 \times 10^{-5} \times 10 \times 10^{-2} \text{ N}$$

$$= 2 \times 10^{-5} \text{ N}$$

Direction of F As the current in the two parallel wires is flowing in the same direction, so the force will be attractive in nature.

Q 8. A closely wound solenoid 80 cm long has 5 layers of winding of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of B inside the solenoid near its centre.

Sol. Length of the long solenoid

$$l = 80 \text{ cm} = 80 \times 10^{-2} \text{ m}$$

Number of turns in each layer $n' = 400$

Number of layers in the solenoid $n = 5$

Total number of turns in the solenoid N

$$= n' \times n = 400 \times 5 = 2000$$

Let, B be the magnitude of the magnetic field inside the solenoid near its centre.

Then,

$$B = \frac{\mu_0 Ni}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 2000 \times 8}{80 \times 10^{-2}}$$

$$= 8\pi \times 10^{-3} \text{ T}$$

Q 9. A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30° with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil?

Sol. Using the formula,

$$\tau = NIA\ B \sin\theta, \text{ we get}$$

$$\tau = 20 \times 12 \times (10 \times 10^{-2})^2 \times 0.80 \times \sin 30°$$

$$= 20 \times 12 \times 10^{-2} \times 0.80 \times \frac{1}{2}$$

$$= 0.96 \text{ N - m}$$

Q 10. In a chamber, a uniform magnetic field of 6.5 G ($1\,\text{G} = 10^{-4}$ T) is maintained. An electron is shot into the field with a speed of 4.8×10^6 ms^{-1} normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. ($e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg)

Sol. The direction of the force is perpendicular to **v** as well as to **B**. So, this force will only change the direction of motion of the electron without affecting its speed and this force provides the necessarily centripetal force to the moving electron, so the electron will move in a circular path. If r be the radius of the circular path followed by the electron, then

$$evB \sin 90° = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{Be}$$

or

$$= \frac{9.1 \times 10^{-31} \times 4.8 \times 10^6}{1.6 \times 10^{-19} \times 6.5 \times 10^{-4}}$$

$$= 4.2 \times 10^{-2} \text{ m}$$

$$= 4.2 \text{ cm}$$

Q 11. In obtain the frequency of revolution of the electron in its circular orbit, does the answer depend on the speed of the electron? Explain.

Sol. Frequency, $f = \frac{v}{2\pi r}$

Substituting the value of r, we get

$$f = \frac{eB}{2\pi m} \qquad \ldots(i)$$

$$= \frac{1.6 \times 10^{-19} \times 6.5 \times 10^{-4}}{2 \times 3.14 \times 9.1 \times 10^{-31}}$$

$$= 18.18 \times 10^{+6} \text{ Hz}$$

$$= 18.18 \text{ MHz}$$

From Eq. (i), we see that the expression of frequency of revolution of the electron does not have the speed of the electron. So, frequency does not depend on the speed of the electron.

Q 12. (a) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60° with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

(b) Would your answer change, if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are unaltered).

Sol. (a) Magnitude of torque acting on the current carrying coil due to the magnetic field is given by,

$$\tau = NIBA \sin\theta$$

$$= 30 \times 6 \times 1 \times \pi \times (8 \times 10^{-2})^2 \times \sin 60°$$

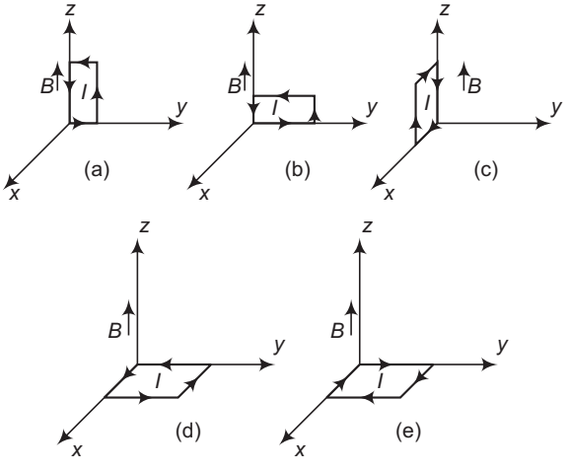
$$\approx 3.133 \text{ N-m}$$

To prevent the coil from rotating, a torque equal and opposite to τ has to be applied.

$\therefore$ Torque required, $\tau = 3.133$ N-m.

(b) No, since the torque on a planar loop is independent of its shape in case the area of the loop is kept same hence the torque will remain unchanged.

Q 13. A uniform magnetic field of 3000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop in the different cases shown in figure. What is the force in each case? Which case corresponds to stable equilibrium?



Sol. Given, B = uniform magnetic field = 3000 G along z-axis

$$= 3000 \times 10^{-4} \text{ T} = 0.3 \text{ T}$$

$\therefore$ A = area of rectangular loop

$$= l \times b = 10 \times 5 = 50 \text{ cm}^2 = 50 \times 10^{-4} \text{ m}^2$$

Torque on the loop is given by

$$\tau = IAB \sin\theta$$

where, θ is the angle between the normal to the plane of the loop and the direction of magnetic field.

(a) Here, $\theta = 90°$

$$\therefore \quad \tau = 0.3 \times 12 \times 50 \times 10^{-4} \times \sin 90°$$

$$= 1.8 \times 10^{-2} \text{ N-m}$$

and acts along negative y-direction.

$$\tau = \mathbf{M} \times \mathbf{B}$$

(b) $\theta = 90°$

$$\therefore \quad \tau = 0.3 \times 12 \times 50 \times 10^{-4} \times \sin 90°$$

$$= 1.8 \times 10^{-2} \text{ N-m}$$

and acts along y'-axis or y-axis

(c) $\theta = 90°$

$$\therefore \quad \tau = 0.3 \times 12 \times 50 \times 10^{-4} \times \sin 90°$$

$$= 1.8 \times 10^{-2} \text{ N-m}$$

and acts along $-x$ direction.

(d) Here $\theta = 0°$

$$\therefore \quad \tau = BIA \sin 0° = 0$$

(e) $\theta = 180°, \therefore \tau = 0$

Net force on a planar loop in a uniform magnetic field is always zero, so force is zero in each case.

Case (d) corresponds to stable equilibrium **M** is aligned with **B**.

Q 14. A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the

(a) total torque on the coil,
(b) total force on the coil,
(c) average force on each electron in the coil due to the magnetic field?

(The coil is made of copper wire of cross-sectional area 10^{-5} m^2 and the free electron density in copper is given to be about 10^{29} m^{-3}).

Sol. (a) Let, n = number of turns in the circular coil = 20

r = radius of the circular coil

$= 10 \text{ cm} = 10 \times 10^{-2}$ m

B = uniform magnetic field = 0.10 T

θ = angle between B and normal to the plane of coil = 0°

I = current in the coil = 5.0 A

A = area of the coil = πr^2

$$= \frac{22}{7} \times (0.10)^2 \text{ m}^2$$

(a) Using the formula,

$$\tau = nBIA \sin\theta$$

We get, $\tau = 20 \times 5 \times 0.10 \times \frac{22}{7} \times (0.10)^2 \times \sin 0°$

$$= 0$$

(b) Total force on a planar current loop in a uniform magnetic field is always zero.

(c) Force on an electron is given by

$$F = Bev_d = Be\left(\frac{I}{neA}\right) = \frac{BI}{nA} \qquad (\because I = neAv_d)$$

where, $n = 10^{29}$ m^{-3} = free electron density in copper.

A' = area of cross-section of copper wire $= 10^{-5}$ m^2

$$\therefore \quad F = \frac{0.10 \times 5}{10^{29} \times 10^{-5}} = 5 \times 10^{-25} \text{ N}$$

Objective Problems

[Level 1]

Lorentz Force and Path of Charged Particle

1. A strong magnetic field is applied on a stationary electron, then the electron
(a) moves in the direction of the field
(b) moves in an opposite direction of the field
(c) remains stationary
(d) starts spinning

2. Lorentz force can be calculated by using the formula
(a) $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ (b) $\mathbf{F} = q(\mathbf{E} - \mathbf{v} \times \mathbf{B})$
(c) $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \cdot \mathbf{B})$ (d) $\mathbf{F} = q(\mathbf{E} \times \mathbf{v} + \mathbf{B})$

3. A proton and a deuteron both having the same kinetic energy, enter perpendicularly into a uniform magnetic field B. For motion of proton and deutron on circular path of radius R_p and R_d respectively, the correct statement is
(a) $R_d = \sqrt{2}R_p$ (b) $R_d = R_p/\sqrt{2}$
(c) $R_d = R_p$ (d) $R_d = 2R_p$

4. A proton and an electron both moving with the same velocity v enter into a region of magnetic field directed perpendicular to the velocity of the particles. They will now move in circular orbits such that
(a) their time periods will be same
(b) the time period for proton will be higher
(c) the time period for electron will be higher
(d) their orbital radii will be same

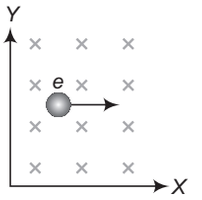
5. An electron is moving on a circular path of radius r with speed v in a transverse magnetic field B. The e/m for it will be
(a) $\frac{v}{Br}$ (b) $\frac{B}{rv}$
(c) Bvr (d) $\frac{vr}{B}$

6. A charged particle of mass m and charge q describes circular motion of radius r in a uniform magnetic field of strength B. The frequency of revolution is
(a) $\frac{Bq}{2\pi m}$ (b) $\frac{Bq}{2\pi rm}$
(c) $\frac{2\pi m}{Bq}$ (d) $\frac{Bm}{2\pi q}$

7. An electron and a proton with equal momentum enter perpendicularly into a uniform magnetic field, then
(a) the path of proton shall be more curved than that of electron
(b) the path of proton shall be less curved than that of electron
(c) both are equally curved
(d) path of both will be straight line

8. Two ions having masses in the ratio 1 : 1 and charges 1 : 2 are projected into uniform magnetic field perpendicular to the field with speeds in the ratio 2 : 3. The ratio of the radii of circular paths along which the two particles move is
(a) 4 : 3 (b) 2 : 3
(c) 3 : 1 (d) 1 : 4

9. In the given figure, the electron enters into the magnetic field. It deflects in . . . direction.



(a) +ve X-direction (b) –ve X-direction
(c) +ve Y-direction (d) –ve Y-direction

10. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is
(a) $\frac{2\pi qB}{m}$ (b) $\frac{2\pi m}{qB}$
(c) $\frac{2\pi mq}{B}$ (d) $\frac{qB}{2\pi m}$

11. A beam of electrons is accelerated through a potential difference V. It is then passed normally through a uniform magnetic field where it moves in a circle of radius r. It would have moved in a circle of radius $2r$, if it were initially accelerated through a potential difference of
(a) $\sqrt{2}V$ (b) $2V$ (c) $2\sqrt{2}V$ (d) $4V$

12. A proton is moving along the negative direction of x-axis in a magnetic field directed along the positive direction of y-axis. The proton will be deflected along the negative direction of
(a) x-axis (b) y-axis
(c) z-axis (d) None of these

13. A proton, a deuteron and an α-particle, having the same kinetic energy, are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote, respectively the radii of the trajectories of these particles, then
(a) $r_\alpha = r_p < r_d$
(b) $r_\alpha > r_d > r_p$
(c) $r_\alpha = r_d > r_p$
(d) $r_p = r_d = r_\alpha$

14. A charged particle travels along a straight line with a speed v in a region where both electric field **E** and magnetic field **B** are present. It follows that
(a) $|\mathbf{E}| = |\mathbf{B}|$ and the two fields are perpendicular
(b) $|\mathbf{E}| = v\,|\mathbf{B}|$ and the two fields are perpendicular
(c) $|\mathbf{B}| = v\,|\mathbf{E}|$ and the two fields are parallel
(d) $|\mathbf{B}| = v\,|\mathbf{E}|$ and the two fields are perpendicular

15. A particle of charge q and velocity v passes undeflected through a space with non-zero electric field E and magnetic field B. The undeflecting conditions will hold if
(a) signs of both q and E are reversed
(b) signs of both q and B are reversed
(c) both B and E are changed in magnitude, but keeping the product of B and E fixed
(d) both B and E are doubled in magnitude

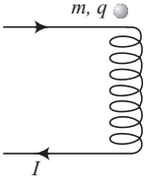
16. Which particles will have minimum frequency of revolution when projected with the same velocity perpendicular to a magnetic field?
(a) Li^+ (b) Electron
(c) Proton (d) He^+

17. A proton of energy 8 eV is moving in a circular path in uniform magnetic field. The energy of an α-particle moving in the same magnetic field and along the same path will be
(a) 4 eV (b) 2 eV
(c) 8 eV (d) 6 eV

18. A positively charged particle falling freely under gravity enters a region having a uniform horizontal magnetic field pointing North to South. The particle will be deflected towards
(a) West (b) North
(c) East (d) South

19. When a charged particle enters in a uniform magnetic field, its kinetic energy
(a) remains constant (b) increases
(c) decreases (d) becomes zero

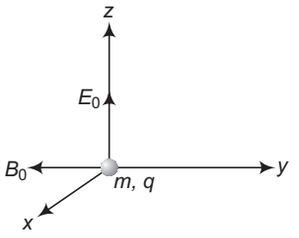
20. A long solenoid carrying a current I is placed with its axis vertical as shown in the figure. A particle of mass m and charge q is released from the top of the solenoid. Its acceleration is (g being acceleration due to gravity)



(a) greater than g (b) less than g
(c) equal to g (d) None of these

21. If a charged particle is describing a circle of radius r in a magnetic field with a time period T. Then,
(a) $T^2 \propto r^3$ (b) $T^2 \propto r$ (c) $T \propto r^2$ (d) $T \propto r^0$

22. A particle of mass m and charge q is released from rest at the origin as shown in the figure. The speed of the particle when it has travelled a distance d along the z-axis is given by



(a) $\sqrt{\dfrac{2qE_0d}{m}}$ (b) $q\sqrt{\dfrac{E_0d}{B_0}}$
(c) $\sqrt{\dfrac{m}{qE_0d}}$ (d) $\sqrt{\dfrac{2m}{qE_0d}}$

23. A particle of mass m and charge Q moving with a velocity **v** enters a region of uniform magnetic field of induction **B**. Then, its path in the region is
(a) always circular
(b) circular if $\mathbf{v} \times \mathbf{B} = 0$
(c) circular if $\mathbf{v} \cdot \mathbf{B} = 0$
(d) None of the above

24. A beam of protons with a velocity of 4×10^5 ms^{-1} enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. The radius of helical path taken by proton beam is
(a) 0.036 m (b) 0.012 m
(c) 0.024 m (d) 0.048 m

25. A particle of mass m and charge q moves with a constant velocity v along the positive x-direction. It enters a region containing a uniform magnetic field **B** directed along the negative z-direction, extending from $x = a$ to $x = b$. The minimum value of v required, so that the particle can just enter the region $x > b$ is
(a) qbB/m (b) $q(b - a)\,B/m$
(c) qaB/m (d) $q(b + a)\,B/2m$

26. A proton of mass 1.67×10^{-27} kg and charge 1.6×10^{-19} C is projected in x-y plane with a speed of 2×10^6 m/s at an angle of 60° to the x-axis. If a uniform magnetic field of 0.14 T is applied along the y-axis, the path of the proton is
(a) a circle of radius 0.2 m and time period $\pi \times 10^{-7}$ s
(b) a circle of radius 0.1 m and time period $2\pi \times 10^{-7}$ s
(c) a helix of radius 0.07 m and time period 0.5×10^{-6} s
(d) a helix of radius 0.14 m and time period 1.0×10^{-7} s

27. If electron velocity is $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ and it is subjected to a magnetic field $4\hat{\mathbf{k}}$, then
(a) speed of electron will change
(b) path of electron will change
(c) Both (a) and (b)
(d) None of the above

28. Which of the following particles will experience maximum magnetic force (magnitude) when projected with the same velocity perpendicular to a magnetic field?
(a) Electron (b) Proton
(c) He^+ (d) Li^{2+}

29. Which of the following particles will describe the smallest circle when projected with the same velocity perpendicular to a magnetic field?
(a) Electron (b) Proton
(c) He^+ (d) Li^+

30. An electric current i enters and leaves a uniform circular wire of radius a through diametrically opposite points. A charged particle q moving along the axis of the circular wire passes through its centre at speed v. The magnetic force acting on the particle when it passes through the centre has a magnitude
(a) $qv\dfrac{\mu_0 i}{2a}$ (b) $qv\dfrac{\mu_0 i}{2\pi a}$
(c) $qv\dfrac{\mu_0 i}{a}$ (d) zero

31. A particle is projected in a plane perpendicular to a uniform magnetic field. The area bounded by the path described by the particle is proportional to
(a) the velocity (b) the momentum
(c) the kinetic energy (d) None of these

32. A proton moves at a speed $v = 2\times10^6$ in a region of constant magnetic field of magnitude $B = 0.05$ T. The direction of the proton when it enters this field is $\theta = 30°$ to the field. When you look along the direction of the magnetic field, the path is a circle projected on a plane perpendicular to the magnetic field. How far will the proton move along the direction of B when two projected circles have been completed?
(a) 4.35 m (b) 0.209 m
(c) 2.82 m (d) 2.41 m

33. The path of a charged particle moving in a magnetic field can be a
(a) straight line (b) circle
(c) parabola (d) helix

34. If a charged particle at rest experiences no electromagnetic force the
(a) electric field must be zero
(b) magnetic field must be zero
(c) electric field may or may not be zero
(d) magnetic field may or may not be zero

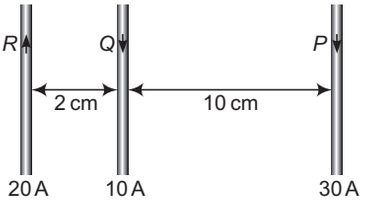
35. If a charged particle projected in a gravity-free room deflects
(a) there must be an electric field
(b) there must be a magnetic field
(c) both fields cannot be zero
(d) both fields can be non-zero

36. An electron is moving along the positive X-axis. You want to apply a magnetic field for a short time, so that the electron may reverse its direction and move parallel to the negative X-axis. This can be done by applying the magnetic field along
(a) Y-axis (b) Z-axis
(c) Y-axis only (d) Z-axis only

Magnetic Force on Current Carrying Wire and Magnetic Dipole

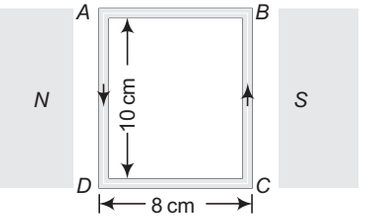
37. Two wires of same length are shaped into a square and a circle. If they carry same current, then the ratio of the magnetic moment is
(a) $2:\pi$ (b) $\pi:2$
(c) $\pi:4$ (d) $4:\pi$

38. Three long, straight and parallel wires carrying currents are arranged as shown in figure. The force experienced by 10 cm length of wire Q is



(a) 1.4×10^{-4} N towards the right
(b) 1.4×10^{-4} N towards the left
(c) 2.6×10^{-4} N towards the right
(d) 2.6×10^{-4} N towards the left

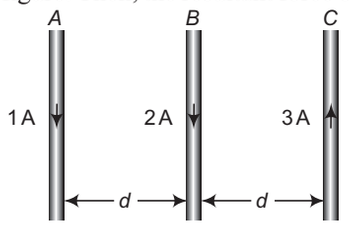
39. A 100 turns coil shown in figure carries a current of 2 A in a magnetic field $B = 0.2\ \text{Wb/m}^2$. The torque acting on the coil is



(a) 0.32 N-m tending to rotate the side AD out of the page
(b) 0.32 N-m tending to rotate the side AD into the page
(c) 0.0032 N-m tending to rotate the side AD out of the page
(d) 0.0032 N-m tending to rotate the side AD into the page

40. In hydrogen atom, the electron is making 6.6×10^{15} rev / s around the nucleus in an orbit of radius 0.528 Å. The magnetic moment (A-m^2) will be
(a) 1×10^{-15} (b) 1×10^{-10}
(c) 1×10^{-23} (d) 1×10^{-27}

41. Three long straight wires *A, B* and *C* are carrying current as shown figure. Then, the resultant force on *B* is directed



(a) towards *A*
(b) towards *C*
(c) perpendicular to the plane of paper and outward
(d) perpendicular to the plane of paper and inward

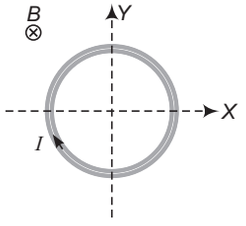
42. Two long conductors, separated by a distance *d* carry currents I_1 and I_2 in the same direction. They exert a force *F* on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to 3*d*. The new value of the force between them is

(a) $-F/3$ (b) $F/3$
(c) $2F/3$ (d) $-2F/3$

43. A circular coil of 20 turns and radius 10 cm is placed in uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in coil is 5 A, then the torque acting on the coil will be

(a) 31.4 N-m (b) 3.14 N-m
(c) 0.314 N-m (d) zero

44. A conducting loop carrying a current *I* is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have a tendency to

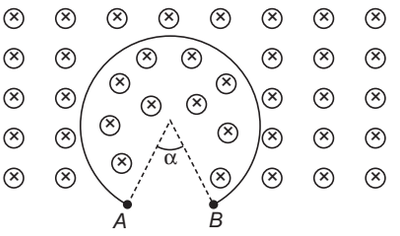


(a) contract
(b) expand
(c) move towards +ve *x*-axis
(d) move towards –ve *x*-axis

45. If a current is passed through a spring, then the spring will

(a) expand
(b) compress
(c) remain same
(d) None of the above

46. A circular loop which is in the form of a major arc of a circle is kept in the horizontal plane and a constant magnetic field *B* is applied in the vertical direction such that the magnetic lines of forces go into the plane. If *R* is the radius of circle and it carries a current *I* in the clockwise direction, then the force on the loop will be

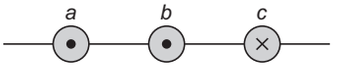


(a) $BIR \tan\alpha$ (b) $2\,BIR\cos(\alpha/2)$
(c) $2\,BIR\sin(\alpha/2)$ (d) None of these

47. A wire of length 2 m carrying a current of 1 A is bend to form a circle. The magnetic moment of the coil is (in A-m^2)

(a) 2π (b) $\pi/2$ (c) $\pi/4$ (d) $1/\pi$

48. Figure shows three long straight wires parallel and equally spaced with identical currents. Then, the force acting on each wire due to the other is



(a) $F_a > F_b > F_c$ (b) $F_b > F_c > F_a$
(c) $F_c > F_a > F_b$ (d) $F_b > F_a > F_c$

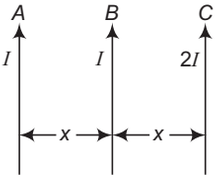
49. In two parallel wires *A* and *B*, 10 and 2 A current respectively is flowing in opposite directions. The distance between the wires is 10 cm. If the wire *A* is of infinite length and wire *B* is of 2 m length, then the force acting on the wire *B* will be

(a) 8×10^{-5} N (b) 4×10^{-5} N
(c) 4×10^{-7} N (d) 8×10^{-7} N

50. A current carrying loop is placed in a uniform magnetic field. The torque acting on it does not depend on

(a) shape of the loop
(b) area of the loop
(c) number of turns in the loop
(d) strength of the current

51. *A, B* and *C* are parallel conductors of equal length carrying currents *I, I* and 2*I* respectively. Distance between *A* and *B* is *x*. Distance between *B* and *C* is also *x*. F_1 is the force exerted by *B* on *A*. F_2 is the force exerted by *C* on *A*. Choose the correct answer.

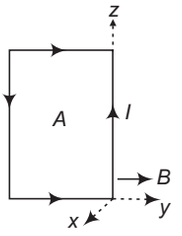


(a) $F_1 = 2F_2$ (b) $F_2 = 2F_1$ (c) $F_1 = F_2$ (d) $F_1 = -F_2$

52. The ratio of magnitude of magnetic field at the centre of a circular current carrying wire to its magnetic moment is *N*. If the current and radius both were doubled the ratio will become

(a) $N/8$ (b) $N/4$
(c) $2N$ (d) $4N$

53. The rectangular coil of area A is in a field B. Find the torque about the z-axis when the coil lies in the position shown and carries a current I,



(a) IAB in negative z-axis
(b) IAB in positive z-axis
(c) $2IAB$ in positive z-axis
(d) $2IAB$ in negative z-axis

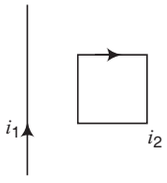
54. The force between two long parallel wires A and B carrying current is 0.004 N/m. The conductors are 0.01 m apart. If the current in conductor A is twice that of conductor B, then the current in the conductor B would be

(a) 5 A (b) 50 A
(c) 10 A (d) 100 A

55. A current of 10 A is flowing in a wire of length 1.5 m. A force of 15 N acts on it when it is placed in a uniform magnetic field of 2T. The angle between the magnetic field and the direction of the current is

(a) 30° (b) 45°
(c) 60° (d) 90°

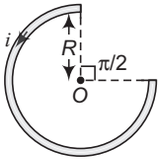
56. Consider the situation shown in figure. The straight wire is fixed but the loop can move under magnetic force. The loop will



(a) remain stationary (b) move towards the wire
(c) move away from the wire (d) rotate about the wire

Biot-Savart's Law and Ampere's Circuital Law

57. A current i ampere flows in a circular arc of wire whose radius is R, which subtend an angle $3\pi/2$ radian at its centre. The magnetic induction B at the centre is



(a) $\frac{\mu_0 i}{R}$ (b) $\frac{\mu_0 i}{2R}$ (c) $\frac{2\mu_0 i}{R}$ (d) $\frac{3\mu_0 i}{8R}$

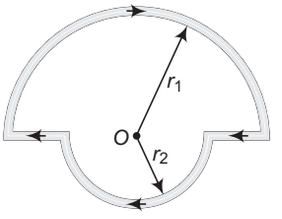
58. The strength of the magnetic field at a distance r near a long straight current carrying wire is B. The field at a distance $\frac{r}{2}$ will be

(a) $\frac{B}{2}$ (b) $\frac{B}{4}$
(c) $2B$ (d) $4B$

59. A current of 0.1 A circulates around a coil of 100 turns and having a radius equal to 5 cm. The magnetic field set up at the centre of the coil is ($\mu_0 = 4\pi \times 10^{-7}$ Wb/A-m)

(a) $4\pi \times 10^{-5}$ T (b) $8\pi \times 10^{-5}$ T
(c) 4×10^{-5} T (d) 2×10^{-5} T

60. In the figure shown there are two semicircles of radii r_1 and r_2 in which a current i is flowing. The magnetic induction at the centre O will be



(a) $\frac{\mu_0 i}{4}(r_1 + r_2)$ (b) $\frac{\mu_0 i}{4}(r_1 - r_2)$
(c) $\frac{\mu_0 i}{4}\left[\frac{r_1 + r_2}{r_1 r_2}\right]$ (d) $\frac{\mu_0 i}{4}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$

61. A particle carrying a charge equal to 100 times the charge on an electron is rotating one rotation per second in a circular path of radius 0.8 m. The value of the magnetic field produced at the centre will be (μ_0 = permeability for vacuum)

(a) $\frac{10^{-7}}{\mu_0}$ (b) $10^{-17}\mu_0$
(c) $10^{-6}\mu_0$ (d) $10^{-7}\mu_0$

62. In hydrogen atom, an electron is revolving in the orbit of radius 0.53 Å with 6.6×10^{15} rps. Magnetic field produced at the centre of the orbit is

(a) 0.125 Wb/m^2 (b) 1.25 Wb/m^2
(c) 12.5 Wb/m^2 (d) 125 Wb/m^2

63. A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle θ at the centre. The value of the magnetic induction at the centre due to the current in the ring is

(a) proportional to $2(180° - \theta)$
(b) inversely proportional to r
(c) zero, only if $\theta = 180°$
(d) zero for all values of θ

64. An arc of a circle of radius R subtends an angle $\frac{\pi}{2}$ at the centre. It carries a current i. The magnetic field at the centre will be

(a) $\frac{\mu_0 i}{2R}$ (b) $\frac{\mu_0 i}{8R}$
(c) $\frac{\mu_0 i}{4R}$ (d) $\frac{2\mu_0 i}{5R}$

65. Which of the following gives the value of magnetic field due to a small current element according to, Biot-Savart's law?

(a) $\frac{i\Delta l \sin\theta}{r^2}$ (b) $\frac{\mu_0}{4\pi}\frac{i\Delta l \sin\theta}{r}$
(c) $\frac{\mu_0}{4\pi}\frac{i\Delta l \sin\theta}{r^2}$ (d) $\frac{\mu_0}{4\pi}\frac{i\Delta l \sin\theta}{r^3}$

66. Magnetic field due to a ring having n turns at a distance x on its axis is proportional to (if r = radius of ring)

(a) $\frac{r}{(x^2+r^2)}$ (b) $\frac{r}{(x^2+r^2)^{3/2}}$
(c) $\frac{nr^2}{(x^2+r^2)^{3/2}}$ (d) $\frac{n^2r^2}{(x^2+r^2)^{3/2}}$

67. When a certain length of wire is turned into one circular loop, the magnetic induction at the centre of coil due to some current flowing is B_1. If the same wire is turned into three loops to make a circular coil, the magnetic induction at the centre of this coil for the same current will be

(a) B_1 (b) $9B_1$
(c) $3B_1$ (d) $27B_1$

68. The ratio of the magnetic field at centre of a current carrying coil of the radius a and at a distance a from centre of the coil and perpendicular to the axis of coil is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$
(c) $\frac{1}{2\sqrt{2}}$ (d) $2\sqrt{2}$

69. A current flows in a conductor from East to West. The direction of the magnetic field at a point above the conductor is

(a) towards North (b) towards South
(c) towards East (d) towards West

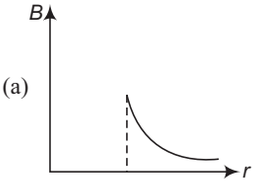
70. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be

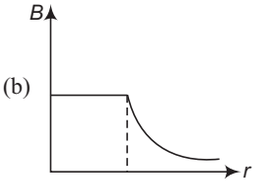
(a) nB (b) n^2B
(c) $3\,nB$ (d) n^3B

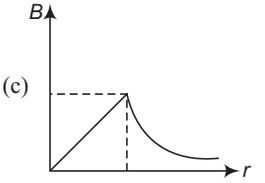
71. The current is flowing in South direction along a power line. The direction of magnetic field above the power line (neglecting earth's field) is

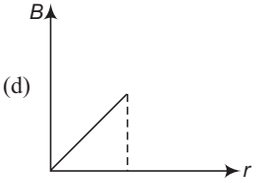
(a) South (b) East (c) North (d) West

72. A long thin hollow metallic cylinder of radius R has a current i ampere. The magnetic induction B away from the axis at a distance r from the axis varies as shown in

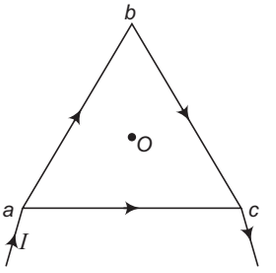
(a) 

(b) 

(c) 

(d) 

73. An equilateral triangle of side length l is formed from a piece of wire of uniform resistance. The current I is fed as shown in the figure. Then, the magnitude of the magnetic field at its centre O is



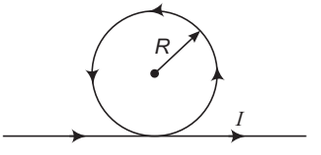
(a) $\frac{\sqrt{3}\mu_0 I}{2\pi l}$ (b) $\frac{3\sqrt{3}\mu_0 I}{2\pi l}$ (c) $\frac{\mu_0 I}{2\pi l}$ (d) zero

74. A straight thin conductor is bent as shown in the adjoining figure. It carries a current i ampere. The radius of the semicircular arc is r metre. The magnetic induction at the centre of semicircular arc is



(a) $\frac{\mu_0 i}{4\pi r}$ (b) $\frac{\mu_0 i}{4r}$ (c) $\frac{\mu_0 i}{8\pi r}$ (d) zero

75. An infinitely long conductor is bent into a circle as shown in figure. It carries a current I ampere and the radius of loop is R metre. The magnetic induction at the centre of loop is

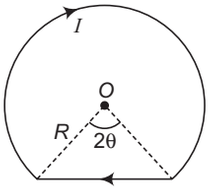


(a) $\frac{\mu_0 2I}{4\pi R}(\pi+1)$ (b) $\frac{\mu_0 2I}{4\pi R}(\pi-1)$
(c) $\frac{\mu_0 I}{8\pi R}(\pi+1)$ (d) zero

76. A circular current carrying coil has a radius R. The distance from the centre of the coil on the axis of the coil, where the magnetic induction is 1/8th of its value at the centre of coil is

(a) $\sqrt{3}R$ (b) $R/\sqrt{3}$ (c) $(2/\sqrt{3})R$ (d) $R/2\sqrt{3}$

77. A current I flows through a closed loop shown in figure. The magnetic field at the centre O is



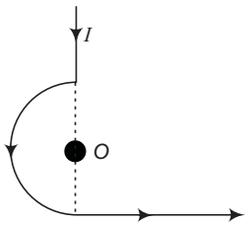
(a) $\frac{\mu_0 I}{2\pi R}(\pi - \theta + \tan\theta)$ (b) $\frac{\mu_0 I}{2\pi R}(\pi - \theta + \sin\theta)$

(c) $\frac{\mu_0 I}{2\pi R}(\theta + \sin\theta)$ (d) None of these

78. Two identical coils carrying equal currents, have a common centre, and their planes are at right angles to each other. Find the ratio of the magnitudes of the resultant magnetic field at the centre and the field due to one coil alone.

(a) 2 : 1 (b) 1 : 1 (c) $1:\sqrt{2}$ (d) $\sqrt{2}:1$

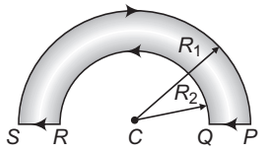
79. Magnetic field induction at the point O due to current flowing in an infinite wire shaped as shown in the figure is



(a) $\frac{\mu_0 I}{4\pi R}$ (b) $\frac{\mu_0 I}{4R} - \frac{\mu_0 I}{4\pi R}$

(c) $\frac{\mu_0 I}{4R} + \frac{\mu_0 I}{2\pi R}$ (d) $\frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R}$

80. The wire loop $PQRSP$ formed by joining two semicircular wires of radii R_1 and R_2 carries a current I as shown. The magnitude of magnetic induction at the centre C is



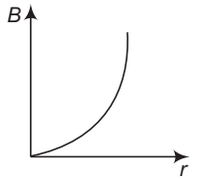
(a) $\frac{\mu_0 I}{4}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$ (b) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

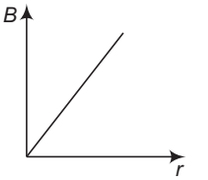
(c) $\mu_0 I\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$ (d) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1}\right)$

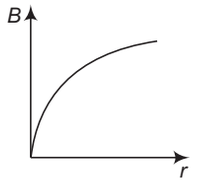
81. The magnetic field at the centre of a circular coil of radius r carrying current I is B_1. The field at the centre of another coil of radius $2r$ carrying same current I is B_2. The ratio B_1/B_2 is

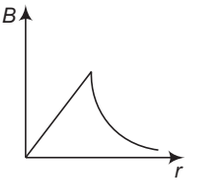
(a) 1/2 (b) 1 (c) 2 (d) 4

82. Which of the following figure shows the magnetic flux density B at a distance r from a long straight rod carrying a steady current i ?

(a)


(b)


(c)


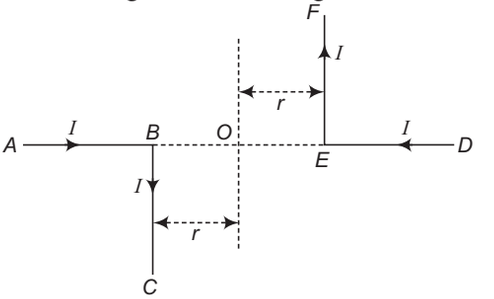
(d)


83. The magnetic induction at the centre of a current carrying circular coil of radius r is

(a) directly proportional to r (b) inversely proportional to r

(c) directly proportional to r^2 (d) inversely proportional to r^2

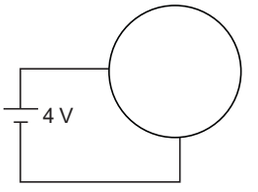
84. Two long thin wires ABC and DEF are arranged as shown. The magnitude of the magnetic field at O is



(a) $\frac{\mu_0 I}{4\pi r}$ (b) $\frac{\mu_0 I}{2\pi r}$

(c) $\frac{\mu_0 I}{2\sqrt{2}\pi r}$ (d) zero

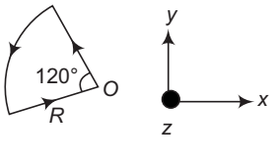
85. A circular conductor, of uniform resistance per unit length, is connected to a battery of 4 V. The total resistance of the conductor is 4 Ω. The net magnetic field at the centre of the conductor is



(a) $\frac{\mu_0}{2}$ (b) $\frac{8\mu_0}{3}$

(c) $2\mu_0$ (d) zero

86. The magnetic field **B** at O due to current carrying loop placed in the x-y plane as shown in the figure is



(a) $\frac{\mu_0 I}{R}\hat{\mathbf{k}}$ (b) $\frac{\mu_0 I}{6R}\hat{\mathbf{k}}$ (c) $\frac{\mu_0 I}{6R}(-\hat{\mathbf{k}})$ (d) zero

87. If the current is flowing clockwise in a circular coil, the direction of magnetic lines of force inside the coil is
(a) towards you
(b) away from you
(c) towards the centre along the radius
(d) away from the centre along the radius

88. Two concentric coils of 10 turns each are situated in the same plane. Their radii are 2 cm and 4 cm and they carry respectively 0.2 and 0.3 A current in opposite directions. The magnetic field (in Wb/m^2) at the centre is
(a) $(35/4)\,\mu_0$ (b) $(\mu_0/80)$
(c) $(7/80)\,\mu_0$ (d) $(25/2)\,\mu_0$

89. A given length of a wire carries a steady current. It is bent first to form a circular plane coil of one turn. If a loop of same length is now bent more sharply to give a double loop of smaller radius, the magnetic field at the centre caused by the same current is
(a) a quarter of its first value
(b) unaltered
(c) four times of its first value
(d) two times of its first value

90. Two infinitely long, thin, insulated, straight wires lie in the x-y plane along the x and y-axes respectively. Each wire carries a current I, respectively in the positive x-direction and positive y-direction. The magnetic field will be zero at all points on the straight line
(a) $y = x$ (b) $y = -x$
(c) $y = x - 1$ (d) $y = -x + 1$

91. There are two identical concentric coils of suitable radii X and Y with their planes at right angles to each other. The coil X lies in the horizontal plane while coil Y lies in the vertical plane. If the coil X carries a current of 1 A, then what value of current in coil Y be passed, so that the resultant field at the centre of the coils just balances the earth's magnetic field of 10^{-5} T inclined at 30° with the vertical?
(a) 1 A (b) $\sqrt{3}$ A (c) 2 A (d) $(1/\sqrt{3})$ A

92. Two parallel, long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the currents are in the same direction, the magnetic field at a point midway between the wires is $10\,\mu T$. If the direction of i_2 is reversed, the field becomes $30\,\mu T$. The ratio i_1/i_2 is
(a) 4 (b) 3 (c) 2 (d) 1

93. In a coaxial, straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero
(a) outside the cable
(b) inside the inner conductor
(c) inside the outer conductor
(d) in between the two conductors

94. A long, straight wire of radius R carries a current distributed uniformly over its cross-section. The magnitude of the magnetic field is
(a) maximum at the axis of the wire
(b) minimum at the axis of the wire
(c) maximum at the surface of the wire
(d) minimum at the surface of the wire

95. A hollow tube is carrying an electric current along its length distributed uniformly over its surface. The magnetic field
(a) increases linearly from the axis to the surface
(b) is constant inside the tube
(c) is zero at the axis
(d) is zero just outside the tube

Miscellaneous Problems

96. Magnetic effect of current was discovered by
(a) Faraday (b) Oersted
(c) Ampere (d) Bohr

97. One tesla is equal to
(a) 10^7 gauss (b) 10^{-4} gauss
(c) 10^4 gauss (d) 10^{-8} gauss

98. A magnetic field can be produced by
(a) a moving charge (b) a changing electric field
(c) Both (a) and (b) (d) None of these

99. Unit of magnetic permeability is
(a) A/m (b) A/m^2
(c) H (d) H/m

100. A moving charge will gain energy due to the application of
(a) electric field (b) magnetic field
(c) Both (a) and (b) (d) None of these

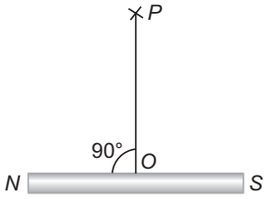
101. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on
(a) ω and q (b) ω, q and m
(c) q and m (d) ω and m

102. Two proton beams moving in same direction will
(a) attract each other
(b) repel each other
(c) Neither attract nor repel
(d) None of the above

103. Cyclotron is a device which is used to
(a) measure of the charge (b) measure the voltage
(c) accelerating protons (d) accelerating electrons

104. The dimensional formula of magnetic induction is
(a) $[MT^4A^{-1}]$ (b) $[MT^{-2}A^{-1}]$
(c) $[MLA^{-2}]$ (d) $[MT^3A]$

105. The magnetic field at P due to the magnet N-S is
(a) along OP
(b) along PO
(c) parallel to N-S
(d) parallel to S-N



106. In Thomson experiment of finding e/m for electrons, beam of electron is replaced by that of muons (particle of same charge as of electrons but mass 208 times that of electrons). No deflection condition in this case is satisfied if
(a) B is increased 208 times (b) E is increased 208 times
(c) B is increased 14.4 times (d) None of these

107. Electrons can be accelerated to a very high energy by means of
(a) betatron (b) cyclotron
(c) thyrotron (d) None of these

108. The maximum energy of a deuteron coming out a cyclotron is 20 MeV. The maximum energy of proton that can be obtained from this accelerator is
(a) 10 MeV
(b) 20 MeV
(c) 30 MeV
(d) 40 MeV

109. Let ε_0 denote the dimensional formula of the permittivity of the vacuum, and μ_0 that of the permeability of the vacuum. If $M =$ mass, $L =$ length, $T =$ time and $I =$ electric current
(a) $\varepsilon_0 = [M^{-1}L^{-3}T^2I]$
(b) $\varepsilon_0 = [M^{-1}L^{-3}T^4\ I^2]$
(c) $\mu_0 = [MLT^{-2}I^{-2}]$
(d) $\mu_0 = [ML^2T^{-1}I]$

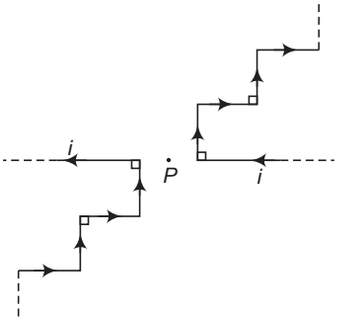
110. Consider three quantities $x = E/B$, $y = \sqrt{1/\mu_0\varepsilon_0}$ and $z = \dfrac{l}{CR}$. Here, l is the length of a wire, C is a capacitance and R is a resistance. All other symbols have standard meanings.
(a) x, y have the same dimensions
(b) y, z have the same dimensions
(c) z, x have the same dimensions
(d) None of the three pairs have the same dimensions

[Level 2]

Only One Correct Option

1. A charged particle P leaves the origin with speed $v = v_0$, at some inclination with the x-axis. There is a uniform magnetic field B along the x-axis. P strikes a fixed target T on the x-axis for a minimum value of $B = B_0$. P will also strike T if
(a) $B = 2B_0,\ v = 2v_0$ (b) $B = 2B_0,\ v = v_0$
(c) Both are correct (d) Both are wrong

2. Two infinitely long conductors carrying equal currents are shaped as shown. The short sections are all of equal lengths. The point P is located symmetrically with respect to the two conductors. The magnetic field at P due to any one conductor is B. The total field at P is



(a) zero (b) B (c) $\sqrt{2}\ B$ (d) $2B$

3. An ionized gas contains both positive and negative ions initially at rest. If it is subjected simultaneously to an electric field along the $+x$ direction and a magnetic field along the $+z$ direction, then
(a) positive ions deflect towards $+y$-direction and negative ions $-y$-direction
(b) all ions deflect towards $+y$-direction
(c) all ions deflect towards $-y$-direction
(d) positive ions deflect towards $-y$-direction and negative ions towards $+y$-direction

4. Two protons are projected simultaneously from a fixed point with the same velocity v into a region where there exists a uniform magnetic field. The magnetic field strength at B and it is perpendicular to the initial direction of v. One proton starts at time $t = 0$ and another proton at $t = \dfrac{\pi m}{2qB}$. The separation between them at time $t = \dfrac{\pi m}{qB}$ [where, m and q are the mass and charge of proton], will be approximately
(a) $\dfrac{2mv}{qB}$ (b) $\dfrac{\sqrt{2}\ mv}{qB}$ (c) $\dfrac{mv}{qB}$ (d) $\dfrac{mv}{2qB}$

5. A disc of radius R rotates with constant angular velocity ω about its own axis. Surface charge density of this disc varies as $\sigma = \alpha r^2$ where, r is the distance from the centre of disc. Determine the magnetic field intensity at the centre of disc

(a) $\mu_0 \alpha \omega R^3$ (b) $\frac{\mu_0 \alpha \omega R^3}{6}$

(c) $\frac{\mu_0 \alpha \omega R^3}{8}$ (d) $\frac{\mu_0 \alpha \omega R^3}{3}$

6. A particle of mass m and having a positive charge q is projected from origin with speed v_0 along the positive x-axis in a magnetic field $\mathbf{B} = -B_0 \hat{\mathbf{k}}$ where, B_0 is a positive constant. If the particle passes through $(0, y, 0)$, then y is equal to

(a) $-\frac{2mv_0}{qB_0}$ (b) $\frac{mv_0}{qB}$

(c) $-\frac{mv_0}{qB}$ (d) $\frac{2mv_0}{qB_0}$

7. A rigid circular loop of radius r and mass m lies in the xy-plane on a flat table and has a current i flowing in it. At this particular place, the earth's magnetic field is $\mathbf{B} = B_x \hat{\mathbf{i}} + B_z \hat{\mathbf{k}}$. The value of i so that the loop starts tilting is

(a) $\frac{mg}{\pi r \sqrt{B_x^2 + B_z^2}}$ (b) $\frac{mg}{\pi r B_x}$

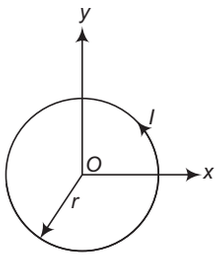
(c) $\frac{mg}{\pi r B_z}$ (d) $\frac{mg}{\pi r \sqrt{B_x B_z}}$

8. Two circular coils 1 and 2 are made from the same wire but the radius of the 1st coil is twice that of the 2nd coil. What is the ratio of potential difference applied across them, so that the magnetic field at their centres is the same?

(a) 3 (b) 4

(c) 6 (d) 2

9. A circular loop of mass m and radius r is lying in a horizontal (X-Y plane) table as shown in figure. A uniform magnetic field B is applied parallel to x-axis. The current I in the loop, so that its one edge just lifts from the table, is



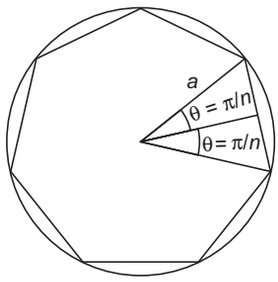
(a) $\frac{mg}{\pi r^2 B}$ (b) $\frac{mg}{\pi r B}$

(c) $\frac{mg}{2\pi r B}$ (d) $\frac{\pi r B}{mgl}$

10. A charged particle with specific charge (charge per unit mass $= \frac{q}{m} = s$) moves undeflected through a region of space containing mutually perpendicular and uniform electric and magnetic fields E and B. When the electric field E is switched off, the particle will move in a particular path of radius

(a) $\frac{E}{Bs}$ (b) $\frac{Es}{B}$ (c) $\frac{Es}{B^2}$ (d) $\frac{E}{B^2 s}$

11. A wire bent in the form of a regular polygon of n sides, is inscribed in a circle of radius a. If i ampere is the current flowing in the wire, then the magnetic flux density (magnetic field) at the centre of the circle is



(a) $\frac{\mu_0 i}{2\pi a} \tan\left(\frac{\pi}{n}\right)$ (b) $\frac{\mu_0 ni}{2\pi a} \tan\left(\frac{\pi}{n}\right)$

(c) $\frac{2}{\pi} \frac{ni}{a} \mu_0 \tan\left(\frac{\pi}{n}\right)$ (d) $\frac{ni}{2a} \mu_0 \tan\left(\frac{\pi}{n}\right)$

12. A circular flexible loop of wire of radius r carrying a current I is placed in a uniform magnetic field B perpendicular to the plane of the circle so that, wire comes under tension. If B is doubled, tension in the loop

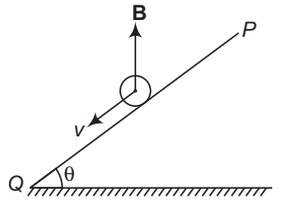
(a) remains unchanged (b) is doubled

(c) is halved (d) becomes 4 times

13. An electron moves in a circular orbit with a uniform speed v. It produces a magnetic field B at the centre of the circle. The radius of the circle is proportional to

(a) $\frac{B}{v}$ (b) $\frac{v}{B}$ (c) $\sqrt{\frac{v}{B}}$ (d) $\sqrt{\frac{B}{v}}$

14. A conducting rod of length L and mass m is moving down a smooth inclined plane of inclination θ with constant speed v. A current I is flowing in the conductor perpendicular to the paper inwards. A vertically upward magnetic field B exists in space there. The magnitude of magnetic field **B** is



(a) $\frac{mg}{IL} \sin\theta$ (b) $\frac{mg}{IL} \cos\theta$ (c) $\frac{mg}{IL} \tan\theta$ (d) $\frac{mg}{IL} \sec\theta$

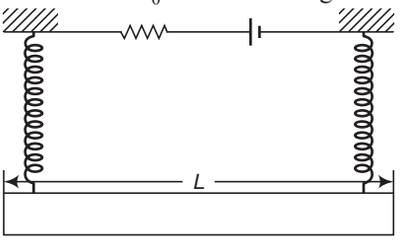
15. The magnetic field existing in a region is given by,

$$\mathbf{B} = B_0\left[1+\frac{x}{l}\right]\hat{\mathbf{k}}$$

A square loop of edge l and carrying current I is placed with its edges parallel to the x-y axis. The magnitude of the net magnetic force experienced by the loop is

(a) $2B_0Il$ (b) zero (c) B_0Il (d) $4B_0Il$

16. A straight rod of mass m and length L is suspended from the identical springs as shown in figure. The spring is stretched a distance x_0 due to the weight of the wire.



The circuit has total resistance R. When the magnetic field perpendicular to the plane of paper is switched on, springs are observed to extend further by the same distance. The magnetic field strength is

(a) $\frac{2mgR}{LE}$ (b) $\frac{mgR}{LE}$

(c) $\frac{mgR}{2LE}$ (d) $\frac{mgR}{E}$

17. A charge q is moving with a velocity $\mathbf{v}_1 = 1\hat{\mathbf{i}}$ m/s at a point in a magnetic field and experiences a force $\mathbf{F} = q[-\hat{\mathbf{j}} + 1\hat{\mathbf{k}}]$ N. If the charge is moving with a velocity $\mathbf{v}_2 = 2\,\hat{\mathbf{j}}$ m/s at the same point, it experiences a force $\mathbf{F}_2 = q(1\hat{\mathbf{i}} - 1\hat{\mathbf{k}})$ N. The magnetic induction $\mathbf{B}$ at that point is

(a) $(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\ \mathrm{Wb/m^2}$ (b) $(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})\mathrm{Wb/m^2}$

(c) $(-\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})\mathrm{Wb/m^2}$ (d) $(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})\mathrm{Wb/m^2}$

18. A large metal sheet carries an electric current along its surface. Current per unit length is λ.



Magnetic field induction near the metal sheet is

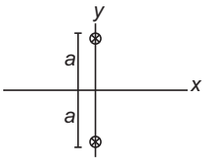
(a) $\lambda\mu_0$ (b) $\frac{\lambda\mu_0}{2}$ (c) $\frac{\lambda\mu_0}{2\pi}$ (d) $\frac{\mu_0}{2\pi\lambda}$

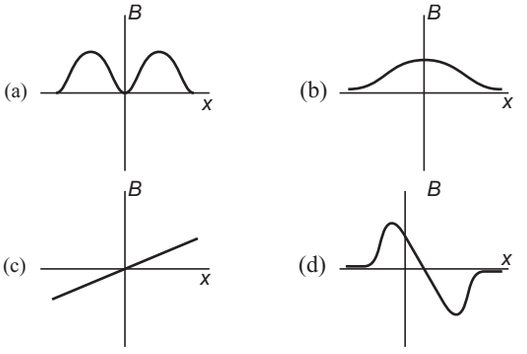
19. A square frame of side 1 m carries a current I, produces a magnetic field B at its centre. The same current is passed through a circular coil having the same perimeter as the square. The magnetic field at the centre of the circular coil is B'. The ratio B/B' is

(a) $\frac{8}{\pi^2}$ (b) $\frac{8\sqrt{2}}{\pi^2}$

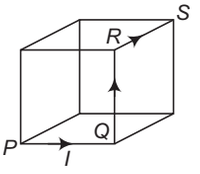
(c) $\frac{16}{\pi^2}$ (d) $\frac{16}{\sqrt{2}\pi^2}$

20. Two straight infinitely long current carrying wires are kept along z-axis at the coordinates $(0, a, 0)$ and $(0, -a, 0)$, respectively as shown in the figure. The current in each of the wire is equal and along negative z-axis (into the plane of the paper). The variation of magnetic field on the x-axis will be approximately





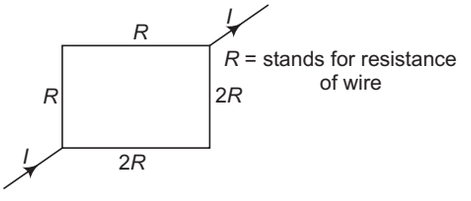
21. A wires $PQRS$ carrying a current I runs along three edges of a cube of side l as shown. There exists a uniform magnetic field of magnitude B along one of the sides of the cube. The magnitude of the force acting on the wire is



(a) zero (b) $\sqrt{3}\,IB$

(c) $\sqrt{2}IlB$ (d) $2IlB$

22. The magnetic field at the centre of square of side a is



(a) $\frac{\sqrt{2}\,\mu_0}{\pi a}$ (b) $\frac{\sqrt{2}\,\mu_0 I}{3\pi a}$

(c) $\frac{2}{3}\frac{\mu_0 I}{a}$ (d) zero

23. A particle of specific charge $q/m = \pi$ C/kg is projected from the origin towards positive x-axis with a velocity of 10 m/s in a uniform magnetic field $\mathbf{B} = -2\hat{\mathbf{k}}$ T. The velocity $\mathbf{v}$ of the particular after time $t = \frac{1}{6}$ s will be

(a) $(5\hat{\mathbf{i}} + 5\sqrt{3}\,\hat{\mathbf{j}})$ m/s (b) $10\hat{\mathbf{j}}$ m/s

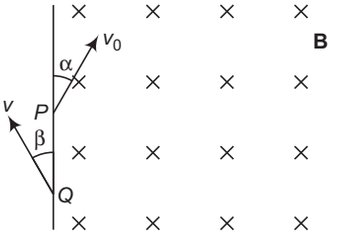
(c) $(5\sqrt{3}\,\hat{\mathbf{i}} + 5\hat{\mathbf{j}})$ m/s (d) $-10\hat{\mathbf{j}}$ m/s

24. A charged particle enters into a uniform magnetic field with velocity v_0 perpendicular to it, the length of magnetic field is $x = \frac{\sqrt{3}}{2} R$, where R is the radius of the circular path of the particle in the field. The magnitude of change in velocity of the particle when it comes out of the field is

(a) $2v_0$ (b) $\frac{v_0}{2}$

(c) $\frac{\sqrt{3}v_0}{2}$ (d) v_0

25. A particle of charge $-q$ and mass m enters a uniform magnetic field **B** (perpendicular to paper inwards) at P with a speed v_0 at an angle α and leaves the field at Q with speed v at angle β as shown in the figure. Then,



(a) $\alpha = \beta$

(b) $v = v_0$

(c) $PQ = \frac{2mv_0 \sin\alpha}{Bq}$

(d) particle remains in the field for time, $t = \frac{2m(\pi - \alpha)}{Bq}$

26. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If **E** and **B** represent the electric and magnetic fields, respectively, then this region of space may have

(a) $E = 0, B = 0$

(b) $E = 0, B \neq 0$

(c) $E \neq 0, B = 0$

(d) $E \neq 0, B \neq 0$

27. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular path of radius R_1 and R_2, respectively. The ratio of mass of X to that of Y is

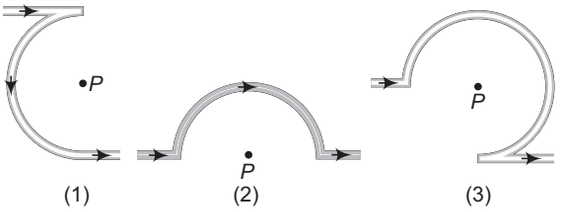
(a) $\left(\frac{R_1}{R_2}\right)^{1/2}$

(b) $\frac{R_2}{R_1}$

(c) $\left(\frac{R_1}{R_2}\right)^2$

(d) $\frac{R_1}{R_2}$

28. Figure here shows three cases : in all cases the circular path has radius r and straight ones are infinitely long. For same current the magnetic field at the centre P in cases 1, 2, 3 have the ratio



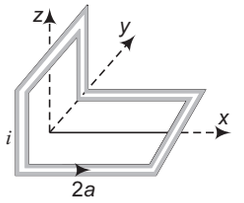
(a) $\left(-\frac{\pi}{2}\right) : \left(\frac{\pi}{2}\right) : \left(\frac{3\pi}{4} - \frac{1}{2}\right)$

(b) $\left(-\frac{\pi}{2} + 1\right) : \left(\frac{\pi}{2} + 1\right) : \left(\frac{3\pi}{4} + \frac{1}{2}\right)$

(c) $-\frac{\pi}{2} : \frac{\pi}{2} : 3\frac{\pi}{4}$

(d) $\left(-\frac{\pi}{2} - 1\right) : \left(\frac{\pi}{2} - \frac{1}{4}\right) : \left(\frac{3\pi}{4} + \frac{1}{2}\right)$

29. A non-planar loop of conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length $2a$. The magnetic field due to this loop at the point P (a, 0, a) points in the direction



(a) $\frac{1}{\sqrt{2}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$ (b) $\frac{1}{\sqrt{3}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}} + \hat{\mathbf{i}})$

(c) $\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ (d) $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{k}})$

30. A long straight wire along the z-axis carries a current I in the negative z-direction. The magnetic vector field **B** at a point having coordinates (x, y) in the $z = 0$ plane is

(a) $\frac{\mu_0 I(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})}{2\pi(x^2 + y^2)}$ (b) $\frac{\mu_0 I(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})}{2\pi(x^2 + y^2)}$

(c) $\frac{\mu_0 I(x\hat{\mathbf{j}} - y\hat{\mathbf{i}})}{2\pi(x^2 + y^2)}$ (d) $\frac{\mu_0 I(x\hat{\mathbf{i}} - y\hat{\mathbf{j}})}{2\pi(x^2 + y^2)}$

31. H^+, He^+ and O^{2+} ions having same kinetic energy pass through a region of space filled with uniform magnetic field B directed perpendicular to the velocity of ions. The masses of the ions H^+, He^+ and O^{2+} are respectively in the ratio 1 : 4 : 16. As a result

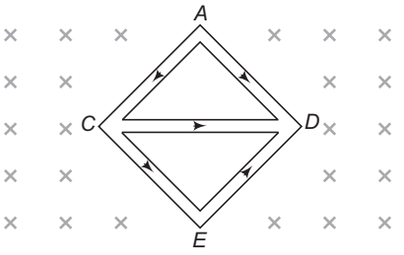
(a) H^+ ions will be deflected most

(b) O^{2+} ions will be deflected least

(c) He^+ and O^{2+} ions will suffer same deflection

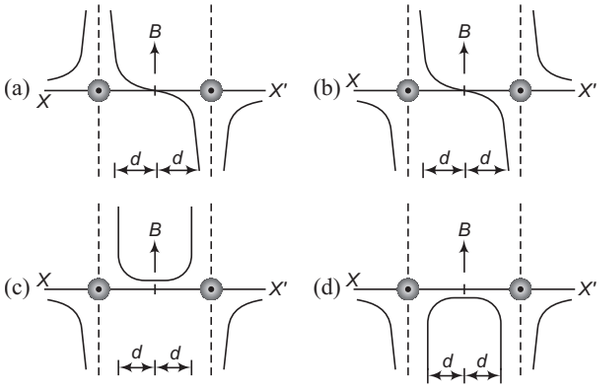
(d) all ions will suffer the same deflection

32. Same current $i = 2$ A is flowing in a wire frame as shown in figure. The frame is a combination of two equilateral triangles *ACD* and *CDE* of side 1 m.It is placed in uniform magnetic field $B = 4$T acting perpendicular to the plane of frame. The magnitude of magnetic force acting on the frame is



(a) 24 N (b) zero (c) 16 N (d) 8 N

33. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper, as shown. The variation of the magnetic field *B* along the line *XX′* is given by



34. Two charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{k}}$

(a) They have equal *z*-components of momenta
(b) They must have equal charges
(c) They necessarily represent a particle, anti-particle pair
(d) The charge to mass ratio satisfy

$$\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0$$

35. In a cyclotron, a charged particle
(a) undergoes acceleration all the time
(b) speeds up between the dees because of the magnetic field
(c) speeds up in a dee
(d) slows down within a dee and speeds up between dees

36. A circular current loop of magnetic moment *M* is in an arbitrary orientation in an external magnetic field **B.** The work done to rotate the loop by 30° about an axis perpendicular to its plane is

(a) MB (b) $\sqrt{3}\,\frac{MB}{2}$ (c) $\frac{MB}{2}$ (d) zero

More than One Correct Options

1. Two circular coils of radii 5 cm and 10 cm carry currents of 2 A. The coils have 50 and 100 turns respectively and are placed in such a way that their planes as well as their centres coincide. Magnitude of magnetic field at the common centre of coils is
(a) $8\pi \times 10^{-4}$ T if currents in the coils are in same sense
(b) $4\pi \times 10^{-4}$ T if currents in the coils are in opposite sense
(c) zero if currents in the coils are in opposite sense
(d) $8\pi \times 10^{-4}$ T if currents in the coils are in opposite sense

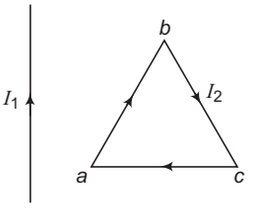
2. A charged particle enters into a gravity free space occupied by an electric field **E** and magnetic field **B** and it comes out without any change in velocity. Then, the possible cases may be
(a) $\mathbf{E} = 0$ and $\mathbf{B} \neq 0$ (b) $\mathbf{E} \neq 0$ and $\mathbf{B} = 0$
(c) $\mathbf{E} \neq 0$ and $\mathbf{B} \neq 0$ (d) $\mathbf{E} = 0$, $\mathbf{B} = 0$

3. A charged particle of unit mass and unit charge moves with velocity $\mathbf{v} = (8\hat{\mathbf{i}} + 6\hat{\mathbf{j}})$ m/s in a magnetic field of $\mathbf{B} = 2\hat{\mathbf{k}}$ T. Choose the correct alternative (s).
(a) The path of the particle may be $x^2 + y^2 - 4x - 21 = 0$
(b) The path of the particle may be $x^2 + y^2 = 25$
(c) The path of the particle may be $y^2 + z^2 = 25$
(d) The time period of the particle will be 3.14 s

4. When a current carrying coil is placed in a uniform magnetic field with its magnetic moment antiparallel to the field
(a) torque on it is maximum
(b) torque on it is zero
(c) potential energy is maximum
(d) dipole is in unstable equilibrium

5. If a long cylindrical conductor carries a steady current parallel to its length
(a) the electric field along the axis is zero
(b) the magnetic field along the axis is zero
(c) the magnetic field outside the conductor is zero
(d) the electric field outside the conductor is zero

6. An infinitely long straight wire is carrying a current I_1. Adjacent to it there is another equilateral triangular wire having current I_2. Choose the wrong options.



(a) Net force on loop is leftwards
(b) Net force on loop is rightwards
(c) Net force on loop is upwards
(d) Net force on loop is downwards

7. A charged particle is moving along positive y-axis in uniform electric and magnetic fields,

$$\mathbf{E} = E_0\hat{\mathbf{k}} \quad \text{and} \quad \mathbf{B} = B_0\hat{\mathbf{i}}$$

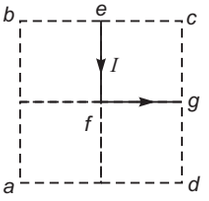
where, E_0 and B_0 are positive constants. Choose the correct options.

(a) Particle may be deflected towards positive z-axis
(b) Particle may be deflected towards negative z-axis
(c) Particle may pass undeflected
(d) Kinetic energy of particle may remain constant

8. A charged particle revolves in circular path in uniform magnetic field after accelerating by a potential difference of V volts. Choose the correct options if V is doubled

(a) kinetic energy of particle will become two times
(b) radius in circular path will become two times
(c) radius in circular path will become $\sqrt{2}$ times
(d) angular velocity will remain unchanged

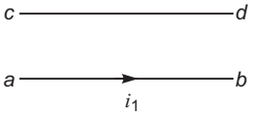
9. $abcd$ is a square. There is a current I in wire efg as shown.



Choose the correct options.

(a) Net magnetic field at a is inwards
(b) Net magnetic field at b is zero
(c) Net magnetic field at c is outwards
(d) Net magnetic field at d is inwards

10. There are two wires ab and cd in a vertical plane as shown in figure. Direction of current in wire ab is rightwards. Choose the correct options.



(a) If wire ab is fixed, then wire cd can be kept in equilibrium by the current in cd in leftward direction
(b) Equilibrium of wire cd will be stable equilibrium
(c) If wire cd is fixed, then wire ab can be kept in equilibrium by flowing current in cd in rightward direction
(d) Equilibrium of wire ab will be stable equilibrium

11. Consider a wire carrying a steady current, I placed in a uniform magnetic field **B** perpendicular to its length. Consider the charges inside the wire. It is known that magnetic forces do no work. This implies that,

(a) motion of charges inside the conductor is unaffected by **B**, since they do not absorb energy
(b) some charges inside the wire move to the surface as a result of **B**
(c) if the wire moves under the influence of **B**, no work is done by the force
(d) if the wire moves under the influence of **B**, no work is done by the magnetic force on the ions, assumed fixed within the wire.

Assertion and Reason

Directions (Q. Nos. 1-20) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** In a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{k}}$, if velocity of a charged particle is $v_0\hat{\mathbf{i}}$ at $t = 0$, then it can have the velocity $v_0\hat{\mathbf{j}}$ at some other instant.

Reason In uniform magnetic field acceleration of a charged particle is always zero.

2. **Assertion** A charged particle moves perpendicular to a uniform magnetic field then its momentum remains constant.

Reason Magnetic force acts perpendicular to the velocity of the particle.

3. **Assertion** A beam of electron can pass undeflected through a region of **E** and **B**.

Reason Force on moving charged particle due to magnetic field may be zero in some cases.

4. **Assertion** If the path of a charged particle in a region of uniform electric and magnetic field is not a circle, then its kinetic energy may remain constant.

Reason In a combined electric and magnetic field region, a moving charge experiences a net force $\mathbf{F} = q\,\mathbf{E} + q\,(\mathbf{v} \times \mathbf{B})$, where symbols have their usual meanings.

5. **Assertion** Flux of magnetic field B through a closed surface is equal to zero.

Reason Magnetic field lines are closed curves. They do not have beginning or end.

6. **Assertion** A flexible wire loop of irregular shape carrying current when placed in a uniform external magnetic field acquires a circular shape.

Reason For a given perimeter circular shape is having the greatest area.

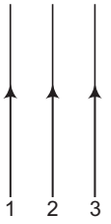
7. **Assertion** If a charged particle is projected in a region where, **B** is perpendicular to velocity of projection, then the net force acting on the particle is independent of its mass.

Reason The particle is performing uniform circular motion and force acting on it is $\frac{mv^2}{r}$.

8. Assertion In non-uniform magnetic field speed of a charged particle varies.

Reason Work done by magnetic force on a charged particle is always zero.

9. Assertion Three infinitely long current carrying wires have equal currents and they are equally spaced. The magnitude of magnetic force on all three is same.



Reason Net force on wire-2 is zero.

10. Assertion Two infinitely long wires A and B carry unequal currents both in inward direction.



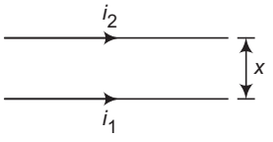
Then, there is only one point (excluding the points at infinity) where net magnetic field is zero.

Reason That point lies between points A and B.

11. Assertion Path of a charged particle in uniform magnetic field cannot be a parabola if no other forces (other than magnetic force) are acting on the particle.

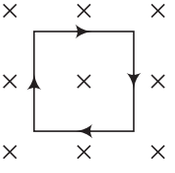
Reason For parabolic path a constant acceleration is required.

12. Assertion Upper wire shown in figure is fixed. At a certain distance x, lower wire can remain in equilibrium.



Reason The above equilibrium of lower wire is stable equilibrium.

13. Assertion Net torque in the current carrying loop placed in a uniform magnetic field (pointing inwards) is zero.



Reason Magnetic moment is inwards.

14. Assertion A charged particle is rotating in a circle. Then, magnetic field at centre of circle and magnetic moment produced by motion of charged particle are parallel to each other.



Reason $\mathbf{M}$ and $\mathbf{B}$ are always parallel to each other.

15. Assertion If a charged particle enters from outside in uniform magnetic field then it will keep on rotating in a circular path.

Reason Magnetic force is always perpendicular to velocity vector.

16. Assertion Magnetic field and electric field are present in a region. Net force on a charged particle in this region is zero if

$$\mathbf{E} = \mathbf{B} \times \mathbf{v}$$

Reason $\frac{E}{B}$ has the dimensions of velocity.

17. Assertion An α-particle and a deutron having same kinetic energy enter in a uniform magnetic field perpendicular to the field. Then radius of circular path of α-particle will be more.

Reason $\frac{q}{m}$ ratio of an α-particle is more than the $\frac{q}{m}$ ratio of a deutron.

18. Assertion A charged particle is rotating in a circular path in uniform magnetic field. Then, plane of circle is perpendicular to the magnetic field.

Reason Circular motion is a two dimensional motion.

19. Assertion At the centre of a circular current carrying loop I_1, there is an infinitely long straight current carrying wire I_2 perpendicular to the plane of the circle. Then, magnetic force of attraction between two is zero.

Reason Magnetic field of I_1 at centre is inwards, parallel to I_2.

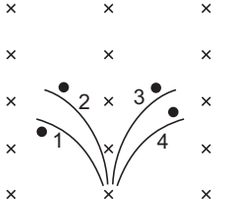
20. Assertion If velocity of charged in a uniform magnetic field at some instant is $(a_1\hat{\mathbf{i}} - a_2\hat{\mathbf{j}})$ and at some other instant is $(b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}})$, then

$$a_1^2 + a_2^2 = b_1^2 + b_2^2$$

Reason Magnetic force cannot change velocity of a charged particle.

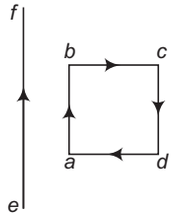
Match the Columns

1. Four particles α-particle, deutron and electron and a Cl^- ion enter in a transverse magnetic field perpendicular to it with same kinetic energy. Their paths are as shown in figure. Now, match the following two columns.



Column I	Column II
(A) Deutron	(p) Path-1
(B) α-particle	(q) Path-2
(C) Electron	(r) Path-3
(D) Cl^-	(s) Path-4

2. A square current carrying loop *abcd* is placed near an infinitely long another current carrying wire *ef*. Now, match the following two columns.



Column I	Column II
(A) Net force on *bc* and *da*	(p) Zero
(B) Net force on *ab* and *cd*	(q) Non-zero
(C) Net force on complete loop *abcd*	(r) Rightwards
(D) Net force on *ab*	(s) Leftwards

3. A charged particle enters in a uniform magnetic field perpendicular to it. Now, match the following two columns.

Column I	Column II
(A) Speed of the particle	(p) is constant
(B) Velocity of the particle	(q) is not constant
(C) Acceleration of the particle	(r) is maximum
(D) Force on the particle	(s) is minimum

4. A charged particle is rotating in uniform circular motion in a uniform magnetic field. Let $r =$ radius of circle, $v =$ speed of particle, $K =$ kinetic energy, $a =$ magnitude of acceleration, $P =$ magnitude of linear momentum, $\frac{q}{m} = \alpha =$ specific charge and $\omega =$ angular speed. Then, match the following two columns.

Column I	Column II
(A) If *v* is doubled	(p) *r* will become two times
(B) If *K* is doubled	(q) ω will become two times
(C) If *P* is doubled	(r) *a* will become two times
(D) If α is doubled	(s) None

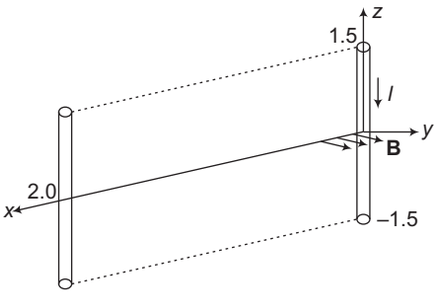
Entrance Gallery

2014

1. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed *u* between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude *I* in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the current *I* in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$, and the value of $\frac{R_1}{R_2}$ is **[JEE Advanced]**

(a) 3 : 1 (b) 5 : 1 (c) 9 : 1 (d) 4 : 3

2. A conductor lies along the *z*-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-a_z$ direction (see figure). For a field $\mathbf{B} = 3.0 \times 10^{-4} e^{-0.2x} a_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ in 5×10^{-3} s. Assume parallel motion along the *x*-axis. **[JEE Main]**



(a) 1.57 W (b) 2.97 W (c) 14.85 W (d) 29.7 W

3. If a magnetic dipole of moment *M* situated in the direction of a magnetic field *B* is rotated by 180°, then the amount of work done is **[Kerala CEE]**

(a) MB (b) $2MB$
(c) $\frac{MB}{\sqrt{2}}$ (d) zero
(e) $\sqrt{MB}$

4. The magnitude of the magnetic field inside a long solenoid is increased by **[Kerala CEE]**

(a) decreasing its radius
(b) decreasing the current through it
(c) increasing its area of cross-section
(d) introducing a medium of higher permeability
(e) decreasing the number of turns in it

5. If the radius of the dees of cyclotron is *r*, then the kinetic energy of a proton of mass *m* accelerated by the cyclotron at an oscillating frequency ν is **[Kerala CEE]**

(a) $4\pi^2 m^2 v^2 r^2$ (b) $4\pi^2 m v^2 r^2$
(c) $2\pi^2 m v^2 r^2$ (d) $\pi^2 m v^2 r^2$
(e) $\pi^2 m^2 v^2 r^2$

6. A solenoid has length 0.4 cm, radius 1 cm and 400 turns of wire. If a current of 5A is passed through this solenoid, what is the magnetic field inside the solenoid? **[Karnataka CET]**

(a) 6.28×10^{-4} T (b) 6.28×10^{-3} T
(c) 6.28×10^{-7} T (d) 628×10^{-6} T

7. A charged particle experiences magnetic force in the presence of magnetic field. Which of the following statements is correct? **[Karnataka CET]**

(a) The particle is moving and magnetic field is perpendicular to the velocity
(b) The particle is moving and magnetic field is parallel to the velocity
(c) The particle is stationary and magnetic field is perpendicular
(d) The particle is stationary and magnetic field is parallel

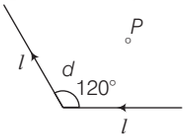
8. If a velocity has both perpendicular and parallel components while moving through a magnetic field, what is the path followed by a charged particle? **[Karnataka CET]**

(a) Circular (b) Elliptical
(c) Linear (d) Helical

9. An electron in a circular orbit of radius 0.05 nm performs 10^{16} revolutions per second. The magnetic moment due to this rotation of electron is (in A-m^2) **[WB JEE]**

(a) 2.16×10^{-23} (b) 3.21×10^{-22}
(c) 3.21×10^{-24} (d) 1.26×10^{-23}

10. A long conducting wire carrying a current I is bent at 120° (see figure). The magnetic field B at a point P on the right bisector of bending angle at a distance d from the bend is (μ_0 is the permeability of free space) **[WB JEE]**

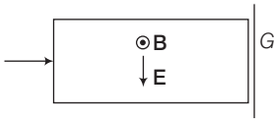


(a) $\frac{3\mu_0 I}{2\pi d}$ (b) $\frac{\mu_0 I}{2\pi d}$
(c) $\frac{\mu_0 I}{\sqrt{3}\,\pi d}$ (d) $\frac{\sqrt{3}\,\mu_0 I}{2\pi d}$

11. A proton of mass m and charge q is moving in a plane with kinetic energy E. If there exists a uniform magnetic field B, perpendicular to the plane of the motion, the proton will move in a circular path of radius **[WB JEE]**

(a) $\frac{2Em}{qB}$ (b) $\frac{\sqrt{2Em}}{qB}$
(c) $\frac{\sqrt{Em}}{2qB}$ (d) $\sqrt{\frac{2Eq}{mB}}$

12. A stream of electrons and protons are directed towards a narrow slit in a screen (see figure).The intervening region has a uniform electric field **E** (vertically downwards) and a uniform magnetic field **B** (out of the plane of the figure) as shown. Then, **[WB JEE]**



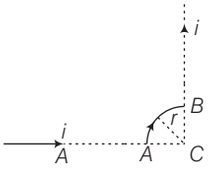
(a) electrons and protons with speed $\frac{|\mathbf{E}|}{|\mathbf{B}|}$ will pass through the slit
(b) protons with speed $\frac{|\mathbf{E}|}{|\mathbf{B}|}$ will pass through the slit, electrons of the same speed will not
(c) neither electrons nor protons will go through the slit irrespective of their speed
(d) electrons will always be deflected upwards irrespective of their speed

13. A straight wire of mass 300 g and length 2.5 m carries a current of 3.5 A. It is suspended in mid air by a uniform horizontal magnetic field B. What is the magnitude of the magnetic field? **[J&K CET]**

(a) 0.654 T (b) 0.336 T (c) 1.576 T (d) 0.939 T

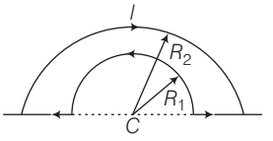
2013

14. A wire carrying current i is shaped as shown. Section AB is a quarter circle of radius r. The magnetic field is directed **[Karnataka CET]**



(a) perpendicular to the plane of the paper and directed into the paper
(b) an angle $\pi/4$ to the plane of the paper
(c) along the bisector of the angle ACB away from AB
(d) along the bisector of ACB towards AB

15. The wire loop formed by joining two semicircular sections of radii R_1 and R_2 and centre C, carries a current I as shown in the figure. The magnetic field at C has a magnitude **[Karnataka CET]**



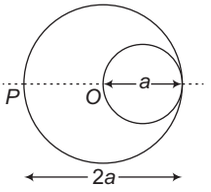
(a) $\frac{\mu_0 I}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (b) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$
(c) $\frac{\mu_0 I}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$ (d) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

16. A direct current I flows along the length of an infinitely long straight thin walled pipe, then the magnetic field is
(a) uniform throughout the pipe but not zero **[OJEE]**
(b) zero only along the axis of the pipe
(c) zero at any point inside the pipe
(d) maximum at the centre and minimum at the edge

17. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius $2R$. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement (s) is (are) **[J&K CET]**
(a) In the region $0 < r < R$, the magnetic field is non-zero
(b) In the region $R < r < 2R$, the magnetic field is along the common axis
(c) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centered on the axis
(d) In the region $r > 2R$, the magnetic field is non-zero

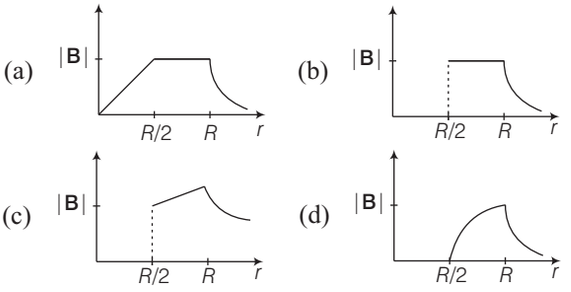
2012

18. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is **[IIT JEE]**



(a) 5 (b) 6 (c) 7 (d) 4

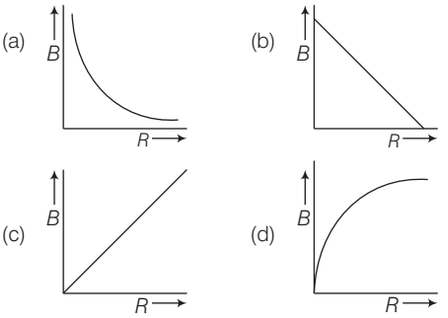
19. An infinitely long hollow conducting cylinder with inner radius $R/2$ and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\mathbf{B}|$ as a function of the radial distance r from the axis is best represented by **[IIT JEE]**



20. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\mathbf{E} = E_0\hat{\mathbf{j}}$ and $\mathbf{B} = B_0\hat{\mathbf{j}}$. At time $t = 0$, this charge has velocity $\mathbf{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time $t > 0$? **[IIT JEE]**

(a) If $\theta = 0°$, the charge moves in a circular path in the x-z plane
(b) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis
(c) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis
(d) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis

21. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation, a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc, then the variation of the magnetic induction at the centre of the disc will be represented by the figure. **[AIEEE]**

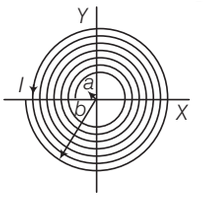


22. An α-particle and a proton moving with the same kinetic energy enter a region of uniform magnetic field at right angles to the field. The ratio of the radii of the paths of α-particle to that of the proton is **[Karnataka CET]**

(a) 1 : 8 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4

2011

23. A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the X-Y plane and a steady current I flows through the wire. The Z-component of the magnetic field at the centre of the spiral is **[IIT JEE]**



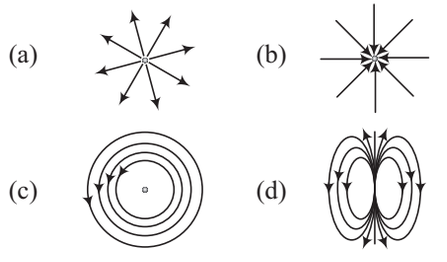
(a) $\frac{\mu_0 N I}{2(b-a)} \ln\left(\frac{b}{a}\right)$ (b) $\frac{\mu_0 N I}{2(b-a)} \ln\left(\frac{b+a}{b-a}\right)$

(c) $\frac{\mu_0 N I}{2b} \ln\left(\frac{b}{a}\right)$ (d) $\frac{\mu_0 N I}{2b} \ln\left(\frac{b+a}{b-a}\right)$

24. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true? **[IIT JEE]**

(a) They will never come out of the magnetic field region
(b) They will come out travelling along parallel paths
(c) They will come out at the same time
(d) They will come out at different times

25. Which of the field pattern given in the figure is valid for electric field as well as for magnetic field? **[IIT JEE]**



26. A current I flow in an infinitely long wire cross-section in the form of a semi-circular ring of radius R. The magnitude of the magnetic induction along its axis is **[AIEEE]**

(a) $\frac{\mu_0 I}{2\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi R}$

(c) $\frac{\mu_0 I}{4\pi R}$ (d) $\frac{\mu_0 I}{\pi^2 R}$

27. The force between two parallel current carrying wires is independent of **[Kerala CEE]**

(a) their distance of separation
(b) the length of the wires
(c) the magnitude of currents
(d) the radii of the wires
(e) the medium in which they are placed

28. An electron moving around the nucleus with an angular momentum l has a magnetic moment **[Kerala CEE]**

(a) $\frac{e}{m}l$ (b) $\frac{e}{2m}l$

(c) $\frac{2e}{m}l$ (d) $\frac{e}{2\pi m}l$

(e) $\frac{e}{4\pi m}l$

29. A thin circular disc of radius R is uniformly charged with density $\sigma > 0$ per unit area. The disc rotates about its axis with a uniform angular speed ω. The magnetic moment of the disc is **[Karnataka CET]**

(a) $2\pi R^4\sigma\omega$ (b) $\pi R^4\sigma\omega$

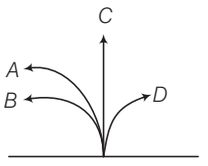
(c) $\frac{\pi R^4}{2}\sigma\omega$ (d) $\frac{\pi R^4}{4}\sigma\omega$

30. A particle of charge e and mass m moves with a velocity v in a magnetic field B applied perpendicular to the motion of the particle. The radius r of its path in the field is **[Karnataka CET]**

(a) $\frac{Bv}{em}$ (b) $\frac{ev}{Bm}$

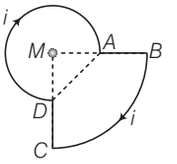
(c) $\frac{Be}{mv}$ (d) $\frac{mv}{Be}$

31. A neutron, a proton, an electron and an α-particle enter a region of uniform magnetic field with the same velocities. The magnetic field is perpendicular and directed into the plane of the paper. The tracks of the particles are labelled in the figure. The electron follows the track **[Karnataka CET]**



(a) D (b) C
(c) B (d) A

32. A current i is flowing through the loop. The direction of the current and the shape of the loop are as shown in the figure.



The magnetic field at the centre of the loop is $\frac{\mu_0 i}{R}$ times ($MA = R, MB = 2R, \angle DMA = 90°$) **[Karnataka CET]**

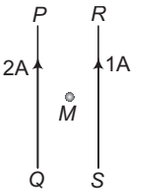
(a) $\frac{5}{16}$, but out of the plane of the paper

(b) $\frac{5}{16}$, but into the plane of the paper

(c) $\frac{7}{16}$, but out of the plane of the paper

(d) $\frac{7}{16}$, but into the plane of the paper

33. PQ and RS are long parallel conductors separated by certain distance. M is the mid-point between them (see the figure) The net magnetic field at M is B. Now, the current 2 A is switched off. The field at M now becomes **[Karnataka CET]**



(a) $2B$ (b) B

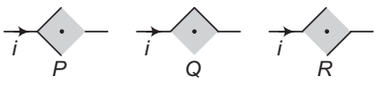
(c) $\frac{B}{2}$ (d) $3B$

34. Magnetic field at the centre of a circular coil of radius R due to i flowing through it is B. The magnetic field at a point along the axis at distance R from the centre is **[Karnataka CET]**

(a) $\frac{B}{2}$ (b) $\frac{B}{4}$

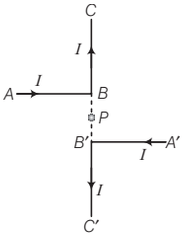
(c) $\frac{B}{\sqrt{8}}$ (d) $\sqrt{8B}$

35. Two thick wires and two thin wires, all of same material and same length, form a square in three different ways P, Q and R as shown in the figure. With correct connections shown, the magnetic field due to the current flow, at the centre of the loop will be zero in case of **[Karnataka CET]**



(a) Q and R (b) P only
(c) P and Q (d) P and R

36. Current through ABC and $A'B'C'$ is I. What is the magnetic field at P? $BP = PB' = r$ (Here $C'B'PBC$ are collinear) **[WB JEE]**



(a) $B = \frac{1}{4\pi}\frac{2I}{r}$ (b) $B = \frac{\mu_0}{4\pi}\left(\frac{2I}{r}\right)$

(c) $B = \frac{\mu_0}{4\pi}\left(\frac{I}{r}\right)$ (d) zero

37. The magnetic field at the point of intersection of diagonals of a square wire loop of side L carrying a current I is **[WB JEE]**

(a) $\frac{\mu_0 I}{\pi L}$ (b) $\frac{2\mu_0 I}{\pi L}$

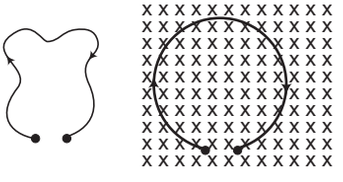
(c) $\frac{\sqrt{2}\mu_0 I}{\pi L}$ (d) $\frac{2\sqrt{2}\mu_0 I}{\pi L}$

38. Toroid is **[MHT CET]**

(a) ring shaped closed solenoid
(b) rectangular shaped solenoid
(c) ring shaped open solenoid
(d) square shaped solenoid

2010

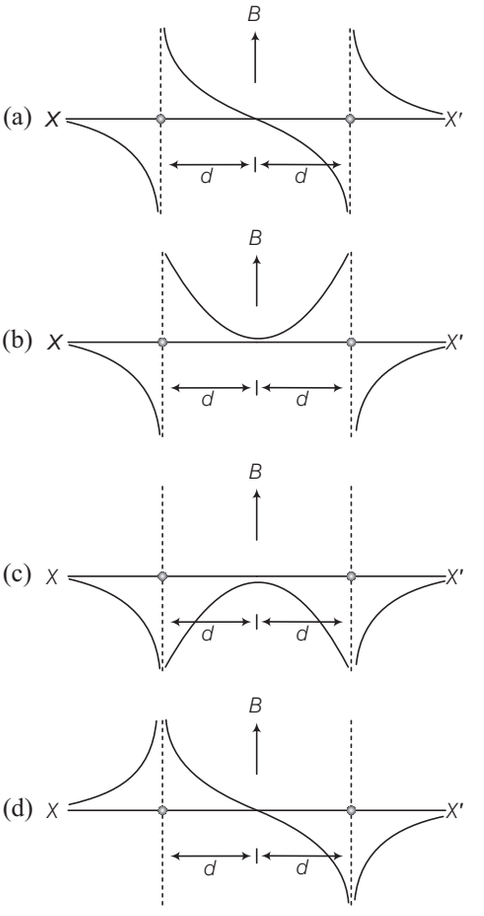
39. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is **[IIT JEE]**



(a) IBL (b) $\frac{IBL}{\pi}$

(c) $\frac{IBL}{2\pi}$ (d) $\frac{IBL}{4\pi}$

40. Two long parallel wires are at a distance $2d$ apart. They carry steady equal current flowing out of the plane of the paper as shown. The variation of the magnetic field along the line XX' is given by **[AIEEE]**



41. A charge + Q is moving upwards vertically. It enters a magnetic field directed to North. The force on the charge will be towards **[OJEE]**

(a) North (b) South
(c) East (d) West

42. Two particles of masses m_a and m_b and same charge are projected in a perpendicular magnetic field. They travel along circular paths of radius r_a and r_b such that $r_a > r_b$. Then which is true? **[OJEE]**

(a) $m_a v_a > m_b v_b$
(b) $m_a > m_b$ and $v_a > v_b$
(c) $m_a = m_b$ and $v_a > v_b$
(d) $m_b v_b > m_a v_a$

43. A circular loop and a square loop are formed from the same wire and the same current is passed through them. Find the ratio of their dipole moments. **[MHT CET]**

(a) 4π (b) $\frac{4}{\pi}$

(c) $\frac{2}{\pi}$ (d) 2π

Answers

Level 1

Objective Problems

1. (c)	**2.** (a)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (c)	**8.** (a)	**9.** (d)	**10.** (b)
11. (d)	**12.** (c)	**13.** (a)	**14.** (b)	**15.** (c)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (c)
21. (d)	**22.** (a)	**23.** (c)	**24.** (b)	**25.** (b)	**26.** (c)	**27.** (b)	**28.** (d)	**29.** (a)	**30.** (d)
31. (c)	**32.** (a)	**33.** (a,b,d)	**34.** (a,d)	**35.** (c,d)	**36.** (a,b)	**37.** (c)	**38.** (c)	**39.** (a)	**40.** (c)
41. (a)	**42.** (c)	**43.** (d)	**44.** (b)	**45.** (b)	**46.** (c)	**47.** (d)	**48.** (b)	**49.** (a)	**50.** (a)
51. (c)	**52.** (a)	**53.** (b)	**54.** (c)	**55.** (a)	**56.** (b)	**57.** (d)	**58.** (c)	**59.** (a)	**60.** (c)
61. (b)	**62.** (c)	**63.** (d)	**64.** (b)	**65.** (c)	**66.** (c)	**67.** (b)	**68.** (d)	**69.** (a)	**70.** (b)
71. (d)	**72.** (a)	**73.** (d)	**74.** (b)	**75.** (a)	**76.** (a)	**77.** (a)	**78.** (d)	**79.** (d)	**80.** (a)
81. (c)	**82.** (d)	**83.** (b)	**84.** (b)	**85.** (d)	**86.** (b)	**87.** (b)	**88.** (d)	**89.** (d)	**90.** (a)
91. (d)	**92.** (c)	**93.** (a)	**94.** (b,c)	**95.** (b,c)	**96.** (b)	**97.** (c)	**98.** (d)	**99.** (d)	**100.** (a)
101. (c)	**102.** (b)	**103.** (c)	**104.** (b)	**105.** (c)	**106.** (c)	**107.** (a)	**108.** (d)	**109.** (b,c)	**110.** (a,b,c)

Level 2

Only One Correct Option

1. (c)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (d)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (d)
11. (b)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (d)
21. (c)	**22.** (b)	**23.** (a)	**24.** (d)	**25.** (all)	**26.** (a,b.d)	**27.** (c)	**28.** (a)	**29.** (d)	**30.** (a)
31. (a,c)	**32.** (a)	**33.** (b)	**34.** (d)	**35.** (a)	**36.** (a)				

More than One Correct Options

1. (a,c)	**2.** (a,c,d)	**3.** (a,b,d)	**4.** (b,c,d)	**5.** (b,d)	**6.** (all)	**7.** (all)	**8.** (a,c,d)	**9.** (a,c,d)	**10.** (a,b,c)
11. (b,d)									

Assertion and Reason

1. (c)	**2.** (d)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (d)	**10.** (b)
11. (a,b)	**12.** (c)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (b)	**17.** (e)	**18.** (b)	**19.** (a)	**20.** (c)

Match the Columns

1. (A→q, B→p, C→s, D→r) **2.** (A→p, B→q,s, C→q,s, D→q,s) **3.** (A→p, B→q, C→q, D→q,r)
4. (A→p, B→q, C→p, D→q)

Entrance Gallery

1. (a)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (b)	**7.** (a)	**8.** (d)	**9.** (d)	**10.** (d)
11. (b)	**12.** (d)	**13.** (b)	**14.** (a)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (c)	**20.** (c,d)
21. (b)	**22.** (b)	**23.** (a)	**24.** (b,d)	**25.** (c)	**26.** (d)	**27.** (d)	**28.** (b)	**29.** (d)	**30.** (d)
31. (a)	**32.** (d)	**33.** (b)	**34.** (c)	**35.** (d)	**36.** (b)	**37.** (d)	**38.** (a)	**39.** (c)	**40.** (a)
41. (d)	**42.** (a)	**43.** (b)							

Solutions

Level 1 : Objective Problems

1. On static charge there is no effect of magnetic field, so electron remains stationary.

2. $\mathbf{F}_{\text{net}} = \mathbf{F}_e + \mathbf{F}_m = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$

$= q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

3. For charged particle on circular path,

$$\frac{mv^2}{r} = qvB$$

$$\therefore \quad r = \frac{mv}{qB}$$

For proton,

$$R_p = \frac{mv}{qB} = \frac{\sqrt{2m_pE}}{qB}$$

For deutron,

$$R_d = \frac{\sqrt{2m_dE}}{qB}$$

$$\therefore \quad \frac{R_d}{R_p} = \sqrt{\frac{m_d}{m_p}} = \sqrt{2}$$

$$\therefore \quad R_d = \sqrt{2}R_p$$

4. When charged particles move in circular path into a region of magnetic field, then time period of charged particles is given by ,

$$T = \frac{2\pi m}{qB}$$

$\therefore \quad T \propto m$ (q and B are same)

$\because \quad m_p > m_e$

$\therefore$ Time period of proton > Time period of electron.

5. We know that, $r = \dfrac{mv}{qB}$

For electron, $r = \dfrac{mv}{eB}$

$$\therefore \quad \frac{e}{m} = \frac{v}{Br}$$

6. For charged particle in uniform magnetic field, time period is given by

$$T = \frac{2\pi m}{qB}$$

$\therefore$ Frequency, $\nu = \dfrac{1}{T} = \dfrac{qB}{2\pi m}$

7. Radius of circular path of charged particle is given by,

$$r = \frac{mv}{qB} = \frac{p}{qB}$$

Since, electron and proton both have same momentum, therefore the circular path of both will have the same radius.

8. $r = \dfrac{mv}{qB} \quad \Rightarrow \quad \dfrac{r_1}{r_2} = \dfrac{m_1 v_1}{m_2 v_2} \times \dfrac{q_2}{q_1}$

$$= \frac{1 \times 2}{1 \times 3} \times \frac{2}{1} = \frac{4}{3}$$

9. From Fleming's left hand rule, the electron deflects, in negative Y-direction.

10. $T = \dfrac{2\pi m}{qB}$.

11. $r = \dfrac{\sqrt{2qVm}}{Bq}$ or $r \propto \sqrt{V}$,

where, V = potential difference

12. $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

13. $r = \dfrac{\sqrt{2Km}}{Bq}$ or $r \propto \dfrac{\sqrt{m}}{q}$

$$\left(\frac{\sqrt{m}}{q}\right)_\alpha : \left(\frac{\sqrt{m}}{q}\right)_p : \left(\frac{\sqrt{m}}{q}\right)_d = \frac{\sqrt{4}}{2} : \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1}$$

$$= 1 : 1 : \sqrt{2}$$

14. $F_e = F_m$, $qE = Bqv$

$$\therefore \quad E = Bv$$

16. $f = \dfrac{Bq}{2\pi m}$, i.e. $f \propto \dfrac{q}{m}$

As, q is same for all given particles, $f \propto \dfrac{1}{m}$. Mass of Li^+ is maximum therefore, frequency will be minimum.

17. $r = \dfrac{\sqrt{2Km}}{Bq}$ or $K \propto \dfrac{q^2}{m}$ (for same radius)

19. Work done by magnetic force is zero.

21. $T = \dfrac{2\pi m}{Bq}$ or T is independent of r.

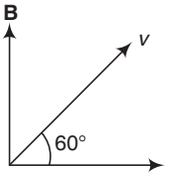
23. Path is circular when $\mathbf{v} \perp \mathbf{B}$ or $\mathbf{v} \cdot \mathbf{B} = 0$.

24. $r = \dfrac{Mv \sin 60°}{Bq} = \dfrac{(1.67 \times 10^{-27}) \times (4 \times 10^5) \sin 60°}{0.3 \times 1.6 \times 10^{-19}}$

$$= 0.012 \text{ m}$$

25. $(b - a) = r = \dfrac{mv}{Bq} \quad \Rightarrow \quad \therefore \quad v_{\min} = \dfrac{(b-a)\,Bq}{m}$

26. $r = \dfrac{mv \cos 60°}{Bq}$



$$= \frac{(1.67 \times 10^{-27})\,(2 \times 10^6) \cos 60°}{0.14 \times 1.6 \times 10^{-19}} = 0.07 \text{ m}$$

and $T = \dfrac{2\pi m}{Bq} = \dfrac{(2\pi)\,(1.67 \times 10^{-27})}{0.14 \times 1.6 \times 10^{-19}}$

$$= 0.5 \times 10^{-6} \text{ s}$$

27. Velocity is in x-y plane and magnetic field along z-axis. Therefore, path of the electron will be a circle. Magnetic force cannot change the speed of a particle.

28. $F = Bqv$

$\therefore \quad F \propto q$

29. $r = \dfrac{mv}{Bq}$, $r \propto m$ (q and v are same)

30. Velocity vector is parallel to magnetic field vector.

31. $r = \dfrac{\sqrt{2\,Km}}{Bq}$

$\therefore \quad A = \pi r^2 = \dfrac{2\pi Km}{B^2q^2}$ or $A \propto K$

32. Distance = 2 (pitch)

$$= 2\left(\frac{2\pi m}{Bq}\right)v\cos\theta$$

$$= \frac{4\times\pi\times 1.6\times 10^{-27}\times 2\times 10^{6}\times\cos 30^\circ}{0.05\times 1.6\times 10^{-19}} = 4.35 \text{ m}$$

37. Let length of each wire is L.

For square length of each side is L / 4

Area of square $= \left(\dfrac{L}{4}\right)^2 = \dfrac{L^2}{16}$

For circle,

$$L = 2\pi r$$

$\therefore \quad r = \dfrac{L}{2\pi}$

Area of circle $= \pi r^2 = \pi\left(\dfrac{L}{2\pi}\right)^2 = \dfrac{L^2}{4\pi}$

As, magnetic movement, $M = iA$

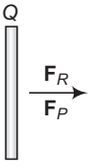
$\therefore \quad \dfrac{M_{\text{square}}}{M_{\text{circle}}} = \dfrac{A_{\text{square}}}{A_{\text{circle}}}$

$$= \frac{L^2/16}{L^2/4\pi} = \frac{\pi}{4}$$

38. Force on wire Q due to wire R,

$$F_R = \frac{4\pi\times 10^{-7}}{4\pi}\times\frac{2\times 20\times 10}{0.02}\times 0.1$$

$= 20\times 10^{-5}$ N (towards right)



Force on wire Q due to wire P

$$F_p = \frac{4\pi\times 10^{-7}}{4\pi}\times\frac{2\times 30\times 10}{0.1}\times 0.1$$

$= 6\times 10^{-5}$ N (towards right)

Net force on Q,

$$F = F_R + F_P$$
$$= 20\times 10^{-5} + 6\times 10^{-5}$$
$$= 26\times 10^{-5} \text{ N}$$
$$= 2.6\times 10^{-4} \text{ N} \quad \text{(towards right)}$$

39. Torque acting on the coil,

$$\tau = NiBA$$
$$= 100\times 0.2\times 2\times (0.08\times 0.1)$$
$$= 0.32 \text{ N-m}$$

Direction of torque τ is given by the vector $\mathbf{M}\times\mathbf{B}$.

40. Current, $i = qv$

$$= 1.6\times 10^{-19}\times 6.6\times 10^{15}$$
$$= 10.5\times 10^{-4} \text{ A}$$

Area, $A = \pi R^2$

$$= 3.14\times (0.528)^2\times 10^{-20} \text{ m}^2$$

$\therefore$ Magnetic movement,

$$M = iA$$
$$= 10.5\times 10^{-4}\times 3.14\times (0.528)^2\times 10^{-20}$$
$$= 10\times 10^{-24} \text{ A-m}^2 = 1\times 10^{-23} \text{ A-m}^2$$

41. Magnetic force on B from both the wires is towards A.

42. Force between two conductors,

$$F \propto \frac{I_1 I_2}{d}$$

When I_1 is changed to $2\,I_1$ and d is changed to $3d$.

$\therefore \quad F' \propto \dfrac{(2\,I_1)(I_2)}{3d} \propto \dfrac{2}{3}F$

As direction of current is reversed so, $F' = -2F/3$.

43. Torque acting on the coil is given by

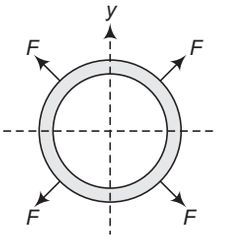
$$\tau = NiBA\sin\theta$$

As magnetic field is normal to plane of coil.

So, $\theta = 0^\circ$

$\therefore \quad \tau = 0$

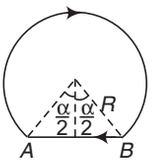
44. Applying Fleming's left hand rule, we see that, if magnetic field is perpendicular to paper inwards and current in the loop is clockwise, the magnetic force F on each element of loop is radially outwards and loop has a tendency to expand outwards.



Also, when a current carrying loop placed in uniform magnetic field, then net force on it is zero and loop cannot have translation motion.

46. $AB = 2R\sin\dfrac{\alpha}{2}$

$\therefore$ Force $= IB\left(2R\sin\dfrac{\alpha}{2}\right) = 2BIR\sin\left(\dfrac{\alpha}{2}\right)$



47. $2\pi R = 2.0$

$\therefore \quad R = \dfrac{1.0}{\pi}$ m

$M = iA = (i)(\pi R^2)$

$$= \frac{1}{\pi}\text{ A-m}^2 = (1)(\pi)\left(\frac{1.00}{\pi}\right)^2$$
$$= \frac{1}{\pi}\text{ A-m}^2$$

49. $F = ilB = \dfrac{(2)(2)(2\times 10^{-7})(10)}{0.1} = 8\times 10^{-5}$ N

52. $N = \dfrac{\left(\dfrac{\mu_0 i}{2R}\right)}{i(\pi R^2)} = \left(\dfrac{\mu_0}{2\pi}\right)\left(\dfrac{1}{R^3}\right)$

or $N \propto \dfrac{1}{R^3}$ and independent of current.

54. $\frac{F}{l}=\frac{\mu_0}{2\pi}\frac{i_A i_B}{r}$ (Given, $i_A=2i_B$)

$\therefore \quad (0.004)=\frac{2\times10^{-7}}{0.01}(2i_B^2)$

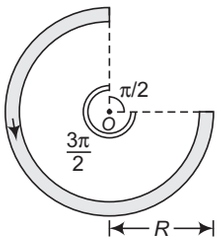
$\therefore \quad i_B=10\,\text{A}$

55. $F=ilB\sin\theta$

$$\sin\theta=\frac{F}{ilB}=\frac{15}{10\times1.5\times2}=\frac{1}{2}$$

$\therefore \quad \theta=30°$

57. Magnetic field induction at O is



$B=\frac{3}{4}$ (due to whole circle)

$=\frac{3}{4}\left(\frac{\mu_0 i}{2R}\right)=\frac{3\mu_0 i}{8R}$

58. Magnetic field at distance r near a straight current carrying wire is given by

$$B=\frac{\mu_0}{4\pi}\cdot\frac{2i}{r}$$

$\Rightarrow \quad B\propto\frac{1}{r}$

$\therefore \quad \frac{B_1}{B_2}=\frac{r_2}{r_1}$

or $\quad \frac{B}{B_2}=\frac{r/2}{r}$

$\therefore \quad B_2=2B$

59. Magnetic field at the centre of circular coil of N turns is given by,

$$B=\frac{\mu_0 Ni}{2r}=\frac{4\pi\times10^{-7}\times100\times0.1}{2\times5\times10^{-2}}=4\pi\times10^{-5}\,\text{T}$$

60. The direction of magnetic induction due to both semicircular parts will be perpendicular to the paper and inwards.

Also, $B=B_1+B_2=\frac{\mu_0 i}{4r_1}+\frac{\mu_0 i}{4r_2}=\frac{\mu_0 i}{4}\left(\frac{r_1+r_2}{r_1 r_2}\right)$

61. Current produced in wire

$$i=\frac{q}{t}=\frac{100\times e}{1}$$

$\therefore \quad i=100e$

Magnetic field at the centre of circular path

$$=\frac{\mu_0}{4\pi}\cdot\frac{2\pi i}{r}=\frac{\mu_0}{4\pi}\cdot\frac{2\pi\times100e}{r}=\frac{\mu_0\times200\times1.6\times10^{-19}}{4\times0.8}=10^{-17}\mu_0$$

62. Magnetic field produced at the centre of orbit

$$B=\frac{\mu_0}{4\pi}\frac{2\pi i}{r}$$

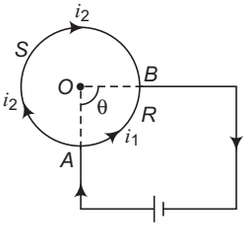
$\because \quad i=\frac{q}{t}=qv$

$\therefore \quad B=\frac{\mu_0}{4\pi}\frac{2\pi(qv)}{r}$

$$=\frac{4\pi\times10^{-7}\times2\times3.14\times1.6\times10^{-19}\times6.6\times10^{15}}{4\pi\times0.53\times10^{-10}}$$

$=12.5\,\text{Wb/m}^2$

63. Let l_1, l_2 be the lengths of two parts ARB and ASB of the ring, and ρ be resistance per unit length of the conductor,



The resistance of the portion ARB will be $R_1=l_1\rho$

The resistance of the portion ASB will be $R_2=l_2\rho$

Let i_1, i_2 be the currents in ARB and ASB respectively.

Since, these parts are in parallel, the potential difference across their ends will be same.

$\therefore \quad i_1R_1=i_2R_2$

or $\quad i_1 l_1\rho=i_2 l_2\rho$

or $\quad i_1 l_1=i_2 l_2$ …(i)

According to Biot-Savart's law, the magnetic fields at the centre O due to current through circular ring ARB and ASB will be

$$B_1=\frac{\mu_0}{4\pi}\frac{i_1 l_1\sin90°}{r^2}$$

and $\quad B_2=\frac{\mu_0}{4\pi}\frac{i_2 l_2\sin90°}{r^2}$

Since, $\quad i_1 l_1=i_2 l_2$

$\therefore \quad B_1=B_2$

According to right hand rule, the directions of B_1 and B_2 are opposite to each other, hence the resultant magnetic field at O will be zero.

64. Magnetic field at the centre of arc is given by,

$$B=\frac{\mu_0}{4\pi}\frac{\theta i}{r}=\frac{\mu_0}{4\pi}\times\frac{\pi}{2}\times\frac{i}{R}=\frac{\mu_0 i}{8R}$$

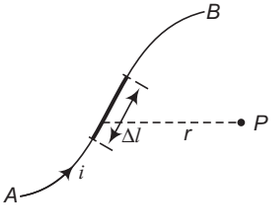
65. Let AB be a conductor carrying a current i, consider small line element Δl of the conductor, due to which the magnetic field dB, is produced, at P, then strength of magnetic field (dB) is given by

(i) $dB \propto i$

(ii) $dB \propto \Delta l$

(iii) $dB \propto \sin\theta$

(iv) $dB \propto \dfrac{1}{r^2}$



$$\therefore \quad dB \propto \frac{i\Delta l \sin\theta}{r^2}$$

or, $$dB = \frac{\mu_0}{4\pi}\frac{i\,\Delta l \sin\theta}{r^2}$$

66. Magnetic field due to a ring having n turns at a distance x and its axis is given by,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi n i r^2}{(x^2+r^2)^{3/2}}$$

$$\therefore \quad B \propto \frac{nr^2}{(x^2+r^2)^{3/2}}$$

67. When wire is turned into n circular loops, then magnetic induction is

$$B' = n^2 B = (3)^2 B_1 = 9B_1$$

68. Ratio of magnetic field at the centre of a current carrying coil and at a distance a from the centre is

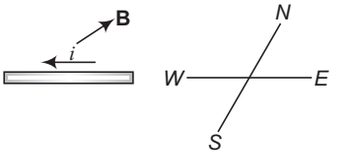
$$\frac{B_c}{B_a} = \left(1+\frac{x^2}{a^2}\right)^{3/2}$$

where, $$x = a$$

$$\therefore \quad \frac{B_c}{B_a} = \left(1+\frac{a^2}{a^2}\right)^{3/2} = 2^{3/2}$$

$$= 2\sqrt{2}$$

69.



According to Fleming's left hand rule, direction of magnetic induction will be towards North.

70. Magnetic field at the centre of (single) one turn loop is given by

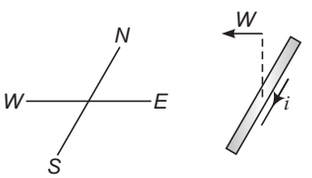
$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi i}{r}$$

and magnetic field at the centre of n-turn loop.

$$B_n = \left(\frac{\mu_0}{4\pi}\cdot\frac{2\pi i}{r/n}\right)\times n$$

$$\therefore \quad B_n = n^2 B$$

71.

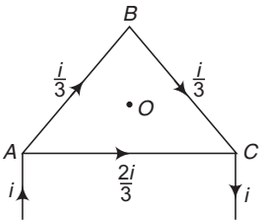


From Fleming's left hand rule, the direction of current is towards West.

72. For hollow metallic cylinder, magnetic field inside is zero while, outside and magnetic field is inversely proportional to distance from centre of cylinder.

$$B_{in} = 0 \quad \Rightarrow \quad B_{out} = \frac{\mu_0 i}{2\pi r}$$

73. Distribution of current will be as shown in below.



At point O, $\quad \mathbf{B}_{abc} = -\mathbf{B}_{ac}$

$\therefore$ Net magnetic field at O is zero.

74. Magnetic field due to straight wires will be zero.

75. B = (due to circular wire) + (due to straight wire)

$$= \frac{\mu_0 I}{2R} + \frac{\mu_0}{2\pi}\frac{I}{R} = \frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right) = \frac{\mu_0 I}{2\pi R}(\pi) = \frac{2\mu_0 I}{4\pi R}(\pi)$$

76. $$\frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}} = \frac{1}{8}\left[\frac{\mu_0 I}{2R}\right]$$

$$\therefore \quad \frac{R^2}{(x^2+R^2)^{3/2}} = \frac{1}{8R}$$

or $$(x^2+R^2)^{3/2} = 8R^3 \quad \text{or} \quad x^2+R^2 = 4R^2$$

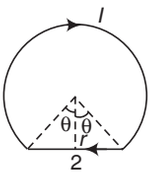
$$\therefore \quad x = \sqrt{3}R$$

77. $B_0 = B_1 + B_2$

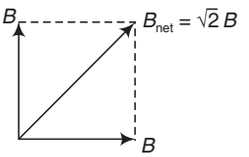
$$= \frac{\mu_0 I}{2R}\left[\frac{2\pi - 2\theta}{2\pi}\right] + \frac{\mu_0 I}{4\pi r}[\sin\theta + \sin\theta]$$

$$= \frac{\mu_0 I}{2R}\left[\frac{\pi-\theta}{\pi}\right] + \frac{\mu_0 I}{4\pi(R\cos\theta)}(2\sin\theta)$$

$$= \frac{\mu_0 I}{2\pi R}[\pi - \theta + \tan\theta]$$



78.



79. Magnetic field at O due to wire 1 will be zero.

$$B_{net} = B_2 + B_3$$

81. $B_1 = \dfrac{\mu_0 I}{2r}$, $B_2 = \dfrac{\mu_0 I}{4r}$ $\quad \Rightarrow \quad \therefore \dfrac{B_1}{B_2} = 2$

82. Inside the rod, $B \propto r$ and outside it $B \propto \dfrac{1}{r}$.

83. $B = \dfrac{\mu_0 N i}{2r}$

84. At O, due to wire AB and DE, magnetic field will be zero. The combined effect of BC and DF is equivalent to that of an infinitely long wire.

88. $$B_{net} = \frac{\mu_0 N_1 i_1}{2R_1} - \frac{\mu_0 N_2 i_2}{2R_2}$$

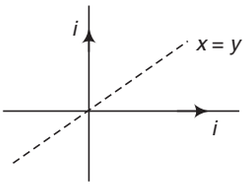
$$= \frac{\mu_0}{2}\left[\frac{10\times 0.2}{2\times 10^{-2}} - \frac{10\times 0.3}{4.0\times 10^{-2}}\right] = \left(\frac{25}{2}\right)\mu_0$$

89. $B = \dfrac{\mu_0 N_i}{2R}$

or $B \propto \dfrac{N}{2R}$

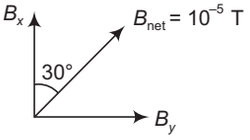
Number of turns will become two times while radius remains half.

90. Along the line $x = y$, magnetic fields due to two wires will be equal and opposite.



91. $\dfrac{B_y}{B_x} = \tan 30°$

$\therefore \quad B_y = \dfrac{B_x}{\sqrt{3}}$



$\therefore \quad i_y = \dfrac{i_x}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}$ A

92. $\dfrac{\mu_0}{2\pi r}\left(\dfrac{1}{i_2} - \dfrac{1}{i_i}\right) = 10 \times 10^{-6}$ ...(i)

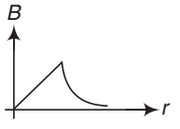
$\dfrac{\mu_0}{2\pi r}\left(\dfrac{1}{i_2} + \dfrac{1}{i_1}\right) = 30 \times 10^{-6}$ ...(ii)

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{i_1 + i_2}{i_1 - i_2} = \frac{3}{1}$$

Solving this equation, we get, $\dfrac{i_1}{i_2} = 2$

94. B varies with r as shown in figure.



96. It was first discovered by the famous Danish scientist Hans Christion Oersted in 1920 that an electric current through the wire deflects the magnetic needle held near the wire. By his famous experiment Oersted corelated that a magnetic field is established around the conductor carrying current.

97. 1 Tesla = 1 Wb/m^2 = 1 N/A-m =10^4 Gauss or Oersted.

98. A moving charge and a changing electric field both can produce magnetic field.

99. Unit of magnetic permeability is H/m.

As, SI unit of magnetic permeability is weber amp^{-1} m^{-1}. But Wb/A is unit of self-inductance or henry. $\left(L = \dfrac{\phi}{i}\right)$

$\therefore$ Unit of magnetic permeability is H/m.

100. When a charged particle moves in both electric and magnetic field, then it gains energy in electric field only because in magnetic field energy remains constant.

104. $B = \dfrac{F}{Il} = \dfrac{[MLT^{-2}]}{[AL]} = [MT^{-2}A^{-1}]$

106. Under no deflection condition.

$$\left(\frac{e}{m}\right) \propto \frac{1}{B^2} \quad \text{or} \quad B \propto \sqrt{m}$$

110. All the three x, y and z have the dimensions of velocity.

Level 2 : Only One Correct Option

1. For minimum value of B, it is colliding after one pitch. It will collide the target again, if pitch is halved or remains same.

$$p = (v\cos\theta)\left(\frac{2\pi m}{Bq}\right)$$

or $\quad p \propto \dfrac{v}{B}$

2. All magnetic fields are cancelling each other.

3. If q is positive, due to electric field, their velocity will become towards positive x-axis.

Now, $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

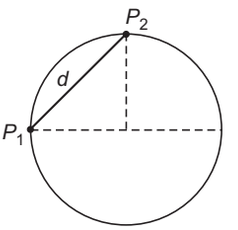
$\mathbf{v} \rightarrow$ towards $\hat{\mathbf{i}}$

$\mathbf{B} \rightarrow$ towards $\hat{\mathbf{k}}$

$\therefore \quad \mathbf{F}_m \rightarrow$ towards $\hat{\mathbf{j}}$

Similarly, we can see with $-q$ charge.

4. Given time $t = \dfrac{\pi m}{Bq} = \dfrac{T}{2}$ of proton



$$d = \sqrt{2}r = \frac{\sqrt{2}mv}{Bq}$$

5. $dA = (2\pi r)\, dr$

$$dq = \sigma \cdot dA = (2\pi\alpha r^3) \cdot dr$$

$$i = (dq)\, f = (dq)\,\frac{\omega}{2\pi} = (\alpha\omega r^3)\, dr$$

$$dB = \frac{\mu_0 i}{2r} = \frac{\alpha\mu_0\omega r^2 dr}{2}$$

$\therefore \quad B = \int_0^R dB = \dfrac{\mu_0\alpha\omega R^3}{6}$

6. $y = 2$ (Radius)

7. $(mg)\,(R) = MB_x = I(\pi r^2)\, B_x$

$\therefore \quad i = \dfrac{mg}{\pi r B_x}$

8. At the centre of coil-1.

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{2\pi i_1}{r_1} \quad \text{...(i)}$$

At the centre of coil-2

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{2\pi i_2}{r_2} \quad \text{...(ii)}$$

but $B_1 = B_2$

$$\therefore \quad \frac{\mu_0}{4\pi}\frac{2\pi i_1}{r_1} = \frac{\mu_0}{4\pi}\frac{2\pi i_2}{r_2}$$

or $$\frac{i_1}{r_1} = \frac{i_2}{r_2}$$

As $r_1 = 2r_2$

$$\therefore \quad \frac{i_1}{2r_2} = \frac{i_2}{r_2}$$

or $$i_1 = 2i_2$$

Now, ratio of potential differences

$$\frac{V_2}{V_1} = \frac{i_2 \times r_2}{i_1 \times r_1} = \frac{i_2 \times r_2}{2i_2 \times 2r_2} = \frac{1}{4}$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{4}{1}$$

9. $(mgr) = MB = (i)(\pi r^2)\, B$

$$\therefore \quad i = \frac{mg}{\pi r B}$$

10. $v = \dfrac{E}{B}$

$$r = \frac{mv}{Bq} = \frac{E/B}{B \cdot s} = \frac{E}{B^2 s}$$

11. $r = a\cos\dfrac{\pi}{n}$



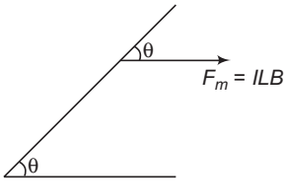
$$B = n\left[\frac{\mu_0}{4\pi}\cdot\frac{i}{a\cos\pi/n}\left(\sin\frac{\pi}{n} + \sin\frac{\pi}{n}\right)\right]$$

$$= \frac{\mu_0 ni}{2\pi a}\tan\left(\frac{\pi}{n}\right)$$

13. $B = \dfrac{\mu_0 i}{2R} = \dfrac{\mu_0 (ef)}{2R} = \dfrac{(\mu_0)(e)\left(\dfrac{v}{2\pi R}\right)}{R}$

or $$R = \sqrt{\frac{v}{B}}$$

14. $mg\sin\theta = ILB\cos\theta$



$$\therefore \quad B = \frac{mg}{IL}\tan\theta$$

15. $\mathbf{F}_{bc} + \mathbf{F}_{da} = 0$

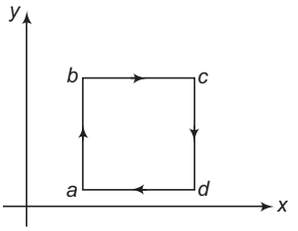
$$\mathbf{F}_{ab} = IlB_0\left(1 + \frac{x}{l}\right)$$

$$= IB_0 l + IB_0 x \quad \text{(towards right)}$$

$$\mathrm{F}_{cd} = IlB_0\left[1 + \frac{x+l}{l}\right]$$

$$= 2IB_0 l + IB_0 x \quad \text{(towards left)}$$

$$\therefore \quad \mathbf{F}_{\text{net}} = F_{cd} - F_{ab} = IB_0 l \quad \text{(towards left)}$$



16. $2Kx_0 = mg = ILB = \left(\dfrac{E}{R}\right)\cdot LB$

$$\therefore \quad B = \frac{mgR}{EL}$$

17. Let the magnetic field is,

$$\mathbf{B} = B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} + B_3\hat{\mathbf{k}}$$

Applying, $\mathbf{F}_m = q(\mathbf{v}\times\mathbf{B})$ two times, we have

$$q[-\hat{\mathbf{j}} + \hat{\mathbf{k}}] = q[(\hat{\mathbf{i}})\times(B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} + B_3\hat{\mathbf{k}})] = q[B_2\hat{\mathbf{k}} - B_3\hat{\mathbf{j}}]$$

Comparing two sides, we get,

$$B_2 = 1 \text{ and } B_3 = 1$$

Further,

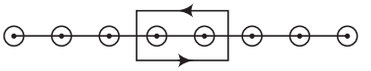
$$q[\hat{\mathbf{i}} - \hat{\mathbf{k}}] = q[(\hat{\mathbf{j}})\times(B_1\hat{\mathbf{i}} + B_2\hat{\mathbf{j}} + B_3\hat{\mathbf{k}})] = q[-B_1\hat{\mathbf{k}} + B_3\hat{\mathbf{i}}]$$

Again comparing, we get

$$B_1 = 1 \text{ and } B_3 = 1$$

$$\therefore \quad \mathbf{B} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\ \text{Wb/m}^2$$

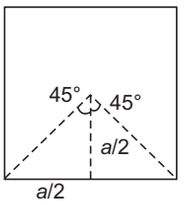
18. Applying Ampere's circuital law in the closed loop as shown in figure,



$$2Bl = \mu_0(\lambda \cdot l)$$

$$\therefore \quad B = \frac{\mu_0\lambda}{2}$$

19. $l = 2\pi R \Rightarrow R = \dfrac{l}{2\pi}$



$$l = 4a \quad \Rightarrow \quad a = \frac{l}{4}$$

$$B' = \frac{\mu_0 i}{2R} = \frac{\mu_0 i}{2\cdot\dfrac{l}{2\pi}} = \frac{\mu_0\pi i}{l}$$

$$B = 4\left[\frac{\mu_0}{4\pi}\cdot\frac{i}{a/2}(\sin 45^\circ + \sin 45^\circ)\right]$$

$$= \frac{2\sqrt{2}\,\mu_0 i}{\pi a} = \frac{8\sqrt{2}\,\mu_0 i}{\pi l}$$

$$\therefore \quad \frac{B}{B'} = \frac{8\sqrt{2}}{\pi^2}$$

20. At centre $B = 0$. At $x \to \alpha, B \to 0$ between zero and zero there will be a maximum value.

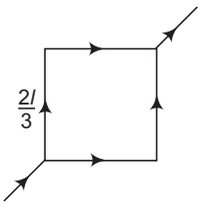
Along x-axis for $x > 0, B_{net}$ is downwards.

Along x-axis for $x < 0, B_{net}$ is upwards.

21. $F_{net} = \sqrt{F^2 + F^2} = \sqrt{2}\, F = \sqrt{2}\, IlB$

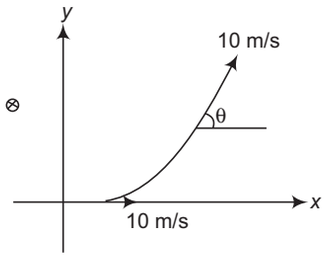
22. $B = 2\left[\frac{\mu_0}{4\pi}\frac{2I/3}{a/2}(\sin 45° + \sin 45°) - \frac{\mu_0}{4\pi}\cdot\frac{I/3}{a/2}(\sin 45° + \sin 45°)\right]$

$= \frac{\sqrt{2}\mu_0 I}{3\pi a}$



23. $\theta = \omega t = \left(\frac{Bq}{m}\right)t$

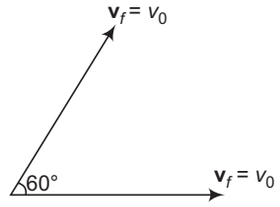
$= 2 \times \pi \times \frac{1}{6} = \frac{\pi}{3}$ or 60°



$\mathbf{v} = 10\cos 60°\,\hat{\mathbf{i}} + 10\sin 60°\,\hat{\mathbf{j}}$

$= (5\hat{\mathbf{i}} + 5\sqrt{3}\,\hat{\mathbf{j}})$ m/s

24. Deviation, $\theta = \sin^{-1}\left(\frac{L}{R}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$



$|\Delta \mathbf{v}| = 2v_0 \sin\frac{\theta}{2}$

$= 2v_0 \sin 30° = v_0$

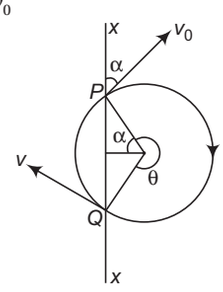
25. $PQ = 2r\sin\alpha$

$= \frac{2mv_0}{Bq}\sin\alpha$

$\theta = (2\pi - 2\alpha)$

$\therefore \quad t = \frac{\theta}{\omega} = \frac{2(\pi - \alpha)}{(Bq/m)}$

$= \frac{2m(\pi - \alpha)}{Bq}$



26. As proton is moving with constant velocity so acceleration is zero.

So, $E = 0$ and $B = 0$.

When $E = 0$ but $B \neq 0$, but parallel to the motion of proton, there will be no net force acting on proton.

When $E \neq 0$ and $B \neq 0$ but electric force and magnetic force cancel each other, then also velocity will remain unchanged.

27. $r = \frac{\sqrt{2mk}}{qB}$

$\therefore \quad r \propto \sqrt{m}$

$\therefore \quad \frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$

28. For first figure magnetic field at O due to straight part (a),

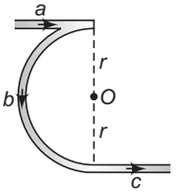
$$B_a = \frac{\mu_0}{4\pi}\cdot\frac{i}{r}$$

Magnetic field at O due to circular part (b)

$$B_b = \frac{\mu_0}{4\pi}\frac{\pi i}{r}$$

Magnetic field at O due to straight part (c)

$$B_c = \frac{\mu_0}{4\pi}\frac{i}{r}$$

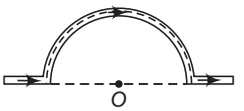


So, net magnetic field at the centre of first figure,

$$B_1 = B_b + B_c - B_a = \frac{\mu_0}{4\pi}\cdot\frac{\pi i}{r} \quad \ldots.(i)$$

For second figure,

magnetic field at O due to two straight and one circular part is



$$B_2 = \frac{\mu_0}{4\pi}\frac{\pi i}{r} \quad \ldots(ii)$$

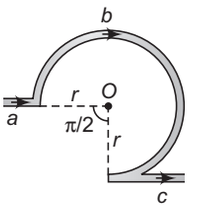
For third figure magnetic field at O due to straight part (a)

$$B_a = 0$$

Magnetic field at O due to circular part (b)

$$B_b = \frac{\mu_0}{4\pi}\frac{(2\pi - \pi/2)i}{r}$$

Magnetic field at C due to straight part (c)



$$B_c = \frac{\mu_0}{4\pi}\cdot\frac{i}{r}$$

So, net magnetic field at centre of third figure

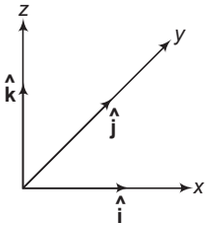
$$B_3 = \frac{\mu_0}{4\pi}\cdot\frac{i}{r}\left(\frac{3\pi}{2} - 1\right) \quad \ldots(iii)$$

From Eqs. (i), (ii) and (iii), we obtain

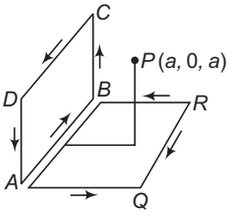
$$B_1 : B_2 : B_3 = \pi : \pi : \left(\frac{3\pi}{2} - 1\right)$$

$$= \left(-\frac{\pi}{2}\right) : \left(\frac{\pi}{2}\right) : \left(\frac{3\pi}{4} - \frac{1}{2}\right)$$

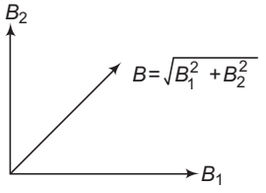
29.



We can consider the non-planar loop as made by two loops *ABCDA* and *AQRBA* as shown in figure.



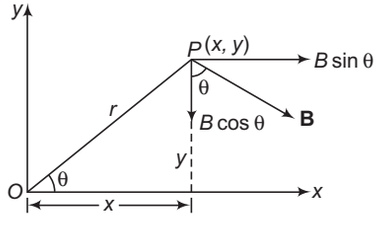
Magnetic field due to loop *ABCDA* will be along *x*-axis and due to loop *AQRBA* along *z*-axis, magnitude of magnetic field due to both loops will be equal.



Therefore, direction of resultant magnetic field at *P* will be along $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{k}})$.

30. In the $z = 0$ plane, the situation is as follows.

Here, $P(x, y)$ is the point and $r = \sqrt{x^2 + y^2}$



Magnetic field at *P* is perpendicular to *OP*, it is shown in figure.

So, $\mathbf{B} = B\sin\theta\,\hat{\mathbf{i}} - B\cos\theta\,\hat{\mathbf{j}}$

where, $B = \frac{\mu_0}{2\pi}\frac{I}{r}$

$$\sin\theta = \frac{y}{r}, \cos\theta = \frac{x}{r}$$

$\therefore$ $\mathbf{B} = \frac{\mu_0 I}{2\pi r}\left(\frac{y}{r}\hat{\mathbf{i}} - \frac{x}{y}\hat{\mathbf{j}}\right)$

$$= \frac{\mu_0}{2\pi}\frac{I(y\hat{\mathbf{i}} - x\hat{\mathbf{j}})}{(x^2 + y^2)}$$

31. $r = \frac{\sqrt{2mk}}{qB}$

$\therefore$ $r \propto \frac{\sqrt{m}}{q}$

$\therefore$ $r_H : r_{He} : r_O = \frac{\sqrt{1}}{1} : \frac{\sqrt{4}}{1} : \frac{\sqrt{16}}{2}$

$= 1 : 2 : 2$

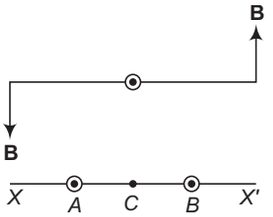
32. $\mathbf{F}_{CAD} = \mathbf{F}_{CD} = \mathbf{F}_{CED}$

$\therefore$ Net force on frame $= 3\mathbf{F}_{CD}$

$= 3(Bil) = 3 \times 2 \times 1 \times 4$

$= 24\,\text{N}$

33. If direction of current is out of paper, then magnetic field at points to the right of wire will be upwards and to the left will be downward. Now magnetic field at *C* is zero. The field in the region *BX′* will be upwards (posture), because all points lying in this region are to the right of both the wires.



Again, magnetic field in the region *AX* will be downwards (–ve). The field in the region *AC* will be upwards (positive) because points are closer to *A* compared to *B*. Similarly, magnetic field in the region *BC* will be downward (negative). Curve in graph (b) satisfy these conditions.

34. For given pitch *d* correspond to charge particle, we have

$$\frac{q}{m} = \frac{2\pi v\cos\theta}{qB} = \text{constant}$$

Since, charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field **B**, LHS for two particles should be same and of opposite sign. Therefore,

$$\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0$$

35. The charged particle undergoes acceleration as

(i) speeds up between the dees because of the oscillating electric field and

(ii) speed remain the same inside the dees because of the magnetic field but direction undergoes change continuously.

36. The rotation of the loop by 30° about an axis perpendicular to its plane make no change in the angle made by axis of the loop with the direction of magnetic field, therefore, the work done to rotate the loop is zero.

More than One Correct Options

1. $B_1 = \frac{\mu_0 N_1 i_1}{2R_1} = \frac{(4\pi \times 10^{-7})(50)(2)}{2(5 \times 10^{-2})} = 4\pi \times 10^{-4}\ \text{T}$

$B_2 = \frac{\mu_0 N_2 i_2}{2R_2} = \frac{(4\pi \times 10^{-7})(100)(2)}{(2)(10 \times 10^{-2})} = 4\pi \times 10^{-4}\ \text{T}$

When currents are in the same direction

$$B_{net} = B_1 + B_2$$

When currents are in the opposite direction

$$B_{net} = B_1 - B_2$$

2. *(a)* $\mathbf{v}$ is parallel on anti-parallel to $\mathbf{B}$.

(c) $q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = 0$

or $\mathbf{E} = -(\mathbf{v} \times \mathbf{B}) = (\mathbf{B} \times \mathbf{v})$

3. $r = \dfrac{mV}{Bq} = \dfrac{(1)(10)}{(2)(1)} = 5\,\text{m}$

$$T = \frac{2\pi m}{Bq}$$

$$= \frac{(2)(\pi)(1)}{(2)(1)} = \pi = 3.14\ \text{s}$$

Plane of circle is perpendicular to $\mathbf{B}$, i.e. X-Y plane.

4. $\theta = 180°$

$\tau = MB \sin\theta = 0$

$U = -MB\cos\theta = +MB = \text{maximum}$

5. If current flow in a conduction,

$E \neq 0$ (for inside points)

$E = 0$ (for outside points)

$B = \dfrac{\mu_0}{2\pi}\dfrac{i}{R^2} r$ (for inside points)

$B = 0$ at $r = 0$, i.e. at centre

$B = \dfrac{\mu_0}{2\pi}\dfrac{i}{r}$ for outside points.

6. $\mathbf{F}_{ab} = \text{upwards}$

$\mathbf{F}_{abc} = \text{leftwards}$

$\therefore$ Net force on loop is neither purely leftwards or rightwards or upwards or downwards.

7. $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

Depending on sign of q, $\mathbf{F}_m$, may be along positive z-axis or along negative z-axis.

$\mathbf{F}_e = q\mathbf{E}$

Again, depending on the value of q it may be along positive z-axis or along negative z-axis.

If q is positive, $\mathbf{v} \times \mathbf{B}$ and $\mathbf{F}_m$ comes along negative z-axis also. But $\mathbf{F}_e$ comes along positive z-axis. So, it may also pass undefeated.

8. $\text{KE} = qV$ or $\text{KE} \propto V$,

$$r = \frac{\sqrt{2qVm}}{Bq}$$

or $r \propto \sqrt{V}$

$$T = \frac{2\pi m}{Bq}$$

or T is independent of V.

9. Point a lies to the right hand side of ef and fg. Hence, both wires produce inward magnetic field. Hence, net magnetic field is inwards.

Same logic can be applied for other points also.

11. Magnetic forces on a wire carrying a steady current, I placed in a uniform magnetic field B, a wire carrying a steady current, I placed in a uniform magnetic field $\mathbf{B}$ perpendicular to its length is given by

$$F = IlB$$

The direction of force is given by Flemings left hand rule and F is perpendicular to the direction of magnetic field $\mathbf{B}$. Therefore, work done by the magnetic force on the ions is zero.

Assertion and Reason

1. Acceleration of charged particle in uniform field is non-zero, but speed remains constant. In given condition particle will rotate in a circular path in x-y plane. Therefore, if velocity is $v_0\hat{\mathbf{i}}$ at some instant, it can have the value $v_0\hat{\mathbf{j}}$ at other instant also.

2. Velocity will change but speed will remain constant. Hence, momentum will change.

3. $\mathbf{F}_m = 0$, if $\theta = 0°$ or $180°$.

Further, if $\mathbf{F}_e + \mathbf{F}_m = 0$

or $q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = 0$

path is undeflected.

4. Kinetic energy will remain constant, if $\mathbf{F}_e + \mathbf{F}_m = 0$

6. It may or may not take a circular shape depending upon the direction of $\mathbf{B}$.

7. Path will be uniform circular, only if field is uniform.

9. Wire-2 is equally attracted by wire-1 and 3.

10. To the right of B, both fields are downwards and to the left of A both fields are upwards.

12. Magnetic force of attraction on lower wire is upwards while its weight is downwards. At a certain distance x these two forces are equal. If the lower wire is displaced upwards from this position, then magnetic force will increase but weight will remain same. Therefore, net force is upwards or equilibrium is unstable.

13. $\mathbf{M}$ and $\mathbf{B}$ are parallel (both inwards). Therefore, $\tau = 0$.

14. If a positive charged particle is rotating in a circle in clockwise direction, then equivalent current is also clockwise. Therefore, at centre $\mathbf{B}$ and $\mathbf{M}$ are both inwards. Hence, they are parallel. But at some other point they may not be parallel.

16. $q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = 0$

$\therefore$ $\mathbf{E} = -(\mathbf{v} \times \mathbf{B}) = (\mathbf{B} \times \mathbf{v})$

17. $r = \dfrac{\sqrt{2Km}}{Bq}$ or $r \propto \dfrac{\sqrt{m}}{q}$

$$\left(\frac{\sqrt{m}}{q}\right) = \frac{\sqrt{4}}{2} = 1$$

$$\left(\frac{\sqrt{m}}{q}\right)_d = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$\therefore$ $r_d > r_\alpha$

Further, $\left(\dfrac{q}{m}\right)_\alpha = \dfrac{2}{4} = \dfrac{1}{2}$ and $\left(\dfrac{q}{m}\right)_d = \dfrac{1}{2}$

$\therefore$ $\left(\dfrac{q}{m}\right)_\alpha = \left(\dfrac{q}{m}\right)_d$

20. Magnetic force can change the velocity but not the speed. Speeds at two instants must be same.

Match the Columns

1. $r = \dfrac{\sqrt{2Km}}{Bq}$

or $r \propto \dfrac{\sqrt{m}}{q}$

2. Magnetic field over the loop from the straight wire is perpendicular to paper inwards.

4. $r = \dfrac{mv}{Bq} = \dfrac{P}{Bq} = \dfrac{\sqrt{2Km}}{Bq}$

$$\omega = \frac{Bq}{m}$$

Entrance Gallery

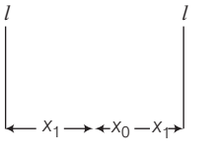
1. $B_2 = \dfrac{\mu_0 I}{2\pi x_1} + \dfrac{\mu_0 I}{2\pi (x_0 - x_1)}$

(when currents are in opposite directions)

$$B_1 = \frac{\mu_0 I}{2\pi x_1} - \frac{\mu_0 I}{2\pi (x_0 - x_1)}$$

(when currents are in same direction)

Substituting, $x_1 = \dfrac{x_0}{3}\left(\text{as } \dfrac{x_0}{x_1} = 3\right)$



$$B_1 = \frac{3\mu_0 I}{2\pi x_0} - \frac{3\mu_0 I}{4\pi x_0} = \frac{3\mu_0 I}{4\pi x_0}$$

$$R_1 = \frac{mv}{qB_1} \text{ and } B_2 = \frac{9\mu_0 I}{4\pi x_0}$$

$$R_2 = \frac{mv}{qB_2}, \frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{9}{3} = 3$$

2. Force exerted on a current carrying conductor

$$F_{ext} = BIL$$

$$\text{Average power} = \frac{\text{work done}}{\text{time taken}}$$

$$P = \frac{1}{t}\int_0^2 F_{ext} \cdot dx = \frac{1}{t}\int_0^2 B(x)\, IL\, dx$$

$$= \frac{1}{5\times 10^{-3}}\int_0^2 3\times 10^{-4} e^{-0.2x} \times 10 \times 3\, dx$$

$$= 9[1 - e^{-0.4}] = 9\left[1 - \frac{1}{e^{0.4}}\right] = 2.967 \approx 2.97 \text{ W}$$

3. If the magnet be rotated from an initial orientation $\theta_1 = 0°$ to final orientation $\theta_2 = 180°$, the total work done

$$W = \int_{\theta_1}^{\theta_2} MB \sin d\theta$$

$$= MB[-\cos\theta]_{\theta_1}^{\theta_2} = MB(\cos\theta_1 - \cos\theta_2)$$

where, B = magnetic field induction

M = magnetic moment

$= MB[+\cos 0° - \cos 180°]$

$= MB[1 + 1] = 2MB$

4. By field along the axis of a solenoid,

$$B = \mu_0 ni$$

where, B = magnetic field

μ_0 = permeability of free space

n = the number of turns per unit length

i = current

So, according to the formula,

The magnitude of the magnetic field inside a long solenoid is increased by introducing a medium of higher permeability.

5. Given, r = the outside radius of the dees

v = the speed of the particle when travelling in a path of this radius

m = the mass of proton

Kinetic energy of particles accelerated in a cyclotron,

$$K = 2\pi^2 m v^2 r^2$$

6. The magnetic field inside the solenoid is given by

$$B = \mu_0 n i_0$$

where, n = number of turns per unit length,

i = current in coil

where, $n = \dfrac{N}{L}\cdot r = \dfrac{400}{0.4\times 10^{-2}} \times 1 \times 10^{-2}$

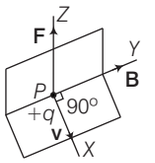
$n = 1000$

$\mu_0 = 4\pi \times 10^{-7}$ T-m/A

$\therefore \quad B = 4 \times 3.14 \times 1000 \times 5 \times 10^{-7}$

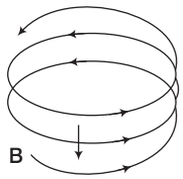
$= 62800 \times 10^{-7} = 6.28 \times 10^{-3}$ T

7. By Fleming's left hand rule,



So, the particle is moving and magnetic field is perpendicular to the velocity.

8. Suppose, the velocity **v** of the particle entering the field **B**, instead of being perpendicular to **B**, makes an angle θ with it. Then, **v** may be resolved into two components, $v_{\parallel} = v\cos\theta$ parallel to **B** and $v_{\perp} = v\sin\theta$ perpendicular to **B**.The component $v_{\parallel}$ gives a linear path and the component $v_{\perp}$ gives a circular path to the particle. The resultant of these two is a helical path whose axis is parallel to the magnetic field.



9. Given, $r = 0.05 \text{ nm} = 0.05 \times 10^{-9}$ m $\quad (\because 1 \text{ nm} = 10^{-9} \text{ m})$

$n = 10^{16}$ revolutions

$e = 1.6 \times 10^{-19}$ C

We know that,

the magnetic moment, $M = Ai$

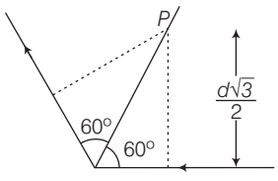
$$M = \pi r^2 \times ne$$

$$M = 3.14 \times (0.05 \times 10^{-9})^2 \times 10^{16} \times 1.6 \times 10^{-19}$$

$$M = 0.1256 \times 10^{-18} \times 10^{16} \times 10^{-19}$$

or $\quad M = 1.26 \times 10^{-23}$ A-m^2

10. We know that,



$$B_{net} = \left[\frac{\mu_0 I}{4\pi r}(\sin\theta_1 + \sin\theta_2)\right]$$

$$= 2\left[\frac{\mu_0}{4\pi} \times \frac{I}{\frac{d\sqrt{3}}{2}} \times (\sin 90° + \sin 30°)\right]$$

$$= 2\left[\frac{\mu_0}{4\pi} \times \frac{2I}{d\sqrt{3}} \times \left(1 + \frac{1}{2}\right)\right]$$

$$= 2\left[\frac{\mu_0}{4\pi} \times \frac{2I}{d\sqrt{3}} \times \frac{3}{2}\right] = \frac{\sqrt{3}\mu_0 I}{2\pi d}$$

11. Given, kinetic energy = E, mass = m, magnetic field = B, charge = q

We know that,

$$F = qvB\sin\theta$$

(motion of a charged particle in a uniform magnetic field)

If $\theta = 90°$,

then $F = qvB$ ...(i)

We also know that (centripetal force),

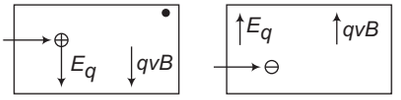
$$F = \frac{mv^2}{r} \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$qvB = \frac{mv^2}{r}, r = \frac{mv}{qB}$$

$$r = \frac{m\sqrt{\frac{2E}{m}}}{qB} \Rightarrow r = \frac{\sqrt{2Em}}{qB} \qquad \left[\because E = \frac{1}{2}mv^2, v = \sqrt{\frac{2E}{m}}\right]$$

12. Neither electrons nor protons will go through the slit irrespective of their speed.



and electrons will always be deflected upwards irrespective of their speed.

13. Here, weight of the wire is balanced by the external magnetic force.

So, $Bil = mg, B = \frac{mg}{il} = \frac{300 \times 10^{-3} \times 10}{3.5 \times 2.5} = 0.34$ T

14. By Grip rule magnetic field due to straight segment of current at C = zero.

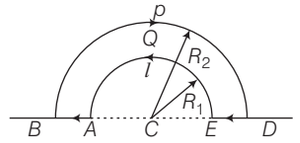
Magnetic field due to quarter circle at

$$C = \frac{\mu_0 I}{8r}$$

vertically down to the paper.

15. Magnetic field at C due to AB and DE = zero.

Magnetic field due to semicircular arc,



$$BPD = \frac{\mu_0 I}{4\pi R_2} \quad \text{(downward)}$$

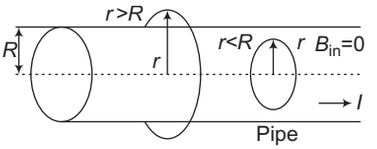
Magnetic field due to semicircular arc,

$$EQA = \frac{\mu_0 I}{4R_1} \quad \text{(upward)}$$

$$B_{net} = \left(\frac{\mu_0 I}{4R_1} - \frac{\mu_0 I}{4R_2}\right) \quad \text{(upward)}$$

16. Required arrangement is shown in figure. According to Ampere's circuital law,

$$B_{outside} = \frac{\mu_0}{4\pi}\frac{2I}{r}$$



For an internal point, $r < R$

$$B_{internal} = \frac{\mu_0(0)}{2\pi r} = 0$$

For a point on pipe, $r = R$

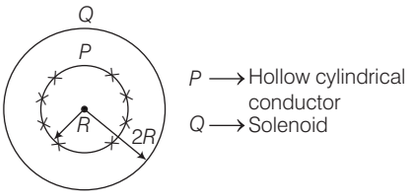
$$B = \frac{\mu_0 I}{2\pi r}$$

For an external point, $r < R$

$$B_{external} = \frac{\mu_0 I}{2\pi r}$$

Hence, magnetic field is zero only inside the pipe at any point,

17.



In the region, $0 < r < R$

$$B_P = 0$$

$B_Q \neq 0$, along the axis

$\therefore$ $B_{net} \neq 0$

In the region, $R < r < 2R$

$B_P \neq 0$, tangential to the circle of radius r, centred on the axis.

$B_Q \neq 0$, along the axis.

$\therefore B_{net} \neq 0$, neither in the directions mentioned in options (b) or (c).

In region, $r > 2R$

$B_P \neq 0$ and $B_Q \neq 0$

$\therefore$ $B_{net} \neq 0$

18. Magnetic field $B_R = B_T - B_C$

where, R = remaining portion

T = total portion and

C = cavity

Magnetic field at point P,

$$B_R = \frac{\mu_0 I_T}{2a\pi} - \frac{\mu_0 I_C}{2(3a/2)\pi} \quad ...(i)$$

We have, $I_T = J(\pi a^2)$

and $I_C = J\left(\frac{\pi a^2}{4}\right)$

Substituting the values in Eq. (i), we have

$$B_R = \frac{\mu_0}{a\pi}\left[\frac{I_T}{2} - \frac{I_C}{3}\right] = \frac{\mu_0}{a\pi}\left[\frac{\pi a^2 J}{2} - \frac{\pi a^2 J}{12}\right] = \frac{5\mu_0 aJ}{12}$$

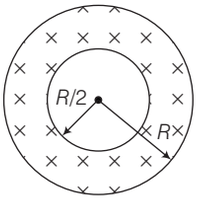
According to question, magnetic field at the point P

$$B = \frac{N}{12}\mu_0 aJ \quad \Rightarrow \quad \therefore \quad N = 5$$

19. r = distance of a point from centre.

For $r < R/2$, Using Ampere's circuital law,

$$\int \mathbf{B}\cdot d\mathbf{l} \text{ or } Bl = \mu_0 (I_{\text{in}})$$



or $$B(2\pi r) = \mu_0 (I_{\text{in}}) \quad \text{or} \quad B = \frac{\mu_0}{2\pi}\frac{I_{\text{in}}}{r}$$

Since, $$I_{\text{in}} = 0$$

$\therefore$ $$B = 0$$

For $$\frac{R}{2} \le r \le R,$$

$$I_{\text{in}} = \left[\pi r^2 - \pi\left(\frac{R}{2}\right)^2\right]\sigma$$

where, σ = current per unit area.

Substituting in Eq. (i), we have

$$B = \frac{\mu_0}{2\pi}\frac{\left[\pi r^2 - \pi\frac{R^2}{4}\right]\sigma}{r} = \frac{\mu_0\sigma}{2r}\left(r^2 - \frac{R^2}{4}\right)$$

At $$r = \frac{R}{2}, \quad B = 0$$

At $$r = R, \quad B = \frac{3\mu_0\sigma R}{8}$$

For $r \ge R$,

$$I_{\text{in}} = I_{\text{total}} = I \text{ (say)}$$

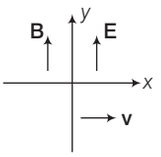
Therefore, substituting in Eq. (i), we have

$$B = \frac{\mu_0}{2\pi}\cdot\frac{I}{r} \text{ or } B \propto \frac{1}{r}$$

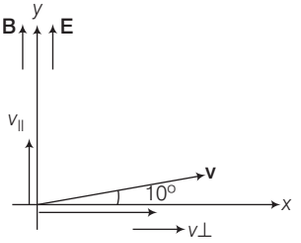
Hence, option (c) is correct.

20. For options (a) and (b),

Magnetic field will rotate the particle in a circular path (in x-y plane or perpendicular to **B**). Electric field will exert a constant force on the particle in positive y-direction. Therefore, resultant path is neither purely circular nor helical.

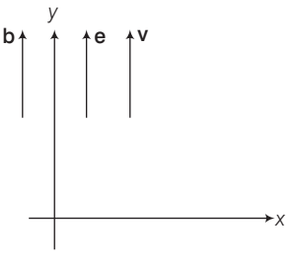


(c)



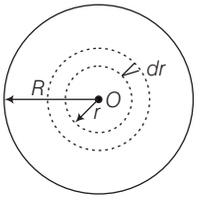
$v_\perp$ and **B** will rotate the particle in a circular path in x-y plane (or perpendicular to **B**). Further $v_{||}$ and **E** will move the particle (with increasing speed) along positive y-axis (or along the axis of above circular path). Therefore, the resultant path is helical with increasing pitch, along the y-axis (or along **B** and **E**). Therefore, option (c) is correct.

(d)



Magnetic force is zero, as θ between **B** and **v** is zero. But electric force will act in y-direction. Therefore, motion is 1-dimensional and uniformly accelerated (towards positive y-direction).

21. Taking an elemental ring of radius r and thickness dr, we obtain magnetic field at the centre of the ring,



$$dB = \frac{\mu_0}{2r}di = \frac{\mu_0}{2r}\cdot\frac{dq}{T} = \frac{\mu_0}{2r}\cdot\frac{Q}{\pi R^2}\cdot 2\pi r\, dr = \frac{\mu_0 Q\omega}{2\pi R^2}dr$$

So, net magnetic field at the centre of disk,

$$B = \int_0^R dB = \frac{\mu_0\omega Q}{2\pi R^2}\int_0^R dr = \frac{\mu_0\omega Q}{2\pi R}$$

i.e. $$B \propto \frac{1}{R}$$

Hence, option (b) is correct

22. Radius of circular path of charged particle in uniform magnetic field,

$$r = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB}$$

$\therefore$ $$r_1 : r_2 = \frac{\sqrt{m_1}}{q_1} : \frac{\sqrt{m_2}}{q_2} = \frac{\sqrt{4}}{1} : \frac{\sqrt{1}}{1} \Rightarrow 2:1$$

23. If we take a small strip of dr at distance r from centre, then number of turns in this strip would be

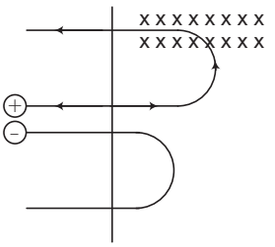
$$dN = \left(\frac{N}{b-a}\right)dr$$

Magnetic field due to this element at the centre of the coil will be

$$dB = \frac{\mu_0 (dN) I}{2r} = \frac{\mu_0 NI}{2(b-a)}\frac{dr}{r}$$

$$B = \int_{r=a}^{r=b} dB = \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

24. $r = \dfrac{mv}{Bq}$ or $r \propto m$



$$r_e < r_p \text{ as } m_e < m_p$$

Further $$T = \frac{2\pi m}{Bq}$$

$$T \propto m \quad \Rightarrow \quad \therefore \quad T_e < T_p$$

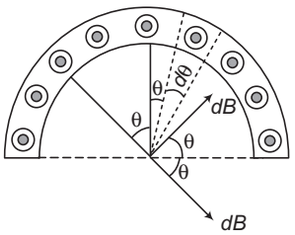
$$t_e = \frac{T_e}{2} \text{ and } t_p = \frac{T_p}{2}$$

or $$t_e < t_p$$

25. Induced electric field lines (produced by change in magnetic field) and magnetic field lines form closed loops.

26. Consider the wire to be made up of large number of thin wires of infinite length. Consider such wire of thickness dl subtending an angle $d\theta$ at centre.

Current through this wire,



$$dl = \frac{d\theta}{\pi} I$$

$\therefore$ Magnetic field at centre due to this portion,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{2dI}{R} = \frac{\mu_0 I}{2\pi^2 R} d\theta$$

Net magnetic field at the centre,

$$B = \int_{-\pi/2}^{\pi/2} dB \cos\theta = \frac{\mu_0 I}{2\pi^2 R}$$

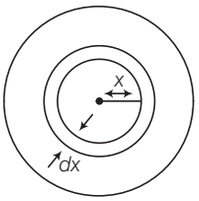
$\Rightarrow$ $$\int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta = \frac{\mu_0 I}{\pi^2 R}$$

27. The force between two parallel current carrying wires is independent of the radii of the wires.

28. $\dfrac{\mu}{l} = \dfrac{1}{2}\dfrac{e}{m} \quad \Rightarrow \quad \mu = \dfrac{1}{2}\dfrac{el}{m} = \dfrac{el}{2m}$

29. Let us consider the disc to be made up of large number of concentric elementary rings.

Consider one such ring of radius x and thickness dx.



Charge on this elementary ring,

$$dq = \sigma \times 2\pi x dx = 2\pi\sigma x dx$$

Current associated with this elementary ring,

$$dI = \frac{dq}{dt} = dr \times f = \sigma\omega x dx$$

Magnetic moment of this elementary ring,

$$dM = dI\pi x^2 = \pi\sigma\omega x^3 dx$$

$\therefore$ Magnetic moment of the entire disc,

$$M = \int_0^R dM = \pi\sigma\omega \int_0^R x^3 dx = \frac{1}{4}\pi R^4 \sigma\omega$$

30. $r = \dfrac{mv}{Bq} = \dfrac{mv}{Be}$

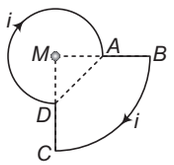
31. $r = \dfrac{mv}{Bq}$

r is least when $\left(\dfrac{m}{q}\right)$ is least.

$\left(\dfrac{m}{q}\right)$ is least for electron, i.e. plane is D.

32. Magnetic field at the centre due to the curved portion

$$DA = \frac{\mu_0 i}{4\pi R}\left(\frac{3\pi}{2}\right)$$



According to right hand screw rule, the magnetic field will be into the plane of paper.

(b) Magnetic field at M due to AB is zero.

(c) Magnetic field at the centre due to the curved portion BC is $\dfrac{\mu_0 i}{4\pi 2R}\left(\dfrac{\pi}{2}\right)$. According to right hand screw rule, the magnetic field will be into the plane of paper.

(d) Magnetic field at M due to DC is zero.

Hence, the resultant magnetic field at M

$$= \frac{3\mu_0 i}{8R} + 0 + \frac{\mu_0 i}{16R} + 0$$

$$= \frac{7\mu_0 i}{16R}$$

33. Magnetic field at mid-point M in first case is $B = B_{PQ} - B_{RS}$

($\therefore B_{PQ}$ and B_{RS} are in opposite directions)

$$= \frac{4\mu_0}{4\pi d} - \frac{2\mu_0}{4\pi d}$$

$\Rightarrow$ $$B = \frac{2\mu_0}{4\pi d}$$

When the current 2 A is switched off, the net magnetic field at M is due to current 1A.

$$B' = \frac{\mu_0 \times 2 \times 1}{4\pi d} = B$$

34. The magnetic field at a point along the axis at distance R from the centre of a circular coil of radius R carrying i is given by

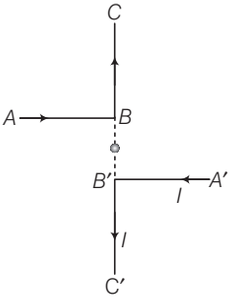
$$B_A = \frac{\mu_0 2\pi i R^2}{4\pi(R^2 + R^2)^{3/2}}$$

$$= \frac{\mu_0 i}{2\sqrt{8}R}$$

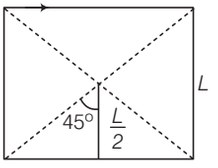
$$= \frac{B}{\sqrt{8}} \qquad \left[B_{\text{centre}} = B = \frac{\mu_0 i}{2R}\right]$$

35. In P and R, current divides equally in two halves, because of equal resistances in the two halves. Due to equal currents in two halves the magnetic field at the centre will be zero.

36. Magnetic field, $B = 2\left[\frac{\mu_0 I}{4\pi r}\right]$

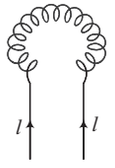


37. $B = 4\left[\frac{\mu_0}{4\pi}\frac{I}{\left(\frac{1}{2}\right)}(\sin 45° + \sin 45°)\right]$



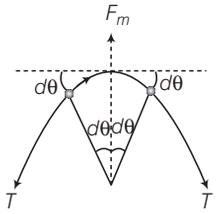
$$B = \frac{\mu_0}{4\pi}\frac{2I}{L}\cdot\frac{2}{\sqrt{2}} \quad \Rightarrow \quad B = \frac{\mu_0}{\pi}\frac{2\sqrt{2}I}{L}$$

38. Toroid is ring shaped closed solenoid.



39. $L = 2\pi R$

$$\therefore \qquad R = \frac{L}{2\pi}$$



$$2T\sin(d\theta) = F_m$$

For small angles, $\sin(d\theta) \approx d\theta$

$$\therefore \qquad 2T(d\theta) = I(dL)B\sin 90°$$

$$= I(2R\cdot d\theta)\cdot B$$

$$\therefore \qquad T = IRB = \frac{ILB}{2\pi}$$

40. The magnetic field in between, because of each will be in opposite direction.

$$B_{\text{in between}} = \frac{\mu_0 i}{2\pi x}\hat{\mathbf{j}} - \frac{\mu_0 i}{2\pi(2d-x)}(-\hat{\mathbf{j}})$$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{1}{x} - \frac{1}{2d-x}\right](\hat{\mathbf{j}})$$

At $\quad x = d,\ B_{\text{in between}} = 0$

For $\quad x < d,\ B_{\text{in between}} = (\hat{\mathbf{j}})$

For $\quad x > d,\ B_{\text{in between}} = (-\hat{\mathbf{j}})$

Towards x, net magnetic field will add up and direction will be $(-\hat{\mathbf{j}})$.

Towards x', net magnetic field will add up and direction will be $(\hat{\mathbf{j}})$.

41. According to Fleming's right hand rule, the force on the charge will be towards West.

42. Radius of circular path, $r_a = \frac{m_a v_a}{qB}$ and $r_b = \frac{m_b v_b}{qB}$

According to question,

$$r_a > r_b$$

$$\therefore \qquad \frac{m_a v_a}{qB} > \frac{m_b v_b}{qB}$$

or $\qquad m_a v_a > m_b v_b$

43. Suppose, the length of wire is l.

When, wire formed in circular loop then radius of loop,

$$r = \frac{l}{2\pi} \qquad (\because l = 2\pi r)$$

Magnetic dipole, $M_1 = iA = i\pi r^2 = i\pi \times \left(\frac{l}{2\pi}\right)^2$

$$= i \times \frac{l^2}{4\pi}$$

When, wire formed in square loop then the side of loop.

$$a = \frac{l}{4}$$

Magnet dipole, $M_2 = iA = i \times a^2 = i \times \frac{l^2}{16}$

$$\frac{M_1}{M_2} = \frac{4}{\pi}$$

21

Magnetism

21.1 Magnetic Poles and Bar Magnets

In electricity, the isolated charge q is the simplest structure that can exist. If two such charges of opposite sign are placed near each other, they form an electric dipole characterised by an electric dipole moment **p**. In magnetism, isolated magnetic poles which would correspond to isolated electric charges do not exist. The simplest magnetic structure is the **magnetic dipole,** characterised by a magnetic dipole moment **M**. A current loop, a bar magnet and a solenoid of finite length are examples of magnetic dipoles.

When a magnetic dipole is placed in an external magnetic field **B**, a magnetic torque τ acts on it, which is given by,

$$\tau = \mathbf{M} \times \mathbf{B}$$

Alternatively, we can measure **B** due to the dipole at a point along its axis a (large) distance r from its centre, by the expression,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$

A bar magnet might be viewed as two poles (North and South) separated by a some distance. However, all attempts to isolate these poles fail. If a magnet is broken, the fragments prove to be dipoles and not isolated poles. If we break up a magnet into the electrons and nuclei that make up its atoms, it will be found that even these elementary particles are magnetic dipoles.

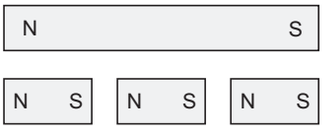


Fig. 21.1 If a bar magnet is broken, each fragment becomes a small dipole

Each current carrying loop is just like a magnetic dipole, whose magnetic dipole moment is given by,

$$\mathbf{M} = ni\mathbf{A}$$



Fig. 21.2

Chapter Snapshot

- Magnetic Poles and Bar Magnets
- Earth's Magnetism
- Vibration Magnetometer
- Magnetic Induction and Magnetic Materials
- Some Important Terms Used in Magnetism
- Properties of Magnetic Materials
- Explanation of Paramagnetism, Diamagnetism and Ferromagnetism

Here, n is the number of turns in the loop, i is the current and **A** represents the area vector of the current loop.

The behaviour of a current loop can be described by the following hypothetical model:

(i) There are two magnetic charges; positive magnetic charge and negative magnetic charge. We call the positive magnetic charge a North pole and the negative magnetic charge as the South pole. Every pole has a pole strengths m. The unit of pole strength is A-m.

(ii) A magnetic charge placed in a magnetic field experiences a force, $\mathbf{F} = m\mathbf{B}$

The force on positive magnetic charge is along the field and a force on a negative magnetic charge is opposite to the field.

(iii) A magnetic dipole is formed when a negative magnetic charge $-m$ and a positive magnetic charge $+m$ are placed at a small separation d. The magnetic dipole moment is, $M = md$

The direction of **M** is from $-m$ to $+m$.

Geometrical Length and Magnetic Length

In the case of a bar magnet, the poles appear at points which are slightly inside the two ends. The distance between the locations of the assumed poles is called the **magnetic length** of the magnet. The distance between the ends is called the **geometrical length.**

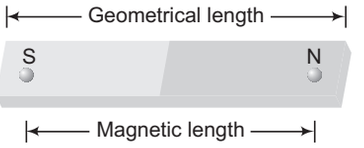


Fig. 21.3

The magnetic length of a bar magnet is written as $2l$. If m be the pole strength and $2l$ the magnetic length of a bar magnet, then its magnetic moment is,

$$M = 2ml$$

Extra Knowledge Points

- Current coil loop, solenoid, etc. are just like a magnetic dipoles, whose dipole moment M is equal to niA. Direction of **M** is from South pole (S) to North pole (N).
- The behaviour of a magnetic dipole (may be a bar magnet also) is similar to the behaviour of an electric dipole. The only difference is that the electric dipole moment **p** is replaced by magnetic dipole moment **M** and the constant $\frac{1}{4\pi\varepsilon_0}$ is replaced by $\frac{\mu_0}{4\pi}$.
- Table 21.1 makes a comparison between an electric dipole and a magnetic dipole.

Table 21.1

S. No.	Physical quantity to be compared	Electric dipole	Magnetic dipole
1.	Dipole moment	$p = q\ (2l)$	$M = m\ (2l)$
2.	Direction of dipole moment	From negative charge to the positive charge	From South to North pole
3.	Net force in uniform field	0	0
4.	Net torque in uniform field	$\tau = \mathrm{p} \times \mathrm{E}$	$\tau = \mathbf{M} \times \mathbf{B}$
5.	Field at far away point on the axis	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3}$ (along p)	$\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$ (along **M**)
6.	Field at far away point on perpendicular bisector	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{P}{r^3}$ (opposite to p)	$\frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$ (opposite to **M**)
7.	Potential energy	$U_\theta = -\mathrm{p} \cdot \mathrm{E} = -pE\cos\theta$	$U_\theta = -\mathbf{M} \cdot \mathbf{B} = -MB\cos\theta$
8.	Work done in rotating the dipole	$W_{\theta_1 - \theta_2} = pE\ (\cos\theta_1 - \cos\theta_2)$	$W_{\theta_1 - \theta_2} = MB\ (\cos\theta_1 - \cos\theta_2)$

In the table θ is the angle between field (**E** or **B**) and dipole moment (**p** or **M**).

Example 21.1 *Calculate the magnetic induction at a point* $1\ \text{Å}$ *away from a proton, measured along its axis of spin. The magnetic moment of the proton is* $1.4 \times 10^{-26}\ A\text{-}m^2$.

Sol. On the axis of a magnetic dipole, magnetic induction is given by,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$

Substituting the values, we get

$$B = \frac{(10^{-7})\,(2)\,(1.4 \times 10^{-26})}{(10^{-10})^3}$$

$$= 2.8 \times 10^{-3}\ \text{T}$$

$$= 2.8\ \text{mT}$$

Example 21.2 *A bar magnet of magnetic moment* 2.0 *A-m*^{2} *is free to rotate about a vertical axis through its centre. The magnet is released from rest from the East-West position. Find the kinetic energy of the magnet as it takes the North-South position. The horizontal component of the earth's magnetic field is* $B = 25\,\mu T$. *Earth's magnetic field is from South to North.*

Sol. Gain in kinetic energy = loss in potential energy

Thus, $\text{KE} = U_i - U_f$

As $U = -MB\cos\theta$

$\therefore \quad \text{KE} = -MB\cos\left(\frac{\pi}{2}\right) - (-MB\cos 0°)$

$= MB$

Substituting the values, we have

$\text{KE} = (2.0)\ (25\times 10^{-6})\ \text{J}$

$= 50\,\mu\text{J}$

21.2 Earth's Magnetism

Our earth behaves as it has a powerful magnet within it. The value of magnetic field on the surface of the earth is a few tenths of a gauss ($1\,\text{G} = 10^{-4}$ T). The earth's South magnetic pole is located near the North geographic pole and the earth's North magnetic pole is located near the South geographic pole. In fact, the configuration of the earth's magnetic field is very much like the one that would be achieved by burying a gigantic bar magnet deep in the interior of the earth.

The axis of the earth's magnet makes an angle of 11.5° with the earth's rotational axis.

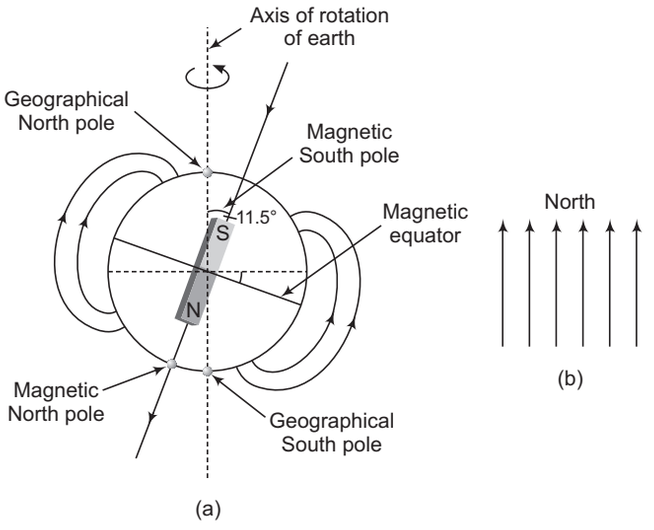


Fig. 21.4

Theories Regarding the Origin of Earth's Magnetism

First Theory Gilbert for the first time in 1600, gave the idea that there is a powerful magnet within the earth at its centre. Later on this theory was denied because the temperature in the interior of the earth is so high that it is impossible to retain its magnetism.

Second Theory The second theory was put forward by Grover in 1849. He put the view that the earth magnetism is due to electric currents flowing near the outer surface of the earth. Hot air, rising from the region near equator, goes towards North and South hemispheres and become electrified. These currents magnetise the ferromagnetic material near the outer surface of the earth.

Third Theory There are many conducting materials including iron and nickel in the molten state within the central core of the earth. Conventional currents are generated in this semifluid core due to the earth's rotation about its axis. Due to these currents, magnetism is generated within the earth.

Till date not a single theory can explain all events regarding the earth's magnetism.

Elements of Earth's Magnetism

There are three elements of earth's magnetism.

(i) **Angle of declination (α)** At any point (say P) on earth's surface the longitude determines the North-South direction. The vertical plane in the direction of longitude or the vertical plane passing through the line joining the geographical North and South poles is called the **geographical meridian.** At point P, there also exists the magnetic field **B**. A vertical plane in the direction of **B** is called **magnetic meridian.**

At any place, the acute angle between the magnetic meridian and the geographical meridian is called **angle of declination** α.

(ii) **Angle of dip** (θ) The angle of dip (θ) at a place is the angle between the direction of the earth's magnetic field and the horizontal at that place.

Angle of dip at some place can be measured from a magnetic needle free to rotate in a vertical plane about a horizontal axis passing through centre of gravity of the needle. At earth's magnetic poles, the magnetic field of the earth is vertical, i.e. angle of dip is 90°, the freely suspended magnetic needle is vertical there. At magnetic equator field is horizontal or angle of dip is 0°. The needle is horizontal. In Northern hemisphere, the North pole of the magnetic needle inclines downwards, whereas in the Southern hemisphere, the South pole of the needle inclines downwards.

(iii) **Horizontal component of earth's magnetic field** Let B_e be the net magnetic field at some point. H and V be the horizontal and vertical components of B_e. Let θ is the angle of dip at the some place, then we can see that,

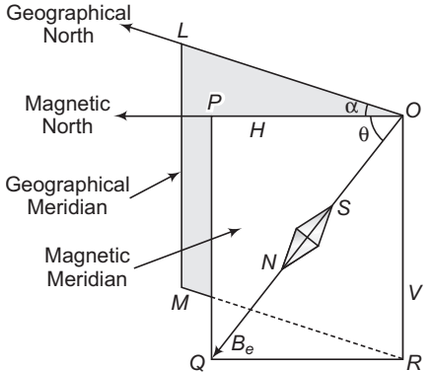


Fig. 21.5

$$H = B_e \cos\theta \quad \ldots(i)$$

and $$V = B_e \sin\theta \quad \ldots(ii)$$

Squaring and adding Eqs. (i) and (ii), we get

$$B_e = \sqrt{H^2 + V^2}$$

Further, dividing Eq. (ii) by Eq. (i), we get

$$\theta = \tan^{-1}\left(\frac{V}{H}\right)$$

By knowing H and θ at some place we can find B_e and V at that place.

Neutral Points

When a magnet is placed at some point on the earth's surface, there are some points where horizontal component of the earth's magnetic field is just equal and opposite to the field due to the magnet. Such points are called neutral points. If a magnetic compass is placed at a neutral point, no force acts on it and it may set in any direction.

Suppose a small bar magnet is placed such that North pole of the magnet is towards the magnetic South pole of the earth then neutral points are obtained both sides on the axis of the magnet. If distance of each neutral point from the middle point of a magnet be r, and the magnitude of the magnetic moment of the magnet be M, then

$$\frac{\mu_0}{4\pi}\cdot\frac{2M}{r^3} = H$$

When North pole of bar magnet is towards the magnetic North pole of the earth, the neutral points are obtained on perpendicular bisectors of the magnet. Let r be the distance of neutral points from centre, then

$$\frac{\mu_0}{4\pi}\cdot\frac{M}{r^3} = H$$

Example 21.3 *In the magnetic meridian of a certain place, the horizontal component of the earth's magnetic field is* 0.26 *G and the dip angle is* 60°. *Find*
(a) vertical component of the earth's magnetic field.
(b) the net magnetic field at this place.

Sol. Given $H = 0.26$ G and $\theta = 60°$

(a) $\tan\theta = \frac{V}{H}$

$\therefore$ $V = H\tan\theta = (0.26)\tan 60° = 0.45$ G

(b) $H = B_e\cos\theta$

$\therefore$ $B_e = \frac{H}{\cos\theta} = \frac{0.26}{\cos 60°} = 0.52$ G

Example 21.4 *A magnetic needle suspended in a vertical plane at* 30° *from the magnetic meridian makes an angle of* 45° *with the horizontal. Find the true angle of dip.*

Sol. In a vertical plane at 30° from the magnetic meridian, the horizontal component is,

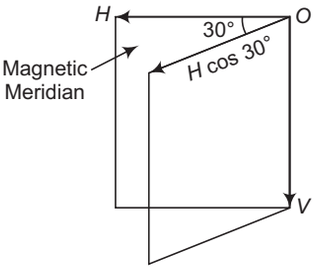


Fig. 21.6

$$H' = H\cos 30°$$

While vertical component is still V. Therefore, apparent dip will be given by

$$\tan\theta' = \frac{V}{H'} = \frac{V}{H\cos 30°}$$

but $\frac{V}{H} = \tan\theta'$ (where, θ = true angle of dip)

$\therefore$ $\tan\theta' = \frac{\tan\theta}{\cos 30°}$

$\therefore$ $\theta = \tan^{-1}[\tan\theta'\cos 30°]$

$= \tan^{-1}[(\tan 45°)(\cos 30°)]$

$\approx 41°$

21.3 Vibration Magnetometer

Vibration magnetometer is an instrument which is used for following two purposes:

(i) To find magnetic moment of a bar magnet.

(ii) To compare magnetic fields of two magnets.

The construction of a vibration magnetometer is as shown in figure.The magnet shown in figure is free to rotate in a horizontal plane. The magnet stays parallel to the horizontal component of the earth's magnetic field. If the

magnet is now displaced through an angle θ, a restoring torque of magnitude $MH \sin\theta$ acts on it and the magnet starts oscillating. From the theory of simple harmonic motion we can find the time period of oscillations of the magnet.

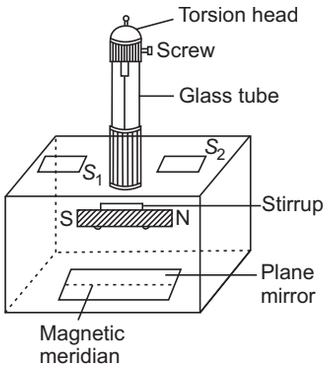


Fig. 21.7

Restoring torque in displaced position is,

$$\tau = -MH \sin\theta \qquad \ldots(i)$$

Here, M = magnetic moment of the magnet and

H = horizontal component of the earth's magnetic field.

Negative sign shows the restoring nature of torque. Now since, $\tau = I\alpha$ and $\sin\theta \approx \theta$ for small angular displacement.

Thus, Eq. (i) can be written as,

$$I\alpha = -MH\theta$$

Since, α is proportional to $-\theta$. Therefore, motion is simple harmonic in nature, time period of which will be given by,

$$T = 2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|} = 2\pi\sqrt{\frac{I}{MH}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{I}{MH}} \qquad \ldots(ii)$$

In the expression of T, I is the moment of inertia of the magnet about its axis of vibration.

(i) **Measurement of magnetic moment** By finding time period T of vibrations of the given magnet we can calculate magnetic moment M by the relation,

$$M = \frac{4\pi^2 I}{T^2 H}$$

(ii) **Comparison of two magnetic fields** Suppose we wish to compare the magnetic fields B_1 and B_2 at some point P due to two magnets. For this vibration, magnetometer is so placed that the centre of its magnet lies on P. Now, one of the given magnets is placed at some known distance from P in the magnetic meridian, such that point P lies on its axial line and its North pole points North. In this position, the fields B_1 at P produced by the magnet will be in the direction of H. Hence, the magnet suspended in the magnetometer will vibrate in the resultant magnetic field $(H + B_1)$. Its period of vibration is noted, say it is T_1, then

$$T_1 = 2\pi\sqrt{\frac{I}{M\,(H + B_1)}}$$

Now, the first magnet is replaced by the second magnet and the second magnet is placed in the same position and again the time period is noted. If the field produced at P due to this magnet be B_2 and the new time period be T_2, then

$$T_2 = 2\pi\sqrt{\frac{I}{M(H + B_2)}}$$

Finally, the time period of the magnetometer under the influence of the earth's magnetic field alone is determined. Let it be T, then

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

Solving above three equations for T, T_1 and T_2 we can show that

$$\frac{B_1}{B_2} = \frac{(T^2 - T_1^2)\,T_2^2}{(T^2 - T_2^2)\,T_1^2}$$

Example 21.5 *A short bar magnet is placed with its North pole pointing North. The neutral point is* 10 *cm away from the centre of the magnet. If* $H = 0.4$ *G, calculate the magnetic moment of the magnet.*

Sol. When North pole of the magnet points towards magnetic North, null point is obtained on perpendicular bisector of the magnet. Simultaneously, magnetic field due to the bar magnet should be equal to the horizontal component of the earth's magnetic field H.

Thus, $$H = \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3}$$

or $$M = \frac{Hr^3}{(\mu_0/4\pi)}$$

Substituting the values, we have

$$M = \frac{(0.4\times 10^{-4})\,(10\times 10^{-2})^3}{10^{-7}} = 0.4 \text{ A-m}^2$$

Example 21.6 *A magnetic needle performs* 20 *oscillations per minute in a horizontal plane. If the angle of dip be* 30°*, then how many oscillations per minute will this needle perform in vertical North-South plane and in vertical East-West plane?*

Sol. In horizontal plane, the magnetic needle oscillates in horizontal component H.

$$\therefore \quad T = 2\pi\sqrt{\frac{I}{MH}}$$

In the vertical North-South plane (magnetic meridian), the needle oscillates in the total earth's magnetic field B_e, and in vertical East-West plane (plane perpendicular to the

magnetic meridian) it oscillates only in the earth's vertical component *V*. If its time period be T_1 and T_2, then

$$T_1 = 2\pi\sqrt{\frac{I}{MB_e}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{I}{MV}}$$

From above equations, we can find

$$\frac{T_1^2}{T^2} = \frac{H}{B_e}$$

or $$\frac{n_1^2}{n^2} = \frac{B_e}{H}$$

Similarly $$\frac{n_2^2}{n^2} = \frac{V}{H}$$

Further, $$\frac{B_e}{H} = \sec\theta = \sec 30° = \frac{2}{\sqrt{3}}$$

and $$\frac{V}{H} = \tan\theta = \tan 30° = \frac{1}{\sqrt{3}}$$

$\therefore$ $$n_1^2 = (n)^2\left(\frac{B_e}{H}\right) = (20)^2\left(\frac{2}{\sqrt{3}}\right)$$

or $$n_1 = 21.5 \text{ oscillation/min}$$

and $$n_2^2 = (n)^2\left(\frac{V}{H}\right) = (20)^2\left(\frac{1}{\sqrt{3}}\right)$$

$\therefore$ $$n_2 = 15.2 \text{ oscillation/min}$$

21.4 Magnetic Induction and Magnetic Materials

We know that the electric lines of force change when a dielectric is placed between the parallel plates of a capacitor. Experiments show that magnetic lines also get modified due to the presence of certain materials in the magnetic field.

Few substances such as O_2, air, platinum, aluminium etc., show a very small increase in the magnetic flux passing through them, when placed in a magnetic field. Such substances are called **paramagnetic substances**. Few other substances such as H_2, H_2O, Cu, Zn, Sb, etc. show a very small decrease in flux and are said to be **diamagnetic**. There are other substances like Fe, Co, etc. through which the flux increases to a larger value and are known as **ferromagnetic substances**.

Magnetisation of Matter

A material body is consisting of large number of atoms and thus large number of electrons. Each electron produces orbital and spin magnetic moments and can be assumed as magnetic dipoles. In the absence of any external magnetic field, the dipoles of individual atoms are randomly oriented and the magnetic moments thus cancel.

When we apply an external magnetic field to a substance, two processes may occur.

(i) All atoms which have non-zero magnetic moment are aligned along the magnetic field.

(ii) If the atom has a zero magnetic moment, the applied magnetic field distorts the electron orbit and thus, induces magnetic moment in opposite direction.

In diatomic substances, the individual atoms do not have a magnetic moment by its own. When an external field is applied the second process occurs. The induced magnetic moment is thus setup in the direction opposite to **B**. In this case, the magnetic flux density in the interior of the body will be less than that of the external field **B**.

In paramagnetic substances, the constituent atoms have intrinsic magnetic moments. When an external magnetic field is applied, both of the above processes occur and the resultant magnetic moment is always in the direction of magnetic field **B** as the first effect predominates over the second.

21.5 Some Important Terms Used in Magnetism

Magnetic Induction (**B**)

When a piece of any substance is placed in an external magnetic field the substance becomes magnetised. If an iron bar is placed in a uniform magnetic field, the magnetised bar produces its own magnetic field in the same direction as those of the original field inside the bar, but in opposite direction outside the bar. This results in a concentration of the lines of force within the bar. The magnetic flux density within the bar is increased whereas it becomes weak at certain places outside the bar.

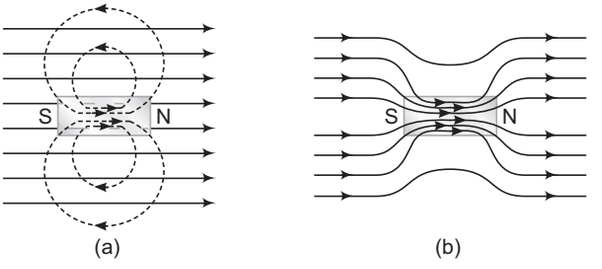


Fig. 21.8

The number of magnetic lines of induction inside a magnetic substance crossing unit area normal to their direction is called the magnitude of magnetic induction or magnetic flux density inside the substance. It is denoted by **B**. The SI unit of **B** is tesla (T) or weber/metre^2 (Wb/m^2). The CGS unit is gauss (G).

$$1\,\text{Wb/m}^2 = 1\,\text{T} = 10^4\,\text{G}$$

Intensity of Magnetisation (**I**)

Intensity of magnetisation (**I**) is defined as the magnetic moment per unit volume of the magnetised substance. This basically represents the extent to which the substance is magnetised. Thus,

$$\mathbf{I} = \frac{M}{V}$$

The SI unit of **I** is ampere/metre (A/m).

Magnetic Intensity or Magnetic Field Strength (H)

When a substance is placed in an external magnetic field, the actual magnetic field inside the substance is the sum of the external field and the field due to its magnetisation.

The capability of the magnetising field to magnetise the substance is expressed by means of a vector **H**, called the magnetic intensity of the field. It is defined through the vector relation,

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{I}$$

The SI unit of **H** is same as that of **I**, i.e. ampere/metre (A/m). The CGS unit is oersted.

Magnetic Permeability (μ)

It is defined as the ratio of the magnetic induction **B** inside the magnetised substance to the magnetic intensity **H** of the magnetising field, i.e.

$$\mu = \frac{\mathbf{B}}{\mathbf{H}}$$

It is basically a measure of conduction of magnetic lines of force through it. The SI unit of magnetic permeability is weber/ampere-metre (Wb/A-m).

Relative Magnetic Permeability (μ_r)

It is the ratio of the magnetic permeability μ of the substance to the permeability of free space.

Thus, $$\mu_r = \frac{\mu}{\mu_0}$$

μ_r is a pure ratio, hence dimensionless. For vacuum its value is 1.

μ_r can also be defined as the ratio of the magnetic field B in the substance when placed in magnetic field B_0. Thus,

$$\mu_r = \frac{B}{B_0}$$

For paramagnetic substance, $\mu_r > 1$,
For diamagnetic substance, $\mu_r < 1$ and
For ferromagnetic substance, $\mu_r >> 1$.

Magnetic Susceptibility (χ_m)

We know that both diamagnetic and paramagnetic substances develop a magnetic moment depending on the applied field. Magnetic susceptibility is a measure of how easily a substance is magnetised in a magnetising field. For paramagnetic and diamagnetic substances, **I**, **H** and χ_m are related by the equation,

$$\mathbf{I} = \chi_m \mathbf{H}$$

or $$\chi_m = \frac{\mathbf{I}}{\mathbf{H}}$$

Thus, the magnetic susceptibility χ_m may be defined as the ratio of the intensity of magnetisation to the magnetic intensity of the magnetising field.

Since, **I** and **H** have the same units, χ_m is unitless. It is a pure number.

By doing simple calculation, we can prove that μ_r and χ_m are related by,

$$\mu_r = 1 + \chi_m$$

For paramagnetic substances χ_m is slightly positive. For diamagnetic, it is slightly negative and for ferromagnetic substances χ_m is positive and very large.

21.6 Properties of Magnetic Materials

As discussed earlier, all substances (whether solid, liquid or gaseous) may be classified into three categories in terms of their magnetic properties (i) paramagnetic, (ii) diamagnetic and (iii) ferromagnetic.

Paramagnetic Substances

Examples of such substances are platinum, aluminium, chromium, manganese, $CuSO_4$ solution, etc. They have the following properties:

(i) The substances when placed in a magnetic field, acquire a feeble magnetisation in the same sense as the applied field. Thus, the magnetic inductance inside the substance is slightly greater than outside to it.

(ii) In a uniform magnetic field, these substances rotate until their longest axes are parallel to the field.

(iii) These substances are attracted towards regions of stronger magnetic field when placed in a non-uniform magnetic field.

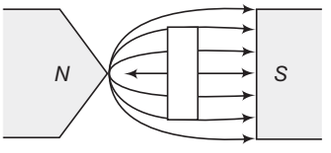


Fig. 21.9

Figure shows a strong electromagnet in which one of the pole pieces is sharply pointed while the other is flat. Magnetic field is much stronger near the pointed pole than near the flat pole. If a small piece of paramagnetic material is suspended in this region, a force can be observed in the direction of arrow.

(iv) If a paramagnetic liquid is filled in a narrow U-tube and one limb is placed in between the pole pieces of an electromagnet such that the level of the liquid is in line with the field, then the liquid will rise in the limb as the field is switched ON.

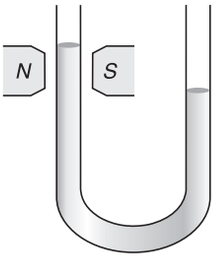


Fig. 21.10

(v) For paramagnetic substances, the relative permeability μ_r is slightly greater than one.

(vi) At a given temperature, the magnetic susceptibility χ_m does not change with the magnetising field. However, it varies inversely as the absolute temperature. As temperature increases χ_m decreases. At some higher temperature χ_m becomes negative and the substance becomes diamagnetic.

Diamagnetic Substances

Examples of such substances are bismuth, antimony, gold, quartz, water, alcohol, etc. They have following properties:

(i) These substances when placed in a magnetic field, acquire feeble magnetisation in a direction opposite to that of the applied field. Thus, the lines of induction inside the substance is smaller than that outside to it.

(ii) In a uniform field, these substances rotate until their longest axes are normal to the field.

(iii) In a non-uniform field, these substances move from stronger to weaker parts of the field.

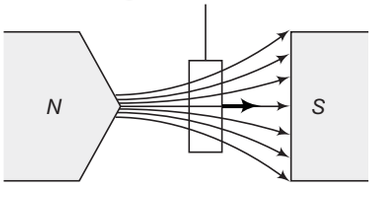


Fig. 21.11

(iv) If a diamagnetic liquid is filled in a narrow U-tube and one limb is placed in between the pole of an electromagnet, the level of liquid depresses when the field is switched ON.

(v) The relative permeability μ_r is slightly less than 1.

(vi) The susceptibility χ_m of such substances is always negative. It is constant and does not vary with field or the temperature.

Ferromagnetic Substances

Examples of such substances are iron, nickel, steel, cobalt and their alloys. These substances resemble to a higher degree with paramagnetic substances as regard their behaviour. They have following additional properties:

(i) These substances are strongly magnetised by even a weak magnetic field.

(ii) The relative permeability is very large and is of the order of hundreds and thousands.

(iii) The susceptibility is positive and very large.

(iv) Susceptibility remains constant for very small values of **H** increases for larger values of **H** and then decreases for very large values of **H**.

(v) Susceptibility decreases steadily with the rise of temperature. Above a certain temperature known as **Curie temperature**, the ferromagnetic substances become paramagnetic. It is 1000°C for iron, 770°C for steel, 360°C for nickel and 1150°C for cobalt.

21.7 Explanation of Paramagnetism, Diamagnetism and Ferromagnetism

There are three properties of atoms that give rise to magnetic dipole moment :

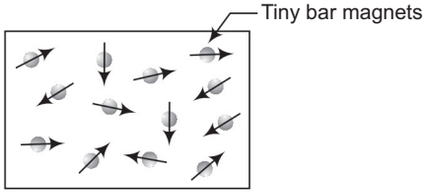
1. The electrons moving around the nucleus in the orbits act as small current loops and contribute magnetic moments.
2. The spinning electron has an intrinsic magnetic dipole moment.
3. The nucleus contribute to magnetic moment due to the motion of charge within the nucleus. The magnitude of nuclear moments is about 10^{-3} times that of electronic moments or the spin magnetic moments as the latter two are of the same order. Still most of the magnetic moment of an atom is produced by electron spin, the net contribution of the orbital revolution is very small.

   This is because most of the electrons pair off in such a way that they produce equal and opposite orbital magnetic moment and they cancel out. Although the electrons also try to pair up with their opposite spins but in case of spin motion of an electron it is not always possible to form equal and opposite pairs.

Paramagnetism

The property of paramagnetism is found in those substances whose atoms or molecules have an excess of electrons spinning in the same direction.

Hence, atoms of paramagnetic substances have a permanent magnetic moment and behave like tiny bar magnets. In the absence of external magnetic field, the atomic magnets are randomly oriented and net magnetic moment is thus, zero.



In the absence of external magnetic field

Fig. 21.12

When paramagnetic substance is placed in an external magnetic field, then each atomic magnet experiences a torque which tends to turn the magnet in the direction of the field. The atomic magnets are thus, aligned in the direction of the field. Thus, the whole substance is magnetised in the direction of the external magnetic field.

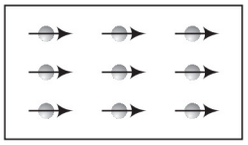


Fig. 21.13

As the temperature of substance is increased the thermal agitation disturbs the magnetic alignment of the atoms. Thus, we can say that paramagnetism is temperature dependent.

Curie's law According to Curie's law magnetic susceptibility of a paramagnetic substance is inversely proportional to absolute temperature T.

$$\chi_m \propto \frac{1}{T}$$

The exact law is beyond the scope of our course.

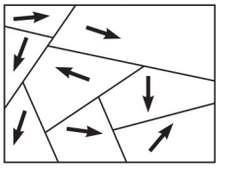
Diamagnetism

The property of diamagnetism is generally found in those substances whose atoms (or molecules) have even number of electrons which form pairs. The net magnetic moment of an atom of a diamagnetic substance is thus zero. When a diamagnetic substance is placed in an external magnetic field, the spin motion of electrons is so modified that the electrons which produce the magnetic moments in the direction of external field slow down while the electrons which produce magnetic moments in opposite direction get accelerated. Thus, a net magnetic moment is induced in the opposite direction of applied magnetic field. Hence, the substance is magnetised opposite to the external field.

- That diamagnetism is temperature independent.

Ferromagnetism

Iron like elements and their alloys are known as ferromagnetic substances. The susceptibility of these substances is in several thousands. Like paramagnetic substances, atoms of ferromagnetic substances have a permanent magnetic moment and behave like tiny magnets. But in ferromagnetic substances the atoms form innumerable small effective regions called domains.



Unmagnetised

Fig. 21.14

The size of the domain vary from about $10^{-6}\ cm^3$ to $10^{-2}\ cm^3$. Each domain has 10^{17} to 10^{21} atoms whose magnetic moments are aligned in the same direction. In an unmagnetised ferromagnetic specimen, the domains are oriented randomly, so that their resultant magnetic moment is zero.

When the specimen is placed in a magnetic field the resultant magnetisation may increase in two different ways.

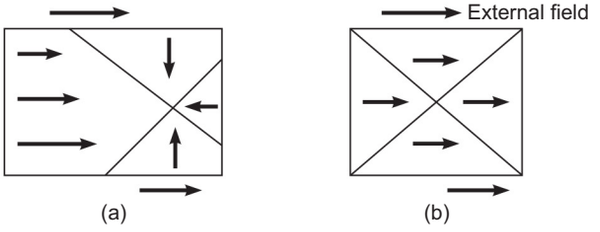


Fig. 21.15

(a) The domains which are oriented favourably w.r.t. the field increase in size. Whereas those oriented opposite to the external field are reduced.

(b) The domains rotate towards the field direction.

- If the external field is weak, specimen gets magnetised by the first method and if the field is strong they get magnetised by the second method.

Hysteresis : Retentivity and Coercivity

The distinguishing characteristic of a ferromagnetic material is not that it can be strongly magnetised but that the intensity of magnetisation **I** is not directly proportional to the magnetising field **H**. If a gradually increasing magnetic field **H** is applied to an unmagnetised piece of iron, its magnetisation increases non-linearly until it reaches a maximum.

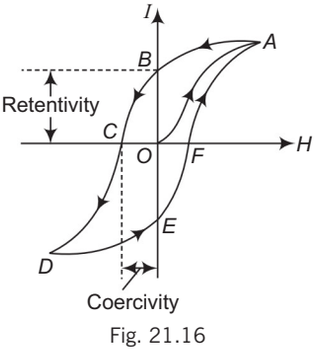


Fig. 21.16

If **I** is plotted against **H**, a curve like *OA* is obtained. This curve is known as magnetisation curve. At this stage all the dipoles are aligned and **I** has reached to a maximum or saturated value. If the magnetic field **H** is now decreased, the **I** does not return along magnetisation curve but follows path *AB*. At $\mathbf{H} = 0$, **I** does not come to its zero value but its value is still near the saturated value. The value of **I** at this point (i.e. *OB*) is known as **remanence, remanent magnetisation or retentivity**. The value of **I** at this point is known as residual induction. On applying a reverse field the value of **I** finally becomes zero. The abscissa *OC* represents the reversed magnetic field needed to demagnetise the specimen.

This is known as **coercivity** of the material.

If the reverse field is further increased, a reverse magnetisation is set up which quickly reaches the saturation value. This is shown as *CD*. If **H** is now taken back from its negative saturation value to its original positive saturation value, a similar curve *DEFA* will be traced. The whole graph *ABCDEFA* thus, forms a closed loop, usually known as **hysteresis loop**. The whole process described above and the property of the iron characterised by it are called **hysteresis**. The energy lost per unit volume of a substance in a complete cycle is equal to the area. Thus, we can conclude following three points from the above discussion:

(i) The retentivity of a substance is a measure of the magnetisation remaining in the substance when the magnetising field is removed.

(ii) The coercivity of a substance is a measure of the reverse magnetising field required to destroy the residual magnetism of the substance.

(iii) The energy loss per unit volume of a substance in a complete cycle of magnetisation is equal to the area of the hysteresis loop.

Demagnetisation It is clear from the hysteresis loop that the intensity of magnetisation **I** does not reduce to zero on removing the magnetising field **H** ·**I** is zero, when the magnetising field **H** is equal to the coercive field.

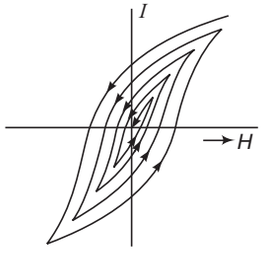


Fig. 21.17

At these points, the magnetic induction is not zero and the specimen is not demagnetised. To demagnetise a substance, it is subjected to several cycles is magnetisation, each time with decreasing magnetising field and finally the field is reduced to zero. In this way, the size of the hysteresis curve goes on decreasing and the area finally reduces to zero.

Demagnetisation is best obtained by placing the specimen in an alternating field of continuously diminishing amplitude. It is also obtained by heating. Ferromagnetic materials become practically non-magnetic at sufficiently high temperatures.

Magnetic Properties of Soft Iron and Steel

A comparison of the magnetic properties of ferromagnetic substances can be made by the comparison of the shapes and sizes of their hysteresis loops.

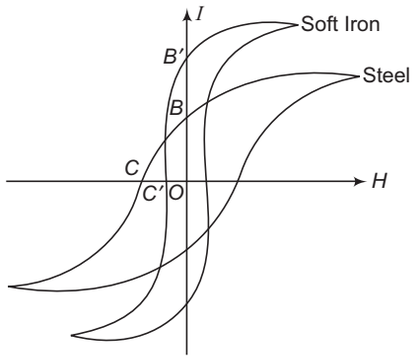


Fig. 21.18

Following three conclusions can be drawn from their hysteresis loops

(i) Retentivity of soft iron is more than the retentivity of steel.

(ii) Coercivity of soft iron is less than the coercivity of steel.

(iii) Area of hysteresis loop (i.e. hysteresis loss) in soft iron is smaller than that in steel.

Choice of Magnetic Materials

The choice of a magnetic material for different uses is decided from the hysteresis curve of a specimen of the material.

(i) **Permanent magnets** The materials for a permanent magnet should have

- high retentivity (so that the magnet is strong) and
- high coercivity (so that the magnetising is not wiped out by stray magnetic fields). As the material in this case is never put to cyclic changes of magnetisation, hence hysteresis is immaterial. From the point of view of these facts **steel** is more suitable for the construction of permanent magnets than soft iron.

Modern permanent magnets are made of cobalt-steel, alloys ticonal.

(ii) **Electromagnets** The materials for the construction of electromagnets should have,
- high initial permeability,
- low hysteresis loss.

From the view point of these facts, **soft iron** is an ideal material for this purpose.

(iii) **Transformer cores and telephone diaphragms** As the magnetic material used in these cases is subjected to cyclic changes. Thus, the essential requirements for the selection of the material are :
- high initial permeability,
- low hysteresis loss to prevent the breakdown.

Electromagnet

As we know that a current carrying solenoid behaves like a bar magnet. If we place a soft iron rod in the solenoid, the magnetism of the solenoid increases hundreds of times and the solenoid is called an electromagnet. It is a temporary magnet.

An electromagnet is made by winding closely a number of turns of insulated copper wire over a soft iron straight rod or a horse shoe rod. On passing current through this solenoid, a magnetic field is produced in the space within the solenoid.

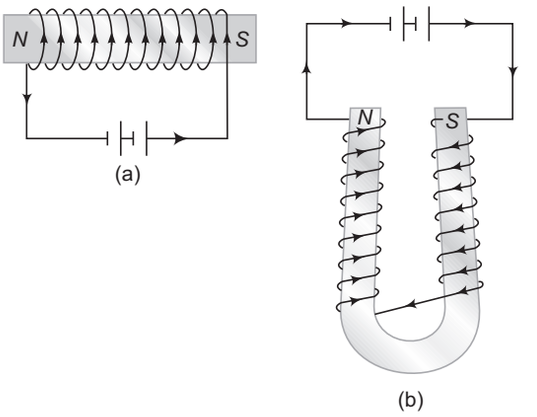


Fig. 21.19

Applications of Electromagnets

(i) Electromagnets are used in electric bell, transformer, telephone diaphragms, etc.

(ii) In medical field they are used in extracting bullets from the human body.

(iii) Large electromagnets are used in cranes for lifting and transferring big machines and parts.

Chapter Summary with Formulae

(i) Every current carrying loop is like a bar magnet (or a magnetic dipole).

(ii) Magnetic field due to a bar magnet at a point lying on its axis is

$$B = \frac{\mu_0}{4\pi}\frac{2M}{r^3} \qquad (\text{along } \mathbf{M})$$

where, $r >>$ size of bar magnet.

(iii) Magnetic field due to a bar magnet at a point on equatorial line is

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3} \qquad (\text{opposite to } \mathbf{M})$$

Again, $r >>$ size of bar magnet.

(iv) Magnetic dipole (or a bar magnet) in uniform magnetic field :

(a) Net force = 0

(b) Net torque, $\tau = \mathbf{M} \times \mathbf{B}$

(c) Magnitude of torque, $\tau = MB\sin\theta$

(d) Magnetic potential energy, $U = -\mathbf{M}\cdot\mathbf{B} = -MB\cos\theta$

(e) Work done in rotating the dipole,

$$W_{\theta_1 - \theta_2} = MB(\cos\theta_1 - \cos\theta_2)$$

(f) At $\theta = 0^o$, $F = 0$, $\tau = 0$, $U = -MB$ = minimum

this is called stable equilibrium position.

(g) At $\theta = 180^o$, $F = 0$, $\tau = 0$, $U = +MB$ = maximum

this is called unstable equilibrium position.

In the above equations, θ is the angle between $\mathbf{M}$ and $\mathbf{B}$.

(v) Time period of small oscillations of a magnetic needle in uniform magnetic field

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

or

$$B = \frac{4\pi^2 I}{MT^2}$$

(vi) Gauss's law for magnetism is

$$\oint \mathbf{B}\cdot d\mathbf{S} = 0$$

or net magnetic flux through any closed surface is zero.

(vii) **Earth's Magnetism**

(a) The value of magnetic field on the surface of the earth is of the order of 10^{-5} T.

(b) The axis of the earth makes an angle of approximately 11.5^o with the earth's rotational axis.

(c) At any point, the vertical plane passing through the line joining the geographical North and South pole is called the geographical meridian.

(d) At any point, a vertical plane in the direction of the earth's magnetic field is called magnetic meridian.

(e) At any place, the acute angle between the magnetic meridian and geographical meridian is called angle of declination α.

(f) The angle of dip (θ) at any place is the angle between the direction of the earth's magnetic field and the horizontal.

(g) $H = B_e\cos\theta$, $V = B_e\sin\theta$

$$B_e = \sqrt{H^2 + V^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{V}{H}\right)$$

Here, B_e = total earth's magnetic field at some points.

H = Horizontal component of the earth's magnetic field at that point

and V = vertical component of the earth's magnetic field at that point.

(viii) **Important terms in Magnetism**

(a) Intensity of magnetisation (or magnetisation)

$\mathbf{I}$ = Magnetic moment per unit volume of magnetised substance

or

$$I = \frac{M}{V}$$

(b) Magnetic intensity (or magnetic field strength)

Capability of the magnetising field to magnetise a substance is expressed by magnetic intensity vector $\mathbf{H}$.

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{I}$$

or

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{I})$$

(c) Magnetic permeability

$$\mu = \frac{B}{H} \quad \text{or} \quad \mathbf{B} = \mu\mathbf{H}$$

(d) Relative magnetic permeability

$$\mu_r = \frac{\mu}{\mu_0} = \frac{B}{B_0}$$

(e) Magnetic susceptibility

$$\chi_m = \frac{I}{H} \quad \text{or} \quad \mathbf{I} = \chi_m\mathbf{H}$$

(f) $\mu_r = 1 + \chi_m$

(ix) Paramagnetic, diamagnetic and ferromagnetic substances

(a) For paramagnetic substances $\mu_r > 1$ and χ_m is slightly positive.

(b) For diamagnetic substances, $\mu_r < 1$ and χ_m is slightly negative.

(c) For ferromagnetic substances $\mu_r >> 1$ and χ_m are positive and very large.

(x) Curie's law magnetic susceptibility of a paramagnetic substance

$$\chi_m \propto \frac{1}{T}$$

(xi) Retentivity of soft iron is more than the retentivity of steel. But its coercivity and hysteresis loss are less.

(xii) For permanent magnets, we require high retentivity and coercivity. So, steel is more suitable than soft iron.

(xiii) For electromagnets we require high initial permeability and low hysteresis loss. So, soft iron is an ideal material.

Additional Examples

Example 1. *If magnetic monopoles existed, how would the Gauss's law of magnetism be modified?*

Sol. Gauss's law in magnetism states that total magnetic flux passing through a closed surface is zero. or

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

In electrostatics right hand side of the above equation is $\frac{q_{in}}{\varepsilon_0}$.

In magnetism if monopoles would have existed then right hand side of the above equation would be equal to $\mu_0 (m_{in})$ or

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 (m_{in})$$

where, m_{in} is the net magnetic charge (or monopole) enclosed by that surface.

Example 2. *Magnetic field lines show the direction along which a small magnetic needle aligns in equilibrium. Do the magnetic field lines also represent the direction of force on a moving charged particle at some point?*

Sol. No, from the equation $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$, we can see that magnetic force F is always perpendicular to magnetic field $\mathbf{B}$ (from the property of cross product). Therefore, it is also misleading to call magnetic field lines as lines of force.

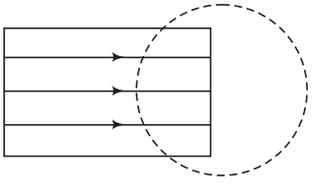
Example 3. *Does a bar magnet exert a torque on itself due to its own field? Does one element of a current carrying wire exert a force on another element of the same wire?*

Sol. No, bar magnet does not exert any force (or torque) on itself due to its own field. One element of a current carrying wire can exert a force on another element of the same wire. For example, when a current is passed through a spring, it is compressed, due to attraction between different rounds of the spring having currents in same direction.

Straight current carrying wire is a special case. One part of it does not exert any force on its other part.

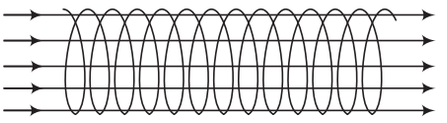
Example 4. *Magnetic field lines are entirely confined within the core of a toroid, but not within a straight solenoid. Why?*

Sol. If field lines were entirely confined between two ends of a straight solenoid, the magnetic flux through the closed dotted surface (shown in figure) would be non-zero.

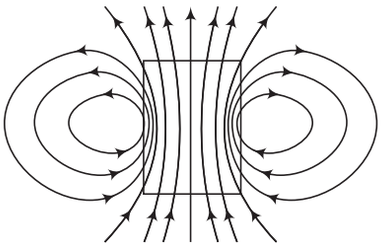


But magnetic flux from any closed surface must be zero (Gauss's law of magnetism). In toroid total flux from a closed surface becomes zero, because it has no ends.

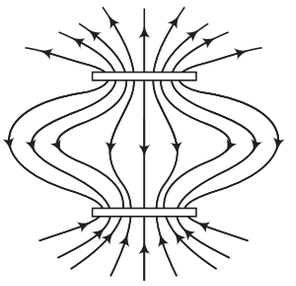
Example 5. *The diagrams given in figure show magnetic field lines (thick lines in the figure). Point out which one is wrong which one of them may describe electrostatic field lines correctly.*



(a)



(b)



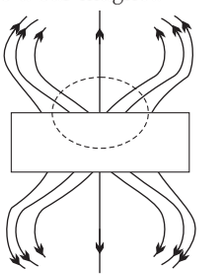
(c)

Sol. (a) Wrong. Field lines of a solenoid at its ends and outside cannot be so completely straight and confined. This violates Ampere's circular law. The lines should curve out at both ends and meet eventually to form closed loops.

These type of lines show uniform electric field. Almost uniform electric field can be seen between the plates of capacitor.

(b) Right. These are the field lines due to a bar magnet.

(c) Wrong. These field lines cannot possibly represent magnetic field lines.



Net flux through a closed surface (shown by dotted lines) surrounding any one plate is not zero.

The given field lines show the electric field lines around a positively charged upper plate and a negatively charged lower plate.

Example 6. *The magnetising field of* 20 *CGS units produces a flux of* 2400 *CGS units in a bar of iron of cross-section* $0.2\ cm^2$. *Calculate the permeability and susceptibility of the bar.*

Sol. Magnetic field

$$B = \frac{\phi}{A} = \frac{2400 \times 10^{-8}}{0.2 \times 10^{-4}} = 1.20\ \text{Wb/m}^2$$

(i) The permeability of the bar material,

$$\mu = \frac{B}{H} = \frac{1.2}{20/4\pi \times 10^{-3}} = 7.54 \times 10^{-4}\ \text{H/m}$$

(ii) The magnetic susceptibility and permeability of a material are related with each other as,

$$\mu = \mu_0 (1 + \chi_m)$$

or $$\chi_m = \frac{\mu}{\mu_0} - 1 = \frac{7.54 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 599$$

Example 7. *The electron in hydrogen atom moves with a speed of* 2.2×10^6 *m/s in an orbit of radius* 5.3×10^{-11} *cm. Find the magnetic moment of the orbiting electron.*

Sol. Frequency of revolution,

$$f = \frac{v}{2\pi r}$$

The moving charge is equivalent to a current loop, given by

$$i = f \times e \quad \text{or} \quad i = \frac{ev}{2\pi r}$$

If A be the area of the orbit, then the magnetic moment of the orbiting electron is,

$$M = iA = \left(\frac{ev}{2\pi r}\right)(\pi r^2) = \frac{evr}{2}$$

Putting the values, we have

$$M = \frac{(1.6 \times 10^{-19})\ (2.2 \times 10^6)\ (5.3 \times 10^{-11})}{2}$$

$$= 9.3 \times 10^{-24}\ \text{A-m}^2$$

Example 8. *A compass needle of magnetic moment* $60\ A\text{-}m^2$ *is pointing geographical North at a certain place. It experiences a torque of* 1.2×10^{-3} *N-m. The horizontal component of the earth's magnetic field at that place is* $40\ \mu Wb/m^2$. *What is the angle of declination at that place?*

Sol. A compass needle in stable equilibrium position points towards magnetic North, i.e. along the horizontal component H of Earth's magnetic field. When it is turned through the angle of declination α, so as to point geographical North then it experiences a torque of magnitude $MH \sin\alpha$.

$\therefore$ $MH \sin\alpha = 1.2 \times 10^{-3}$ N-m (given)

Here, $M = 60\ \text{A-m}^2$, $H = 40 \times 10^{-6}\ \text{Wb/m}^2$

$\therefore$ $$\sin\alpha = \frac{1.2 \times 10^{-3}}{60 \times 40 \times 10^{-6}} = 0.5$$

$\therefore$ $\alpha = 30°$

Example 9. *A small bar magnet having a magnetic moment of* $9 \times 10^{-3}\ A\text{-}m^2$ *is suspended at its centre of gravity by a light torsionless string at a distance of* $10^{-2} m$ *vertically above a long, straight horizontal wire carrying a current of* 1.0 *A from East to West. Find the frequency of oscillation of the magnet about its equilibrium position. The moment of inertia of the magnet is* $6 \times 10^{-9}\ kg\text{-}m^2$. $(H = 3 \times 10^{-5}\ T)$.

Sol. The magnetic moment of the bar magnet is,

$$M = 9 \times 10^{-3}\ \text{A-m}^2$$

The magnitude of the magnetic field at the location of the magnet due to current carrying wire is,

$$B = \frac{\mu_0}{2\pi}\frac{i}{r} = \frac{(2 \times 10^{-7})\ (1.0)}{10^{-2}}$$

$$= 2 \times 10^{-5}\ \text{Wb/m}^2 \qquad (S \text{ to } N)$$

The earth's horizontal magnetic field is,

$$H = 3 \times 10^{-5}\ \text{T} = 3 \times 10^{-5}\ \text{Wb/m}^2$$

$\therefore$ $$B + H = 5 \times 10^{-5}\ \text{Wb/m}^2$$

The frequency of oscillation will be,

$$f = \frac{1}{2\pi}\sqrt{\frac{M(B+H)}{I}} \qquad \text{(Here, } I \text{ = moment of inertia)}$$

$$= \frac{1}{2\pi}\sqrt{\frac{(9 \times 10^{-3}) \times (5 \times 10^{-5})}{6 \times 10^{-9}}} \qquad (S \text{ to } N)$$

$$= 1.38\ \text{Hz}$$

Example 10. *A magnet performs* 15 *oscillations per minute in a horizontal plane where angle of dip is* 60° *and the earth's total field is* 0.5 *G. At another place where total field is* 0.6 *G, the magnet performs* 20 *oscillations per minute. What is the angle of dip at this place?*

Sol. As, $H = B\cos\theta$

where, H = horizontal component of the earth's magnetic field,

B = total earth's magnetic field

and θ = angle of dip.

Then, $H_1 = B_1 \cos\theta_1$

and $H_2 = B_2 \cos\theta_2$

Further, $$T_1 = 2\pi\sqrt{\frac{I}{MH_1}} = 2\pi\sqrt{\frac{T}{MB_1\cos\theta_1}}$$

and $$T_2 = 2\pi\sqrt{\frac{I}{MH_2}} = 2\pi\sqrt{\frac{I}{MB_2\cos\theta_2}}$$

$\therefore$ $$\frac{T_1^2}{T_2^2} = \frac{B_2\cos\theta_2}{B_1\cos\theta_1}$$

$\therefore$ $$\cos\theta_2 = \frac{B_1}{B_2} \times \frac{T_1^2}{T_2^2}\cos\theta_1 = \frac{B_1}{B_2} \times \left(\frac{f_2}{f_1}\right)^2 \cos\theta_1$$

$$= \left(\frac{0.5}{0.6}\right)\left(\frac{20}{15}\right)^2 \cos 60° = 0.74$$

$\therefore$ $$\theta_2 = \cos^{-1}(0.74) = 42.2°$$

NCERT Selected Questions

Q 1. Answer the following questions regarding the earth's magnetism :

(a) A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.

(b) The angle of dip at a location in Southern India is about 18°. Would you expect a greater or smaller dip angle in Britain?

(c) If you made a map of magnetic field lines at Melbourne in Australia, would the lines seen to go into the ground or come out of the ground?

(d) In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic North or South pole?

(e) The earth's field, it is claimed, roughly, approximates the field due to a dipole of magnetic moment 8×10^{22} J T^{-1} located at its centre. Check the order of magnitude of this number in some way.

(f) Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all?

Sol. (a) The three independent quantities conventionally used to specify the earth's magnetic field are magnetic declination (θ), horizontal component of the earth's magnetic field ($\mathbf{B}_H$) and magnetic dip (δ).

(b) Great Britain will have a greater dip angle, because it is located closer to the magnetic North pole.

(c) The magnetic lines of force will seem to come out of ground at Melbourne, because it is situated in Southern hemisphere where North pole of the earth's magnetic field lies.

(d) At the magnetic poles earth's magnetic field is exactly vertical and as the compass needle is free to rotate in horizontal plane, so the compass may point in any direction there.

(e) The magnetic field at a point on equatorial line of magnetic dipole (assuming the earth has magnetic dipole) is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$$

Taking $M = 8 \times 10^{22}$ JT^{-1} and $r = 6.4 \times 10^6$ m = radius of the earth.

$$\therefore \quad B = 10^{-7} \times \frac{8 \times 10^{22}}{(6.4 \times 10^6)^3}$$

$$= 0.3 \times 10^{-4} \text{ T} = 0.3 \text{ G}$$

The above value is almost same as the earth's magnetic field. This checks the magnitude of the dipole moment.

(f) Yes, this is possible. The earth's magnetic field is only approximately a dipole field. Thus, local N-S poles may exist oriented in different directions. This is possible due to deposits or magnetised materials.

Q 2. Answer the following questions:

(a) The earth's magnetic field varies from point to point in space. Does it also change with time? If so, on what time scale does it change appreciably?

(b) The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why?

(c) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the battery (i.e. the source of energy) to sustain these currents?

(d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

(e) The earth's field departs from its dipole shape substantially at large distances (greater than about 30000 km). What agencies may be responsible for this distortion?

(f) Interstellar space has an extremely weak magnetic field of the order of 10^{-12} T. Can such a weak field be of any significant consequence? Explain.

Sol. (a) Yes, earth's magnetic field changes with time for say, annual changes, daily changes, secular changes with period of about 960 yr. Magnetic storms are considered as irregular changes. The time for an appreciable change is roughly few hundred years.

(b) Molten iron cannot retain magnetism, because it is above its Curie temperature.

(c) Radioactivity in the interior of the earth but it is not certain.

(d) Earth's magnetic field gets weakly recorded in certain rocks during solidification. Analysis of this rock magnetism gives us idea about geomagnetic history.

(e) If the distance is large, the magnetic field gets modified by the field produced by motion of ions in the earth's ionosphere.

(f) The deflection of a charged particle moving in a magnetic field is given by

$$Bev = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{Be}$$

If B is low, r is high, i.e. radius of curvature of path is very large. Thus, if distance is very large, the charged particle is hardly noticeable.

Q 3. A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.25 T experiences a torque of magnitudes equal to 4.5×10^{-2} J. What is the magnitude of magnetic moment of the magnet?

Sol. $\tau = MB\sin\theta$

$$\therefore \quad M = \frac{\tau}{B\sin\theta} = \frac{4.5\times10^{-2}}{0.25\times\sin 30°}$$

$$= \frac{4.5\times10^{-2}}{0.25\times\left(\frac{1}{2}\right)} = 36\times10^{-2}\ \text{JT}^{-1}$$

$$= 0.36\ \text{JT}^{-1}$$

Q 4. A short bar magnet of magnetic moment $\mathbf{M} = 0.32\ \text{J T}^{-1}$ is placed in a uniform magnetic field 0.15 T. If the bar magnet is free to rotate in the plane of the field, which orientation would correspond to its (a) stable and (b) unstable equilibrium? What is the potential energy of the magnet in each case?

Sol. (a) When **M** is parallel to **B**, the magnet is in stable equilibrium,

$$\therefore \quad \theta = 0°$$

$\therefore$ Potential energy in this case is given by

$$U = -\mathbf{M}\cdot\mathbf{B} = -MB\cos\theta$$

$$= -0.32\times0.15\times1$$

$$= -0.048\ \text{J}$$

(b) When **M** is anti-parallel to **B**, the magnet will be in unstable equilibrium,

$$\therefore \quad \theta = 180°$$

Thus, potential energy in this case is given by

$$U = -\mathbf{M}\cdot\mathbf{B} = -MB\cos180°$$

$$= -0.32\times0.15\times(-1)$$

$$= +0.048\ \text{J}$$

Q 5. A closely wound solenoid of 800 turns and area of cross-section $2.5\times10^{4}\ \text{m}^2$ carries a current of 3.0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment?

Sol. $M = NIA$

$$= 800\times3.0\times2.5\times10^{-4}$$

$$= 0.60\ \text{J T}^{-1}$$

Which acts along the axis of the solenoid in the direction related to the sense of flow of current according to right handed screw rule.

Q 6. If the solenoid in previous example 5 is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of torque on the solenoid when its axis makes an angle of 30° with the direction of applied field?

Sol. $\tau = MB\sin\theta$

$$= 0.60\times0.25\times\sin 30°$$

$$= 0.6\times0.25\times\frac{1}{2}$$

$$= 0.075\ \text{N-m}.$$

Q 7. A bar magnet of magnetic moment $1.5\ \text{J T}^{-1}$ lies aligned with the direction of a uniform magnetic field of 0.22 T.

(a) What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment, (i) normal to the field direction (ii) opposite to the field direction?

(b) What is the torque on the magnet in cases (i) and (ii)?

Sol. $\theta_1 = 0°$ as the dipole is aligned with **B**

(a) (i) When the bar magnet is displaced to perpendicular position, then $\theta_2 = 90°$.

$$W = MB\,(\cos\theta_1 - \cos\theta_2), \text{we get}$$

$$W = 1.5\times0.22\,(\cos0° - \cos90°)$$

$$= 1.5\times0.22\,(1-0)$$

$$= 0.33\ \text{J}$$

(ii) When the magnet is displaced so as to align **M** opposite to **B**, then

$$\theta_2 = 180°$$

$$\therefore \quad W' = MB\,(\cos\theta_1 - \cos\theta_2)$$

$$= 1.5\times0.22\,(\cos0° - \cos180°)$$

$$= 0.33\,[1-(-1)]$$

$$= 0.33\times2$$

$$= 0.66\ \text{J}$$

(b) (i) Let τ_1 be the torque on the magnet when it is aligned perpendicular to **B**, then using the relation,

$$\tau = MB\sin\theta, \text{ we get}$$

$$\tau_1 = MB\sin\theta_1$$

$$\tau_1 = 1.5\times0.22\times\sin 90° \qquad (\because \theta_1 = 90°)$$

$$= 0.33\times(1) = 0.33\ \text{N-m}$$

(ii) Let τ_2 be the torque in case (ii)

Here, $\theta_2 = 180°$

$$\therefore \quad \tau_2 = MB\sin\theta_2$$

$$= 1.5\times0.22\times\sin180°$$

$$= 0$$

Q 8. A closely wound solenoid of 2000 turns and area of cross-section $1.6\times10^{-4}\ \text{m}^2$, carrying a current of 4.0 A, is suspended through its centre allowing it to turn in a horizontal plane.

(a) What is the magnetic moment associated with the solenoid?

(b) What is the force and torque on the solenoid, if a uniform horizontal magnetic field of $7.5\times10^{-2}\text{T}$ is set up at an angle of 30° with the axis of the solenoid?

Sol. (a) Let M = magnetic moment of the solenoid. Then,

$$M = NIA, \text{ we get}$$
$$M = 2000 \times 4.0 \times 1.6 \times 10^{-4}$$
$$= 1.28 \text{ JT}^{-1}$$

The direction of **M** is along the axis of the solenoid in the direction related to the sense of current according to right handed screw rule.

(b) Here, $\theta = 30°$

$$\mathbf{B} = 7.5 \times 10^{-2} \text{ T}$$

The solenoid behaves as a bar magnet placed in a uniform magnetic field, so the force is

$$\therefore \qquad F = 0$$

Using the relation, $\tau = MB \sin\theta$, we get

$$\tau = 1.28 \times 7.5 \times 10^{-2} \times \sin 30°$$
$$= 1.28 \times 7.5 \times 10^{-2} \times \frac{1}{2}$$
$$= 0.048 \text{ J}$$

The direction of the torque is such that it tends to align the axis of the solenoid (i.e. magnetic moment **M**) along **B**.

Q 9. A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A rests with its plane normal to an external field of magnitude 5.0×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of 2.0 s^{-1}. What is the moment of inertia of the coil about its axis of rotation?

Sol. A = area of cross-section of the coil

$$= \pi r^2$$
$$= \pi\ (0.01)^2 = 10^{-4}\ \pi \text{m}^2$$

If M be the magnetic moment of the coil, then

$$M = NIA = 16 \times 0.75 \times 10^{-4}\ \pi \text{ A-m}^2$$

Also, let T be the time period of oscillation of the coil.

Then, $$T = \frac{1}{f} = \frac{1}{2} = 0.5 \text{ s}$$

Also, we know that

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

or $$T^2 = 4\pi^2 \cdot \frac{I}{MB_H}$$

$$\therefore \qquad I = \frac{MB_H T^2}{4\pi^2}$$
$$= \frac{16 \times 0.75 \times 10^{-4}\ \pi \times 5.0 \times 10^{-2} \times (0.5)^2}{4\pi^2}$$
$$\approx 1.2 \times 10^{-4} \text{ kg-m}^2$$

Q 10. A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its North tip pointing down at 22° with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.35 G. Determine the magnitude of the earth's magnetic field at the plane.

Sol. $B_H = B\cos\delta$

$$\therefore \qquad B = \frac{B_H}{\cos\delta} = \frac{0.35}{\cos 22°} = \frac{0.35}{0.9272}$$
$$= 0.38 \text{ G}$$

Q 11. At a certain location in Africa, a compass points 12° West of the geographic North. The North tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.

Sol. Using the relation, $B_H = B\cos\delta$, we get the magnitude of **B** is given by.

$$B = \frac{B_H}{\cos\delta} = \frac{0.16}{\cos 60°} = \frac{0.16}{\left(\frac{1}{2}\right)}$$
$$= 0.16 \times 2 = 0.32 \text{ G}$$

or $$B = 0.32 \times 10^{-4} \text{ T} \qquad (\because 1\text{G} = 10^{-4} \text{ T})$$

Direction of B The earth's field lies in a vertical plane 12° West of geographic meridian at an angle of 60° above the horizontal line.

Q 12. A short bar magnet has a magnetic moment of 0.48 JT^{-1}. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of magnet on (a) the axis (b) the equatorial lines (normal bisector) of the magnet.

Sol. (a) **When the point lies on the axis**

Let B_1 be the magnetic field at P.

$$\therefore \qquad B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$
$$= 10^{-7} \times \frac{2 \times 0.48}{(0.1)^3}$$
$$= 0.96 \times 10^{-4} \text{ T} \quad \text{along } S\text{-}N \text{ direction}$$

(b) Let $\mathbf{B}_2$ be the magnetic field at point P on the equatorial line.

$$\therefore \qquad B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$$
$$= 10^{-7} \times \frac{0.48}{(0.1)^3}$$
$$= 0.48 \times 10^{-4} \text{ T}$$
$$= 0.48 \text{ G along } N\text{-}S \text{ direction.}$$

***Q* 13.** Answer the following questions :

(a) Why does a paramagnetic sample display greater magnetisation (for the same magnetising field) when cooled?

(b) Why is diamagnetism in contrast, almost independent of temperature?

(c) If a toroid uses bismuth for its core, will the field in the core be (slightly) greater or (slightly) less than when the core is empty?

(d) Is the permeability of a ferromagnetic material independent of the magnetic field? If not, is it more for lower or higher fields?

(e) Magnetic field lines are always nearly normal to the surface of a ferromagnetic at every point. (This fact is analogous to the static electric field lines being normal to the surface of a conductor at every point.) Why?

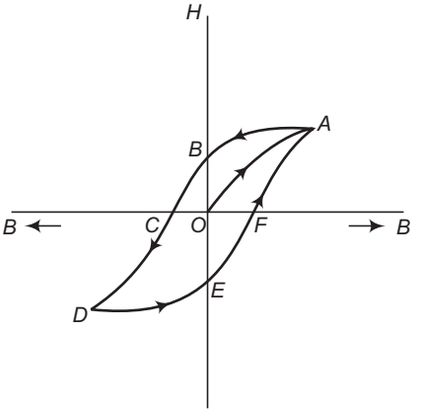
(f) Would the maximum possible magnetisation of a paramagnetic sample be of the same order of magnitude as the magnetisation of a ferromagnetic substance.

Sol. (a) On cooling, the tendency of thermal agitations to disrupt the alignment of magnetic dipoles decreases in case of paramagnetic materials. Thus, they display greater amount of magnetism.

(b) On placing a sample of diamagnetic material in magnetic field, the magnetisation (induced dipole moment) is opposite to direction of magnetising field. Thus, it is not affected by temperature.

(c) The field in the core will be slightly less than when the core is empty as bismuth is diamagnetic.

(d) No, because permeability of ferromagnetic material is dependent on applied magnetic field. Graph for B and H is shown. B is large for smaller value of H, thus permeability μ $(= B/H)$ is greater for lower fields.



(e) The field lines meet the material normally as $\mu_r >> 1$.

(f) Yes. A paramagnetic substance sample with saturated magnetism will have same order of magnetisation of a ferromagnet. However, a saturated magnetisation will require very high field. There will be a minor difference in the strength of the atomic dipoles of paramagnetic and ferromagnetic materials.

***Q* 14.** Answer the following questions :

(a) Explain qualitatively on the basis of domain picture, the irreversibility in the magnetisation curve of a ferromagnet.

(b) The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetisation, which piece will dissipate greater heat energy?

(c) A system displaying a hysteresis loop such as a ferromagnet is a device for storing memory? Explain the meaning of this statement.

(d) What kind of ferromagnetic material is used for coating magnetic tapes in a cassette player or for building memory stores in a modern computer?

(e) A certain region of space is to be shielded from magnetic fields. Suggest a method.

Sol. (a) In a ferromagnetic material, atoms form a very large number of small effective regions called domains. Each domain has a linear dimension ≈ 1000 Å. Within each domain, a special interaction called exchange coupling renders dipole moments of all atoms in a particular direction.

Thus, each domain is a strong magnet without any external magnetic field but the ferromagnetic substance does not behave as a magnet in the absence of external magnetic field as the magnetic moments of different domains are randomly oriented.

(b) The carbon steel piece will dissipate greater heat energy as heat lost per cycle is proportional to the area of the hysteresis loop.

(c) The ferromagnetic substance shows that it remains magnetised even after the removal of external magnetic field. It means that magnetism is stored as a memory in the ferromagnet. Thus, a system displaying hysteresis is a device for storing memory.

(d) Ceramics are used for coating magnetic tapes in cassette players or for building. Memory stores in modern computers. Ceramics are double oxides of barium and iron. Ceramics are also called ferrites.

(e) It is done by surrounding the region with soft iron rings. Magnetic field lines will be drawn into rings and enclosed space will be free of magnetic fields. But this shielding is only approximate, unlike the perfect electric shielding or a cavity in a conductor placed in an external electric field.

Objective Problems

[Level 1]

1. Unit of magnetic flux density (or magnetic induction) is
(a) tesla
(b) weber/metre^2
(c) newton/ampere-metre
(d) All of the above

2. A magnet is placed in iron powder and then taken out, then maximum iron powder is at
(a) some distance away from North pole
(b) some distance away from South pole
(c) the middle of the magnet
(d) the end of the magnet

3. A magnet of magnetic moment M and pole strength m is cut in two equal parts along the axis of magnet, then magnetic moment of each part will be
(a) M (b) $M/2$
(c) $M/4$ (d) $2M$

4. If a magnet of pole strength m is divided into four parts such that the length and width of each part is half that of initial one, then the pole strength of each part will be
(a) $m/4$ (b) $m/2$
(c) $m/8$ (d) $4m$

5. The magnetic field at a distance d from a short bar magnet in longitudinal and transverse positions are in the ratio
(a) 1:1 (b) 2:3
(c) 2:1 (d) 3:2

6. Which of the following is the most suitable material for making permanent magnet?
(a) Steel (b) Soft iron
(c) Copper (d) Nickel

7. If a bar magnet of magnetic moment M is freely suspended in a uniform magnetic field of strength B, the work done in rotating the magnet through an angle θ is
(a) $MB(1-\sin\theta)$ (b) $MB\sin\theta$
(c) $MB\cos\theta$ (d) $MB(1-\cos\theta)$

8. A magnet of magnetic moment M is situated with its axis along the direction of a magnetic field of strength B. The work done is rotating it by an angle of 180° will be
(a) $-MB$ (b) $+MB$
(c) zero (d) $+2MB$

9. Magnetic field intensity is defined as
(a) magnetic moment per unit volume
(b) magnetic induction force acting on a unit magnetic pole
(c) number of lines of force crossing per unit area
(d) number of lines of force crossing per unit volume

10. A magnetic needle kept in a non-uniform magnetic field experiences
(a) a force and a torque
(b) a force but not a torque
(c) a torque but not a force
(d) neither a torque nor a force

11. Two identical thin bar magnets each of length l and pole strength m are placed at right angles to each other with North pole of one touching South pole of the other. Magnetic moment of the system is
(a) ml (b) $2ml$
(c) $\sqrt{2}ml$ (d) $\frac{1}{2}ml$

12. What happens to the force between magnetic poles when their pole strength and the distance between them are both doubled?
(a) Force increases to two times the previous value
(b) No change
(c) Force decreases to half the previous value
(d) Force increases to four times the previous value

13. The work done in turning a magnet of magnetic moment M by an angle of 90° from the meridian is n times the corresponding work done to turn it through an angle of 60°, where n is given by
(a) 1/2 (b) 2 (c) 1/4 (d) 1

14. Magnetic lines of force due to a bar magnet do not intersect, because
(a) a point always has a single net magnetic field
(b) the lines have similar charges and so repel each other
(c) the lines always diverge from a single point
(d) None of the above

15. The unit of magnetic moment is
(a) Wb/m (b) Wb-m^2
(c) A-m (d) A-m^2

16. A permanent magnet
(a) attracts all substances
(b) attracts only magnetic substances
(c) attracts magnetic substances and repels all non-magnetic substances
(d) attracts non-magnetic substances and repels magnetic substances

17. The SI unit of magnetic permeability is
(a) Am^{-1}
(b) Am
(c) Hm^{-1}
(d) No unit, it is a dimensionless number

18. A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.16 T experiences a torque of magnitude 0.032 J. The magnetic moment of the bar magnet will be
(a) 0.23 J/T (b) 0.40 J/T
(c) 0.80 J/T (d) zero

19. A magnet of magnetic moment $50\hat{\mathbf{i}}$ A-m^2 is placed along the x-axis in a magnetic field $\mathbf{B} = (0.5\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})$ T. The torque acting on the magnet is
(a) $175\hat{\mathbf{k}}$ N-m (b) $150\hat{\mathbf{k}}$ N-m
(c) $75\hat{\mathbf{k}}$ N-m (d) $25\sqrt{37}\,\hat{\mathbf{k}}$ N-m

20. A bar magnet of magnetic moment 3.0 A-m^2 is placed in a uniform magnetic induction field of 2×10^{-5} T. If each pole of the magnet experiences a force of 6×10^{-4} N, the length of the magnet is
(a) 0.5 m (b) 0.3 m
(c) 0.2 m (d) 0.1 m

21. A bar magnet when placed at an angle of 30° to the direction of magnetic field induction of 5×10^{-2} T, experiences a moment of couple 25×10^{-6} N-m. If the length of the magnet is 5 cm, its pole strength is
(a) 2×10^{-2} A-m (b) 5×10^{-2}A-m
(c) 2 A-m (d) 5 A-m

22. If a magnet is hanged with its magnetic axis, then it stops in
(a) magnetic meridian (b) geometric meridian
(c) angle of dip (d) None of these

23. A magnet of magnetic moment 2 J/T is aligned in the direction of magnetic field of 0.1 T. What is the net work done to bring the magnet normal to the magnetic field?
(a) 0.1 J (b) 0.2 J
(c) 1 J (d) 2 J

24. Earth's magnetic field always has a horizontal component except at
(a) equator (b) magnetic poles
(c) a latitude of 60° (d) an altitude of 60°

25. A dip needle in a plane perpendicular to magnetic meridian will remain
(a) vertical
(b) horizontal
(c) in any direction
(d) at an angle of dip to the horizontal

26. The angle between the magnetic meridian and geographical meridian is called
(a) angle of dip
(b) angle of declination
(c) magnetic moment
(d) power of magnetic field

27. If the angles of dip at two places are 30° and 45° respectively, then the ratio of horizontal components of the earth's magnetic field at the two places will be
(a) $\sqrt{3} : \sqrt{2}$ (b) $1 : \sqrt{2}$
(c) $1 : \sqrt{3}$ (d) 1 : 2

28. A line passing through places having zero value of magnetic dip is called
(a) isoclinic line
(b) agonic line
(c) isogonic line
(d) aclinic line

29. A dip circle is at right angles to the magnetic meridian. What will be the apparent dip?
(a) 0° (b) 30°
(c) 60° (d) 90°

30. The earth's magnetic field at a certain place has a horizontal component 0.3 G and the total strength 0.5 G. The angle of dip is
(a) $\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{3}{4}\right)$
(c) $\tan^{-1}\left(\frac{4}{3}\right)$ (d) $\sin^{-1}\left(\frac{3}{5}\right)$

31. When the *N*-pole of a bar magnet points towards the South and *S*-pole towards the North, the null points are at the
(a) magnetic axis
(b) magnetic centre
(c) perpendicular divider of magnetic axis
(d) *N* and *S* poles

32. Lines which represent places of constant angle of dip are called
(a) isobaric lines (b) isogonic lines
(c) isoclinic lines (d) isodynamic lines

33. Vibration magnetometer is used for comparing
(a) magnetic fields (b) earth's field
(c) magnetic moments (d) All of these

34. Two magnets of same size and mass make respectively 10 and 15 oscillations per minute at certain place. The ratio of their magnetic moments is
(a) 4 : 9 (b) 9 : 4
(c) 2 : 3 (d) 3 : 2

35. When 2 A current is passed through a tangent galvanometer, it gives a deflection of 30°. For 60° deflection, the current must be
(a) 1 A (b) $2\sqrt{3}$ A
(c) 4 A (d) 6 A

36. The permanent magnet is made from which one of the following substances?
(a) Diamagnetic (b) Paramagnetic
(c) Ferromagnetic (d) Electromagnetic

37. The only property possessed by ferromagnetic substance is
(a) hysteresis
(b) susceptibility
(c) directional property
(d) attracting magnetic substances

38. Substances in which the magnetic moment of a single atom is not zero, is known as
(a) diamagnetism (b) ferromagnetism
(c) paramagnetism (d) ferrimagnetism

39. A dip circle is taken to geomagnetic equator. The needle is allowed to move in a vertical plane perpendicular to the magnetic meridian. The needle will stay in
(a) horizontal direction only
(b) vertical direction only
(c) any direction except vertical and horizontal
(d) None of the above

40. Which of the following statements are true about the magnetic susceptibility χ_m of paramagnetic substance?
(a) Value of χ_m is inversely proportional to the absolute temperature of the sample
(b) χ_m is positive at all temperature
(c) χ_m is negative at all temperature
(d) χ_m does not depend on the temperature of the sample

41. An example of a diamagnetic substance is
(a) aluminium (b) copper
(c) iron (d) nickel

42. Magnetic permeability is maximum for
(a) diamagnetic substance (b) paramagnetic substance
(c) ferromagnetic substance (d) All of these

43. If a diamagnetic solution is poured into a U-tube and one arm of this U-tube placed between the poles of a strong magnet with the meniscus in a line with the field, then the level of the solution will
(a) rise (b) fall
(c) oscillate slowly (d) remain as such

44. The relative permeability is represented by μ_r and the susceptibility is denoted by χ for a magnetic substance. Then, for a paramagnetic substance
(a) $\mu_r < 1, \chi < 0$ (b) $\mu_r < 1, \chi > 0$
(c) $\mu_r > 1, \chi < 0$ (d) $\mu_r > 1, \chi > 0$

45. Which of the following statements is true?
(a) Diamagnetism is temperature dependent
(b) Paramagnetism is temperature dependent
(c) Paramagnetism is temperature independent
(d) None of the above

46. Identify the paramagnetic substance
(a) Iron (b) Aluminium
(c) Nickel (d) Hydrogen

47. The universal property of all substances is
(a) diamagnetism (b) ferromagnetism
(c) paramagnetism (d) All of these

48. Which one of the following is a non-magnetic substance?
(a) Iron (b) Nickel (c) Cobalt (d) Brass

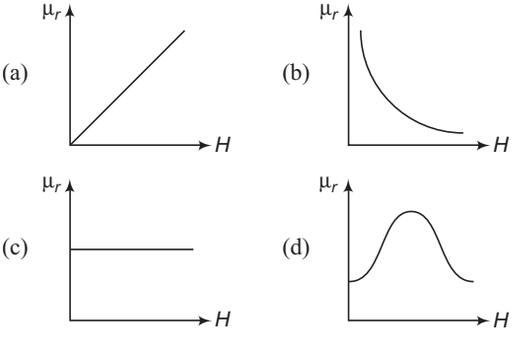
49. Liquid oxygen remains suspended between two pole faces of a magnet, because it is
(a) diamagnetic (b) paramagnetic
(c) ferromagnetic (d) anti-ferromagnetic

50. The materials suitable for making electromagnets should have
(a) high retentivity and high coercivity
(b) low retentivity and low coercivity
(c) high retentivity and low coercivity
(d) low retentivity and high coercivity

51. When a piece of a ferromagnetic substance is put in a uniform magnetic field, the flux density inside it is four times the flux density away from the piece. The magnetic permeability of the material is
(a) 1 (b) 2 (c) 3 (d) 4

52. Which of the following is diamagnetic?
(a) Aluminium (b) Quartz
(c) Nickel (d) Bismuth

53. For ferromagnetic material, the relative permeability (μ_r), *versus* magnetic intensity (H) has the following shape.



54. Two bar magnets of the same mass, length and breadth having magnetic moments M and $2M$ are joined together pole to pole and suspended in a vibration magnetometer. The time period of oscillation is 3 s. If now the polarity of one of the magnets is reversed, the time period of oscillation will be
(a) $\sqrt{3}$ s (b) $3\sqrt{3}$ s
(c) 3 s (d) 6 s

55. All the magnetic materials loss their magnetic properties when
(a) dipped in water
(b) dipped in oil
(c) brought near a piece of iron
(d) strongly heated

56. Which of the following is most suitable for the core of electromagnets?
(a) Iron (b) Steel
(c) Soft iron (d) Cu-Ni alloy

57. At the magnetic North pole of the earth, the values of the horizontal component H and the angle of dip θ are
(a) $H = 0, \theta = 45°$ (b) $H = B_e, \theta = 0°$
(c) $H = 0, \theta = 90°$ (d) $H = B_e, \theta = 90°$

58. A thin rectangular magnet suspended freely has a period of oscillation 4 s. If it is broken into two halves each having half their initial length, then when suspended similarly, the time period of oscillation of each part will be
(a) 4 s (b) 2 s (c) 1 s (d) $2\sqrt{2}$ s

59. The unit of pole strength is
(a) A-m (b) A-m^{-1} (c) A-m^{-2} (d) A-m^2

60. Which of the following properties make soft iron the suitable for core of transformers?
(a) Low hysteresis loss, low permeability
(b) Low hysteresis loss, high permeability
(c) High hysteresis loss, high permeability
(d) High hysteresis loss, low permeability

61. When the magnetic inclination (dip) was measured at various places on the earth, in one of the following countries it was found to be zero. Which one was it?
(a) Pakistan (b) Brazil
(c) Scotland (d) Canada

62. At a certain place, the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.50 oersted. The earth's total magnetic field (in oersted) is
(a) $\sqrt{3}$ (b) 1
(c) $1/\sqrt{3}$ (d) 1/2

63. A dip circle is so that its needle moves freely in the magnetic meridian. In this position, the angle of dip is 40°. Now the dip circle is rotated so that the plane in which the needle moves makes an angle of 30° with the magnetic meridian. In this position, the needle will dip by an angle
(a) 40° (b) 30°
(c) more than 40° (d) less than 40°

64. At a neutral point
(a) field of magnet is zero
(b) field of the earth is zero
(c) field of magnet is perpendicular to field to the earth
(d) None of the above

65. A ferromagnetic material is heated above its Curie temperature. Which one is a correct statement?
(a) Ferromagnetic domains are perfectly arranged
(b) Ferromagnetic domains become random
(c) Ferromagnetic domains are not influenced
(d) Ferromagnetic material changes into diamagnetic material

66. Which magnetic materials have negative susceptibility?
(a) Diamagnetic materials
(b) Paramagnetic materials
(c) Ferromagnetic materials
(d) All of the above

67. Two magnets are held together in a vibration magnetometer and are allowed to oscillate in the earth's magnetic field with like poles together. 12 oscillations per minute are made but for unlike poles together only 4 oscillations per minute are executed. The ratio of their magnetic moments is
(a) 3 : 1 (b) 1 : 3
(c) 3 : 5 (d) 5 : 4

68. A clinic lines are the lines joining places of
(a) zero dip (b) equal dip
(c) zero declination (d) equal declination

69. Two magnets have the same length and the same pole strength. But one of the magnets has a small hole at its centre. Then,
(a) both have equal magnetic moment
(b) one with hole has smaller magnetic moment
(c) one with hole has larger magnetic moment
(d) one with hole loses magnetism through the hole

70. Magnetic length is
(a) lesser than geometric length
(b) equal to geometric length
(c) greater than geometric length
(d) None of the above

71. In a deflection magnetometer, the needle is short and the pointer is long, because the
(a) needle cannot be made long
(b) circular scale cannot be made short
(c) needle must be in a uniform field
(d) pointer must be in a non-uniform field

72. The tangents deflection produced in $\tan A$ and $\tan B$ positions by a short magnet at equal distances are in the ratio
(a) 1 : 1
(b) 2 : 1
(c) 1 : 2
(d) None of the above

73. Hysteresis loss for steel is ..... that for iron.
(a) lesser than (b) equal to
(c) greater than (d) Either (b) or (c)

74. The relation connecting B, H and I in SI system is
(a) $B = H + I$ (b) $B = H - I$
(c) $B = \mu_0 (H + I)$ (d) $B = \mu_0 (H - I)$

75. Permeability is defined as the ratio between
(a) magnetic induction and susceptibility
(b) magnetic induction and magnetising field
(c) magnetising field and magnetic induction
(d) magnetising field and susceptibility

76. Above the Curie temperature, the susceptibility of a ferromagnetic substance varies
(a) directly as the absolute temperature
(b) inversely as the absolute temperature
(c) inversely as the square of absolute temperature
(d) directly as the square of absolute temperature

77. Hysteresis is exhibited by a ..... substance.
(a) paramagnetic (b) ferromagnetic
(c) diamagnetic (d) All of these

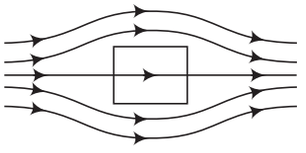
78. Which of the following materials has got the maximum retentivity?
(a) Copper (b) Zinc
(c) Soft iron (d) Hard iron

79. Magnetic meridian is a
(a) point (b) horizontal plane
(c) vertical plane (d) line along *N*-*S*

80. If the current is doubled, the deflection is also doubled in
(a) a tangent galvanometer (b) a moving coil galvanometer
(c) both (a) and (b) (d) None of these

81. The permeability of paramagnetic substance is
(a) slightly more than vacuum
(b) slightly less than vacuum
(c) much more than vacuum
(d) None of the above

82. The given figure represents a material which is



(a) paramagnetic (b) diamagnetic
(c) ferromagnetic (d) None of these

83. A bar magnet is oscillating in the earth's magnetic field with time period T. If its mass is increased four times, then its time period will be
(a) $4T$ (b) $2T$
(c) T (d) $T/2$

84. The temperature at which a ferromagnetic material becomes paramagnetic is called a
(a) neutral temperature (b) Curie temperature
(c) inversion temperature (d) critical temperature

85. When a diamagnetic substance is brought near North or South pole of a bar magnet, it is
(a) attracted by the poles
(b) repelled by the poles
(c) attracted by the North pole and repelled by the South pole
(d) repelled by the North pole and attracted by the South pole

86. The area enclosed by a hysteresis loop is a measure of
(a) retentivity (b) susceptibility
(c) permeability (d) energy loss per cycle

87. Angle of dip at the equator is
(a) 0° (b) 30°
(c) 60° (d) 90°

88. A long magnetic needle of length $2L$, magnetic moment M and pole strength m units is broken into two at the middle. The magnetic moment and pole strength of each piece will be
(a) $(M/2), m/2$ (b) $M, m/2$
(c) $M/2, m$ (d) M, m

89. Magnetic field is measured by
(a) pyrometer (b) hydrometer
(c) thermometer (d) fluxmeter

90. The unit of magnetic susceptibility is
(a) H (b) Wb/m
(c) A/m (d) None of these

91. The ratio of the magnetic field due to small bar magnet in end on position to broad side on position is (at equal distance from the magnet)
(a) 1/4 (b) 1/2
(c) 1 (d) 2

92. The magnetic field on the axis of a short bar magnet at a distance of 10 cm is 0.2 oersted. What will be the field at a point, distant 5 cm on the line perpendicular to the axis and passing through the magnet?
(a) 0.025 oersted (b) 0.2 oersted
(c) 0.4 oersted (d) 0.8 oersted

93. The arms of a deflection magnetometer in the tan B position are placed
(a) East-West (b) North-South
(c) North-East (d) South-West

94. A magnetic needle, suspended horizontally by an unspun silk fibre, oscillates in the horizontal plane, because of a restoring force originating mainly from
(a) the torsion of the silk fibre
(b) the force of gravity
(c) the horizontal component of the earth's magnetic field
(d) All the above factors

95. Due to the earth's magnetic field charged cosmic ray particles
(a) require greater kinetic energy to reach the equator than pole
(b) require less kinetic energy to reach the equator than pole
(c) can never reach the pole
(d) can never reach the equator

96. The angle between the earth's magnetic and the earth's geographic axis is
(a) zero (b) 11.5°
(c) 23° (d) None of these

97. Earth's magnetic field always has a horizontal component except at
(a) magnetic equator (b) magnetic pole
(c) geographical North pole (d) everywhere

98. A paramagnetic liquid is filled in a glass U-tube of which one limb is placed between the pole pieces of an electromagnet. When the field is switched on the liquid in the limb which is in the field will
(a) rise (b) fall
(c) remain stationary (d) first rise and then fall

99. A vibration magnetometer is placed at the South pole, then the time period will be
(a) zero
(b) infinity
(c) same as at magnetic equator
(d) same as at any other place on earth

100. Susceptibility is positive and large for a
(a) paramagnetic substance
(b) ferromagnetic substance
(c) non-magnetic substance
(d) diamagnetic substance

101. A compass needle which is allowed to move in a horizontal plane is taken to a geomagnetic pole. It
(a) stay in North-South direction only
(b) stay in East-West direction only
(c) become rigid showing no movement
(d) stay in any position

[Level 2]

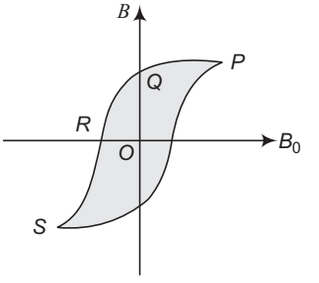
Only One Correct Option

1. A magnet makes 40 oscillations per minute at a place having magnetic field intensity of 0.1×10^{-5} T. At another place, it takes 2.5 s to complete one vibration. The value of the earth's horizontal field at that place is
(a) 0.25×10^{-6} T (b) 0.36×10^{-6} T
(c) 0.66×10^{-8} T (d) 1.2×10^{-6} T

2. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be
(a) 2 s (b) $\frac{2}{3}$ s
(c) $2\sqrt{3}$ s (d) $\frac{2}{\sqrt{3}}$ s

3. A magnet oscillating in a horizontal plane has a time period of 2 s at a place where the angle of dip is 30° and 3 s at another place, where the angle of dip is 60°. The ratio of resultant magnetic fields at the two places is
(a) $\frac{4\sqrt{3}}{7}$
(b) $\frac{4}{9\sqrt{3}}$
(c) $\frac{9}{4\sqrt{3}}$
(d) $\frac{9}{\sqrt{3}}$

4. The figure illustrates how B, the flux density inside a sample of unmagnetised ferromagnetic material varies with B_0, the magnetic flux density in which the sample is kept. For the sample to be suitable for making a permanent magnet



(a) OQ should be large, OR should be small
(b) OQ and OR should both be large
(c) OQ should be small and OR should be large
(d) OQ and OR should both be small

5. The magnetic moment of a steel wire is M. If it is bent into a half circular arc, then what is its new magnetic moment?
(a) M (b) $\frac{2M}{\pi}$ (c) $\frac{M}{2\pi}$ (d) $\frac{M}{\pi}$

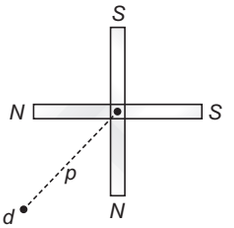
6. A circular coil of radius 20 cm and 20 turns of wire is mounted vertically with its plane in magnetic meridian. A small magnetic needle is placed at the centre of the coil and it is deflected through 45° when a current is passed through the coil. Horizontal component of the earth's field is 0.37×10^{-4} T. The current in coil is
(a) 0.6 A (b) 6 A
(c) 6×10^{-3} A (d) 0.06 A

7. Two tangent galvanometers having coils of the same radius are connected in series. A current flowing in them produces deflections of 60° and 45° respectively. The ratio of the number of turns in the coils is
(a) $4\sqrt{3}$ (b) $\frac{(\sqrt{3}+1)}{1}$
(c) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (d) $\frac{\sqrt{3}}{1}$

8. A dip needle lies initially in the magnetic meridian when it shows an angle of dip θ at a place. The dip circle is rotated through an angle x in the horizontal plane and then it shows an angle of dip θ′. Then, $\frac{\tan\theta'}{\tan\theta}$ is

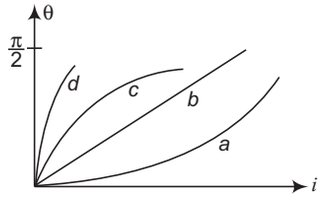
(a) $1/\cos x$ (b) $1/\sin x$
(c) $1/\tan x$ (d) $\cos x$

9. Two short magnets of equal dipole moments M are fastened perpendicularly at their centres (figure). The magnitude of the magnetic field at a distance d from the centre on the bisector of the right angle is



(a) $\frac{\mu_0}{4\pi}\frac{M}{d^3}$ (b) $\frac{\mu_0}{4\pi}\frac{\sqrt{2}\,M}{d^3}$
(c) $\frac{\mu_0}{4\pi}\frac{2\sqrt{2}\,M}{d^3}$ (d) $\frac{\mu_0}{4\pi}\frac{2M}{d^3}$

10. Which of the four graphs may best represent the current-deflection relation in a tangent galvanometer?



(a) d (b) a
(c) c (d) b

11. In a permanent magnet at room temperature,
(a) magnetic moment of each molecule is zero
(b) the individual molecules have non-zero magnetic moment which are all perfectly aligned
(c) domains are partially aligned
(d) domains are all perfectly aligned

More than One Correct Options

1. S is the surface of a lump of magnetic material.
(a) Lines of **B** are necessarily continuous across S
(b) Some lines of **B** must be discontinuous across S
(c) Lines of **H** are necessarily continuous across S
(d) Lines of **H** cannot all be continuous across S

2. The primary origin (*s)* of magnetism lies in
(a) atomic currents
(b) Pauli exclusion principle
(c) polar nature of molecules
(d) intrinsic spin of electron

3. Essential difference between electrostatic shielding by a conducting shell and magnetostatic shielding is due to
(a) electrostatic field lines can end on charges and conductors have free charges
(b) lines of **B** can also end but conductors cannot end them
(c) lines of **B** cannot end on any material and perfect shielding is not possible
(d) shells of high permeability materials can be used to divert lines of **B** from the interior region

4. Let the magnetic field on the earth be modelled by that of a point magnetic dipole at the centre of the earth. The angle of dip at a point on the geographical equator
(a) is always zero
(b) can be zero at specific points
(c) can be positive or negative
(d) is bounded

Assertion and Reason

Directions (Q. Nos. 1-20) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*
(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** The poles of a magnet cannot be separated by breaking it into two pieces.
Reason The magnetic moment will be reduced to half when a magnet is broken into two equal pieces.

2. **Assertion** Horizontal component of the earth's magnetic field (H) has been chosen as a magnetic element instead of the vertical component (V).
Reason Most of our experiments are performed in horizontal configuration. So, H is more relevant.

3. **Assertion** At neutral point, a compass needle may point out in any arbitrary direction.
Reason Magnetic field of the earth is balanced by field due to magnet at the neutral point.

4. **Assertion** χ_m - T graph for a diamagnetic material is a straight line parallel to T-axis.
Reason This is because susceptibility of a diamagnetic material is not affected by temperature.

5. **Assertion** Time period of vibration of a pair of magnets in sum position is always smaller than in difference position.
Reason $T = 2\pi\sqrt{I/MH}$, where symbols have their standard meaning.

6. **Assertion** Magnetism is relativistic.

Reason When we move along with the charge, so that there is no motion relative to us, we find no magnetic field associated with the charge.

7. **Assertion** Steel is not attracted by a magnet.

Reason Steel is not a magnetic substance.

8. **Assertion** When radius of a circular wire carrying current is doubled, its magnetic moment becomes four times.

Reason Magnetic moment is directly proportional to area of the loop.

9. **Assertion** It is not necessary that every magnet has one North pole and one-South pole.

Reason It is a basic fact that magnetic poles occur in pairs.

10. **Assertion** Relative magnetic permeability has no units and no dimensions.

Reason $\mu_r = \mu / \mu_0$, where the symbols have their standard meaning.

11. **Assertion** The net magnetic flux coming out of a closed surface is always zero.

Reason Unlike poles of equal strength exist together.

12. **Assertion** To protect any instrument from external magnetic field, it is put inside an iron box.

Reason Iron is a magnetic substance.

13. **Assertion** A magnet suspended freely in a uniform magnetic field experiences no net force, but a torque that tends to align the magnet along the field.

Reason Net force $mB - mB = 0$. But the forces on North and South poles being equal, unlike and parallel make up a couple that tends to align the magnet, along the field.

14. **Assertion** If one arm of a U-tube containing a diamagnetic solution is placed in between the poles of a strong magnet with the level in line with the field, the level of the solution falls.

Reason Diamagnetic substances are repelled by the magnetic field.

15. **Assertion** Basic difference between an electric line and magnetic line of force is that former is discontinuous and the latter is continuous or endless.

Reason No, electric lines of forces exist inside a charged conductor but magnetic lines do exist inside a magnet.

16. **Assertion** The earth's magnetic field is due to iron present in its core.

Reason At a high temperature magnet loses its magnetic property or magnetism.

17. **Assertion** The ferromagnetic substances do not obey Curie's law.

Reason At Curie point ferromagnetic substances start behaving as a paramagnetic substances.

18. **Assertion** The properties of paramagnetic and ferromagnetic substances are not affected by heating.

Reason As temperature rises, the alignment of molecular magnets gradually decreases.

19. **Assertion** A soft iron core is used in a moving coil galvanometer to increase the strength of magnetic field.

Reason From soft iron more number of the magnetic lines of force passes.

20. **Assertion** Earth′s magnetic field inside a closed iron box is less as compared to the outside.

Reason The magnetic permeability of iron is low.

Entrance Gallery

2014

1. The coercivity of a small magnet, where the ferromagnet gets demagnetised is 3×10^3 A/m. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetised when inside the solenoid is **[JEE Main]**

(a) 30 mA (b) 60 mA
(c) 3 A (d) 6 A

2. A susceptibility of a certain magnetic material is 400. What is the class of the magnetic material? **[Karnataka CET]**

(a) Diamagnetic (b) Paramagnetic
(c) Ferromagnetic (d) Ferroelectric

3. The intensity of magnetisation of a bar magnet is 5.0×10^4 Am^{-1}. The magnetic length and the area of cross-section of the magnet are 12 cm and 1 cm^2 respectively. The magnitude of magnetic moment of this bar magnet is (in SI unit) **[WB JEE]**

(a) 0.6 (b) 1.3
(c) 1.24 (d) 2.4

4. The angle between the true geographic North and the North shown by a compass needle is called as **[J&K CET]**

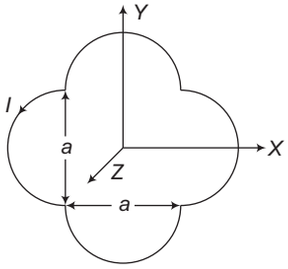
(a) inclination (b) magnetic declination
(c) angle of meridian (d) magnetic pole

5. The magnetic needle has magnetic moment 8.7×10^{-2} A-m^2 and moment of inertia 11.5×10^{-6} kg-m^2. It performs 10 complete oscillations in 6.70 s, what is the magnitude of the magnetic field? **[J&K CET]**

(a) 0.012 T
(b) 0.120 T
(c) 1.200 T
(d) 2.10 T

2013

6. A loop carrying current I lies in the XY - plane as shown in the figure. The unit vector $\hat{\mathbf{k}}$ is coming out of the plane of the paper. The magnetic moment of the current loop is **[JEE Advanced]**



(a) $a^2 I\ \hat{\mathbf{k}}$ (b) $\left(\frac{\pi}{2}+1\right) a^2 I\ \hat{\mathbf{k}}$

(c) $-\left(\frac{\pi}{2}+1\right) a^2 I\ \hat{\mathbf{k}}$ (d) $(2\pi+1)\, a^2 I\ \hat{\mathbf{k}}$

7. A straight strip has magnetic moment M. If bent as a semicircle, the magnetic moment of the strip will be **[OJEE]**

(a) $\frac{2M}{\pi}$ (b) $\frac{2\pi}{M}$ (c) $\frac{2\pi}{3M}$ (d) $\frac{2\pi^2}{M}$

8. Ferromagnetic materials used in a transformer must have **[OJEE]**

(a) low permeability and high hysterisis loss
(b) high permeability and low hysterisis loss
(c) high permeability and high hysterisis loss
(d) low permeability and low hysterisis loss

2012

9. A vibration magnetometer consists of two identical bar magnets placed one over the other such that they are mutually perpendicular and bisect each other. The time period of combination is 4 s. If one of the magnets is removed, find the period of other **[Karnataka CET]**

(a) 5 s (b) 3.36 s
(c) 4.36 s (d) 5.36 s

2011

10. Resultant force acting on a diamagnetic material in a magnetic field is in direction **[IIT JEE]**

(a) from stronger to the weaker part of the magnetic field
(b) from weaker to the stronger part of the magnetic field
(c) perpendicular to the magnetic field
(d) in the direction making 60° to the magnetic field

11. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The torque required to keep the needle in this position will be **[Kerala CEE]**

(a) $2W$ (b) W
(c) $\frac{W}{\sqrt{2}}$ (d) $\frac{W}{\sqrt{3}}$
(e) $\sqrt{3}W$

12. Two identical magnetic dipoles of magnetic moment $2\,\text{A-m}^2$ are placed at a separation of 2 m with their axis perpendicular to each other in air. The resultant magnetic field at a mid point between the dipoles is **[Kerala CEE]**

(a) $4\sqrt{5}\times10^{-5}$ T (b) $2\sqrt{5}\times10^{-5}$ T
(c) $4\sqrt{5}\times10^{-7}$ T (d) $2\sqrt{5}\times10^{-7}$ T
(e) $4\sqrt{2}\times10^{-7}$ T

2010

13. If the magnet is cut into four equal parts such that their lengths and breadths are equal. Pole strength of each part is **[MHT CET]**

(a) m
(b) $m/2$
(c) $m/4$
(d) $m/8$

14. On applying an external magnetic field, to a ferromagnetic substance domains **[MHT CET]**

(a) align in the direction of magnetic field
(b) align in the direction opposite to magnetic field
(c) remain unaffected
(d) None of the above

Answers

Level 1

Objective Problems

1. (d)	**2.** (d)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (b)	**10.** (a)
11. (c)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (d)
21. (a)	**22.** (a)	**23.** (b)	**24.** (b)	**25.** (a)	**26.** (b)	**27.** (a)	**28.** (d)	**29.** (d)	**30.** (c)
31. (a)	**32.** (c)	**33.** (d)	**34.** (a)	**35.** (d)	**36.** (c)	**37.** (a)	**38.** (c)	**39.** (d)	**40.** (a,b)
41. (b)	**42.** (c)	**43.** (b)	**44.** (d)	**45.** (b)	**46.** (b)	**47.** (a)	**48.** (d)	**49.** (b)	**50.** (c)
51. (d)	**52.** (d)	**53.** (d)	**54.** (b)	**55.** (d)	**56.** (c)	**57.** (c)	**58.** (b)	**59.** (a)	**60.** (b)
61. (b)	**62.** (c)	**63.** (c)	**64.** (d)	**65.** (b)	**66.** (a)	**67.** (d)	**68.** (a)	**69.** (b)	**70.** (a)
71. (c)	**72.** (b)	**73.** (c)	**74.** (c)	**75.** (b)	**76.** (b)	**77.** (b)	**78.** (c)	**79.** (c)	**80.** (b)
81. (a)	**82.** (b)	**83.** (b)	**84.** (b)	**85.** (b)	**86.** (d)	**87.** (a)	**88.** (c)	**89.** (d)	**90.** (d)
91. (d)	**92.** (d)	**93.** (b)	**94.** (c)	**95.** (a)	**96.** (b)	**97.** (b)	**98.** (a)	**99.** (b)	**100.** (b)
101. (c)									

Level 2

Only One Correct Option

1. (b)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (a)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (c)
11. (d)									

More than One Correct Options

1. (a,d)	**2.** (a,d)	**3.** (a,c,d)	**4.** (b,c,d)

Assertion and Reason

1. (b)	**2.** (a)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (a)	**9.** (d)	**10.** (a)
11. (a)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (b)	**16.** (d)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (c)

Entrance Gallery

1. (c)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (a)
11. (e)	**12.** (d)	**13.** (b)	**14.** (a)						

Solutions

Level 1 : Objective Problems

1. SI unit of B

$$= \frac{\text{Unit of } \phi}{\text{Unit of } \Delta A}$$

$$= \frac{\text{Wb}}{\text{m}^2}$$

$$= \text{Tesla}$$

Also, SI unit of B is Wb/m^2 and N/A-m.

2. Magnetic pole strength is stronger at end part of magnet, so maximum iron powder is collected at the end point of magnet.

3. If magnet is cut along the axis of magnet of length l, then new pole strength becomes half, i.e. $m' = \frac{m}{2}$ and new length remains same i.e. $l' = l$.



$\therefore$ New magnetic moment

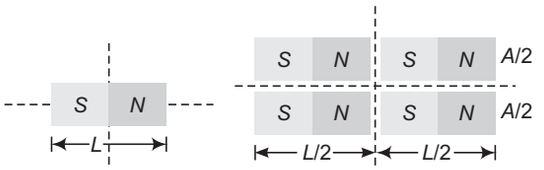
$$M' = \frac{m}{2} \times l = \frac{ml}{2} = \frac{M}{2}$$

If non-magnet is cut perpendicular to the axis of magnet, then new pole strength $m' = m$ and new length $l' = l/2$.

$\therefore$ New magnetic moment,

$$M' = m \times \frac{l}{2} = \frac{ml}{2} = \frac{M}{2}$$

4.



The situation is summerised in figure, we see that magnetic pole strength of each part becomes half, i.e. $m/2$.

5. For longitudinal positions,

$$B_1 \propto \frac{2M}{d^3}$$

For transverse positions,

$$B_2 \propto \frac{M}{d^3}$$

$\therefore$ $$\frac{B_1}{B_2} = 2:1$$

6. Steel has more retentivity and coercivity, so it is used for making permanent magnet.

7. Work done in rotating the magnet through an angle θ from initial position (i.e. $\theta_1 = 0°$) is given by

$$W = MB(\cos\theta_1 - \cos\theta)$$

$$= MB(\cos 0° - \cos\theta)$$

$$= MB(1 - \cos\theta)$$

8. Work done in rotating the magnet from angle θ_1 to angle θ_2 is given by

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

Here, $\theta_1 = 0°$ and $\theta_2 = 180°$

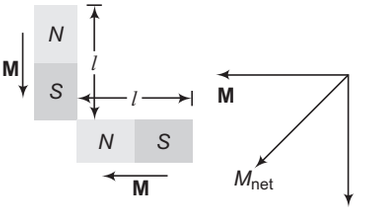
$$W = MB(\cos 0° - \cos 180°)$$

$$= 2MB$$

9. Magnetic moment per unit volume is intensity of magnetisation and number of lines of force crossing per unit area is intensity of magnetic field. So, right option is (b).

10. In non-uniform magnetic field a magnetic needle experiences a force and a torque.

11. Net magnetic moment of the system is given by



$$M = \sqrt{M^2 + M^2} = \sqrt{2}M$$

$$= \sqrt{2}\, ml$$

12. Let pole strengths be m_1 and m_2 and distance between them is r, then force between magnetic poles

$$F \propto \frac{m_1 m_2}{r^2}$$

In second case,

$$F' \propto \frac{(2m_1)(2m_2)}{(2r)^2}$$

$$\propto \frac{m_1 m_2}{r^2}$$

$\therefore$ $$F' = F$$

13. In first case,

$$W_1 = MB(\cos 0° - \cos 90°)$$

$$= MB(1 - 0) = MB$$

In second case,

$$W_2 = MB(\cos 0° - \cos 60°)$$

$$= MB\left(1 - \frac{1}{2}\right) = \frac{MB}{2}$$

$\therefore$ $$\frac{W_1}{W_2} = 2$$

$\Rightarrow$ $$W_1 = 2W_2$$

$\therefore$ $$n = 2$$

14. Magnetic lines of force due to a bar magnet do not intersect because if they intersect then it means, there are two directions of magnetic field intensity which is impossible.

15. Magnetic moment

$$M = NiA$$

Its unit = A-m^2

16. A permanent magnet has large retentivity and coercivity so it attracts only magnetic substances.

17. Magnetic permeability

$$\mu = \frac{B}{H}$$

Its SI unit is $\frac{\text{Wb/m}^2}{\text{A/m}} = \frac{\text{V} \times \text{s}}{\text{A} \times \text{m}} = \frac{\Omega \times \text{s}}{\text{m}} = \frac{\text{H}}{\text{m}}$

18. Magnetic moment of the bar magnet is given by

$$\tau = MB\sin\theta$$

$$\therefore \quad M = \frac{\tau}{B\sin\theta}$$

$$= \frac{0.032}{0.16 \times \sin 30^\circ}$$

$$= \frac{0.032 \times 2}{0.16}$$

$$= 0.4 \text{ J/T}$$

19. Torque is a vector quantity and it is given by

$$\tau = \mathbf{M} \times \mathbf{B}$$

$$= (50\hat{\mathbf{i}}) \times [0.5\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}}]$$

$$= 150(\hat{\mathbf{i}} \times \hat{\mathbf{j}})$$

$$= 150\hat{\mathbf{k}} \text{ N-m}$$

20. Force experienced by each pole of bar magnet is given by

$$F = mB$$

where, m is pole strength of bar magnet.

But $M = mL$

$$\therefore \quad F = \frac{M}{L} \times B$$

$$\Rightarrow \quad 6 \times 10^{-4} = \frac{3}{L} \times 2 \times 10^{-5}$$

$$\therefore \quad L = 0.1 \text{ m}$$

22. A freely hanged magnet stays with its magnetic axis parallel to magnetic meridian.

23. Work done in rotating the magnet in uniform magnetic field is given by

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

Here, $\theta_1 = 0^\circ, \theta_2 = 90^\circ$

$$\therefore \quad W = 2 \times 0.1(\cos 0^\circ - \cos 90^\circ)$$

$$= 2 \times 0.1(1 - 0)$$

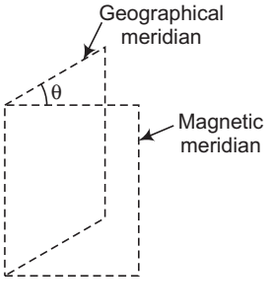
$$= 0.2 \text{ J}$$

24. At magnetic poles, angle of dip is 90°, so horizontal component of the earth

$$B_H = B\cos\theta = B\cos 90^\circ = 0^\circ$$

25. A dip needle in a plane perpendicular to magnetic meridian will remain always vertical.

26. Angle between magnetic meridian and geographical meridian is called angle of declination.



27. $$\frac{(B_H)_1}{(B_H)_2} = \frac{B\cos\theta_1}{B\cos\theta_2} = \frac{\cos\theta_1}{\cos\theta_2} = \frac{\cos 30^\circ}{\cos 45^\circ}$$

$$= \frac{\sqrt{3}}{2} \times \sqrt{2} = \frac{\sqrt{3}}{\sqrt{2}}$$

28. A line passing through places having zero value of magnetic dip is called aclinic line.

30. Total strength of magnetic field

$$B^2 = B_H^2 + B_V^2$$

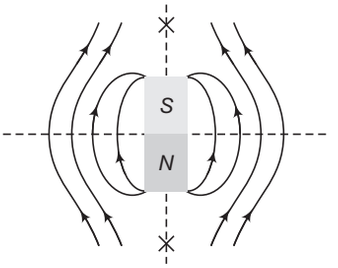
$$\therefore \quad B_V = \sqrt{B^2 - B_H^2}$$

$$= \sqrt{(0.5)^2 - (0.3)^2} = 0.4$$

$$\tan\phi = \frac{B_V}{B_H} = \frac{0.4}{0.3} = \frac{4}{3}$$

$$\therefore \quad \phi = \tan^{-1}\left(\frac{4}{3}\right)$$

31. If the North pole of a magnet points South, the fields of the magnet and the earth will point in opposite directions along the axis of the magnet, so two neutral points are obtained which are equidistant from the magnet on its axis.



32. Lines which represent places of constant angle of dip are called isoclinic lines. The line joining places of zero dip is called aclinic line (or magnetic equator). At all places upon this line a freely suspended magnet will remain horizontal.

33. Vibration magnetometer is simple instrument which is used to find out the magnetic moment of a magnet or to compare the magnetic moments of two bar magnets. Here the magnet is made to oscillate and its time period is calculated, hence the instrument is called vibration magnetometer. It's time period is given by

$$T = 2\pi\sqrt{\left(\frac{I}{MH}\right)}$$

34. Time period of vibrating magnet is given by

$$T = 2\pi\sqrt{\left(\frac{I}{MH}\right)}$$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\left(\frac{M_2}{M_1}\right)}$$

$$\therefore \quad \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$$

$$= \frac{(60/15)^2}{(60/10)^2} = \frac{4}{9}$$

35. For a tangent galvanometer, if I A current flows through coil then this current is proportional to angle of deflection (of the needle).

$$I \propto \tan\theta$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

$$\Rightarrow \quad \frac{2}{I_2} = \frac{\tan 30^\circ}{\tan 60^\circ}$$

$$\therefore \quad I_2 = 6 \text{ A}$$

36. If a magnet retains its attracting power for a long time, it is said to be permanent, otherwise temporary. Permanent magnets are made of ferromagnetic substances.

37. Some peculiar properties of ferromagnetic materials are commonly displayed by curve of **B** against **H** which is called *B* -*H* curve or hysteresis in *B* loop. Dia and para do not show these properties.

51. Given, $B = 4B_0$

$$\therefore \quad \frac{B}{B_0} = 4$$

$$\Rightarrow \quad \mu_r = 4 \qquad \left(\because \mu_r = \frac{B}{B_0}\right)$$

54. $T \propto \frac{1}{\sqrt{M}}$

$$M_1 = 3M \text{ and } M_2 = M$$

$$\therefore \quad T_2 = \sqrt{3}\,T_1 = 3\sqrt{3}\ \text{s}$$

58. $T \propto \sqrt{\frac{I}{M}}$

When length is halved, I will become $\frac{1}{8}$ times $\left(I = \frac{ml^2}{12}\right)$ while M will become half.

62. $H = B_e \cos\theta$

$$\frac{1}{2} = B_e \frac{\sqrt{3}}{2}$$

$$\therefore \quad B_e = \frac{1}{\sqrt{3}} \text{ oersted}$$

63. Apparent dip,

$$\theta' = \tan^{-1}\left(\frac{V'}{H'}\right) = \tan^{-1}\left(\frac{V}{H\cos 30^\circ}\right) = \tan^{-1}\left(\frac{2}{\sqrt{3}}\cdot\theta\right)$$

Here, θ = true, dip = 40°

$$\therefore \quad \theta' > \theta$$

67. $\frac{f_1}{f_2} = \frac{12}{4} = \frac{3}{1}$

$$\therefore \quad \frac{T_1}{T_2} = \frac{1}{3}$$

Further, $T \propto \frac{1}{\sqrt{M}}$

$$\therefore \quad \sqrt{\frac{M_1 - M_2}{M_1 + M_2}} = \frac{1}{3}$$

This equation gives,

$$\frac{M_1}{M_2} = \frac{5}{4}$$

76. Above the Curie temperature, ferromagnetic substance becomes paramagnetic in nature.

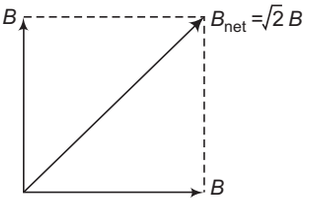
83. $T \propto \sqrt{I}$

By increasing mass to four times, the moment of inertia will also increase four times.

92. $\frac{B}{0.2} = \frac{M/(5)^3}{2M/(10)^3} = \frac{(10/5)^3}{2}$

$$\therefore \quad B = 0.8 \text{ oersted}$$

101. $B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$



$$\therefore \quad B_{\text{net}} = \frac{\mu_0}{4\pi}\frac{\sqrt{2}M}{d^3}$$

Level 2 : Only One Correct Option

1. Time period, $T = 2\pi\sqrt{\frac{I}{MB}}$

$$\therefore \quad T \propto \frac{1}{\sqrt{B}}$$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{B_2}{B_1}}$$

$$\Rightarrow \quad \frac{60/40}{2.5} = \sqrt{\frac{B_2}{0.1\times 10^{-5}}}$$

$$\therefore \quad B_2 = 0.36\times 10^{-6}\ \text{T}$$

2. Initially, the time period of the magnet

$$T = 2\pi\sqrt{\left(\frac{I}{MB}\right)}$$

For each part of magnet, its moment of inertia is $\frac{I}{27}$ and its magnetic moment = $M/3$.

When magnet is cut along its length into three equal parts, then placed on each other, then new moment of inertia of system

$$I' = \frac{I}{27}\times 3 = \frac{I}{9}$$

and magnetic moment of system

$$M' = \frac{M}{3}\times 3 = M$$

$\therefore$ Time period of system

$$T' = 2\pi\sqrt{\frac{I'}{M'B}} = \frac{1}{3}\times 2\pi\sqrt{\frac{1}{B}}$$

$$= \frac{T}{3} = \frac{2}{3}\ \text{s}$$

3. Time period, $T \propto \frac{1}{\sqrt{B_H}} = \frac{1}{\sqrt{B\cos\theta}}$

$$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{B_2\cos\theta_2}{B_1\cos\theta_1}}$$

$$\therefore \quad \frac{B_1}{B_2} = \frac{T_2^2}{T_1^2}\times\frac{\cos\theta_2}{\cos\theta_1} = \left(\frac{3}{2}\right)^2\times\frac{\cos 60^\circ}{\cos 30^\circ}$$

$$\therefore \quad \frac{B_1}{B_2} = \frac{9}{4\sqrt{3}}$$

5. $L = \pi R$

$$\therefore \quad R = \frac{L}{\pi}$$

New effective length will become, $L' = 2R = \frac{2L}{\pi}$

6. The needle is deflected through 45° means magnetic field at the centre of circular coil is equal (in magnitude) to the horizontal component of the earth's magnetic field.

$$\frac{\mu_0 Ni}{2R} = H$$

$$\therefore \quad i = \frac{2RH}{\mu_0 N}$$

$$= \frac{2 \times 0.2 \times 0.37 \times 10^{-4}}{(4\pi \times 10^{-7})(20)} \approx 0.6 \text{ A}$$

7. In series current is same.

$$\frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

or

$$\frac{N_1}{N_2} = \frac{\tan 60°}{\tan 45°}$$

$$= \sqrt{3}$$

8. $\tan\theta' = \frac{V'}{H'} = \frac{V'}{H\cos x} = \frac{\tan\theta}{\cos x}$

$$\therefore \quad \frac{\tan\theta'}{\tan\theta} = \frac{1}{\cos x}$$

11. As we know a permanent magnet is a substance which at room temperature retain ferromagnetic property for a long period of time.

The individual atoms in a ferromagnetic material possess a dipole moment as in a paramagnetic material.

However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called domain. Thus, we can say that in a permanent magnet at room temperature, domains are all perfectly aligned.

More than One Correct Options

1. Magnetic field lines for magnetic induction (**B**) form continuous lines. So, lines of **B** are necessarily continuous across *S*.

Also, magnetic intensity (**H**) varies for inside and outside the lump. So, lines of **H** cannot all be continuous across *S*.

2. The primary origin of magnetism lies in the fact that the electrons are revolving and spinning about nucleus of an atom, which gives rise to current called atomic current.

This atomic currents gives rise to magnetism. The revolving and spinning about nucleus of an atom is called intrinsic spin of electron.

3. Electrostatic shielding is the phenomenon to block the effects of an electric field. The conducting shell can block the effects of an external field on its internal content or the effect of an internal field on the outside environment.

Magnetostatic shielding is done by using an enclosure made of a high permeability magnetic material to prevent a static magnetic field outside the enclosure from reaching objects inside it or to confine a magnetic field within the enclosure.

4. If the total magnetic field of the earth is modelled by a point magnetic dipole at the centre, then it is in the same plane of geographical equator, thus the angle of dip at a point on the geographical equator is bounded in a range from positive to negative value.

Entrance Gallery

1. For solenoid, the magnetic field needed to be magnetised the magnet.

$$B = \mu_0 nI$$

where, $n = 100$, $l = 10 \text{ cm} = \frac{10}{100} \text{ m} = 0.1 \text{ m}$

$$\Rightarrow \quad 3 \times 10^3 = \frac{100}{0.1} \times I$$

$$\Rightarrow \quad I = 3 \text{ A}$$

2. The class of the magnetic material is ferromagnetic, because $\chi_m > 10^2$.

3. We know that, intensity of magnetisation, $I = \frac{M}{V}$

where, M = magnetic moment, V = volume

So, $M = IV = 5.0 \times 10^4 \times \frac{12}{100} \times \frac{1}{(100)^2}$

$$= 60 \times 10^4 \times 10^{-6} = 0.6 \text{ A-m}^2$$

4. It is called magnetic declination.

5. Given, $M = 8.7 \times 10^{-2}$ A-m^2, $I = 11.5 \times 10^{-6}$ kg-m^2

and $T = 0.67$ s

$\Rightarrow$ From the formula,

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$$\Rightarrow \quad B = \frac{4\pi^2 \times I}{M T^2}$$

$$= \frac{4\pi^2 \times 11.5 \times 10^{-6}}{8.7 \times 10^{-2} \times (0.67)^2}$$

$$= \frac{453.5 \times 10^{-6}}{10^{-2} \times 3.9}$$

$$= 116.28 \times 10^{-4}$$

$$= 0.0116 = 0.012 \text{ T}$$

6. Area of the given loop is

A = (area of two circles of radius $\frac{a}{2}$ and area of a square of side a)

$$= 2\pi\left(\frac{a}{2}\right)^2 + a^2$$

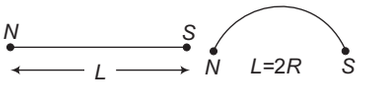
$$= \left(\frac{\pi}{2} + 1\right) a^2$$

$$|\mathbf{M}| = IA = \left(\frac{\pi}{2} + 1\right) a^2 I$$

From screw law, direction of **M** is outwards or in positive Z-direction.

$$\therefore \quad \mathbf{M} = \left(\frac{\pi}{2} + 1\right) a^2 I \, \hat{\mathbf{k}}$$

7. On bending a rod its pole strength remain unchanged, whereas its magnetic moment changes.



New magnetic moment,

$$M' = m(2R) = m\left(\frac{2L}{\pi}\right) = \frac{2M}{\pi}.$$

8. Ferromagnetic materials used in a transformer must have high permeability and low hysterisis loss.

9. As magnets are perpendicular to each other, the resultant magnetic moment

$$= \sqrt{M^2 + M^2} = \sqrt{2}M$$

$$\therefore \quad T_1 = 2\pi\sqrt{\frac{2I}{\sqrt{2}MH}} \qquad \ldots(\text{i})$$

In the second case, $T_2 = 2\pi\sqrt{\frac{I}{MH}}$ …(ii)

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{1}{(2)^{1/4}}$$

$$\therefore \quad T_2 = \frac{4}{(2)^{1/4}} = 3.36 \text{ s} \qquad (\text{given, } T_1 = 4\,\text{s})$$

10. Resultant force acting on a diamagnetic material in a magnetic field is in direction from stronger to the weaker part of the magnetic field.

11. We know that,

$$W = MB(1 - \cos\theta)$$
$$= MB(1 - \cos 60°)$$
$$= MB \times \frac{1}{2}$$
$$\tau = MB\sin\theta$$
$$= MB\sin 60°$$
$$\tau = \sqrt{3}W \qquad [\because MB = 2W]$$

12. Resultant magnetic field,

$$R = \sqrt{P^2 + Q^2}$$

$$B = \left[\left\{\frac{2P}{(d/2)^3}\right\}^2 + \left\{\frac{P}{(d/2)^3}\right\}^2\right]^{1/2} \times 10^{-7}$$

$$= \sqrt{(4^2 + 4)} \times 10^{-7} \qquad (\because \text{ given, } d = 2\text{m})$$
$$= \sqrt{20} \times 10^{-7}$$
$$= 2\sqrt{5} \times 10^{-7} \text{ T}$$

13. If magnet cut into four equal parts and their lengths and breadth are equal, then

Pole strength for each part, $m' = \frac{m}{2}$

14. On applying magnetic field, domains of ferromagnetic substances align themselves in the direction of magnetic field.

22

Electromagnetic Induction

22.1 Introduction

Almost every modern device has electric circuits at its heart. We learned in the chapter of current electricity that an electromagnetic force (emf) is required for a current to flow in a circuit. But for most of the electric devices used in industry, the source of emf is not a battery but an electrical generating station. In these stations, other forms of energy are converted into electric energy. For example, in a hydroelectric plant gravitational potential energy is converted into electric energy. Similarly, in a nuclear plant nuclear energy is converted into electric energy.

But how this conversion is done? Or what is the physics behind this? The branch of physics, known as **electromagnetic induction** gives the answer to all these queries. If the **magnetic flux** (ϕ_B) through a circuit changes, an emf and a current are induced in the circuit. Electromagnetic induction was discovered in 1830. The central principle of electromagnetic induction is **Faraday's law**. This law relates induced emf to change in magnetic flux in any loop including a closed circuit. We will also discuss Lenz's law, which helps us to predict the directions of induced emf and current.

Chapter Snapshot

- Introduction
- Magnetic Field Lines and Magnetic Flux
- Faraday's Law
- Lenz's Law
- Motional Electromotive Force
- Self-inductance and Inductors
- Mutual Inductance
- Growth and Decay of Current in an *L-R* Circuit : Growth of Current
- Oscillations in *L-C* Circuit
- Some Applications of Electromagnetic Induction

22.2 Magnetic Field Lines and Magnetic Flux

Let us first discuss the concept of magnetic field lines and magnetic flux. We can represent any magnetic field by **magnetic field lines**. Unlike the electric lines of force it is wrong to call them magnetic lines of force, because they do not point in the direction of the force on a charge. The force on a moving charged particle is always perpendicular to the magnetic field (or magnetic field lines) at the particle's position.

The idea of magnetic field lines is same as for the electric field lines. The magnetic field at any point is tangential to the field line at that point. Where the field lines are close, the magnitude of field is large, where the field lines are far apart, the field magnitude is small. Also, because the direction of **B** at each point is unique, field lines never intersect. Unlike the electric field lines, magnetic lines form a closed loop.

Magnetic Flux

The flux associated with a magnetic field is defined in a similar manner to that used to define electric flux. Consider an element of area $d\mathbf{S}$ on an arbitrary shaped surface as shown in figure. If the magnetic field at this element is **B**, the magnetic flux through the element is

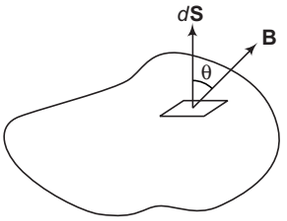


Fig. 22.1

$$d\phi_B = \mathbf{B} \cdot d\mathbf{S} = B ds \cos\theta$$

Here, $d\mathbf{S}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area dS and θ is the angle between **B** and $d\mathbf{S}$ at that element. In general, $d\phi_B$ varies from element to element. The total magnetic flux through the surface is the sum of the contributions from the individual area elements.

$$\therefore \qquad \phi_B = \int B dS \cos\theta = \int \mathbf{B} \cdot d\mathbf{S}$$

Note down the following points regarding the magnetic flux :

(i) Magnetic flux is a scalar quantity (dot product of two vectors is a scalar quantity).

(ii) The SI unit of magnetic flux is tesla-metre^2 (1T-m^2). This unit is called weber (1Wb).

$$1\text{Wb} = 1\ \text{T-m}^2 = 1\ \text{N-m/A}$$

The CGS unit of flux is Maxwell (Mx).

$$1\ \text{Wb} = 10^8\ \text{Mx}$$

Thus, unit of magnetic field is also Wb/m^2.

(1Wb/m^2),

or $$1\ \text{T} = 1\ \text{Wb/m}^2$$

(iii) In the special case in which **B** is uniform over a plane surface with total area S,

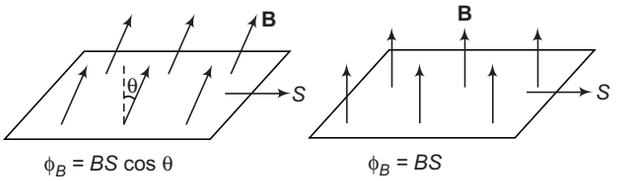


Fig. 22.2

$$\phi_B = BS\cos\theta$$

If **B** is perpendicular to the surface, then $\cos\theta = 1$ and

$$\phi_B = BS$$

22.3 Faraday's Law

This law states that the induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop.

$$e = -\frac{d\phi_B}{dt}$$

If a circuit is a coil consisting of N loops all of the same area and if ϕ_B is the flux through one loop, an emf is induced in every loop, thus the total induced emf in the coil is given by the expression,

$$e = -N\frac{d\phi_B}{dt}$$

The negative sign in the above equations is of important physical significance, which we will discuss in Art. 22.4.

Note down the following points regarding the Faraday's law :

(i) As we have seen induced emf is produced only when there is a change in magnetic flux passing through a loop. The flux passing through the loop is given by

$$\phi = BS\cos\theta$$

This flux can be changed in several ways:

(a) The magnitude of **B** can change with time. In the problems, if magnetic field is given a function of time, it implies that the magnetic field is changing. Thus,

$$B = B(t)$$

(b) The current producing the magnetic field can change with time. For this the current can be given as a function of time. Hence,

$$i = i(t)$$

(c) The area enclosed by the loop can change with time. This can be done by pulling a loop inside (or outside) a magnetic field. By doing so, the area enclosed by loop (hatched area) can be changed.

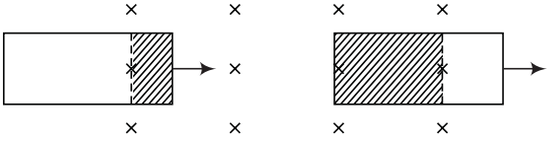


Fig. 22.3

(d) The angle θ between **B** and the normal to the loop can change with time.This can be done by rotating a loop in a magnetic field.

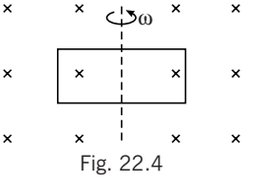


Fig. 22.4

(e) Any combination of the above can occur.

(ii) When the magnetic flux passing through a loop is changed an induced emf and hence, an induced current is produced in the circuit. If R is the resistance of the circuit, then induced current is given by

$$i = \frac{e}{R} = \frac{1}{R}\left(\frac{-d\phi_B}{dt}\right)$$

Current starts flowing in the circuit, means flow of charge takes place. Charge flown in the circuit in time dt will be given by

$$dq = idt = \frac{1}{R}(-d\phi_B)$$

Thus, for a time interval Δt we can write,

$$e = -\frac{\Delta\phi_B}{\Delta t}, \quad i = \frac{1}{R}\left(\frac{-\Delta\phi_B}{\Delta t}\right) \text{ and } \Delta q = \frac{1}{R}(-\Delta\phi_B)$$

From these equations, we can see that e and i are inversely proportional to Δt while Δq is independent of Δt. It depends on magnitude of change in flux, not the time taken in it. To understand it let us take an example.

Example 22.1 *A square loop ACDE of area* $20\ cm^2$ *and resistance* $5\ \Omega$ *is rotated in a magnetic field* $\mathbf{B} = 2\ T$ *through* $180°$*,*
(a) in $0.01\ s$ *and*
(b) in $0.02\ s$*.*
Find the magnitude of e, i and Δq *in both the cases.*

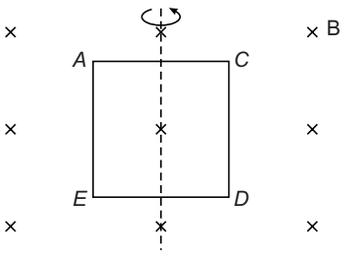


Fig. 22.5

Sol. Let us take the area vector S perpendicular to plane of loop inwards. So, initially, dS ↑↑ B and when it is rotated by 180°, S ↑↓ B. Hence, initial flux passing through the loop,

$$\phi_i = BS\cos 0° = (2)(20 \times 10^{-4})(1) = 4.0 \times 10^{-3} \text{ Wb}$$

Flux passing through the loop when it is rotated by 180°,

$$\phi_f = BS\cos 180° = (2)(20 \times 10^{-4})(-1)$$
$$= -4.0 \times 10^{-3} \text{ Wb}$$

Therefore, change in flux,

$$\Delta\phi_B = \phi_f - \phi_i = -8.0 \times 10^{-3} \text{ Wb}$$

(a) Given $\Delta t = 0.01$ s, $R = 5\ \Omega$

$$\therefore \quad |e| = \left|-\frac{\Delta\phi_B}{\Delta t}\right|$$

$$= \frac{8.0 \times 10^{-3}}{0.01} = 0.8 \text{ V} \quad \text{or} \quad i = \frac{|e|}{R} = \frac{0.8}{5} = 0.16 \text{ A}$$

and $\Delta q = i\Delta t = 0.16 \times 0.01 = 1.6 \times 10^{-3}$ C

(b) $\Delta t = 0.02$ s

$$\therefore \quad |e| = \left|-\frac{\Delta\phi_B}{\Delta t}\right| = \frac{8.0 \times 10^{-3}}{0.02} = 0.4 \text{ V}$$

$$i = \frac{|e|}{R} = \frac{0.4}{5} = 0.08 \text{ A}$$

and $\Delta q = i\Delta t = (0.08)(0.02)$
$= 1.6 \times 10^{-3}$ C

Time interval Δt in part (b) is two times, the time interval in part (a), so, e and i are half while Δq is same.

(iii) **Direction of induced current** The direction of induced current in a loop may be obtained using equations,

$$e = -\frac{d\phi_B}{dt} \quad \text{or} \quad i = -\frac{1}{R}\cdot\frac{d\phi_B}{dt}$$

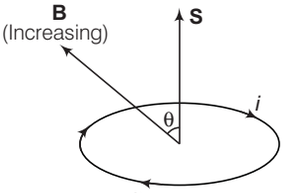
The steps to decide the direction are as follows :

(a) Define a positive direction for the area vector **S.**

(b) From the directions of **S** and the magnetic field **B**, determine the sign of ϕ_B and its rate of change $\frac{d\phi_B}{dt}$.

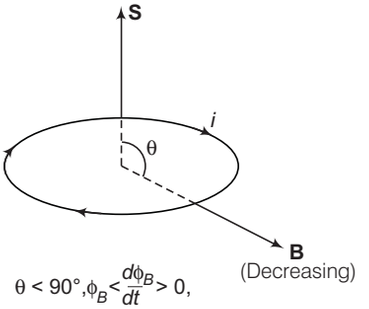
(c) Determine the sign of the induced current. If $\frac{d\phi_B}{dt}$ is positive, i.e. flux is increasing, sign of induced current is negative and *vice-versa*.

(d) Finally, determine the direction of induced current using your right hand. Curl the fingers of your right hand around **S**, with your right thumb in the direction of **S.** If the induced current is positive, it is in the same direction as your curled fingers and if it is negative, it is in the opposite direction.



$\theta < 90°, \phi_B > \frac{d\phi_B}{dt} > 0$,
So, induced current is negative.

Fig. 22.6



$\theta < 90°, \phi_B < \frac{d\phi_B}{dt} > 0$,
So induced current is negative.

Fig. 22.7

22.4 Lenz's Law

The negative sign in Faraday's equations of electromagnetic induction describes, the direction in which the induced emf drives current around a circuit. However, that direction is most easily determined with the help of Lenz's law. Lenz's law is not an independent principle. It gives the same result as the sign rules we introduced in connection with Faraday's law, but is often easier to use. This law states that

The direction of any magnetic induction effect is such as to oppose the cause of the effect.

Later, we will see that Lenz's law is directly related to **energy conservation**. Let us take few simple examples based on Lenz's law.

Example 22.2 *A bar magnet is freely falling along the axis of a circular loop as shown in figure. State whether its acceleration a is equal to, greater than or less than the acceleration due to gravity g.*

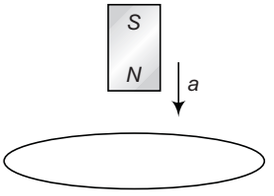


Fig. 22.8

Sol. $a < g$. Because according to Lenz's law, whatever may be the direction of induced current, it will oppose the cause. Here, the cause is the free fall of magnet and so the induced current will oppose it and the acceleration of magnet will be less than the acceleration due to gravity g. This can be understood in a different manner. When the magnet falls downwards with its North pole downwards. The magnetic field lines passing through the coil in the downward direction increase. Since, the induced current opposes this, the upper side of the coil will become North pole, so that field lines of coil's magnetic field are upwards. Now, like poles repel each other. Hence, $a < g$.

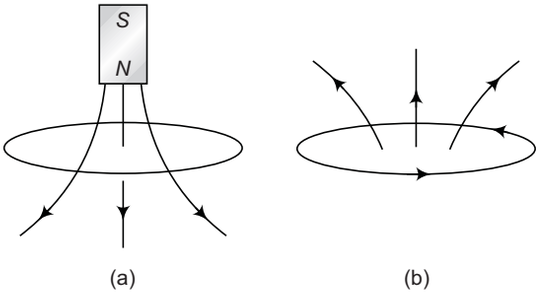


Fig.22.9

Example 22.3 *A bar magnet is brought near a solenoid as shown in figure. Will the solenoid attract or repel the magnet?*

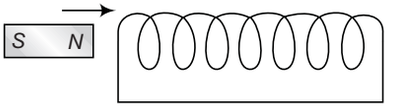


Fig. 22.10

Sol. When the magnet is brought near the solenoid, according to Lenz's law, both repel each other. On the other hand, if the magnet is moved away from the solenoid, it attracts the magnet. When the magnet is brought near the solenoid, the nearer side becomes the same pole and when it is moved away it becomes the opposite pole as shown in figure.

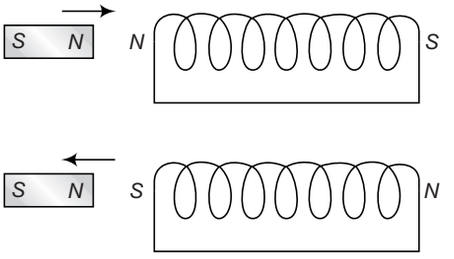
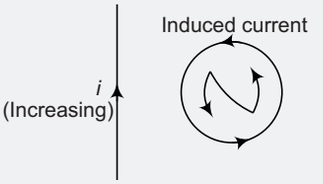


Fig. 22.11

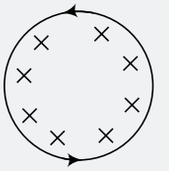
It can also be understood by increasing or decreasing field lines as in example 22.2.

Extra Knowledge Points

- To apply Lenz's law, you can remember **RIN** or ⊗ **IN** (when the loop lies on the plane of paper).



- **RIN** In RIN, R stands for right, I stands for increasing and N for North pole (anti-clockwise). It means, if a loop is placed on the right side of a straight current carrying conductor and the current i in the conductor is increasing, then induced current in the loop is anti-clockwise (↺).
- ⊗ **IN** In ⊗ IN suppose, the magnetic field in the loop is perpendicular to paper inwards (⊗) and this field is increasing, then induced current in the loop is anti-clockwise (↺).



Example 22.4 *A circular loop is placed near a current carrying conductor as shown. Find the direction of induced current, if the current in the wire is decreasing.*

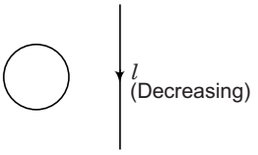


Fig. 22.12

Sol. **Note** Here, we apply RIN (discussed in Extra Knowledge Points). In this case, loop is placed to the right of current carrying wire (not to the left as it appears, because if you move in the direction of current, loop lies to the right). Now the current is decreasing, therefore induced current in the loop is clockwise (↻).



Fig. 22.13

Example 22.5 *A current carrying straight wire passes inside a triangular coil as shown in figure. The current in the wire is perpendicular to paper inwards. Find the direction of the induced current in the loop, if current in the wire is increased.*

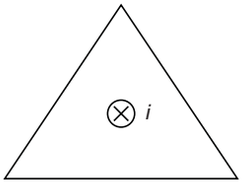
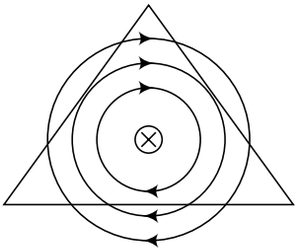


Fig. 22.14

Sol. Magnetic field lines round the current carrying wire are as shown in figure. Since, the lines are tangential to the loop ($\theta = 90°$), the flux passing through the loop is zero, whether the current is increased or decreased. Change in flux is zero. Therefore, induced current in the loop will be zero.


Fig. 22.15

22.5 Motional Electromotive Force

So far we considered the cases in which an emf is induced in a stationary circuit placed in a magnetic field when the field changes with time. In this section we describe what is called **motional emf**, which is the emf induced in a conductor moving through a constant magnetic field.

The straight conductor of length l shown in figure is moving through a uniform magnetic field directed into the page. For simplicity we assume that the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent. The electrons in the conductor experience a force

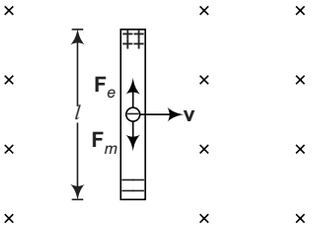


Fig. 22.16

$$\mathbf{F}_m = -e(\mathbf{v} \times \mathbf{B})$$

Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end. As a result of this charge separation, an electric field is produced inside the conductor. The charges accumulate at both ends until the downward magnetic force evB is balanced by the upward electric force eE. At this point, electrons stop moving. The condition for equilibrium requires that

$$eE = evB$$

or $$E = vB$$

The electric field produced in the conductor (once the electrons stop moving and E is constant) is related to the potential difference across the ends of the conductor according to the relationship

$$\Delta V = El = Blv$$

$$\therefore \quad \Delta V = Blv$$

where, the upper end is at a higher electric potential than the lower end. Thus, a potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field.

Now, suppose the moving rod slides along a stationary U-shaped conductor, forming a complete circuit. No magnetic force acts on the charges in the stationary U-shaped conductor, but there is an electric field caused by the charge accumulations at a and b. Under the action of this field a counter-clockwise current is established around this complete circuit. The moving rod becomes a source of electromotive force. Within it, positive charge moves from lower to higher potential and in the remainder of the circuit, charge moves from higher to lower potential. We call this a **motional electromagnetic force** denoted by e, we can write

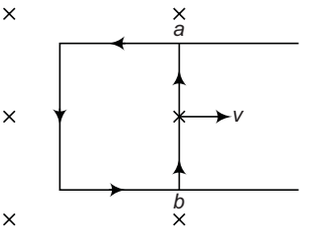


Fig. 22.17

$$e = Bvl$$

If R is the resistance of the circuit, then current in the circuit is,

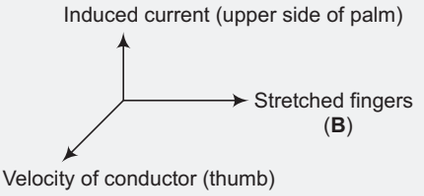
$$i = \frac{e}{R} = \frac{Bvl}{R}$$

Extra Knowledge Points

- The direction of motional emf or current can be given by right hand rule.

  Stretch your right hand.

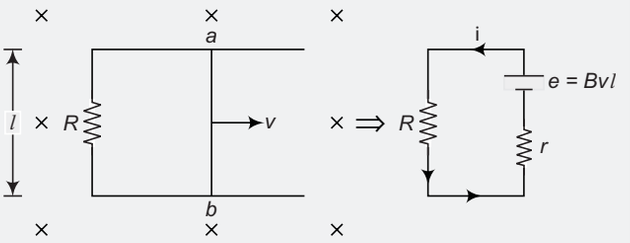
  The stretched fingers point in the direction of magnetic field.



  Thumb is along the velocity of conductor. The upper side of palm is at higher potential and lower side on lower potential. If the circuit is closed the induced current within the conductor is along perpendicular to palm upwards.

- In the figure shown, we can replace the moving rod *ab* by a battery of emf *Bvl* with the positive terminal at *a* and the negative terminal at *b*. The resistance *r* of the rod *ab* may be treated as the internal resistance of the battery. Hence, the current in the circuit is,

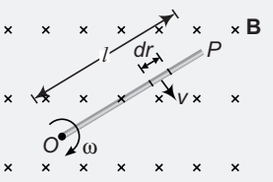
$$i = \frac{e}{R + r}$$



  or $$i = \frac{Bvl}{R + r}$$

- **Motional emf induced in a rotating bar**

  A conducting rod of length *l* rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field **B** is directed perpendicular to the plane of rotation as shown in figure. Consider a segment of rod of length *dr* at a distance *r* from *O*. This segment has a velocity,



$$v = r\,\omega$$

  The induced emf in this segment is,

$$de = Bvdr = B(r\,\omega)dr$$

  Because every segment of the rod is moving perpendicularto **B**, an emf *de* of the same form is generated across each. Summing the emfs induced across all segments, which are in series, gives the total emf across the rod.

$$\therefore \quad e = \int_0^l de$$

$$= \int_0^l Br\,\omega dr = \frac{B\omega l^2}{2}$$

$$\therefore \quad e = \frac{B\omega l^2}{2}$$

  From right hand rule, we can see that *P* is at higher potential then *O*. Thus,

$$V_P - V_O = \frac{B\omega l^2}{2}$$

Example 22.6 *Two parallel rails with negligible resistance are* 10.0 *cm apart. They are connected by* 5.0 Ω *resistor. The circuit also contains two metal rods having resistances of* 10.0 Ω *and* 15.0 Ω *along the rails. The rods are pulled away from the resistor at constant speeds* 4.00 *m/s and* 2.00 *m/s, respectively. A uniform magnetic field of magnitude* 0.01 *T is applied perpendicular to the plane of the rails. Determine the current in the* 5.0 Ω *resistor.*

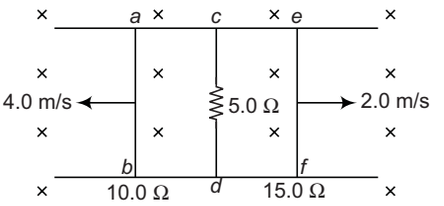


Fig. 22.18

How to Proceed Here, two conductors are moving in uniform magnetic field. So, we will use the motional approach. The rod *ab* will act as a source of emf,

$$e_1 = Bvl = (0.01)\,(4.0)\,(0.1)$$
$$= 4 \times 10^{-3}\ \text{V}$$

and internal resistance $r_1 = 10.0\,\Omega$

Similarly, rod *ef* will also act as a source of emf,

$$e_2 = (0.01)\,(2.0)\,(0.1)$$
$$= 2.0 \times 10^{-3}\ \text{V}$$

and internal resistance $r_2 = 15.0\,\Omega$

From right hand rule, we can see that

$$V_b > V_a$$

and $$V_e > V_f$$

Now, either by applying Kirchhoff's laws or applying principle of superposition (discussed in the chapter of current electricity) we can find current through 5.0 Ω resistor. We will here use the superposition principle. You solve it by using Kirchhoff's laws.

Sol. In the figures $R = 5.0\,\Omega$, $r_1 = 10\,\Omega$, $r_2 = 15\,\Omega$, $e_1 = 4 \times 10^{-3}$ V and $e_2 = 2 \times 10^{-3}$ V.

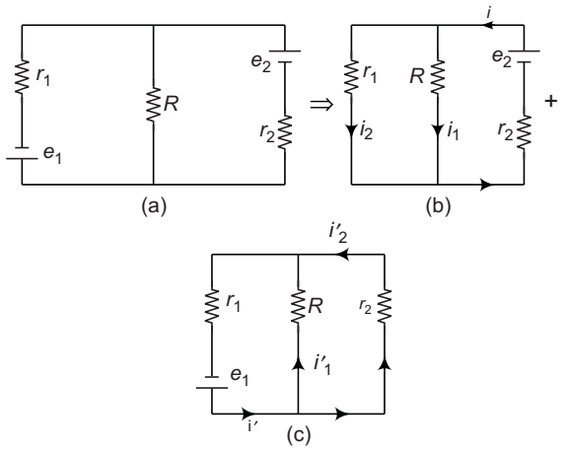


Fig. 22.19

Refer figure (b) Net resistance of the circuit

$$= r_2 + \frac{Rr_1}{R + r_1} = 15 + \frac{10 \times 5}{10 + 5} = \frac{55}{3}\ \Omega$$

$\therefore$ Current $(i) = \dfrac{e_2}{\text{Net resistance}}$

$$= \frac{2 \times 10^{-3}}{55/3} = \frac{6}{55} \times 10^{-3}\ \text{A}$$

$\therefore$ Current through R,

$$i_1 = \left(\frac{r_1}{R + r_1}\right) i$$

$$= \left(\frac{10}{10 + 5}\right)\left(\frac{6}{55} \times 10^{-3}\right)\text{A}$$

$$= \frac{4}{55} \times 10^{-3}\ \text{A}$$

$$= \frac{4}{55}\ \text{mA}$$

Refer figure (c) Net resistance of the circuit

$$= r_1 + \frac{Rr_2}{R + r_2}$$

$$= 10 + \frac{5 \times 15}{5 + 15} = \frac{55}{4}\ \Omega$$

$\therefore$ Current, $i' = \dfrac{e_1}{\text{Net resistance}}$

$$= \frac{4 \times 10^{-3}}{55/4}$$

$$= \frac{16}{55} \times 10^{-3}\ \text{A}$$

$\therefore$ Current through R, $i_1' = \left(\dfrac{r_2}{R + r_2}\right) i'$

$$= \left(\frac{15}{15 + 5}\right)\left(\frac{16}{55}\right) \times 10^{-3}\ \text{A}$$

$$= \frac{12}{55}\ \text{mA}$$

From superposition principle, net current through 5.0 Ω resistor is

$$i_1' - i_1 = \frac{8}{55}\ \text{mA from } d \text{ to } c.$$

22.6 Self-inductance and Inductors

Consider a single isolated circuit. When a current is present in the circuit, it sets up a magnetic field that causes a magnetic flux through the same circuit. This flux changes as the current in the circuit is changed. According to Faraday's law any change in flux in a circuit produces an induced emf in it. Such an emf is called a **self induced emf**. The name is so called, because the source of this induced emf is the change of current in the same circuit.

According to Lenz's law, the self-induced emf always opposes the change in the current that caused the emf and so tends to make it more difficult for variations in current to occur. We will here like to define a term **self-inductance** of a circuit which is of great importance in our proceeding discussions. It can be defined in following two ways :

First Definition Suppose a circuit includes a coil with N turns of wire. It carries a current i. The total flux $(N\phi_B)$ linked with the coil is directly proportional to the current (i) in the coil, i.e.

$$N\phi_B \propto i$$

When the proportionality sign is removed a constant L comes in picture, which depends on the dimensions and number of turns in the coil. This constant is called self-inductance. Thus,

$$N\phi_B = Li \quad \text{or} \quad L = \frac{N\phi_B}{i}$$

From here, we can define self-inductance (L) of any circuit as, **the total flux per unit current**. The SI unit of self inductance is henry (H).

Second Definition If a current i is passed in a circuit and it is changed with a rate di/dt, the induced emf e produced in the circuit is directly proportional to the rate of change of current. Thus,

$$e \propto \frac{di}{dt}$$

When the proportionality constant is removed the same constant L again comes here.

Hence, $$e = -L\frac{di}{dt}$$

The negative sign here is a reflection of Lenz's law. It says that the self-induced emf in a circuit opposes any change in the current in that circuit. From the above equation,

$$L = \left|\frac{-e}{di/dt}\right|$$

This equation states that, **the self-inductance of a circuit is the magnitude of self-induced emf per unit rate of change of current**.

A circuit or part of a circuit, that is designed to have a particular inductance is called an **inductor**. The usual symbol for an inductor is,



Fig. 22.20

Thus, an inductor is a circuit element which opposes the change in current through it. It may be a circular coil, solenoid, etc.

Significance of Self-inductance and Inductor

Like capacitors and resistors, inductors are among the circuit elements of modern electronics. Their purpose is to oppose any variations in the current through the circuit. In a DC circuit an inductor helps to maintain a steady state current despite fluctuations in the applied emf. In an AC circuit, an inductor tends to suppress variations of the current that are more rapid than desired. An inductor plays a dormant role in a circuit so far as current is constant. It becomes active when current changes in the circuit.

Every inductor has some self-inductance which depends on the size, shape and the number of turns, etc. For N turns close together, it is always proportional to N^2. It also depends on the magnetic properties of the material enclosed by the circuit. When the current passing through it is changed an emf of magnitude $L\, di/dt$ is induced across it. Later in this article, we will explore the method of finding the self-inductance of an inductor.

Potential Difference across an Inductor

We can find the direction of self-induced emf across an inductor from Lenz's law.

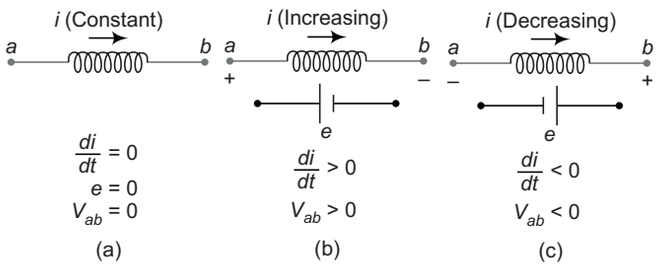


Fig. 22.21

The induced emf is produced whenever there is a change in the current in the inductor. This emf always acts to oppose this change. Figure shows three cases. Assume that the inductor has negligible resistance, so the PD, $V_{ab} = V_a - V_b$ between the inductor terminals a and b is equal in magnitude to the self-induced emf.

Refer figure (a) The current is constant and there is no self-induced emf. Hence, $V_{ab} = 0$.

Refer figure (b) The current is increasing, so $\frac{di}{dt}$ is positive. The induced emf e must oppose the increasing current, so it must be in the sense from b to a, a becomes the higher potential terminal and V_{ab} is positive. The direction of the emf is analogous to a battery with a as its positive terminal.

Refer figure (c) The current is decreasing and $\frac{di}{dt}$ is negative. The self-induced emf e opposes this decrease and V_{ab} is negative. This is analogous to a battery with b, as its positive terminal.

In each case, we can write the PD, V_{ab} as

$$V_{ab} = -e = L\frac{di}{dt}$$

The circuit behaviour of an inductor is quite different from that of a resistor. While a resistor opposes the current i, an inductor opposes the change (di/dt) in the current.

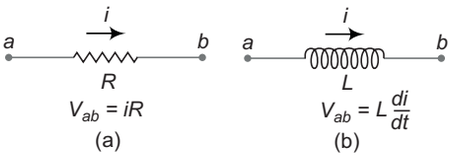


Fig. 22.22

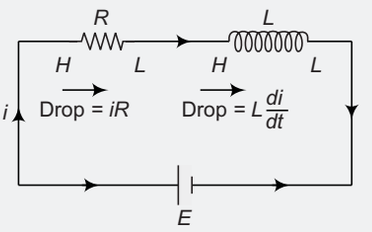
Extra Knowledge Points

- **Kirchhoff's second law with an inductor**

  In Kirchhoff's second law (Loop rule) when we go through an inductor in the same direction as the assumed current, we encounter a voltage drop equal to $L\, di/dt$, where di/dt is to be substituted with sign.

  For example in the loop shown in figure, Kirchhoff's second law gives the equation.

$$E - iR - L\frac{di}{dt} = 0$$



Example 22.7 *The inductor shown in figure has inductance* $0.54\ H$ *and carries a current in the direction shown that is decreasing at a uniform rate* $\frac{di}{dt} = -0.03\ A/s$.

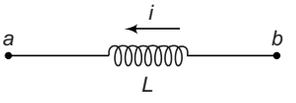


Fig. 22.23

(a) Find the self-induced emf.
(b) Which end of the inductor a or b is at a higher potential?

Sol. (a) Self-induced emf,

$$e = -L\frac{di}{dt} = (-0.54)\,(-0.03)\ \text{V}$$
$$= 1.62\times 10^{-2}\ \text{V}$$

(b) $V_{ba} = L\frac{di}{dt} = -1.62\times 10^{-2}$ V

Since, $V_{ba}\ (= V_b - V_a)$ is negative. It implies that $V_a > V_b$ or *a* is at higher potential.

Example 22.8 *In the circuit diagram shown in figure, $R = 10\,\Omega$, $L = 5H$, $E = 20\,V$, $i = 2\,A$. This current is decreasing at a rate of $-1.0\,A/s$. Find V_{ab} at this instant.*

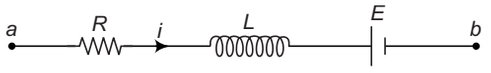


Fig. 22.24

Sol. PD across inductor,

$$V_L = L\frac{di}{dt} = (5)\,(-1.0) = -5\ \text{V}$$

Now, $V_a - iR - V_L - E = V_b$

$$\therefore\quad V_{ab} = V_a - V_b = E + iR + V_L$$
$$= 20 + (2)(10) - 5 = 35\ \text{V}$$

As the current is decreasing the inductor can be replaced by a source of emf $e = \left|L\cdot\frac{di}{dt}\right| = 5$ V in such a manner that this emf supports the decreasing current, or it sends the current in the circuit in the same direction as the existing current. So, positive terminal of this source is towards *b*. Thus, the given circuit can be drawn as:

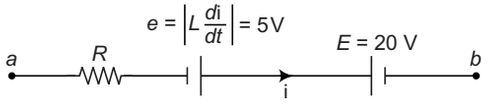


Fig. 22.25

Now, we can find V_{ab}.

Method of Finding Self-inductance of a Circuit

We use the equation, $L = N\phi_B / i$ to calculate the inductance of given circuit.

A good approach for calculating, the self-inductance of a circuit consists of the following steps :

(a) Assume that there is a current *i* flowing through the circuit (we can call the circuit as inductor).
(b) Determine the magnetic field **B** produced by the current.
(c) Obtain the magnetic flux ϕ_B.
(d) With the flux known, the self-inductance can be found from $L = N\phi_B / i$.

To demonstrate this procedure we now calculate the self-inductance of two inductors.

Inductance of a Solenoid

Let us find the inductance of a uniformly wound solenoid having N turns and length l. Assume that l is much longer than the radius of the windings and that the core of the solenoid is air.

We can assume that the interior magnetic field due to a current i is uniform and given by equation

$$B = \mu_0 ni = \mu_0\left(\frac{N}{l}\right)i$$

where, $n = \frac{N}{l}$ is the number of turns per unit length.

The magnetic flux through each turn is,

$$\phi_B = BS = \mu_0\frac{NS}{l}i$$

Here, S is the cross-sectional area of the solenoid. Now,

$$L = \frac{N\phi_B}{i} = \frac{N}{i}\left(\frac{\mu_0 NSi}{l}\right) = \frac{\mu_0 N^2 S}{l}$$

$$\therefore\quad L = \frac{\mu_0 N^2 S}{l}$$

This result shows that L depends on dimensions (S, l) and is proportional to the square of the number of turns.

$$L \propto N^2$$

Because $N = nl$, we can also express the result in the form,

$$L = \mu_0\frac{(nl)^2}{l}S = \mu_0 n^2 Sl = \mu_0 n^2 V$$

or $$L = \mu_0 n^2 V$$

Here, $V = Sl$ is the volume of the solenoid.

Energy Stored in an Inductor

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field.

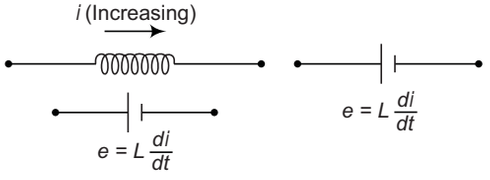


Fig. 22.26

An increasing current in an inductor causes an emf between its terminals.

The work done per unit time is power.

$$P = \frac{dW}{dt} = -ei = -Li\frac{di}{dt}$$

From $dW = -dU$ or $\frac{dW}{dt} = -\frac{dU}{dt}$

we have, $\frac{dU}{dt} = Li\frac{di}{dt}$ or $dU = Li\,di$

The total energy U supplied while the current increases from zero to a final value i is,

$$U = L\int_0^i i\,di = \frac{1}{2}Li^2 \quad \Rightarrow \quad \therefore \quad U = \frac{1}{2}Li^2$$

Example 22.9 *(a) Calculate the inductance of an air core solenoid containing* 300 *turns, if the length of the solenoid is* 25.0 *cm and its cross-sectional area is* 4.00 cm^2.

(b) Calculate the self induced emf in the solenoid, if the current through it is decreasing at the rate of 50.0 *A/s.*

Sol. (a) The inductance of a solenoid is given by,

$$L = \frac{\mu_0 N^2 S}{l}$$

Substituting the values, we have

$$L = \frac{(4\pi \times 10^{-7})(300)^2\ (4.00 \times 10^{-4})}{(25.0 \times 10^{-2})}\text{H}$$

$$= 1.81 \times 10^{-4}\ \text{H}$$

(b) $e = -L\frac{di}{dt}$

Here, $\frac{di}{dt} = -50.0$ A/s

$\therefore$ $e = -(1.81 \times 10^{-4})(-50.0)$

$= 9.05 \times 10^{-3}$ V

or $e = 9.05$ mV

Example 22.10 *What inductance would be needed to store* 1.0 *kWh of energy in a coil carrying a* 200 *A current?* $(1\,kWh = 3.6 \times 10^6\ J)$

Sol. We have, i = 200 A

and U = 1 kWh = 3.6×10^6 J

$\therefore$ $L = \frac{2U}{i^2}$ $\left(U = \frac{1}{2}Li^2\right)$

$= \frac{2(3.6 \times 10^6)}{(200)^2} = 180$ H

22.7 Mutual Inductance

We have already considered in the previous chapters, the magnetic interaction between two wires carrying steady currents. The current in one wire causes a magnetic field, which exerts a force on the current in the second wire.

An additional interaction arises between two circuits, when there is a changing current in one of the circuits.

Consider two neighbouring coils of wire as shown in figure. A current flowing in coil 1 produces magnetic field and hence, a magnetic flux through coil 2. If the current in coil 1 changes, the flux through coil 2 changes as well. According to Faraday's law, this induces an emf in coil 2. In this way, a change in the current in one circuit can induce a current in a second circuit. This phenomenon is known as **mutual induction**. Like the self-inductance (L), two circuits has mutual inductance (M).

It also have two definitions as under:

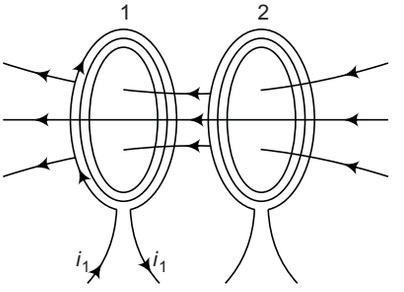


Fig. 22.27

First definition Suppose, the circuit 1 has a current i_1 flowing in it. Then, total flux $N_2\phi_{B_2}$ linked with circuit 2 is proportional to the current in 1. Thus,

$$N_2\,\phi_{B_2} \propto i_1$$

or $$N_2\phi_{B_2} = Mi_1$$

Here, the proportionality constant M is known as the mutual inductance M of the two circuits.

Thus, $$M = \frac{N_2\phi_{B_2}}{i_1}$$

From this expression M can be defined as the **total flux $N_2\phi_{B_2}$ linked with circuit 2 per unit current in circuit 1.**

Second definition If we change the current in circuit 1 at a rate di_1/dt, an induced emf e_2 is developed in circuit 1, which is proportional to the rate di_1/dt. Thus,

$$e_2 \propto di_1/dt \quad \text{or} \quad e_2 = -Mdi_1/dt$$

Here, the proportionality constant is again M. Negative sign indicates that e_2 is in such a direction that it opposes any change in the current in circuit 1. From the above equation,

$$M = \left|\frac{-e_2}{di_1/dt}\right|$$

This equation states that, **the mutual inductance of two circuits is the magnitude of induced emf e_2 per unit rate of change of current di_1/dt.**

Note down the following points regarding the mutual inductance :

1. The SI unit of mutual inductance is **henry** (H).
2. M depends upon closeness of the two circuits, their orientations and sizes and the number of turns, etc.

3. **Reciprocity theorem** $M_{21} = M_{12} = M$

$$e_2 = -M(di_1/dt) \quad \text{and} \quad e_1 = -M(di_2/dt)$$

$$M_{12} = \frac{N_2\phi_{B_2}}{i_1} \quad \text{and} \quad M_{21} = \frac{N_1\phi_{B_1}}{i_2}$$

4. A good approach for calculating the mutual inductance of two circuits consists of the following steps :
   (a) Assume anyone of the circuits as primary (first) and the other as secondary (second).
   (b) Suppose a current i_1 flows through the primary circuit.
   (c) Determine the magnetic field **B** produced by the current i_1.
   (d) Obtain the magnetic flux ϕ_{B_2}.
   (e) With the flux known, the mutual inductance can be found from,

$$M = \frac{N_2\,\phi_{B_2}}{i_1}$$

Mutual Inductance of a Solenoid Surrounded by a Coil

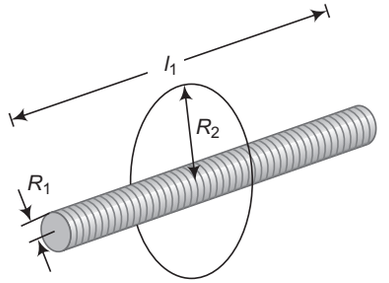


Fig. 22.28

Figure shows a coil of N_2 turns and radius R_2 surrounding a long solenoid of length l_1, radius R_1 and number of turns N_1.

To calculate M between them, let us assume a current i_1 in solenoid.

There is no magnetic field outside the solenoid and the field inside has magnitude,

$$B = \mu_0\left(\frac{N_1}{l_1}\right)i_1$$

and is directed parallel to the solenoid's axis. The magnetic flux ϕ_{B_2} through the surrounding coil is, therefore,

$$\phi_{B_2} = B\,(\pi R_1^2) = \frac{\mu_0 N_1 i_1}{l_1}\pi R_1^2$$

Now, $$M = \frac{N_2\phi_{B_2}}{i_1} = \left(\frac{N_2}{i_1}\right)\left(\frac{\mu_0 N_1 i_1}{l_1}\right)\pi R_1^2$$

$$= \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1} \quad \therefore \quad M = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}$$

Notice that M is independent of the radius R_2 of the surrounding coil. This is because solenoid's magnetic field is confined to its interior. In principle, we can also calculate M by finding the magnetic flux through the solenoid produced by the current in the surrounding coil. This approach is much more difficult, because ϕ_{B_1} is so complicated. However, since $M_{12} = M_{21}$, we do know the result of this calculation.

Coefficient of Coupling of the Two Circuits

Let us now calculate mutual inductance between two circuits in terms of the self-inductance of each circuit alone.

Let us first consider a case when the total flux associated with one coil links with the other, i.e. a case of maximum flux linkage. Consider two coils placed adjacent to each other,

$$M_{12} = \frac{N_2\phi_{B_2}}{i_1} \quad \text{and} \quad M_{21} = \frac{N_1\phi_{B_1}}{i_2}$$

Similarly, $$L_1 = \frac{N_1\phi_{B_1}}{i_1} \quad \text{and} \quad L_2 = \frac{N_2\phi_{B_2}}{i_2}$$

If all the flux of coil 2 links coil 1 and *vice-versa*, then

$$\phi_{B_2} = \phi_{B_1}$$

Since, $$M_{12} = M_{21} = M,$$

Hence, we have $$M_{12}M_{21} = M^2$$

$$= \frac{N_1 N_2 \phi_{B_1}\phi_{B_2}}{i_1 i_2} = L_1 L_2$$

$$\therefore \quad M_{\max} = \sqrt{L_1 L_2}$$

This is the maximum possible value of M as the total flux associated with one coil links with the other. In general only a fraction K_2 (<1) of ϕ_{B_2} passes through the coil 1. Similarly, a fraction K_1 (<1) of ϕ_{B_1} passes through coil 2.

Hence, $$\phi_{B_1} = K_2\phi_{B_2}$$

and $$\phi_{B_2} = K_1\phi_{B_1}$$

$$\therefore \quad M_{21}M_{12} = M^2$$

$$= \frac{N_1 N_2 K_1 K_2 \phi_{B_1}\phi_{B_2}}{i_1 i_2}$$

$$= K_1 K_2 L_1 L_2$$

or $$M = K\sqrt{L_1 L_2} \qquad (K \le 1)$$

Here, $K = \sqrt{K_1 K_2}$ is a number depending on the geometry of the coils and their relative closeness having value between 0 and 1.

Combination of Inductances

In Series If several inductances are in series so that there are no interactions through mutual inductance.

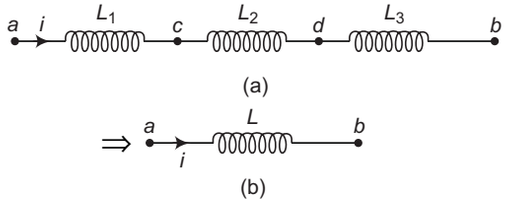


Fig. 22.29

Refer figure (a)

$$V_a - V_c = L_1 \frac{di}{dt}$$

$$V_c - V_d = L_2 \frac{di}{dt} \quad \text{and} \quad V_d - V_b = L_3 \frac{di}{dt}$$

Adding all these equations, we have

$$V_a - V_b = (L_1 + L_2 + L_3)\frac{di}{dt} \qquad \ldots(i)$$

Refer figure (b)

$$V_a - V_b = L\frac{di}{dt} \qquad \ldots(ii)$$

Here, L = equivalent inductance.

From Eqs. (i) and (ii), we have

$$L = L_1 + L_2 + L_3$$

In parallel

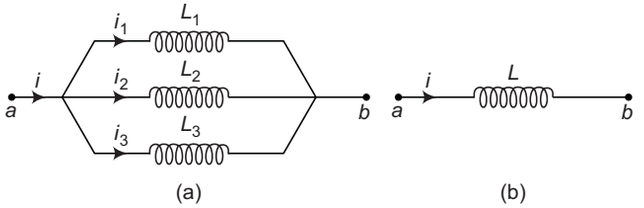


Fig. 22.30

Refer figure (a)

$$i = i_1 + i_2 + i_3$$

or $$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt}$$

or $$\frac{di}{dt} = \frac{V_a - V_b}{L_1} + \frac{V_a - V_b}{L_2} + \frac{V_a - V_b}{L_3} \qquad \ldots(i)$$

Refer figure (b)

$$\frac{di}{dt} = \frac{V_a - V_b}{L} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

If the flux from one inductance links another, mutual inductance term becomes important. This mutual interaction may increase or decrease, the flux due to the self induction. The equivalent inductance of the pair of coils in series is,

$$L = L_1 + L_2 \pm 2M$$

Example 22.11 *A straight solenoid has* 50 *turns per cm in primary and* 200 *turns in the secondary. The area of cross- section of the solenoid is* 4 cm^2. *Calculate the mutual inductance.*

Sol. The magnetic field at any point inside the straight solenoid of primary with n_1 turns per unit length carrying a current i_1 is given by the relation,

$$B = \mu_0 n_1 i_1$$

The magnetic flux through the secondary of N_2 turns each of area S is given as,

$$N_2\phi_2 = N_2(BS) = \mu_0 n_1 N_2 i_1 S$$

$$\therefore \quad M = \frac{N_2\phi_2}{i_1} = \mu_0 n_1 N_2 S$$

Substituting the values,

$$M = (4\pi \times 10^{-7})\left(\frac{50}{10^{-2}}\right)(200)\,(4 \times 10^{-4})$$

$$= 5.0 \times 10^{-4} \text{ H}$$

22.8 Growth and Decay of Current in an *L-R* Circuit : Growth of Current

Let us consider a circuit consisting of a battery of emf E, a coil of self-inductance L and a resistor R. The resistor R may be a separate circuit element or it may be the resistance of the inductor windings. By closing switch S_1, we connect R and L in series with constant emf E. Let i be the current at some time t after switch S_1 is closed and di/dt be its rate of change at that time. Applying Kirchhoff's loop rule strating at the negative terminal and proceeding counter-clockwise around the loop.

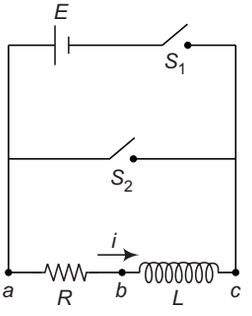


Fig. 22.31

$$E - V_{ab} - V_{bc} = 0 \quad \text{or} \quad E - iR - L\frac{di}{dt} = 0$$

$$\therefore \quad \int_0^i \frac{di}{E - iR} = \int_0^t \frac{dt}{L}$$

or $$i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

By letting $E/R = i_0$ and $L/R = \tau_L$, the above expression reduces to

$$i = i_0\,(1 - e^{-t/\tau_L})$$

Here, $i_0 = E/R$ is the current at $t = \infty$. It is also called the steady state current or the maximum current in the circuit.

And $\tau_L = \dfrac{L}{R}$ is called **time constant** of the *L-R* circuit. At a time equal to one time constant, the current has risen to $(1 - 1/e)$ or about 63% of its final value i_0.

The *i-t* graph is as shown in figure.

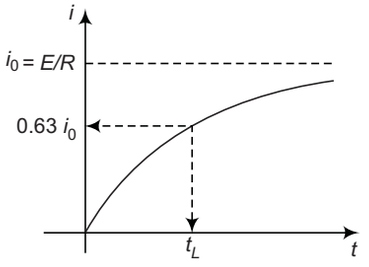


Fig. 22.32

Note that the final current i_0 does not depend on the inductance *L*, it is the same as it would be if the resistance *R* alone were connected to the source with emf *E*.

Let us have an insight into the behaviour of an *L-R* circuit from energy considerations.

The instantaneous rate at which the source delivers energy to the circuit ($P = Ei$) is equal to the instantaneous rate at which energy is dissipated in the resistor ($= i^2R$) plus the rate at which energy is stored in the inductor ($= iV_{bc} = Li\, di/dt$) or $\dfrac{d}{dt}\left(\dfrac{1}{2}Li^2\right) = Li\dfrac{di}{dt}$. Thus,

$$Ei = i^2R + Li\frac{di}{dt}$$

Decay of Current

Now, suppose switch S_1 in the circuit shown in figure has been closed for a long time and that the current has reached its steady state value i_0. Resetting our stopwatch to redefine the initial time we close switch S_2 at time $t = 0$ and at the same time we should open the switch S_1 to by pass the battery. The current through *L* and *R* does not instantaneously go to zero but decays exponentially. To apply Kirchhoff's loop rule and to find current in the circuit at time *t*, let us draw the circuit once more.

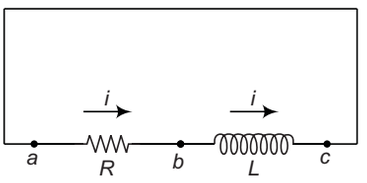


Fig. 22.33

Applying loop rule we have,

$$(V_a - V_b) + (V_b - V_c) = 0 \qquad (\text{as } V_a = V_c)$$

or $$iR + L\left(\frac{di}{dt}\right) = 0$$

Note Do not bother about the sign of $\dfrac{di}{dt}$.

$$\therefore \quad \frac{di}{i} = -\frac{R}{L}dt$$

$$\therefore \quad \int_{i_0}^{i}\frac{di}{i} = -\frac{R}{L}\int_0^t dt$$

$$\therefore \quad i = i_0 e^{-t/\tau_L}$$

where, $\tau_L = \dfrac{L}{R}$, is the time for current to decrease to $1/e$ or about 37% of its original value. The *i-t* graph is as shown in figure.

The energy that is needed to maintain the current during this decay is provided by energy stored in the magnetic field. Thus, the rate at which energy is dissipated in the resistor = rate at which the stored energy decreases in magnetic field of inductor

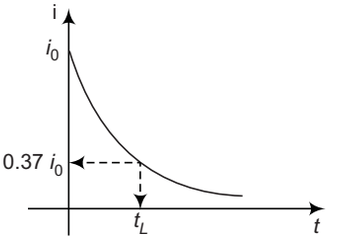


Fig. 22.34

or $$i^2R = -\frac{dU}{dt} = -\frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\left(-\frac{di}{dt}\right)$$

or $$i^2R = Li\left(\frac{-di}{dt}\right)$$

Example 22.12 *A coil of resistance* 20 Ω *and inductance* 0.5 *H is switched to DC* 200 *V supply. Calculate the rate of increase of current*
(a) at the instant of closing the switch and
(b) after one time constant.
(c) Find the steady state current in the circuit.

Sol. (a) This is the case of growth of current in an *L-R* circuit. Hence, current at time *t* is given by,

$$i = i_0(1 - e^{-t/\tau_L})$$

Rate of increase of current,

$$\frac{di}{dt} = \frac{i_0}{\tau_L}e^{-t/\tau_L}$$

At $t = 0$, $$\frac{di}{dt} = \frac{i_0}{\tau_L} = \frac{E/R}{L/R} = \frac{E}{L}$$

Substituting the value, we have

$$\frac{di}{dt} = \frac{200}{0.5} = 400 \text{ A/s}$$

(b) At $t = \tau_L$,

$$\frac{di}{dt} = (400)e^{-1}$$
$$= (0.37)\ (400)$$
$$= 148\ \text{A/s}$$

(c) The steady state current in the circuit,

$$i_0 = \frac{E}{R} = \frac{200}{20} = 10\ \text{A}$$

22.9 Oscillations in *L-C* Circuit

If a charged capacitor C is short circuited through an inductor L, the charge and current in the circuit start oscillating simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat. We also assume an idealised situation in which energy is not radiated away from the circuit. With these idealisations-zero resistance and no radiation, the oscillations in the circuit persist indefinitely and the energy is transferred from the capacitor's electric field to the inductor's magnetic field and back. The total energy associated with the circuit is constant. This is analogous to the transfer of energy in an oscillating mechanical system from potential energy to kinetic energy and back, with constant total energy. Later we will see that this analogy goes much further.

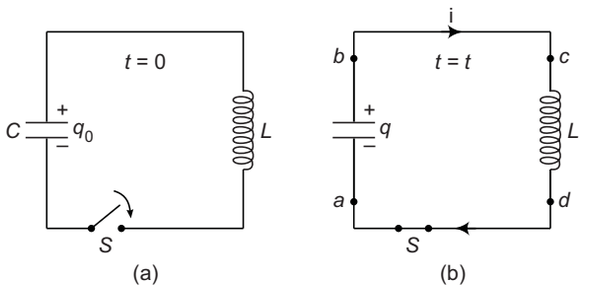


Fig. 22.35

Let us now derive an equation for the oscillations in an *L-C* circuit.

Refer figure (a) A capacitor is charged to a PD

$$V_0 = q_0 C$$

Here, q_0 is the maximum charge on the capacitor. At time $t = 0$, it is connected to an inductor through a switch S. At time $t = 0$, switch S is closed.

Refer figure (b) When the switch is closed, the capacitor starts discharging. Let at time t charge on the capacitor is $q\ (< q_0)$ and since, it is further decreasing there is a current i in the circuit in the direction shown in figure. Later we will see that, as the charge is oscillating there may be a situation when q will be increasing, but in that case direction of current is also reversed and the equation remains unchanged.

The potential difference across capacitor = potential difference across inductor, or

$$V_b - V_a = V_c - V_d$$

$$\therefore \quad \frac{q}{C} = L\left(\frac{di}{dt}\right) \qquad \ldots(\text{i})$$

Now, as the charge is decreasing,

$$i = \left(\frac{-dq}{dt}\right)$$

or

$$\frac{di}{dt} = -\frac{d^2q}{dt^2}$$

Substituting in Eq. (i), we get

$$\frac{q}{C} = -L\left(\frac{d^2q}{dt^2}\right)$$

or

$$\frac{d^2q}{dt^2} = -\left(\frac{1}{LC}\right)q \qquad \ldots(\text{ii})$$

This is the standard equation of simple harmonic motion $\left(\frac{d^2x}{dt^2} = -\omega^2 x\right)$.

Here

$$\omega = \frac{1}{\sqrt{LC}} \qquad \ldots(\text{iii})$$

The general solution of Eq. (ii), is

$$q = q_0 \cos(\omega t \pm \phi)$$

For example in our case $\phi = 0$

as $q = q_0$ at $t = 0$.

Hence, $q = q_0 \cos \omega t \qquad \ldots(\text{iv})$

Thus, we can say that charge in the circuit oscillates simple harmonically with angular frequency given by Eq. (iii). Thus,

$$\omega = \frac{1}{\sqrt{LC}},$$

$$f = \frac{\omega}{2\pi}$$

$$= \frac{1}{2\pi\sqrt{LC}}$$

and

$$T = \frac{1}{f}$$

$$= 2\pi\sqrt{LC}$$

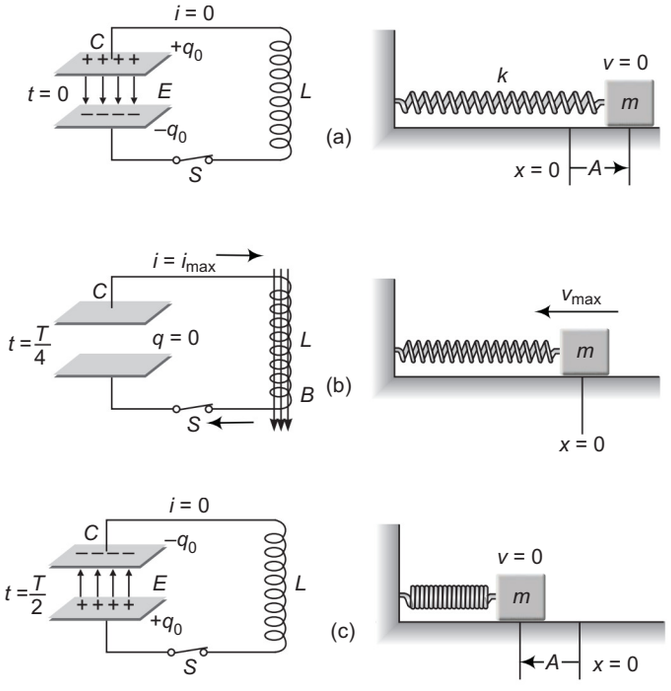
The oscillations of the *L-C* circuit are an electromagnetic analog to the mechanical oscillations of a block-spring system.

Table below shows a comparison of oscillations of a mass spring system and an *L-C* circuit.

Table 22.1

S. No.	Mass spring system	Inductor-capacitor circuit
1.	Displacement (x)	Charge (q)
2.	Velocity (v)	Current (i)
3.	Acceleration (a)	Rate of change of current $\left(\frac{di}{dt}\right)$
4.	$\frac{d^2x}{dt^2} = -\omega^2 x$, where, $\omega = \sqrt{\frac{k}{m}}$	$\frac{d^2q}{dt^2} = -\omega^2 q$, where, $\omega = \frac{1}{\sqrt{LC}}$
5.	$x = A\sin(\omega t \pm \phi)$ or $x = A\cos(\omega t \pm \phi)$	$q = q_0 \sin(\omega t \pm \phi)$ or $q = q_0 \cos(\omega t \pm \phi)$
6.	$v = \frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$	$i = \frac{dq}{dt} = \omega\sqrt{q_0^2 - q^2}$
7.	$a = \frac{dv}{dt} = -\omega^2 x$	Rate of change of current $= \frac{di}{dt} = -\omega^2 q$
8.	Kinetic energy $= \frac{1}{2}mv^2$	Magnetic energy $= \frac{1}{2}Li^2$
9.	Potential energy $= \frac{1}{2}kx^2$	Potential energy $= \frac{1}{2}\frac{q^2}{C}$
10.	$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant} = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$	$\frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C} = \text{constant} = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}Li_{max}^2$
11.	$\lvert v_{max}\rvert = A\omega$	$i_{max} = q_0\omega$
12.	$\lvert a_{max}\rvert = \omega^2 A$	$\left\lvert\left(\frac{di}{dt}\right)_{max}\right\rvert = \omega^2 q_0$
13.	$\frac{1}{k}$	C
14.	m	L

A graphical description of the energy transfer between the inductor and the capacitor in an L-C circuit is shown in figure. The right side of the figure shows the analogous energy transfer in the oscillating block-spring system.



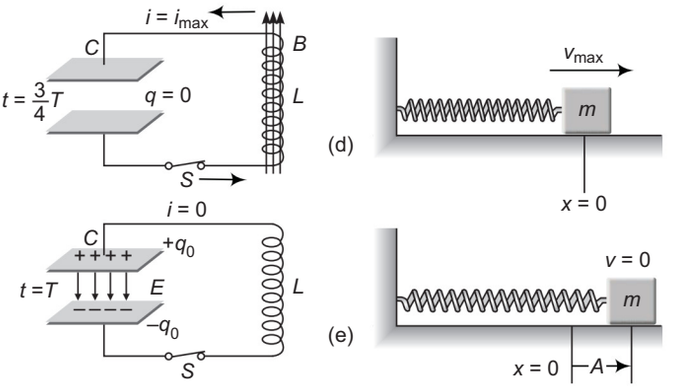


Fig. 22.36

- In L-C oscillations, q, i and $\frac{di}{dt}$ all oscillate simple harmonically with same angular frequency ω. But the phase difference between q and i or between i and $\frac{di}{dt}$ is $\frac{\pi}{2}$, while that between i and $\frac{di}{dt}$ is π. Their amplitudes are q_0, $q_0\omega$ and $\omega^2 q_0$ respectively. So, now suppose

$$q = q_0 \cos\omega t,$$

then $i = \frac{dq}{dt} = -q_0\omega\sin\omega t$ and $\frac{di}{dt} = -q_0\omega^2\cos\omega t$

Similarly,potential energy across capacitor (U_C) and across inductor (U_L) also oscillate with double, the frequency 2ω but not simple harmonically. The different graphs are as shown in Fig. 22.37.

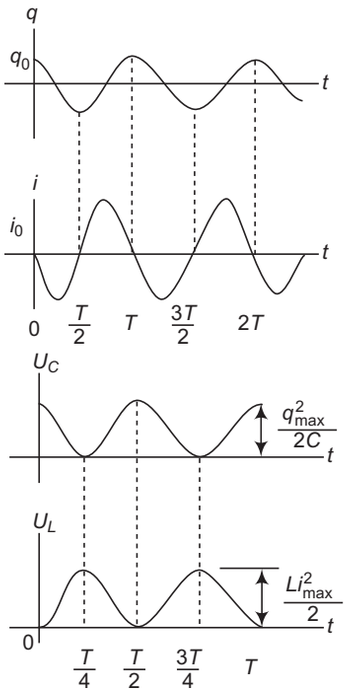


Fig. 22.37

Example 22.13 *A capacitor of capacitance* $25\,\mu F$ *is charged to* $300\,V$. *It is then connected across a* $10\,mH$ *inductor. The resistance in the circuit is negligible.*

(a) Find the frequency of oscillation of the circuit.

(b) Find the potential difference across capacitor and magnitude of circuit current $1.2\,ms$ *after the inductor and capacitor are connected.*

(c) Find the magnetic energy and electric energy at $t = 0$ *and* $t = 1.2\,ms$.

Sol. (a) The frequency of oscillation of the circuit is,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Substituting the given values we have,

$$f = \frac{1}{2\pi\sqrt{(10\times10^{-3})(25\times10^{-6})}} = 318.3 \text{ Hz}$$

(b) Charge across the capacitor at time t will be,

$$q = q_0\cos\omega t$$

and $$i = -q_0\omega\sin\omega t$$

Here, $$q_0 = CV_0 = (25\times10^{-6})(300)$$

$$= 7.5\times10^{-3} \text{ C}$$

Now, charge in the capacitor after $t = 1.2\times10^{-3}$ s is,

$$q = (7.5\times10^{-3})\cos(2\pi\times318.3)(1.2\times10^{-3}) \text{ C}$$

$$= -5.53\times10^{-3} \text{ C}$$

$\therefore$ PD across capacitor,

$$V = \frac{|q|}{C} = \frac{5.53\times10^{-3}}{25\times10^{-6}} = 221.2 \text{ V}$$

The magnitude of current in the circuit at

$t = 1.2\times10^{-3}$ s is,

$$|i| = q_0\omega\sin\omega t$$

$$= (7.5\times10^{-3})(2p)(318.3)$$

$$\sin(2\pi\times318.3)(1.2\times10^{-3}) \text{ A}$$

$$= 10.13 \text{ A}$$

(c) **At** $t = 0$

Current in the circuit is zero. Hence, $U_L = 0$

Charge in the capacitor is maximum.

Hence, $U_C = \frac{1}{2}\frac{q_0^2}{C}$ or $U_C = \frac{1}{2}\times\frac{(7.5\times10^{-3})^2}{(25\times10^{-6})}$

$$= 1.125 \text{ J}$$

$\therefore$ Total energy $E = U_L + U_C = 1.125$ J

At $t = 1.2$ ms

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(10\times10^{-3})(10.13)^2 = 0.513 \text{ J}$$

$\therefore$ $U_C = E - U_L = 1.125 - 0.513 = 0.612$ J

Otherwise U_C can be calculated as,

$$U_C = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\times\frac{(5.53\times10^{-3})^2}{(25\times10^{-6})}$$

$$= 0.612 \text{ J}$$

22.10 Some Applications of Electromagnetic Induction

Eddy Currents

When a changing magnetic flux is applied to a piece of conducting material, circulating currents called eddy currents are induced in the material. These eddy currents often have large magnitudes and heat up the conductor.

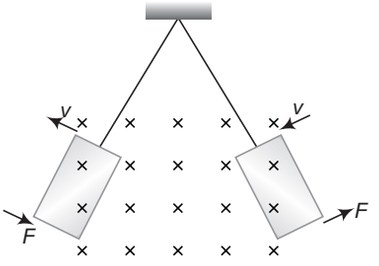


Fig. 22.38

When a metal plate is allowed to swing through a strong magnetic field, then in entering or leaving the field the eddy currents are set up in the plate which opposes the motion as shown in figure. The kinetic energy dissipates in the form of heat. The slowing down of the plate is called the **electromagnetic damping.**

The electromagnetic damping is used to damp the oscillations of a galvanometer coil or chemical balance and in braking electric trains. Otherwise the eddy currents are often undesirable. To reduce the eddy currents some slots are cut into moving metallic parts of machinary. These slots intercept the conducting paths and decreases, the magnitudes of the induced currents.

Back Emf of Motors

An electric motor converts electrical energy into mechanical energy and is based on the fact that a current carrying coil in a uniform magnetic field experiences a torque. As the coil rotates in the magnetic field, the flux linked with the rotating coil will change and hence, an emf called back emf is produced in the coil.

When the motor is first turned on, the coil is at rest and so there is no back emf. The start up current can be quite large. To reduce start up current a resistance called **starter** is put in series with the motor for a short period when the motor is started. As the rotation rate increases the back emf increases and hence, the current reduces.

Electric Generator or Dynamo

A dynamo converts mechanical energy (rotational kinetic energy) into electrical energy. It consists of a coil rotating in a magnetic field. Due to rotation of the coil magnetic flux linked with it changes, so an emf is induced in the coil.

Suppose at time $t = 0$, plane of coil is perpendicular to the magnetic field.

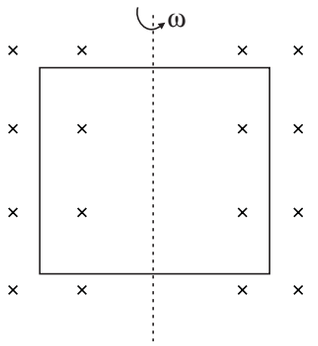


Fig. 22.39

The flux linked with it at any time t will be given by

$$\phi = NBA \cos \omega t$$

(N = number of turns in the coil)

$$\therefore \quad e = -\frac{d\phi}{dt} = NBA\omega \sin \omega t$$

or $$e = e_0 \sin \omega t$$

where, $$e_0 = NBA\,\omega$$

Transformer

It is a device which is either used to increase or decrease, the voltage in AC circuits through mutual induction.

A transformer consists of two coils wound on the same core.

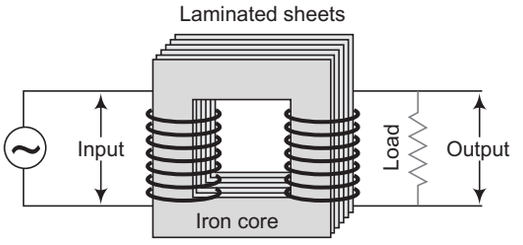


Fig. 22.40

The coil connected to input is called primary while the other connected to output is called secondary coil. An alterning current passing through the primary creates a continuously changing flux through the core. This changing flux induces an alternating emf in the secondary.

As magnetic lines of force are closed curves, the flux per turn of primary must be equal to flux per turn of the secondary. Therefore,

$$\frac{\phi_P}{N_P} = \frac{\phi_S}{N_S}$$

or $$\frac{1}{N_P} \cdot \frac{d\phi_P}{dt} = \frac{1}{N_S} \cdot \frac{d\phi_S}{dt} \qquad \left(\text{as } e \propto \frac{d\phi}{dt}\right)$$

$$\therefore \quad \frac{e_S}{e_P} = \frac{N_S}{N_P}$$

In an ideal transformer, there is no loss of power. Hence,

$$e_i = \text{constant} \quad \Rightarrow \quad \therefore \quad \frac{e_S}{e_P} = \frac{N_S}{N_P} = \frac{i_P}{i_S}$$

Regarding a transformer following are few important points :

(i) In **step-up** transformer $N_S > N_P$. It increases voltage and reduces current.

(ii) In **step-down** transformer, $N_P > N_S$. It increases current and reduces voltage.

(iii) It works only on AC.

(iv) A transformer cannot increase (or decrease) voltage and current simultaneously. As, ei = constant.

(v) Some power is always lost due to eddy currents, hysteresis, etc.

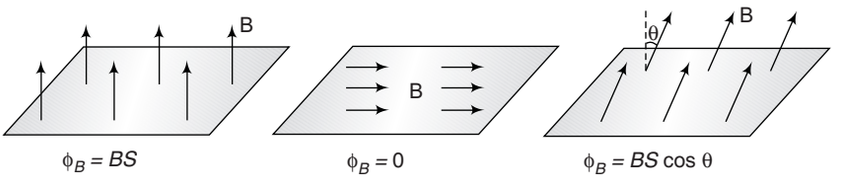
Chapter Summary with Formulae

(i) **Magnetic Flux**

(a) $d\phi_B = \mathbf{B}\cdot\mathbf{dS} = BdS\cos\theta$ (b) $\phi_B = \int d\phi_B = \int \mathbf{B}\cdot\mathbf{dS} = \int BdS\cos\theta$

(c) In uniform magnetic field $\phi_B = BS\cos\theta$

(d)



(ii) **Faraday's Law**

(a) Induced emf, $e = -\dfrac{d\phi_B}{dt}$ or $-\dfrac{d}{dt}(N\phi_B)$ or $-N\cdot\dfrac{d\phi_B}{dt}$

(b) Average induced emf, $e = -\dfrac{\Delta\phi_B}{\Delta t} = -N\dfrac{\Delta\phi_B}{\Delta t}$

(c) Induced current in closed loop, $i = \dfrac{e}{R} = \dfrac{-d\phi_B/dt}{R}$

(d) Flow of charge due to induction, $\Delta q = i(\Delta t) = \dfrac{-\Delta\phi_B}{R}$

(e) In the above equations negative sign indicates that induced effects always oppose the cause, due to which they are produced (Lenz's law). Therefore, their directions (e.g. direction of induced current) are accordingly.

(iii) **Motional emf**

(a) $e = Bvl$ (b) $e = \dfrac{B\omega l^2}{2}$

(iv) **Self Inductance**

(a) $L = \left(\dfrac{-e}{di/dt}\right) = \dfrac{N\phi_B}{i}$ (b) $e = -L(di/dt)$

(c) The self inductance of a long solenoid, the core of which consists of a magnetic material of permeability μ_r is given by

$$L = \mu_r\mu_0 n^2 Al$$

where, n = number of turns per unit length.

(d) Self inductance of a circular coil is $L = \dfrac{\mu_0\pi N^2 R}{2}$

(e) Two or more than two inductors (without considering their mutual interaction)

$L = L_1 + L_2$ (in series)

$\dfrac{1}{L} = \dfrac{1}{L_1} + \dfrac{1}{L_2}$ (in parallel)

(v) **Mutual Inductance**

(a) $M = \left(\dfrac{-e_1}{di_2/dt}\right)$ or $\left(\dfrac{-e_2}{di_1/dt}\right)$ (b) $M = \left(\dfrac{N_1\phi_1}{i_2}\right)$ or $\left(\dfrac{N_2\phi_2}{i_1}\right)$

(c) $M_{12} = M_{21}$

(d) Resultant inductance of two inductors after considering their mutual interaction is given by :

$$L = L_1 + L_2 \pm 2M$$

(vi) **AC Generator**

(a) In an AC generator, mechanical energy is converted into electrical energy by virtue of electromagnetic induction.

(b) If a coil of N turns and area A is rotated in a uniform magnetic field with constant angular velocity ω, then the emf produced is

$$e = NBA\omega\sin\omega t$$

Here, we have assumed that at time $t = 0$, the plane of the coil is perpendicular to the field or angle between $\mathbf{B}$ and $\mathbf{S}$ (of the coil) is 0°.

(c) Maximum value of emf is $e_0 = NBA\omega$

(d) $\omega = 2\pi f$

Additional Examples

Example 1. *Show that* $\frac{L}{R}$ *has the dimensions of time.*

Sol. $U_B = \frac{1}{2}Li^2$ = magnetic energy

$H = i^2Rt$ = heat dissipated across a resistance.

From the above two equations, we can see that, U_B and H have the same dimensions (of energy)

$$\therefore \quad \left[\frac{1}{2}Li^2\right] = [i^2Rt]$$

or $$\left[\frac{L}{R}\right] = [t] \text{ or } [T]$$ Hence Proved.

Example 2. *Show that* $\sqrt{LC}$ *has dimensions of time.*

Sol. $U_B = \frac{1}{2}Li^2$ = magnetic energy and

$U_C = \frac{1}{2}\frac{q^2}{C}$ = electrostatic energy

From the above two equations we can see that $\frac{1}{2}Li^2$ and $\frac{1}{2}\frac{q^2}{C}$ have the same dimensions.

$$\therefore \quad \left[\frac{1}{2}Li^2\right] = \left[\frac{1}{2}\frac{q^2}{C}\right]$$

or $$[LC] = [q^2/i^2]$$

But $$[q/i] = [t]$$

$$\therefore \quad [LC] = [t^2]$$

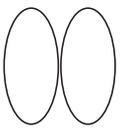
or $$[\sqrt{LC}] = [t] = [T]$$ Hence Proved.

Example 3. *Two circular loops are placed with their centres separated by a fixed distance. How would you orient the loops to have*

(a) the largest mutual inductance?

(b) the smallest mutual inductance?

Sol. (a) For largest mutual inductance, the coils should be placed face to face as shown in figure.



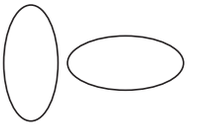
By doing this maximum flux will pass through a coil when current is passed through its adjacent coil. For same current i_1

Now, $$M = \frac{N_2\phi_2}{i_1}$$

or $$M \propto N_2\phi_2$$

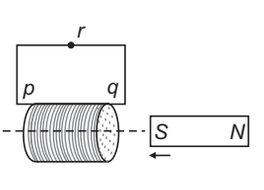
More the total flux passing through a coil (due to a current in it's adjacent coil), the more is the mutual inductance.

(b) By placing the coils as shown in figure, least flux will pass through a coil due to the current of its adjacent coil.

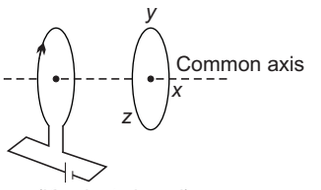


Therefore, mutual inductance will be minimum.

Example 4. *Predict the direction of induced current in the situations described by the following figures.*



(a)



(Key just closed)

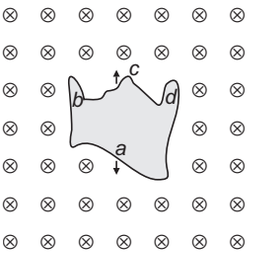
(b)

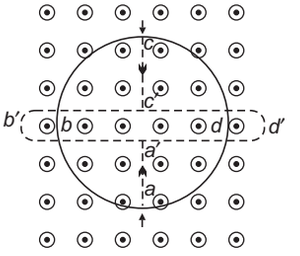
Sol. **Figure (a)** South pole of magnet is approaching towards the solenoid. Therefore, due to induced current right hand side (near side) of the solenoid should also become South pole. Because according to Lenz's law induced effects should repel the magnet (as it is approaching).

To become South pole, current from right hand side should be clockwise (↻). Clockwise on solenoid means path of induced current should be *qrpq*.

Figure (b) Just after closing, the switch current in the shown direction will increase. To oppose its effect induced current in adjacent coil should be in opposite direction. Hence, direction of induced current should be *xyzx*.

Example 5. *Use Lenz's law to determine the direction of induced current in the situations described by figure.*





(a) A wire of irregular shape turning into a circular shape.

(b) A circular loop being deformed into a narrow straight wire.

Sol. (a) For given perimeter area of circle is maximum. Hence, from the given loop magnetic field (or magnetic flux) is increasing in ⊗ direction. Therefore, induced current should produce magnetic field. For producing magnetic field current from our side should be anti-clockwise or in the direction of *adcba*.

(b) In this case area of loop is decreasing. Hence magnetic field (or magnetic flux) is decreasing in direction. Therefore, induced current should produce magnetic field. For producing magnetic field current from our side should be anti-clockwise or in the direction of *a' d' c' b' a'*.

Example 6. *A coil consists of* 200 *turns of wire having a total resistance of* 2.0 Ω. *Each turn is a square of side* 18 *cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from* 0 *to* 0.5 *T in* 0.80 *s, what is the magnitude of induced emf and current in the coil while the field is changing?*

Sol. From the Faraday's law,

Induced emf $$|e| = \frac{Nd\phi}{dt} = (N.S)\frac{\Delta B}{\Delta t}$$

$$= \frac{(200)\,(18\times 10^{-2})^2\,(0.5-0)}{0.8}$$

$$= 4.05 \text{ V}$$

Induced current $$i = \frac{|e|}{R} = \frac{4.05}{2} \approx 2.0 \text{ A}$$

Example 7. *The magnetic flux threading a metal ring varieswith time t according to* $\phi_B = 3(at^3 - bt^2)$ *T-m*^{2} *with* $a = 2.00\,s^{-3}$ *and* $b = 6.00\,s^{-2}$. *The resistance of the ring is* 3.0 Ω. *Determine the maximum current induced in the ring during the interval from* $t = 0$ *to* $t = 2.0\,s$.

Sol. Given, $\phi_B = 3(at^3 - bt^2)$

$$\therefore \quad |e| = \left|\frac{d\phi_B}{dt}\right| = 9at^2 - 6bt$$

$\therefore$ Induced current

$$i = \frac{|e|}{R} = \frac{9at^2 - 6bt}{3}$$

$$= 3at^2 - 2bt$$

For current to be maximum,

$$\frac{di}{dt} = 0$$

$$\therefore \quad 6at - 2b = 0 \quad \text{or} \quad t = \frac{b}{3a}$$

i.e. at $t = \frac{b}{3a}$, current is maximum

This maximum current is,

$$i_{\max} = 3a\left(\frac{b}{3a}\right)^2 - 2b\left(\frac{b}{3a}\right)$$

$$= \frac{b^2}{3a} - \frac{2b^2}{3a}$$

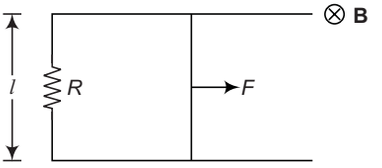
Magnitude of this maximum current will be

$$i_{\max} = \frac{b^2}{3a}$$

Substituting the given values of a and b, we have

$$i_{\max} = \frac{(6)^2}{3(2)} = 6.0 \text{ A}$$

Example 8. *Figure shows the top view of a rod that can slide without friction. The resistor is* 6.0 Ω *and a* 2.5 *T magnetic field is directed perpendicularly downward into the paper. Let* $l = 1.20\,m$.



(a) Calculate the force F required to move the rod to the right at a constant speed of 2.0 m/s.

(b) At what rate is energy delivered to the resistor?

(c) Show that this rate is equal to the rate of work done by the applied force.

Sol. The motional emf in the rod, $e = Bvl$

or $e = (2.5)\,(2.0)\,(1.2)V = 0.6$ V

The current in the circuit $i = \frac{e}{R} = \frac{6.0}{6.0} = 1.0$ A

(a) The magnitude of force F required will be equal to the magnetic force acting on the rod, which opposes the motion.

$\therefore \quad F = F_m = ilB$

or $F = (1.0)\,(1.2)\,(2.5)$ N

$= 3$ N

(b) Rate by which energy is delivered to the resistor is,

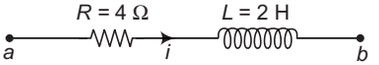
$$P_1 = i^2R = (1)^2\,(6.0) = 6 \text{ W}$$

(c) The rate by which work is done by the applied force is,

$$P_2 = F \cdot v = (3)\,(2.0) = 6 \text{ W}$$

$$P_1 = P_2$$

Example 9. *In the figure shown* $i = 10e^{-4t}$ *A. Find* V_L *and* V_{ab}.



Sol. $V_L = +L\frac{di}{dt} = (2)\frac{d}{dt}(10e^{-4t})$

$$= -80\,e^{-4t}$$

Further, $V_a - iR - L\frac{di}{dt} = V_b$

$$\therefore \quad V_a - V_b = iR + L\frac{di}{dt}$$

or $V_{ab} = (10e^{-4t})\,(4) - 80e^{-4t}$

$$= -40\,e^{-4t}$$

Example 10. *(a) What is the magnetic flux through one turn of a solenoid of self inductance* 8.0×10^{-5} *H, when a current of* 3.0 *A flows through it? Assume that the solenoid has* 1000 *turns and is wound from wire of diameter* 1.0 *mm.*
(b) What is the cross-sectional area of the solenoid?

Sol. Given,

$L = 8.0 \times 10^{-5}$ H, $i = 3.0$ A and $N = 1000$

(a) From the relation, $L = \dfrac{N\phi}{i}$

The flux linked with one turn,

$$\phi = \frac{Li}{N} = \frac{(8.0\times 10^{-5})\,(3.0)}{1000} = 2.4 \times 10^{-7}\ \text{Wb}$$

(b) This $\phi = BS = (\mu_0 ni)\,(S)$

Here, n = number of turns per unit length

$$= \frac{N}{l} = \frac{N}{N.d} = \frac{1}{d}$$

$$\therefore \qquad \phi = \frac{\mu_0 iS}{d}$$

or
$$S = \frac{\phi d}{\mu_0 i}$$

$$= \frac{(2.4\times10^{-7})\ (1.0\times10^{-3})}{(4\pi\times10^{-7})\ (3.0)}$$

$$= 6.37\times10^{-5}\ \text{m}^2$$

Example 11. *A* 10 *H inductor carries a current of* 20 *A. How much ice at* 0°C *could be melted by the energy stored in the magnetic field of the inductor? Latent heat of ice is* $22.6\times10^3\ J/kg$.

Sol. Energy stored is $\frac{1}{2}Li^2$. This energy is completely used in melting the ice. Hence, $\frac{1}{2}Li^2 = mL_f$

Here, L_f = latent heat of fusion

Hence, mass of ice melted, $m = \dfrac{Li^2}{2L_f}$

Substituting the values we have,

$$m = \frac{(10)\ (20)^2}{2\ (2.26\times10^3)}$$

$$= 0.88\ \text{kg}$$

Example 12. *Two solenoids A and B spaced close to each other and sharing the same cylindrical axis have* 400 *and* 700 *turns respectively. A current of* 3.50 *A in coil A produced an average flux of* $300\mu T\text{-}m^2$ *through each turn of A and a flux of* $90.0\mu T\text{-}m^2$ *through each turn of B.*
(a) Calculate the mutual inductance of the two solenoids.
(b) What is the self inductance of A?
(c) What emf is induced in B when the current in A increases at the rate of 0.5 *A/s?*

Sol. (a) $M = \dfrac{N_B\phi_B}{i_A} = \dfrac{(700)\ (90\times10^{-6})}{3.5} = 1.8\times10^{-2}$ H

(b) $L_A = \dfrac{N_A\phi_A}{i_A} = \dfrac{(400)\ (300\times10^{-6})}{3.5} = 3.43\times10^{-2}$ H

(c) $e_B = M\left(\dfrac{di_A}{dt}\right) = (1.8\times10^{-2})\ (0.5) = 9.0\times10^{-3}$ V

Example 13. *A sensitive electronic device of resistance* 175 Ω *is to be connected to a source of emf by a switch. The device is designed to operate with a current of* 36 *mA, but to avoid damage to the device, the current can rise to no more than* 4.9 *mA in the first* 58 μs *after the switch is closed. To protect the device it is connected in series with an inductor.*
(a) What emf must the source have?
(b) What inductance is required?
(c) What is the time constant?

Sol. (a) Given $R = 175\ \Omega$ and peak value current

$$i_0 = 36\times10^{-3}\ \text{A}$$

$\therefore$ The applied voltage

$$V = i_0 R = (175)\ (36\times10^{-3})\ \text{V} = 6.3\ \text{V}$$

(b) From the relation,

$$i = i_0\ (1 - e^{-t/\tau_L})$$

we have, $(4.9) = (36)[1 - e^{-t/\tau_L}]$

or
$$e^{-t/\tau_L} = 1 - \frac{4.9}{36} = 0.864$$

$$\therefore \qquad \frac{t}{\tau_L} = -\ln(0.864)$$

$$= 0.146$$

or
$$\frac{t}{L/R} = 0.146$$

$$\therefore \qquad \frac{Rt}{L} = 0.146$$

or
$$L = \frac{Rt}{0.146} = \frac{(175)\ (58\times10^{-6})}{0.146}$$

$$= 7.0\times10^{-2}\ \text{H}$$

(c) Time constant of the circuit,

$$\tau_L = \frac{L}{R} = \frac{7.0\times10^{-2}}{175}$$

$$= 4.0\times10^{-4}$$

Example 14. *For the circuit shown in figure,* $E = 50\,V$, $R_1 = 10\,\Omega$, $R_2 = 20\,\Omega$, $R_3 = 30\,\Omega$ *and* $L = 2.0\,mH$. *Find the current through* R_1 *and* R_2.
(a) Immediately after switch S is closed.
(b) A long time after S is closed.
(c) Immediately after S is reopened.
(d) A long time after S is reopened.

Sol. (a) Resistance offered by inductor immediately after switch is closed will be infinite. Therefore, current through R_3 will be zero and,

$$\text{current through } R_1 = \text{current through } R_2 = \frac{E}{R_1 + R_2}$$

$$= \frac{50}{10 + 20} = \frac{5}{3}\text{ A}$$

(b) After long time of closing the switch, resistance offered by inductor will be zero.

In that case R_2 and R_3 are in parallel, and the resultant of these two is then in series with R_1. Hence,

$$R_{net} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$= 10 + \frac{(20)(30)}{20 + 30}$$

$$= 22\ \Omega$$

Current through the battery (or through R_1)

$$= \frac{E}{R_{net}} = \frac{50}{22}\text{ A}$$

This current will distribute in R_2 and R_3 in inverse ratio of resistance. Hence,

$$\text{Current through } R_2 = \left(\frac{50}{22}\right)\left(\frac{R_3}{R_2 + R_3}\right)$$

$$= \left(\frac{50}{22}\right)\left(\frac{30}{30 + 20}\right) = \frac{15}{11}\text{ A}$$

(c) Immediately after switch is reopened, the current through R_1 will become zero.

But current through R_2 will be equal to the steady state current through R_3. Which is equal to,

$$\left(\frac{50}{22} - \frac{15}{11}\right)\text{A} = 0.91\text{ A}$$

(d) A long after S is reopened, current through all resistors will be zero.

Example 15. *In an L-C circuit $L = 3.3\,H$ and $C = 840\ pF$. At $t = 0$ charge on the capacitor is $105\,\mu C$ and maximum. Compute the following quantities at $t = 2.0\ ms$.*

(a) The energy stored in the capacitor.

(b) The energy stored in the inductor.

(c) The total energy in the circuit.

Sol. Given, $L = 3.3$ H, $C = 840 \times 10^{-12}$ F

and $q_0 = 105 \times 10^{-6}$ C

The angular frequency of L-C oscillations is,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.3 \times 840 \times 10^{-12}}}$$

$$= 1.9 \times 10^4\ \text{rad/s}$$

Charge stored in the capacitor at time t, would be,

$$q = q_0 \cos \omega t$$

(a) At $t = 2 \times 10^{-3}$ s

$$q = (105 \times 10^{-6}) \cos [1.9 \times 10^4]\,[2 \times 10^{-3}]$$

$$= 100.3 \times 10^{-6}\text{ C}$$

$\therefore$ Energy stored in the capacitor

$$U_C = \frac{1}{2}\frac{q^2}{C} = \frac{(100.3 \times 10^{-6})^2}{2 \times 840 \times 10^{-12}}$$

$$= 6.0\text{ J}$$

(b) Total energy in the circuit

$$U = \frac{1}{2}\frac{q_0^2}{C}$$

$$= \frac{(105 \times 10^{-6})^2}{2 \times 840 \times 10^{-12}}$$

$$= 6.56\text{ J}$$

(c) Energy stored in inductor in the given time
= total energy in circuit − energy stored in capacitor
= (6.56 – 6.0) J = 0.56 J

NCERT Selected Questions

Q 1. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

Sol. Here, n = number of turns per unit length of the solenoid

$= 15 \text{ turns cm}^{-1} = 1500 \text{ turns m}^{-1}$

A = area of loop placed inside the solenoid

$= 2.0 \text{ cm}^2 = 2 \times 10^{-4} \text{ m}^2$

$I_1 = 2.0 \text{ A}, I_2 = 4.0 \text{ A}$

$I_2 - I_1 = 4 - 2 = 2 \text{ A}$

$\Delta t = 0.1 \text{ s}$

$\therefore \quad \frac{\Delta I}{\Delta t}$ or $\frac{dI}{dt} = \frac{2}{0.1}$

$= 20 \text{ A/s}$

We know that the magnetic field produced inside the solenoid is given by,

$$B = \mu_0 nI$$

If ϕ be the magnetic flux linked with the loop, then

$$\phi = BA = \mu_0 nIA$$

$\therefore$ Using the relation, $e = -\frac{d\phi}{dt}$, we get

$$e = -\frac{d}{dt}(\phi)$$

$$= -\frac{d}{dt}(\mu_0 nIA) = -\mu_0 nA \cdot \frac{dI}{dt}$$

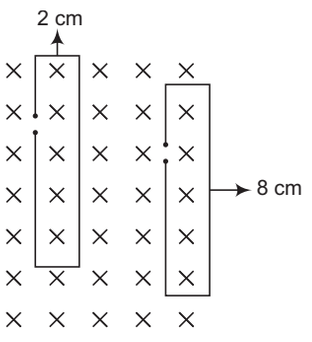
$\therefore$ Magnitude of e is given by

$$e = \mu_0 nA \cdot \frac{dI}{dt}$$

$$= 4\pi \times 10^{-7} \times 1500 \times 2 \times 10^{-4} \times 20$$

$$= 7.54 \times 10^{-6} \text{ V}$$

Q 2. A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s^{-1} in a direction normal to the



(a) longer side,

(b) shorter side of the loop? For how long does the induced voltage last in each case?

Sol. (a) When the velocity is normal to the longer side

Using the relation, $e = Bvl$, we get

$$e = 0.3 \times 8 \times 10^{-2} \times 10^{-2}$$

$$= 2.4 \times 10^{-4} \text{ V}$$

The emf will last in the loop till it does not get out of the magnetic field, i.e. for the time the loop takes to travel a distance equal to the length of the shorter arm.

$$\therefore \quad t = \frac{\text{Length of shorter arm}}{v} = \frac{2 \times 10^{-2}}{10^{-2}} = 2 \text{ s}$$

(b) When the velocity is normal to the shorter arm

$$e = Blv = 0.3 \times 2 \times 10^{-2} \times 10^{-2}$$

$$= 0.6 \times 10^{-4} \text{ V}$$

$$t = \frac{\text{Length of longer arm}}{v} = \frac{8 \times 10^{-2}}{10^{-2}} = 8 \text{ s}$$

Q 3. A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Sol. Using relation,

$$e = \frac{1}{2} B\omega l^2 = \frac{1}{2} \times 0.5 \times 400 \times (1)^2$$

$$= 100 \text{ V}$$

Q 4. A horizontal straight wire 10 m long extending from East to West is falling with a speed of 5.0 ms^{-1}, at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb m^{-2}.

(a) What is the instantaneous value of the emf induced in the wire?

(b) What is the direction of the emf ?

(c) Which end of the wire is at the higher electrical potential?

Sol. (a) Let e be the value of instantaneous induced emf in the wire.

$\therefore$ Using the relation, $e = Blv$, we get

$$e = B_H lv$$

$$= 0.3 \times 10^{-4} \times 10 \times 5$$

$$= 1.5 \times 10^{-3} \text{ V}$$

(b) The induced emf will act in a direction so as to oppose the motion of the falling straight wire, i.e. from West to East.

(c) As the induced emf opposes its cause, i.e. it sets from low to high potential end, thus eastern end is at higher potential as induced emf acts from West to East.

Q 5. Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V is induced, give an estimate of the self inductance of the circuit.

Sol. Using the relation,

$$e = -L\frac{dI}{dt}$$

We get, $L = \dfrac{-e}{\left(\dfrac{dI}{dt}\right)} = \dfrac{-200}{\left(\dfrac{-5}{0.1}\right)} = \dfrac{200\times 0.1}{5} = 4\text{ H}$

Q 6. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A is 0.5 s, what is the change of flux linkage with the other coil?

Sol. If e be the induced emf produced in the other coil, then using the relation,

$$e = -M\frac{dI}{dt}, \text{we get}$$

$$e = -1.5\times\frac{20}{0.5} = -60\text{ V}$$

Also we know that

$$e = -\frac{d\phi}{dt}$$

$$\therefore \quad d\phi = -e\times dt = -(-60)\times 0.5 = 30\text{ Wb}$$

Q 7. A jet plane is travelling towards West at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30°?

Sol. Here, $v = 1800\text{ km/h} = 1800\times\dfrac{5}{18} = 500\text{ ms}^{-1}$ towards West.

δ = angle of dip = 30°

$\therefore$ B_V = vertical component of the earth's magnetic field

$= B\sin\delta = 5\times10^{-4}\times\sin 30°$

$= 5\times10^{-4}\times\dfrac{1}{2} = 2.5\times10^{-4}$ T

B_V is normal to both the wings and the direction of motion. So, if e be the induced emf produced (= voltage difference developed between the ends of the wing), then

$$e = B_V lv$$

$$= 2.5\times10^{-4}\times 25\times 500 \approx 3.1\text{ V}$$

The direction of the wing is immaterial as long as it is horizontal for this answer.

Q 8. Suppose the loop in example 2 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of 0.02 Ts^{-1}. If the cut is joined and the loop has a resistance of 1.5 Ω, how much power is dissipated by the loop as heat? What is the source of this power?

Sol. Here, $A = 8\times 2\text{ cm}^2 = 16\times10^{-4}\text{ m}^2$

where, B_1 = initial value of magnetic field = 0.3 T

$\dfrac{dB}{dt}$ = rate of decrease of the magnetic field

$= 0.02\text{ Ts}^{-1}$

If ϕ be the flux linked with the loop, then

$$\phi = BA$$

Thus the induced emf e produced in the loop is given by,

$$e = \frac{d\phi}{dt} = \frac{d}{dt}(BA)$$

$$= A\frac{dB}{dt}$$

$$= 16\times10^{-4}\times 0.02$$

$$= 3.2\times10^{-5}\text{ V}$$

$\therefore$ The induced current I is given by,

$$I = \frac{e}{R} = \frac{3.2\times10^{-5}}{1.6} = 2\times10^{-5}\text{ A}$$

$$\therefore \quad P = I^2R = (2\times10^{-5})^2\times 1.6$$

$$= 6.4\times10^{-10}\text{ W}$$

The source of this power is the external agency changing the magnetic field with time.

Q 9. An air-cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Sol. Here, l = Length of the solenoid = 30 cm = 30×10^{-2} m

A = Area of cross-section

$= 25\text{ cm}^2 = 25\times10^{-4}\text{ m}^2$

N = Total number of turns = 500

If n be the number of turns per unit length, then

$$n = \frac{N}{l} = \frac{500}{30\times10^{-2}} = \frac{5000}{3}$$

We know that the magnetic field inside the solenoid is given by,

$$B = \mu_0 nI = 4\pi\times10^{-7}\times\frac{5000}{3}\times 2.5$$

$$= 52.36\times10^{-4}\text{ T}$$

If ϕ_1 be the initial magnetic flux linked with the solenoid

Then $\phi_1 = NBA = 500\times 52.36\times10^{-4}\times 25\times10^{-4}$

$$= 65.45\times10^{-4}\text{ Wb}$$

ϕ_2 = final flux = 0

as $I_2 = 0$

$\therefore$ $\Delta\phi$ = change in magnitude flux

$= \phi_2 - \phi_1 = -65.45\times10^{-4}$ Wb

$\therefore$ Average back emf, $E = -\dfrac{\Delta\phi}{\Delta t} = \dfrac{65.45\times10^{-4}}{10^{-3}}$

$= 6.545$ V

Objective Problems

[Level 1]

Induced emf, Induced Current and Induced Flow of Charge

1. In electromagnetic induction, the induced charge in a coil is independent of
(a) change in the flux
(b) time taken to change the flux
(c) resistance in the circuit
(d) None of the above

2. A magnet is brought towards a coil (i) speedly (ii) slowly, then the induced emf/induced charge will be respectively
(a) more in first case/more in first case
(b) more in first case/equal in both cases
(c) less in first case/more in second case
(d) less in first case/equal in both cases

3. In a coil of area 10 cm^2 and 10 turns with a magnetic field directed perpendicular to the plane and is changing at the rate of 10^8 gauss/s. The resistance of the coil is 20 Ω. The current in the coil will be
(a) 5 A (b) 0.5 A
(c) 0.05 A (d) 5×10^8 A

4. A coil having 500 square loops each of side 10 cm is placed normal to a magnetic flux which increases at the rate of 1.0 T/s. The induced emf in volts is
(a) 0.1 (b) 0.5
(c) 1 (d) 5

5. A coil having an area 2 m^2 is placed in a magnetic field which changes from 1 Wb/m^2 to 4 Wb/m^2 in an interval of 2 s. The emf induced in the coil will be
(a) 4 V (b) 3 V
(c) 1.5 V (d) 2 V

6. A coil of 40 Ω resistance, 100 turns and radius 6 mm is connected to ammeter of resistance 160 Ω. Coil is placed perpendicular to the magnetic field. When coil is taken out of the field, 32 μC charge flows through it. The intensity of magnetic field will be
(a) 6.55 T (b) 5.66 T
(c) 2.55 T (d) 0.566 T

7. The magnetic flux linked with a coil, in webers, is given by the equation $\phi = 3t^2 + 4t + 9$. Then, the magnitude of induced emf at $t = 2$ s will be
(a) 2 V (b) 4 V
(c) 8 V (d) 16 V

8. A coil has an area of 0.05 m^2 and it has 800 turns. It is placed perpendicularly in a magnetic field of strength 4×10^{-5} Wb/m^2, it is rotated through 90° in 0.1 s. The average emf induced in the coil is
(a) 0.056 V (b) 0.046 V
(c) 0.026 V (d) 0.016 V

9. In a magnetic field of 0.05 T, area of a coil changes from 101 cm^2 to 100 cm^2 without changing the resistance which is 2 Ω. The amount of charge that flows during this period is
(a) 2.5×10^{-6} C (b) 2×10^{-6} C
(c) 10^{-6} C (d) 8×10^{-6} C

10. If a current of 10 A changes in one second through a coil, and the induced emf is 10 V,then the self-inductance of the coil is
(a) $\frac{2}{5}$ H (b) $\frac{4}{5}$ H
(c) $\frac{5}{4}$ H (d) 1 H

11. The resistance and inductance of series circuit are 5 Ω and 20 H respectively. At the instant of closing the switch, the current is increasing at the rate 4 A/s. The supply voltage is
(a) 20 V (b) 80 V
(c) 120 V (d) 100 V

12. The number of turns in the coil of an AC generator is 5000 and the area of the coil is 0.25 m^2. The coil is rotated at the rate of 100 cycles/s in a magnetic field of 0.2 W/m^2. The peak value of the emf generated is nearly
(a) 786 kV (b) 440 kV
(c) 220 kV (d) 157.1 kV

13. Two coils have a mutual inductance 0.005 H. The current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10$ A and $\omega = 100\pi$ rad/s. The maximum value of emf in the second coil is (in volt)
(a) 2π (b) 5π
(c) π (d) 4π

14. A rectangular coil, rotates about an axis normal to the magnetic field. If E_m is the maximum value of the induced emf, the instantaneous emf, when the plane of the coil makes an angle of 45° with the magnetic field is
(a) $\frac{1}{2} E_m$ (b) $\frac{1}{4} E_m$ (c) $\frac{1}{\sqrt{2}} E_m$ (d) E_m

15. A conducting circular loop is placed in a uniform magnetic field of induction B tesla with its plane normal to the field. Now, the radius of the loop starts shrinking at the rate (dr/dt). Then, the induced emf at the instant when the radius is r, is

(a) $\pi rB(dr/dt)$ (b) $2\pi rB(dr/dt)$
(c) $\pi r^2(dB/dt)$ (d) $(\pi r^2/2)B(dr/dt)$

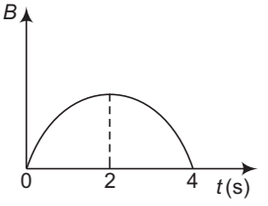
16. A coil having an area A_0 is placed in a magnetic field which changes from B_0 to $4B_0$ in time interval t. The emf induced in the coil will be

(a) $3A_0B_0/t$ (b) $4A_0B_0/t$
(c) $3B_0/A_0t$ (d) $4B_0/A_0t$

17. A physicist works in a laboratory, where the magnetic field is 2 T. She wears a necklace of enclosing area 0.01 m^2 in such a way that the plane of the necklace is normal to the field and is having a resistance $R = 0.01\,\Omega$. Because of power failure, the field decays to 1 T in time 10^{-3} s. Then, what is the total heat produced in her necklace?

(a) 10 J (b) 20 J
(c) 30 J (d) 40 J

18. In a closed loop, which has some inductance but negligible resistance, uniform but time varying magnetic field is applied directed into the plane of the loop. Variation of field with time is shown. Initially current in the loop was zero. Then.



(a) emf induced in the loop is zero at $t = 2$ s
(b) current in the loop will be maximum at $t = 2$ s
(c) direction of emf in the loop will change at $t = 2$ s
(d) None of the above

19. A magnetic field given by $B(t) = 0.2\,t - 0.05t^2$ tesla is directed perpendicular to the plane of a circular coil containing 25 turns of radius 1.8 cm and whose total resistance is 1.5 Ω. The power dissipation at 3 s is approximately

(a) 1.37 μW (b) 7 μW
(c) zero (d) 4 μW

20. A circular ring of diameter 20 cm has a resistance 0.01 Ω. How much charge will flow through the ring, if it is rotated from a position perpendicular to a uniform magnetic field of $B = 2$ T to a position parallel to field?

(a) 4 C (b) 6.28 C
(c) 3.14 C (d) 2.5 C

21. The magnetic flux ϕ (in weber) in a closed circuit of resistance 10 Ω varies with time t (in second) according to equation $\phi = 6t^2 - 5t + 1$. The magnitude of induced current at $t = 0.25$ s is

(a) 1.2 A (b) 0.8 A
(c) 0.6 A (d) 0.2 A

Lenz's Law

22. When the current through a solenoid increases at a constant rate, the induced current

(a) is a constant and is in the direction of the inducing current
(b) is a constant and is opposite to the direction of the inducing current
(c) increase with time and is in the direction of inducing current
(d) increase with time and is opposite to the direction of inducing current

23. The direction of induced emf during electromagnetic induction is given by

(a) Faraday's law (b) Lenz's law
(c) Maxwell's (d) Ampere's law

24. Two different loops are concentric and lie in the same plane. The current in the outer loop is clockwise and increasing with time. The induced current in the inner loop then, is

(a) clockwise
(b) zero
(c) counter clockwise
(d) in a direction that depends on the ratio of the loop radii

25. Lenz's law is due to conservation of

(a) charge (b) momentum
(c) energy (d) current

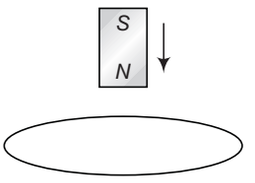
26. The circular loops of equal radii are placed coaxially at some separation. The first is cut and a battery is inserted in between to drive a current in it. The current changes slightly because of the variation in resistance with temperature. During this period, the two loops

(a) attract each other
(b) repel each other
(c) do not exert any force on each other
(d) attract or repel each other depending on the sense of the current

27. A small, conducting circular loop is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid. If the current in the solenoid is varied, the current induced in the loop is

(a) clockwise
(b) anti-clockwise
(c) zero
(d) clockwise or anti-clockwise depending on whether the resistance is increased or decreased

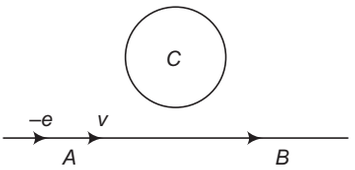
28. The North pole of a magnet is brought near a metallic ring as shown in the figure. The direction of induced current in the ring will be



(a) anti-clockwise
(b) clockwise
(c) first anti-clockwise and then clockwise
(d) first clockwise and then anticlockwise

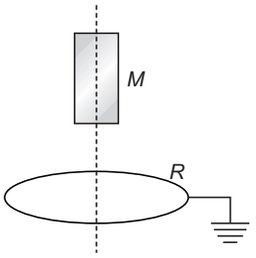
29. When a magnet with its magnetic moment along the axis of a circular coil and directed towards the coil is withdrawn away from the coil, parallel to itself, the current in the coil, as seen by the withdrawing magnet is
(a) zero
(b) clockwise
(c) anti-clockwise
(d) None of the above

30. An electron moves along the line AB which lies in the same plane as a circular loop of conducting wire as shown in figure. What will be the direction of the current induced (if any) in the loop?



(a) No current will be induced
(b) The current will be clockwise
(c) The current will be anti-clockwise
(d) The current will change direction as the electron passes through

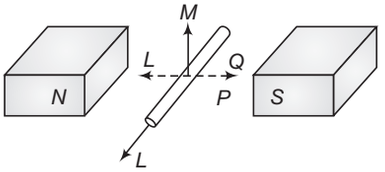
31. A small magnet M is allowed to fall through a fixed horizontal conducting ring R. Let g be the acceleration due to gravity. The acceleration of M will be



(a) $< g$ when it is above R and moving towards R
(b) $> g$ when it is above R and moving towards R
(c) $< g$ when it is below R and moving away from R
(d) $> g$ when it is below R and moving away from R

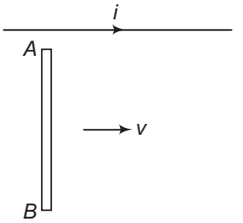
Motional emf

32. An electric potential difference will be induced between ends of the conductor shown in the diagram, when the conductor moves in the direction



(a) P
(b) Q
(c) L
(d) M

33. The current carrying wire and the rod AB are in the same plane. The rod moves parallel to the wire with a velocity v. Which one of the following statements is true about induced emf in the rod?



(a) End A will be at lower potential w.r.t. B
(b) A and B will be at the same potential
(c) There will be no induced emf in the rod
(d) Potential at A will be higher than that at B

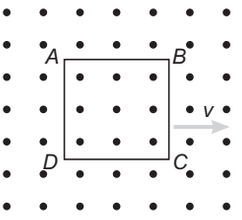
34. A long horizontal metallic rod with length along the East-West direction is falling under gravity. The potential difference between its two ends will
(a) be zero (b) be constant
(c) increase with time (d) decrease with time

35. A wheel with ten metallic spokes each 0.50 m long is rotated with a speed of 120 rev/min in a plane normal to the earth's magnetic field at the plane. If the magnitude of the field is 0.4 G, the induced emf between the axle and the rim of the wheel is equal to
(a) 1.256×10^{-3} V (b) 6.28×10^{-4} V
(c) 1.256×10^{-4} V (d) 6.28×10^{-5} V

36. A metal rod of length 2 m is rotating with an angular velocity of 100 rad/s in a plane perpendicular to a uniform magnetic field of 0.3 T. The potential difference between the ends of the rod is
(a) 30 V (b) 40 V (c) 60 V (d) 600 V

37. An aeroplane in which the distance between the tips of the wings is 50 m is flying horizontally from East to West with the speed of 360 km/h over a place, where the vertical component of the earth's magnetic field is 2.0×10^{-4} Wb-m^{-2}. The potential difference between the tips of the wings would be
(a) 0.1 V (b) 0.01 V (c) 0.2 V (d) 1.0 V

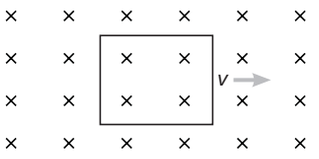
38. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. An electric field is induced



(a) in AD, but not in BC (b) in BC, but not in AD
(c) Neither in AD nor in BC (d) in both AD and BC

39. Consider the following statements.
(A) An emf can be induced by moving a conductor in a magnetic field.
(B) An emf can be induced by changing the magnetic field.
(a) Both A and B are true (b) A is true but B is false
(c) B is true but A is false (d) Both A and B are false

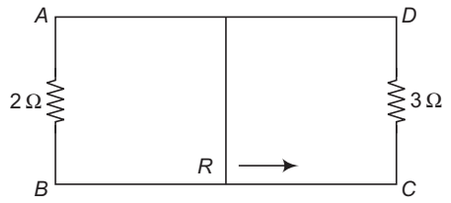
40. A conducting square loop of side l and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A uniform and constant magnetic field B exists along the perpendicular to the plane of the loop as shown in figure. The current induced in the loop is



(a) $\frac{Blv}{R}$ clockwise (b) $\frac{Blv}{R}$ anti-clockwise
(c) $\frac{2Blv}{R}$ anti-clockwise (d) zero

41. A conducting rod is moved with a constant velocity v in a magnetic field. A potential difference appears across the two ends
(a) if $\mathbf{v} \,||\, \mathbf{I}$ (b) if $\mathbf{v} \,||\, \mathbf{B}$
(c) if $\mathbf{I} \,||\, \mathbf{B}$ (d) None of these

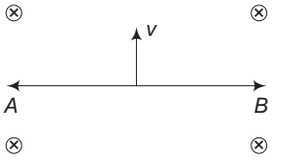
42. A rectangular loop with a sliding connector of length 10 cm is situated in uniform magnetic field perpendicular to plane of loop. The magnetic induction is 0.1 T and resistance of connector (R) is 1Ω . The sides AB and CD have resistances 2Ω and 3 Ω respectively. Find the current in the connector during its motion with constant velocity 1 m/s.



(a) $\frac{1}{242}$ A (b) $\frac{1}{220}$ A
(c) $\frac{1}{55}$ A (d) $\frac{1}{440}$ A

43. Two rails of a railway track, insulated from each other and the ground, are connected to a millivoltmeter. What is the reading of the millivoltmeter when a train travels at a speed of 20 m/s along the track? Given that the vertical component of the earth's magnetic field is 0.2×10^{-4} Wb / m^2 and the rails are separated by 1 m
(a) 4 mV (b) 0.4 mV (c) 80 mV (d) 10 mV

44. A rod AB moves with a uniform velocity v in a uniform magnetic field as shown in figure.



(a) The rod becomes electrically charged
(b) The end A becomes positively charged
(c) The end B becomes positively charged
(d) None of the above

Self and Mutual Inductance

45. A 50 mH coil carries a current of 2 A. The energy stored in joules is
(a) 1 (b) 0.1
(c) 0.05 (d) 0.5

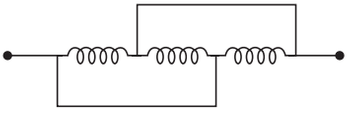
46. Two coils of self inductance L_1 and L_2 are placed closer to each other so that total flux in one coil is completely linked with other. If M is mutual inductance between them, then
(a) $M = L_1 L_2$ (b) $M = L_1 / L_2$
(c) $M = \sqrt{L_1 L_2}$ (d) $M = (L_1 L_2)^2$

47. Two pure inductors each of self inductance L are connected in parallel but are well separated from each other. The total inductance is
(a) $2L$ (b) L
(c) $\frac{L}{2}$ (d) $\frac{L}{4}$

48. The self inductance of solenoid of length L, area of cross-section A and having N turns is
(a) $\frac{\mu_0 N^2 A}{L}$ (b) $\frac{\mu_0 NA}{L}$
(c) $\mu_0 N^2 LA$ (d) $\mu_0 NLA$

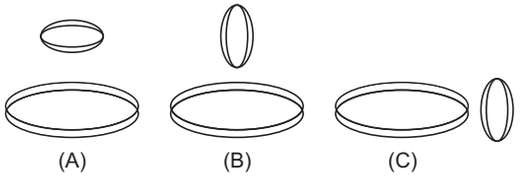
49. The self inductance of a coil is L. Keeping the length and area same, the number of turns in the coil is increased to four times. The self inductance of the coil will now be
(a) $\frac{1}{4}L$ (b) L
(c) $4L$ (d) $16L$

50. All the three inductors have inductance 3.0 H. The equivalent inductance of the circuit is



(a) 1 H (b) 2 H
(c) 3 H (d) 9 H

51. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be



(a) maximum in situation (A)
(b) maximum in situation (B)
(c) maximum in situation (C)
(d) the same in all situations

52. During a current change from 2 A to 4 A in 0.5 s, 8V of emf is developed in a coil. The coefficient of self induction is
(a) 1 H (b) 2 H (c) 4 H (d) 8 H

53. Two inductances connected in parallel are equivalent to a single inductance of 1.5 H, and when connected in series are equivalent to a single inductance of 8 H. The difference in their inductance is
(a) 3 H (b) 7.5 H
(c) 2 H (d) 4 H

54. Two coils X and Y are placed in a circuit such that when the current changes by 2 A in coil X the magnetic flux changes by 0.4 Wb in Y. The value of mutual inductance of the coils is
(a) 0.2 H (b) 5 H
(c) 0.8 H (d) 4 H

55. The mutual inductance between a primary and secondary circuits is 0.5 H. The resistance of the primary and the secondary circuits are 20 Ω and 5 Ω respectively. To generate a current of 0.4 A in the secondary, current in the primary must be changed at the rate of
(a) 4.0 A/s (b) 1.6 A/s
(c) 16.0 A/s (d) 8.0 A/s

56. The coefficient of mutual induction between two circuits is equal to the, emf produced in one circuit, when the current in the second circuit is
(a) kept steady at 1 A
(b) cut-off at 1 A level
(c) changed at the rate of 1 A/s
(d) changed from 1 A/s to 2 A/s

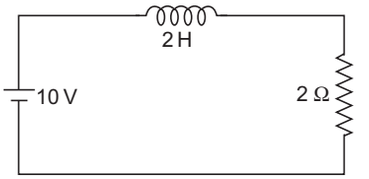
L-R Circuit

57. An ideal coil of 10 H is joined in series with a resistance of 5Ω and a battery of 5 V. 2 s after joining, the current flowing in ampere in the circuit will be
(a) e^{-1} (b) $(1-e^{-1})$
(c) $(1-e)$ (d) e

58. An L-R circuit has a cell of emf E, which is switched on at time $t = 0$. The current in the circuit after a long time will be
(a) zero (b) $\frac{E}{R}$
(c) $\frac{E}{L}$ (d) $\frac{E}{\sqrt{L^2 + R^2}}$

59. An inductance L and a rsistance R are first connected to a battery. After some time the battery is disconnected but L and R remain connected in a closed circuit. Then, the current reduces to 37% of its initial value in
(a) RL s (b) $\frac{R}{L}$ s
(c) $\frac{L}{R}$ s (d) $\frac{1}{LR}$ s

60. In the figure magnetic energy stored in the coil is



(a) zero (b) infinite
(c) 25 J (d) None of these

61. A coil of inductance 300 mH and resistance 2 Ω is conencted to a source of voltage 2 V. The current reaches half of its steady state value in
(a) 0.15 s (b) 0.3 s
(c) 0.05 s (d) 0.1 s

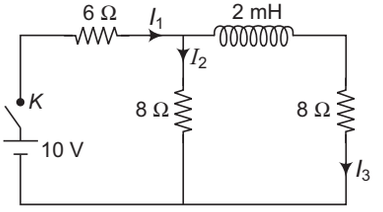
62. An inductor of 2 H and a resistance of 10 Ω are connected in series with a battery of 5 V. The initial rate of change in current is
(a) 0.5 A/s (b) 2.0 A/s
(c) 2.5 A/s (d) 0.25 A/s

63. In an inductor of inductance $L = 100$ mH, a current of $I = 10$ A is flowing. The energy stored in the inductor is
(a) 5 J (b) 10 J
(c) 100 J (d) 1000 J

64. During current growth in an L-R circuit the time constant is the time in which the magnitude of current becomes
(a) I_0 (b) $I_0/2$
(c) $0.63\, I_0$ (d) $0.37\, I_0$

65. An L-R circuit with a battery is connected at $t = 0$. Which of the following quantities is not zero just after the connection?
(a) Current in the circuit
(b) Magnetic field energy
(c) Power delivered by the battery
(d) Emf induced in the inductor

66. In the circuit shown in the figure, what is the value of I_1 just after pressing the key K ?

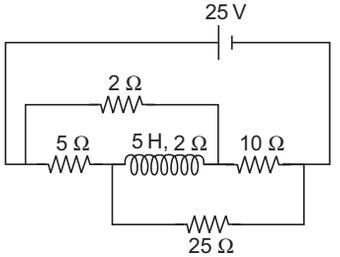


(a) 5/7 A (b) 5/11 A
(c) 1 A (d) None of these

67. The time constant of an inductance coil is 2.0×10^{-3} s. When a 90 Ω resistance is joined in series, the time constant becomes 0.5×10^{-3} s. The inductance and resistance of the coil are
(a) 30 mH, 30 Ω
(b) 30 mH, 60 Ω
(c) 60 mH, 30 Ω
(d) 60 mH, 60 Ω

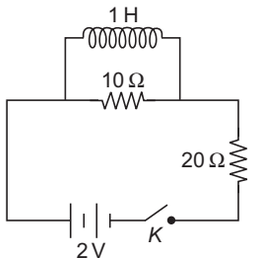
68. An emf of 20 V is applied in a circuit coil containing 5 H inductance and 5 Ω resistance. The ratio of the currents at time $t = \infty$ and $t = \ln(2)$ is
(a) $\frac{1}{e}$ (b) $\frac{1}{2}$
(c) e (d) None of these

69. In the circuit shown, what is the energy stored in the coil at steady state?



(a) 21.3 J (b) 42.6 J
(c) Zero (d) 213 J

70. In the following figure, what is the final value of current in the 10 Ω resistor when the plug of key K is inserted?



(a) (3/10) A (b) (3/20) A
(c) (3/11) A (d) Zero

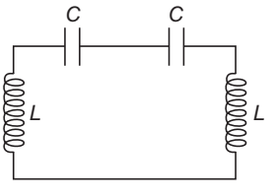
L-C Oscillations

71. The equivalent quantity of mass in an inductor circuit is
(a) charge (b) potential
(c) inductance (d) current

72. If L, C and R represent, the physical quantities inductance, capacitance and resistance respectively, only the following combination does not have the dimensions of frequency
(a) $\frac{1}{RC}$ (b) $\frac{R}{L}$ (c) $\frac{1}{\sqrt{LC}}$ (d) $\frac{C}{L}$

73. A condenser of capacity 20 μF is first charged and then discharged through a 10 mH inductance. Neglecting the resistance of the coil, the frequency of the resulting vibrations will be
(a) 356 cycle/s (b) 35.6 cycle/s
(c) 356×10^3 cycle/s (d) 3.56 cycle/s

74. The natural frequency of the circuit shown in the figure is



(a) $\frac{1}{2\pi\sqrt{LC}}$ (b) $\frac{1}{2\pi\sqrt{2LC}}$
(c) $\frac{2}{2\pi\sqrt{LC}}$ (d) None of these

75. In an L-C circuit,
(a) the energy stored in L as well as in C is magnetic energy
(b) the energy stored in L is magnetic but in C it is electrical
(c) the energy stored in L is electrical but in C it is magnetic
(d) the energy stored in L as well as C is electrical energy

76. The capacitor of an oscillatory circuit is enclosed in a container. When the container is evacuated, the frequency is 10 kHz. When the container is filled with a gas, the frequency changes by 50 Hz. The dielectric constant of the gas is
(a) 2.01 (b) 1.01
(c) 2.001 (d) 1.001

Transformer

77. In a transformer, the coefficient of mutual inductance between the primary and the secondary coil is 0.2 H. When the current changes by 5 A/s in the primary, the induced emf in the secondary will be
(a) 5 V (b) 1 V
(c) 25 V (d) 10 V

78. What is increased in step-down transformer?
(a) Voltage (b) Current
(c) Power (d) Current density

79. The transformation ratio in the step-up transformer is
(a) 1
(b) greater than one
(c) less than one
(d) the ratio greater or less than one depends on the other factors

80. The ratio of secondary to the primary turns in a transformer is 3 : 2. If the power output be P, then the input power neglecting all losses must be equal to
(a) $5P$ (b) $1.5P$
(c) P (d) $\frac{2}{5}P$

81. We can reduce eddy currents in the core of transformer
(a) by increasing the number of turns in secondary coil
(b) by taking laminated core
(c) by making step-down transformer
(d) by using a weak AC at high potential

82. In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4 A, then that in the secondary is
(a) 4 A (b) 2 A
(c) 6 A (d) 10 A

83. Quantity that remains unchanged in a transformer is
(a) voltage (b) current
(c) frequency (d) None of these

84. In a transformer, the number of turns in primary and secondary are 500 and 2000 respectively. If current in primary is 48 A, the current in the secondary is
(a) 12 A (b) 24 A
(c) 48 A (d) 144 A

85. A loss-free transformer having 100 turns in primary is used to transmit 10 kW of power. The input voltage is 200 V and power is transmitted at 5 kV. The currents in the primary and secondary of the transformer are
(a) 2 A and 50 A
(b) 50 A and 2 A
(c) 25 A and 4 A
(d) 12.5 A and 8 A

86. The core used in a transformer and other electromagnetic devices is laminated so that
(a) ratio of voltage in the primary and secondary may be increased
(b) energy loss due to eddy currents may be minimised
(c) the weight of the transformer may be reduced
(d) residual magnetism in the core may be reduced

87. The unit of magnetic flux is
(a) Wb/m^2 (b) Wb (c) H (d) A/m

88. The dimensions of magnetic flux are
(a) $[MLT^{-2}A^{-2}]$
(b) $[ML^2T^{-2}A^{-2}]$
(c) $[ML^2T^{-1}A^{-2}]$
(d) $[ML^2T^{-2}A^{-1}]$

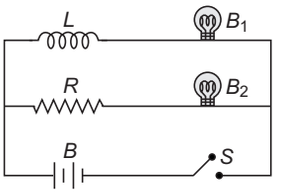
89. The magnetic flux linked with a vector area **A** in a uniform magnetic field B is
(a) $\mathbf{B} \times \mathbf{A}$ (b) AB
(c) $\mathbf{B} \cdot \mathbf{A}$ (d) $\frac{B}{A}$

90. A coil and a bulb are connected in series with a DC source, a soft iron core is, then inserted in the coil, then
(a) intensity of the bulb remains the same
(b) intensity of the bulb decreases
(c) intensity of the bulb increases
(d) the bulb ceases to glow

91. The unit of inductance is
(a) A/V-s (b) J/A
(c) V-s/A (d) V-A/s

92. The SI unit of inductance, the henry, can be written as
(a) weber/ampere
(b) volt-second/ampere
(c) joule/(ampere)^2
(d) ohm-second

93. An inductor L, a resistance R and two identical bulbs, B_1 and B_2 are connected to a battery through a switch S as shown in the figure. Which of the following statements gives the correct description of the happenings, when the switch S is closed?



(a) The bulb B_2 lights up earlier than B_1 and finally both the bulbs shine equally bright
(b) B_1 lights up earlier and finally both the bulbs acquire equal brightness
(c) B_2 lights up earlier and finally B_1 shines brighter than B_2
(d) B_1 and B_2 light up together with equal brightness all the time

94. Which of the following is constructed on the principle of electromagnetic induction?
(a) Galvanometer (b) Electric motor
(c) Generator (d) Voltmeter

95. Dimensions of self inductance are
(a) $[MLT^2A^2]$
(b) $[ML^2T^{-1}A^{-2}]$
(c) $[ML^2T^{-2}A^{-1}]$
(d) $[ML^2T^{-2}A^{-1}]$

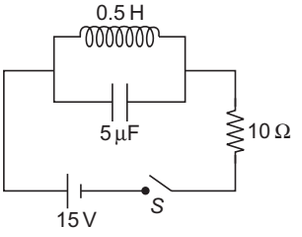
96. Two identical circular loops of metal wires are lying on a table without touching each other. Loop A carries a current which increases with time. In response the loop B
(a) remain stationary
(b) is attracted by loop A
(c) is repelled by loop A
(d) rotates about its centre of mass with CM fixed

97. Two particles each of mass m and charge q are attached to the ends of a light rigid rod of length $2R$. The rod is rotated at constant angular speed about a perpendicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of rod is
(a) $q/2m$ (b) q/m (c) $2q/m$ (d) $q/\pi m$

98. Which one of the following is a correct statement?
(a) Electric field is always conservative
(b) Electric field due to a varying magnetic field is conservative
(c) Electric field is conservative due to electrostatic charges while non-conservative due to a time varying magnetic field
(d) Electric field lines are always closed loops

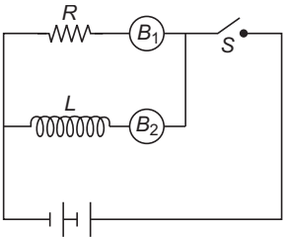
99. Eddy currents are produced when
(a) a metal is kept in varying magnetic field
(b) a metal is kept in a steady magnetic field
(c) a circular coil is placed in a magnetic field
(d) through a circular coil current is passed

100. In the circuit shown in figure, current through the battery at $t = 0$ and $t = \infty$ is



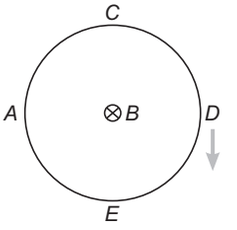
(a) 1.5 A, 1.5 A (b) 0, 0
(c) 1, 1.5 A (d) 1.5 A, 0

101. Figure shows two bulbs B_1 and B_2 resistor R and inductor L, when the switch S is turned off



(a) Both B_1 and B_2 die out promptly
(b) Both B_1 and B_2 die out with some delay
(c) B_2 dies out promptly, but B_1 with some delay
(d) B_1 dies out promptly but B_2 with some delay

102. A vertical conducting ring of radius R falls vertically in horizontal uniform magnetic field of magnitude B. The direction of B is $\otimes$. When the speed of the ring is v



(a) no current flows in the ring
(b) A and D are at the same potential
(c) C and E are at the same potential
(d) the potential difference between A and D is $2BRv$, with D at a higher potential

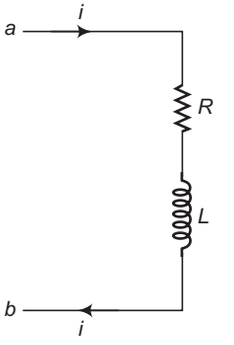
[Level 2]

Only One Correct Option

1. A square loop of side l, resistance R is placed in a uniform magnetic field B acting normally to the plane of the loop. If we attempt to pull it out of the field with a constant velocity v, the power needed is

(a) $BRlv$ (b) $\dfrac{B^2l^2v^2}{R}$
(c) $\dfrac{Bl^2v^2}{R}$ (d) $\dfrac{Bvl}{R}$

2. When the current in the portion of the circuit shown in the figure is 2A and increasing at the rate of 1 A/s, the measured potential difference $V_{ab} = 8$ V. However when the current is 2A and decreasing at the rate of 1 A/s, the measured potential difference $V_{ab} = 4$ V. The values of R and L are



(a) 3 Ω and 2 H respectively
(b) 3 Ω and 3 H respectively
(c) 2 Ω and 1 H respectively
(d) 3 Ω and 1 H respectively

3. Two different coils have self inductances $L_1 = 8\,\text{mH}$ and $L_2 = 2\,\text{mH}$. The current in one coil is increased at a constant rate. The current in second coil is also increased at the same constant rate. At certain instant of time, the power given to the coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then, choose the wrong option.

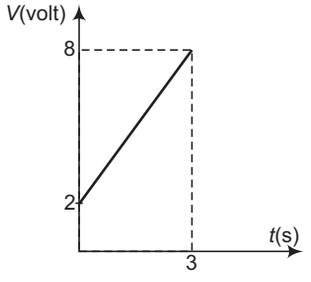
(a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = 4$ (c) $\frac{W_1}{W_2} = \frac{1}{4}$ (d) $\frac{V_1}{V_2} = \frac{1}{4}$

4. An emf of 15V is applied in a circuit coil containing 5H inductance and $10\,\Omega$ resistance. The ratio of the currents at time $t = \infty$ and $t = 1\,\text{s}$ is

(a) $\frac{e^{1/2}}{e^{1/2} - 1}$ (b) $\frac{e^2}{e^2 - 1}$

(c) $1 - e^{-1}$ (d) e^{-1}

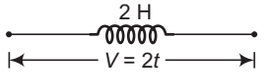
5. A circuit element is placed in a closed box. At time $t = 0$, a constant current generator supplying a current of 1 A is connected across the box. Potential difference across the box varies according to graph shown in the figure. The element in the box is



(a) a resistance of 2 Ω (b) a battery of emf 6 V

(c) an inductance of 2 H (d) a capacitance of 0.5 F

6. A variable voltage $V = 2t$ is applied across an inductor of inductance $L = 2\,\text{H}$ as shown in figure. Then,



(a) current *versus* time graph is a parabola

(b) energy stored in magnetic field at $t = 2\,\text{s}$ is 4 J

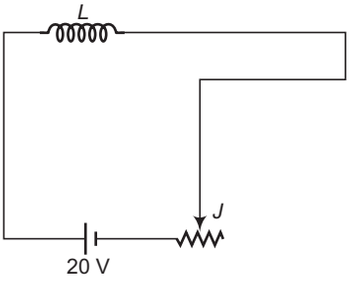
(c) Potential energy at time $t = 1\,\text{s}$ in magnetic field is increasing at a rate of 1 J/s

(d) energy stored in magnetic field is zero all the time

7. A non-conducting ring having charge q uniformly distributed over its circumference is placed on a rough horizontal surface. A vertical time varying magnetic field $B = 4t^2$ is switched on at time $t = 0$. Mass of the ring is m and radius is R. The ring starts rotating after 2 s, the coefficient of friction between the ring and the table is

(a) $\frac{4qmR}{g}$ (b) $\frac{2qmR}{g}$ (c) $\frac{8qR}{mg}$ (d) $\frac{qR}{2mg}$

8. In the circuit shown in figure the jockey J is being pulled towards right so that the resistance in the circuit is increasing. Its a value at some instant is $5\,\Omega$. The current in the circuit at this instant will be



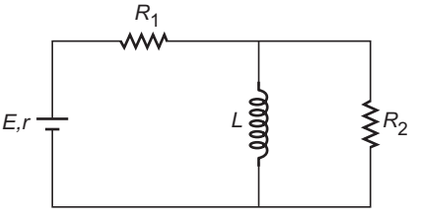
(a) 4 A

(b) less than 4 A

(c) more than 4 A

(d) may be less than or more than 4 A depending on the value of L

9. The value of time constant for the given circuit is



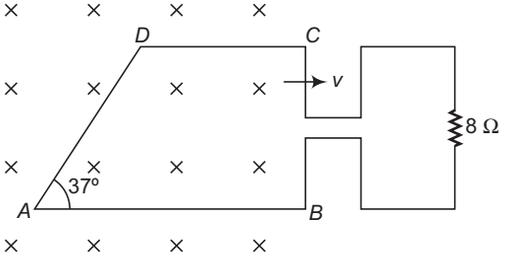
(a) $\frac{L}{R_1 + r + R_2}$

(b) $\frac{L}{(R_1 + r)}$

(c) $\frac{L\,(R_1 + R_2 + r)}{(R_1 + r)\,R_2}$

(d) None of the above

10. The loop $ABCD$ is moving with velocity v towards right. The magnetic field is 4 T. The loop is connected to a resistance of $8\,\Omega$. If steady current of 2 A flows in the loop, then value of v, if loop has resistance of $4\,\Omega$, is (given, $AB = 30\,\text{cm}$, $AD = 30\,\text{cm}$)



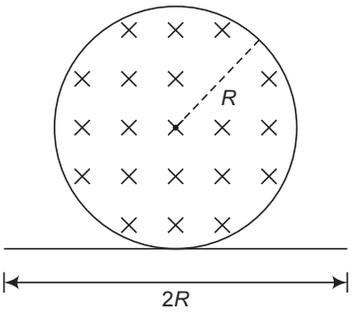
(a) $\frac{50}{3}$ m/s

(b) 20 m/s

(c) 10 m/s

(d) $\frac{100}{3}$ m/s

11. A uniform but time varying magnetic field is present in a circular region of radius R. The magnetic field is perpendicular and into the plane of the loop and the magnitude of field is increasing at a constant rate α. There is a straight conducting rod of length $2R$ placed as shown in figure.

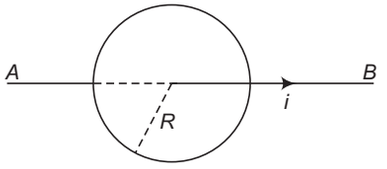


The magnitude of induced emf across the rod is

(a) $\pi R^2\alpha$ (b) $\frac{\pi R^2\alpha}{2}$

(c) $\frac{R^2\alpha}{\sqrt{2}}$ (d) $\frac{\pi R^2\alpha}{4}$

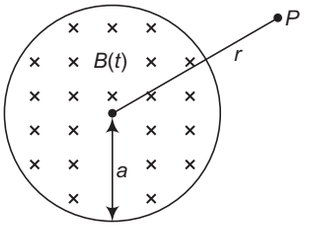
12. An infinitely long conductor AB lies along the axis of a circular loop of radius R. If the current in the conductor AB varies at the rate of x ampere/sec, then the induced emf in the loop is



(a) $\frac{\mu_0 xR}{2}$ (b) $\frac{\mu_0 xR}{4}$

(c) $\frac{\mu_0 \pi xR}{2}$ (d) zero

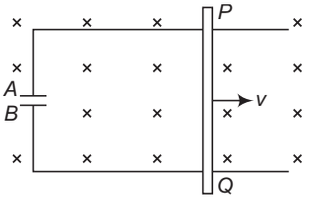
13. A uniform but time-varying magnetic field $B(t)$ exists in a cylindrical region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region



(a) is zero (b) decreases as $\frac{1}{r}$

(c) increases as r (d) decreases as $\frac{1}{r^2}$

14. A conducting rod PQ of length $L = 1.0$ m is moving with a uniform speed $v = 2$ m/s in a uniform magnetic field $B = 4.0$ T directed into the paper. A capacitor of capacity $C = 10\,\mu$F is connected as shown in figure. Then,



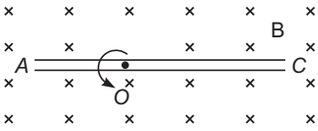
(a) $q_A = +80\,\mu$C and $q_B = -80\,\mu$C

(b) $q_A = -80\,\mu$C and $q_B = +80\,\mu$C

(c) $q_A = 0 = q_B$

(d) Charge stored in the capacitor increases exponentially with time

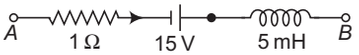
15. A conducting rod AC of length $4l$ is rotated about a point O in a uniform magnetic field B directed into the paper. $AO = l$ and $OC = 3l$. Then,



(a) $V_A - V_O = \frac{B\omega l^2}{2}$ (b) $V_O - V_C = \frac{7}{2}B\omega l^2$

(c) $V_A - V_C = 4B\omega l^2$ (d) $V_C - V_O = \frac{9}{2}B\omega l^2$

16. The network shown in the figure is a part of a complete circuit. If at a certain instant the current i is 5 A and decreasing at the rate of 10^3 A/s then, $V_B - V_A$ is



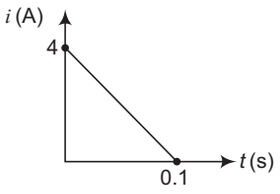
(a) 5 V (b) 10 V

(c) 15 V (d) 20 V

17. The current in an L-R circuit builds upto $\frac{3}{4}$th of its steady state value in 4 s. The time constant of this circuit is

(a) $\frac{1}{\ln 2}$ s (b) $\frac{2}{\ln 2}$ s (c) $\frac{3}{\ln 2}$ s (d) $\frac{4}{\ln 2}$ s

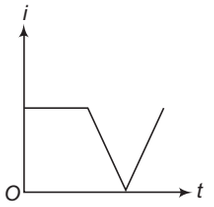
18. Some magnetic flux is changed from a coil of resistance 10 Ω. As a result an induced current is developed in it, which varies with time as shown in figure. The magnitude of change in flux through the coil in Wb is



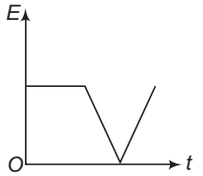
(a) 2 (b) 4

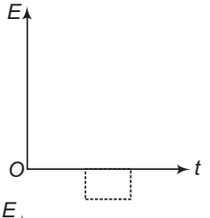
(c) 6 (d) None of these

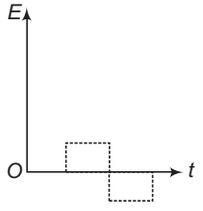
19. The current i in an induction coil varies with time t according to the graph shown

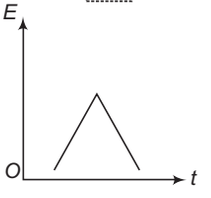


in figure. Which of the following graphs shows the induced emf (E) in the coil with time?

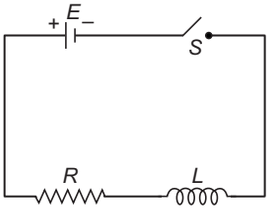
(a)


(b)


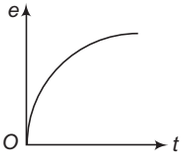
(c)


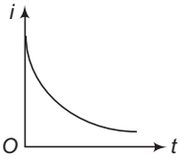
(d)


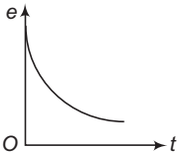
20. Switch S of the circuit shown in figure is closed at $t = 0$. If e denotes the induced

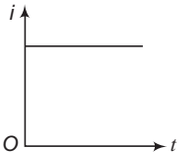


emf in L and i, the current flowing through the circuit at time t, which of the following graphs is correct?

(a)


(b)


(c)


(d)


21. A coil of inductance $L = 50\mu H$ and resistance $= 0.5\ \Omega$ is connected to a battery of emf = 5 V. A resistance of $10\ \Omega$ is connected parallel to the coil. Now at some instant the connection of the battery is switched off. Then, the amount of heat generated in the coil after switching off the battery is

(a) 1.25 mJ (b) 2.5 mJ (c) 0.65 mJ (d) 0.12 mJ

22. A pair of coils of turns n_1 and n_2 are kept close together. Current passing through the first is reduced at rate r, and emf 3 mV is developed across the other coil. If the second coil carries current which is then reduced at the rate $2r$, the emf produced across the first coil will be

(a) $\frac{6n_1}{n_2}$ mV (b) $\frac{6n_2}{n_1}$ mV

(c) 6 mV (d) 3/2 mV

23. An L-C circuit contains a 0.60 H inductor and a 25 μF capacitor. What is the rate of change of the current (in A/s), when the charge on the capacitor is 3.0×10^{-5} C?

(a) 2.0 (b) 4.0

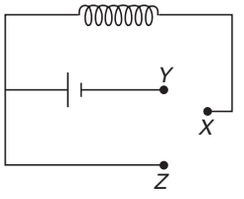
(c) 3.0 (d) 6.0

24. A capacitor of capacity 2 μF is charged to a potential difference of 12 V. It is then connected across an inductor of inductance 0.6 mH. The current in the circuit at a time, when the potential difference across the capacitor is 6.0 V is

(a) 0.6 A (b) 1.2 A

(c) 2.4 A (d) 3.6 A

25. In the circuit shown, the coil has inductance and resistance. When X is joined to Y, the time constant is τ during growth of current. When the steady state is reached, heat is produced in the coil at a rate P. X is now joined to Z,



(a) the total heat produced in the coil is $P\tau$

(b) the total heat produced in the coil is $\frac{1}{2}P\tau$

(c) the total heat produced in the coil is $2P\tau$

(d) the data given is not sufficient to reach a conclusion

26. A loop, made of straight edges has six corners at A (0, 0, 0), B (L, 0, 0), C (L, L, 0), D (0, L, 0), E (0, L, L) and F (0, 0, L). A magnetic field $\mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}})$ T is present in the region. The flux passing through the loop $ABCDEFA$ (in that order) is

(a) B_0L^2 Wb (b) $2B_0L^2$ Wb

(c) $\sqrt{2}B_0L^2$ Wb (d) $4B_0L^2$ Wb

27. The self inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N increases as

(a) l and A increase

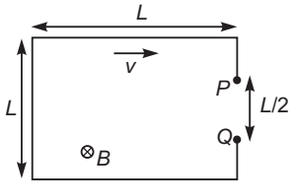
(b) l decreases and A increases

(c) l increases and A decreases

(d) both l and A decrease

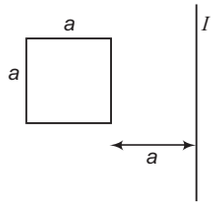
More than One Correct Options

1. The loop shown moves with a velocity v in a uniform magnetic field of magnitude B, directed into the paper. The potential difference between points P and Q is e. Then,



(a) $e = \frac{1}{2} BLv$
(b) $e = BLv$
(c) P is positive w.r.t. Q
(d) Q is positive w.r.t. P

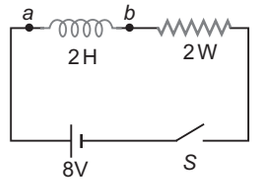
2. In infinitely long wire is placed near a square loop as shown in figure. Choose the correct options.



(a) The mutual inductance between the two is $\frac{\mu_0 a}{2\pi} \ln(2)$
(b) The mutual inductance between the two is $\frac{\mu_0 a^2}{2\pi} \ln(2)$
(c) If a constant current is passed in the straight wire in upward direction and loop is brought close to the wire, then induced current in the loop is clockwise
(d) In the above condition, induced current in the loop is anti-clockwise

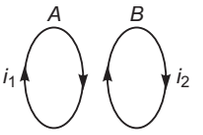
3. Choose the correct options.
(a) SI unit of magnetic flux is H-A
(b) SI unit of coefficient of self inductance is J/A
(c) SI unit of coefficient of self inductance is $\frac{\text{V-s}}{\text{A}}$
(d) SI unit of magnetic induction is Wb

4. In the circuit shown in figure, circuit is closed at time $t = 0$. At time $t = \ln(2)$ s



(a) rate of energy supplied by the battery is 16 J/s
(b) rate of heat dissipated across resistance is 8 J/s
(c) rate of heat dissipated across resistance is 16 J/s
(d) $V_a - V_b = 4$ V

5. Two circular coils are placed adjacent to each other. Their planes are parallel and currents through them i_1 and i_2 are in same directions. Choose the correct options.

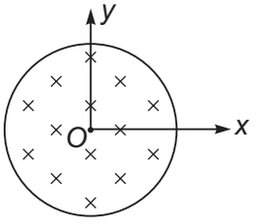


(a) When A is brought near B, current i_2 will decrease
(b) In the above process, current i_2 will increase
(c) When current i_1 is increased, current i_2 will decrease
(d) In the above process, current i_2 will increase

6. A coil of area $2\,\text{m}^2$ and resistance $4\,\Omega$ is placed perpendicular to a uniform magnetic field of 4 T. The loop is rotated by 90° in 0.1 s. Choose the correct options.
(a) Average induced emf in the coil is 8 V
(b) Average induced current in the circuit is 20 A
(c) 2 C charge will flow in the coil in above period
(d) Heat produced in the coil in the above period can't be determined from the given data

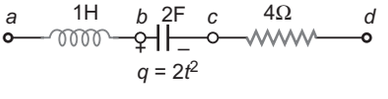
7. In L-C oscillations
(a) time period of oscillations is $\frac{2\pi}{\sqrt{LC}}$
(b) maximum current in circuit is $\frac{q_0}{\sqrt{LC}}$
(c) maximum rate of change of current in circuit is $\frac{q_0}{LC}$
(d) maximum potential difference across the inductor is $\frac{q_0}{2C}$
where, q_0 is maximum charge on capacitor

8. Magnetic field in a cylindrical region of radius R in inward direction is as shown in figure.



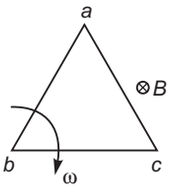
(a) an electron will experience no force kept at $(2R, 0, 0)$ if magnetic field increases with time
(b) in the above situation, electron will experience the force in negative y-axis
(c) If a proton is kept at $\left(0, \frac{R}{2}, 0\right)$ and magnetic field is decreasing, then it will experience the force in positive x-direction
(d) if a proton is kept at $(-R, 0, 0)$ and magnetic field is increasing, then it will experience force in negative y-axis

9. In the figure shown, q is in coulomb and t in second. At time $t = 1$ s



(a) $V_a - V_b = 4$ V
(b) $V_b - V_c = 1$ V
(c) $V_c - V_d = 16$ V
(d) $V_a - V_d = 20$ V

10. An equilateral triangular conducting frame is rotated with angular velocity ω in uniform magnetic field B as shown. Side of triangle is l. Choose the correct options.



(a) $V_a - V_c = 0$ (b) $V_a - V_c = \frac{B\omega l^2}{2}$

(c) $V_a - V_b = \frac{B\omega l^2}{2}$ (d) $V_c - V_b = -\frac{B\omega l^2}{2}$

11. A metal plate is getting heated. It can be because
(a) a direct current is passing through the plate
(b) it is placed in a time varying magnetic field
(c) it is placed in a space varying magnetic field, but does not vary with time
(d) a current (either direct or alternating) is passing through the plate

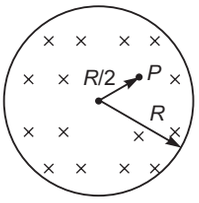
12. An emf is produced in a coil, which is not connected to an external voltage source. This can be due to
(a) the coil being in a time varying magnetic field
(b) the coil moving in a time varying magnetic field
(c) the coil moving in a constant magnetic field
(d) the coil is stationary in external spatially varying magnetic field, which does not change with time

13. A circular coil expands radially in a region of magnetic field and no electromotive force is produced in the coil. This can be because
(a) the magnetic field is constant
(b) the magnetic field is in the same plane as the circular coil and it may or may not vary
(c) the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably
(d) there is a constant magnetic field in the perpendicular (to the plane of the coil) direction

Comprehension Based Questions

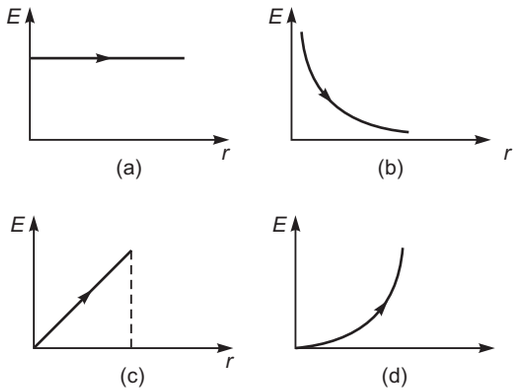
Passage I (Q. 1 to 3)

A uniform but time varying magnetic field $B = (2t^3 + 24t)\,T$ is present in a cylindrical region of radius $R = 2.5\,cm$ as shown in figure.



1. The force on an electron at P at $t = 2.0\,s$ is
(a) 96×10^{-21}N (b) 48×10^{-21}N
(c) 24×10^{-21} N (d) zero

2. The variation of electric field at any instant as a function of distance measured from the centre of cylinder in first problem is



3. In the previous problem, the direction of circular electric lines at $t = 1\,s$ is
(a) clockwise
(b) anti-clockwise
(c) no current is induced
(d) cannot be predicted

Passage II (Q. 4 to 7)

A thin non-conducting ring of mass m, radius a carrying a charge q can rotate freely about its own axis which is vertical. At the initial moment the ring was at rest in horizontal position and no magnetic field was present. At instant $t = 0$, a uniform magnetic field is switched on which is vertically downward and increases with time according to the law $B = B_0 t$. Neglecting magnetism induced due to rotational motion of ring.

4. The magnitude of induced emf on the closed surface of ring will be
(a) $\pi a^2 B_0$ (b) $2a^2 B_0$
(c) zero (d) $\frac{1}{2}\pi a^2 B_0$

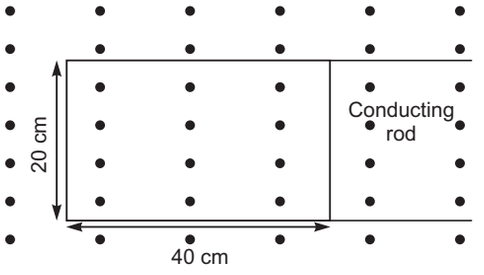
5. The magnitude of an electric field on the circumference of the ring is
(a) aB_0 (b) $2aB_0$
(c) $\frac{1}{2}aB_0$ (d) zero

6. Angular acceleration of ring is
(a) $\frac{qB_0}{2m}$ (b) $\frac{qB_0}{4m}$
(c) $\frac{qB_0}{m}$ (d) $\frac{2qB_0}{m}$

7. Find instantaneous power developed by electric force acting on the ring at $t = 1\,s$
(a) $\frac{2q^2B_0^2a^2}{14m}$ (b) $\frac{q^2B_0^2a^2}{8m}$
(c) $\frac{3q^2B_0^2a^2}{m}$ (d) $\frac{q^2B_0^2a^2}{4m}$

Passage III (Q. 8 to 10)

Figure shows a conducting rod of negligible resistance that can slide on smooth U-shaped rail made of wire of resistance $1\,\Omega/m$. *Position of the conducting rod at* $t = 0$ *is shown. A time t dependent magnetic field* $B = 2t$ *tesla is switched on at* $t = 0$.



8. The current in the loop at $t = 0$ due to induced emf is

(a) 0.16 A, clockwise (b) 0.08 A, clockwise
(c) 0.16 A, anti-clockwise (d) zero

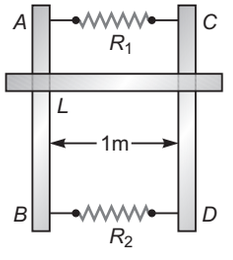
9. At $t = 0$, when the magnetic field is switched on the conducting rod is moved to the left at constant speed 5 cm/s by some external means. At $t = 2$ s, net induced emf has magnitude

(a) 0.12 V (b) 0.08 V (c) 0.04 V (d) 0.02 V

10. The magnitude of the force required to move the conducting rod at constant speed 5 cm/s at the same instant $t = 2$ s, is equal to

(a) 0.096 N (b) 0.12 N
(c) 0.08 N (d) 0.064 N

Passage IV (Q. 11 to 13)



Two parallel vertical metallic rails AB and CD are separated by $1\,m$. *They are connected at the two ends by resistances* R_1 *and* R_2 *as shown in the figure. A horizontal metallic bar L of mass* $0.2\,kg$ *slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of* $0.6\,T$ *perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in* R_1 *and* R_2 *are* $0.76\,W$ *and* $1.2\,W$ *respectively* ($g = 9.8\,m/s^2$)

11. The terminal velocity of the bar L will be

(a) 2 m/s (b) 3 m/s
(c) 1 m/s (d) None of these

12. The value of R_1 is

(a) 0.47 Ω (b) 0.82 Ω
(c) 0.12 Ω (d) None of these

13. The value of R_2 is

(a) 0.6 Ω (b) 0.5 Ω (c) 0.4 Ω (d) 0.3 Ω

Assertion and Reason

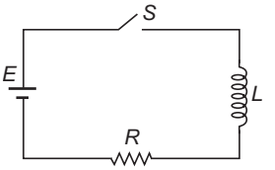
Directions (Q. Nos. 1-20) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** Two concentric conducting rings of different radii are placed in space. The mutual inductance of both the rings is maximum, if the rings are coplanar.

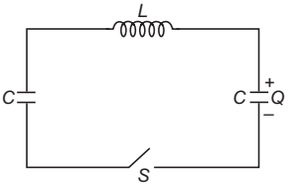
Reason For two coaxial conducting rings of different radii, the magnitude of magnetic flux in one ring due to current in other ring is maximum when both rings are coplanar.

2. **Assertion** In the figure, just after closing the switch the potential drop across inductor is maximum.



Reason The rate of change of current just after closing the switch is maximum.

3. **Assertion** In the circuit shown when switch S is closed, current in the circuit will be maximum, when electrostatic energy of capacitors becomes zero.



Reason In the given circuit of L-C oscillations when charges are equally distributed in the two capacitors, the current in the circuit will be maximum.

4. **Assertion** Magnetic flux and the electric flux have the same units.

Reason Flux passing through a surface gives an idea about the field lines crossing that surface.

5. **Assertion** A rectangular loop is placed near an infinitely long current carrying straight wire. The magnetic field of the straight wire is non-uniform. Therefore, magnetic flux through the loop can be obtained by integration.

Reason No induced emf will be developed in the loop, if current through straight wire is constant.

6. **Assertion** If a starlight wire is moved in a magnetic field, no emf will be induced across its two ends as the circuit is not closed.

Reason Since, the circuit is not closed induced current will be zero.

7. **Assertion** If a charged particle is placed in a time varying magnetic field it will experience a force.

Reason Time varying field will produce an electric field.

8. **Assertion** In an *L*-*C* oscillations potential difference across the capacitor is always equal to the potential difference across the inductor.

Reason Both are always in parallel, hence the potential difference is same.

9. **Assertion** When two magnetise are brought closer to each other they will always repel each other.

Reason According to Lenz law induced effects always opposes the cause.

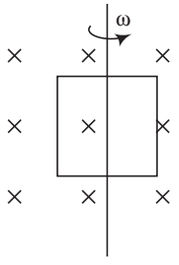
10. **Assertion** Mutual inductance of two coils depends on the distance between the coils and their orientation.

Reason It does not depend on the magnetic material filled between the coils.

11. **Assertion** If a loop is placed in a non-uniform (w.r.t. position) magnetic field, then induced emf is produced in the loop.

Reason In a non-uniform magnetic field magnetic flux passing through the loop will change. Therefore, induced emf is produced.

12. **Assertion** A conducting loop shown in figure is rotated in a uniform magnetic field with constant angular velocity ω as shown in figure. At time $t = 0$, plane of the loop is perpendicular to the magnetic field. Induced emf produced in the loop is maximum, when plane of loop is parallel to magnetic field.



Reason When plane of loop is parallel to magnetic field, magnetic flux passing through the loop is zero.

13. **Assertion** If a magnet is brought closer to a current carrying loop along its axis, current always decreases in the loop.

Reason Magnet is repelled by the loop.

14. **Assertion** If current shown in the figure is increasing, then



$$V_A > V_B$$

Reason If current passing through an inductor is constant, then both ends of the inductor are at same potential.

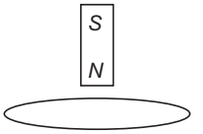
15. **Assertion** Coefficient of self-iniduction of an inductor dependes upon the rate of change of current passing through it.

Reason From $e = -L\dfrac{di}{dt}$

We can see that, $L = \dfrac{-e}{(di/dt)}$

or $L \propto \dfrac{1}{(di/dt)}$

16. **Assertion** A magnet is dropped along the axis of a circular conducting loop as shown in figure. Then, acceleration of magnet is always less than *g*.



Reason When magnet is above the loop it will repet the magnet and when it is below the loop it will attract the magnet.

17. **Assertion** In *L* - *C* oscillations current in the circuit is maximum at the instant charge in the capacitor is zero.

Reason At this instant whole of the energy is stored as magnetic energy.

18. **Assertion** If two inductors are in parallel, then current in them distributes in inverse ratio of their inductance.

Reason In parallel, potential difference remains constant.

19. **Assertion** If current passing through a circular loop is doubled, then magnetic flux linked with the circular loop will also become two times.

Reason No flux will link through the coil by its own current.

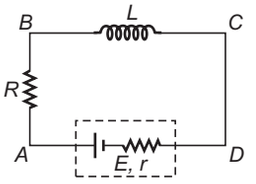
20. **Assertion** An induced emf of 2 V is developed in a circular loop, if current in the loop is changed at a rate of 4 A/s. If 4 A of current is passed through this loop, then flux linked with this coil will be 2 Wb.

Reason Flux linked with the coil is

$$|\phi| = \left|\frac{e}{di/dt}\right| i$$

Match the Columns

1. In the circuit shown in figure, $E = 10\,\text{V}$, $r = 1\,\Omega$, $R = 4\,\Omega$ and $L = 5\,\text{H}$. Switch of the circuit is closed at time $t = 0$. Then, match the following two columns.



Column I	Column II
(A) V_{AD} at $t = 0$	(p) 4 V
(B) V_{BC} at $t = 0$	(q) 8 V
(C) V_{AB} at $t = \alpha$	(r) 0 V
(D) V_{AD} at $t = \alpha$	(s) 10 V

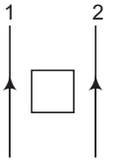
2. In an L-C oscillations, maximum charge is q_0, inductance is L and capacitance is C. Now match the following two columns. (Given that $q_0 = 4\,\text{C}, L = 2\,\text{H}$ and $C = 1\,\text{F}$)

Column I	Column II
(A) Maximum current in the circuit	(p) 2 SI unit
(B) Maximum rate of change of current in the circuit	(q) $2\sqrt{2}$ SI unit
(C) Maximum magnetic potential energy	(r) 16 SI unit
(D) Angular frequency of oscillations	(s) $\frac{1}{\sqrt{2}}$ SI unit

3. Match the following columns.

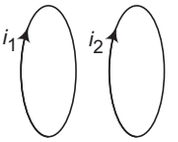
Column I	Column II
(A) Tesla	(p) $[ML^2A^{-2}T^{-2}]$
(B) Weber	(q) $[MLA^{-2}T^{-1}]$
(C) Weber/m^2	(r) $[MA^{-1}T^{-2}]$
(D) Henry	(s) None

4. A square loop is symmetrically placed between two infinitely long current carrying wires in the same direction. Magnitude of currents in both the wires are same. Now match the following two columns.



Column I	Column II
(A) Loop is moved towards right	(p) Induced current in the loop is clockwise
(B) Loop is moved towards left	(q) Induced current in the loop is anti-clockwise
(C) Wire-1 is moved towards left	(r) Induced current in the loop is zero.
(D) Wire-2 is moved towards right	(s) Induced current in the loop is non-zero.

5. Two coaxial identical circular current carrying loops are shown in figure currents in them are in the same directions. Now match the following two columns.



Column I	Column II
(A) Current i_1 is increased	(p) Loops will attract each other
(B) Current i_2 is decreased	(q) Loops will repel each other
(C) Loop-1 is moved towards loop-2	(r) Current i_1 will increase
(D) Loop-2 is moved away from loop-1	(s) Current i_2 will increase

Entrance Gallery

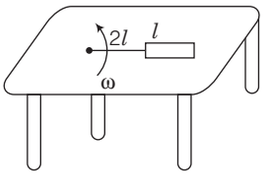
2014

1. A solenoid of inductance 2 H carries a current 1 A. What is the magnetic energy stored in a solenoid ? **[Karnataka CET]**

(a) 2 J (b) 1 J
(c) 4 J (d) 5 J

2013

2. A metallic rod of length l is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field B in the region, the emf induced across the ends of the rod is **[JEE Main]**



(a) $\frac{2B\omega l^3}{2}$
(b) $\frac{3B\omega l^3}{2}$
(c) $\frac{4B\omega l^2}{2}$
(d) $\frac{5B\omega l^2}{2}$

3. A step-down transformer has 50 turns on secondary and 1000 turns on primary winding. If a transformer is connected to 220 V and 1 A AC source, what is output current of the transformer ? **[Karnataka CET]**

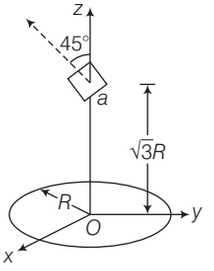
(a) $\frac{1}{20}$ A (b) 20 A (c) 100 A (d) 2 A

2012

4. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement (s) is (are) **[IIT JEE]**

(a) the emf induced in the loop is zero, if the current is constant
(b) the emf induced in the loop is finite, if the current is constant
(c) the emf induced in the loop is zero, if the current decreases at a steady rate
(d) the emf induced in the loop is finite, if the current decreases at a steady rate

5. A circular wire loop of radius R is placed in the x-y plane centred at the origin O. A square loop of side a ($a << R$) having two turns is placed with its centre at $z = \sqrt{3}\,R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the values of p is **[IIT JEE]**



(a) 7 (b) 8
(c) 6 (d) 9

2011

6. A boat is moving due East in a region where the earth's magnetic field is 5.0×10^{-5} $NA^{-1}m^{-1}$ due North and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is **[AIEEE]**

(a) 0.75 mV
(b) 0.50 mV
(c) 0.15 mV
(d) 1 mV

7. A horizontal straight wire 20 m long extending from East to West is falling with a speed of 5.0 ms^{-1}, at right angles to the horizontal component of the earth's magnetic field 0.030×10^{-4} Wbm^{-2}. The instantaneous value of the emf induced in the wire will be **[AIEEE]**

(a) 6.0 mV
(b) 3 mV
(c) 4.5 mV
(d) 1.5 mV

8. The core of a transformer is laminated to reduce **[Kerala CEE]**

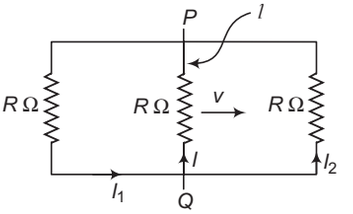
(a) flux leakage
(b) output power
(c) hysteresis
(d) copper loss
(e) eddy current

9. A transformer works on the principle of **[Karnataka CET]**

(a) magnetic effect of the electrical current
(b) mutual induction
(c) electrical inertia
(d) self induction

2010

10. A rectangular loop has a sliding connector PQ of length l and resistance $R\,\Omega$ and it is moving with a speed v as shown in the figure. The set up is placed in an uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are **[AIEEE]**



(a) $I_1 = -I_2 = \frac{Blv}{R}$, $I = \frac{2Blv}{R}$
(b) $I_1 = I_2 = \frac{Blv}{3R}$, $I = \frac{2Blv}{3R}$
(c) $I_1 = I_2 = I = \frac{Blv}{R}$
(d) $I_1 = I_2 = \frac{Blv}{6R}$, $I = \frac{Blv}{3R}$

11. A transformer of efficiency 90% draws an input power of 4kW. An electrical appliance connected across the secondary draws a current of 6 A. The impedance of the device is **[Kerala CEE]**

(a) 60 Ω
(b) 50 Ω
(c) 80 Ω
(d) 100 Ω
(e) 120 Ω

12. There is an uniform magnetic field directed perpendicular and into the plane of the paper. An irregular shaped conducting loop is slowly changing into a circular loop in the plane of the paper. Then, **[Karnataka CET]**

(a) current is induced in the loop in the anti-clockwise direction
(b) current is induced in the loop in the clockwise direction
(c) AC is induced in the loop
(d) no current is induced in the loop

Answers

Level 1

Objective Problems

1. (b)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (d)
11. (b)	**12.** (d)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (a)	**18.** (a,c)	**19.** (d)	**20.** (b)
21. (d)	**22.** (b)	**23.** (b)	**24.** (c)	**25.** (c)	**26.** (a)	**27.** (c)	**28.** (a)	**29.** (b)	**30.** (d)
31. (a,c)	**32.** (d)	**33.** (d)	**34.** (c)	**35.** (d)	**36.** (c)	**37.** (d)	**38.** (d)	**39.** (a)	**40.** (d)
41. (d)	**42.** (b)	**43.** (b)	**44.** (b)	**45.** (b)	**46.** (c)	**47.** (c)	**48.** (a)	**49.** (d)	**50.** (a)
51. (a)	**52.** (b)	**53.** (d)	**54.** (a)	**55.** (a)	**56.** (c)	**57.** (b)	**58.** (b)	**59.** (c)	**60.** (c)
61. (d)	**62.** (c)	**63.** (a)	**64.** (c)	**65.** (a)	**66.** (a)	**67.** (c)	**68.** (b)	**69.** (c)	**70.** (d)
71. (c)	**72.** (d)	**73.** (a)	**74.** (a)	**75.** (b)	**76.** (b)	**77.** (b)	**78.** (b)	**79.** (b)	**80.** (c)
81. (b)	**82.** (b)	**83.** (c)	**84.** (a)	**85.** (b)	**86.** (b)	**87.** (b)	**88.** (d)	**89.** (c)	**90.** (b)
91. (c)	**92.** (All)	**93.** (c)	**94.** (c)	**95.** (c)	**96.** (c)	**97.** (a)	**98.** (c)	**99.** (a)	**100.** (a)
101. (b)	**102.** (a,c,d)								

Level 2

Only One Correct Option

1. (b)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (d)	**6.** (a,b,c)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (d)
11. (d)	**12.** (d)	**13.** (b)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (c)
21. (d)	**22.** (c)	**23.** (a)	**24.** (a)	**25.** (b)	**26.** (b)	**27.** (b)			

More than One Correct Options

1. (a,c)	**2.** (a,c)	**3.** (a,c)	**4.** (a,d)	**5.** (a,c)	**6.** (b,c,d)	**7.** (b,c)	**8.** (b,c,d)	**9.** (a,b,c)	**10.** (a,c)
11. (a,b,d)	**12.** (a,b,c)	**13.** (b,c)							

Comprehension Based Questions

1. (b)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (c)
11. (c)	**12.** (a)	**13.** (d)							

Assertion and Reason

1. (a)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (b)	**8.** (a)	**9.** (a,b)	**10.** (a)
11. (e)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (d)	**16.** (a)	**17.** (a,b)	**18.** (b)	**19.** (c)	**20.** (a)

Match the Columns

1. (A→s; B→s; C→q; D→q)
2. (A→q; B→p; C→r; D→s)
3. (A→r; B→s; C→r; D→p)
4. (A→p,s; B→q,s; C→p,q; D→q,s)
5. (A→r; B→p,r; C→q; D→p,r,s)

Entrance Gallery

1. (b)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (e)	**9.** (b)	**10.** (b)
11. (d)	**12.** (a)								

Solutions

Level 1 : Objective Problems

1. Induced emf, $e = -\frac{d\phi}{dt}$

but $e = iR$

and $i = \frac{dq}{dt}$

$\therefore \quad \frac{dq}{dt} R = -\frac{d\phi}{dt}$

$\therefore \quad dq = -\frac{d\phi}{R}$

So, induced charge in a coil is independent of time taken to change the flux.

2. The magnitude of induced emf is directly proportional to the rate of change of magnetic flux. Also, induced charge $dq = -\frac{d\phi}{R}$ which is independent of time.

3. Induced current is given by,

$$i = \frac{e}{R} = -\frac{N(d\phi/dt)}{R} = -\frac{N}{R}\left(\frac{d\phi}{dt}\right)$$

$$= \frac{10 \times 10^8 \times 10^{-4} \times 10^{-4} \times 10}{20}$$

$$= 5 \text{ A}$$

4. $e = -N\frac{d\phi}{dt}$

$$|e| = N\frac{d\phi}{dt}$$

$$= N\frac{d}{dt}(BA\cos\theta)$$

$$= N\frac{dB}{dt} \cdot A\cos\theta$$

$$= 500 \times 1 \times (10 \times 10^{-2})^2 \cos 0°$$

$$= 5 \text{ V}$$

5. $e = -\frac{d\phi}{dt}$

$$|e| = \frac{d}{dt}(BA)$$

$$= A\frac{dB}{dt}$$

$$= 2 \times \frac{(4-1)}{2} = 3\text{V}$$

6. $\Delta q = -\frac{N}{R}\Delta\phi$

$\therefore \quad 32 \times 10^{-6} = -\frac{100}{(160+40)}(0 - B) \times \pi \times (6 \times 10^{-3})^2 \times \cos 0°$

$\therefore \quad B = 0.566 \text{ T}$

7. $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(3t^2 + 4t + 9)$

$$= -(6t + 4)$$

$$= -[6 \times (2) + 4] = -16$$

$\therefore \quad |e| = 16 \text{ V}$

8. Induced emf,

$$e = \frac{-NBA(\cos\theta_2 - \cos\theta_1)}{\Delta t}$$

$$= -\frac{800 \times 4 \times 10^{-5} \times 0.05(\cos 90° - \cos 0°)}{0.1}$$

$$= 0.016 \text{ V}$$

9. Magnetic flux,

$$\phi = B \cdot A$$

Change in flux, $d\phi = B \cdot dA$

$$= 0.05(101 - 100)10^{-4}$$

$$= 5 \times 10^{-6} \text{ Wb}$$

$\therefore \quad$ Charge, $dq = \frac{d\phi}{R} = \frac{5 \times 10^{-6}}{2} = 2.5 \times 10^{-6}$ C

10. Self inductance of coil,

$$|e| = L\frac{di}{dt}$$

$\therefore \quad 10 = L \times \frac{10}{1}$

$\therefore \quad L = 1 \text{ H}$

11. For growth of current in *L*-*R* circuit current is given by,

$$i = i_0(1 - e^{-Rt/L})$$

$\therefore \quad \frac{di}{dt} = -i_0\left(-\frac{R}{L}\right)e^{-Rt/L}$

$$= \frac{i_0 R}{L}e^{-Rt/L}$$

At $\frac{di}{dt} = \frac{i_0 R}{L} = \frac{E}{L}$

$\therefore \quad E = 20 \times 4 = 80 \text{ V}$

12. Peak value of emf generated in generator is given by

$$e_0 = \omega NBA$$

$$= (2\pi\nu)\, NBA$$

$$= 2 \times 3.14 \times 1000 \times 5000 \times 0.2 \times 0.25$$

$$= 157.1 \text{ kV}$$

13. Value of induced emf in coil,

$$e = M\frac{di}{dt}$$

$$= 0.005 \times \frac{d}{dt}(i_0 \sin \omega t)$$

$$= 0.005 \times i_0 \omega \cos \omega t$$

$\therefore \quad e_{\max} = 0.005 \times 10 \times 100\pi$

$$= 5\pi$$

16. $|e| = \frac{\Delta\phi}{\Delta t} = \frac{4B_0A_0 - B_0A_0}{t} = \frac{3B_0A_0}{t}$

17. $|e| = \frac{\Delta\phi}{\Delta t} = \frac{(2-1)\,(0.01)}{10^{-3}} = 10 \text{ V}$

$$i = \frac{e}{R} = 10^3 \text{ A}$$

$$H = eit = (10)\,(10^3)\,(10^{-3})$$

$$= 10 \text{ J}$$

18. $e = -\frac{d\phi}{dt} = -A \cdot \left(\frac{dB}{dt}\right) = -A$ (slope of B-t graph)

At $t = 2$, slope is zero and it changes its sign.

19. $\phi = NAB = (25)(\pi)(1.8 \times 10^{-2})^2 \ [0.2t - 0.05t^2]$

$= 0.025\ (0.2t - 0.05t^2)$

$e = \left|\frac{d\phi}{dt}\right| = |0.025\ (0.2 - 0.05t^2)|$

At $t = 3$ s, $e = 0.0025$ V

$$i = \frac{e}{R} = 0.0016 \text{ A}$$

$\therefore \quad P = ei$

$= 4 \times 10^{-6}$ W $= 4\mu$W

20. $|\Delta q| = \frac{\Delta\phi}{R} = \frac{(2)\left(\frac{\pi}{4}\right)(0.2)^2}{0.01} = 6.28$ C

21. $e = \left|\frac{d\phi}{dt}\right| = 12t - 5$

At $t = 0.25$ s

$$e = 2 \text{ V}$$

$\therefore \quad i = \frac{e}{R} = 0.2$ A

24. According to Lenz's law, the induced current will be in such a direction, so that it opposes the change due to which it is produced.

27. $\phi = 0$. Therefore, $\Delta\phi = 0$.

28. The magnet gets repelled. Therefore, North pole is formed on the top side.

31. In both the cases (a) and (c) the induced current in the ring will be in such a direction that it attracts the magnet.

32. **B**, **v** and **I** should be mutually perpendicular.

34. Induced emf

$$e = Bvl$$

$\therefore \quad e \propto v$

Also, $v = 0 + gt$

$\therefore \quad e \propto gt$

So, potential difference between its two ends will increase with time.

35. Induced emf between the axle and the rim,

$$e = \frac{1}{2} Bl^2\omega = \pi l^2 fB \ (\text{as } \omega = 2\pi f)$$

$$= 0.4 \times 10^{-4} \times (0.5)^2 \times (3.14) \times \frac{120}{60}$$

$$= 6.28 \times 10^{-5} \text{ V}$$

36. Induced emf, when rod rotates in a vertical plane perpendicular to magnetic field

$$e = \frac{1}{2} Bl^2\omega$$

$$= \frac{1}{2} \times 0.3 \times (2)^2 \times 100$$

$$= 60 \text{ V}$$

37. Potential difference

$$= Bvl = (2.0 \times 10^{-4})\left(360 \times \frac{5}{18}\right)(50) = 1.0 \text{ V}$$

42. $e = Bvl = (0.1)\ (1)\ (0.1) = \frac{1}{100}$ V

Net resistance $= 1 + \frac{2 \times 3}{2 + 3} = \frac{11}{5}\ \Omega$

$$\therefore \quad i = \left(\frac{\frac{1}{100}}{11/5}\right) = \frac{1}{220} \text{ A}$$

43. Potential difference $= Bvl = (0.2 \times 10^{-4})\ (20)\ (1)$ V $= 0.4$ mV

45. Energy stored in the coil,

$$E = \frac{1}{2} Li^2 = \frac{1}{2} \times 50 \times 10^{-3} \times 4$$

$$= 0.1 \text{ J}$$

46. Mutual inductance between two coils,

$$M = -\frac{e_2}{(di_1/dt)} = -\frac{e_1}{(di_2/dt)}$$

Also, $e_1 = -L_1 \frac{di_1}{dt}$

$$e_2 = -L_2 \frac{di_2}{dt}$$

$\therefore \quad M^2 = \frac{e_1 e_2}{\left(\frac{di_1}{dt}\right)\left(\frac{di_2}{dt}\right)} = L_1 L_2$

$\Rightarrow \quad M = \sqrt{L_1 L_2}$

47. Equivalent inductance,

$$\frac{1}{L_{eq}} = \frac{1}{L} + \frac{1}{L}$$

$\therefore \quad L_{eq} = \frac{L}{2}$

48. Self inductance of solenoid is given by,

$$\frac{\mu_0 N^2 A}{L}$$

49. Self-inductance of coil is directly proportional to square of number of turns in the coil, i.e.

$$L \propto N^2$$

$\therefore \quad \frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$

$\therefore \quad \frac{L}{L_2} = \frac{N^2}{(4N)^2}$

$\therefore \quad L_2 = 16L$

50. In the given circuit, three inductances are in parallel, their equivalent inductance,

$$\frac{1}{L_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \text{ or } L_{eq} = \frac{3}{3} = 1\text{H}$$

51. Mutual inductance between two coils depends on their degree of flux linkage, i.e. the fraction of flux linked with one coil which is when some current passes through the other coil. In figure (A) two coils with their planes are parallel. In this situation, maximum flux passes.

52. $L = \left|\frac{e}{di/dt}\right| = \frac{8}{(2/0.5)} = 2$ H

53. $L_P = \dfrac{L_1 L_2}{L_1 + L_2}$ and $L_S = L_1 + L_2$

54. $M = \dfrac{\Delta\phi_2}{\Delta i_1}$, because $\phi_2 = Mi_1$

$\therefore \quad \Delta\phi_2 = M(\Delta i_1)$

55. $e_2 = M\left(\dfrac{di_1}{dt}\right) = i_2 R_2$

$\therefore \quad \left(\dfrac{di_1}{dt}\right) = \dfrac{i_2 R_2}{M} = \dfrac{(0.4)\,(5)}{0.5} = 4\ \text{A/s}$

57. Growth of current in the circuit is given by

$$i = i_0(1 - e^{-Rt/L})$$

where, i_0 is peak value of current and

$$i_0 = \frac{5}{5} = 1\ \text{A}$$

$\therefore \quad i = 1\,(1 - e^{-5\times 2/10})$

$= (1 - e^{-1})\ \text{A}$

58. In case of growth of current in a *L*-*R* circuit, the current in the circuit grows exponentially with time 0 to the maximum value $i_0 = E / R$.

59. In one time constant current reduces to $\dfrac{1}{e}$ times.

60. Current in the circuit $i = V / R = 10 / 2 = 5\text{A}$

$\therefore$ Magnetic energy stored in the coil

$$U = \frac{1}{2}Li^2$$

$$= \frac{1}{2}\times 2\times (5)^2$$

$$= 25\ \text{J}$$

61. During growth of current in the coil

$$i = i_0(1 - e^{-Rt/L})$$

For $\quad i = \dfrac{i_0}{2}$

$$\frac{i_0}{2} = i_0\,(1 - e^{-Rt/L})$$

$\therefore \quad t = 0.693\,\dfrac{L}{R}$

$$= 0.693 \times \frac{300\times 10^{-3}}{2} = 0.1\ \text{s}$$

62. Growth of current in the circuit is

$$i = i_0(1 - e^{-Rt/L})$$

$\Rightarrow \quad \dfrac{di}{dt} = \dfrac{d}{dt}i_0 - \dfrac{d}{dt}i_0 e^{-Rt/L}$

$\therefore \quad \dfrac{di}{dt} = 0 - i_0\left(-\dfrac{R}{L}\right)e^{-Rt/L} = \dfrac{i_0 R}{L}e^{-Rt/L}$

Initially, $t = 0$,

$\therefore \quad \dfrac{di}{dt} = \dfrac{i_0 R}{L}$

$$= \frac{E}{L} = \frac{5}{2} = 2.5\ \text{A/s}$$

63. $E = \dfrac{1}{2}Li^2$

66. Just after pressing the key *K*, the inductor offers infinite resistance.

67. $\dfrac{L}{R} = 2.0\times 10^{-3}$ s

and $\quad \dfrac{L}{R+90} = 0.5\times 10^{-3}$ s

Solving these two equations, we get

$$L = 60\ \text{mH} \quad \text{and} \quad R = 30\ \Omega$$

68. $t_{1/2} = (\ln 2)\tau_2$

where, $\quad \tau_1 = \dfrac{L}{R} = 1\,\text{sec}$

$$t_{1/2} = (\ln 2)\,\text{sec}$$

In half life, half of the current grows in the circuit.

69. Four resistances form a balanced Wheatstone bridge.

70. The whole current will pass through the inductor.

72. $\tau_c = CR, \tau_L = \dfrac{L}{R}$ and $\omega = \dfrac{1}{\sqrt{LC}}$

73. $\omega = \dfrac{1}{\sqrt{LC}}$

$\therefore \quad f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{20\times 10^{-6}\times 10\times 10^{-3}}}$

$= 356$ cycles/s

74. $C_{\text{net}} = C/2$ (in series) and $L_{\text{net}} = 2L$ (in series)

76. $\omega = \dfrac{1}{\sqrt{LC}}$ or $f \propto \dfrac{1}{\sqrt{C}}$

$\therefore \quad \dfrac{f_2}{f_1} = \sqrt{\dfrac{C_1}{C_2}}$

$= \dfrac{10^4 - 50}{10^4} = \sqrt{\dfrac{1}{K}}$ (K = dielectric constant)

$\therefore \quad K = 1.01$

77. Induced emf in the circuit

$$e = M\frac{di}{dt} = (0.2)(5) = 1\ \text{V}$$

78. In step down transformer

$$V_p > V_s$$

but, $\quad \dfrac{V_p}{V_s} = \dfrac{i_s}{i_p}$

$\therefore \quad i_s > i_p$

So, current in the secondary coil is greater than the primary.

79. Transformation ratio

$$k = \frac{N_s}{N_p} = \frac{V_s}{V_p}$$

For step up transformer

$$N_s > N_p \quad \text{i.e.} \quad V_s > V_p$$

Hence, $k > 1$

80. If we neglect all losses, then input power = output power.

82. $\dfrac{N_p}{N_s} = \dfrac{V_p}{V_s} = \dfrac{i_s}{i_p}$

$\therefore \quad i_s = \dfrac{N_p i_p}{N_s} = \dfrac{4\times 140}{280} = 2\ \text{A}$

84. $\dfrac{N_1}{N_2} = \dfrac{i_2}{i_1}$

$\therefore \quad i_2 = \left(\dfrac{N_1}{N_2}\right)i_1 = \left(\dfrac{500}{2000}\right)\times 48 = 12\ \text{A}$

86. Power, (i.e. Pi) remains constant.

89. Magnetic flux

$$\phi = [B][A]$$

$$= \left[\frac{F}{iL}\right][A] \qquad (\text{as } F = BiL\sin\theta)$$

i.e. $$\phi = \left[\frac{\mathrm{MLT^{-2}}}{\mathrm{AL}}\right][\mathrm{L^2}]$$

$$= [\mathrm{ML^2T^{-2}A^{-1}}]$$

90. Magnetic flux linked with a vector area **A** in a uniform magnetic field **B** is given by,

$$\phi = \mathbf{B}\cdot\mathbf{A}$$

91. In such situation, Li = constant.

92. Induced emf, $e = L\dfrac{di}{dt}$

$\therefore$ $$L = \frac{e}{(di/dt)}$$

$\Rightarrow$ $$L = \frac{\mathrm{V\text{-}s}}{\mathrm{A}}$$

93. From $U = \dfrac{1}{2}Li^2$, $[L] = \left[\dfrac{U}{i^2}\right]$

and $$L = \frac{e}{di/dt}$$

$$L = \phi/i,\ [L] = [\phi/i][L] = [edt/di]$$

94. In L-R circuit current grows exponentially.

97. Apply Lenz's law.

98. It is a standard result.

101. At $t = 0$, current will flow from the capacitor and at $t = \infty$ current flows from the inductor.

Level 2 : Only One Correct Option

1. $P = F\cdot v = Fm\cdot v$

$$= il\,B\cdot v$$

$$= \left(\frac{Bvl}{R}\right)\cdot lB\cdot v$$

$$= \frac{B^2l^2v^2}{R}$$

2. $V_a - iR - L\dfrac{di}{dt} = V_b$

$\therefore$ $$V_a - V_b = iR + L\frac{di}{dt}$$

According to given conditions, we obtain

$$8 = 2R + L \qquad \ldots(\text{i})$$

$$4 = 2R - L$$

Solving these two equations, we get

$$R = 3\,\Omega$$

and $$L = 2\,\mathrm{H}$$

3. $V = L\dfrac{di}{dt}$ or $V \propto L$ $\left(\text{as } \dfrac{di}{dt} \to \text{same}\right)$

$\therefore$ $$\frac{V_1}{V_2} = \frac{L_1}{L_2} = 4$$

$$P = Vi = \text{constant}$$

$\therefore$ $$i \propto \frac{1}{V}$$

or $$\frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{1}{4}$$

$$W = \frac{1}{2}Li^2$$

$\therefore$ $$\frac{W_1}{W_2} = \frac{L_1}{L_2}\left(\frac{i_1}{i_2}\right)^2$$

$$= (4)\left(\frac{1}{16}\right) = \frac{1}{4}$$

4. $\tau_L = \dfrac{L}{R} = \dfrac{1}{2}\,\mathrm{s}$

$$i = i_0\,(1 - e^{-t/\tau_L})$$

$\therefore$ $$\frac{i_0}{i} = \frac{1}{1 - e^{-t/\tau_L}}$$

Substituting, $t = 1\,\mathrm{s}$ and $\tau_L = \dfrac{1}{2}\,\mathrm{s}$

We get, $$\frac{i_0}{i} = \frac{e^2}{e^2 - 1}$$

5. For a capacitor,

$i = \left(\dfrac{dq}{dt}\right) = C\cdot\left(\dfrac{dV}{dt}\right)$, i.e. i = constant, if $\dfrac{dV}{dt}$ = constant

$\therefore$ $$C = \frac{i}{\left(\frac{dV}{dt}\right)} = \frac{1}{2} = 0.5\,\mathrm{F}$$

6. $V = 2t$

$\therefore$ $$L\cdot\frac{di}{dt} = 2t$$

or $$2\cdot\frac{di}{dt} = 2t$$

or $$\frac{di}{dt} = t$$

or $$(di) = t\,(dt)$$

Integrating, we get

$$i = \frac{t^2}{2}$$

i.e. i-t graph is a parabola.

At $t = 2\,\mathrm{s}$, $i = 2\,\mathrm{A}$

$\therefore$ $$U = \frac{1}{2}Li^2 = \frac{1}{2}\times 2\times 4 = 4\,\mathrm{J}$$

$$\frac{dU}{dt} = Li\left(\frac{di}{dt}\right)$$

$$= (2)\left(\frac{t^2}{2}\right)(t) = t^3$$

At $t = 1\,\mathrm{s}$, $\dfrac{dU}{dt} = 1\,\mathrm{J/s}$

7. $El = \dfrac{d\phi}{dt}$

or $$E(2\pi R) = \pi R^2\cdot\frac{dB}{dt} = \pi R^2\,(8t)$$

$\therefore$ $$E = 4Rt$$

$$F = qE = 4qRt \qquad (\text{tangential})$$

$$\tau_F = 4qR^2t$$

$$\tau_f = (\mu mgR)$$

When $\tau_F > \tau_f$, ring will start rotating.

At 2 s,

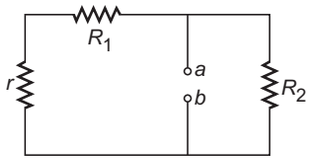
$$8qR^3 = \mu mgR$$

$$\therefore \quad \mu = \frac{8qR}{mg}$$

8. Since, you are decreasing current in the circuit by increasing resistance of the circuit.

Induced emf across inductor will support 20 V battery. Hence, net emf of the circuit is greater than 20 V or current in the circuit is more than 4 A.

9. After short circuiting the battery.



$$R_{ab} = \frac{R_2 (R_1 + r)}{R_2 + R_1 + r}$$

$$\therefore \quad \tau_L = \frac{L}{R_{\text{net}}} = \frac{L(R_1 + R_2 + r)}{(R_1 + r) R_2}$$

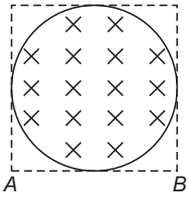
10. $I = \dfrac{\text{Potential difference}}{\text{Resistance}}$

$$\therefore \quad \text{Potential difference} = 2 \times 12 = 24 \text{ V} = B v l$$

Here, $l = AD \sin 37° = 0.3 \times \frac{3}{5} = 0.18$ m

$$\therefore \quad v = \frac{24}{Bl} = \frac{24}{4 \times 1.08} = \frac{100}{3} \text{ m/s}$$

11. $V_{ba} = \int \mathbf{E} \cdot d\mathbf{l} = \frac{1}{4}\left(\frac{d\phi}{dt}\right)$

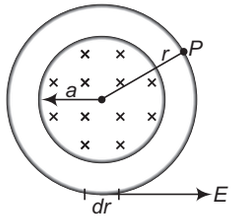


$$= \frac{1}{4}(\pi R^2) \cdot \frac{dB}{dt} = \frac{\pi R^2 \alpha}{4}$$

12. Magnetic lines on circular loop, due top current in wire AB are tangnetial to its plane.

$$\therefore \quad \phi = 0 \text{ or } \frac{d\phi}{dt} = 0 \text{ or induced emf} = 0.$$

13.



Draw a concentric circle of radius r. The induced electric field (E) at any point on the circle is equal to that at P.

For this circle, induced emf

$$e = \oint \mathbf{E} \cdot d\mathbf{l} = \left|\frac{d\phi}{dt}\right| = A\left|\frac{dB}{dt}\right|$$

$$\therefore \quad E = \oint dl = \pi a^2 \left|\frac{dB}{dt}\right| \quad (\text{but, } \oint dl = 2\pi r)$$

$$\therefore \quad E \times (2\pi r) = \pi a^2 \left|\frac{dB}{dt}\right|$$

$$\therefore \quad E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right|$$

$$\Rightarrow \quad E \propto \frac{1}{r}$$

14. According to Fleming's right hand rule P is at higher potential and Q is at lower potential. Therefore, A is positively charged and B is negatively charged.

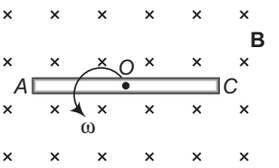
Also, charge

$$Q = CV = C(Bvl)$$
$$= 10 \times 10^{-6} \times 4 \times 2 \times 1 = 80 \mu\text{C}$$

$$\therefore \quad q_A = 80 \mu\text{C}$$

and $\quad q_B = -80 \mu\text{C}$

15. For rotating rod, induced emf



$$e = \frac{1}{2} Bl^2 \omega$$

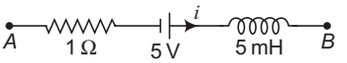
For part AO, $\quad e_{OA} = e_O - e_A = \frac{1}{2} Bl^2 \omega$

For part OC,

$$e_{OC} = e_O - e_C = \frac{1}{2} B(3l)^2 \omega$$

$$\therefore \quad e_A - e_C = 4Bl^2 \omega$$

16. Applying Kirchhoff's second law to above circuit,



$$V_A - iR + 15 - L\frac{di}{dt} = V_B$$

or $\quad V_A - 5 \times 1 + 15 - (5 \times 10^{-3})(-10^3) = V_B$

$$\therefore \quad V_B - V_A = 15 \text{ V}$$

17. In L-R circuit, for growth of current,

$$i = i_0(1 - e^{-Rt/L})$$

or $\quad \frac{3}{4} i_0 = i_0(1 - e^{-t/\tau})$

(where, $\tau = \frac{L}{R}$ = time constant)

$$\therefore \quad e^{-t/\tau} = 1 - \frac{3}{4} = \frac{1}{4}$$

$$e^{t/\tau} = 4 \text{ or } \frac{t}{\tau} = \log 4$$

$$\tau = \frac{t}{\log 4} = \frac{4}{2 \log 2}$$

$$\tau = \frac{2}{\log 2} \text{ s}$$

18. Charge induced in coil

$$dq = \frac{d\phi}{R} = i\,dt = \text{Area under } i\text{-}t \text{ graph}$$

$$\therefore \quad d\phi = (\text{Area under } i\text{-}t \text{ graph})R$$

$$= \frac{1}{2} \times 4 \times 0.1 \times 10 = 2 \text{ Wb}$$

19. $E = -L\dfrac{di}{dt}$

Induced emf $\propto$ – Slope of i-t graph.

20. At $t = 0$, induced emf e is maximum and is equal to applied emf, but current i is zero.

After lapse of time, current through the circuit increases but induced emf decreases.

21. $U = \dfrac{1}{2}Li_0^2 = \dfrac{1}{2} \times 50 \times 10^{-6}\left(\dfrac{5}{0.5}\right)^2 \text{J} = 2.5 \text{ mJ}$

Now, after switching off the battery this energy is dissipated in coil (resistance = 0.5) and resistance (10 Ω) in the ratio of their resistances ($H = i^2Rt$ or $H \propto R$). Therefore, heat generated in the coil.

$$H = 2.5\left(\frac{0.5}{0.5 + 10}\right)\text{mJ}$$

$$= 0.12 \text{ mJ}$$

22. $M_{12} = M_{21}$, when rate of charge of current is doubled, induced emf also become two times.

23. $\dfrac{di}{dt} = \omega^2 q = \dfrac{q}{LC} = \dfrac{3.0 \times 10^{-5}}{0.6 \times 25 \times 10^{-6}} = 2 \text{ A/s}$

24. $q = (2\mu\text{F})\ (6.0 \text{ V}) = 12\mu\text{C}$,

$q_0 = (2\mu\text{F})\ (12 \text{ V}) = 24\mu\text{C}$,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.6 \times 10^{-3} \times 2 \times 10^{-6}}}$$

$$= 2.9 \times 10^4 \text{ rad/s}$$

$$i = \omega\sqrt{q_0^2 - q^2}$$

$$= (2.9 \times 10^4)\ (10^{-6})\sqrt{(24)^2 - (12)^2}$$

$$= 0.6 \text{ A}$$

25. $P = (i_0)^2 \cdot R$ i.e. $(i_0)^2 = \dfrac{P}{R}$

$$U = \frac{1}{2}Li_0^2 = \frac{1}{2}(\tau R)\left(\frac{P}{R}\right) = \frac{1}{2}P\tau$$

26. Here, $A = L^2\,\hat{\mathbf{k}}$ and $\mathbf{B} = B_0\,(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ T

$$\phi = \mathbf{B}.\mathbf{A} = B_0(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}).L^2\hat{\mathbf{k}} = 4B_0L^2 \text{ Wb}$$

27. The self-inductance of a long solenoid of cross-sectional area A and length l, having n turns per unit length, filled the inside of the solenoid with a material of relative permeability (*e.g.*, soft iron, which has a high value of relative permeability) is given by

$$L = \mu_r\mu_0 n^2 Al$$

where, $n = N/l$

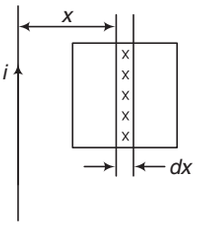
More than One Correct Options

1. $e = Bvl$

where, $l = \dfrac{L}{2}$

For polarity of this motional emf we can use right hand rule.

2. ***(a)***



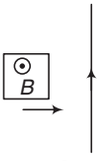
$$B_x = \frac{\mu_0}{2\pi}\frac{i}{x}$$

$$d\phi = (B_x)\,dS = \left(\frac{\mu_0}{2\pi}\frac{i}{x}\right)(adx)$$

$$\phi = \int_a^{2a} d\phi = \frac{\mu_0 ia}{2\pi}\ln 2$$

$$M = \frac{\phi}{i} = \frac{\mu_0 a}{2\pi}\ln 2$$

(c)



Wire produces ⊙ magnetic field over the loop. If the loop is brought closer to the wire ⊙ magnetic field passing through the loop increases. Hence, induced current produces ⊗ magnetic field so, induced current is clockwise.

3. ***(a)*** $L = \dfrac{N\phi}{i}$

$$\therefore \quad \phi = \frac{Li}{N}$$

So, SI unit of flux is H-A.

(c) $L = \dfrac{-e}{\Delta i/\Delta t} = -\dfrac{-e\Delta t}{\Delta i}$

Hence, SI unit of L is $\dfrac{\text{V - s}}{\text{A}}$.

4. $\tau_L = \dfrac{L}{R} = \dfrac{2}{2} = 1 \text{ s}$

$$t_{1/2} = (\ln 2)\,\tau_L = (\ln 2) \text{ s}$$

Hence, the given time is half-life time.

$$\therefore \quad i = \frac{i_0}{2} = \frac{8/2}{2} = 2 \text{ A}$$

Rate of energy supplied by battery $= Ei = 8 \times 2 = 16$ J/s

$$P_R = i^2R = (2)^2(2) = 8 \text{ J/s}$$

$$V_a - V_b = E - iR = 8 - 2 \times 2 = 4 \text{V}$$

5. According to Lenz's law, induced effects always oppose the change i_1 and i_2 both are in same direction. Hence, magnetic lines from B due to both currents are from right to left. By bringing A closer to B or increasing i_1 right to left magnetic field from B will increase. So, i_2 should decrease.

6. $\phi_i = BS\cos 0° = (4)\,(2) = 8 \text{ Wb}$

$$\phi_f = BS\cos 90° = 0$$

$$\Delta\phi = 8 \text{ Wb}$$

$$|e| = \frac{\Delta\phi}{\Delta t} = \frac{8}{0.1} = 80 \text{ V}$$

$$i = \frac{|e|}{R} = 20 \text{ A}$$

$$\Delta q = \frac{\Delta\phi}{R} = 2 \text{ C}$$

This current is not constant. So, we cannot find the heat generated unless current function with time is not known.

7. $i_{\max} = \omega q_0 = \left(\frac{1}{\sqrt{LC}}\right) q_0$

$\left(\frac{di}{dt}\right)_{\max} = \omega^2 q_0 = \left(\frac{1}{LC}\right) q_0$

8. If ⊗ magnetic field increase than induced electric lines are anti-clockwise. If ⊗ magnetic field decreases, then induced electric lines are clockwise (both inside and outside the cylindrical region).

On positive charge, force is in the direction of **E**. On negative charge, force is in the opposite direction of **E**.

9. $q = 2t^2$

$$i = \frac{dq}{dt} = 4t$$

$$\frac{di}{dt} = 4\ \text{A/s}$$

At $t = 1$ s, $q = 2$ C, $i = 4$ A

and $\frac{di}{dt} = 4$ A/s

$$V_a - V_b = L\frac{di}{dt} = 1 \times 4 = 4\ \text{V}$$

$$V_b - V_c = \frac{q}{C} = \frac{2}{2} = 1\ \text{V}$$

$$V_c - V_d = iR = 4 \times 4 = 16\ \text{V}$$

$V_a - V_d$ is summation of above three i.e. 21 V.

10. $V_a - V_c = 0$ as **I** is parallel to **v**.

$V_a - V_b = V_c - V_b = \frac{B\omega l^2}{2}$

11. A metal plate is getting heated when a DC or AC current is passed through the plate, known as heating effect of current. Also, when metal plate is subjected to time varying magnetic field, the magnetic flux linked with the plate changes and eddy currents comes into existence which make the plate hot.

12. Here, magnetic flux linked with the isolated coil change when the coil being in a time varying magnetic field, the coil moving in a constant magnetic field or in time varying magnetic field.

13. When circular coil expands radially in a region of magnetic field such that the magnetic field is in the same plane as the circular coil or the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably in such a way that the cross product of magnetic field and surface area of plane of coil remain constant at every instant.

Comprehension Based Questions

1. $\frac{dB}{dt} = (6t^2 + 24)\ \text{T/s}$

At $t = 2$ s, $\frac{dB}{dt} = (48)\ \text{T/s}$

$$El = \frac{d\phi}{dt} = S\left(\frac{dB}{dt}\right) \quad \text{or} \quad E(2\pi r) = \pi r^2 \left(\frac{dB}{dt}\right)$$

$$\therefore \quad E = \frac{r}{2} \cdot \frac{dB}{dt} \qquad \ldots(\text{i})$$

$$F = qE = \frac{qr}{2}\frac{dB}{dt}$$

$$= \frac{(1.6 \times 10^{-19})\ (1.25 \times 10^{-2})}{2}\ (48)$$

$$= 48 \times 10^{-21}\ \text{N}$$

2. From Eq. (i) of above problem we can see that,

$$E \propto r$$

i.e. E - r graph is a straight line passing through origin.

3. ⊗ Magnetic field is increasing. Hence, ⊙ magnetic field is produced by a conducting circular loop placed there. For producting. Magnetic field induced current should be anti-clockwise. Direction of induced circular electric lines are also anti-clockwise.

4. $|e| = \left|\frac{d\phi}{dt}\right| = S \cdot \frac{dB}{dt} = (\pi a^2)\ B_0$

5. $El = \frac{d\phi}{dt}$

$$\therefore \quad E(2\pi a) = (\pi a^2) B_0$$

or $$E = \frac{1}{2} a B_0$$

6. $F = qE = \frac{1}{2} qaB_0$

$$\tau = F \cdot a = \frac{1}{2} qa^2 B_0$$

$$\alpha = \frac{\tau}{I} = \frac{\left(\frac{1}{2} qa^2 B_0\right)}{ma^2}$$

$$= \frac{2qB_0}{2m}$$

7. $\omega = \alpha t = \left(\frac{qB_0}{2m}\right) t$

$$P = \tau \cdot \omega = \left(\frac{1}{2} qa^2 B_0\right)\left(\frac{qB_0}{2m}\right)$$

$$= \frac{q^2 B_0^2 a^2}{4m}$$

8. $|e| = \frac{d\phi}{dt} = S \cdot \frac{dB}{dt}$

$$= (0.2 \times 0.4)\ (2) = 0.16\ \text{V}$$

$$i = \frac{|e|}{R} = \frac{0.16}{(1)\ (40 + 40 + 20) \times 10^{-2}}$$

$$= 0.16\ \text{A}$$

⊙ Magnetic field passing through the loop is increasing. So, induced current should produce ⊗ magnetic field. Hence, induced current is clockwise.

9. At $t = 2$ s, rod will move 10 cm. Hence 40 cm side will become 30 cm.

$$|e| = e_1\ (\text{say}) = S\left(\frac{dB}{dt}\right)$$

$$(0.2 \times 0.3)\ (2) = 0.12\ \text{V}$$

At $t = 2$ s, $B = 4$ T

$$\therefore \quad e_2 = Bvl$$

$$= (4)\ (5 \times 10^{-2})\ (0.2)$$

$$= 0.04\ \text{V}$$

$$\therefore \quad e_{\text{net}} = e_1 - e_2 = 0.08\ \text{V}$$

10. $i = \frac{e_{\text{net}}}{R} = \frac{0.08}{(1)\ (30 + 30 + 20) \times 10^{-2}}$

$$= 0.1\ \text{A}$$

$$F = ilB$$

$$= (0.1)\ (0.2)\ (4)$$

$$= 0.08\ \text{N}$$

11 to 13.

At terminal velocity

$$iLB = mg$$

$$\therefore \quad i = \frac{mb}{LB} = \frac{0.2 \times 98}{1 \times 0.6}$$

$$i = 3.27 \text{ A} \qquad \ldots(\text{i})$$

$$e = Bv L \qquad (v = \text{terminal velocity})$$

$$= (0.6)\,(v)\,(1)$$

$$e = 0.6\,v$$

$$P_{R_1} = \frac{e^2}{R_1}$$

$$\therefore \quad 0.76 = \frac{0.36\,v^2}{R_1} \qquad \ldots(\text{ii})$$

$$P_{R_2} = \frac{e^2}{R_2}$$

$$\therefore \quad 1.2 = \frac{0.36\,v^2}{R_2} \qquad \ldots(\text{iii})$$

R_1 and R_2 are in parallel.

$$\therefore \quad R_{\text{net}} = \frac{R_1 R_2}{R_1 + R_2} \qquad \ldots(\text{iv})$$

$$i = \frac{e}{R_{\text{net}}} \qquad \ldots(\text{v})$$

Solving these five equations, we can get the results.

Assertion and Reason

3. Since, the capacities of two capacitors are same, in equilibrium charges on them should be same.

4. Electric flux = ES and magnetic flux = BS. Both have different units.

5. Although flux is obtained by integration but that remains constant with time if current through the straight wire is constant.

11. In non-uniform magnetic field magnetic flux will be obtained by integration, but it will not vary with time.

12. $\phi = BA\cos\omega t$

At $t = 0$, $\phi =$ maximum

$$|e| = \left|\frac{d\phi}{dt}\right| = B\,\omega A \sin\omega t$$

$|e|_{\max}$ when $\omega t = 90°$, i.e. loop is rotated 90°, or it is parallel to magnetic field. At this instant $\phi = 0$.

13. According to Lenz's law, induction effects always oppose the cause.

14. If left to right current is increasing, then induced battery $\left(\text{emf} = L \cdot \frac{di}{dt}\right)$ will produce right to left current, i.e. its positive terminal is on left hand side or $V_A > V_B$.

Further, $i =$ constant $\Rightarrow \frac{di}{dt} = 0$ or $V_L = 0$.

15. L depends upon the dimensions of the inductor and the magnetic material filled in between them.

16. Lenz's law

18. $V = L\frac{di}{dt} =$ constant

$$\therefore \quad L\,(di) = \text{constant}$$

$$\Rightarrow \quad Li = \text{constant}$$

or $$i \propto \frac{1}{L}$$

19. $\phi = \boldsymbol{Li} \;\Rightarrow\; \phi \propto \boldsymbol{i}$.

20. $L = \frac{-e}{di/dt}$ and $\phi = Li$

Entrance Gallery

1. By energy stored in an inductor,

$U = \frac{1}{2}Li_0^2$ (L = inductance of inductor, i_0 = current)

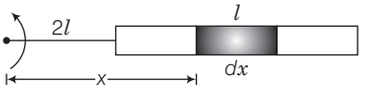
where, $L = 2$ H, $i_0 = 1$ A

$$\therefore \quad U = \frac{1}{2} \times 2 \times 1^2 = 1 \text{ J}$$

2. Emf induced

$$e = \int_{2l}^{3l} (\omega x)\,B\,dx = B\omega\,\frac{[(3l)^2 - (2l)^2]}{2}$$

$$= \frac{5Bl^2\omega}{2}$$



3. We know that

Power in the secondary coil = Power in the primary coil

$$V_S \times N_P = V_P \times N_S$$

or $$\frac{N_S}{N_P} = \frac{V_S}{V_P}$$

(V_S = Voltage across secondary coil)

(V_P = Voltage across primary coil)

(N_P = Number of turns is primary coil)

(N_S = Number of turns in secondary coil)

where, $N_S = 50$ turn, $N_P = 1000$ turn, $V_P = 220$ V, $V_S = ?$

$$\frac{50}{1000} = \frac{V_S}{220}$$

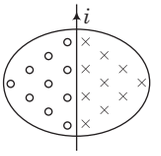
$$\Rightarrow \quad V_S = \frac{50 \times 220}{1000} = 11\,V$$

But $$V_S I_S = V_P I_P$$

$$\Rightarrow \quad 11 \times I_S = 220 \times 1$$

$$I_S = \frac{220}{11} = 20 \text{ A}$$

4. Due to the current in the straight wire, net magnetic flux from the circular loop is zero. Because, in half of the circle, magnetic fields are inwards and in other half, magnetic fields are outwards. Therefore, change in current will not cause any change in magnetic flux from the loop. Therefore, induced emf under all conditions through the circular loop is zero.



5. If I current flows through the circular loop, then magnetic flux at the location of square loop is

$$B = \frac{\mu_0 I R^2}{2(R^2 + Z^2)^{3/2}}$$

Substituting the value of $Z = (\sqrt{3}R)$, we have

$$B = \frac{\mu_0 I}{16R}$$

Now, total flux through the square loop is

$$\phi_T = NBA\cos\theta = (2)\left(\frac{\mu_0 I}{16R}\right)a^2 \cos 45°$$

Mutual inductance, $M = \frac{\phi_T}{I} = \frac{\mu_0 a^2}{2^{7/2} R}$ …(i)

$$M' = \frac{\mu_0 a^2}{2^{P/2} R} \quad \text{…(ii)}$$

On comparing the Eqs. (i) and (ii), we have

$$\frac{p}{2} = \frac{7}{2}$$

$$\Rightarrow \quad p = 7$$

6. Induced emf, $e = B_H lv = 5.0 \times 10^{-5} \times 2 \times 1.50$

$$= 0.15 \times 10^{-3} \text{ V}$$

$$= 0.15 \text{ mV}$$

7. Induced emf, $e = B_H lv = 0.30 \times 10^{-4} \times 20 \times 5.0 = 3 \text{ mV}$

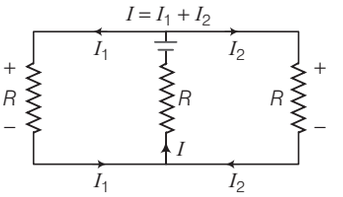
8. After lamination of the core of a transformer eddy current reduces. Because transformer are basically two coils of wire wrapped around a core of iron. They work by induction. Induction works when current flows in one conductor and the magnetic field forms around that. Conductor sweeps the other conductor and induces the voltage. If we did not laminate core, the core would provide a place for the magnetic lines to produce current and that current in the core would heat and the core really fast and waste energy. So, by laminating the cores, we break up the current paths with in core and limited eddy current.

9. A transformer works on the principle of mutual induction.

10. A moving conductor is equivalent to a battery of emf

$$= vBl \quad \text{(motional emf)}$$

Equivalent circui is shown in below figure.



Applying Kirchhoff's law,

$$I_1 R + IR - vBl = 0 \quad \text{…(i)}$$

$$I_2 R + IR - vBl = 0 \quad \text{…(ii)}$$

Adding Eqs. (i) and (ii), we get

$$2IR + IR = 2vBl$$

$$I = \frac{2vBl}{3R}$$

$$\therefore \quad I_1 = I_2 = \frac{vBl}{3R}$$

11. Given, efficiency, $\eta = 90\%$

Input power, $P_i = 4$ kW

Output power, $P_0 = 0.9 \times 4 = 3.6$ kW

As, $P = I^2 R$

$$\therefore \quad R = \frac{3.6 \text{ kW}}{36}$$

$$= \frac{3.6 \times 10^3}{36}$$

$$= 100\,\Omega \quad [\because Z = R]$$

12. As, the shape of the loop is changing and hence the flux linked with the loop changes. There will an induced emf and hence induced current in the coil. Applying right hand screw rule, we get induced current in anti-clockwise direction.

23

Alternating Current

23.1 Introduction

A century ago, one of the great technological debates was whether the electrical distribution system should be AC or DC. Thomas Edison favoured Direct Current (DC), i.e. steady current that does not vary with time. George Westinghouse favoured Alternating Current (AC), with sinusoidally varying voltages and currents. He argued that transformers can be used to step the voltage up or down with AC but not with DC. Low voltages are safer for consumer use, but high voltages and correspondingly low currents are best for long distances power transmission to minimize i^2R losses in the cables. Eventually, Westinghouse prevailed and most present day household and industrial power distribution systems operate with alternating current.

A time varying current or voltage may be periodic and non-periodic. In case of periodic current or voltage, the current or voltage is said to be alternating, if its amplitude is constant and alternate half cycle is positive and half negative. If the current or voltage varies periodically as sine or cosine function of time, the current or voltage is said to be sinusoidal and is what we usually mean by it.

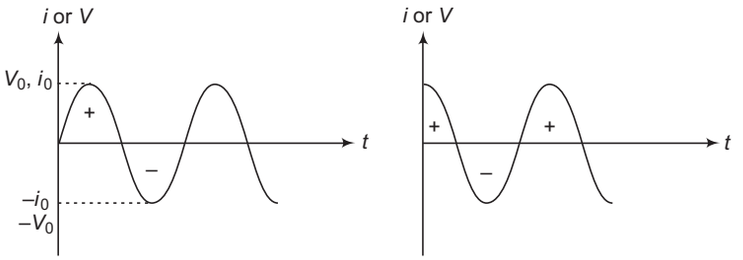


Fig. 23.1

Chapter Snapshot

- Introduction
- Alternating Current and Phasors
- Current and Potential Relations
- Phasor Algebra
- Series *L*-*R* Circuit
- Series *C*-*R* Circuit
- Series *L*-*C*-*R* Circuit
- Parallel Circuit (Rejector Circuit)
- Power in an AC Circuit
- Choking Coil
- Transformer

23.2 Alternating Current and Phasors

The basic principle of the AC generator is a direct consequence of Faraday's law of induction. When a conducting loop is rotated in a magnetic field at constant angular frequency ω, a sinusoidal voltage (emf) is induced in the loop.

This instantaneous voltage is given by

$$V = V_0 \sin \omega t \qquad \ldots(i)$$

The usual circuit diagram symbol for an AC source is shown in Fig. 23.2.



Fig 23.2

In Eq. (i), V_0 is the maximum output voltage of the AC generator or the **voltage amplitude** and ω is the **angular frequency**, equal to 2π times the frequency f.

$$\omega = 2\pi f$$

The frequency of AC in India is 50 Hz, i.e.

$$f = 50 \text{ Hz}$$

So, $$\omega = 2\pi f \approx 314 \text{ rad/s}$$

The time of one cycle is known as **time period** T, the number of cycles per second the frequency f.

$$T = \frac{1}{f} \quad \text{or} \quad T = \frac{2\pi}{\omega}$$

A sinusoidal current might be described as,

$$i = i_0 \sin \omega t$$

If an alternating current is passed through an ordinary ammeter or voltmeter, it will record the mean value for the complete cycle, as the quantity to be measured varies with time. The **average value** of current for one cycle is

$$\langle i \rangle_{\text{one cycle}} = (i_{\text{av}})_T = \frac{\int_0^T i\,dt}{\int_0^T dt} = \frac{\int_0^{2\pi/\omega} (i_0 \sin \omega t)\,dt}{\int_0^{2\pi/\omega} dt} = 0$$

Thus, $$\langle i \rangle_{\text{one cycle}} = 0$$

Similarly, the average value of the voltage (or emf) for one cycle is zero. $\langle V \rangle_{\text{one cycle}} = 0$

Since these averages for the whole cycle are zero, the DC instrument will indicate zero deflection. In AC, the average value of current is defined as its average taken over half the cycle. Hence,

$$\langle i \rangle_{\text{Half cycle}} = (i_{\text{av}})_{T/2} = \frac{\int_0^{T/2} i\,dt}{\int_0^{T/2} dt} = \frac{\int_0^{\pi/\omega} (i_0 \sin \omega t)\,dt}{\int_0^{\pi/\omega} dt} = \frac{2}{\pi} i_0$$

This is sometimes simply written as, i_{av}.

Hence, $$i_{\text{av}} = \langle i \rangle_{\text{Half cycle}} = \frac{2}{\pi} i_0 \approx 0.637\, i_0$$

Similarly, $$V_{\text{av}} = \frac{2}{\pi} V_0 \approx 0.637 V_0$$

A DC meter can be used in an AC circuit, if it is connected in the full wave rectifier circuit. The average value of the rectified current is the same as the average current in any half cycle, i.e. $\frac{2}{\pi}$ times the maximum current i_0.

A more useful way to describe a quantity is the **root mean square (rms)** value. We square the instantaneous current, take the average (mean) value of i^2 and finally take the square root of that average. This procedure defines the root-mean-square current denoted as i_{rms}. Even when i is negative, i^2 is always positive so, i_{rms} is never zero (unless i is zero at every instant). Hence,

$$\langle i^2 \rangle_{\text{one cycle}} = \frac{\int_0^T i^2\,dt}{\int_0^T dt} = \frac{\int_0^{2\pi/\omega} (i_0^2 \sin^2 \omega t)\,dt}{\int_0^{2\pi/\omega} dt} = \frac{i_0^2}{2}$$

$$\therefore \qquad i_{\text{rms}} = \sqrt{\langle i^2 \rangle}_{\text{one cycle}} = \frac{i_0}{\sqrt{2}} \approx 0.707\, i_0$$

Thus, $$i_{\text{rms}} = \frac{i_0}{\sqrt{2}} \approx 0.707 i_0$$

Similarly, we get

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \approx 0.707 V_0$$

The square root of the mean square value is called the **virtual value** and is the value given by AC instruments.

Thus, when we speak of our house hold power supply as 220 V AC, this means that the rms voltage is 220 V and its voltage amplitude is given by

$$V_0 = \sqrt{2}\, V_{\text{rms}} = 311 \text{ V}$$

Form Factor

The ratio, $$\frac{\text{rms value}}{\text{average value}} = \frac{V_0/\sqrt{2}}{2V_0/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$

is known as form factor.

The different values i_0, i_{av} and i_{rms} are shown in Fig. 23.3.

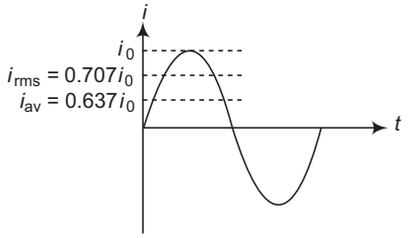


Fig. 23.3

Note Points

- The average value of $\sin \omega t$, $\cos \omega t$, $\sin 2\omega t$, $\cos 2\omega t$, etc, is zero because it is positive half of the time and negative rest half of the time. Thus,

  $\langle \sin \omega t \rangle = \langle \cos \omega t \rangle = \langle \sin 2\omega t \rangle = \langle \cos 2\omega t \rangle = 0$

  ∴ If $i = i_0 \sin \omega t$

  Then $\langle i \rangle = \langle i_0 \sin \omega t \rangle = i_0 \langle \sin \omega t \rangle = 0$

- The average value of $\sin^2\omega t$ and $\cos^2\omega t$ is $\frac{1}{2}$

  or $\langle \sin^2\omega t \rangle = \langle \cos^2\omega t \rangle = \frac{1}{2}$

  $\therefore$ If $i^2 = i_0^2 \sin^2\omega t$

  then $\langle i^2 \rangle = \langle i_0^2 \sin^2\omega t \rangle = i_0^2 \langle \sin^2\omega t \rangle = \frac{i_0^2}{2}$

- Like SHM, general expressions of current/voltage in and sinusoidal AC are,

  $i = i_0 \sin(\omega t \pm \phi), V = V_0 \sin(\omega t \pm \phi)$

  or $i = i_0 \cos(\omega t \pm \phi)$ and $V = V_0 \cos(\omega t \pm \phi)$

Phasors

If an AC generator is connected to a series circuit containing resistors, inductors and capacitors, we want to know the amplitude and time characteristics of the alternating current. To simplify our analysis of circuits containing two or more elements, we use graphical constructions called **phasor diagrams.** In these constructions, alternating (sinusoidal) quantities, such as current and voltage are rotating vectors called **phasors.**

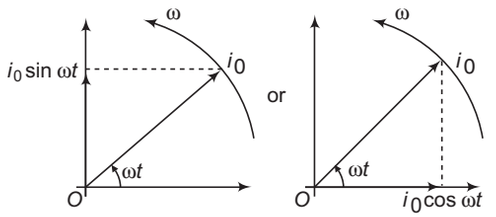


Fig. 23.4

In these diagrams, the instantaneous value of a quantity that varies sinusoidally with time is represented by the projection onto a vertical axis (if it is a sine function) or onto a horizontal axis (if it is a cosine function) of a vector with a length equal to the amplitude (i_0) of the quantity. The vector rotates counter clockwise with constant angular velocity ω.

A phasor is not a real physical quantity with a direction in space, such as velocity, momentum or electric field. Rather, it is a geometric entity that helps us to describe and analyse physical quantities that vary sinusoidally with time.

Example 23.1 *If the current in an AC circuit is represented by the equation,*

$$i = 5 \sin(300t - \pi/4),$$

Here, t is in second and is in ampere. Calculate,

(a) peak and rms value of current.

(b) frequency of AC.

(c) average current.

Sol. (a) As in case of AC, $i = i_0 \sin(\omega t \pm \phi)$

$\therefore$ The peak value $i_0 = 5$ A

and $i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

$= 3.535$ A

(b) Angular frequency

$\omega = 300$ rad / s

$\therefore$ $f = \frac{\omega}{2\pi} = \frac{300}{2\pi} \approx 47.75$ Hz

(c) $i_{av} = \left(\frac{2}{\pi}\right) i_0 = \left(\frac{2}{\pi}\right)(5) = 3.18$ A

23.3 Current and Potential Relations

In this section, we will derive voltage current relations for individual circuit elements carrying a sinusoidal current. We will consider resistors, inductors and capacitors.

Resistor in an AC Circuit

Consider a resistor with resistance R through which there is a sinusoidal current given by,

$$i = i_0 \sin\omega t \quad \ldots(i)$$

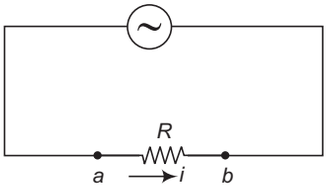


Fig. 23.5

Here, i_0 is the current amplitude (maximum current). From Ohm's law, the instantaneous potential difference between points a and b is,

$$V_R = iR = (i_0 R)\sin\omega t$$

We can write as,

$$i_0 R = V_0, \quad \text{(the voltage amplitude)}$$

$\therefore$ $$V_R = V_0 \sin\omega t \quad \ldots(ii)$$

From Eqs. (i) and (ii), we can see that current and voltage are in phase if only resistance is in the circuit. Fig.23.6 shows graphs of i and V_R as functions of time.

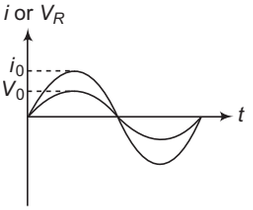


Fig. 23.6

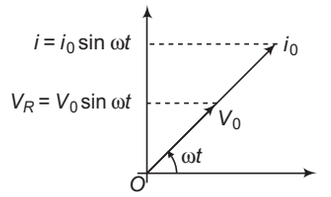


Fig. 23.7

The corresponding phasor diagram is also shown in Fig. 23.7.

Because i and V_R are in phase and have the same frequency, the current and voltage phasors rotate together, they are parallel at each instant. Their projection on vertical axis represent the instantaneous current and voltage respectively.

- Direction of an alternating current is not shown in a circuit, as it keeps on changing. In the figure, direction of instantaneous current is only shown.

Capacitor in an AC Circuit

If a capacitor of capacitance C is connected across the alternating source, the instantaneous charge on the capacitor,

$$q = CV_C = CV_0 \sin \omega t$$

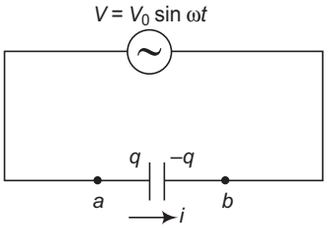


Fig. 23.8

and the instantaneous current i passing through it, is given by

$$i = \frac{dq}{dt} = CV_0 \omega \cos \omega t = \frac{V_0}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

or $$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Here, $$V_0 = \frac{i_0}{\omega C}$$

This relation shows that the quantity $\frac{1}{\omega C}$ is the effective, AC resistance or the **capacitive reactance** of the capacitor and is represented as X_C. It has unit as ohm. Thus,

$$X_C = \frac{1}{\omega C}$$

It is clear that the **current leads the voltage by 90°** or the potential drop across the capacitor lags the current passing it by 90°.

Fig. 23.9 shows V and i as functions of time t.

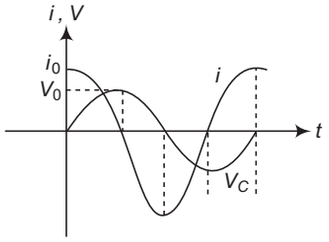


Fig. 23.9

The phasor diagram in Fig. 23.10 shows that voltage phasor is behind the current phasor by a quarter cycle or 90°.

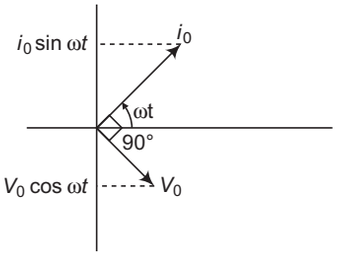


Fig. 23.10

Inductor in an AC Circuit

Consider a pure inductor of self inductance L and zero resistance connected to an alternating source. Again we assume that an instantaneous current $i = i_0 \sin \omega t$ flows through the inductor. Although there is no resistance, there is a potential difference V_L between the inductor terminals a and b because the current varies with time, giving rise to a self induced emf.

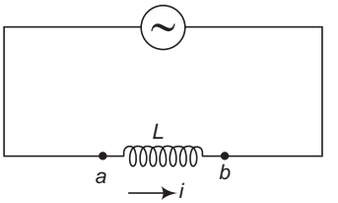


Fig. 23.11

$$V_L = V_{ab} = -(\text{induced emf})$$

$$= -\left(-L\frac{di}{dt}\right) \quad \text{or} \quad V_L = L \cdot \frac{di}{dt} = L\, i_0 \omega \cos \omega t$$

or $$V_L = V_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \ldots(i)$$

where, $$V_0 = i_0 (\omega L) \qquad \ldots(ii)$$

or $$i_0 = \frac{V_0}{\omega L}$$

$\therefore$ $$i = \frac{V_0}{\omega L} \sin(\omega t) \qquad \ldots(iii)$$

Eq. (iii) shows that effective AC resistance, i.e. **inductive reactance of inductor**,

$$X_L = \omega L$$

and the maximum current,

$$i_0 = \frac{V_0}{X_L}$$

The unit of X_L is also ohm.

From Eqs. (i) and (iii), we see that **the voltage across the inductor leads the current passing through it by 90°**.

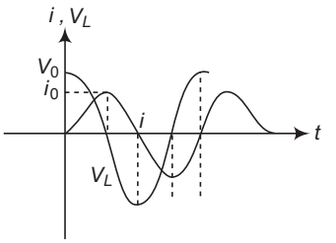


Fig.23.12

Fig. 23.12 shows V_L and i as functions of time.

Phasor diagram in Fig. 23.13 shows that V_L leads the current i by 90°.

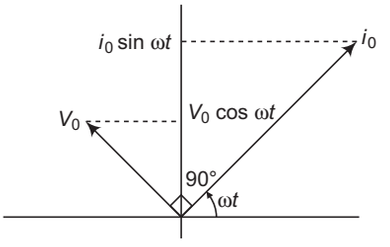


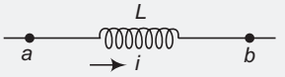
Fig. 23.13

Extra Knowledge Points

- **Circuit elements with AC**

Circuit elements	Amplitude relation	Circuit quantity	Phase of V
Resistor	$V_0 = i_0 R$	R	in phase with i
Capacitor	$V_0 = i_0 X_C$	$X_C = \frac{1}{\omega C}$	lags i by 90°
Inductor	$V_0 = i_0 X_L$	$X_L = \omega L$	leads i by 90°

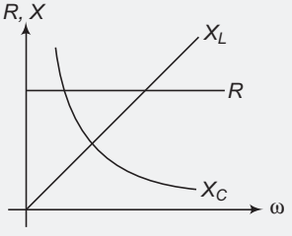
- In DC, $\omega = 0$, therefore, $X_L = 0$ and $X_C = \infty$
- The potential of point a w.r.t. point b is given by



$V_L = +L\frac{di}{dt}$, the negative of the induced emf. This expression gives the correct sign of V_L in all cases.

- If an oscillating voltage of a given amplitude V_0 is applied across an inductor the resulting current will have a smaller amplitude i_0 for larger value of ω. Since X_L is proportional to frequency, a high frequency voltage applied to the inductor gives only a small current while a lower frequency voltage of the same amplitude gives rise to a larger current. Inductors are used in some circuit applications, such as power supplies and radio interference filters to block high frequencies while permitting lower frequencies to pass through. A circuit device that uses an inductor for this purpose is called a low pass filter.
- The capacitive reactance of a capacitor is inversely proportional both to the capacitance C and to angular frequency ω. The greater the capacitance and the higher the frequency, the smaller is the capacitive reactance X_C. Capacitors tend to pass high frequency current and to block low frequency current, just the opposite of inductors.

  A device that passes signals of high frequency is called a high pass filter.
- Figure shows the graphs of R, X_L and X_C as functions of angular frequency ω.



- Remember that we can write, $V_R = iR$, $(V_0)_R = i_0 R$, $(V_0)_L = i_0 X_L$ and $(V_0)_C = i_0 X_C$ but can't write (for instantaneous voltages).

$$V_L = iX_L \quad \text{or} \quad V_C = iX_C$$

- This is because there is a 90° phase difference between the voltage and current in both an inductor and a capacitor.

Example 23.2 *A 100 Ω resistance is connected in series with a 4 H inductor. The voltage across the resistor is,* $V_R = (2.0\,V)\sin(10^3\,rad/s)$

(a) Find the expression of circuit current.
(b) Find the inductive reactance.
(c) Derive an expression for the voltage across the inductor.

Sol. (a) $i = \frac{V_R}{R} = \frac{(2.0\text{ V})\sin(10^3\text{ rad/s})t}{100}$

$= (2.0\times 10^{-2}\text{ A})\sin(10^3\text{ rad/s})t$

(b) $X_L = \omega L = (10^3\text{ rad/s})(4\text{ H})$

$= 4.0\times 10^3\ \Omega$

(c) The amplitude of voltage across inductor,

$V_0 = i_0 X_L$

$= (2.0\times 10^{-2}\text{ A})(4.0\times 10^3\ \Omega)$

$= 80\text{ V}$

In an AC voltage across the inductor leads the current by 90° or $\frac{\pi}{2}$ rad. Hence,

$$V_L = V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= (80\text{ V})\sin\left\{(10^3\text{ rad/s})t + \frac{\pi}{2}\text{ rad}\right\}$$

The amplitude of voltage across the resistor (= 2.0 V) is not same as the amplitude of the voltage across the inductor (= 80 V), even though the amplitude of the current through both devices is the same.

23.4 Phasor Algebra

The complex quantities normally employed in AC circuit analysis, can be added and subtracted like coplanar vectors. Such coplanar vectors which represent sinusoidally time varying quantities are known as **phasors**.

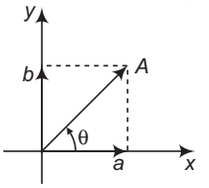


Fig. 23.14

In Cartesian form, a phasor A can be written as,

$$A = a + jb$$

where, a is the x-component and b is the y-component of phasor A.

The magnitude of A is,

$$|A| = \sqrt{a^2 + b^2}$$

and the angle between the direction of phasor A and the positive x-axis is,

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

When a given phasor A, the direction of which is along the x-axis is multiplied by the operator j, a new phasor jA is obtained which will be 90° anti-clockwise from A, i.e. along y-axis. If the operator j is multiplied now to the phasor jA, a new phasor j^2A is obtained which is along negative x-axis and having same magnitude as of A. Thus,

$$j^2 A = -A$$

$$\therefore \quad j^2 = -1 \quad \text{or} \quad j = \sqrt{-1}$$

Now, using the j operator, let us discuss different circuits of an AC.

23.5 Series *L-R* Circuit

As we know potential difference across a resistance in AC is in phase with current and it leads in phase by 90° with current across the inductor.

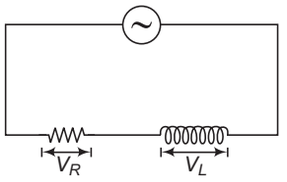


Fig. 23.15

Suppose in phasor diagram current is taken along positive x-direction. Then, V_R is also along positive x-direction and V_L along positive y-direction, so, we can write

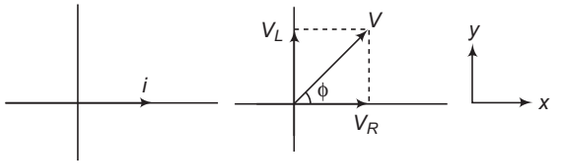


Fig. 23.16

$$\begin{aligned} V &= V_R + jV_L \\ &= iR + j(iX_L) \\ &= iR + j(i\omega L) \qquad (\text{as } X_L = \omega L) \\ &= iZ \end{aligned}$$

where, $Z = R + jX_L = R + j(\omega L)$ is called as impedance of the circuit. Impedance plays the same role in AC circuits as the ohmic resistance does in DC circuits. The modulus of impedance is,

$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

The potential difference leads the current by an angle,

$$\phi = \tan^{-1} \frac{|V_L|}{|V_R|}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) \quad \text{or} \quad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

23.6 Series *C-R* Circuit

Potential difference across a capacitor in AC lags in phase by 90° with the current in the circuit.

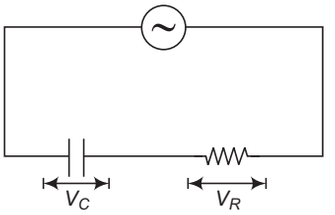


Fig. 23.17

Suppose in phasor diagram current is taken along positive x-direction. Then, V_R is also along positive x-direction but V_C is along negative y-direction. So we can write,

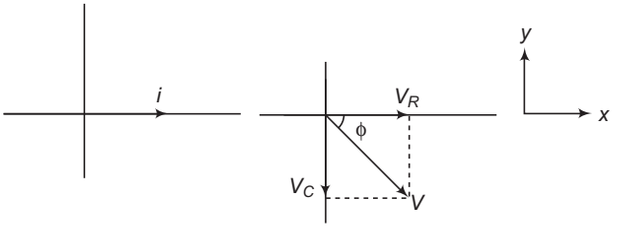


Fig. 23.18

$$\begin{aligned} V &= V_R - jV_C \\ &= iR - j(iX_C) \\ &= iR - j\left(\frac{i}{\omega C}\right) \\ &= iZ \end{aligned}$$

where, impedance is,

$$Z = R - j\left(\frac{1}{\omega C}\right)$$

The modulus of impedance is,

$$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

and the potential difference lags the current by an angle,

$$\phi = \tan^{-1}\left|\frac{V_C}{V_R}\right|$$

$$= \tan^{-1}\frac{X_C}{R}$$

$$= \tan^{-1}\left(\frac{1/\omega C}{R}\right)$$

or $$\phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

23.7 Series *L-C-R* Circuit

Potential difference across an inductor leads the current by 90° in phase while that across a capacitor, it lags in phase by 90°.

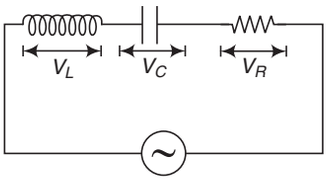


Fig. 23.19

Suppose in a phasor diagram current is taken along positive x-direction. Then, V_R is along positive x-direction, V_L along positive y-direction and V_C along negative y-direction.

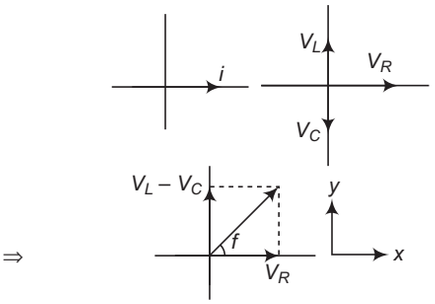


Fig. 23.20

So, we can write,

$$\begin{aligned} V &= V_R + jV_L - jV_C \\ &= iR + j(iX_L) - j(iX_C) \\ &= iR + j[i(X_L - X_C)] \\ &= iZ \end{aligned}$$

Here impedance is,

$$\begin{aligned} Z &= R + j(X_L - X_C) \\ &= R + j\left(\omega L - \frac{1}{\omega C}\right) \end{aligned}$$

The modulus of impedance is,

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

and the potential difference leads the current by an angle,

$$\phi = \tan^{-1}\left|\frac{V_L - V_C}{V_R}\right|$$

$$= \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

or $$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

Extra Knowledge Points

- Let us take the most general case of a series *L-C-R* circuit in an AC.

$$|Z| = \sqrt{R^2 + (X_L \sim X_C)^2}$$

If $X_L = X_C$ or $\omega L = \frac{1}{\omega C}$

or $\omega = \frac{1}{\sqrt{LC}}$ or $f = \frac{1}{2\pi\sqrt{LC}}$

The modulus of impedance, $|Z| = R$ and the current is in phase with voltage, i.e. if $V = V_0 \sin\omega t$ then, $i = i_0 \sin\omega t$

where, $i_0 = \frac{V_0}{|Z|} = \frac{V_0}{R}$

Such a condition is known as **resonance** and frequency known as **resonance frequency** and is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

The current in such a case is maximum.

If $X_L > X_C$, then the modulus of the impedance

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

and the voltage leads the current by an angle given by

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

i.e. if $V = V_0 \sin\omega t$ then,

$$i = i_0 \sin(\omega t - \phi)$$

where, $i_0 = \frac{V_0}{|Z|}$

If $X_C > X_L$, then the modulus of the impedance is,

$$|Z| = \sqrt{R^2 + (X_C - X_L)^2}$$

and the current leads the voltage by an angle given by,

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

i.e. if $V = V_0 \sin\omega t$ then,

$$i = i_0 \sin(\omega t + \phi)$$

where, $i_0 = \frac{V_0}{|Z|}$

- $i_0 = \frac{V_0}{|Z|}$, $i_{rms} = \frac{V_{rms}}{|Z|}$

But in general, $i \neq \frac{V}{|Z|}$.

- In *L-C-R*, circuit whenever voltage across various elements is asked, find rms values unless stated in the question for the peak or instantaneous value.

The rms values are,

$$V_R = i_{rms}R$$
$$V_L = i_{rms}X_L$$

and $$V_C = i_{rms}X_C$$

- The peak values can be obtained by multiplying the rms values by $\sqrt{2}$. The instantaneous values across different elements is rarely asked.

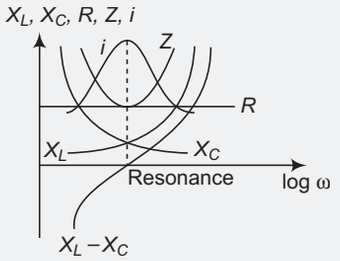
- **Voltage magnification in series resonance circuit**

  At resonance $\left(f = \frac{1}{2\pi\sqrt{LC}}\right)$ the potential difference across the inductor and the capacitor are equal and 180° out of phase and therefore, cancel out. Hence, the applied emf is merely to overcome the resistance opposition only. If an inductance or capacitance of very large reactance (X_L or X_C) is connected with $X_L = X_C$ (at resonance) then, potential difference across them increases to a very high value. The ratio is known as voltage magnification and is given by,

  $$\frac{\text{Potential Difference across inductance (or capacitance)}}{\text{Applied emf}} = \frac{i_{rms}(\omega L)}{i_{rms}(R)} = \frac{\omega L}{R} \text{ or } \frac{i_{rms}\left(\frac{1}{\omega C}\right)}{i_{rms}(R)} = \frac{1}{\omega CR}$$

  This ratio is greater than unity.

- **Response curves of series circuit** The impedance of an *L-C-R* circuit depends on the frequency. The dependance is shown in figure. The frequency is taken on logarithmic scale, because of its wide range.



  From the figure we can see that at resonance,

  (i) $X_L = X_C$ or $\omega = \frac{1}{\sqrt{LC}}$

  (ii) $Z = Z_{min} = R$ and

  (iii) i is maximum.

- Here by Z we mean the modulus of Z and i means i_{rms}.

- **Acceptor circuit** If the frequency of the AC supply can be varied (e.g. in radio or television signal) then in series *L-C-R* circuit, at a frequency $f = 1/2\pi\sqrt{LC}$ maximum current flows in the circuit and have a maximum potential difference across its inductance (or capacitance). This is the method by which a radio or television set is tuned at a particular frequency. The circuit is known as acceptor circuit.

Example 23.3 *An alternating emf* 200 *virtual volts at* 50 *Hz is connected to a circuit of resistance* 1 Ω *and inductance* 0.01 *H. What is the phase difference between the current and the emf in the circuit? Also, find the virtual current in the circuit.*

Sol. In case of an AC, the voltage leads the current in phase by an angle,

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

where, $X_L = \omega L = (2\pi fL)$

$= (2\pi)(50)(0.01) = \pi\ \Omega$

and $R = 1\ \Omega$

$\therefore \quad \phi = \tan^{-1}(\pi) \approx 72.3°$

Further, $i_{rms} = \frac{V_{rms}}{|Z|} = \frac{V_{rms}}{\sqrt{R^2 + X_L^2}}$

Substituting the values we have,

$$i_{rms} = \frac{200}{\sqrt{(1)^2 + (\pi)^2}} = 60.67 \text{ A}$$

Example 23.4 *A resistance and inductance are connected in series across a voltage,*

$$V = 283 \sin 314t$$

The current is found to be $4 \sin\left(314t - \frac{p}{4}\right)$. *Find the values of the inductance and resistance.*

Sol. In *L-R* series circuit current lags the voltage by an angle,

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Here, $\phi = \frac{\pi}{4}$

$\therefore \quad X_L = R$

or $\omega L = R \qquad (\omega = 314 \text{ rad/s})$

$\therefore \quad 314\, L = R \qquad \ldots(i)$

Further, $V_0 = i_0 |Z|$

$\therefore \quad 283 = 4\sqrt{R^2 + X_L^2}$

or $R^2 + (\omega L)^2 = \left(\frac{283}{4}\right)^2 = 5005.56$

or $2R^2 = 5005.56 \qquad (\text{as } \omega L = R)$

$\therefore \quad R = 50\ \Omega$

and from Eq. (i), $L = 0.16$ H

Example 23.5 *Find the voltage across the various elements, i.e. resistance, capacitance and inductance which are in series and having values* 1000 Ω, 1 μ*F and* 2.0 *H respectively. Given emf as,*

$$V = 100\sqrt{2}\ sin\ 1000\, t \text{ V}$$

Sol. The rms value of voltage across the source,

$$V_{rms} = \frac{100\sqrt{2}}{\sqrt{2}} = 100 \text{ V}$$

$\omega = 1000$ rad /s

$$\therefore \quad i_{rms} = \frac{V_{rms}}{|Z|} = \frac{V_{rms}}{\sqrt{R^2 + (X_L \sim X_C)^2}} = \frac{V_{rms}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$= \frac{100}{\sqrt{(1000)^2 + \left(1000 \times 2 - \frac{1}{1000 \times 1 \times 10^{-6}}\right)^2}}$$

$= 0.0707$ A

The current will be same everywhere in the circuit, therefore,

Potential difference across resistor

$$V_R = i_{rms} R = 0.0707 \times 1000 = 70.7 \text{ V}$$

Potential difference across inductor

$$V_L = i_{rms} X_L = 0.0707 \times 1000 \times 2 = 141.4 \text{ V}$$

and potential difference across capacitor,

$$V_C = i_{rms} X_C = 0.0707 \times \frac{1}{1 \times 1000 \times 10^{-6}} = 70.7 \text{ V}$$

- The rms voltages do not add directly as,

$$V_R + V_L + V_C = 282.8 \text{ V}$$

which is not the source voltage 100 V. The reason is that these voltages are not in phase and can be added by vector or by phasor algebra.

23.8 Parallel Circuit (Rejector Circuit)

Let us consider an alternating source connected across an inductance L in parallel with a capacitor C.

The resistance in series with the inductance is R and with the capacitor is zero. (Every inductor has some non-zero internal resistance while in case of a capacitor it can be neglected).

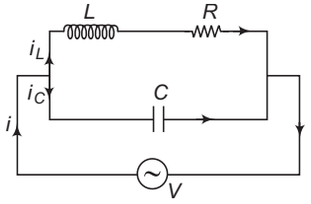


Fig. 23.21

Let the instantaneous value of emf applied be V and the corresponding currents are i, i_L and i_C. Then,

$$i = i_L + i_C$$

or
$$\frac{V}{Z} = \frac{V}{R + j\omega L} + \frac{V}{-j/\omega C}$$

$$= \frac{V}{R + j\omega L} - \frac{(\omega C)V}{j}$$

$$= \frac{V}{R + j\omega L} - \frac{j(\omega C)V}{j^2}$$

$$= \frac{V}{R + j\omega L} + j(\omega C)V \qquad (\text{as } j^2 = -1)$$

$$\therefore \quad \frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$\frac{1}{Z}$ is known as **admittance** (Y). Therefore,

$$Y = \frac{1}{Z} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$= \frac{R + j(\omega C R^2 + \omega^3 L^2 C - \omega L)}{R^2 + \omega^2 L^2}$$

$\therefore$ The magnitude of the admittance,

$$Y = |Y| = \frac{\sqrt{R^2 + (\omega C R^2 + \omega^3 L^2 C - \omega L)^2}}{R^2 + \omega^2 L^2}$$

The admittance will be minimum, when

$$\omega C R^2 + \omega^3 L^2 C - \omega L = 0 \quad \text{or} \quad \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

It gives the condition of resonance and the corresponding frequency,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

is known as resonance frequency. At resonance frequency admittance is minimum or the impedance is maximum. Thus, the parallel circuit does not allow this frequency from the source to pass in the circuit. Due to this reason the circuit with such a frequency is known as **rejector circuit**.

- If $R = 0$, resonance frequency is $\frac{1}{2\pi\sqrt{LC}}$ same as resonance frequency in acceptor circuit.

At resonance, the reactive component of Y vanishes or Y is real. The reciprocal of the admittance is called the **parallel resistor** or the **dynamic resistance**. The dynamic resistance is thus, reciprocal of the real part of the admittance.

$$\therefore \quad \text{Dynamic resistance} = \frac{R^2 + \omega^2 L^2}{R}$$

Substituting, $\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$ (resonance frequency)

We have, dynamic resistance $= \frac{L}{CR}$

$\therefore$ Peak current through the supply $= \frac{V_0}{L/CR} = \frac{V_0 CR}{L}$

The peak current through capacitor $= \frac{V_0}{1/\omega C} = \omega C V_0$.

The ratio of the peak current through capacitor and through the supply is known as ***Q*-factor**.

Thus, Q-factor $= \frac{V_0 \omega C}{V_0 CR/L} = \frac{\omega L}{R}$

This is basically the measure of current magnification. The rejector circuit at resonance exhibits current magnification of $\frac{\omega L}{R}$, similar to the voltage magnification of the same ratio exhibited by the series acceptor circuit at resonance.

- At resonance the current through the supply and voltage are in phase, while the current through the capacitor leads the voltage by 90°.

23.9 Power in an AC Circuit

In case of a steady current the rate of doing work is given by,

$$P = Vi$$

In an alternating circuit current and voltage both vary with time and also they differ in time. So, we cannot use $P = Vi$ for the power generated.

Suppose in an AC the voltage is leading the current by an angle ϕ. Then, we can write

$$V = V_0 \sin \omega t$$

and $$i = i_0 \sin (\omega t - \phi)$$

The instantaneous value of power in that case is,

$$P = Vi = V_0 i_0 \sin \omega t \sin (\omega t - \phi)$$

or $$P = V_0 i_0 \left[\sin^2 \omega t \cos \phi - \frac{1}{2} \sin 2\omega t \sin \phi \right] \quad \ldots(i)$$

Now, the average rate of doing work (power) in one cycle will be,

$$\langle P \rangle_{\text{one cycle}} = \frac{\int_0^{T = 2\pi/\omega} P dt}{\int_0^{T = 2\pi/\omega} dt} \quad \ldots(ii)$$

Substituting the value of P from Eq. (i) in Eq. (ii) and then integrating it with proper limits, we get

$$\langle P \rangle_{\text{one cycle}} = \frac{1}{2} V_0 i_0 \cos \phi$$

$$= \frac{V_0}{\sqrt{2}} \cdot \frac{i_0}{\sqrt{2}} \cos \phi$$

or $$\langle P \rangle_{\text{one cycle}} = V_{\text{rms}} i_{\text{rms}} \cos \phi$$

Here, the term $\cos \phi$ is known as **power factor**.

It is said to be leading, if current leads voltage, lagging if current lags voltage. Thus, a power factor of 0.5 lagging means current lags the voltage by 60° (as $\cos^{-1} 0.5 = 60°$). The product of V_{rms} and i_{rms} gives the **apparent power**. While the **true power** is obtained by multiplying the apparent power by the power factor $\cos \phi$. Thus,

Apparent power $= V_{\text{rms}} \times i_{\text{rms}}$

and true power = apparent power × power factor

For $\phi = 0°$, the current and voltage are in phase. The power is thus, maximum ($= V_{\text{rms}} \times i_{\text{rms}}$). For $\phi = 90°$, the power is zero. The current is then stated **wattless**. Such a case will arise when resistance in the circuit is zero.

The circuit is purely inductive or capacitive. The case is similar to that of a frictionless pendulum, where the total work done by gravity upo3n the pendulum in a cycle is zero.

23.10 Choking Coil

Let us consider a choke coil (used in tube lights) of large inductance L and low resistance R. The power factor for such a coil is given by

$$\cos \phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \approx \frac{R}{\omega L} \qquad (\text{as } R << \omega L)$$

As $R << \omega L$, $\cos \phi$ is very small. Thus, the power absorbed by the coil $V_{\text{rms}}\, i_{\text{rms}} \cos \phi$ is very small. On account of its large impedance $Z = \sqrt{R^2 + \omega^2 L^2}$, the current passing through the coil is very small. Such a coil is used in AC circuits for the purpose of adjusting current to any required value without waste of energy.

The only loss of energy is due to hysteresis in the iron core, which is much less than the loss of energy in the resistance that can also reduce the current, if placed instead of the choke coil.

Example 23.6 *A capacitor of capacitance* 250 *pF is connected in parallel with a choke coil having inductance of* 1.6×10^{-2} *H and resistance* 20 Ω. *Calculate*
(a) the resonance frequency and
(b) the circuit impedance at resonance.

Sol. (a) The resonance frequency of a rejector *L-C-R* circuit is given by,

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{1}{(1.6 \times 10^{-2})(250 \times 10^{-12})} - \frac{(20)^2}{(1.6 \times 10^{-2})^2}}$$

$$= 7.96 \times 10^4 \text{ Hz}$$

(b) The circuit impedance at resonance is given by,

$$Z = \frac{L}{CR} = \frac{1.6 \times 10^{-2}}{(250 \times 10^{-12})(20)} = 3.2 \times 10^6 \ \Omega$$

Example 23.7 *An AC circuit consists of a* 220 Ω *resistance and a* 0.7 *H choke. Find the power absorbed from* 220 *V and* 50 *Hz source connected in this circuit, if the resistance and choke are joined,*
(a) in series
(b) in parallel.

Sol. (a) In series, the impedance of the circuit is

$$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi f L)^2}$$

$$= \sqrt{(220)^2 + (2 \times 3.14 \times 50 \times 0.7)^2}$$

$$= 311 \ \Omega$$

$$\therefore \quad i_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{220}{311} = 0.707 \text{ A}$$

and $$\cos \phi = \frac{R}{Z} = \frac{220}{311} = 0.707$$

∴ The power absorbed in the circuit,

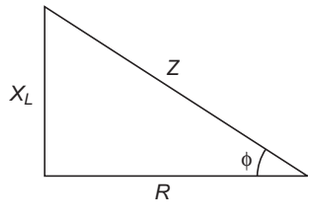


Fig. 23.22

$$P = V_{rms}\, i_{rms} \cos\phi$$
$$= (220)\,(0.707)\,(0.707)\ \text{W}$$
$$= 110.08\ \text{W}$$

(b) When the resistance and choke are in parallel, the entire power is absorbed in resistance, as the choke (having zero resistance) absorbs no power.

$$\therefore \quad P = \frac{V_{rms}^2}{R} = \frac{(220)^2}{220} = 220\ \text{W}$$

23.11 Transformer

It is a device which is either used to increase or decrease the voltage in AC circuits through mutual induction. A transformer consists of two coils wound on the same core.

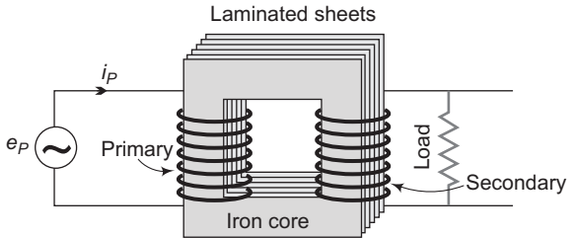


Fig. 23.23

The coil connected to input is called primary coil while the other connected to output is called secondary coil. An alternating current passing through the primary coil creates a continuously changing flux through the core. This changing flux induces an alternating emf in the secondary coil.

The value of this emf depends on the number of turns in the secondary coil.

Let us consider an ideal transformer in which the primary coil has negligible resistance and all the flux (say ϕ) in the core links both primary and secondary windings.

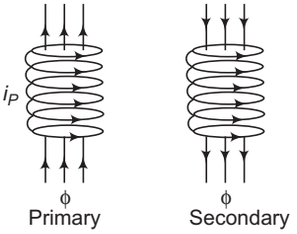


Fig. 23.24

Suppose that there are N_P turns in primary coil and N_S turns in secondary coil . An alternating emf e_P is applied across the primary coil which produces a current i_P in the primary coil. Due to this current i_P a magnetic flux ϕ is linked with both the coils as shown above. Since, applied voltage (e_P) in primary coil is alternating, the current i_P is also alternating. Due to this variable current i_P magnetic flux ϕ becomes variable and emf is induced in both the coils primary and secondary. The emf induced in the primary is $-N_P \cdot \frac{d\phi}{dt}$ and in secondary is $-N_S \frac{d\phi}{dt}$.

where, $-N_S \frac{d\phi}{dt} = e_S$ is the output voltage obtained to us.

Since, resistance of primary circuit has been neglected, Kirchhoff 's loop law applied to the primary circuit gives,

$$e_P - N_P \frac{d\phi}{dt} = 0 \quad \text{or} \quad \frac{d\phi}{dt} = \frac{e_P}{N_P} \qquad \text{...(i)}$$

Output voltage (across secondary coil) obtained to us is

$$e_S = -N_S \frac{d\phi}{dt}$$

Substituting value of $\frac{d\phi}{dt}$ from Eq. (i), we get

$$e_S = -\frac{N_S}{N_P} \cdot e_P$$

Ignoring the minus sign which shows that e_S is 180° out of phase with e_P we get,

$$\frac{e_S}{e_P} = \frac{N_S}{N_P}$$

In an ideal transformer, there is no loss of power. Hence,

$$ei = \text{constant}$$
$$\therefore \quad \frac{e_S}{e_P} = \frac{N_S}{N_P} = \frac{i_P}{i_S}$$

Regarding a transformer following are few important points:

1. In **step-up** transformer $N_S > N_P$. It increases voltage and reduces current.
2. In **step-down** transformer, $N_P > N_S$. It increases current and reduces voltage.
3. It works only on AC.
4. A transformer cannot increase (or decrease) voltage and current simultaneously. As, ei = constant.
5. In actual transformers, there is always some flux leakage, i.e. not all the flux due to the primary coil passes through the secondary coil. This loss of flux can be reduced by winding the primary and secondary coils one over the other.

6. Further, in actual transformers, the windings of primary and secondary coils have some finite (non-zero) resistance. So energy is lost in these windings in the form of heat. These are minimized by using thick wires.

7. Alternating magnetic flux also induces eddy currents in the iron core and causes loss of energy in the form of heat. This can be reduced by using laminated core.

8. The magnetisation of the core is repeatedly reversed by the alternating magnetic field.

Therefore, core of the transformer is made of a material which has a low hysteresis loss. (e.g. soft iron).

9. The large scale transmission and distribution of electrical energy over long distances is done with the use of transformers. The voltage output of the generator is first stepped-up (so that current is reduced and consequently, the i^2R loss is cut down). It is then transmitted over long distances to an area sub-station near the consumers. There the voltage is stepped down at distributing sub-stations and utility poles before a power supply of 240 V reaches our homes.

Example 23.8 *A radio set operates at* $6\,V$ *DC. A transformer with* 18 *turns in the secondary coil is used to step down the input* $220\,V$ *AC emf to* $6\,V$ *AC emf. This AC emf is then rectified by another circuit to give* $6\,V$ *DC which is fed to the radio. Find the number of turns in the primary coil.*

Sol. Using the equation,

$$\frac{e_S}{e_P} = \frac{N_S}{N_P}$$

$$\therefore \quad N_P = \frac{e_P}{e_S} N_S$$

Substituting the values we have,

$$N_P = \frac{220}{6} \times 18$$

$$= 660$$

Example 23.9 *A transformer has* 50 *turns in the primary coil and* 100 *in the secondary coil. If the primary coil is connected to a* $220\,V$ *DC supply, what will be the voltage across the secondary coil*?

Sol. Transformer does not work with DC supply. Therefore, voltage across secondary will be zero.

Chapter Summary with Formulae

(i) Frequency of AC in India is 50 Hz.

(ii) The AC is converted into DC with the help of rectifier while DC is converted into AC with the help of inverter.

(iii) An AC cannot produce electroplating or electrolysis.

(iv) The AC is measured by hot wire ammeter.

(v) An AC can be stepped up or down with the help of a transformer.

(vi) An AC can be transmitted over long distances without much power loss.

(vii) An AC can be regulated by using choke coil without any significant waste of energy.

(viii) In an AC (sinusoidal) current or voltage can have following four values.

(a) instantaneous value

(b) peak value (i_0 or V_0)

(c) rms value (i_{rms} or V_{rms})

(d) average value : In full cycle, average value is zero, while in half cycle it is non-zero.

- In sinusoidal AC even the average value in half cycle can also be zero. It depends on the time interval over which half, average value is desired.

(e) $i_{\text{rms}} = \frac{i_0}{\sqrt{2}}, V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$

(f) $\langle i \rangle_{\text{positive half cycle}} = \frac{2}{\pi} i_0 \approx 0.636\ i_0$. Similarly, $\langle V \rangle_{\text{positive half cycle}} = \frac{2}{\pi} V_0 \approx 0.636\ V_0$

(ix) **In a series *L*-*C*-*R* AC circuit**

(a) Capacitive reactance, $X_C = \frac{1}{\omega C}$

(b) Inductive reactance, $X_L = \omega L$

(c) Impedance, $Z = \sqrt{R^2 + (X_C - X_L)^2}$

(d) If $X_C > X_L$, current leads and if $X_L > X_C$, voltage leads by an angle ϕ given by,

$$\cos\phi = \frac{R}{Z} \quad \text{or} \quad \tan\phi = \frac{X_C \sim X_L}{R}$$

(e) Instantaneous power = instantaneous current $\times$ instantaneous voltage

(f) Average power = $V_{\text{rms}}\ i_{\text{rms}} \cos\phi$, where, $\cos\phi = \frac{R}{Z}$ = power factor.

(g) $i_0 = \frac{V_0}{Z}$ or $i_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$

(h) $(V_C)_{\text{rms}} = (i_{\text{rms}})\, X_C, (V_L)_{\text{rms}} = (i_{\text{rms}})\, X_L$ and $(V_R)_{\text{rms}} = (i_{\text{rms}})\, R$

(i) $V = \sqrt{V_R^2 + (V_C - V_L)^2}$, where, V is the rms value of applied voltage.

V_R is the rms value of voltage across resistance. V_C across capacitor and V_L across inductor, etc.

(j) At $\omega = \omega_r = \frac{1}{\sqrt{LC}}$, $X_C = X_L$ and Z has the minimum value equal to R. Power factor in this case is 1.

(k) At $\omega > \omega_r$, $X_L > X_C$, voltage leads the current function and circuit is inductive.

(l) At $\omega < \omega_r$, $X_C > X_L$, current leads the voltage function and circuit is capacitive.

(x) **Quality factor**, $Q = \frac{\omega_r L}{R}$

It is a measure of sharpness of resonance. If value of Q is large, sharpness is more and it is more selective.

(xi) **Transformer**

(a) $\frac{e_S}{e_P} = \frac{N_S}{N_P} = \frac{i_P}{i_S}$

(b) ei = constant or, $e_S\ i_S = e_P\ i_P$

(c) In step up transformer, $N_S > N_P$. Therefore, $e_S > e_P$. It increases voltage and reduces current.

(d) In step down transformer, $N_S < N_P$. Therefore, $e_S < e_P$. It reduces voltage and increases current.

(e) It works only an AC.

Additional Examples

Example 1. *What idea do you get from the power factor of an L-C-R series circuit?*

Sol. The power factor is a measure of how close the circuit is to the maximum power. Its value lies between 0 and 1.

$$0 \le \text{Power factor} \le 1$$

Example 2. *A step up transformer changes a low-voltage into a high-voltage. Does it violate the law of conservation of energy.*

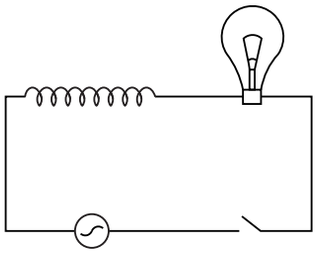
Sol. No, it does not violate the law of conservation of energy. The current is reduced by the same proportion.

ei or power = constant

Example 3. *At an airport, a person is made to walk through the doorway of a metal detector, for security reasons. If she/he is carrying anything made of metal, the metal detector emits a sound. On what principle does this detector work?*

Sol. Metal detector in airports is based on the principle of resonance in AC circuits. A metal detector consists of a coil of many turns. This coil is connected to a capacitor. When we walk through the metal detector (with some metal with us) impedance of the circuit changes. (Because when a metal rod is kept between a coil its self inductance changes). Due to change in impedance (actually it decreases) there is significant change in current in the circuit. This change in current causes a sound as an alarm.

Example 4. *A light bulb and an open coil inductor are connected to an AC source through a key as shown in figure.*



The switch is closed and after sometime, an iron rod is inserted into the interior of the inductor. The glow of the light bulb (a) increases, (b) decreases, (c) is unchanged, as the iron rod is inserted. Give your answer with reasons.

Sol. Correct answer is (b).

By inserting an iron rod inside the coil self inductance of the coil increases. Therefore, total impedance of the circuit increases. Hence, current decreases. Therefore, the glow of the light bulb decreases.

Example 5. *A lamp is connected in series with a capacitor. Predict your observations for* DC *and* AC *connections. What happens in each case, if the capacitance of the capacitor is reduced?*

Sol. With a DC source, current in a capacitor wire becomes zero after it is fully charged. Therefore, lamp will not shine after this. There will be no change even if C is reduced .

With an AC source, current in the circuit with capacitor is non-zero. Therefore, lamp shines. If C is reduced, then capacitive reactance $\left(X_C = \dfrac{1}{\omega C}\right)$ will increase. Therefore, impedance of the circuit will increase. Hence, current will decrease or the bulb will shine less brightly than before.

Example 6. *A sinusoidal voltage of frequency* $60\,Hz$ *and peak value* $150\,V$ *is applied to a series L-R circuit, where* $R = 20\,\Omega$ *and* $L = 40\,mH$.
(a) Compute T, ω, X_L, Z *and* ϕ.
(b) Compute the amplitudes of current, V_R *and* V_L.

Sol. (a) $T = \dfrac{1}{f} = \dfrac{1}{60}$ s

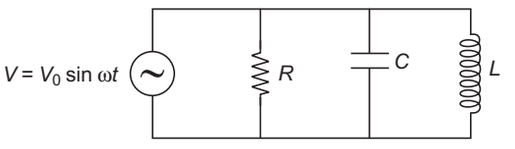
$$\omega = 2\pi f = 377 \text{ rad/s}$$
$$X_L = \omega L = (377)\,(0.040)$$
$$= 15.08\,\Omega$$
$$Z = \sqrt{X_L^2 + R^2} = 25.05\,\Omega$$
$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$
$$= \tan^{-1}\,(0.754) = 37°$$

(b) Amplitudes (maximum value) are,

$$i_0 = \frac{V_0}{Z} = \frac{150}{25.05} \approx 6\text{A}$$
$$(V_0)_R = i_0 R = 120 \text{ V}$$
$$(V_0)_L = i_0 X_L = 90.5 \text{ V}$$

- $V_0 = \sqrt{(V_0)_R^2 + (V_0)_L^2}$

Example 7. *For the circuit shown in figure, find the instantaneous current through each element.*



Sol. The three current equations are

$$V = i_R R,\ V = L\frac{di_L}{dt}$$

and $$\frac{dV}{dt} = \frac{1}{C} i_C \qquad \text{...(i)}$$

The steady state solutions of Eq. (i) are

$$i_R = \frac{V_0}{R}\sin\omega t \equiv (i_0)_R \sin\omega t$$

$$i_L = -\frac{V_0}{\omega L}\cos\omega t$$

$$\equiv -\frac{V_0}{X_L}\cos\omega t \equiv -(i_0)_L \cos\omega t$$

and $i_C = V_0\omega C\cos\omega t$

$$\equiv \frac{V_0}{X_C}\cos\omega t \equiv (i_0)_C \cos\omega t$$

where, the reactances X_L and X_C are as defined.

Example 8. *An L-C-R series circuit with* $100\,\Omega$ *resistance is connected to an AC source of* $200\,V$ *and angular frequency* $300\,rad/s$. *When only the capacitance is removed, the current lags behind the voltage by* $60°$. *When only the inductance is removed, the current leads the voltage by* $60°$. *Calculate the current and the power dissipated in the L-C-R circuit.*

Sol. When capacitance is removed,

$$\tan\phi = \frac{X_L}{R}$$

or $$\tan 60° = \frac{X_L}{R}$$

$\therefore$ $$X_L = \sqrt{3}R \quad ...(i)$$

When inductance is removed,

$$\tan\phi = \frac{X_C}{R}$$

or $$\tan 60° = \frac{X_C}{R}$$

$\therefore$ $$X_C = \sqrt{3}\,R \quad ...(ii)$$

From Eqs. (i) and (ii), we see that, $X_C = X_L$

So, the *L-C-R* circuit is in resonance.

Hence, $$Z = R$$

$\therefore$ $$i_{rms} = \frac{V_{rms}}{Z} = \frac{200}{100} = 2\text{A}$$

$$\langle P\rangle = V_{rms}\, i_{rms}\cos\phi$$

At resonance current and voltage are in phase or $\phi = 0°$

$\therefore$ $$\langle P\rangle = (200)(2)(1)$$

$$= 400\text{ W}$$

Example 9. *A series L-C-R circuit containing a resistance of* $120\,\Omega$ *has angular frequency* $4\times10^5\,rad/s$. *At resonance the voltages across resistance and inductance are* $60\,V$ *and* $40\,V$ *respectively. Find the values of L and C. At what angular frequency the current in the circuit lags the voltage by* $\pi/4$ *?*

Sol. At resonance, $X_L - X_C = 0$

and $$Z = R = 120\ \Omega$$

$\therefore$ $$i_{rms} = \frac{(V_R)_{rms}}{R} = \frac{60}{120} = \frac{1}{2}\text{ A}$$

Also, $$i_{rms} = \frac{(V_L)_{rms}}{\omega L}$$

$\therefore$ $$L = \frac{(V_L)_{rms}}{\omega i_{rms}}$$

$$= \frac{40}{(4\times10^5)\left(\frac{1}{2}\right)}$$

$$= 2.0\times10^{-4}\text{ H}$$

$$= 0.2\text{ mH}$$

The resonance frequency is given by,

$$\omega = \frac{1}{\sqrt{LC}}$$

or $$C = \frac{1}{\omega^2 L}$$

Substituting the values, we have

$$C = \frac{1}{(4\times10^5)^2\ (2.0\times10^{-4})}$$

$$= 3.125\times10^{-8}\text{ F}$$

Current lags the voltage by 45°, when

$$\tan 45° = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Substituting the values of *L, C, R* and tan 45° we get,

$$\omega = 8\times10^5\text{ rad/s}$$

Example 10. *A current of* $4\,A$ *flows in a coil when connected to a* $12\,V$ *DC source. If the same coil is connected to a* $12\,V$, $50\,rad/s$ *AC source, a current of* $2.4\,A$ *flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a* $2500\,\mu F$ *capacitor is connected in series with the coil.*

Sol. (i) A coil consists of an inductance (*L*) and a resistance (*R*). In DC only resistance is effective. Hence,

$$R = \frac{V}{i} = \frac{12}{4} = 3\ \Omega$$

In AC, $$i_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + \omega^2 L^2}}$$

$\therefore$ $$L^2 = \frac{1}{\omega^2}\left[\left(\frac{V_{rms}}{i_{rms}}\right)^2 - R^2\right]$$

$\therefore$ $$L = \frac{1}{\omega}\sqrt{\left(\frac{V_{rms}}{i_{rms}}\right)^2 - R^2}$$

Substituting the values, we have

$$L = \frac{1}{50}\sqrt{\left(\frac{12}{2.4}\right)^2 - (3)^2}$$

$$= 0.08\text{ H}$$

(ii) When capacitor is connected to the circuit, the impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Here, $$R = 3\ \Omega$$

$$X_L = \omega L$$

$$= (50)\ (0.08)$$
$$= 4\ \Omega$$

and $$X_C = \frac{1}{\omega C}$$

$$= \frac{1}{(50)\,(2500 \times 10^{-6})} = 8\ \Omega$$

$$\therefore \quad Z = \sqrt{(3)^2 + (4-8)^2} = 5\ \Omega$$

Now, $$\langle P \rangle = V_{\text{rms}}\, i_{\text{rms}} \cos\phi$$

$$= V_{\text{rms}} \times \frac{V_{\text{rms}}}{Z} \times \frac{R}{Z}$$

$$= \left(\frac{V_{\text{rms}}}{Z}\right)^2 \times R$$

Substituting the values, we have

$$\langle P \rangle = \left(\frac{12}{5}\right)^2 \times 3$$

$$= 17.28\ \text{W}$$

Example 11. *A choke coil is needed to operate an arc lamp at* $160\,V$ *(rms) and* $50\,Hz$. *The lamp has an effective resistance of* $5\ \Omega$ *when running at* $10\,A$*(rms). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on* $160\,V$ *(DC), what additional resistance is required? Compare the power loses in both cases.*

Sol. For lamp,

$$(V_{\text{rms}})_R = (i_{\text{rms}})\,(R) = 10 \times 5 = 50\ \text{V}$$

In series,

$$(V_{\text{rms}})^2 = (V_{\text{rms}})_R^2 + (V_{\text{rms}})_L^2$$

$$\therefore \quad (V_{\text{rms}})_L = \sqrt{(V_{\text{rms}})^2 - (V_{\text{rms}})_R^2}$$

$$= \sqrt{(160)^2 - (50)^2}$$

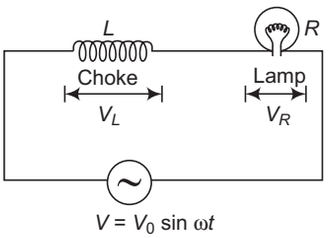
$$= 152\ \text{V}$$

As $$(V_{\text{rms}})_L = (i_{\text{rms}})\,X_L = (i_{\text{rms}})\,(2\pi f L)$$

$$\therefore \quad L = \frac{(V_{\text{rms}})_L}{(2\pi f)\,(i_{\text{rms}})}$$

Substituting the values,

$$L = \frac{152}{(2\pi)\,(50)\,(10)} = 4.84 \times 10^{-2}\ \text{H}$$



Now, when the lamp is operated at 160 V DC and instead of choke let an additional resistance R' is put in series with it then,

$$V = i\,(R + R') \quad \text{or} \quad 160 = 10\,(5 + R')$$

$$\therefore \quad R' = 11\ \Omega$$

In case of AC, as the choke has no resistance, power loss in choke is zero.

In case of DC, the loss in additional resistance R' is,

$$P = i^2 R' = (10)^2\,(11) = 1100\ \text{W}$$

NCERT Selected Questions

Q 1. A 100 Ω resistor is connected to a 220 V, 50 Hz AC supply.

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

Sol. (a) $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{220}{100} = 2.2 \text{ A}$

(b) We know that the power dissipated in an AC circuit is given by

$$P = V_{\text{rms}}\, I_{\text{rms}} \cos \phi$$

where, ϕ is the phase difference between current and voltage.

In a circuit containing resistor only, $\phi = 0°$, thus $\cos \phi = \cos 0° = 1$

$$\therefore \quad P = V_{\text{rms}}\, I_{\text{rms}} = 220 \times 2.2 = 484 \text{ W}$$

Q 2. (a) The peak voltage of an AC supply is 300 V. What is the rms voltage?

(b) The rms value of current in an AC circuit is 10 A. What is the peak current?

Sol. (a) Here, peak value of AC supply, $E_0 = 300$ V

$I_{\text{rms}} =$ rms value of current $= 10$ A

$\therefore$ Using the relation, $E_{\text{rms}} = \dfrac{E_0}{\sqrt{2}}$, we get

$$E_{\text{rms}} = \frac{300}{\sqrt{2}} = 212.1 \text{ V}$$

(b) Let I_0 be the peak value of the current.

Then, $\quad I_{\text{rms}} = \dfrac{I_0}{\sqrt{2}}$

$$\therefore \quad I_0 = \sqrt{2}\, I_{\text{rms}} = 1.414 \times 10 = 14.14 \text{ A}$$

Q 3. A 44 mH inductor is connected to 220 V, 50 Hz AC supply. Determine the rms value of the current in the circuit.

Sol. $I_{\text{rms}} = \dfrac{E_{\text{rms}}}{X_L}$

where, $X_L = \omega L = 2\pi f L$ is the reactance of the inductor

$$= 2\pi \times 50 \times 44 \times 10^{-3} = 13.82 \ \Omega$$

$$\therefore \quad I_{\text{rms}} = \frac{220}{13.82} = 15.9 \text{ A}$$

Q 4. A 60 μF capacitor is connected to a 110 V, 60 Hz AC supply. Determine the rms value of the current in the circuit.

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{X_C}$$

where, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$$= \frac{1}{2 \times \pi \times 60 \times 60 \times 10^{-6}} \ \Omega$$

$$\therefore \quad I_{\text{rms}} = 110 \times 2 \times 3.14 \times 3600 \times 10^{-6} = 2.49 \text{ A}$$

Q 5. In Q. 3 and 4, what is the net power absorbed by each circuit over a complete cycle? Explain your answer.

Sol. The net power absorbed by each cicuit in Q. 3 and 4 is zero.

Explanation We know that the power absorbed in an AC circuit is given by

$$P_{\text{av}} = E_{\text{rms}}\, I_{\text{rms}} \cos \phi$$

where, $\cos \phi$ is power factor.

For a pure inductor and pure capacitor circuit, $\phi = \dfrac{\pi}{2}$

$$\therefore \quad \cos \phi = \cos \frac{\pi}{2} = 0$$

Thus, $P_{\text{av}} = 0$ in each case.

Q 6. A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

Sol. Angular frequency of free oscillations of L-C oscillations,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}} = 1.11 \times 10^3 \text{ rad s}^{-1}$$

Q 7. Suppose, the initial charge on the capacitor in Q.6 is 6 mC. What is the total energy stored in the circuit initially? What is the total energy at later time?

Sol. Total energy, $\quad U = \dfrac{1}{2} \cdot \dfrac{Q_i^2}{C}$

$$= \frac{1}{2} \times \frac{(6 \times 10^{-3})^2}{30 \times 10^{-6}} = 0.6 \text{ J}$$

As, there is no resistance in the circuit, thus there is no loss of energy, hence the total energy will remain conserved during the L-C oscillations.

Thus, total energy at the later time = 0.6 J.

Q 8. A series *L*-*C*-*R* circuit with $R = 20\ \Omega$, $L = 1.5$ H and $C = 35\ \mu$F is connected to a variable frequency 200 V, AC supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Sol. $Z = R = 20\ \Omega$

Since, the *L*-*C*-*R* circuit is resistive, so the phase angle between the current and voltage is zero.

i.e. $\phi = 0°$ and $I_{rms} = \dfrac{E_{rms}}{Z} = \dfrac{200}{20} = 10$ A

P = average power transferred per cycle.

$\therefore \quad P = I_{rms}\, E_{rms} \cos 0° = 10 \times 200 = 2000$ W

$= 2.0$ kW

Q 9. A radio can tune over the frequency range of a portion of MW broadcast band (800 kHz to 1200 kHz). If its *L*-*C* circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?

Sol. For tuning, the natural frequency, i.e. the frequency of free oscillations of the *L*-*C* circuit should be equal to the frequency of the radiowave.

$$f = \frac{1}{2\pi\sqrt{LC}}, \text{we get } f_1 = \frac{1}{2\pi\sqrt{LC_1}}$$

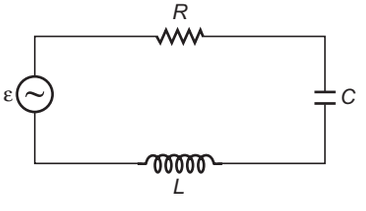
or $C_1 = \dfrac{1}{4\pi^2 L f_1^2}$ or $C_1 = \dfrac{1}{4 \times 9.87 \times 2 \times 10^{-4} \times (8 \times 10^5)^2}$

$= 197.8 \times 10^{-12}$ F $= 197.8$ pF

Similarly, $C_2 = \dfrac{1}{4\pi^2 L f_2^2} = \dfrac{1}{4 \times 9.87 \times 2 \times 10^{-4} \times (12 \times 10^5)^2}$

or $\quad C_2 = 87.95 \times 10^{-12}\text{F} = 87.95$ pF

Q 10. Figure here shows a series *L*-*C*-*R* circuit connected to a variable frequency 230 V source. $L = 5.0$ H, $C = 80\ \mu$F and $R = 40\ \Omega$.



(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the *L*-*C* combination is zero at the resonating frequency.

Sol. (a) If ω_0 be the resonant angular frequency = source frequency at resonance, then

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50 \text{ rad s}^{-1}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{50}{2 \times 3.142} = 7.96 \text{ Hz}$$

(b) At resonance

$$Z = R = 40\ \Omega$$

$$I_{rms} = \frac{E_{rms}}{Z} = \frac{230}{40} = 5.75 \text{ A}$$

and $\quad I_0 = \dfrac{E_0}{Z} = \dfrac{\sqrt{2}\, E_{rms}}{Z} = \dfrac{\sqrt{2} \times 230}{40} = 8.13$ A

(c) The rms potential drop across R is given by

$$V_{rms} = I_{rms}\, R = 5.75 \times 40 = 230 \text{ V}$$

The rms potential drop across L is given by

$$V_{L\,rms} = I_{rms}\, X_L = I_{rms}\, (\omega_0 L)$$

$$= 5.75 \times 50 \times 5 = 1437.5 \text{ V}$$

The rms potential drop across C is given by

$$V_{rms} = I_{rms}\, X_C = I_{rms} \times \frac{1}{\omega_0 C}$$

$$= 5.75 \times \frac{1}{50 \times 80 \times 10^{-6}} = 1437.5 \text{ V}$$

The rms potential drop across *L*-*C* is given by

$$V_{LC} = V_L - V_C = 1437.5 - 1437.5 = 0$$

Q 11. A 100 μF capacitor in series with a 40 Ω resistance is connected to 110 V, 60 Hz supply.

(a) What is the maximum current in the circuit?

(b) What is the time lag between the current maximum and the voltage maximum?

Sol. (a) $E_0 = \sqrt{2}\, E_{rms} = \sqrt{2} \times 110$ V

$\therefore\ I_0$ = maximum current in the circuit is given by

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}}$$

or $\quad I_0 = \dfrac{\sqrt{2} \times 110}{\sqrt{(40)^2 + \left(\dfrac{1}{120\pi \times 10^{-4}}\right)^2}} = 3.24$ A

(b) In an *R*-*C* circuit, the voltage lags behind the current by the phase angle ϕ given by

$$\tan\phi = \frac{X_C}{R} = \frac{1}{\omega CR} = \frac{1}{120\pi \times 10^{-4} \times 40}$$

$$= 0.6628 = \tan 33.5°$$

$$\therefore \quad \phi = 33.5° = 33.5 \times \frac{\pi}{180} \text{ rad}$$

$\therefore$ Time between the current maximum and the voltage maximum is given by

$$t = \frac{\phi}{\omega} = \frac{\dfrac{33.5\,\pi}{180}}{120\pi}$$

$$= 1.55 \times 10^{-3} \text{ s} = 1.55 \text{ ms}$$

Objective Problems

[Level 1]

Resistance, Impedance, Phase Difference and Resonance

1. Voltage and current in an AC circuit are given by $V = 5\sin\left(100\pi t - \frac{\pi}{6}\right)$ and $I = 4\sin\left(100\pi t + \frac{\pi}{6}\right)$, then

(a) voltage leads the current by 30°
(b) current leads the voltage by 30°
(c) current leads the voltage by 60°
(d) voltage leads the current by 60°

2. An inductance of 1 mH, a condenser of 10 μF and a resistance of 50 Ω are connected in series. The reactances of inductor and condensers are same. The reactance of either of them will be

(a) 100 Ω (b) 30 Ω (c) 3.2 Ω (d) 10 Ω

3. In *L*-*C*-*R* circuit, the capacitance is changed from *C* to 4*C*. For the same resonant frequency, the inductance should be changed from *L* to

(a) 2*L* (b) *L*/2 (c) *L*/4 (d) 4*L*

4. A 10 Ω resistance, 5 mH coil and 10 μF capacitor are joined in series. When a suitable frequency alternating current source is joined to this combination, the circuit resonates. If the resistance is halved, the resonance frequency

(a) is halved (b) is doubled
(c) remains unchanged (d) in quadrupled

5. The reactance of a 25 μF capacitor at the AC frequency of 4000 Hz is

(a) $\frac{5}{\pi}\Omega$ (b) $\sqrt{\frac{5}{\pi}}\Omega$
(c) 10 Ω (d) $\sqrt{10}\,\Omega$

6. The frequency for which a 5 μF capacitor has a reactance of $\frac{1}{1000}\Omega$ is given by

(a) $\frac{100}{\pi}$ MHz (b) $\frac{1000}{\pi}$ Hz
(c) $\frac{1}{1000}$ Hz (d) 1000 Hz

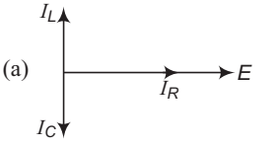
7. The resonant frequency of a circuit is *f*. If the capacitance is made 4 times the initial value, then the resonant frequency will become

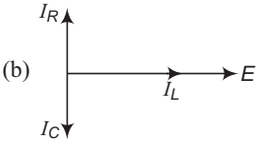
(a) *f*/2 *(b) 2f*
(c) *f* (d) *f*/4

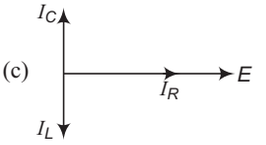
8. The capacity of a pure capacitor is 1 F. In DC circuits, its effective resistance will be

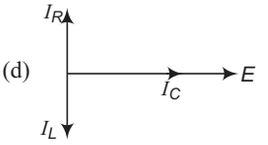
(a) zero (b) infinite
(c) 1 Ω (d) 1/2 Ω

9. An alternating emf is applied across a parallel combination of a resistance *R,* capacitance *C* and an inductance *L*. If I_R, I_L and I_C are the currents through *R, L* and *C* respectively, then the diagram which correctly represents, the phase relationship among I_R, I_L, I_C and source emf *E,* is given by

(a) 

(b) 

(c) 

(d) 

10. In an AC circuit containing only capacitance, the current

(a) leads the voltage by 180°
(b) remains in phase with the voltage
(c) leads the voltage by 90°
(d) lags the voltage by 90°

11. An *L*-*C*-*R* series circuit is connected to a source of alternating current. At resonance the applied voltage and current flowing through the circuit will have a phase difference of

(a) zero (b) π/4 (c) π/2 (d) π

12. An inductance and a resistance are connected in series with an AC potential. In this circuit

(a) the current and the potential difference across the resistance lead the potential difference across the inductance by phase angle π/2
(b) the current and the potential difference across the resistance lag behind potential difference across the inductance by an angle $\frac{\pi}{2}$
(c) the current and the potential difference across the resistance lag behind the potential difference across the inductance by an angle π.
(d) the potential difference across the resistance lags behind the potential difference across the inductance by an angle $\frac{\pi}{2}$ but the current in the resistance leads the potential difference across inductance by $\frac{\pi}{2}$

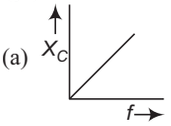
13. The resistance of a coil for DC is 5 Ω. In case of AC, the resistance will
(a) remain 5 Ω (b) decrease
(c) increase (d) be zero

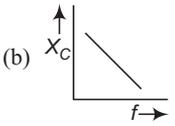
14. What will be the approximate resistance offered by a capacitor of 10 μF and frequency 100 Hz?
(a) 160 Ω (b) 1600 Ω
(c) 16 Ω (d) None of these

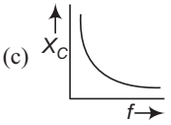
15. In *L*-*R* circuit, resistance is 8 Ω and inductive reactance is 6 Ω, then impedance is
(a) 2 Ω (b) 14 Ω (c) 4 Ω (d) 10 Ω

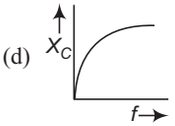
16. In an AC circuit, the current lags behind the voltage by $\frac{\pi}{3}$. The components in the circuit may be
(a) *R* and *L* (b) *L* and *C* (c) *R* and *C* (d) only *R*

17. The reactance of a coil when used in the domestic AC power supply (220 V, 50 cycles per second) is 50 Ω. The inductance of the coil is nearly
(a) 2.2 H (b) 1.6 H (c) 0.22 H (d) 0.16 H

18. Which of the following curves correctly represent the variation of capacitive reactance (X_C) with frequency (f)?
(a) 

(b) 

(c) 

(d) 

19. A series *L*-*C*-*R* circuit is operated at resonance. Then
(a) voltage across *R* is minimum
(b) impedance is minimum
(c) impedance is maximum
(d) current amplitude is minimum

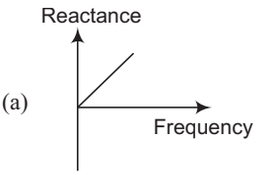
20. The value of current at resonance in a series *L*-*C*-*R* circuit is affected by the value of
(a) *R* only (b) *C* only (c) *L* only (d) *L*,*C* and *R*

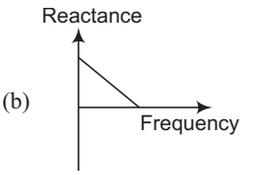
21. A coil having an inductance of 1/x H is connected in series with a resistance of 300 Ω. If 20 V and a 200 Hz source are impressed across the combination, the value of the tangent of the phase angle between the voltage and the current is
(a) $\frac{5}{4}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

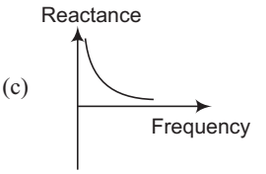
22. In an AC circuit, a resistance of *R* Ω is connected in series with an inductance *L*. If phase angle between voltage and current be 45°, the value of inductive reactance will be
(a) *R* /4
(b) *R* /2
(c) *R*
(d) cannot be found with the given data

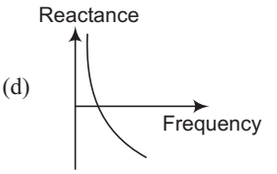
23. In a circuit containing *R* and *L*, as the frequency of the impressed AC increases, the impedance of the circuit
(a) decreases
(b) increases
(c) remains unchanged
(d) first increases and then decreases

24. With increase in frequency of an AC supply, the impedance of an *L*-*C*-*R* series circuit
(a) remains constant
(b) increases
(c) decreases
(d) decreases at first, becomes minimum and then increases

25. Which of the shown graphs may represent the reactance of a series *L*-*C* combination?
(a) 

(b) 

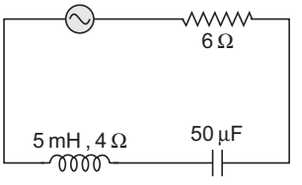
(c) 

(d) 

Current, Voltage, RMS and Average Values

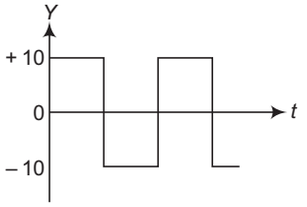
26. An alternating voltage is connected in series with a resistance *R* and an inductance *L*. If the potential drop across the resistance is 200 V and across the inductance is 150 V, then the applied voltage is
(a) 350 V (b) 250 V
(c) 500 V (d) 300 V

27. An alternating voltage $E = 200\sqrt{2}\sin(100t)$ is connected to a 1 μF capacitor through an AC ammeter. The reading of the ammeter shall be
(a) 10 mA (b) 20 mA
(c) 40 mA (d) 80 mA

28. In the circuit shown, the AC source has voltage $V = 20\cos(\omega t)$ volt with ω = 2000 rad/s, the amplitude of the current will be nearest to


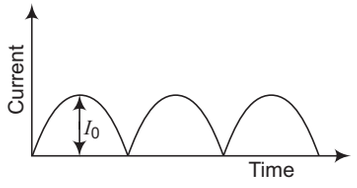
(a) 2 A
(b) 3.3 A
(c) $2/\sqrt{5}$A
(d) $\sqrt{5}$A

29. The rms voltage of the waveform shown is



(a) 10 V (b) 7 V
(c) 6.37 V (d) None of these

30. The output sinusoidal current *versus* time curve of a rectifier is shown in the figure. The average value of output current in this case is



(a) zero (b) $\frac{I_0}{2}$
(c) $\frac{2I_0}{\pi}$ (d) I_0

31. The instantaneous current in an AC circuit is $i = \sqrt{2}\sin(50t + \pi/4)$. The rms value of current is
(a) $\sqrt{2}$ A (b) 50 A
(c) 90 A (d) 1 A

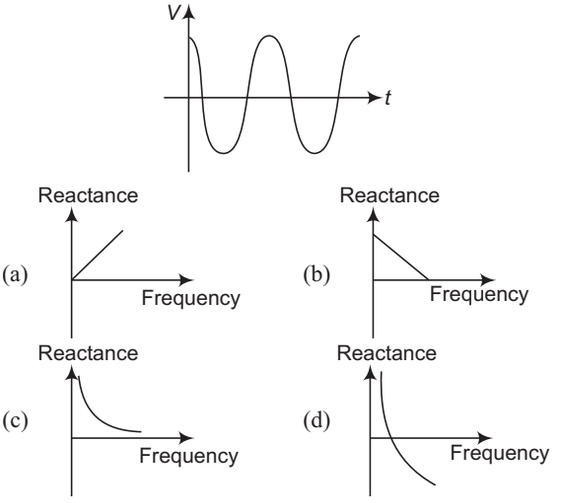
32. A 20V AC is applied to a circuit consisting of a resistance and a coil with a negligible resistance. If the voltage across the resistance is 12 V, the voltage across the coil is
(a) 16 V (b) 10 V
(c) 8 V (d) 6 V

33. An *L*-*R* circuit has $R = 10\,\Omega$ and $L = 2$ H. If 120 V, 60 Hz AC voltage is applied, then current in the circuit will be
(a) 0.32 A (b) 0.16 A
(c) 0.48 A (d) 0.80 A

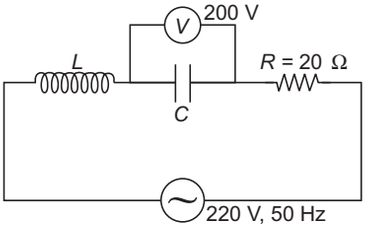
34. In a series *L*-*C*-*R* circuit, the voltage across resistance, capacitance and inductance is 10 V each. If the capacitance is short circuited, the voltage across the inductance will be
(a) $\frac{10}{\sqrt{2}}$ V
(b) 10 V
(c) $10\sqrt{2}$ V
(d) 20 V

35. In an *L*-*C*-*R* series circuit, the AC voltage across *R, L* and *C* come out as 10 V, 10 V and 20 V respectively. The voltage across the entire combination will be
(a) 30 V
(b) $10\sqrt{3}$ V
(c) 20 V
(d) $10\sqrt{2}$ V

36. The voltage across a pure inductor is represented in figure. Which one of the following curves in the figure will represent the current?



37. In the circuit shown rms current is 11 A. The potential difference across the inductor is



(a) 220 V (b) 0 V (c) 300 V (d) 200 V

38. An alternating voltage E (in volt) $= 200\sqrt{2}\sin(100t)$ is connected to a 1 μF capacitor through an AC ammeter. The reading of the ammeter shall be
(a) 10 mA (b) 20 mA (c) 40 mA (d) 80 mA

39. The peak value of an alternating emf E given by

$$E = E_0 \cos \omega t$$

is 10 V and frequency is 50 Hz. At time $t = (1/600)$s, the instantaneous value of emf is
(a) 10 V (b) $5\sqrt{3}$ V (c) 5 V (d) 1 V

40. An alternating voltage is given by

$$e = e_1 \sin \omega t + e_2 \cos \omega t$$

Then, the root mean square value of voltage is given by
(a) $\sqrt{e_1^2 + e_2^2}$ (b) $\sqrt{e_1 e_2}$ (c) $\sqrt{\frac{e_1 e_2}{2}}$ (d) $\sqrt{\frac{e_1^2 + e_2^2}{2}}$

41. An AC voltage is given by

$$E = E_0 \sin \frac{2\pi t}{T}$$

Then the mean value of voltage calculated over any time interval of $T/2$ second
(a) is always zero (b) is never zero
(c) is always $(2E_0/\pi)$ (d) may be zero

Power Consumption and Heat Generation

42. An AC supply gives $30\,V_{\text{rms}}$ which passes through a $10\,\Omega$ resistance. The power dissipated in it is
(a) $90\sqrt{2}$ W (b) 90 W
(c) $45\sqrt{2}$ W (d) 45 W

43. The average power dissipated in a pure inductor of inductance L when an AC current is passing through it, is
(a) $\frac{1}{2}LI^2$ (b) $\frac{1}{4}LI^2$
(c) $2Li^2$ (d) zero

44. In an AC circuit, the power factor
(a) is zero when the circuit contains an ideal resistance only
(b) is unity when the circuit contains an ideal resistance only
(c) is zero when the circuit contains an ideal inductance only
(d) is unity when the circuit contains an ideal inductance only

45. Two identical electric heaters each marked 1000 W, 220 V are connected in series. This combination is connected to an AC supply of 220 V. What will be their combined rate of heating? (Assume that resistance of each heater remains constant)
(a) 2000 W (b) 1000 W (c) 500 W (d) 250 W

46. An alternating potential $V = V_0 \sin \omega t$ is applied across a circuit. As a result the current $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ flows in it. The power consumed in the circuit per cycle is
(a) zero (b) $0.5\,V_0 I_0$
(c) $0.707\,V_0 I_0$ (d) $1.414\,V_0 I_0$

47. In a heating arrangement an alternating current having a peak value of 28 A is used. To produce the same heat energy, if the constant current is used, its magnitude must be
(a) about 14 A (b) about 28 A
(c) about 20 A (d) Cannot say

48. 110 V (rms) is applied across a series circuit having resistance 11 Ω and impedance 22 Ω. The power consumed is
(a) 275 W (b) 366 W (c) 550 W (d) 1100 W

49. An *R*-*L*-*C* circuit containing a 52 Ω resistor, a 230 mH inductor and a $8.8\,\mu$F capacitor is driven by an AC voltage source that has an amplitude of 150 V and frequency $f = 80$ Hz. How much average power is dissipated by this circuit?
(a) 78.6 W (b) Zero (c) 19.6 W (d) 24.8 W

50. In an *L*-*R* circuit, the inductive reactance is equal to the resistance R of the circuit. An emf $E = E_0 \cos(\omega t)$ is applied to the circuit. The power consumed in the circuit is
(a) $\frac{E_0^2}{\sqrt{2}R}$ (b) $\frac{E_0^2}{4R}$ (c) $\frac{E_0^2}{2R}$ (d) $\frac{E_0^2}{8R}$

51. The impedance of a circuit consists of 3 Ω resistance and 4 Ω reactance. The power factor of the circuit is
(a) 0.4 (b) 0.6 (c) 0.8 (d) 1.0

52. In an AC circuit V_0, I_0 and cos θ are voltage amplitude, current amplitude and power factor respectively, the power consumption is
(a) $\frac{1}{2}V_0 I_0 \cos\theta$ (b) $\left(\frac{1}{\sqrt{2}}\right)V_0 I_0 \cos\theta$
(c) $V_0 I_0 \cos\theta$ (d) $\left(\frac{1}{\sqrt{2}}\right)V_0 I_0 \cos\theta$

53. Power factor is one for
(a) pure inductor
(b) pure capacitor
(c) pure resistor
(d) Either an inductor or a capacitor

54. A direct current of 2 A and an alternating current having a maximum value of 2 A flow through two identical resistances. The ratio of heat produced in the two resistances will be
(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 4 : 1

55. The power factor of a circuit is
(a) Z/R (b) R/Z
(c) R/X (d) X/R

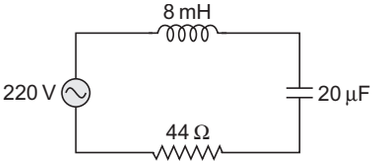
Miscellaneous Problems

56. The process by which AC is converted into DC is known as
(a) purification (b) amplification
(c) rectification (d) current amplification

57. A choke coil has
(a) high inductance and low resistance
(b) low inductance and high resistance
(c) high inductance and high resistance
(d) low inductance and low resistance

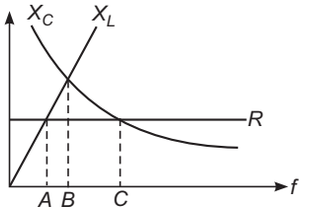
58. *L*, *C* and *R* denote inductance, capacitance and resistance respectively. Pick out the combination which does not have the dimensions of frequency
(a) $\frac{1}{RC}$ (b) $\frac{R}{L}$ (c) $\frac{1}{\sqrt{LC}}$ (d) $\frac{C}{L}$

59. For the series *L*-*C*-*R* circuit shown in the figure, what is the resonance frequency and the current at the resonating frequency?



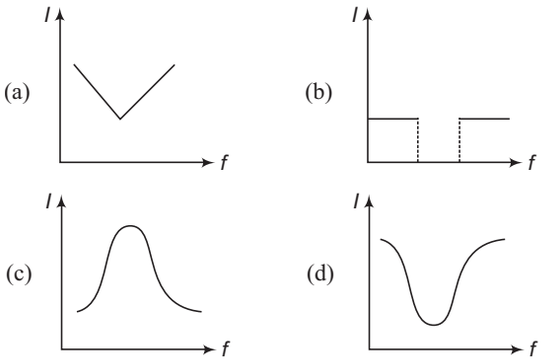
(a) 2500 rad s^{-1} and $5\sqrt{2}$ A
(b) 2500 rad s^{-1} and 5 A
(c) 2500 rad s^{-1} and $\frac{5}{\sqrt{2}}$ A
(d) 25 rad s^{-1} and $5\sqrt{2}$ A

60. The figure shows variation of R, X_L and X_C with frequency f in a series L, C, R circuit. Then for what frequency point, the circuit is inductive?

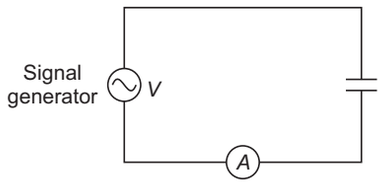


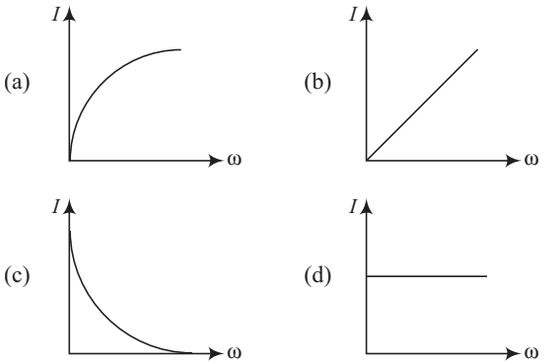
(a) A
(b) B
(c) C
(d) All points

61. An AC source of variable frequency f is connected to an L-C-R series circuit. Which one of the graphs in figure represents the variation of current I in the circuit with frequency f?



62. A constant voltage at different frequencies is applied across a capacitance C as shown in the figure. Which of the following graphs correctly depicts the variation of current with frequency?





63. Current and voltage in AC are

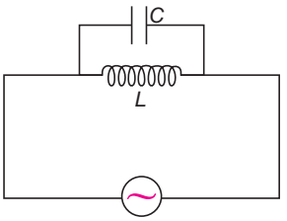
$$i = i_0 \sin(\omega t - \pi/4)$$

and $$V = V_0 \sin(\omega t + \pi/4),$$

Then,

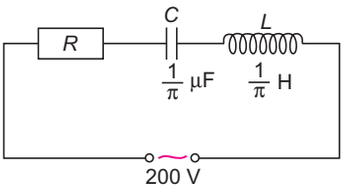
(a) $X_L > X_C$
(b) $R = 0$
(c) Both are correct
(d) Both are wrong

64. For the circuit shown in the figure, the current through the inductor is 1.6 A, while the current through the condenser is 0.4 A. Then,



(a) the current drawn from the source is $I = 2\sqrt{2}$A
(b) $I = 1.65$ A
(c) $I = 1.2$ A
(d) $I = 2.0$ A

65. In the circuit shown in figure, the supply has a constant rms value V but variable frequency f. The frequency at which the voltage drop across R is maximum is



(a) 100 Hz
(b) 500 Hz
(c) 300 Hz
(d) None of the above

66. Alternating current is transmitted to distant places at

(a) high voltage and low current
(b) high voltage and high current
(c) low voltage and low current
(d) low voltage and high current

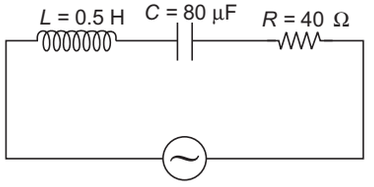
67. The frequency of a sinusoidal wave $E = 0.40\cos[2000t + 0.80]$ would be

(a) 1000 π Hz
(b) 2000 Hz
(c) 20 Hz
(d) 1000/π Hz

68. A 220 V AC is more dangerous than 220 V DC, because

(a) the AC attracts
(b) the DC repels
(c) the body offers less resistance to AC
(d) peak voltage for AC is much larger than 220 V

69. In the given figure, a series *L-C-R* circuit is connected to a variable frequency source of 230 V. The impedance and amplitude of the current at the resonating frequency will be



(a) 20 Ω and 4.2 A (b) 30 Ω and 6.9 A
(c) 25 Ω and 5.8 A (d) 40 Ω and 5.75 A

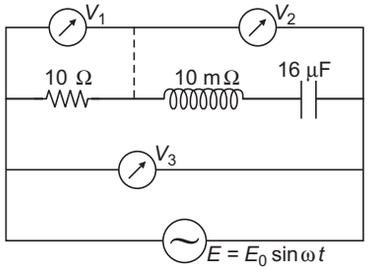
70. Using an AC voltmeter, the potential difference in the electrical line in a house is read to be 234 V. If the line frequency is known to be 50 cycles per sec, the equation for the line voltage is

(a) 165 sin ($200\pi t$) (b) 234 sin ($100\pi t$)
(c) 331 sin ($100\pi t$) (d) 440 sin ($200\pi t$)

71. The reciprocal of impedance is called

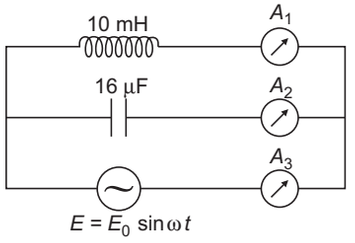
(a) reactance (b) admittance
(c) inductance (d) conductance

72. In figure, which voltmeter reads zero, when ω is equal to the resonant frequency of series *L-C-R* circuit?



(a) V_1 (b) V_2
(c) V_3 (d) None of these

73. An inductor of 10 mH and a capacitor of 16 μF are connected in the circuit as shown in figure. The frequency of the power supply is equal to the resonant frequency of the circuit. Which ammeter will read zero ampere?



(a) A_1 (b) A_2
(c) A_3 (d) None of these

74. Frequency of AC mains in India is

(a) 30 cps (b) 50 cps
(c) 60 cps (d) 120 cps

75. Ohm's law expressed as, $E = IR$

(a) can never be applied to AC
(b) applies to AC in the same manner as to DC
(c) always applies to AC circuits when Z is substituted for R
(d) tells us that $E_{\text{eff}} = 0.707\,(E_{\max})$ for AC

76. A choke coil and capacitor are connected in series and the current through the combination is maximum for AC of frequency n. If they are connected in parallel, at what frequency is the current through the combination minimum?

(a) n (b) $n/2$
(c) $2n$ (d) None of these

77. The rms value of an AC of 50 Hz is 10 A. The time taken by an alternating current in reaching from zero to maximum value and the peak value of current will be

(a) 2×10^{-2} s and 14.14 A (b) 1×10^{-2} s and 7.07 A
(c) 5×10^{-3} s and 7.07 A (d) 5×10^{-3} s and 14.14 A

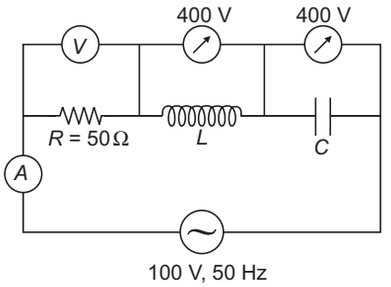
78. A capacitor is a perfect insulator for

(a) direct current
(b) alternating current
(c) direct as well as alternating current
(d) None of the above

79. In non-resonant circuit, what will be the nature of the circuit for frequencies higher than the resonant frequency?

(a) Resistive
(b) Capacitive
(c) Inductive
(d) None of the above

80. In the series *L-C-R* circuit, the voltmeter and ammeter readings are



(a) $V = 100$ V, $I = 2$ A
(b) $V = 100$ V, $I = 5$ A
(c) $V = 1000$ V, $I = 2$ A
(d) $V = 300$ V, $I = 1$ A

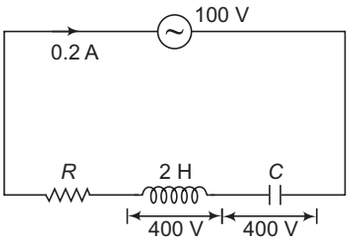
81. An AC source is connected to a capacitor. The current in the circuit is I. Now, a dielectric slab is inserted into the capacitor, then the new current is

(a) equal to I
(b) more than I
(c) less than I
(d) may be more than or less than I

[Level 2]

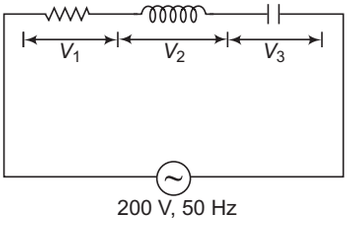
Only One Correct Option

1. Which of the following options is correct?



(a) $R = 400\ \Omega, C = 0.5\ \mu F$ (b) $R = 500\ \Omega, C = 0.5\ \mu F$
(c) $R = 500\ \Omega, C = 1\ \mu F$ (d) $R = 400\ \Omega, C = 0.1\ \mu F$

2. For series *L-C-R* AC circuit shown in figure, the readings of V_1 and V_3 are same and each equal to 100 V. Then,



(a) the reading V_2 is 200 V
(b) the reading of V_2 is 0
(c) the circuit is in resonant mode and resonant frequency is 50 Hz
(d) The inductive and capacitive reactance are equal

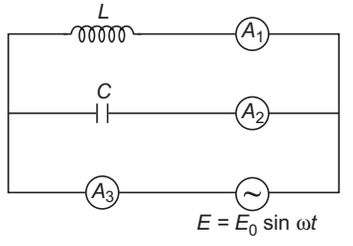
3. When an AC voltage, of variable frequency is applied to series *L-C-R* circuit, the current in the circuit is the same at 4 kHz and 9 kHz. The current in the circuit is maximum at

(a) 5 kHz (b) 6.5 kHz (c) 4.2 kHz (d) 6 kHz

4. When 100 V DC is applied across a solenoid, a current of 1 A flows in it. When 100 V AC is applied across the same solenoid the current drops to 0.5 A. If the frequency of the AC source is 50 Hz, the impedance and inductance of the solenoid are

(a) 200 Ω and 0.55 H (b) 100 Ω and 0.86 H
(c) 200 Ω and 1.0 H (d) 1100 Ω and 0.93 H

5. An inductor *L* and a capacitor *C* are connected in the circuit as shown in the figure. Which ammeter will read zero ampere?



(a) A_1 (b) A_2
(c) A_3 (d) None of these

6. An ideal choke takes a current of 8 A when connected to an AC source of 100 V and 50 Hz. A pure resistor under the same conditions takes a current of 10 A. If two are connected in series to an AC supply of 100 V and 40 Hz, then the current in the series combination of above resistor and inductor is

(a) 10 A (b) 5 A (c) $5\sqrt{2}$ A (d) $10\sqrt{2}$ A

7. An AC source is connected with a resistance *(R)* and an uncharged capacitance *(C)*, in series. The potential difference across the resistor is in phase with the initial potential difference across the capacitor for the first time at the instant (assume that at $t = 0$, emf is zero).

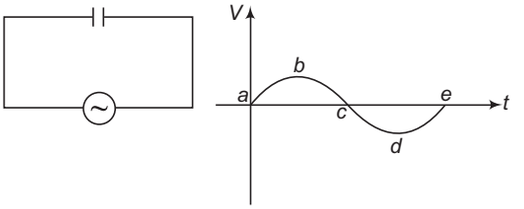
(a) $\frac{\pi}{4\omega}$ (b) $\frac{2\pi}{\omega}$ (c) $\frac{\pi}{2\omega}$ (d) $\frac{3\pi}{2\omega}$

where, ω is the angular frequency.

8. Current through an AC series *L-C-R* circuit is 2 A, if operated at resonant frequency and 1 A if operated at 50% less than resonant frequency. The current (in A) if the frequency is 100% more than the resonant frequency, is

(a) $\sqrt{2}$ (b) 1
(c) $\sqrt{3}$ (d) Data insufficient

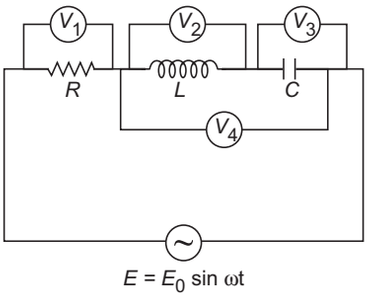
9. For an AC circuit containing capacitor only, the applied AC voltage waveform is shown in figure.



For this situation mark the correct statement(s).

(a) As *V* increases from *a* to *b*, the charging of capacitor takes place
(b) As *V* increases from *a* to *b*, the current in circuit decreases from maximum to zero value
(c) As *V* decreases from *b* to *c*, the capacitor discharges
(d) As *V* decreases from *b* to *c* charging of capacitor takes place

10. In the given figure, which voltmeter will read zero voltage at resonant frequency?



(a) V_1 (b) V_2 (c) V_3 (d) V_4

11. A coil having an inductance of $\frac{1}{\pi}$ henry is connected in series with a resistance of 300 Ω. If 20 V 200 Hz AC source is applied across the combination, the phase angle between the voltage and the current is

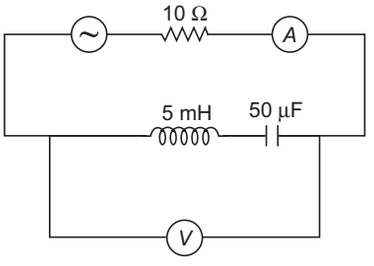
(a) $\tan^{-1}\frac{5}{4}$

(b) $\tan^{-1}\frac{4}{5}$

(c) $\tan^{-1}\frac{3}{4}$

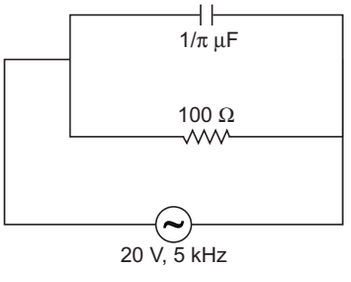
(d) $\tan^{-1}\frac{4}{3}$

12. In the circuit shown in figure, the AC source gives a voltage $V = 20\cos(2000t)$. Neglecting source resistance, the voltmeter and ammeter readings will be (approximately)



(a) 4 V, 2.0 A
(b) 0 V, 2A
(c) 0 V, 1.4 A
(d) 8 V, 2.0 A

13. A signal generator supplies a sine wave of 20 V, 5 kHz to the circuit shown in the figure. Then, choose the wrong statement.

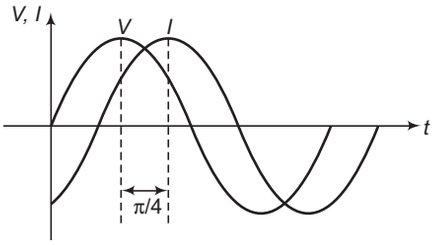


(a) The current in the resistive branch is 0.2 A
(b) The current in the capacitive branch is 0.126 A
(c) Total line current is ≈ 0.283 A
(d) Current in both the branches is same

14. A complex current wave is given by $i = (5 + 5\sin 100\pi t)$ A. Its average value over one time period is given as

(a) 10 A
(b) 5 A
(c) $\sqrt{50}$ A
(d) zero

15. An AC voltage $V = V_0 \sin 100t$ is applied to the circuit, the phase difference between current and voltage is found to be $\frac{\pi}{4}$, then



(a) $R = 100\ \Omega, C = 1\ \mu F$ (b) $R = 1 k\Omega, C = 10\ \mu F$
(c) $R = 10\ k\Omega, L = 1$ H (d) $R = 1\ k\Omega, L = 10$ H

16. When an alternating voltage of 220 V is applied across a device *P*, a current of 0.25 A flows through the circuit and it leads the applied voltage by an angle $\pi/2$ radian. When the same voltage source is connected across another device *Q*, the same current is observed in the circuit but in phase with the applied voltage. What is the current when the same source is connected across a series combination of *P* and *Q*.

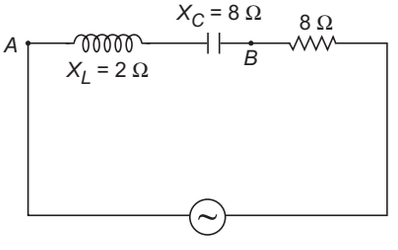
(a) $\frac{1}{4\sqrt{2}}$ A lagging in phase by $\frac{\pi}{4}$ with voltage

(b) $\frac{1}{4\sqrt{2}}$ A leading in phase by $\frac{\pi}{4}$ with voltage

(c) $\frac{1}{\sqrt{2}}$ A leading in phase by $\frac{\pi}{4}$ with voltage

(d) $\frac{1}{\sqrt{2}}$ A leading in phase by $\frac{\pi}{6}$ with voltage

17. An inductor ($X_L = 2\Omega$) a capacitor ($X_C = 8\Omega$) and a resistance ($R = 8\Omega$) are connected in series with an AC source. The voltage output of AC source is given by $V = 10\cos(100\pi t)$.



The instantaneous potential difference between points *A* and *B*, when the applied voltage is 3/5th of the maximum value of applied voltage is

(a) 0 V (b) 6 V
(c) 8 V (d) None of these

18. A group of electric lamps having a total power rating of 1000 W is supplied by an AC voltage $E = 200\sin(310t + 60°)$. Then, the rms value of the circuit current is

(a) 10 A (b) $5\sqrt{2}$ A (c) 20 A (d) $10\sqrt{2}$ A

19. One 10 V, 60 W bulb is to be connected to 100 V line. The required self inductance of indication coil will be ($f = 50$ Hz)

(a) 0.052 H (b) 2.42 H
(c) 16.2 H (d) 16.2 mH

20. An alternating voltage $V = 30\sin 50t + 40\cos 50t$ is applied to a resistor of resistance 10 Ω. The rms value of current through resistor is

(a) $\frac{5}{\sqrt{2}}$ A (b) $\frac{10}{\sqrt{2}}$ A
(c) 5 A (d) None of these

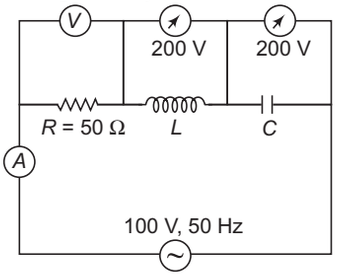
21. An alternating voltage $V = 140\sin 50t$ is applied to a resistor of 10 Ω. This voltage produces ΔH heat in the resistor in time Δt. To produce the same heat in the same time, required DC current is

(a) 14 A
(b) about 20 A
(c) about 10 A
(d) None of the above

22. An AC circuit consists of a resistance and a choke coil in series. The resistance is of 220 Ω and choke coils is of 0.7 H. The power absorbed from 220 V and 50 Hz, source connected with the circuit, is

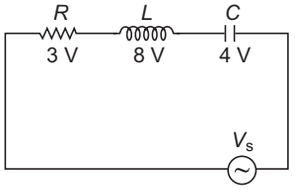
(a) 55 W
(b) 110 W
(c) 220 W
(d) 440 W

23. In the series *L*-*C*-*R* circuit, the voltmeter and ammeter readings are respectively



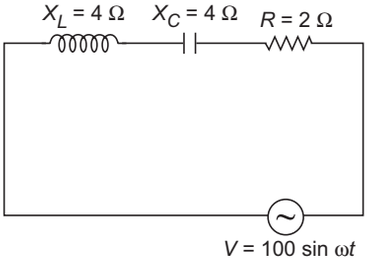
(a) $V = 200$ V, $I = 4$ A
(b) $V = 150$ V, $I = 2$ A
(c) $V = 100$ V, $I = 5$ A
(d) $V = 100$ V, $I = 2$ A

24. Current in resistance is 1 A, then



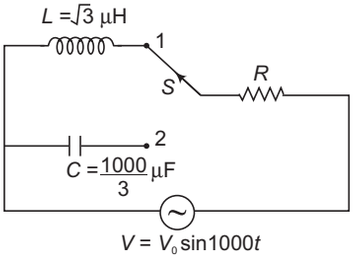
(a) $V_s = 5$ V
(b) impedance of network is 5Ω
(c) power factor of given circuit is (0.6) lagging (current is lagging)
(d) All of the above

25. Which of the following statement is correct regarding the AC circuit shown in the adjacent figure?



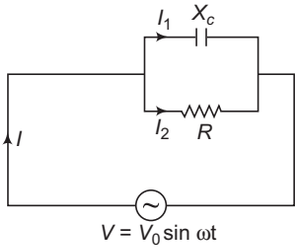
(a) The rms value of current through the circuit is $i_{rms} = 5\sqrt{2}$ A
(b) The phase difference between source emf and current is $\phi = \cos^{-1}\left(\frac{1}{3}\right)$
(c) Average power dissipated in the circuit is 500 W
(d) None of the above

26. In the given AC circuit, when switch *S* is at position 1, the source emf leads current by $\pi/6$. Now, if the switch is at position 2, then



(a) current leads source emf by $\frac{\pi}{4}$
(b) current leads source emf by $\frac{\pi}{3}$
(c) source emf lead current by $\frac{\pi}{4}$
(d) source emf leads current by $\frac{\pi}{3}$

27. In the given AC circuit



(a) current I_2 and V are is same phase
(b) current I_2 leads I_1 by 90°
(c) current I leads I_2 by $\theta < 90°$
(d) current I leads I_1 by $\theta < 90°$

28. Two coils have a mutual inductance 0.005 H. The current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10$ A and $\omega = 100\pi$ rad/s. The maximum value of emf in the second coil is (in volt)

(a) 2π (b) 5π (c) π (d) 4π

29. In a certain circuit current changes with time according to $i = 2\sqrt{t}$. The root mean square value of current between $t = 2$ to $t = 4$ s will be

(a) 3 A (b) $3\sqrt{3}$ A (c) $2\sqrt{3}$ A (d) $\sqrt{3}$ A

30. An AC source of angular frequency ω is fed across a resistor r and a capacitor C in series. The current registered is I. If now the frequency of source is changed to $\frac{\omega}{3}$ (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency ω.

(a) $\sqrt{\frac{3}{5}}$ (b) $\sqrt{\frac{2}{5}}$ (c) $\sqrt{\frac{1}{5}}$ (d) $\sqrt{\frac{4}{5}}$

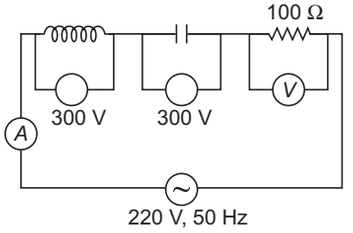
31. An *L-C-R* series circuit with a resistance of 100 Ω is connected to an AC source of 200 V (rms) and angular frequency 300 rad/s. When only the capacitor is removed, the current lags behind the voltage by 60°. When only the inductor is removed the current leads the voltage by 60°. The average power dissipated in original *L-C-R* circuit is

(a) 50 W (b) 100 W (c) 200 W (d) 400 W

32. A virtual current of 4 A and 50 Hz flows in an AC circuit containing a coil. The power consumed in the coil is 240 W. If the virtual voltage across the coil is 100 V. Its inductance will be

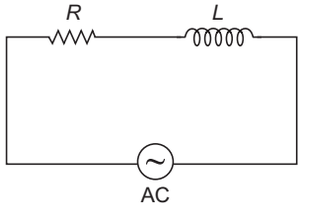
(a) $\frac{1}{3\pi}$ H (b) $\frac{1}{5\pi}$ H (c) $\frac{1}{7\pi}$ H (d) $\frac{1}{9\pi}$ H

33. In the circuit shown below, what will be the reading of the voltmeter and ammeter?



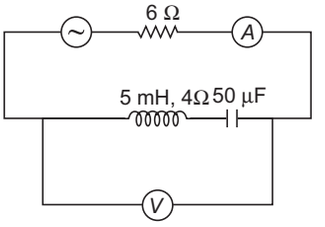
(a) 200 V, 1 A (b) 800 V, 2 A
(c) 220 V, 2 A (d) 220 V, 2.2 A

34. A circuit contains resistance R and an inductance L in series. An alternating voltage $V = V_0 \sin \omega t$ is applied across it. The currents in R and L respectively will be



(a) $I_R = I_0 \cos \omega t, I_L = I_0 \cos \omega t$
(b) $I_R = -I_0 \sin \omega t, I_L = I_0 \cos \omega t$
(c) $I_R = I_0 \sin \omega t, I_L = -I_0 \cos \omega t$
(d) None of the above

35. In the circuit shown in figure, the AC source gives a voltage $V = 20 \cos (2000\, t)$. Neglecting source resistance, the voltmeter and ammeter readings will be

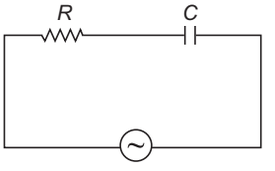


(a) 1.68 V, 0.47 A (b) 0 V, 0.47 A
(c) 5.6 V, 1.4 A (d) 0 V, 1.4 A

36. A pure resistive circuit element X, when connected to an AC supply of peak voltage 200 V, gives a peak current of 5 A. A second circuit element Y, when connected to the same AC supply also gives the same value of peak current but the current lags behind by 90°. If the series combination of X and Y is connected to the same supply, what will be the rms value of current?

(a) $\frac{10}{\sqrt{2}}$ A
(b) $\frac{5}{\sqrt{2}}$ A
(c) $\frac{5}{2}$ A
(d) 5 A

37. A 50 Hz, AC source of 20 V is connected across R and C as shown in figure. The voltage across R is 12 V. The voltage across C is



(a) 8 V
(b) 16 V
(c) 10 V
(d) not possible to determine unless value of R and C are given

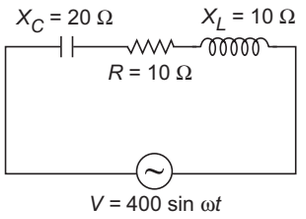
38. A coil, a capacitor and an AC source of rms voltage 24 V are connected in series. By varying the frequency of the source, a maximum rms current of 6 A is observed. If this coil is connected to a battery of emf 12 V and internal resistance 4 Ω, the maximum current through it will be

(a) 2.4 A (b) 1.8 A
(c) 1.5 A (d) 1.2 A

39. In series *L-C-R* circuit voltage drop across resistance is 8 V and across inductor is 6 V and across capacitor is 12 V. Then,

(a) voltage of the source will be leading in the circuit
(b) voltage drop across each element will be less than the applied voltage
(c) power factor of the circuit will be 3/4
(d) None of the above

40. In the *L-C-R* circuit shown in figure,



(a) current will lead the voltage
(b) rms value of current is 20 A
(c) power factor of the circuit is $1/\sqrt{2}$
(d) voltage drop across resistance is 200 V

41. If the rms current in a 50 Hz AC circuit is 5 A, the value of the current 1/300 s after its value becomes zero is
(a) $5\sqrt{2}$ A (b) $5\sqrt{3/2}$ A (c) 5 / 6 A (d) $5/\sqrt{2}$ A

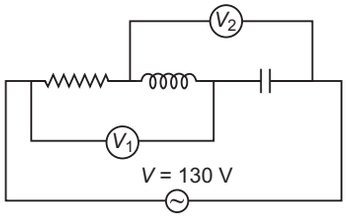
42. When a voltage measuring device is connected to AC mains, the meter shows the steady input voltage of 220 V. This means
(a) input voltage cannot be AC voltage, but a DC voltage
(b) maximum input voltage is 220 V
(c) the meter reads not v but $< v^2 >$ and is calibrated to read $\sqrt{< v^2 >}$
(d) the pointer of the meter is stuck by some mechanical defect

43. To reduce the resonant frequency in an *L-C-R* series circuit with a generator
(a) the generator frequency should be reduced
(b) another capacitor should be added in parallel to the first
(c) the iron core of the inductor should be removed
(d) dielectric in the capacitor should be removed

44. An inductor of reactance 1Ω and a resistor of 2Ω are connected in series to the terminals of a 6V (rms) AC source. The power dissipated in the circuit is
(a) 8 W (b) 12 W
(c) 14.4 W (d) 18 W

More than One Correct Options

1. In a *R-L-C* series circuit shown, the readings of voltmeters V_1 and V_2 are 100 V and 120 V. Choose the correct statement(s).



(a) Voltage across resistor, inductor and capacitor are 50 V, 86.6 V and 206.6 V, respectively
(b) Voltage across resistor, inductor and capacitor are 10 V, 90 V and 30 V, respectively
(c) Power factor of the circuit is 5/13
(d) Circuit is capacitive in nature

2. Current in an AC circuit is given by $i = 3\sin\omega t + 4\cos\omega t$, then
(a) rms value of current is 5 A
(b) mean value of this current in positive one-half period will be $\frac{6}{\pi}$
(c) if voltage applied is $V = V_m \sin\omega t$, then the circuit must be containing resistance and capacitance
(d) if voltage applied is $V = V_m \cos\omega t$, then the circuit may contain resistance and inductance only

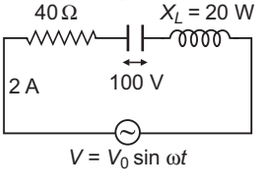
3. A tube light of 60 V, 60 W rating is connected across an AC source of 100 V and 50 Hz frequency. Then
(a) an inductance of $\frac{2}{5\pi}$ H may be connected in series
(b) a capacitor of $\frac{250}{\pi}\mu$F may be connected in series to it
(c) an inductor of $\frac{4}{5\pi}$ H may be connected in series
(d) a resistance of 40 Ω may be connected in series

4. In an AC circuit, the power factor
(a) is unity when the circuit contains an ideal resistance only
(b) is unity when the circuit contains an ideal inductance only
(c) is zero when the circuit contains an ideal resistance only
(d) is zero when the circuit contains an ideal inductance only

5. In an AC series circuit given that, $R = 10\,\Omega$, $X_L = 20\,\Omega$ and $X_C = 10\,\Omega$. Then, choose the correct options.
(a) Voltage function will lead the current function
(b) Total impedance of the circuit is $10\sqrt{2}$ Ω
(c) Phase angle between voltage function and current function is 45°
(d) Power factor of circuit is $\frac{1}{\sqrt{2}}$

6. In the above problem further choose the correct options.
(a) The given values are at frequency less than the resonance frequency
(b) The given values are at frequency more than the resonance frequency
(c) If frequency is increased from the given value, impedance of the circuit will increase
(d) If frequency is decreased from the given value, current in the circuit may increase or decrease

7. In the circuit shown in figure,



(a) $V_R = 80$ V (b) $X_C = 50\,\Omega$ (c) $V_L = 40$ V (d) $V_0 = 100$ V

8. In *L-C-R* series AC circuit
(a) If R is increased current will decrease
(b) If L is increased current will decrease
(c) If C is increased current will increase
(d) If C is increased current will decrease

9. As the frequency of an AC circuit increases, the current first increases and then decreases. What combination of circuit elements is most likely to comprise the circuit?
(a) Inductor and capacitor
(b) Resistor and inductor
(c) Resistor and capacitor
(d) Resistor, inductor and capacitor

10. In an alternating current circuit consisting of elements in series, the current increases on increasing the frequency of supply. Which of the following elements are likely to constitute the circuit?
(a) Only resistor (b) Resistor and an inductor
(c) Resistor and a capacitor (d) Only a capacitor

11. Electrical energy is transmitted over large distances at high alternating voltages. Which of the following statements is (are) correct?
(a) For a given power level, there is a lower current
(b) Lower current implies less power loss
(c) Transmission lines can be made thinner
(d) It is easy to reduce the voltage at the receiving end using step-down transformers

12. When an AC voltage of 220 V is applied to the capacitor C
(a) the maximum voltage between plates is 220 V
(b) the current is in phase with the applied voltage
(c) the charge on the plates is in phase with the applied voltage
(d) power delivered to the capacitor is zero

13. The line that draws power supply to your house from street has
(a) zero average current
(b) 220 V average voltage
(c) voltage and current out of phase by 90°
(d) voltage and current possibly differing in phase ϕ such that $|\phi| < \frac{\pi}{2}$

Comprehension Based Question

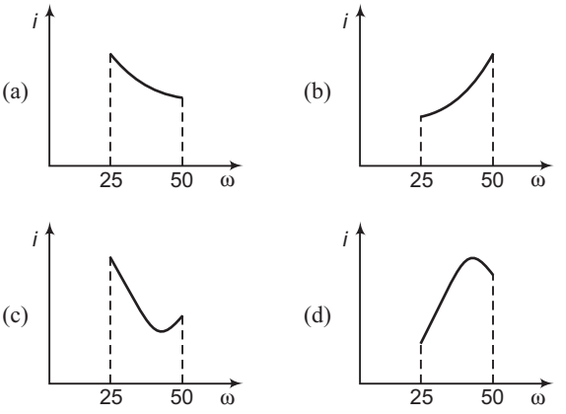
Passage I (Q. 1 to 3)

A student in a lab took a coil and connected it to a 12 *V DC source. He measures the steady state current in the circuit to be* 4 *A. He then replaced the* 12 *V DC source by a* 12 *V,* $(\omega = 50\, rad/s)$ *AC source and observes that the reading in the AC ammeter is* 2.4 *A. He then decides to connect a* 2500 μF *capacitor in series with the coil and calculate the average power developed in the circuit. Further he also decides to study the variation in current in the circuit (with the capacitor and the battery in series). Based on the readings taken by the student answer the following questions.*

1. The value of resistance of the coil calculated by the student is
(a) 3 Ω (b) 4 Ω
(c) 5 Ω (d) 8 Ω

2. The power developed in the circuit, when the capacitor of 2500 μF is connected in series with the coil is
(a) 28.8 W
(b) 23.04 W
(c) 17.28 W
(d) 9.6 W

3. Which of the following graph roughly matches the variations of current in the circuit (with the coil and capacitor connected in the series), when the angular frequency is decreased from 50 rad/s to 25 rad/s?



Passage II (Q. 4 to 6)

It is known to all of you that the impedance of a circuit is dependent on the frequency of source. In order to study the effect of frequency on the impedance, a student in a lab took 2 *impedance boxes P and Q and connected them in series with an AC source of variable frequency. The emf of the source is constant at* 10*V. Box P contains a capacitance of* 1μF *in series with a resistance of* 32 Ω*. And the box Q has a coil of self inductance* 4.9 *mH and a resistance of* 68 Ω *in series. He adjusted the frequency so that the maximum current flows in P and Q. Based on his experimental set up and the reading by him at various moment answer the following questions.*

4. The angular frequency for which he detects maximum current in the circuit is
(a) $10^5/7$ rad/s
(b) 10^4 rad/s
(c) 10^5 rad/s
(d) $10^4/7$ rad/s

5. Impedance of box P at the above frequency is
(a) 70 Ω
(b) 77 Ω
(c) 90 Ω
(d) 100 Ω

6. Power factor of the circuit at maximum current is
(a) 1/2
(b) 1
(c) zero
(d) $1/\sqrt{2}$

Assertion and Reason

Directions (Q. Nos. 1-15) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** At resonance power factor of *L-C-R* series circuit is 1.
Reason At resonance $X_C = X_L$.

2. **Assertion** At frequency greater than resonance frequency circuit is inductive in nature.
Reason $X_L \propto \omega$.

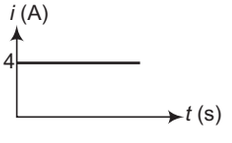
3. **Assertion** In series, *L-C-R* voltage across capacitor is always less than the applied voltage.
Reason In series *L-C-R* circuit,
$$V = \sqrt{V^2 + (V_L + V_C)^2}$$

4. **Assertion** In one complete cycle power is consumed only across a resistance in series *L-C-R* circuit.
Reason Average power consumed across an inductor or a capacitor is zero.

5. **Assertion** If an inductor coil is connected in DC source, the current supplied by it is I_1. If the same coil is connected with an AC source of same voltage. Then, current is I_2. Then, $I_2 < I_1$.
Reason In AC circuit, inductor coil offers more resistance.

6. **Assertion** Current *versus* time graph is as shown in figure, rms value of current is 4 A.
Reason For a constant current, rms current is equal to that constant value.

7. **Assertion** In an AC, only capacitor circuit instantaneous power at any instant of time is zero.



Reason Phase difference between current function and voltage function is 90°.

8. **Assertion** When a ferromagnetic rod is inserted inside an inductor, then current in *L-C-R,* alternating circuit will decrease.
Reason By inserting the ferromagnetic rod inside the inductor, coefficient of self induction and hence the net impedance will increases.

9. **Assertion** A capacitor is not connected in a DC circuit.
Reason In DC circuit current through capacitor circuit becomes zero in steady state.

10. **Assertion** Average value of current in half cycle of an AC circuit cannot be zero.
Reason For positive half cycle average value of current is $\frac{2}{\pi} i_0$, where i_0 is the peak value current.
In the time interval from t_1 to t_2 average value of current will be zero.

11. **Assertion** An AC can be transmitted over long distances without much power loss.
Reason An AC can be stepped up or down with the help of a transformer.

12. **Assertion** In *L-C-R* series AC circuit $X_L = X_C = R$ at a given frequency. When frequency is doubled, the impedance of the circuit is $\frac{\sqrt{13}}{2} R$.
Reason The given frequency is resonance frequency.

13. **Assertion** Average power in an AC circuit is given by,
$$P = i_{\text{rms}}^2 R$$
Reason In one full cycle net power is dissipated only along a resistor.

14. **Assertion** Inductive reactance of an inductor in DC circuit is zero.
Reason Angular frequency of DC circuit is zero.

15. **Assertion** At resonance power factor of series *L-C-R* circuit is zero.
Reason At resonance current function and voltage functions are in same phase.

Match the Columns

1. Angular frequency ω in an AC, *L-C-R* series circuit is gradually increased. Then, match the following two columns.

Column I	Column II
(A) Capacitive reactance	(p) will continuously increase
(B) Inductive reactance	(q) will continuously decrease
(C) Resistance	(r) will remain constant
(D) Total impedance	(s) will first decrease then, increase

2. Match the following two columns for *L-C-R* series AC circuit.

Column I	Column II
(A) At resonance frequency	(p) Power factor = 0
(B) No resistance in the circuit	(q) Power factor = 1
(C) Only resistance in the circuit	(r) Circuit is capacitor
(D) Frequency greater than the angular frequency	(s) Circuit is inductive

3. In an AC, series *L-C-R* circuit, $R = X_L = X_C$ applied AC voltage is *V*. Then, match the following two columns.

	Column I		Column II
(A)	V_R	(p)	zero
(B)	V_C	(q)	V
(C)	V_{RL}	(r)	$\sqrt{2}$ V
(D)	V_{CL}	(s)	2 V

4. In an AC, series *L-C-R* circuit, applied voltage is

$$V = \{100\sqrt{2}\sin(\omega t + 45°)\}V$$

Given that, $R = 30\,\Omega$, $X_L = 50\,\Omega$ and $X_C = 10\,\Omega$

Now, match the following two columns.

	Column I		Column II
(A)	Current in the circuit	(p)	120 SI unit
(B)	Power dissipated in the circuit	(q)	60 SI unit
(C)	Potential difference across resistance	(r)	2 SI unit
(D)	Potential difference across inductance	(s)	None

5. In a series *L-C-R,* AC circuit assuming that symbols have their usual meanings match the following two columns.

	Column I		Column II
(A)	If *R* is decreased	(p)	*I* will decrease
(B)	If ω is decreased	(q)	*I* will increase
(C)	If X_L is increased	(r)	*I* will first decrease, then increase
(D)	If *Z* is increased	(s)	Cannot say

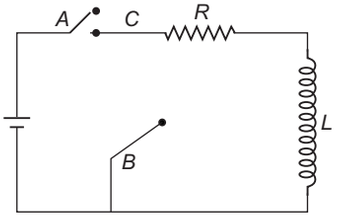
Entrance Gallery

2014

1. The current voltage relation of diode is given by $I = (e^{1000\,V/T} - 1)$ mA, where the applied voltage *V* is in volt and the temperature *T* is in kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA? **[JEE Main]**

(a) 0.2 mA (b) 0.02 mA (c) 0.5 mA (d) 0.05 mA

2. In the circuit shown here, the point *C* is kept connected to the point *A* till the current flowing through the circuit becomes constant. Afterward, suddenly point *C* is disconnected form point *A* and connected to point *B* at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to **[JEE Main]**



(a) $\frac{e}{1-e}$ (b) 1 (c) – 1 (d) $\frac{1-e}{e}$

3. A series *L-C-R* circuit contains inductance 5 mH, capacitance 2 μF and resistance 10 Ω. If a frequency AC source is varied, what is the frequency at which maximum power is dissipated? **[Karnataka CET]**

(a) $\frac{10^5}{\pi}$ Hz (b) $\frac{10^{-5}}{\pi}$ Hz

(c) $\frac{2}{\pi}\times 10^5$ Hz (d) $\frac{5}{\pi}\times 10^3$ Hz

4. The average power dissipated in AC circuit is 2 W. If a current flowing through a circuit is 2 A impedance is 1 Ω, what is the power factor of the AC circuit? **[Karnataka CET]**

(a) 0.5 (b) 1

(c) zero (d) $\frac{1}{\sqrt{2}}$

5. Two different coils have self inductance $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same rate. At a certain instant of time, the power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then, **[J & K CET]**

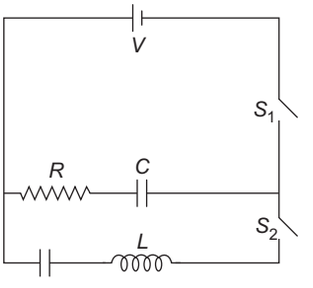
(a) $W_2/W_1 = 8$
(b) $W_2/W_1 = 1/8$
(c) $W_2/W_1 = 4$
(d) $W_2/W_1 = 1/4$

2013

6. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is **[JEE Main]**

(a) 3 V/m (b) 6 V/m
(c) 9 V/m (d) 12 V/m

7. In a *L-C-R* circuit as shown below both switches are open initially. Now, switch S_1 and S_2, are closed (*q* is charge on the capacitor and $\tau = RC$ is capacitance time constant). Which of the following statement is correct? **[JEE Main]**



(a) Work done by the battery is half of the energy dissipated in the resistor
(b) At $t = \tau$, $q = CV/2$
(c) At $t = 2\tau$, $q = CV\ (1 - e^{-2})$
(d) At $t = \frac{\tau}{2}$, $q = CV\ (1 - e^{-1})$

8. A hot wire ammeter reads 10 A in an AC circuit. The peak value of the current will be [Karnataka CET]

(a) $\frac{3}{\pi}$ A (b) $\frac{10}{\sqrt{2}}$ A

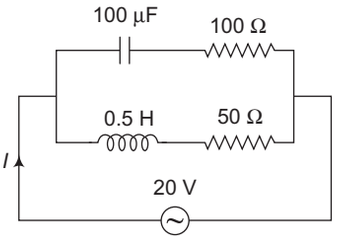
(c) $10\sqrt{2}$ A (d) 6π A

9. An ideal transformer has 500 and 5000 turn in primary and secondary windings respectively. If the primary voltage is connected to a 6 V battery, then the secondary voltage is [OJEE]

(a) zero (b) 60 V (c) 0.6 V (d) 6.0 V

2012

10. In the given circuit, the AC source has $\omega = 100$ rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is (are) [IIT JEE]



(a) the current through the circuit, I is 0.3 A

(b) the current through the circuit, I is $0.3\sqrt{2}$ A

(c) the voltage across 100 Ω resistor = $10\sqrt{2}$ V

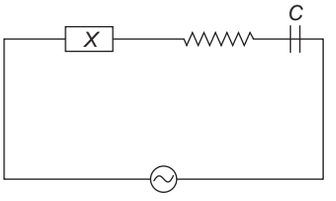
(d) the voltage across 50 Ω resistor = 10 V

11. Alternative emf of $E = 220\sqrt{2}\sin 100\,t$ is applied on a capacitor 1μF, then current is [OJEE]

(a) 22 mA (b) 33 mA

(c) 42 mA (d) 12 mA

12. If $V_{AB} = V_{AC}$, then X is [OJEE]



(a) in resonance with inductance and capacitance

(b) in resonance with resistance and capacitance

(c) in resonance with inductance and resistance

(d) None of the above

2011

13. A series R-C circuit is connected to AC voltage source. Consider two cases (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true? [IIT JEE]

(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$

(c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$

14. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is [AIEEE]

(a) $\frac{\pi}{4}\sqrt{LC}$ (b) $2\pi\sqrt{LC}$

(c) $\sqrt{LC}$ (d) $\pi\sqrt{LC}$

15. If $E = 100\sin(100t)$ volt and $I = 100\sin\left(100t + \frac{\pi}{3}\right)$ mA are the instantaneous values of voltage and current, then the rms values of voltage and current are respectively [Kerala CEE]

(a) 70.7 V, 70.7 mA

(b) 70.7 V, 70.7 A

(c) 141.4 V, 141.4 mA

(d) 141.4 V, 141.4 A

(e) 100 V, 100 mA

16. If E_0 is the peak emf, I_0 is the peak current and ϕ is the phase difference between them, then the average power dissipation in the circuit is [Kerala CEE]

(a) $\frac{1}{2}E_0I_0$

(b) $\frac{E_0I_0}{\sqrt{2}}$

(c) $\frac{1}{2}E_0I_0\sin\phi$

(d) $\frac{1}{2}E_0I_0\cos\phi$

(e) $\frac{1}{2}E_0I_0\tan\phi$

17. In an AC circuit, V and I are given by, $V = 150\sin(150\,t)$ volt and $I = 150\sin\left(150t + \frac{\pi}{3}\right)$ ampere. The power dissipated in the circuit is [Karnataka CET]

(a) zero (b) 5625 W

(c) 150 W (d) 106 W

18. The peak value of an alternating current is 5 A and its frequency is 60 Hz. Find its rms value and time taken to reach the peak value of current starting from zero. [MHT CET]

(a) 3.536 A, 4.167 ms

(b) 3.536 A, 15 ms

(c) 6.07 A, 10 ms

(d) 3.539 A, 4.167 ms

2010

19. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased [IIT JEE]

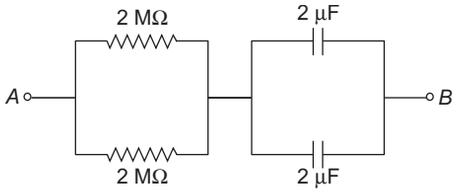
(a) the bulb glows dimmer

(b) the bulb glows brighter

(c) total impedance of the circuit is unchanged

(d) total impedance of the circuit increases

20. At time $t = 0$, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in sec) does the voltage across them become 4 V? (take, $\ln 5 = 1.6$, $\ln 3 = 1.1$) **[IIT JEE]**

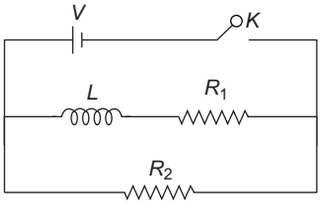


(a) 2 (b) 3 (c) 2.5 (d) $\frac{3}{2}$

21. In a series L-C-R circuit $R = 200\,\Omega$ and the voltage and the frequency of the main supply is 220V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit, the current leads the voltage by 30°. The power dissipated in the L-C-R circuit is **[AIEEE]**

(a) 305 W (b) 210 W
(c) zero (d) 242 W

22. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is **[AIEEE]**



(a) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$
(b) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = \infty$
(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$
(d) $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

23. In a series resonant R-L-C circuit, the voltage across R is 100 V and the value of $R = 1000\,\Omega$. The capacitance of the capacitor is 2×10^{-6} F, angular frequency of AC is 200 rad s^{-1}. Then, the potential difference across the inductance coil is **[Karnataka CET]**

(a) 100 V (b) 40 V (c) 250 V (d) 400 V

24. A capacitor and an inductance coil are connected in separate AC circuits with a bulb glowing in both the circuits. The bulb glows more brightly when **[Karnataka CET]**

(a) an iron rod is introduced into the inductance coil
(b) the number of turns in the inductance coil is increased
(c) separation between the plates of the capacitor is increased
(d) a dielectric is introduced into the gap between the plates of the capacitor

25. An ideal choke draws a current of 8 A, when connected to an AC supply of 100 V, 50 Hz. A pure resistor draws a current of 10 A when connected to the same source. The ideal choke and the resistor are connected in series and then connected to the AC source of 150 V, 40 Hz. The current in the circuit becomes **[Karnataka CET]**

(a) $\frac{15}{\sqrt{2}}$ A (b) 8 A (c) 18 A (d) 10 A

Answers

Level 1

Objective Problems

1. (d)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (c)
11. (a)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (d)	**16.** (a)	**17.** (d)	**18.** (c)	**19.** (b)	**20.** (a)
21. (d)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (d)	**26.** (b)	**27.** (b)	**28.** (a)	**29.** (a)	**30.** (c)
31. (d)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (d)	**36.** (d)	**37.** (d)	**38.** (b)	**39.** (b)	**40.** (d)
41. (d)	**42.** (b)	**43.** (d)	**44.** (b,c)	**45.** (c)	**46.** (a)	**47.** (c)	**48.** (a)	**49.** (a)	**50.** (b)
51. (b)	**52.** (a)	**53.** (c)	**54.** (c)	**55.** (b)	**56.** (c)	**57.** (a)	**58.** (d)	**59.** (b)	**60.** (c)
61. (c)	**62.** (b)	**63.** (c)	**64.** (c)	**65.** (b)	**66.** (a)	**67.** (d)	**68.** (d)	**69.** (d)	**70.** (c)
71. (b)	**72.** (b)	**73.** (c)	**74.** (b)	**75.** (c)	**76.** (a)	**77.** (d)	**78.** (a)	**79.** (c)	**80.** (d)
81. (b)									

Level 2

Only One Correct Option

1. (b)	**2.** (a,c,d)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (b)	**9.** (a,b,c)	**10.** (d)
11. (d)	**12.** (c)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (b)	**19.** (a)	**20.** (a)
21. (c)	**22.** (b)	**23.** (d)	**24.** (d)	**25.** (d)	**26.** (a)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (a)
31. (d)	**32.** (b)	**33.** (d)	**34.** (d)	**35.** (c)	**36.** (c)	**37.** (b)	**38.** (c)	**39.** (d)	**40.** (a,b,c,d)
41. (b)	**42.** (c)	**43.** (b)	**44.** (c)						

More than One Correct Options

1. (a,c,d)	**2.** (c,d)	**3.** (c,d)	**4.** (a,d)	**5.** (all)	**6.** (b,c,d)	**7.** (a,b,c)	**8.** (a)	**9.** (a,d)	**10.** (c,d)
11. (a,b,d)	**12.** (c,d)	**13.** (a,d)							

Comprehension Based Questions

1. (a)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (b)

Assertion and Reason

1. (a,b)	**2.** (a,b)	**3.** (d)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (d)	**8.** (e)	**9.** (d)	**10.** (d)
11. (a)	**12.** (b)	**13.** (a,b)	**14.** (a,b)	**15.** (d)					

Match the Columns

1. (A→q, B→p, C→r, D→s) **2.** (A→q, B→p, C→q, D→r, E→s) **3.** (A→q, B→q, C→r, D→s)
4. (A→r, B→p, C→q, D→s) **5.** (A→q, B→s, C→s, D→p)

Entrance Gallery

1. (a)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (a)	**10.** (a)
11. (a)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (a)	**16.** (d)	**17.** (b)	**18.** (a)	**19.** (b)	**20.** (a)
21. (d)	**22.** (d)	**23.** (c)	**24.** (d)	**25.** (a)					

Solutions

Level 1 : Objective Problems

1. Phase different between current and voltage,

$$\Delta\phi = \phi_2 - \phi_1 = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{3}$$

2. According to problem,

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$\therefore \quad \omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{10^{-3} \times 10 \times 10^{-6}}}$$

$$= \frac{1}{\sqrt{10^{-8}}} = 10^4$$

$$\therefore \quad X_L = \omega L = 10^4 \times 10^{-3} = 10\,\Omega$$

3. For same resonant frequency,

$$\frac{\omega_1}{\omega_2} = \sqrt{\frac{L_2 C_2}{L_1 C_1}} \Rightarrow \frac{\omega}{\omega} = \sqrt{\frac{L_2 4C}{LC}}$$

$$1 = 2\sqrt{\frac{L_2}{L}} \Rightarrow \therefore \quad L_2 = \frac{L}{4}$$

4. Resonant frequency

$$f = \frac{1}{2\pi\sqrt{LC}}$$

does not depend on resistance.

5. Capacitive reactance,

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2\pi \times 4000 \times 25 \times 10^{-6}} = \frac{5}{\pi}\,\Omega$$

6. Capacitive reactance

$$X_C = \frac{1}{2\pi\nu C}$$

$$\Rightarrow \quad \frac{1}{1000} = \frac{1}{2\pi \times \nu \times 5 \times 10^{-6}}$$

$$\therefore \quad \nu = \frac{100}{\pi}\,\text{MHz}$$

7. Resonant frequency,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \quad f \propto \frac{1}{\sqrt{C}} \quad \text{or} \quad \frac{f_1}{f_2} = \sqrt{\frac{C_2}{C_1}}$$

$$\Rightarrow \quad \frac{f}{f_2} = \sqrt{\frac{4C}{C}} \Rightarrow f_2 = \frac{f}{2}$$

8. In DC circuits $f = 0$,

$$\therefore \quad X_C = \frac{1}{2\pi(0)\,C} = \frac{1}{0} = \infty$$

9. Current in inductance I_L lags behind, current in resistance I_R is in phase with voltage, while current in capacitance I_C leads by a phase of $\pi/2$.

13. In case of AC, $Z = \sqrt{R^2 + (\omega L)^2}$, while in case of DC it is only R.

14. $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 100 \times 10^{-5}} = 160\,\Omega$

17. $X_L = (2\pi f L)$

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{50}{(2\pi)(50)} = 0.16\,\text{H}$$

23. Impedance, $Z = \sqrt{R^2 + (2\pi f L)^2}$

As f increases, Z will increase.

26. Applied voltage, $V = \sqrt{V_R^2 + V_L^2}$

$$\therefore \quad V = \sqrt{(200)^2 + (150)^2}$$

$$= 250\,\text{V}$$

27. Reading of ammeter $= i_{\text{rms}}$

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{V_0}{\sqrt{2}} \times \omega C = \frac{200\sqrt{2} \times 100 \times (1 \times 10^{-6})}{\sqrt{2}}$$

$$= 2 \times 10^{-2}\,\text{A} = 20\,\text{mA}$$

28. Total resistance of the circuit,

$$R = 6 + 4 = 10\,\Omega$$

Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\,\Omega$$

Inductive reactance,

$$X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\,\Omega$$

$$\therefore \quad Z = \sqrt{R^2 + (X_L - X_C)^2} = 10\,\Omega$$

Amplitude of current

$$= i_0 = \frac{V_0}{Z} = \frac{20}{10} = 2\,\text{A}$$

29. $V_{\text{rms}} = \sqrt{V_{\text{av}}} = \sqrt{\frac{1}{T}\int_0^T V^2\,dt} = \sqrt{\frac{1}{T}\int_0^T 10^2\,dt} = 10\,\text{V}$

30. Average value of output current is given by

$$I_{\text{av}} = \frac{\int_0^{T/2} I\,dt}{\int_0^{T/2} dt} = \frac{\int_0^{T/2} I_0 \sin\omega t\,dt}{T/2}$$

$$= \frac{2I_0}{T}\left[-\frac{\cos\omega t}{\omega}\right]_0^{T/2} = \frac{2I_0}{T}\left[-\frac{\cos(\omega T/2)}{\omega} + \frac{\cos 0°}{\omega}\right]$$

$$= \frac{2I_0}{\omega T}[-\cos\pi + \cos 0°] = \frac{2I_0}{2\pi}[1+1] = \frac{2I_0}{\pi}$$

31. $i_{\text{rms}} = \frac{i_0}{\sqrt{2}}$

32.

$$V^2 = V_R^2 + V_L^2$$

$$\therefore \quad V_L = \sqrt{V^2 - V_R^2}$$

$$= \sqrt{(20^2 - (12)^2}$$

$$= 16\,\text{V}$$

33. $$i=\frac{V}{Z}=\frac{V}{\sqrt{R^2+(2\pi fL)^2}}$$
$$=\frac{120}{\sqrt{(10)^2+(2\pi\times60\times2)^2}}$$
$$=0.16\,\text{A}$$

34. $X_C=X_L=R$
$$V=\sqrt{V_R^2+(V_L-V_C)^2}$$
but $V_L=V_C$
$\therefore$ $V=V_R=10$
When capacitor is short circuited, $V_C=0$
and $V_R=V_L$ (as $R=X_L$)
$\therefore$ $V=\sqrt{V_R^2+V_L^2}$
or $10=\sqrt{2}\,V_L$
$\therefore$ $V_L=\frac{10}{\sqrt{2}}\,\text{V}$

35. $V=\sqrt{V_R^2+(V_L-V_C)^2}$

36. Current lags the voltage by $\frac{\pi}{2}$.

37. $V_C=200\,\text{V}$ (given)
$$V_R=(11)\,(20)=220\,\text{V}$$
$V=220\,\text{V}$ (applied)
$\therefore$ $V_L=200\,\text{V}$
Because, $V=\sqrt{V^2+(V_L-V_C)^2}$

38. $X_C=\frac{1}{\omega C}=\frac{1}{100\times10^{-6}}=10^4\ \Omega$
$$E_{\text{rms}}=200\,\text{V}$$
$\therefore$ $i_{\text{rms}}=\frac{E_{\text{rms}}}{X_C}=\frac{200}{10^4}=20\,\text{mA}$

39. $E=10\cos(2\pi ft)$
$$=10\cos\left[2\pi\times50\times\frac{1}{600}\right]$$
$$=10\cos\frac{\pi}{6}=5\sqrt{3}\,\text{V}$$

40. $e=\sqrt{e_1^2+e_2^2}\sin(\omega t+\phi)$
where, $\cos\phi=\frac{e_1}{\sqrt{e_1^2+e_2^2}}$
and $\sin\phi=\frac{e_2}{\sqrt{e_1^2+e_2^2}}$
i.e. $e_0=\sqrt{e_1^2+e_2^2}$
or $e_{\text{rms}}=\frac{\sqrt{e_1^2+e_2^2}}{\sqrt{2}}$

42. Power dissipated in circuit,
$$P=\frac{V_{\text{rms}}^2}{R}=\frac{(30)^2}{10}=90\,\text{W}$$

43. In case of a pure inductance, $R=0$
$\therefore$ Power factor $=\cos\phi=\frac{R}{Z}=\frac{0}{X}=0$
So, $P_{\text{av}}=E_{\text{rms}}\cdot i_{\text{rms}}\times0=0$

44. Power factor of an AC circuit is given by,
$$\cos\phi=\frac{R}{Z}$$
For circuit containing an ideal resistance only,
$$Z=R$$
$\therefore$ $\cos\phi=1$
For circuit containing an ideal inductance, $R=0$
$\therefore$ $\cos\phi=\frac{0}{X_L}=0$

45. For only a resistance, it hardly matters, whether the source is DC or AC. In series,
$$P=\frac{P_1P_2}{P_1+P_2}$$

47. $i_0=28\,\text{A}\ \Rightarrow\ \therefore\ i_{\text{rms}}=\frac{i_0}{\sqrt{2}}\simeq20\,\text{A}$

48. $P=V_{\text{rms}}\cos\phi$
$$=V_{\text{rms}}\left(\frac{V_{\text{rms}}}{Z}\right)\left(\frac{R}{Z}\right)=\frac{(110)^2\,(11)}{(22)^2}=275\,\text{W}$$

49. $Z=\sqrt{(52)^2+\left[230\times10^{-3}\times2\pi\times80-\frac{1}{2\pi\times80\times8.8\times10^{-6}}\right]^2}$
$=122\ \Omega$
$P=\left(\frac{V_{\text{rms}}}{Z}\right)^2\cdot R=\left(\frac{150}{122}\right)^2(52)=78.6\,\text{W}$

50. $X_L=R$
$\therefore$ $Z=\sqrt{2}R$
$$P=\left(\frac{V_{\text{rms}}}{Z}\right)^2\cdot R=\left(\frac{E_0/\sqrt{2}}{\sqrt{2}R}\right)^2\cdot R=\frac{E_0^2}{4R}$$

51. $Z=\sqrt{(3)^2+(4)^2}=5\,\Omega\ \Rightarrow\ \cos\phi=\frac{R}{Z}$

54. $\frac{H_{\text{DC}}}{H_{\text{AC}}}=\left(\frac{i_{\text{DC}}}{i_{\text{rms}}}\right)^2\ (H\propto i^2)=\left(\frac{2}{\sqrt{2}}\right)^2=2$

57. A choke coil has high inductance and low resistance, due to large inductance L of the coil, the current in the circuit is decreased appreciably. Due to small resistance of the coil, the power loss in the choke, coil is less.

59. Resonance frequency,
$$\omega=\frac{1}{\sqrt{LC}}$$
$$=\frac{1}{\sqrt{8\times10^{-3}\times20\times10^{-6}}}$$
$$=2500\,\text{rad/s}$$
Resonance current,
$$i=\frac{V}{R}=\frac{220}{44}=5\,\text{A}$$

60. At point A, $X_C>X_L$
At point B, $X_C=X_L$
At point C, $X_C<X_L$
So at C, circuit is inductive.

61. At resonance frequency current is maximum.

62. For capacitive circuits,
$$X_C=\frac{1}{\omega C}$$
$\therefore$ $i=\frac{V}{X_C}=V\omega C$
$\Rightarrow$ $i\propto\omega$

63. Phase difference between current and voltage is $\pi/2$, with voltage leading.

64. Current through inductor and through condenser are out of phase.

65. Voltage drop across R is maximum at resonance, i.e. at

$$\omega = \frac{1}{\sqrt{LC}}$$

or
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$
$$= \frac{1}{2\pi\sqrt{\left(\frac{1}{\pi}\right)\left(\frac{1}{\pi}\times 10^{-6}\right)}}$$
$$= 500\ \text{Hz}$$

67. $2\pi f = 2000$

$\therefore$ $$f = \frac{1000}{\pi}\ \text{Hz}$$

70. $V_0 = \sqrt{2}\, V_0 = 331\ \text{V}$

$$\omega = 2\pi f = 100\ \pi$$

81. By introducing the slab C will increase. Therefore, X_C will decrease or I will increase.

Level 2 : Only One Correct Option

1. Circuit is in resonance,

$\therefore$ $V = V_R = IR$

or $$R = \frac{V}{I} = \frac{100}{0.2} = 500\ \Omega$$
$$X_L = \frac{V_L}{I} = \frac{400}{0.2} = 2000\ \Omega = \omega L$$

$\therefore$ $\omega = 1000\ \text{rad/s}$

Further, $$X_L = X_C = 2000\ \Omega = \frac{1}{\omega C}$$

$\therefore$ $$C = \frac{1}{2000\,\omega} = 0.5\ \mu\text{F}$$

2. In resonance, $V_L = V_C$ and $V = V_R$

3. $I_1 = I_2$

$\therefore$ $$\omega_1 L - \frac{1}{\omega_1 C} = \frac{1}{\omega_1 C} - \omega_2 L$$

Solving this, we get

$\frac{1}{\sqrt{LC}}$ = resonance frequency = $\sqrt{\omega_1\omega_2}$

At resonance frequency current will be maximum.

4. Impedance $= \frac{V_{AC}}{I_{AC}} = \frac{100}{0.5} = 200\ \Omega = Z$

$$R = \frac{V_{ac}}{I_{ac}} = \frac{100}{1} = 100\ \Omega$$
$$X_L = \sqrt{Z^2 - R^2}$$
$$= 100\sqrt{3}\ \Omega = (2\pi f L)$$

$\therefore$ $$L = \frac{100\sqrt{3}}{2\pi f}$$
$$= \frac{100\times 1.732}{2\times 3.14\times 50}$$
$$= 0.55\ \text{H}$$

5. I_L and I_C are out of phase. Hence, main current will be zero.

6. $X_L = \frac{100}{8}\ \Omega = 12.5\ \Omega$ with 50 Hz frequency with 40 Hz frequency.

$$X_L' = 12.5\times\frac{40}{50} \qquad (\text{as } X_L \propto \omega)$$
$$= 10\ \Omega$$
$$R = \frac{100}{10} = 10\ \Omega$$

$\therefore$ $$Z = \sqrt{R^2 + (X_L')^2} = 10\sqrt{2}\ \Omega$$
$$I = \frac{100}{10\sqrt{2}} = 5\sqrt{2}\ \text{A}$$

7. Let $V = V_0 \sin\omega t$ $\qquad (\text{as } V = 0 \text{ at } t = 0)$

Then, $V_R = V_0 \sin\omega t$

and $V_C = V_0 \sin(\omega t - \pi/2)$

V and V_R are in same phase. While V_C lays V (or V_R) by 90°.

Now, V_R is in same phase with initial potential difference across the capacitor for the first time when,

$$\omega t = -\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}$$

$\therefore$ $$t = \frac{3\pi}{2\omega}$$

8. At resonance, $X_C = X_L$

If ω is reduced to half,

$$X_C' = 23X_C \qquad \left(\text{as } X_C \propto \frac{1}{\omega}\right)$$

and $$X_L' = \frac{X_L}{2} \qquad (\text{as } X_L \propto \omega)$$

If ω is made two times,

$$X_C' = \frac{X_C}{2} \quad \text{and} \quad X_L' = 2X_L$$

i.e. value of Z will remain unchanged, current should be 1 A.

9. $V = V_0 \sin\omega t$

$$q = CV = CV_0 \sin\omega t = q_0 \sin\omega t$$
$$i = \frac{dq}{dt} = q_0\omega\cos\omega t = i_0 \cos\omega t$$

11. $X_L = 2\pi f L = 2\pi\times 200\times\frac{1}{\pi} = 400\ \Omega$

$$\tan\phi = \frac{X_L}{R} = \frac{4}{3}$$

12. $X_L = \omega L = (2000)(5\times 10^{-3}) = 10\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{(2000)(50\times 10^{-6})} = 10\ \Omega$$

Since, $X_C = X_L$

$\therefore$ $$I = \frac{V}{Z} = \frac{20/\sqrt{2}}{10}$$

or $I = \sqrt{2}\ \text{A}$ and $V = 0$

13. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\times 5\times 10^3\times\frac{1}{\pi}\times 10^{-6}} = 100\ \Omega$

Since, $X_C = R$

$\therefore$ $$I_C = I_R = \frac{20}{100} = 0.2\ \text{A}$$
$$I = \sqrt{(0.2)^2 + (0.2)^2}$$
$$= 0.283\ \text{A}$$

14. $\langle i\rangle_T = \langle 5\rangle_T + \langle 5\sin 100\,\pi t\rangle_T$

$= 5 + 0 = 5\,\text{A}$

15. In the given graph current is leading the voltage by 45°. Therefore, circuit is C-R,

$$\tan\phi = \frac{X_C}{R}$$

or $$X_C = R$$

16. P is a capacitor,

$$X_C = \frac{220}{0.25} = 880\,\Omega$$

Q is resistance,

$$R = \frac{220}{0.25} = 880\,\Omega$$

When P and Q are in series.

$$Z = \sqrt{R^2 + XC^2}$$

$$= 800\sqrt{2}\,\Omega$$

$$\phi = \tan^{-1}\left(\frac{XC}{R}\right) = 45° \quad \text{(current leading)}$$

$$i = \frac{V}{Z} = \frac{1}{4\sqrt{2}}\,\text{A}$$

17. $Z = \sqrt{R^2 + (X_C - X_L)^2} = 10\,\Omega$

$$\cos\phi = \frac{R}{Z} = \frac{4}{5}$$

$\therefore$ $$\phi = 37°$$

$$I_0 = \frac{V_0}{Z} = \frac{10}{10} = 1\,\text{A}$$

$\therefore$ $$I = 1\cos(100\,\pi t + 37°)$$

$$V = 10\cos(100\,\pi t)$$

The applied voltage is $\frac{3}{5}$ th of the maximum applied voltage.

When,

or $$100\,\pi t = 53°$$

$\therefore$ $$I = 1\cos(53° + 37°) = 0$$

$\therefore$ $$V_R = 0$$

$\therefore$ $$V_{AB} = V_{\text{applied}} = \frac{3}{5}\times 10 = 6\,\text{V}$$

18. $P = I_{\text{rms}}^2\,R$

$\therefore$ $$1000 = I_{\text{rms}}^2\,R \qquad \ldots\text{(i)}$$

$$V_{\text{rms}} = I_{\text{rms}}R$$

$\therefore$ $$\frac{200}{\sqrt{2}} = I_{\text{rms}}\,R \qquad \ldots\text{(ii)}$$

Solving these two equations, we get

$$I_{\text{rms}} = 5\sqrt{2}\,\text{A}$$

19. From, $$P = V_i = \frac{V^2}{R}$$

$$i = \frac{P}{V} = 6\,\text{A}$$

$$R = \frac{V^2}{P} = \frac{100}{60} = \frac{5}{3}\,\Omega$$

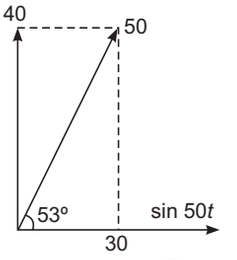
Now, $$i = \frac{V}{Z}$$

$$6 = \frac{100}{\sqrt{\left(\frac{5}{3}\right)^2 + (2\pi fL)^2}}$$

Solving this equation, we get

$$L = 0.052\,\text{H}$$

20. $V = 30\sin 50t + 40\cos 50t = 50\sin(50t + 53°)$



$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{50/\sqrt{2}}{10} = \frac{5}{\sqrt{2}}\,\text{A}$$

21. $I_0 = \frac{140}{10} = 14\,\text{A}$

$\therefore$ $$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \approx 10\,\text{A}$$

$\therefore$ I_{DC} required will be approximately 10 A.

22. $X_L = (2\pi fL) = 2\pi\times 50\times 0.7 = 220\,\Omega$

$\therefore$ $$Z = \sqrt{R^2 + X_L^2} = 220\sqrt{2}\,\Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{220}{220\sqrt{2}} = \frac{1}{\sqrt{2}}\,\text{A}$$

$$P = I_{\text{rms}}^2 \cdot R = \frac{1}{2}\times 220$$

$$= 110\,\text{W}$$

23. $V_L = V_C \Rightarrow X_L = X_C$ or circuit is shown in resonance.

$\therefore$ $$V_{\text{applied}} = V_R = 100\,\text{V}$$

$$I = \frac{V_R}{I} = 2\,\text{A}$$

24. $V_s^2 = (3)^2 + (8-4)^2, V_s = 5\,\text{V}$

Now, $$Z = \frac{V_s}{I} = \frac{5}{1} = 5\,\Omega,$$

Also, $$V_R = IR$$

or $$R = \frac{3}{1} = 3\,\Omega$$

So Power factor $= \frac{R}{Z} = \frac{3}{5} = 0.6$ as $V_L > V_C$.

$\Rightarrow I$ lags V so this is a lagging nature.

25. $X_C = X_L$

$\therefore$ $$Z = R = 2\Omega$$

$$i_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$$

$$= \frac{100/\sqrt{2}}{2} = 25\sqrt{2}\,\text{A}$$

Phase difference will be zero.

Average power

$$= V_{\text{rms}}\,i_{\text{rms}}\cos\phi = \frac{100}{\sqrt{2}}\times 25\sqrt{2}\times\cos 0°$$

$$= 2500\,\text{W}$$

26. In position-1,

$$\tan\frac{\pi}{6} = \frac{\omega L}{R} = \frac{(1000)(\sqrt{3}\times 10^{-3})}{R} \text{ or } \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{R}$$

$\therefore$ $$R = 3\,\Omega$$

In Position-2,

$$\tan\phi = \frac{X_C}{R} = \frac{(1/\omega C)}{R} = \frac{\dfrac{1}{1000\times\dfrac{1000}{3}\times 10^{-6}}}{3} = 1$$

$$\therefore \quad \phi = \frac{\pi}{4}$$

27. The given AC circuit is the combination of two pure parallel circuits with the applied voltage. In which I_2 is in phase with V and I_1 leads V by 90°.

28. Value of induced emf in coil,

$$e = M\frac{di}{dt}$$

$$= 0.005\times\frac{d}{dt}(i_0\sin\omega t)$$

$$= 0.005\times i_0\,\omega\cos\omega t$$

$$\therefore \quad e_{\max} = 0.005\times 10\times 100\pi = 5\pi$$

29. $i_{\text{rms}}^2 = \dfrac{\int i^2\,dt}{\int dt}$

$$= \frac{\int_2^4 (4t)\,dt}{\int_2^4 dt} = \frac{4\int_2^4 t\,dt}{2} = 2\left[\frac{t^2}{2}\right]_2^4 = 12\text{ A}^2$$

$$i_{\text{rms}} = 2\sqrt{3}\text{ A}$$

30. Current is halved, means impedance is doubled.

$$X_C \propto \frac{1}{\omega}$$

$\therefore$ When ω is made $\frac{1}{3}$rd, X_C will become 3 times.

$$Z' = 2Z$$

$$\therefore \quad \sqrt{R^2 + (3X_C)^2} = 2\sqrt{R^2 + X_C^2}$$

$$\therefore \quad \frac{X_C}{R} = \sqrt{\frac{3}{5}}$$

31. Phase angle, $\tan\phi = \dfrac{X_L}{R} = \dfrac{X_C}{R}$

$$\Rightarrow \quad \tan 60° = \frac{X_L}{R} = \frac{X_C}{R}$$

$$\Rightarrow \quad X_L = X_C = \sqrt{3}\,R$$

i.e. $\quad Z = \sqrt{R^2 + (\sqrt{3}\,R - \sqrt{3}\,R)^2}$

$$\Rightarrow \quad Z = R$$

So, average power, $P = \dfrac{V^2}{R} = \dfrac{200\times 200}{100}$

$$= 400\text{ W}$$

32. Power consumed in the coil,

$$P = i_{\text{rms}}^2\,R$$

$$\therefore \quad R = \frac{P}{i_{\text{rms}}^2} = \frac{240}{16} = 15\,\Omega$$

Impedance, $\quad Z = \dfrac{V}{i} = \dfrac{100}{4} = 25\,\Omega$

Now, $\quad X_L = \sqrt{Z^2 - R^2}$

$$= \sqrt{(25)^2 - (15)^2} = 20\,\Omega$$

$$\therefore \quad 2\pi\nu L = 20$$

$$L = \frac{20}{2\pi\times 50} = \frac{1}{5\pi}\text{ Hz}$$

33. Taking, $V^2 = V_R^2 + (V_L - V_C)^2$

According to the figure, we conclude that

$$V_L = V_C$$

$$\therefore \quad V_R = V = 220\text{ V}$$

Reading of voltmeter = 220 V

Reading of ammeter, $i = \dfrac{220}{100} = 2.2\text{ A}$

34. $I_R = I_L = I$

But they have a phase difference $0 < \phi < 90°$, with applied voltage.

35. $Z = \sqrt{10^2 + (X_L - X_C)^2}$

Here, $\quad X_L = 2000\times 5\times 10^{-3} = 10\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{200\times 50\times 10^{-6}} = 10\,\Omega$$

$$\therefore \quad Z = \sqrt{10^2 + 0} = 10\,\Omega$$

Maximum current, $\quad i_0 = \dfrac{V_0}{R} = \dfrac{20}{10}$

$$\therefore \quad i_0 = 2\text{ A}$$

Ammeter reading, $\quad i_{\text{rms}} = \dfrac{2}{\sqrt{2}} = 1.4\text{ A}$

Voltmeter reading, $V_{\text{rms}} = 4\times 1.4 = 5.6\text{ V}$

36. $R = X_L = \dfrac{200}{5} = 40\,\Omega$

$$\therefore \quad Z = \sqrt{R_f^2 + X_L^2} = 40\sqrt{2}\,\Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{200/\sqrt{2}}{40\sqrt{2}} = \frac{5}{2}\text{ A}$$

37. $V_C = \sqrt{V^2 - V_R^2}$

38. Let R be the resistance of coil + capacitor. Then,

$$R = \frac{24}{6} = 4\,\Omega$$

In the second case,

$$i_{\max} = \frac{12}{4+4} = \frac{12}{8} = 1.5\text{ A}$$

39. Since, $V_C > V_L$

$$\therefore \quad X_C > X_L.$$

Hence, current will lead the voltage.

$$V = \sqrt{V^2 + (V_C - V_L)^2}$$

$$= 10\text{ V} < V_C$$

$$\cos\phi = \frac{R}{Z} = \frac{8}{10} = \frac{4}{5}$$

40. $X_C > X_L$. Hence, current will lead the voltage.

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$= 10\sqrt{2}\,\Omega$$

$$\therefore \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{400/\sqrt{2}}{10\sqrt{2}} = 20\text{ A}$$

$$\cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$$

$$V_R = I_{\text{rms}}\,R = (20)\,(10) = 200\text{ V}$$

41. Given, $\nu = 50\text{ Hz}$, $I_{\text{rms}} = 5\text{ A}$

$$t = \frac{1}{300}\text{ s}$$

We have to find $I(t)$

$$I_0 = \text{Peak value} = \sqrt{2},\ I_{\text{rms}} = \sqrt{2}\times 5$$
$$= 5\sqrt{2}\ \text{A}$$
$$I = I_0 \sin \omega t$$
$$= 5\sqrt{2}\sin 2\pi \nu t$$
$$= 5\sqrt{2}\sin 2\pi \times 50 \times \frac{1}{300}$$
$$= 5\sqrt{2}\sin \frac{\pi}{3} = 5\sqrt{2}\times\frac{\sqrt{3}}{2} = 5\sqrt{3/2}\ \text{A}$$

42. The voltmeter connected to AC mains reads mean value ($<v^2>$) and is calibrated in such a way that it gives value of $<v^2>$, which is multiplied by form factor to give rms value.

43. We know that resonant frequency in an L-C-R circuit is given by

$$v_0 = \frac{1}{2\pi\sqrt{LC}}$$

Now to reduce v_0, either we can increase L or we can increase C.

To increase capacitance, we must connect another capacitor parallel to the first.

44. Given, $X_L = 1\Omega,\ R = 2\Omega\ \ E_{\text{rms}} = 6\,\text{V}, P_{\text{av}} = ?$

Average power dissipated in the circuit

$$P_{\text{av}} = E_{\text{rms}}\, I_{\text{rms}} \cos\phi \qquad \ldots(\text{i})$$
$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{E_{\text{rms}}}{Z}$$
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{4+1} = \sqrt{5}$$
$$I_{\text{rms}} = \frac{6}{\sqrt{5}}\text{A}$$
$$\cos\phi = \frac{R}{Z} = \frac{2}{\sqrt{5}}$$
$$P_{\text{av}} = 6\times\frac{6}{\sqrt{5}}\times\frac{2}{\sqrt{5}} \qquad [\text{from Eq. (i)}]$$
$$= \frac{72}{\sqrt{5}\sqrt{5}} = \frac{72}{5} = 14.4\,\text{W}$$

More than One Correct Options

1.
$$\sqrt{V_R^2 + V_L^2} = 100 \qquad \ldots(\text{i})$$
$$V_L \sim V_C = 120 \qquad \ldots(\text{ii})$$
$$\sqrt{V_R^2 + (V_L \sim V_C)^2} = 130 \qquad \ldots(\text{iii})$$

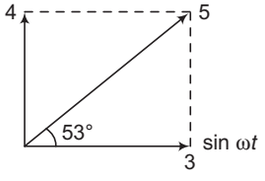
Solving these three equations, we get,

$V_R = 50\,\text{V}, V_L = 86.6\,\text{V}$ and $V_C = 206.6\,\text{V}$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{V_R}{V} = \frac{50}{130} = \frac{5}{13}$

Since, $V_C > V_L$, circuit is capacitive in nature.

2. $i = 5\sin(\omega t + 53°)$



$$i_0 = 5\ \text{A}$$
$$\therefore \quad i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = \frac{5}{\sqrt{2}}\ \text{A}$$

Mean value of current in positive half cycle is

$$\frac{2}{\pi} i_0 = \left(\frac{2}{\pi}\right)(5) = \left(\frac{10}{\pi}\right)\text{A}$$

In $V = V_m \sin\omega t$, current $i = 5\sin(\omega t + 53°)$ leads the voltage function. Hence, circuit is capacitive in nature. Same is the case with part (d).

3. $P_R = V_R\, i$

$$\therefore \quad i = \frac{P_R}{V_R} = \frac{60}{60} = 1\ \text{A}$$

Now, $V_L = \sqrt{V^2 - V_R^2} = \sqrt{(100)^2 - (60)^2}$

$$= 80\,\text{V} = i X_L = i(2\pi f L)$$
$$L = \frac{80}{2\pi f i} = \frac{80}{(2\pi)(50)(1)} = \frac{4}{5\pi}\,\text{H}$$

If we connect another resistance R in series than it should consume 40 V, so that remaining 60 V is used by the tube light.

$$R = \frac{V}{i} = \frac{40}{1} = 40\ \Omega$$

4. Power factor, $\cos\phi = \frac{R}{Z}$

When circuit contains only resistance, then

$$Z = R$$
$$\therefore \quad \cos\phi = 1$$

When circuit contains only inductance, then

$$R = 0$$
$$\therefore \quad \cos\phi = 0$$

5. ***(a)*** $X_L > X_C$, hence voltage function will lead the current function.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(10)^2 + (20-10)^2}$

$$= 10\sqrt{2}\ \Omega$$

(c) $\cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$

Hence, $\phi = 45°$

(d) Power factor, $\cos\phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$

6. ***(b)*** At resonance frequency, (ω_r)

$$X_L > X_C$$

In the given values, $X_L > X_C$. Hence

$$\omega > \omega_r$$

as, $X_L = \omega L$

$\Rightarrow$ $X_L \propto \omega$

and $X_C = \frac{1}{\omega C}$

$\Rightarrow$ $X_C \propto \frac{1}{\omega}$

(c) If frequency is increased from the given value, X_L will further increase. So $X_L - X_C$ will increase. Hence, net impedence will increase.

(d) If frequency is decreased from the given value, then X_C will increase and X_L will decrease.

So, $X_L - X_C$ may be less than the previous value or $X_C - X_L$ may be greater than the previous. So, Z may either increase or decrease. Hence, current may decrease or increase.

7. *(a)* $V_R = IR = 80\ \text{V}$

(b) $X_C = \dfrac{V_C}{I} = \dfrac{100}{2} = 50\ \text{V}$

(c) $V_L = IX_L = 40\ \text{V}$

(d) $V = V_{\text{rms}} = \sqrt{V_R^2 + (V_C - V_L)^2}$

$= \sqrt{(80)^2 + (100-40)^2}$

$= 60\ \text{V}$

$\therefore \quad V_0 = \sqrt{2}\, V_{\text{rms}}$

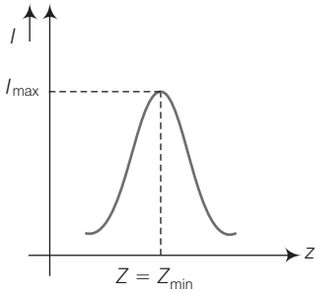
$= 60\sqrt{2}\ \text{V}$

8. $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + \left(\omega L \sim \dfrac{1}{\omega C}\right)^2}}$

By increasing R, current will definitely decrease by change in L or C, current may increase or decrease.

9. Reactance of an inductor of inductance L is, $X_L = 2\pi\nu L$

where, υ is frequency of the AC circuit.



X_C = Reactance of the capacitive circuit

$= \dfrac{1}{2\pi fC}$

On increasing frequency ν, clearly X_L increases and X_C decreases.

For a L-C-R circuit,

Z = Impedance of the circuit

$= \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{R^2 + \left(2\pi\nu L - \dfrac{1}{2\pi\nu C}\right)^2}$

As frequency (ν) increases, Z decreases and at certain value of frequency know as resonant frequency (ν_0), impedance Z is minimum that is $Z_{\min} = R$ current varies inversely with impedance and at $Z_{\min}$ current is maximum.

10. According to the question, the current increases on increasing the frequency of supply. Hence, the reactance of the circuit must be decreases as increasing frequency.

For a capacitive circuit,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

Clearly when frequency increases, X_C decreases.

For R-C circuit, $\quad X = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}$

when frequency increases X decreases.

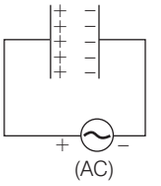
11. We have to transmit energy (power) over large distances at high alternating voltages, so current flowing through the wires will be low because for a given power (P).

$P = E_{\text{rms}}\, I_{\text{rms}}$, I_{rms} is low when E_{rms} is high.

Power loss $= I^2_{\text{rms}}\, R =$ low $\quad (\because I_{\text{rms}}$ is low)

Now, at the receiving end high voltage is reduced by using step-down transformers.

12. When the AC voltage is applied to the capacitor, the plate connected to the positive terminal will be at higher potential and the plate connected to the negative terminal will be at lower potential.



The plate with positive charge will be at higher potential and the plate with negative charge will be at lower potential. So, we can say that the charge is in phase with the applied voltage.

Power applied to a circuit is $P_{\text{av}} = V_{\text{rms}}\, I_{\text{rms}} \cos\phi$

For capacitive circuit, $\phi = 90°$

$\Rightarrow \quad \cos\phi = 0$

$\Rightarrow\ P_{\text{av}} =$ Power delivered $= 0$

13. For house hold supplies, AC currents are used which are having zero average value over a cycle.

The line is having some resistance so power factor $\cos\phi = \dfrac{R}{Z} \neq 0$

so, $\quad \phi \neq \pi/2$

$\Rightarrow \quad \phi < \pi/2$

i.e. phase lies between 0 and $\pi/2$.

Comprehension Based Questions

1. $V_{DC} = I_{DC}\, R$

$\therefore \quad R = \dfrac{V_{\text{DC}}}{I_{\text{DC}}} = \dfrac{12}{4} = 3\ \Omega$

2. $I_{\text{AC}} = \dfrac{V_{\text{AC}}}{Z} = \dfrac{V_{\text{AC}}}{\sqrt{R^2 + X_L^2}}$

$2.4 = \dfrac{12}{\sqrt{(3)^2 + X_L^2}}$

Solving this equation, we get

$X_L = 4\Omega$

$X_C = \dfrac{1}{\omega C}$

$= \dfrac{1}{50 \times 2500 \times 10^{-6}}$

$= 8\Omega$

$Z = \sqrt{R^2 + (X_C - X_L)^2} = 5\Omega$

$\therefore \quad I = \dfrac{V_{DC}}{Z} = \dfrac{12}{5} = 2.4\ \text{A} = I_{\text{rms}}$

$P = I^2_{\text{rms}}\, R = (2.4)^2\ (3)$

$= 17.28\ \text{W}$

3. At given frequency $X_C > X_L$. If ω is further decreased, X_C will increase $\left(\text{as } X_C \propto \dfrac{1}{\omega}\right)$ and X_L will increase (as $X_L \propto \omega$).

Therefore, $X_C - X_L$ and hence Z will increase. So, current will decrease.

4. $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{4.9\times10^{-3}\times10^{-6}}} = \dfrac{10^5}{7}$ rad/s

5. $X_C = \dfrac{1}{\omega C} = \dfrac{1}{\left(\dfrac{10^5}{7}\right)(10^{-6})} = 70\ \Omega$

$$Z_P = \sqrt{R_P^2 + X_C^2}$$
$$= \sqrt{(32)^2 + (70)^2} \approx 77\ \Omega$$

6. At maximum current means at resonance,

$$X_L = X_C, Z = R$$

$\therefore$ Power factor, $\cos\phi = \dfrac{R}{Z} = 1$

Assertion and Reason

1. At resonance, $X_C = X_L$ and $Z = R$.

$\therefore$ Power factor, $\cos\phi = \dfrac{R}{Z} = 1$

2. At $\omega = \omega_r$, $X_L = X_C$

At $\omega > \omega_r$, $X_L > X_C$ as,

$$X_L \propto \omega$$

and $$X_L \propto \frac{1}{\omega}$$

3. $V_R \le V$, but V_C or V_L can be greater than V also.

5. $Z_{DC} = r$

$$Z_{AC} = \sqrt{r^2 + X_L^2}$$

where, r = internal resistance of the coil.

7. If $V = V_0 \sin\omega t$, then current function will lead the voltage function by 90°.

$\therefore$ $$i = i_0 \cos\omega t$$

$\therefore$ Instantaneous power,

$$P = Vi = V_0 i_0 \sin\omega t \cos\omega t$$

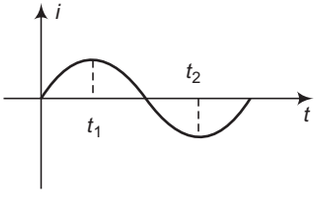
or $P \neq 0$ at all times.

8. $Z = \sqrt{R^2 + (X_L \sim X_C)^2}$

By inserting the ferromagnetic rod, L and therefore X_L will increase. But value of Z and therefore I may increase, decrease or remain same.

9. Capacitor is connected in DC circuit for charging of capacitor.

10.



In the time interval from t_1 to t_2, average value of current will be zero.

12. When frequency is doubled,

$$X_L = 2R \qquad (\text{as } X_L \propto \omega)$$

and $$X_C = \frac{R}{2} \qquad \left(\text{as } X_C \propto \frac{1}{\omega}\right)$$

$\therefore$ $$Z = \sqrt{R^2 + \left(2R - \frac{R}{2}\right)^2} = \frac{\sqrt{13}}{2} R$$

13. $P = V_{rms}\, I_{rms} \cos\phi$

$$= (I_{rms}\, Z)(I_{rms})\left(\frac{R}{Z}\right) = I_{rms}^2\, R$$

14. $X_L = \omega L$

In DC circuit, $\omega = 0$,

$\therefore$ $$X_L = 0$$

15. At resonance, $\phi = 0$

$\therefore$ $$\cos\phi = 1 \quad \text{or} \quad \text{power factor} = 1$$

Match the Columns

1. $X_C = \dfrac{1}{\omega C} \propto \dfrac{1}{\omega}$, $X_L = \omega L \propto L$

R is not a function of ω.

Z is minimum at resonance frequency.

2. Power factor, $\cos\phi = \dfrac{R}{Z}$

At resonance frequency, $X_L = X_C$

At frequency > resonance frequency, $X_L > X_C$ as $X_L \propto \omega$

3. $Z = R$ $\qquad (\text{as } X_L = X_C)$

$\therefore$ $$I = \frac{V}{Z} = \frac{V}{R}$$

Now, $$V_R = IR = V$$
$$V_C = I X_C = V = V_L$$
$$V_{RL} = \sqrt{V^2 + V^2} = \sqrt{2}\ \text{V}$$
$$V_{LC} = V - V = 0$$

4. $Z = \sqrt{R^2 + (X_L - X_C)^2} = 50\ \Omega$

$$I = \frac{V}{Z} = \frac{100\ \text{V}}{50\ \Omega} = 2\ \text{A}$$
$$P = I^2 R = 120\ \text{W}$$
$$V_R = IR = 60\ \text{V}$$
$$V_L = IX_L = 100\ \text{V}$$

5. $I = \dfrac{V}{\sqrt{R^2 + (X_L \sim X_C)^2}}$

Entrance Gallery

1. Given, $I = (e^{1000V/T} - 1)$ mA, $dV = \pm 0.01$ V,

$T = 300$ K and $I = 5$ mA

So, $$I = e^{1000V/T} - 1$$
$$I + 1 = e^{1000V/T}$$

Taking log on both sides, we get

$$\log(I+1) = \log e^{1000V/T}$$
$$\log(I+1) = \frac{1000V}{T}$$

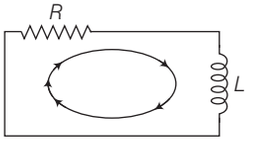
On differentiating, $\dfrac{dI}{I+1} = \dfrac{1000}{T} dV$

$$dI = \frac{1000}{T} \times (I+1)\, dV$$

$\Rightarrow$ $$dI = \frac{1000}{300} \times (5+1) \times 0.01 = 0.2\ \text{mA}$$

So, error in the value of current is 0.2 mA.

2. After connecting C to B (hanging the switch), the circuit will act like an L-R discharging circuit.

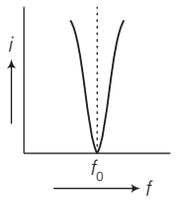


Applying Kirchhoff's loop equation, then we get

$$V_R + V_L = 0 \Rightarrow V_R = -V_L$$

$$\frac{V_R}{V_L} = -1$$

3. We know that,
(resonating frequency is given by)



$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{5\times10^{-3}\times2\times10^{-6}}}$$

where, $L = 5\times10^{-3}$ H and $C = 2\times10^{-6}$ F

$$\therefore \quad f_0 = \frac{1}{2\pi\sqrt{10^{-8}}} = \frac{1}{2\pi\times10^{-4}}$$

$$= \frac{10^4}{2\pi}\text{ Hz} = \frac{5\times10^3}{\pi}\text{ Hz}$$

4. As, average power, $\overline{P} = 2$ W, $i_{rms} = 2$ A and impedance, $Z = 1\ \Omega$
We know that, $\overline{P} = V_{rms}\times i_{rms}\cdot\cos\phi$

$$\overline{P} = i_{rms}^2 Z\cdot\cos\phi \qquad (\because V_{rms} = i_{rms}\cdot Z)$$

$$\cos\phi = \frac{\overline{P}}{i_{rms}^2\cdot Z} = \frac{2}{(2)^2\times1} = 0.5$$

5. Given, $\frac{dI_1}{dt} = \frac{dI_2}{dt}$, $L_1 = 8$ mH and $L_2 = 2$ mH

$\because$ $p_1 = p_2 \Rightarrow \frac{dw_1}{dt} = \frac{dw_2}{dt}$

$$L_1 I_1 \frac{dI_1}{dt} = L_2 I_2 \frac{dI_2}{dt} \qquad \left[\because \frac{dI_1}{dt} = \frac{dI_2}{dt}, \text{given}\right]$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{2\text{ mH}}{8\text{ mH}} = \frac{1}{4}$$

$\because$ We know that, $W = \frac{1}{2}LI^2$

$$\Rightarrow \text{So,} \quad \frac{W_1}{W_2} = \frac{L_1 I_1}{L_2 I_2^2} = \frac{L_1}{L_2}\left(\frac{I_1}{I_2}\right)^2 = \left(\frac{8}{2}\right)\left(\frac{1}{4}\right)^2$$

$$= 4\times\frac{1}{16} = \frac{1}{4}$$

$$\therefore \quad \frac{W_2}{W_1} = 4$$

6. Peak value of electric field is given by

$$E_0 = B_0 c = 20\times10^{-9}\times3\times10^8 = 6\text{ V/m}$$

7. For charging of capacitor, $q = CV(1 - e^{t/\tau})$

at $t = 2\tau$

and $q = CV(1 - e^{-2})$

8. The reading of hot wire ammeter or rms value of current,

$$I_{rms} = 10\text{ A}$$

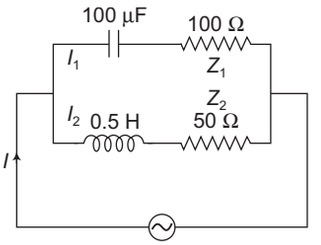
The peak value of the current is given by

$$I_0 = \sqrt{2}\times I_{rms}$$

$$= \sqrt{2}\times10 = 10\sqrt{2}\text{ A}$$

9. Transformer does not work in DC supply (battery supply) so, secondary voltage will be zero.

10.



Circuit 1, $X_C = \frac{1}{\omega C} = 100\ \Omega$

$$\therefore \quad Z_1 = \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}\ \Omega$$

$$\phi_1 = \cos^{-1}\left(\frac{R_1}{Z_1}\right) = 45°$$

In this circuit, current leads the voltage.

$$I_1 = \frac{V}{Z_1} = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}}\text{ A}$$

$$V_{100\,\Omega} = (100)\,I_1 = (100)\,\frac{1}{5\sqrt{2}}\text{ V} = 10\sqrt{2}\text{ V}$$

Circuit 2, $X_L = \omega L = (100)(0.5) = 50\ \Omega$

$$Z_2 = \sqrt{(50)^2 + (50)^2} = 50\sqrt{2}\ \Omega$$

$$\phi_2 = \cos^{-1}\left(\frac{R_2}{Z_2}\right) = 45°$$

In this circuit, voltage leads the current.

$$I_2 = \frac{V}{Z_2} = \frac{20}{50\sqrt{2}} = \frac{\sqrt{2}}{5}\text{ A}$$

$$V_{50\,\Omega} = (50)\,I_2$$

$$= 50\left(\frac{\sqrt{2}}{5}\right) = 10\sqrt{2}\text{ V}$$

Further, I_1 and I_2 have a mutual phase difference of 90°.

$$\therefore \quad I = \sqrt{I_1^2 + I_2^2} = \sqrt{\frac{1}{50} + \frac{4}{50}}$$

$$= \frac{1}{\sqrt{10}}\text{ A} \approx 0.3\text{ A}$$

11. Given, $E = 220\sqrt{2}\sin(100t)$ …(i)

Standard equation, $E = E_0\sin\omega t$. …(ii)

On comparing Eq. (i) and Eq. (ii), we get

$$E_0 = 220\sqrt{2} \quad \text{and} \quad \omega = 100$$

$$E_{\text{rms}} = \frac{220\sqrt{2}}{\sqrt{2}} = 220\ \text{V}$$

Current, $$i_{\text{rms}} = \frac{V}{X_C} = 220 \times \omega C \qquad \left[\because \frac{1}{X_C} = \omega C\right]$$

$$= 220 \times 100 \times 1 \times 10^{-6}$$

$$= 22\ \text{mA}$$

12. At resonance, $V_L = V_C$

So, X is in resonance with inductance and capacitance.

13. Impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{(R)^2 + \left(\frac{1}{\omega C}\right)^2}$

In case (b), capacitance C will be more. Therefore, impedance Z will be less. Hence, current will be more.

14. As initially charge is maximum,

$$q = q_0 \cos\omega t$$

$$\Rightarrow \quad i = \frac{dq}{dt} = -\omega q_0 \sin\omega t$$

Given, $$\frac{1}{2} Li^2 = \frac{q^2}{2C}$$

$$\Rightarrow \quad \frac{1}{2} L(\omega q_0 \sin\omega t)^2 = \frac{(q_0 \cos\omega t)^2}{2C}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad \tan\omega t = 1$$

$$\tan(\omega t) = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC}$$

15. Given, $E = 100\sin 100t$

$$I = 100\sin\left(100t + \frac{\pi}{3}\right)$$

We know that, $E_{\text{rms}} = \frac{E_0}{\sqrt{2}}$ and $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$

Hence, $$E_{\text{rms}} = \frac{100}{\sqrt{2}} = 70.7\ \text{V}$$

$$I_{\text{rms}} = \frac{100}{\sqrt{2}} = 70.7\ \text{mA}$$

16. The average power dissipation in the circuit is $\frac{1}{2} E_0 I_0 \cos\phi$.

17. Power, $P = \frac{1}{2} V_0 I_0 \cos\phi$

$$= 0.5 \times 150 \times 150 \times \cos 60°$$

$$= \frac{22500}{4} = 5625\ \text{W}$$

18. $i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.536\ \text{A}$

Time period, $T = \frac{1}{f} = \frac{1}{60} = 0.0166$

$$\therefore \quad t = \frac{T}{4} = 4.16\ \text{ms}$$

19. Impedance $Z = \sqrt{R^2 + X_C^2}$, $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$, $P = I_{\text{rms}}^2 R$

where, $$X_C = \frac{1}{\omega C}$$

As ω is increased, X_C will decrease or Z will decrease. Hence, I_{rms} or P will increase. Therefore, bulb glows brighter.

20. Voltage across the capacitors will increase from 0 to 10 V exponentially. The voltage at time t will be given by

$$V = 10(1 - e^{-t/\tau_c})$$

Here, $$\tau_c = C_{\text{net}}\ R_{\text{net}} = (1 \times 10^6)(4 \times 10^{-6}) = 4\ \text{s}$$

$$\therefore \quad V = 10(1 - e^{-t/4}) \qquad \ldots(\text{i})$$

Substituting $V = 4$ V in Eq. (i), we have,

$$4 = 10(1 - e^{-t/4}) \quad \text{or} \quad e^{-t/4} = 0.6 = \frac{3}{5}$$

Taking log both sides, we have

$$-\frac{t}{4} = \ln 3 - \ln 5$$

$$t = 4(\ln 5 - \ln 3) = 2\ \text{s}$$

21. The given circuit is under resonance as $X_L = X/C$.

Hence, power dissipated in the circuit is given by

$$P = \frac{V^2}{R} = 242\ \text{W}$$

22. At $t = 0$, inductor behaves like an infinite resistance.

So at $$t = 0,\ i = \frac{V}{R_2}$$

and at $t = \infty$, inductor behaves like a conducting wire,

$$i = \frac{V}{R_{\text{eq}}} = \frac{V(R_1 + R_2)}{R_1 R_2}$$

23. The current in the circuit,

$$i = \frac{V_R}{R} = \frac{100}{1000} = 0.1\ \text{A}$$

At resonance,

$$V_L = V_C = ix\ X_C = \frac{i}{\omega C} = \frac{0.1}{200 \times 2 \times 10^{-6}}$$

$$= 250\ \text{V}$$

24. Current, $i = \frac{V}{\sqrt{R^2 + X_C^2}}$

If a dielectric is introduced into the gap between the plates of capacitor. Its capacitance will increase and hence, impedance of the circuit will decrease. Thus, current and hence, brightness of the bulb increase.

25. Given that resistance, $R = \frac{100}{10} = 10\ \Omega$

and inductive reactance, $X_L = 2\pi fL$

$$\frac{100}{8} = 2\pi \times 50 \times L \quad \Rightarrow \quad L = \frac{1}{8\pi}\ \text{H}$$

$$X_L' = 2\pi f' L = 2\pi \times 40 \times \frac{1}{8\pi} = 10\ \Omega$$

Impedance of the circuit is given by

$$Z = \sqrt{R^2 + X_L'^2}$$

$$= \sqrt{(10)^2 + (10)^2}$$

$$= 10\sqrt{2}\ \Omega$$

Current in the circuit is given by

$$i = \frac{V}{Z} = \frac{150}{10\sqrt{2}} = \frac{15}{\sqrt{2}}\ \text{A}$$

24

Geometric Optics

24.1 Introduction

The branch of physics called **optics** deals with the behaviour of light and other electromagnetic waves. Light is the principal means by which we gain knowledge of the world. Consequently, the nature of light has been the source of one of the longest debates in the history of science.

Electromagnetic radiation with wavelengths in the range of about 4000 Å to 7000 Å, to which eye is sensitive is called **light.**

In the present and next chapter, we investigate the behaviour of a beam of light when it encounters simple optical devices like mirrors, lenses and apertures. Under many circumstances, the wavelength of light is negligible compared with the dimensions of the device as in the case of ordinary mirrors and lenses. A light beam can then be treated as a ray whose propagation is governed by simple geometric rules. The part of optics that deals with such phenomena is known as **geometric optics.** However, if the wavelength is not negligible compared with the dimensions of the device (for example a very narrow slit), the ray approximation becomes invalid and we have to examine the behaviour of light in terms of its wave properties. This study is known as **physical optics.**

Chapter Snapshot

- Introduction
- Nature of Light
- Few General Points of Geometric Optics
- Reflection of Light
- Refraction of Light
- Thin Lenses
- Total Internal Reflection (TIR)
- Refraction through Prism
- Optical Instruments

24.2 Nature of Light

The question whether light is a wave or a particle has a very interesting and long history. Early theories considered light to be a stream of particles which emanated from a source and caused the sensation of vision upon entering the eye. The most influential proponent of this particle theory of light was **Newton**. Using it, he was able to explain the laws of reflection and refraction. The chief proponents of the wave theory of light propagation were **Christiaan Huygens** and **Robert Hooke.** Huygens using his wave theory was also able to explain reflection and refraction. Newton saw the virtues of the wave theory of light particularly as it explained the colours formed by thin films, which Newton studied extensively. However, he rejected the wave theory because of the observed straight line propagation of light. Because of Newton's great reputation and authority, this reluctant rejection of the wave theory of light based on lack of evidence of diffraction was strictly adhered to by Newton's followers. Newton's particle theory of light was accepted for more than a century.

In 1801, **Thomas Young** revived the wave theory of light. He was one of the first to introduce the idea of interference as a wave phenomenon in both light and sound. His observation of interference with light was a clear demonstration of the wave nature of light. Young's work went unnoticed by the scientific community for more than a decade.

Fresnel performed extensive experiments on interference and diffraction and put the wave theory on a mathematical basis. He showed that the rectilinear propagation of light is a result of very short wavelength of visible light. In 1850, **Jean Foucault** measured the speed of light in water and showed that it is less than that in air, thus ruling out Newton's particle theory according to whom the speed of light is more in water.

But the drama was not yet over. The climax came when the wave theory of light failed to explain the photoelectric effect invented by **Albert Einstein** in 1905. He himself explained it on the basis of particle nature of light. An amicable understanding was ultimately reached in accepting that light has **dual nature**. It can behave as particles as well as waves depending on its interaction with the surroundings. Later, it was found that even the well established particles such as electrons also have a dual character and can show interference and diffraction under suitable conditions.

Electromagnetic Waves

A wave travelling along x-axis with a speed v satisfies the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad \ldots(i)$$

Maxwell was able to show that time dependent electric and magnetic fields also satisfy the wave equation. The changing electric and magnetic fields form the basis of electromagnetic waves. In free space, far from the source of the fields, the fields satisfy **Maxwell's wave equations**

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2} \qquad \ldots(ii)$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0\varepsilon_0\frac{\partial^2 B}{\partial t^2} \qquad \ldots(iii)$$

On comparing these with the standard wave equation, we see that the electromagnetic wave speed is

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \qquad \ldots(iv)$$

When the values $\mu_0 = 4\pi \times 10^{-7}$ H/m

and $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m are inserted, we find

$$c = 3 \times 10^8 \text{ m/s}$$

This is the speed of light in vacuum.

The simplest plane wave solutions to Eqs. (ii) and (iii) are

$$E = E_0 \sin(\omega t - kx) \qquad \ldots(v)$$

$$B = B_0 \sin(\omega t - kx) \qquad \ldots(vi)$$

From these equations, we see that at any point E and B are in phase. The electric and magnetic fields in a plane electromagnetic wave are perpendicular to each other and also perpendicular to the direction of propagation of light as shown in figure. They are transverse electromagnetic waves. The magnitudes of the fields are related by

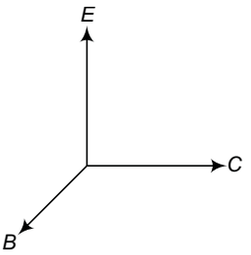


Fig. 24.1

$$c = \frac{E}{B} \quad \text{or} \quad E = cB \qquad \ldots(vii)$$

According to the thinking of the 19th century, the constants μ_0 and ε_0 referred to properties of the **ether,** the medium through which the electromagnetic waves were assumed to propagate. This is not our present thinking. The ether does not exist and electromagnetic waves do not require any medium in which to propagate. However, when they travel through a substance, the fields do interact with charges in the medium. The strength of the interaction is related to the permittivity ε and the permeability μ of the substance. As a result the speed of light in medium is reduced to $\frac{1}{\sqrt{\varepsilon\mu}}$. Hence,

$$v = \frac{1}{\sqrt{\varepsilon\mu}} \qquad \ldots(viii)$$

The ratio of c and v ($< c$) is known as the refractive index of the substance. This is a pure ratio which has a value greater than or equal to one. Thus,

$$\text{Refractive index} = \frac{c}{v} \qquad \ldots(ix)$$

Example 24.1 *The magnetic field of an electromagnetic wave in a substance is given by*

$$B = (2 \times 10^{-6}\ T)\cos[\pi(0.04x + 10^7 t)]$$

Find the refractive index of the substance.

Sol. Comparing the given equation with the standard wave equation

$$B = B_0 \cos(\omega t + kx)$$

We have, $\omega = \pi \times 10^7$ rad/s

and $k = \pi \times (0.04)\ \text{m}^{-1}$

$\therefore$ Speed of electromagnetic wave in this medium is

$$v = \frac{\omega}{k} = 2.5 \times 10^8 \text{ m/s}$$

Now, refractive index of substance $= \frac{c}{v}$

$$= \frac{3 \times 10^8}{2.5 \times 10^8} = 1.2$$

24.3 Few General Points of Geometric Optics

Here are few general points which one consider are important before studying the geometric optics. Students who have never studied the optics before are advised to read this article once more after finishing the present chapter.

1. Normally the object is kept on the left hand side of the optical instrument (mirror, lens etc.), i.e. the ray of light travels from left to right. Sometimes, it may happen that the light is travelling in opposite direction. See the figure.

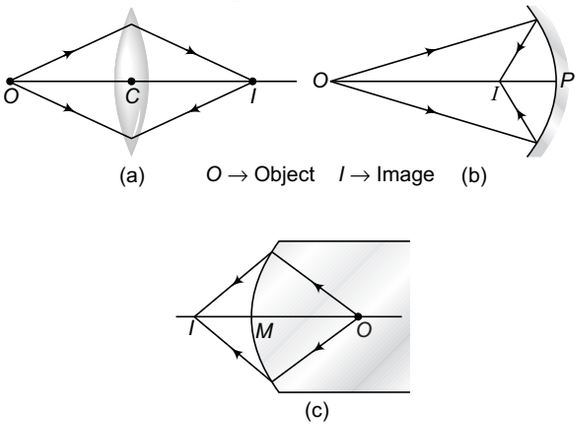


Fig.24.2

In Fig. (a) and (b), light is travelling from left to right and in Fig. (c) it is travelling from right to left.

2. Whenever a silvered surface comes on the path of a ray of light, it returns from there, otherwise it keeps on moving forwards.

3. Sign convention The distances measured along the incident light are taken as positive while the distances against incident light are taken as negative, e.g. in Fig. (a) and (b) the incident light travels from left to right. So, the distances measured in this direction are positive. While in Fig. (c) the incident light travels from right to left. So, in this case this direction will be positive. Distances are measured from pole of the mirror [point P in Fig. (b)], optical centre of the lens [point C in Fig. (a)] and the centre of the refracting surface [point M in Fig. (c)].

It may happen in some problems that sign convention does not remain same for the whole problem.

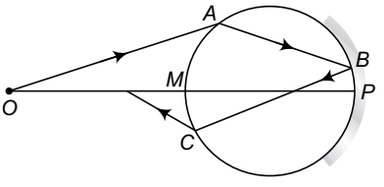


Fig. 24.3

4. Object distance (from P, C or M along the optic axis) is shown by u and image distance by v.

5. Image at infinity means rays after refraction or reflection have become parallel to the optic axis. If a screen is placed directly in between these parallel rays no image will be formed on the screen. But if a converging lens (convex) is placed on the path of the parallel rays and a screen is placed at the focus of the lens, image will be formed on the screen. Sometimes, our eye plays the role of this converging lens and the retina is the screen.

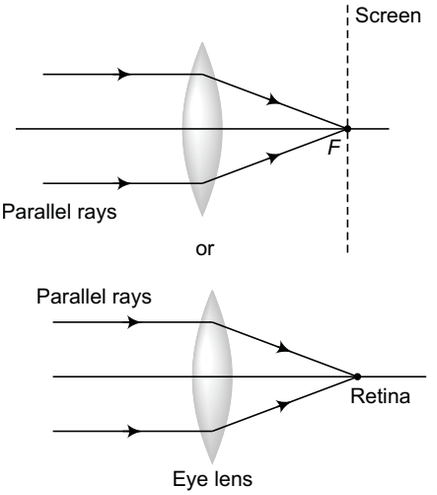


Fig. 24.4

6. Real object, virtual object, real image, virtual image In Fig. (a), object is real while image is virtual. In Fig. (b), object is virtual while its image is real.

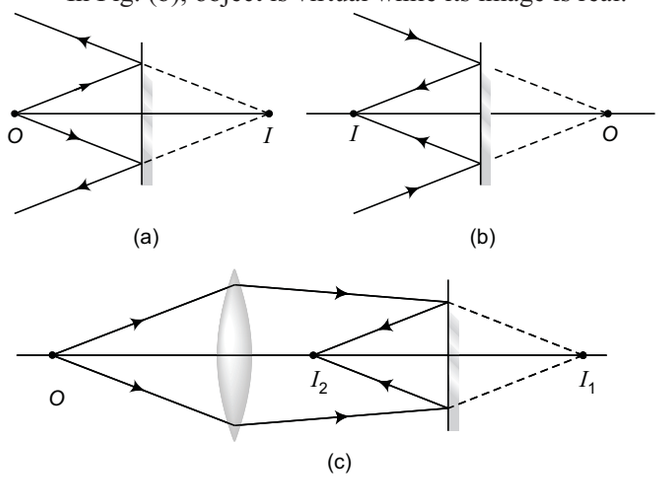


Fig. 24.5

In Fig. (c), the object O is real. Its image formed by the lens (i.e. I_1) is real. But it acts as a virtual object for mirror which forms its real image I_2.

The virtual images cannot be taken on screen. But they can be seen by our eye. Because our eye lens forms their real image on our retina. Thus, if we put a screen at I in Fig. (a) no image will be formed on it. At the same time if we put the screen at I in Fig. (b), the image will be formed.

24.4 Reflection of Light

When waves of any type strike the interface between two optical materials, new waves are generated which move away from the barrier. Experimentally, it is found that the rays corresponding to the incident and reflected waves make equal angles with the normal to the interface and that the reflected ray lies in the plane of incidence formed by the incident ray and the normal. Thus, the two laws of reflection can be summarised as under :

(1) $\angle i = \angle r$

(2) Incident ray, reflected ray and normal lie on the same plane.

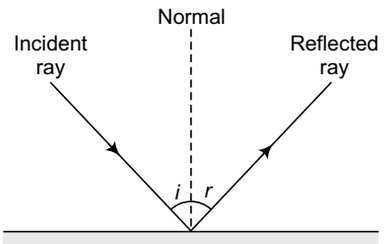


Fig. 24.6

- The above two laws of reflection can be applied to the reflecting surfaces which are not even horizontal. The following figures illustrates the point.

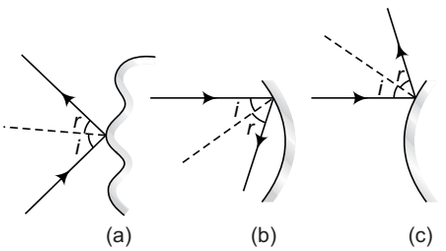


Fig. 24.7

Reflection from a Plane Surface (or Plane Mirror)

Almost everybody is familiar with the image formed by a plane mirror. If the object is real, the image formed by a plane mirror is virtual, erect, of same size and at the same distance from the mirror. The ray diagram of the image of a point object and of an extended object is shown as below:

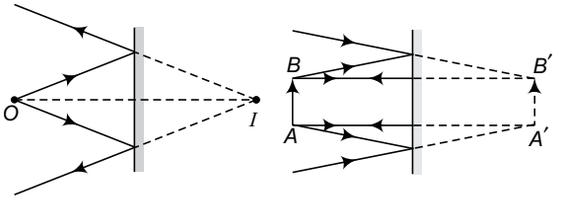
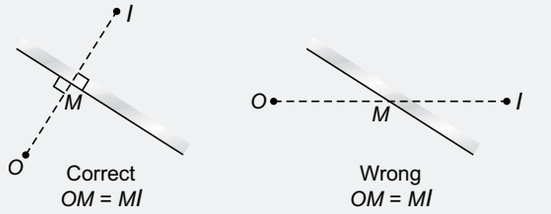


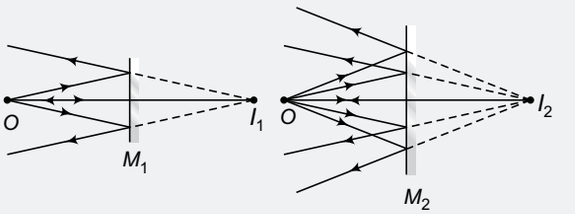
Fig. 24.8

Extra Knowledge Points

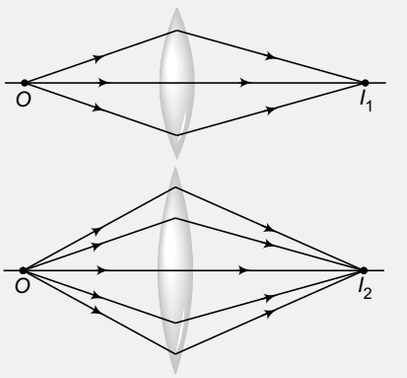
- To find the location of image of an object from an inclined plane mirror, you have to see the perpendicular distance of the object from the mirror.



- Suppose an object O is placed in front of two plane mirrors M_1 and M_2. Size of M_2 is more than that of M_1. In this case the intensity of the images formed by M_2, (i.e. I_2) will be more than that formed by M_1, (i.e. I_1). This is because I_2 is formed from more number of reflected rays. Or we can say it is formed from more light. The same is the case with an image formed by a lens of large aperture.

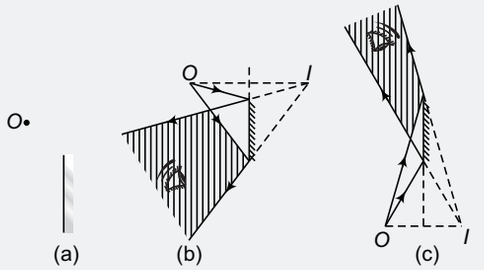


In the figure shown, I_2 will be more intense. This is the reason why we choose a lens of large aperture in telescopes to see the distant objects. You might have heard that the strength of an observatory is measured from the aperture of its lens. Now, you can think if this is the case, then why don't we take a lens of an aperture as large as we wish. Actually, some technical problems arise in casting a lens of large aperture. So, we have some limitations in doing so.



- Field of view of an object for a given mirror. Suppose a point object O is placed in front of a small mirror as shown in figure (a), then a question arises in mind whether this mirror will form the image of this object or not or suppose an elephant is standing in front of a small mirror, will the mirror form the image of the elephant or not.

The answer is yes, it will form. A mirror whatever may be the size of it forms the images of all objects lying in front of it. But every object has its own field of view for the given mirror. The field of view is the region between the extreme reflected rays and depends on the location of the object in front of themirror. If our eye lies in the field of view, then only we can see the image of the object otherwise not. The field of view of an object placed at different locations in front of a plane mirror are shown in figure (b) and (c).

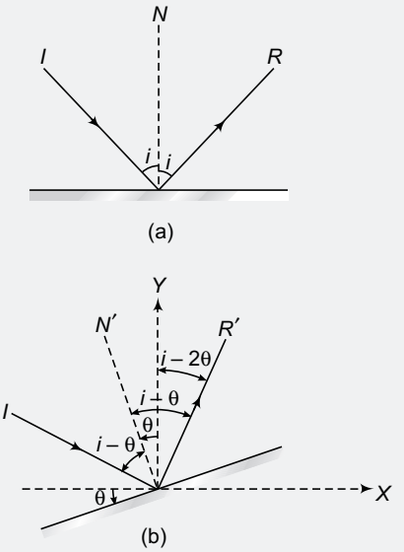


- Suppose a mirror is rotated by an angle θ (say anti-clockwise), keeping the incident ray fixed, then the reflected ray rotates by 2 θ along the same sense, i.e. anticlockwise.

  In Fig. (a), *I* is the incident ray, *N* the normal and *R* the reflected ray.

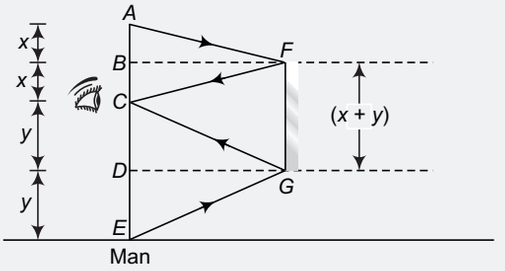
  In Fig. (b), *I* remains as it is. *N* and *R* shift to *N′* and *R′*.

  From the two figures we can see that earlier the reflected ray makes an angle *i* with *y*-axis while after rotating the mirror it makes the angle $(i-2\theta)$. Thus, we may conclude that the reflected ray has been rotated by angle 2θ.



- The minimum length of a plane mirror to see one's full height in it is $\frac{H}{2}$, where *H* is the height of man.

But the mirror should be placed in a fixed position which is shown in figure shown as below :



A ray starting from head (*A*) after reflecting from upper end of the mirror (*F*) reaches the eye at *C*. Similarly, the ray starting from the foot (*E*) after reflecting from the lower end (*G*) also reaches the eye at *C*. In two similar triangles *ABF* and *BFC*, $AB = BC = x$ (say). Similarly in triangles *CDG* and *DGE*,

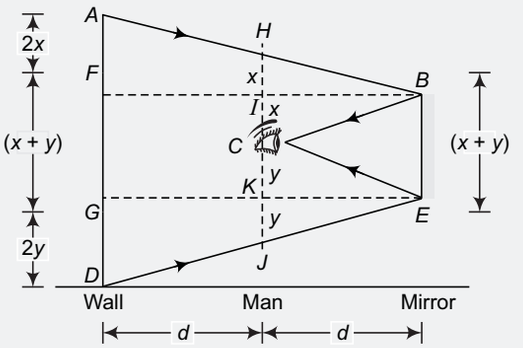
$$CD = DE = y \text{ (say)}$$

Now, we can see that height of the man is $2(x + y)$ and that of mirror is $(x + y)$, i.e. height of the mirror is half the height of the man.

- The mirror can be placed anywhere between the centre line *BF* (of *AC*) and *DG* (of *CE*)
- A man is standing exactly at midway between a wall and a mirror and he want to see the full height of the wall (behind him) in a plane mirror (in front of him). The minimum length of mirror in this case should be $\frac{H}{3}$, where *H* is the height of wall. The ray diagram in this case is drawn in figure.

  In triangles *HBI* and *IBC*, $HI = IC = x$ (say). Now, in triangles *HBI* and *ABF*,

$$\frac{AF}{HI} = \frac{FB}{BI} \text{ or } \frac{AF}{x} = \frac{2d}{d} \text{ or } AF = 2x$$



Similarly, we can prove that $DG = 2y$ if

$$CK = KJ = y$$

Now, we can see that height of the wall is $3(x + y)$ while that of the mirror is $(x + y)$.

Example 24.2 *A point source of light S, placed at a distance L in front of the centre of a mirror of width d, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance* 2*L from it as shown. The greatest distance over which he can see the image of the light source in the mirror is*
(a) $d/2$ *(b)* d *(c)* $2d$ *(d)* $3d$

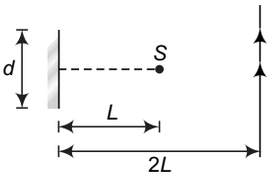


Fig. 24.9

Sol. (d) The ray diagram will be as shown in Fig. 24.10.

$$HI = AB = d$$

$$DS = CD = \frac{d}{2}$$

Since, $AH = 2AD$

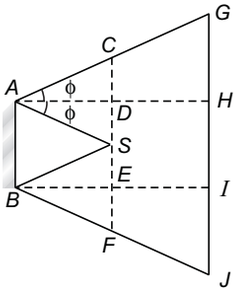


Fig. 24.10

$$\therefore \quad GH = 2\,CD = 2\frac{d}{2} = d$$

Similarly, $IJ = d$

$$\therefore \quad GJ = GH + HI + IJ$$
$$= d + d + d = 3d$$

Example 24.3 *Two plane mirrors* M_1 *and* M_2 *are inclined at angle* θ *as shown. A ray of light* 1*, which is parallel to* M_1 *strikes* M_2 *and after two reflections, the ray* 2 *becomes parallel to* M_2*. Find the angle* θ*.*

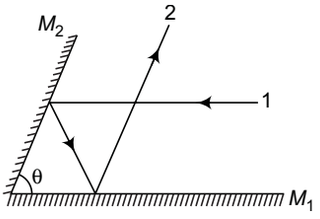


Fig. 24.11

Sol. Different angles are as shown in Fig. 24.12. In triangle *ABC*,

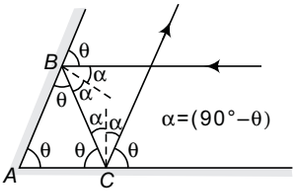


Fig. 24.12

$$\theta + \theta + \theta = 180°$$

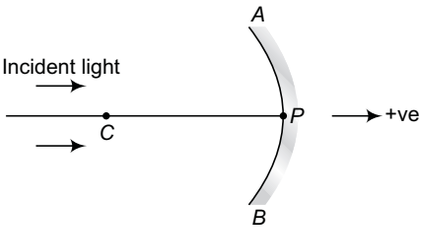
$$\therefore \quad \theta = 60°$$

Reflection from a Spherical Surface

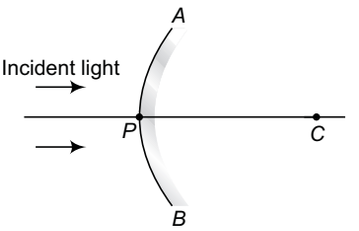
We shall mainly consider the spherical mirrors, i.e. those which are part of a spherical surface.

Terms and Definitions

There are two types of spherical mirrors, concave and convex.



(a) Concave mirror



(b) Convex mirror

Fig. 24.13

Centre *C* of the sphere of which the mirror is a part is called the **centre of curvature** of the mirror and *P* the centre of the mirror surface, is called the **pole**. The line *CP* produced is the **principal axis** and *AB* is the **aperture** of the mirror. The distance *CP* is called the **radius of curvature** (*R*)**.** All distances are measured from point *P*. We can see from the two figures that *R* is positive for convex mirror and negative for concave mirror.

Principal Focus

Observation shows that a narrow beam of rays parallel and near to the principal axis, is reflected from a concave mirror, so that all rays converge to a point *F* on the principal axis. *F* is called the **principal focus** of the mirror and it is a **real** focus, since light actually passes through it. Concave mirrors are also known as **converging mirrors** because of their action on a parallel beam of light. They are used in car head-lights, search-lights and telescopes.

A narrow beam of rays, parallel and near to the principal axis, falling on a convex mirror is reflected to form a divergent beam which appears to come from a point *F* behind the mirror. A convex mirror, thus has a virtual principal focus. It is also called a **diverging mirror**. The distance *FP* is called the **focal length** (*f*) of the mirror. Further, we can see that *f* is negative for a concave mirror

and positive for convex mirror. Later, we will see that $f = R/2$.

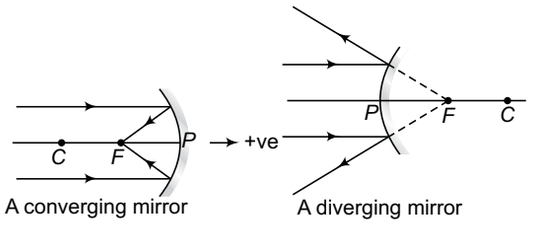


Fig. 24.14

Paraxial rays Rays which are close to the principal axis and make small angles with it, i.e. they are nearly parallel to the axis, are called paraxial rays. Our treatment of spherical mirrors will be restricted to such rays which means we shall consider only mirrors of small aperture. In diagrams, however, they will be made larger for clarity.

Relation between *f* and *R*

A ray *AM* parallel to the principal axis of a concave mirror of small aperture is reflected through the principal focus *F*. If *C* is the centre of curvature, *CM* is the normal to the mirror at *M* because the radius of a spherical surface is perpendicular to the surface. From first law of reflection,

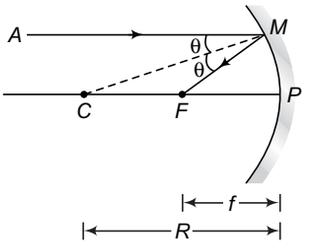


Fig. 24.15

$$\angle i = \angle r$$

or $\angle AMC = \angle CMF = \theta$ (say)

But $\angle AMC = \angle MCF$ (alternate angles)

$\therefore$ $\angle CMF = \angle MCF$

Therefore, ΔFCM is thus isosceles with $FC = FM$. The rays are paraxial and so *M* is very close to *P*. Therefore,

$$FM \approx FP$$

$\therefore$ $$FC = FP$$

or $$FP = \frac{1}{2} CP \quad \text{or} \quad f = \frac{R}{2}$$

Exercise Prove the above relation for convex mirror.

Images formed by Spherical Mirrors

In general position of image and its nature, (i.e. whether it is real or virtual, erect or inverted, magnified or diminished) depend on the distance of object from the mirror.

Information about the image can be obtained either by drawing a **ray diagram** or by calculation using **formulae.**

Ray diagrams We shall consider the small objects and mirrors of small aperture, so that all rays are paraxial. To construct the image of a point object, two of the following four rays are drawn passing through the object. To construct the image of an extended object the image of two end points is only drawn. The image of a point object lying on principal axis is formed on the principal axis itself. The four rays are as under

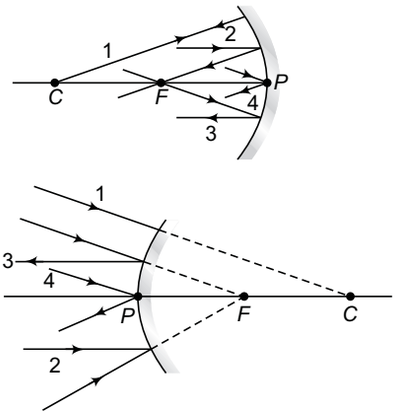


Fig. 24.16

Ray 1. A ray through the centre of curvature which strikes the mirror normally and is reflected back along the same path.

Ray 2. A ray parallel to principal axis after reflection either actually passes through the principal focus *F* or appears to diverge from it.

Ray 3. A ray passing through the principal focus *F* or a ray which appears to converge at *F* is reflected parallel to the principal axis.

Ray 4. A ray striking at pole *P* is reflected symmetrically back in the opposite sides.

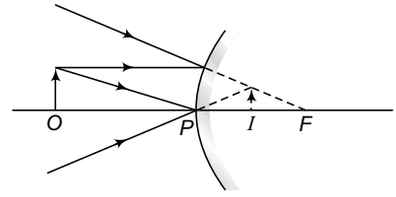


Fig. 24.17

Note Points

- Image formed by convex mirror is always virtual, erect and diminished no matter where the object is. Fig. 24.18 shows that convex mirror gives a wider field of view than a plane mirror, convex mirrors are therefore, used as rear view mirrors in cars or scooters. Although, they make the estimation of distances more difficult but still they are preferred because there is only a small movement of the image for a large movement of the object.

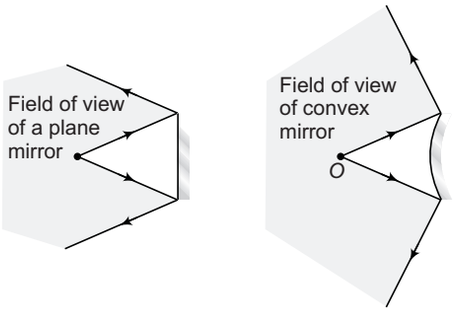


Fig. 24.18

- In case of a concave mirror the image is erect and virtual when the object is placed between *F* and *P*. In all other positions of object, the image is real.

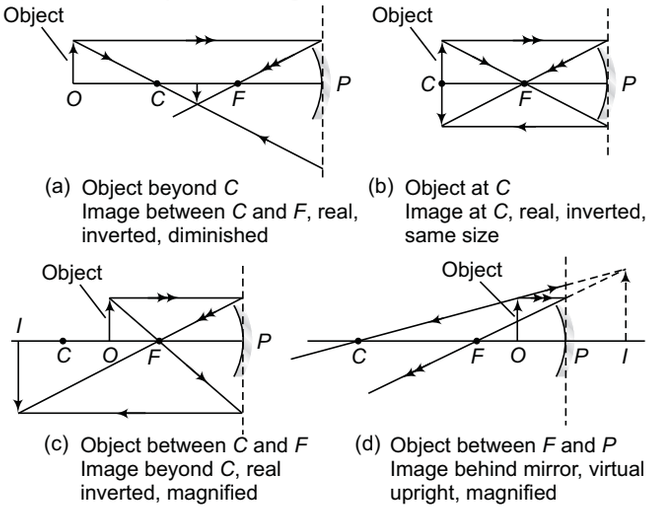


Fig. 24.19

Mirror Formula

In Fig. 24.20 (a) and (b), a ray *OM* from a point object *O* on the principal axis is reflected at *M* so that the angle θ, made by the incident and reflected rays with the normal *CM* are equal. A ray *OP* strikes the mirror normally and is reflected back along *PO*. The intersection *I* of the reflected rays *MI* and *PO* in Fig. (a) gives a **real** point image of *O* and in Fig. (b) gives a **virtual** point image of *O*. Let α, β and γ be the angles as shown. As the rays are paraxial, these angles are small, we can take $\alpha \approx \tan\alpha = \frac{MP}{OP}$, $\beta = \frac{MP}{CP}$ and $\gamma = \frac{MP}{IP}$.

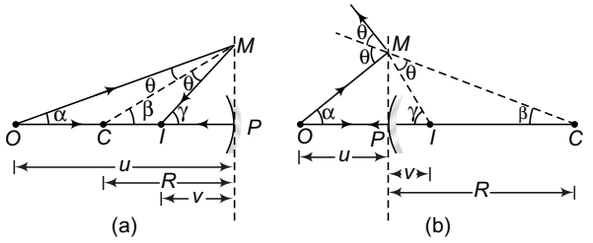


Fig. 24.20

Now, let us take the two figures simultaneously.

Concave	Convex
In triangle CMO, $\beta = \alpha + \theta$ (the exterior angle)	In triangle CMO,
or $\theta = \beta - \alpha$ …(i)	$\theta = \alpha + \beta$ …(iv) (the exterior angle)
In ΔCMI, $\gamma = \beta + \theta$	In ΔCMI, $\gamma = \theta + \beta$
$\therefore$ $\theta = \gamma - \beta$ …(ii)	or $\theta = \gamma - \beta$ …(v)
From Eqs. (i) and (ii), $2\beta = \gamma + \alpha$ …(iii)	From Eqs. (iv) and (v), $2\beta = \gamma - \alpha$ …(vi)
Substituting the values of α, β and γ, we get $\frac{2}{CP} = \frac{1}{IP} + \frac{1}{OP}$ …(A)	Substituting the values of α, β and γ, we get $\frac{2}{CP} = \frac{1}{IP} - \frac{1}{OP}$ …(B)
If we now substitute the values with sign, i.e. $CP = -R$, $IP = -v$ and $OP = -u$ we get, $\frac{2}{R} = \frac{1}{v} + \frac{1}{u}$ or $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ as $\left(f = \frac{R}{2}\right)$	If we now substitute the values with sign, i.e. $CP = +R$, $IP = +v$ and $OP = -u$, we get $\frac{2}{R} = \frac{1}{v} + \frac{1}{u}$ or $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ as $\left(f = \frac{R}{2}\right)$

- Out of *u*, *v* and *f* values of two will be known to us and we will be asked to find the third. In such types of problems, two cases are possible. ***Case* 1** is when signs of all three will be known to us from the given information. Under this condition substitute all three with sign, then answer (i.e. the third quantity) will come without sign. Only numerical value of the unknown comes. ***Case* 2** is when the sign of third unknown quantity is not known to us. Under such situations, substitute only the known quantities with sign. Sign with numerical value will automatically come in the answer.

Magnification

The lateral, transverse or linear magnification *m* is defined as given by

$$m = \frac{\text{Height of image}}{\text{Height of object}} = \frac{I'I}{O'O} = \frac{IP}{OP} \qquad \text{…(i)}$$

(From similar triangles)

Here, $IP = -v$ and $OP = -u$, further object is erect and image is inverted, so we can take $I'I$ as negative and $O'O$ as positive and Eq. (i) will then become

$$\frac{I'I}{O'O} = -\frac{v}{u}$$

or $$m = -\frac{v}{u}$$

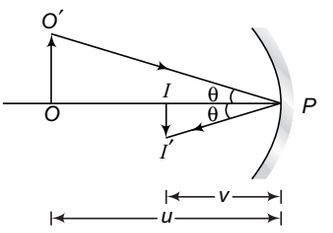


Fig. 24.21

- We have derived $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ and $m = -\frac{v}{u}$ for special cases of the position of object but the same result can be derived for other cases.

Example 24.4 *Find the distance of object from a concave mirror of focal length* 10 *cm, so that image size is four times the size of the object.*

Sol. Concave mirror can form real as well as virtual image. Here nature of image is not given in the question. So, we will consider two possible cases.

***Case* 1 (when image is real)** Real image is formed on the same side of the object, i.e. u, v and f all are negative. So let,

$$u = -x$$

Then, $v = -4x$ as $\left|\frac{v}{u}\right| = |m| = 4$ and $f = -10$ cm

Substituting in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

We have $\frac{1}{-4x} - \frac{1}{x} = \frac{1}{-10}$ or $\frac{5}{4x} = \frac{1}{10}$

$\therefore \quad x = 12.5$ cm

- $|x| > |f|$ and we know that in case of a concave mirror, image is real when object lies beyond F.

***Case* 2 (when image is virtual)** In case of a mirror, image is virtual when it is formed behind the mirror, i.e. u and f are negative while v is positive. So let,

$$u = -y$$

then $v = +4y$ and $f = -10$ cm

Substituting in $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

We have $\frac{1}{4y} - \frac{1}{y} = \frac{1}{-10}$ or $\frac{3}{4y} = \frac{1}{10}$ or $y = 7.5$ cm

- Here $|y| < |f|$, as we know that image is virtual when the object lies between F and P.

Extra Knowledge Points

- For spherical mirrors, $m = -\frac{v}{u}$

  Positive value of m means that v and u are of opposite signs. So, if u is negative, then v is positive and *vice-versa*. Thus, if $m = +2$ for a real object, it means image is virtual, erect and two times greater in size. Similarly, $m = -\frac{1}{2}$ means image is real, inverted and $\frac{1}{2}$ times in size (that of object).

- For concave mirror

S. No.	Position of object	Details of image
1.	At ∞	At F, real, inverted $\|m\| << 1$
2.	Between C and ∞	Between F and C, real, inverted, $\|m\| < 1$
3.	At C	At C, real, inverted, $\|m\| = 1$
4.	Between F and C	Between C and ∞, real, inverted, $\|m\| > 1$
5.	At F	At infinity, real, inverted, $\|m\| >> 1$
6.	Between F and P	Behind the mirror, virtual, erect $\|m\| > 1$

For convex mirror

S. No.	Position of object	Details of image
1.	At infinity	At F, virtual, erect, $\|m\| << 1$
2.	In front of mirror	Between P and F, virtual, erect, $\|m\| < 1$

Example 24.5 *An object ABED is placed in front of a concave mirror beyond centre of curvature C as shown in figure. State the shape of the image.*

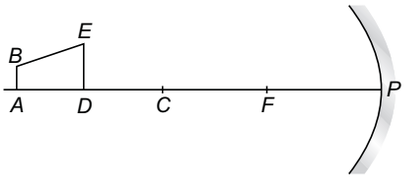


Fig. 24.22

Sol. Object is placed beyond C. Hence, the image will be real and it will lie between C and F. Further u, v and f all are negative, hence the mirror formula will become

$$-\frac{1}{v} - \frac{1}{u} = -\frac{1}{f} \quad \text{or} \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{u - f}{uf} \quad \text{or} \quad v = \frac{f}{1 - \frac{f}{u}}$$

Now $u_{AB} > u_{ED} \Rightarrow v_{AB} < v_{ED}$

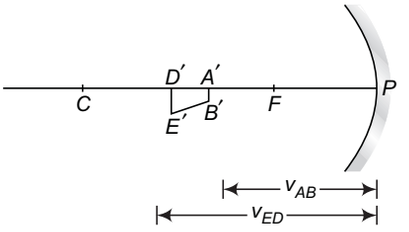


Fig. 24.23

and $|m_{AB}| < |m_{ED}| \qquad \left(\text{as } m = -\frac{v}{u}\right)$

Therefore, shape of the image will be as shown in Fig. 24.23.

Also note that $v_{AB} < u_{AB}$ and $v_{ED} < u_{ED}$,

So, $|m_{AB}| < 1$ and $|m_{ED}| < 1$

24.5 Refraction of Light

When light passes from one medium, say air, to another, say glass, part is reflected back into the first medium and the rest passes into the second medium. When it passes into the second medium, its direction of travel is changed. It either bends towards the normal or away from the normal. This phenomenon is known as **refraction**.

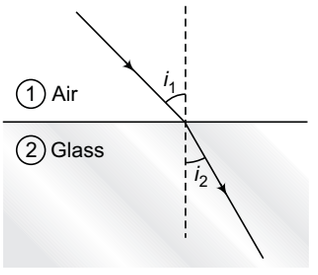


Fig. 24.24

Laws of Refraction

There are two laws of refraction:

1. For two particular media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant, i.e.

$$\frac{\sin i_1}{\sin i_2} = \text{constant} \qquad \ldots(i)$$

This is known as **Snell's law.**

2. The incident ray, the reflected ray and the refracted ray all lie in the same plane.

Note Points

- The constant ratio $\frac{\sin i_1}{\sin i_2}$ is called the **refractive index** for light passing from the first to the second medium (or refractive index of 2 with respect to 1). It is denoted by ${}_1\mu_2$. Thus,

$${}_1\mu_2 = \frac{\sin i_1}{\sin i_2} \qquad \ldots(i)$$

- If medium 1 is a vacuum (or in practice air) we refer ${}_1\mu_2$ as the **absolute refractive index** of medium 2 and denote it by μ_2 or simply μ. (if no other medium is there).
- Now, we can write Snell's law as

$$\mu \sin i = \text{constant} \qquad \ldots(ii)$$

For two media, $\mu_1 \sin i_1 = \mu_2 \sin i_2$

or $$\frac{\mu_2}{\mu_1} = \frac{\sin i_1}{\sin i_2} = {}_1\mu_2 \qquad \ldots(iii)$$

- From Eq. (ii) we can see that $i_1 > i_2$ if $\mu_2 > \mu_1$, i.e. if a ray of light passes from a rarer to a denser medium it bends towards normal and *vice-versa*.

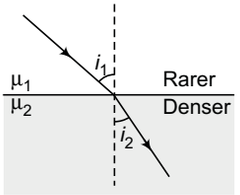


Fig. 24.25

- Eq. (iii) can be written as

$${}_1\mu_2 = \frac{\sin i_1}{\sin i_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\mu_2}{\mu_1} \qquad \ldots(iv)$$

Here v_1 is the speed of light in medium 1 and v_2 in medium 2. Similarly, λ_1 and λ_2 are the corresponding wavelengths.

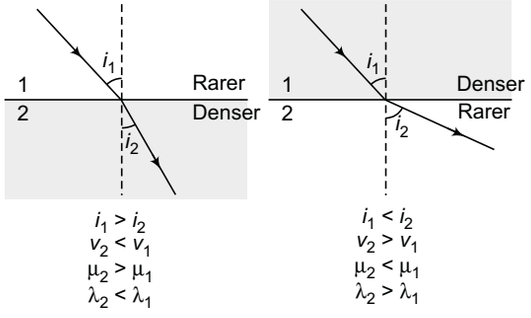


Fig. 24.26

If $\mu_2 > \mu_1$, then $v_1 > v_2$ and $\lambda_1 > \lambda_2$, i.e. in a rarer medium speed and hence, wavelength of light is more.

- In general speed of light in any medium is less than its speed in vacuum. It is convenient to define refractive index μ of a medium as

$$\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \frac{c}{v}$$

- As a ray of light moves from medium 1 to medium 2, its wavelength changes but its frequency remains constant.

$\mu_2 > \mu_1$, $v_1 > v_2$, $\lambda_1 > \lambda_2$

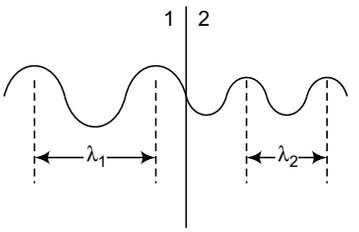


Fig. 24.27

- ${}_1\mu_2 = \frac{\mu_2}{\mu_1}$ and ${}_2\mu_1 = \frac{\mu_1}{\mu_2}$

$\therefore$ ${}_1\mu_2 = \frac{1}{{}_2\mu_1}$

- ${}_1\mu_2 = \frac{\mu_2}{\mu_1}$, ${}_2\mu_3 = \frac{\mu_3}{\mu_2}$ and ${}_3\mu_1 = \frac{\mu_1}{\mu_3}$

$\therefore$ ${}_1\mu_2 \times {}_2\mu_3 \times {}_3\mu_1 = 1$

- Experiments show that if the boundaries of the media are **parallel** the emergent ray *CD* although laterally displaced, is parallel to the incident ray *AB* if $\mu_1 = \mu_5$. We can also directly apply the Snell's law.

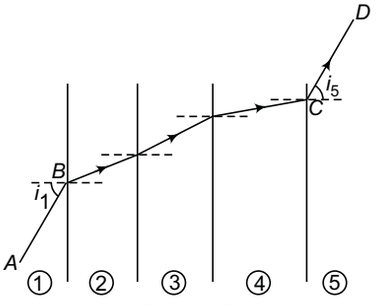


Fig. 24.28

($\mu \sin i$ = constant) in medium 1 and 5, i.e.

$$\mu_1 \sin i_1 = \mu_5 \sin i_5$$

So, $i_1 = i_5$ if $\mu_1 = \mu_5$

If any of the boundary is not parallel, we cannot use this law directly by jumping the intervening media.

Example 24.6 *Refractive index of glass with respect to water is* (9/8)*. Refractive index of glass with respect to air is* (3/2)*. Find the refractive index of water with respect to air.*

Sol. Given, ${}_w\mu_g = 9/8$ and ${}_a\mu_g = 3/2$

As, ${}_a\mu_g \times {}_g\mu_w \times {}_w\mu_a = 1$

$\therefore$ $$\frac{1}{{}_w\mu_a} = {}_a\mu_w = {}_a\mu_g \times {}_g\mu_w = \frac{{}_a\mu_g}{{}_w\mu_g}$$

$\therefore$ $${}_a\mu_w = \frac{3/2}{9/8} = \frac{4}{3}$$

Example 24.7 *(a) Find the speed of light of wavelength* $\lambda = 780\,nm$ *(in air) in a medium of refractive index* $\mu = 1.55$. *(b) What is the wavelength of this light in the given medium?*

Sol. (a) $v = \frac{c}{\mu} = \frac{3.0 \times 10^8}{1.55} = 1.94 \times 10^8$ m/s

(b) $\lambda_{medium} = \frac{\lambda_{air}}{\mu} = \frac{780}{1.55} = 503$ nm

Example 24.8 *A light beam passes from medium* 1 *to medium* 2 . *Show that the emerging beam is parallel to the incident beam.*

Sol. Applying Snell's law at A and B, $\mu_1 \sin i_1 = \mu_2 \sin i_2$

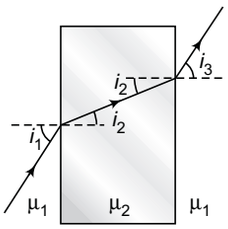


Fig. 24.29

or $$\frac{\mu_1}{\mu_2} = \frac{\sin i_2}{\sin i_1} \quad \ldots(i)$$

Similarly, $$\mu_2 \sin i_2 = \mu_1 \sin i_3$$

$\therefore$ $$\frac{\mu_1}{\mu_2} = \frac{\sin i_2}{\sin i_3} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$i_3 = i_1$$

i.e. the emergent ray is parallel to incident ray.

Single Refraction from a Plane Surface

Following four results can be drawn after refraction from a plane surface.

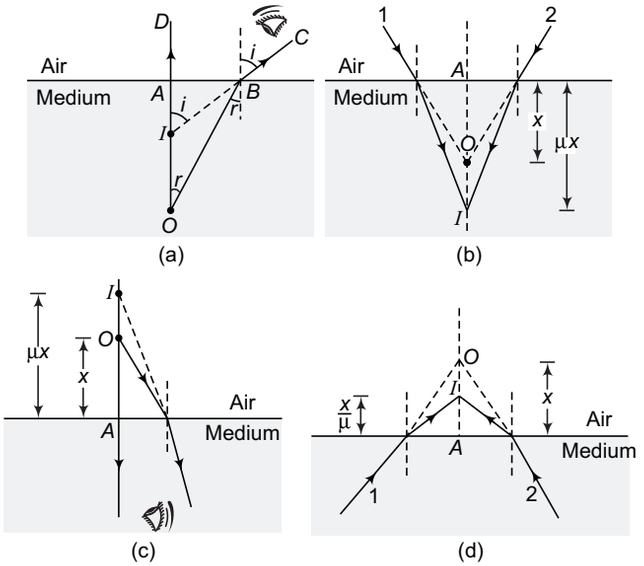


Fig. 24.30

In all the four figures, single refraction is taking place through a plane surface. Refractive index of medium (may be glass, water etc.) is μ. In Fig. (a) and (d), the ray of light is travelling from denser to rare medium and hence, it bends away from the normal. In Fig. (b) and (c), the ray of light is travelling from a rare to a denser medium and hence, it bends towards the normal. Now, let us take the four figures individually.

Refer Fig. (a) Object O is placed at a distance x from A. Ray OA, which falls normally on the plane surface, passes undeviated as AB. Ray OB, which falls at angle r (with the normal) on the plane surface, bends away from the normal and passes as BC in air. Rays AD and BC meet at I after extending these two rays backwards. Let BC makes an angle i ($> r$) with normal.

In the figure, $\angle AOB$ will be r and $\angle AIB$ is i. For normal incidence, (i.e. small angles of i and r)

$$\sin i \approx \tan i = \frac{AB}{AI} \quad \ldots(i)$$

and $$\sin r \approx \tan r = \frac{AB}{AO} \quad \ldots(ii)$$

Dividing Eq. (i) by Eq. (ii), we have

$$\frac{\sin i}{\sin r} = \frac{AO}{AI}$$

or $$\mu = \frac{AO}{AI} \qquad \left(\text{as } \frac{\sin i}{\sin r} = \mu\right)$$

$\therefore$ $$AI = \frac{AO}{\mu} = \frac{x}{\mu}$$

If point O is at a depth of d from a water surface, then the above result is also sometimes written as,

$$d_{\text{apparent}} = \frac{d_{\text{actual}}}{\mu}$$

or the apparent depth is μ times less than the actual depth.

Refer Fig. (b) In the absence of the plane refracting surface the two rays 1 and 2 would had met at O. Proceeding in the similar manner we can prove that after refraction from the plane surface they will now meet a point I where,

$$AI = \mu x$$

Refer Fig. (c) In this case object is at O, a distance $AO = x$ from the plane surface. When seen from inside the medium it will appear at I, where

$$AI = \mu x$$

Refer Fig. (d) The two rays 1 and 2 meeting at O will now meet at I after refraction from the plane surface, where

$$AI = \frac{AO}{\mu} = \frac{x}{\mu}$$

In all the four cases, the change in the value of x is μ times whether it is increasing or decreasing. All the relations can be derived for small angles of incidence as done in part (a).

Exercise. Three immiscible liquids of refractive indices μ_1, μ_2 and μ_3 (with $\mu_3 > \mu_2 > \mu_1$) are filled in a vessel. Their depths are d_1, d_2 and d_3 respectively. Prove that the apparent depth (for normal incidence) when seen from top of the first liquid will be

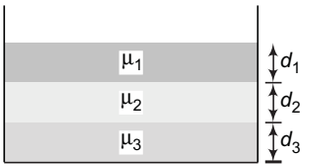


Fig. 24.31

$$d_{app} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3}$$

Shift due to a Glass Slab

(Double Refraction from Plane Surfaces)

(i) **Normal shift** Here, again two cases are possible

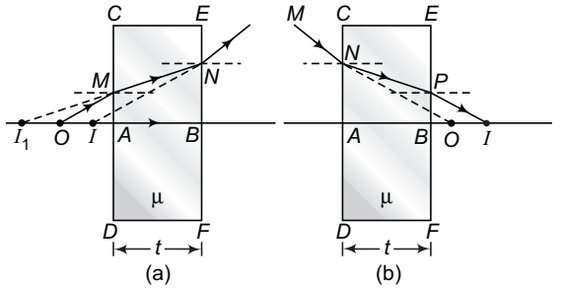


Fig. 24.32

Refer Fig. (a) An object is placed at O. Plane surface CD forms its image (virtual) at I_1. This image acts as object for EF which finally forms the image (virtual) at I. Distance OI is called the normal shift and its value is

$$OI = \left(1 - \frac{1}{\mu}\right) t$$

This can be proved as below :

Let $OA = x$

Then, $AI_1 = \mu x$ (Refraction from CD)

$$BI_1 = \mu x + t$$

$$BI = \frac{BI_1}{\mu} = x + \frac{t}{\mu} \quad \text{(Refraction from } EF\text{)}$$

$\therefore$ $OI = (AB + OA) - BI$

$$= (t + x) - \left(x + \frac{t}{\mu}\right) = \left(1 - \frac{1}{\mu}\right) t$$

Refer Fig. (b) The ray of light which would had met line AB at O will now meet this line at I after two times refraction from the slab. Here,

$$OI = \left(1 - \frac{1}{\mu}\right) t$$

(ii) **Lateral shift** We have already discussed that ray MA is parallel to ray BN. But the emergent ray is displaced laterally by a distance d, which depends on μ, t and i and its value is given by the relation,

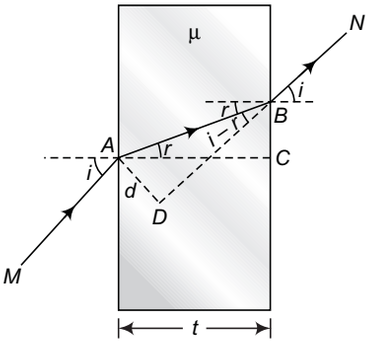


Fig. 24.33

$$d = t\left(1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right) \sin i$$

Proof $AB = \dfrac{AC}{\cos r} = \dfrac{t}{\cos r}$ (as $AC = t$)

Now, $d = AB \sin (i - r)$

$$= \frac{t}{\cos r}[\sin i \cos r - \cos i \sin r]$$

or $d = t\,[\sin i - \cos i \tan r]$ …(i)

Further, $\mu = \dfrac{\sin i}{\sin r}$ or $\sin r = \dfrac{\sin i}{\mu}$

$\therefore$ $\tan r = \dfrac{\sin i}{\sqrt{\mu^2 - \sin^2 i}}$

Substituting in Eq. (i), we get

$$d = \left[1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right] t \sin i$$

Exercise. Show that for small angles of incidence,

$$d = ti\left(\frac{\mu - 1}{\mu}\right)$$

Example 24.9 *A point object O is placed in front of a concave mirror of focal length* 10 *cm. A glass slab of refractive index* $\mu = 3/2$ *and thickness* 6 *cm is inserted between object and mirror. Find the position of final image when the distance x shown in figure is*
(a) 5 *cm,* *(b)* 20 *cm.*

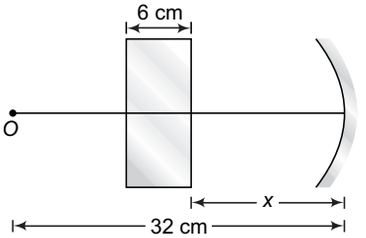


Fig. 24.34

Sol. As we have read in the above article the normal shift produced by a glass slab is,

$$\Delta x = \left(1 - \frac{1}{\mu}\right) t = \left(1 - \frac{2}{3}\right)(6) = 2 \text{ cm}$$

i.e. for the mirror the object is placed at a distance $(32 - \Delta x) = 30$ cm from it. Applying mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ or } \frac{1}{v} - \frac{1}{30} = -\frac{1}{10} \quad \text{or} \quad v = -15 \text{ cm}$$

(a) **When** $x = 5$ cm The light falls on the slab on its return journey as shown. But the slab will again shift it by a distance $\Delta x = 2$ cm. Hence, the final real image is formed at a distance $(15 + 2) = 17$ cm from the mirror.

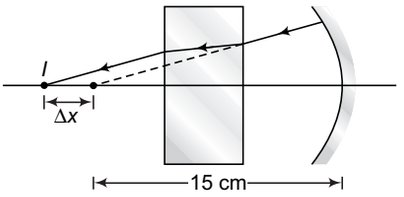


Fig. 24.35

(b) **When** $x = 20$ cm This time also the final image is at a distance 17 cm from the mirror but it is virtual as shown.

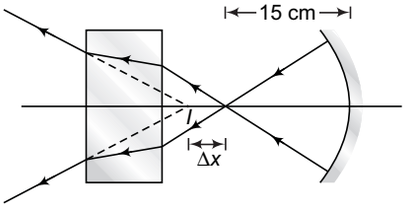


Fig. 24.36

Refraction from a Spherical Surface

Consider two transparent media having indices of refraction μ_1 and μ_2, where the boundary between the two media is a spherical surface of radius R. We assume that $\mu_1 < \mu_2$. Let us consider a single ray leaving point O and focussing at point I. Snell's law applied to this refracted ray gives,

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

Because θ_1 and θ_2 are assumed to be small, we can use the small angle approximation

$\sin\theta \approx \theta$ (angles in radians) and say that

$$\mu_1\theta_1 = \mu_2\theta_2 \qquad \ldots(i)$$

From the geometry shown in the figure,

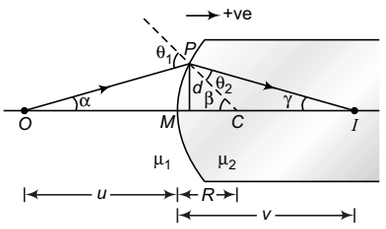


Fig. 24.37

$$\theta_1 = \alpha + \beta \qquad \ldots(ii)$$

and

$$\beta = \theta_2 + \gamma \qquad \ldots(iii)$$

Eqs. (i) and (iii) can be combined to express θ_2 in terms of α and β. Substituting the resulting expression into Eq. (ii), then yields

$$\beta = \frac{\mu_1}{\mu_2}(\alpha + \beta) + \gamma$$

So,

$$\mu_1\alpha + \mu_2\gamma = (\mu_2 - \mu_1)\beta \qquad \ldots(iv)$$

Since, the arc PM (of length S) subtends an angle β at the centre of curvature,

$$\beta = \frac{S}{R}$$

Also in the paraxial approximation,

$$\alpha = \frac{S}{u} \qquad \text{and} \qquad \gamma = \frac{S}{v}$$

Using these expressions in Eq. (iv) with proper signs, we are left with,

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

or

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \qquad \ldots(v)$$

Although the formula (v) is derived for a particular situation, it is valid for all other situations of refraction at a single spherical surface.

Lateral magnification The lateral magnification may be obtained with the help of the adjacent figure, where two rays from the tip of an object of height h_o meet at the corresponding point on an image of height h_i. One ray passes through the centre of curvature of the spherical surface so its direction is unchanged. The path of the second ray is obtained from Snell's law. With the paraxial approximation,

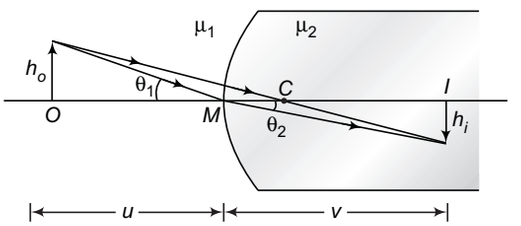


Fig. 24.38

$$\sin\theta_1 \approx \frac{h_o}{u} \quad \text{and} \quad \sin\theta_2 \approx \frac{h_i}{v}$$

Combining these equations with Snell's law then gives,

$$\mu_1\left(\frac{h_o}{u}\right) = \mu_2\left(\frac{h_i}{v}\right)$$

or

$$\frac{h_i}{h_o} = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right) \qquad \ldots(vi)$$

The lateral magnification m is the ratio of the image height to the object height or $\frac{h_i}{h_o}$. We therefore, obtain

$$m = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right) \qquad \ldots(vii)$$

- Here $v = +ve$, $u = -ve$, $h_i = -ve$ and $h_o = +ve$ (distances measured above the axis are taken positive). So, if we put these sign conventions in Eq. (vi), we obtain the same result *viz.*, $m = \frac{\mu_1}{\mu_2}\frac{v}{u}$.

Example 24.10 *A glass sphere of radius $R = 10$ cm is kept inside water. A point object O is placed at 20 cm from A as shown in figure. Find the position and nature of the image when seen from other side of the sphere. Also draw the ray diagram. Given $\mu_g = 3/2$ and $\mu_w = 4/3$.*

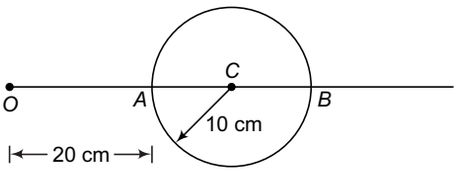


Fig. 24.39

Sol. A ray of light starting from O gets refracted twice. The ray of light is travelling in a direction from left to right. Hence, the distances measured in this direction are taken positive. Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, twice with proper signs.

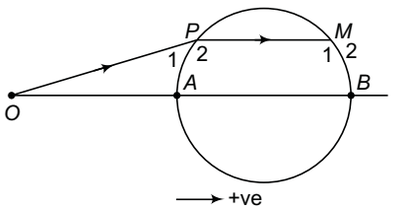


Fig. 24.40

we have, $\frac{3/2}{AI_1} - \frac{4/3}{-20} = \frac{3/2 - 4/3}{10}$ or $AI_1 = -30$ cm

Now, the first image I_1, acts as an object for the second surface, where

$$BI_1 = u = -(30 + 20) = -50 \text{ cm}$$

$$\therefore \quad \frac{4/3}{BI_2} - \frac{3/2}{-50} = \frac{4/3 - 3/2}{-10}$$

$\therefore BI_2 = -100$ cm i.e. the final image I_2 is virtual and is formed at a distance 100 cm (towards left) from B. The ray diagram is as shown.

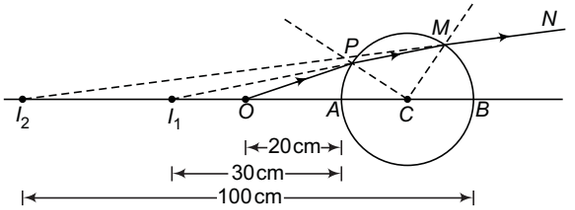


Fig. 24.41

Following points should be noted while drawing the ray diagram :

(i) At P, the ray travels from rare to a denser medium. Hence, it will bend towards normal PC. At M, it travels from a denser to a rarer medium, hence moves away from the normal MC.

(ii) PM ray when extended backwards meets at I_1 and MN ray when extended meets at I_2.

- The refraction formula $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ can also be applied to plane refracting surfaces with $R = \infty$. Let us derive $d_{app} = \frac{d_{actual}}{\mu}$ using this.

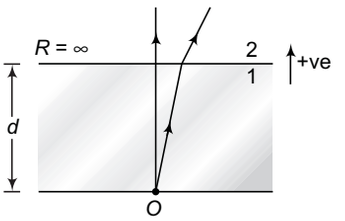


Fig. 24.42

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

With proper sign and values, we have

$$\frac{1}{v} - \frac{\mu}{-d} = \frac{1-\mu}{\infty} = 0 \quad \text{or} \quad v = -\frac{d}{\mu}$$

i.e. image of object O is formed at a distance $\frac{d}{\mu}$ on same side.

or $d_{app} = \frac{d_{actual}}{\mu}$

24.6 Thin Lenses

A lens is one of the most familiar optical devices for a human being. A lens is an optical system with two refracting surfaces. The simplest lens has two spherical surfaces close enough together that we can neglect the distance between them (the thickness of the lens). We call this a **thin lens**.

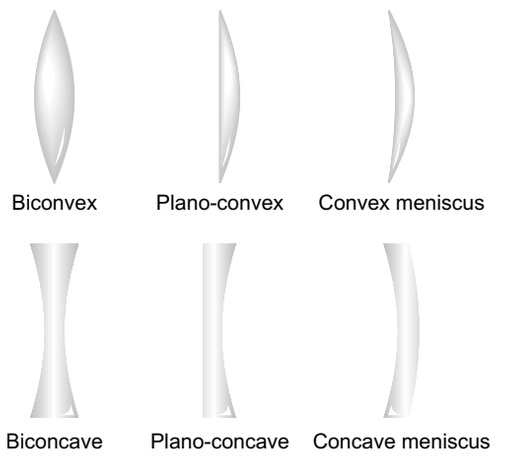


Fig. 24.43. Types of lenses

Lenses are of two basic types **convex** which are thicker in the middle, than at the edges and **concave** for which the reverse holds.

Figure shows examples of both types bounded by spherical or plane surfaces.

As there are two spherical surfaces, there are two centres of curvature C_1 and C_2 and correspondingly two radii of curvature R_1 and R_2.

The line joining C_1 and C_2 is called the **principal axis** of the lens. The centre P of the thin lens which lies on the principal axis, is called the **optical centre**.

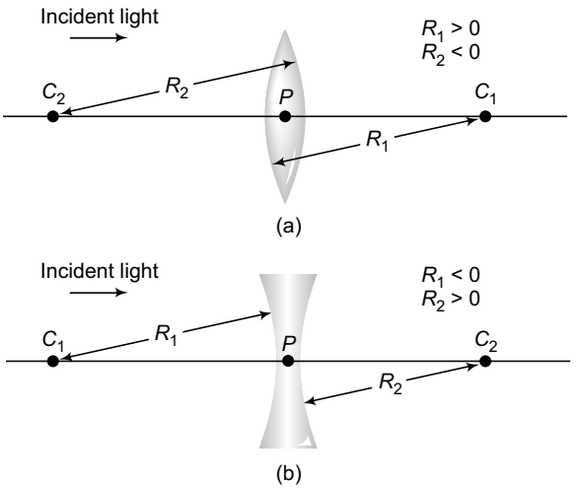


Fig. 24.44 (a) A converging thin lens and (b) A diverging thin lens.

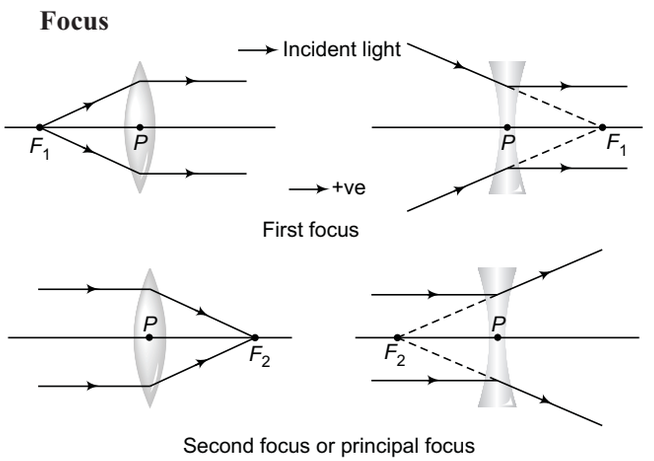


Fig. 24.45

Unlike a mirror, a lens has two foci :

First focus (F_1) It is defined as a point at which if an object (real in case of a convex lens and virtual for concave) is placed, the image of this object is formed at infinity. Or we can say, rays passing through F_1 become parallel to the principal axis after refraction from the lens. The distance PF_1 is the first focal length f_1.

Second focus or principal focus (F_2) A narrow beam of light travelling parallel to the principal axis either converge (in case of a convex lens) or diverge (in case of a concave lens) at a point F_2 after refraction from the lens. This point F_2 is called the second or principal focus. If the rays converge at F_2, the lens is said a converging lens and if they diverge, they are called diverging lens. Distance PF_2 is the second focal length f_2.

Note Points

- From the figure we can see that f_1 is negative for a convex lens and positive for a concave lens. But f_2 is positive for convex lens and negative for concave lens.
- $|f_1| = |f_2|$ if the media on the two sides of a thin lens have same refractive index.
- We are mainly concerned with the second focus f_2. Thus, wherever we write the focal length f, it means the second or principal focal length. Thus, $f = f_2$ and hence, f is positive for a convex lens and negative for a concave lens.

Lens Maker's Formula and Lens Formula

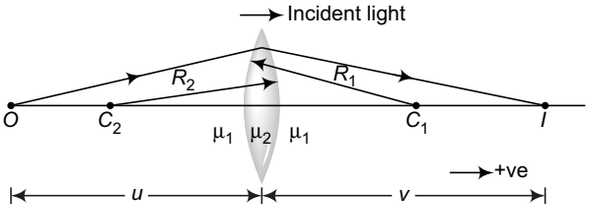


Fig. 24.46

Consider an object O placed at a distance u from a convex lens as shown in figure. Let its image I after two refractions from spherical surfaces of radii R_1 (positive) and R_2 (negative) be formed at a distance v from the lens. Let v_1 be the distance of image formed by refraction from the refracting surface of radius R_1. This image acts as an object for the second surface. Using,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

twice, we have

or $$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad \ldots(i)$$

and $$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{-R_2} \quad \ldots(ii)$$

Adding Eqs. (i) and (ii) and then simplifying, we get

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots(iii)$$

This expression relates the image distance v of the image formed by a thin lens to the object distance u and to the thin lens properties (index of refraction and radii of

curvature). It is valid only for paraxial rays and only when the lens thickness is much less then R_1 and R_2. The **focal length** ***f*** of a thin lens is the image distance that corresponds to an object at infinity. So, putting $u = \infty$ and $v = f$ in the above equation, we have

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots(iv)$$

If the refractive index of the material of the lens is μ and it is placed in air, $\mu_2 = \mu$ and $\mu_1 = 1$, so that Eq. (iv) becomes

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots(v)$$

This is called the **lens maker's formula** because it can be used to determine the values of R_1 and R_2 that are needed for a given refractive index and a desired focal length f.

Combining Eqs. (iii) and (iv), we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \ldots(vi)$$

Which is known as the **lens formula.** Following conclusions can be drawn from Eqs. (iv), (v) and (vi):

1. For a converging lens, R_1 is positive and R_2 is negative. Therefore, $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ in Eq. (v) comes out a positive quantity and if the lens is placed in air, $(\mu - 1)$ is also a positive quantity. Hence, the focal length f of a converging lens comes out to be positive. For a diverging lens however, R_1 is negative and R_2 is positive and the focal length f becomes negative.

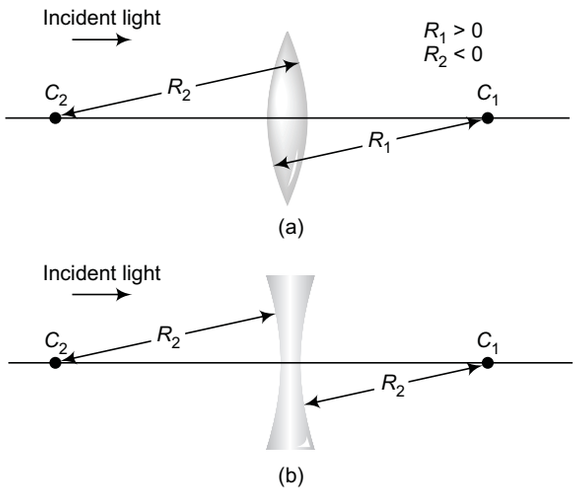


Fig. 24.47

2. Focal length of a mirror $\left(f_M = \frac{R}{2}\right)$ depends only upon the radius of curvature R while that of a lens [Eq. (iv)] depends on μ_1, μ_2, R_1 and R_2. Thus, if a lens and a mirror are immersed in some liquid, the focal length of lens would change while that of the mirror will remain unchanged.

3. Suppose $\mu_2 < \mu_1$ in Eq. (iv), i.e. refractive index of the mezdium (in which lens is placed) is more than the refractive index of the material of the lens, then $\left(\frac{\mu_2}{\mu_1} - 1\right)$ becomes a negative quantity, i.e. the lens changes its behaviour. A converging lens behaves as a diverging lens and *vice-versa*. An air bubble in water seems as a convex lens but behaves as a concave (diverging) lens.

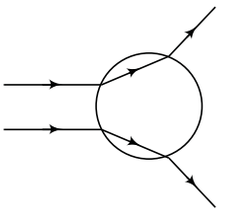


Fig. 24.48 Air bubble in water diverges the parallel beam of light incident on it.

Example 24.11 *Focal length of a convex lens in air is* 10 *cm. Find its focal length in water. Given that* $\mu_g = 3/2$ *and* $\mu_w = 4/3$.

Sol. $$\frac{1}{f_{air}} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots(i)$$

and $$\frac{1}{f_{water}} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{f_{water}}{f_{air}} = \frac{(\mu_g - 1)}{(\mu_g/\mu_w - 1)}$$

Substituting the values, we have

$$f_{water} = \frac{(3/2 - 1)}{\left(\frac{3/2}{4/3} - 1\right)} f_{air} = 4 f_{air} = 4 \times 10 = 40 \text{ cm}$$

Students can remember the result $f_{water} = 4 f_{air}$, if $\mu_g = 3/2$ and $\mu_w = 4/3$.

Images Formed by Thin Lenses

Information as to the position and nature of the image in any case can be obtained either from a ray diagram or by calculation.

(a) Ray diagram To construct the image of a small object perpendicular to the axis of a lens, two of the following three rays are drawn from the top of the object.

1. A ray parallel to the principal axis after refraction passes through the principal focus or appears to diverge from it.

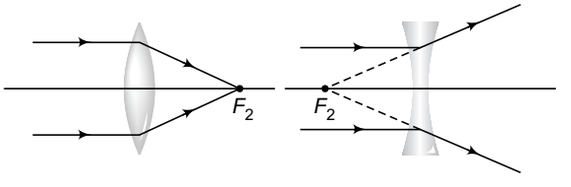


Fig. 24.49

2. A ray through the optical centre P passes undeviated because the middle of the lens acts like a thin parallel-sided slab.

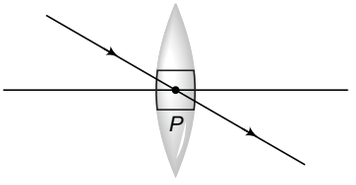


Fig. 24.50

3. A ray passing through the first focus F_1 become parallel to the principal axis after refraction.

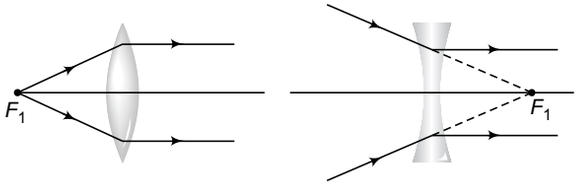
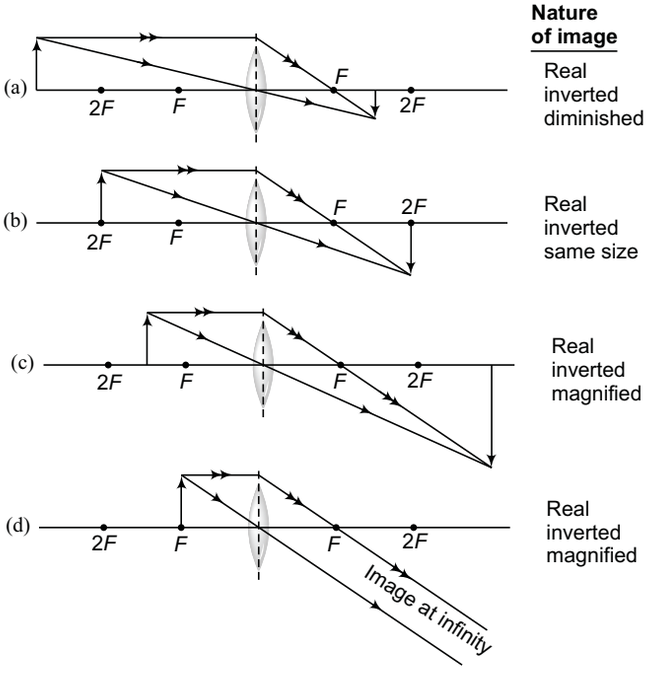


Fig. 24.51

Keeping the above three points in mind we can show that image formed by a concave lens is always virtual, erect and diminished (like a convex mirror) while the nature of image in case of a convex lens depends on the position of object. The ray diagrams for a convex and a concave lens are shown. For a convex lens image is virtual when object lies between F and P. In all other cases it is real.



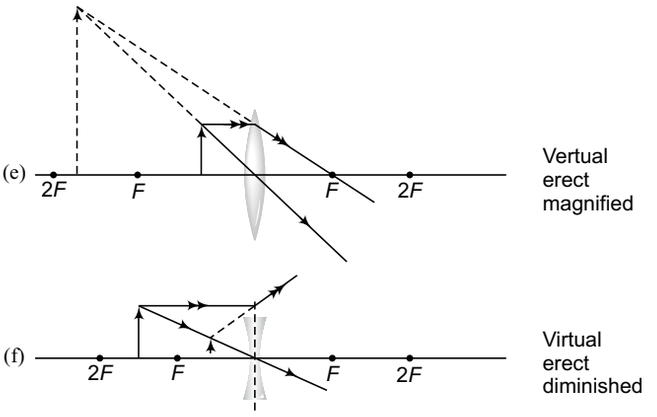


Fig. 24.52 Ray diagrams for a convex lens (*a*-*e*) and a concave lens (*f*).

(b) Lens formula We have already discussed the lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

for finding the position of the lens. While using this formula appropriate sign of u, v and f must be included.

Magnification The lateral, transverse or linear magnification m produced by a lens is defined by,

$$m=\frac{\text{Height of image}}{\text{Height of object}}=\frac{I}{O}$$

A real image II' of an object OO' formed by a convex lens is shown in figure.

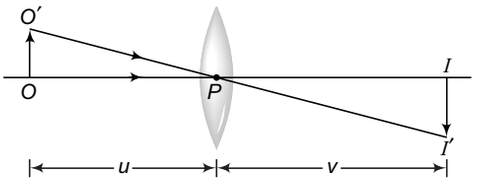


Fig. 24.53

$$\frac{\text{Height of image}}{\text{Height of object}}=\frac{II'}{OO'}=\frac{v}{u}$$

Substituting v and u with proper sign,

$$\frac{II'}{OO'}=\frac{-I}{O}=\frac{v}{-u}$$

or $$\frac{I}{O}=m=\frac{v}{u}$$

Thus, $$m=\frac{v}{u}$$

- Suppose m is positive it means v and u are of same sign, i.e. image and object are on the same side (left side), which implies that the image of a real object is virtual. Thus, $m = +2$, means image is virtual, erect and two times magnified and $|v| = 2|u|$.
  Similarly $m = -\frac{1}{2}$ means image is real, inverted and diminished and $|v| = \frac{1}{2}|u|$.

Example 24.12 *Find the distance of an object from a convex lens if image is two times magnified. Focal length of the lens is* 10 *cm.*

Sol. Convex lens forms both type of images real as well as virtual. Since, nature of the image is not mentioned in the question, we will have to consider both the cases.

When image is real Means v is positive and u is negative with $|v| = 2|u|$. Thus, if

$u = -x$ then $v = 2x$ and $f = 10$ cm

Substituting in $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

we have $\frac{1}{2x} + \frac{1}{x} = \frac{1}{10}$

or $\frac{3}{2x} = \frac{1}{10}$

$\therefore$ $x = 15$ cm

$x = 15$ cm, means object lies between F and $2F$.

When image is virtual Means v and u both are negative. So let,

$u = -y$ then $v = -2y$ and $f = 10$ cm

Substituting in, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

we have, $\frac{1}{-2y} + \frac{1}{y} = \frac{1}{10}$

or $\frac{1}{2y} = \frac{1}{10}$

$\therefore$ $y = 5$ cm

$y = 5$ cm, means object lies between F and P.

Displacement Method to Determine the Focal Length of a Convex Lens

If the distance d between an object and screen is greater than 4 times the focal length of a convex lens, then there are two positions of the lens between the object and the screen at which a sharp image of the object is formed on the screen. This method is called **displacement method** and is used in laboratory to determine the focal length of convex lens.

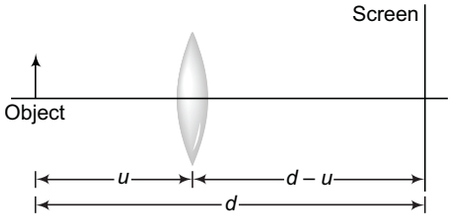


Fig. 24.54

To prove this, let us take an object placed at a distance u from a convex lens of focal length f. The distance of image from the lens $v = (d - u)$. From the lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

we have, $\frac{1}{d-u} - \frac{1}{-u} = \frac{1}{f}$

or $u^2 - du + df = 0$

$\therefore$ $u = \frac{d \pm \sqrt{d(d-4f)}}{2}$

Now, there are following possibilities :

(i) If $d < 4f$, then u is imaginary.
So, physically no position of the lens is possible.

(ii) If $d = 4f$, then $u = \frac{d}{2} = 2f$. So, only one position is possible. From here we can see that the **minimum distance between an object and its real image in case of a convex lens is 4 *f*.**

(iii) If $d > 4f$, there are two positions of lens at distances $\frac{d + \sqrt{d(d-4f)}}{2}$ and $\frac{d - \sqrt{d(d-4f)}}{2}$ for which real image is formed on the screen.

(iv) Suppose I_1 is the image length in one position of the object and I_2 the image length in second position, then object length O is given by,

$$O = \sqrt{I_1 I_2}$$

This can be proved as under

$$|u_1| = \frac{d + \sqrt{d(d-4f)}}{2}$$

$\therefore$ $|v_1| = d - |u_1| = \frac{d - \sqrt{d(d-4f)}}{2}$

$$|u_2| = \frac{d - \sqrt{d(d-4f)}}{2}$$

$\therefore$ $|v_2| = d - |u_2| = \frac{d + \sqrt{d(d-4f)}}{2}$

Now, $|m_1 m_2| = \frac{I_1}{O} \times \frac{I_2}{O} = \frac{|v_1|}{|u_1|} \times \frac{|v_2|}{|u_2|}$

Substituting the values, we get

$\frac{I_1 I_2}{O^2} = 1$ or $O = \sqrt{I_1 I_2}$

(v) Focal length of lens is given by,

$$f = \frac{d^2 - x^2}{4d}$$

Here, x is the distance between two positions of the lens.

Focal Length of Two or More thin Lenses in Contact

Combinations of lenses in contact are used in many optical instruments to improve their performance.

Suppose two lenses of focal lengths f_1 and f_2 are kept in contact and a point object O is placed at a distance u from the combination. The first image (say I_1) after refraction from the first lens is formed at a distance v_1 (whatever may be the sign of v_1) from the combination. This image I_1 acts as an object for the second lens and let v be the distance of the final image from the combination. Applying the lens formula,

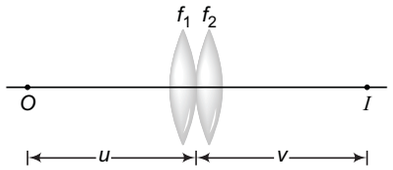


Fig. 24.55

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

For the two lenses, we have

$$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_1} \quad \ldots(i)$$

and

$$\frac{1}{v}-\frac{1}{v_1}=\frac{1}{f_2} \quad \ldots(ii)$$

Adding Eqs. (i) and (ii), we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{F}\ (\text{Say})$$

Here, F is the equivalent focal length of the combination. Thus,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

Similarly for more than two lenses in contact, the equivalent focal length is given by the formula,

$$\frac{1}{F}=\sum_{i=1}^{n}\frac{1}{f_i}$$

- Here f_1, f_2 etc., are to be substituted with sign.

Example 24.13 *A converging lens of focal length 5 cm is placed in contact with a diverging lens of focal length 10 cm. Find the combined focal length of the system.*

Sol. Here, $f_1 = +5$ cm and $f_2 = -10$ cm

Therefore, the combined focal length F is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{5}-\frac{1}{10}=+\frac{1}{10}$$

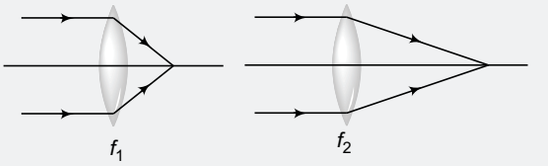
$$\therefore \quad F = +10 \text{ cm}$$

i.e. the combination behaves as a converging lens of focal length 10 cm.

Extra Knowledge Points

- **Power of an optical instrument** By optical power of an instrument (whether it is a lens, mirror or a refractive surface) we mean the ability of the instrument to deviate the path of rays passing through it. If the instrument converges the rays parallel to the principal axis its power is said positive and if it diverges the rays it is said a negative power.

  The shorter the focal length of a lens (or a mirror) the more it converges or diverges light. As shown in the figure,



$$f_1 < f_2$$

and hence the power $P_1 > P_2$, as bending of light in case 1 is more than that of case 2. For a lens,

$$P\,(\text{in dioptre}) = \frac{1}{f\ (\text{metre})}$$

and for a mirror,

$$P\,(\text{in dioptre}) = \frac{-1}{f\ (\text{metre})}$$

- Thus, convex lens and concave mirror have positive power or they are converging in nature. Concave lens and convex mirror have negative power or they are diverging in nature.

Example 24.14 *A spherical convex surface separates object and image space of refractive index 1.0 and 4/3. If radius of curvature of the surface is 10 cm, find its power.*

Sol. Let us see where does the parallel rays converge (or diverge) on the principal axis. Let us call it the focus and the corresponding length, the focal length f. Using $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ with proper values and signs, we have

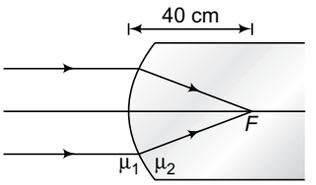


Fig. 24.56

$$\frac{4/3}{f}-\frac{1}{\infty}=\frac{4/3-1}{+10} \quad \text{or} \quad f = 40 \text{ cm} = 0.4 \text{ m}$$

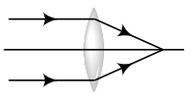
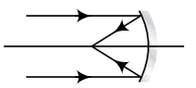
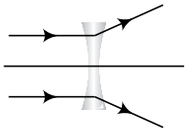
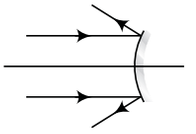
Since, the rays are converging, its power should be positive. Hence,

$$P\,(\text{in dioptre}) = \frac{+1}{f\ (\text{metre})} = \frac{1}{0.4}$$

or

$$P = 2.5 \text{ D}$$

Table 24.1 Following table gives the sign of *P* and *f* for different type of lens and mirror

Nature of lens/mirror	Focal length (*f*)	Power $P_L = \frac{1}{f}, P_M = -\frac{1}{f}$	Converging/diverging	Ray diagram
Convex lens	+ ve	+ ve	converging	
Concave mirror	– ve	+ ve	converging	
Concave lens	– ve	– ve	diverging	
Convex mirror	+ ve	– ve	diverging	

24.7 Total Internal Reflection (TIR)

Figure shows the reflection and refraction of a light ray at the interface between a denser and a rare medium, whose refractive indices are μ_D and μ_R. Angle of incidence in denser medium is *i* and angle of refraction is *r*.

From Snell's law ($\mu \sin i = \text{constant}$), we may write

$$\mu_D \sin i = \mu_R \sin r \quad \text{or} \quad \frac{\mu_D}{\mu_R} \sin i = \sin r$$

The right hand side of this equation is a sine function that has a range from 0 to 1. The left hand side must therefore, have the same range, i.e.

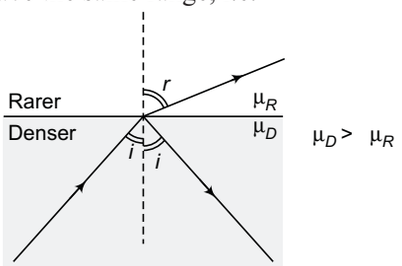


Fig. 24.57

$$1 \geq \frac{\mu_D}{\mu_R} \sin i \geq 0 \quad \text{or} \quad \frac{\mu_R}{\mu_D} \geq \sin i \geq 0$$

or $$\theta_c \geq i \geq 0$$

Here, $$\theta_c = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right) \qquad \ldots(i)$$

is called the **critical angle**. When the angle of incidence exceeds θ_c, no refracted beam is observed and the incident beam is completely reflected at the boundary. This phenomenon known as **Total Internal Reflection** (TIR), only occurs when the light travels from a denser medium to a rare medium. When the rare medium is air,

$$\mu_R = 1 \quad \text{and} \quad \mu_D = \mu$$

$$\therefore \quad \theta_c = \sin^{-1}\left(\frac{1}{\mu}\right) \qquad \ldots(ii)$$

TIR has following applications :

(i) Totally reflecting prisms Refractive index of crown glass is 3/2. Hence,

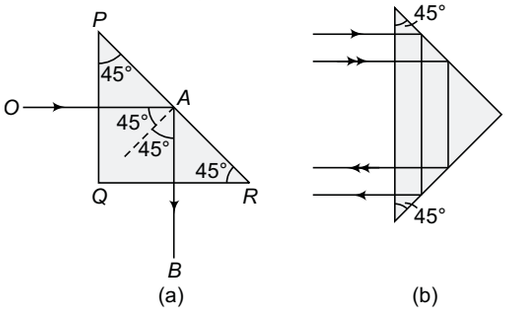


Fig. 24.58 Prism reflectors

$$\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{2}{3}\right) \approx 42°$$

A ray *OA* incident normally on face *PQ* of a crown glass prism suffers TIR at face *PR* since, the angle of incidence in the optically denser medium is 45°. A bright ray *AB* emerges at right angles to face *QR*. The prism thus, reflects the ray through 90°. Light can be reflected through 180° and an erect image can be obtained of an inverted one, if the prism is arranged as shown in Fig. (b).

(ii) Optical fibres Light can be confined within a bent glass rod by TIR and so piped along a twisted path as in figure. The beam is reflected from side to side practically without loss (except for that due to absorption in the glass) and emerges only at the end of the rod, where it strikes the surface almost normally, i.e. at an angle less than the critical angle. A single, very thin, solid glass fibre behaves in the same way and if several thousand are taped together a flexible light pipe is obtained that can be used, i.e. in medicine and engineering to illuminate an inaccessible spot. Optical fibres are now a days used to carry telephone, television and computer signals from one place to the other.

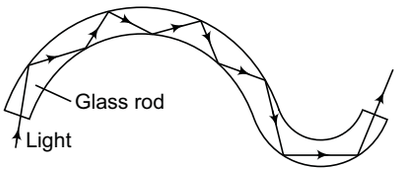


Fig. 24.59 Principle of an optical fibre

As we have seen, $\theta_C = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right)$

Suppose, we have two sets of media 1 and 2 and

$$\left(\frac{\mu_R}{\mu_D}\right)_1 < \left(\frac{\mu_R}{\mu_D}\right)_2 \quad \text{then} \quad (\theta_C)_1 < (\theta_C)_2$$

So, a ray of light has more chances to have TIR in case 1.

Example 24.15 *An isotropic point source is placed at a depth h below the water surface. A floating opaque disc is placed on the surface of water, so that the bulb is not visible from the surface. What is the minimum radius of the disc? Take refractive index of water* $= \mu$.

Sol. As shown in figure light from the source will not emerge out of water, if $i > \theta_c$.

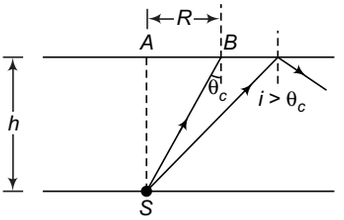


Fig. 24.60

Therefore, minimum radius R corresponds to $i = \theta_c$

In ΔSAB,

$$\frac{R}{h} = \tan\theta_c$$

$$\therefore \quad R = h\tan\theta_c$$

or $$R = \frac{h}{\sqrt{\mu^2 - 1}}$$

$$\sin\theta_C = \frac{1}{\mu}$$

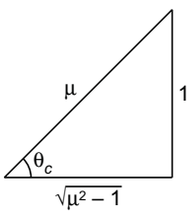


Fig. 24.61

24.8 Refraction through Prism

A prism has two plane surfaces AB and AC inclined to each other as shown in figure. $\angle A$ is called the **angle of prism** or **refracting angle**.

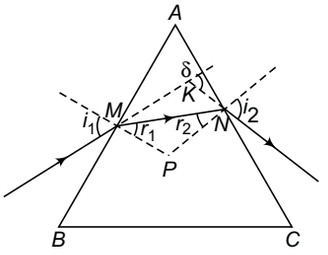


Fig. 24.62

The importance of the prism really depends on the fact that the angle of deviation suffered by light at the first refracting surface, say AB (in 2-dimensional figure) is not cancelled out by the deviation at the second surface AC (as it is in a parallel glass slab), but is added to it. This is why it can be used in a spectrometer, an instrument for analysing light into its component colours.

General Formulae

(i) In quadrilateral $AMPN$,

$$\angle AMP + \angle ANP = 180°$$

$$\therefore \quad A + \angle MPN = 180° \qquad \ldots(i)$$

In triangle MNP,

$$r_1 + r_2 + \angle MPN = 180° \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$r_1 + r_2 = A \qquad \ldots(iii)$$

(ii) **Deviation** Deviation δ means angle between incident ray and emergent ray.

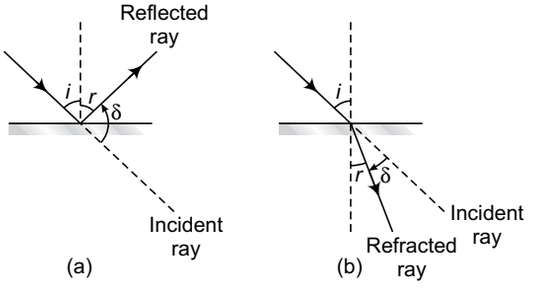


Fig. 24.63

In reflection,

$$\delta = 180° - 2i = 180° - 2r$$

In refraction,

$$\delta = |i - r|$$

In prism a ray of light gets refracted twice one at M and other at N. At M its deviation is $i_1 - r_1$ and at N it is $i_2 - r_2$. These two deviations are added. So, the net deviation is,

$$\begin{aligned}\delta &= (i_1 - r_1) + (i_2 - r_2)\\ &= (i_1 + i_2) - (r_1 + r_2)\\ &= (i_1 + i_2) - A\end{aligned}$$

Thus, $$\delta = (i_1 + i_2) - A \qquad \ldots(iv)$$

(iii) **If A and i_1 are small** The expression for the deviation in this case is basically used for developing the lens theory. Consider a ray falling almost normally in air on a prism of small angle A (less than about 6° or 0.1 rad), so that angle i_1 is small. Now, $\mu = \frac{\sin i_1}{\sin r_1}$, therefore, r_1 will also be small. Since, sine of a small angle is nearly equal to the angle in radians, we have

$$i_1 = \mu r_1$$

Also, $A = r_1 + r_2$ and so if A and r_1 are small r_2 and i_2 will also be small. From $\mu = \frac{\sin i_2}{\sin r_2}$ we can say

$$i_2 = \mu r_2$$

Substituting these values in Eq. (iv), we have

$$\delta = (\mu r_1 + \mu r_2) - A = \mu(r_1 + r_2) - A = \mu A - A$$

or $$\delta = (\mu - 1)\, A \qquad \ldots(v)$$

This expression shows that for a given angle A all rays entering a small angle prism at small angles of incidence suffer the same deviation.

(iv) **Minimum deviation**

It is found that the angle of deviation δ varies with the angle of incidence i_1 of the ray incident on the first refracting face of the prism. The variation is shown in figure and for one angle of incidence it has a minimum value δ_{min}. At this value the ray passes symmetrically through the prism (a fact that can be proved theoretically as well as be shown experimentally), i.e. the angle of emergence of the ray from the second face equals the angle of incidence of the ray on the first face.

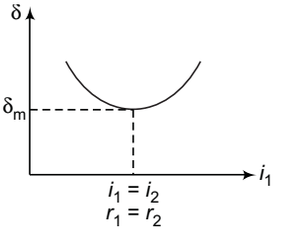


Fig. 24.64

$$i_2 = i_1 = i \qquad \ldots(vi)$$

It therefore, follows that

$$r_1 = r_2 = r \qquad \ldots(vii)$$

From Eqs. (iii) and (vii), we get

$$r = \frac{A}{2}$$

Further at $\delta = \delta_m = (i + i) - A$

or $$i = \frac{A + \delta_m}{2} \qquad \ldots(viii)$$

$$\mu = \frac{\sin i}{\sin r} \text{ or } \mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}} \qquad \ldots(ix)$$

(v) **Condition of no emergence** In this section, we want to find the condition such that a ray of light entering the face AB does not come out of the face AC for any value of angle i_1, i.e. TIR takes place on AC.

$$r_1 + r_2 = A$$

$\therefore$ $$r_2 = A - r_1$$

or $$(r_2)_{min} = A - (r_1)_{max} \qquad \ldots(x)$$

Now, r_1 will be maximum when i_1 is maximum and maximum value of i_1 can be 90°.

Hence,

$$\mu = \frac{\sin(i_1)_{max}}{\sin(r_1)_{max}} = \frac{\sin 90°}{\sin (r_1)_{max}}$$

$\therefore$ $$\sin(r_1)_{max} = \frac{1}{\mu} = \sin\theta_c$$

$\therefore$ $$(r_1)_{max} = \theta_c$$

$\therefore$ From Eq. (x),

$$(r_2)_{min} = A - \theta_c \qquad \ldots(xi)$$

Now, if minimum value of r_2 is greater than θ_c, then obviously all values of r_2 will be greater than θ_c and TIR will take place under all conditions. Thus, the condition of no emergence is,

$$(r_2)_{min} > \theta_c \quad \text{or} \quad A - \theta_c > \theta_c$$

or $$A > \frac{\theta_c}{2} \qquad \ldots(xii)$$

(vi) **Dispersion and deviation of light by a prism** White light is a superposition of waves with wavelengths extending throughout the visible spectrum. The speed of light in vacuum is the same for all wavelengths, but the speed in a material substance is different for different wavelengths. Therefore, the index of refraction of a material depends on wavelength. In most materials the value of refractive index μ decreases with increasing wavelength.

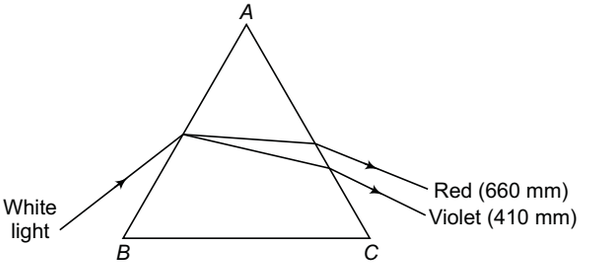


Fig. 24.65

If a beam of white light, which contains all colours, is sent through the prism, it is separated into a spectrum of colours. The spreading of light into its colour components is called **dispersion.**

Dispersive Power

When a beam of white light is passed through a prism of transparent material light of different wavelengths are deviated by different amounts. If δ_r, δ_y and δ_v are the deviations for red, yellow and violet components, then

average deviation is measured by δ_y as yellow light falls in between red and violet. $\delta_v - \delta_r$ is called **angular dispersion.** The **dispersive power** of a material is defined as the ratio of angular dispersion to the average deviation when a white beam of light is passed through it. It is denoted by ω. As we know,

$$\delta = (\mu - 1)\, A$$

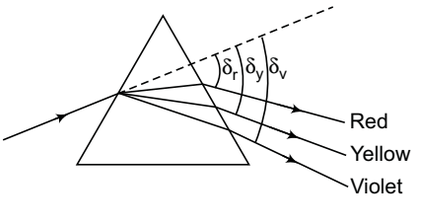


Fig. 24.66

This equation is valid when A and i are small. Suppose, a beam of white light is passed through such a prism, the deviation of red, yellow and violet light are

$$\delta_r = (\mu_r - 1)A\,,$$
$$\delta_y = (\mu_y - 1)A$$
and $$\delta_v = (\mu_v - 1)\, A$$

The angular dispersion is $\delta_v - \delta_r = (\mu_v - \mu_r)\, A$ and the average deviation is $\delta_y = (\mu_y - 1)\, A$. Thus, the dispersive power of the medium is,

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} \qquad \ldots(i)$$

Dispersion without Average Deviation and Average Deviation without Dispersion

Figure shows two prisms of refracting angles A and A' and dispersive powers ω and ω' respectively. They are placed in contact in such a way that the two refracting angles are reversed with respect to each other. A ray of light passes through the combination as shown. The deviation produced by the two prisms are,

$$\delta_1 = (\mu - 1)\, A \quad \text{and} \quad \delta_2 = (\mu' - 1)\, A'$$

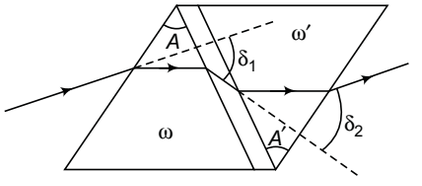


Fig. 24.67

As the two deviations are opposite to each other, the net deviation is,

$$\delta = \delta_1 - \delta_2 = (\mu - 1)A - (\mu' - 1)\, A' \qquad \ldots(ii)$$

Using this equation, the average deviation produced by the combination, if white light is passed is,

$$\delta_y = (\mu_y - 1)A - (\mu'_y - 1)\, A' \qquad \ldots(iii)$$

and the net angular dispersion is,

$$\delta_v - \delta_r = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$$

But as $\mu_v - \mu_y = \omega(\mu_y - 1)$ from Eq. (i), we have

$$\delta_v - \delta_r = (\mu_y - 1)\,\omega A - (\mu'_y - 1)\,\omega'\, A' \qquad \ldots(iv)$$

Dispersion without average deviation

From Eq. (iii), $\delta_y = 0$,

if $$\frac{A}{A'} = \frac{\mu'_y - 1}{\mu_y - 1} \qquad \ldots(v)$$

This is the required condition of dispersion without average deviation. Using this in Eq. (iv), the net angular dispersion produced is given as

$$\delta_v - \delta_r = (\mu_y - 1)A\,(\omega - \omega')$$

Average deviation without dispersion

From Eq. (iv), $\delta_v - \delta_r = 0$, if

$$\frac{A}{A'} = \frac{(\mu'_y - 1)\,\omega'}{(\mu_y - 1)\,\omega} = \frac{\mu'_v - \mu'_r}{\mu_v - \mu_r} \qquad \ldots(vi)$$

This is the required condition of average deviation without dispersion. Using the above condition in Eq. (iii), the net average deviation is

$$\delta_y = (\mu_y - 1)A\left(1 - \frac{\omega}{\omega'}\right)$$

Example 24.16 *An isosceles glass prism has one of its faces coated with silver. A ray of light is incident normally on the other face (which is equal to the silvered face). The ray of light is reflected twice on the same sized faces and then emerges through the base of the prism perpendicularly. Find angles of prism.*

Most of the problems of prisms are easily solved by drawing proper ray diagram and then applying laws of geometry with the basic knowledge of prism formulae.

Sol.

$r_1 = 0$

$\therefore$ $r_2 = A = 180° - 2\theta$ …(i)

$\angle DFE = 180° - 90° - 2r_2$

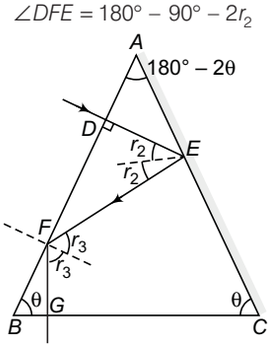


Fig. 24.68

$= 180° - 90° - 360° + 4\theta$

$= 4\theta - 270°$ …(ii)

$\therefore$ $r_3 = 90° - \angle DFE = 360° - 4\theta$ ...(iii)

$\angle BFG = 90° - \theta = 90° - r_3$

or $r_3 = \theta$ ...(iv)

From Eqs. (iii) and (iv), we have

$5\theta = 360°$

$\therefore$ $\theta = 72°$

and $180° - 2\theta = 36°$

$\therefore$ Angles of prism are 72°, 72° and 36°.

24.9 Optical Instruments

Optical instruments are used to assist the eye in viewing an object. Our eye lens has a power to adjust its focal length to see the nearer objects. This process of adjusting focal length is called **accommodation**. However, if the object is brought too close to the eye, the focal length cannot be adjusted to form the image on the retina. Thus, there is a minimum distance for the clear vision of an object. This distance is called **least distance of distinct vision (D).** For normal eye this distance is generally taken to be 25 cm.

Visual angle The size of an object as sensed by us is related to the size of the image formed on the retina.

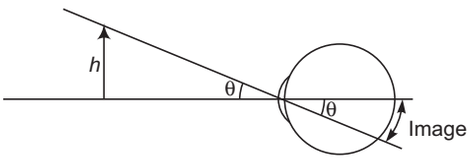


Fig. 24.69

The size of the image on the retina is roughly proportional to the angle subtended by the object on the eye. This angle is known as the visual angle. Optical instruments are used to increase this angle artificially in order to improve the clarity.

Magnifying power (M) Magnifying power is the factor by which the image on the retina can be enlarged by using the microscope or telescope. For a microscope and for a telescope the definition of M is slightly different.

For a microscope,

$$M = \frac{\text{Visual angle formed by final image}}{\text{Visual angle formed by the object when kept at distance } D}$$

For a telescope,

$$M = \frac{\text{Visual angle formed by final image}}{\text{Visual angle subtended by the object directly when seen from naked eye.}}$$

Note that M is different from linear magnification $m\left(=\pm\frac{v}{u}\right)$ which is the ratio of height of image to that of object. While M is the ratio of apparent increase in size of image seen by the eye. Unit of M is X. Thus, we write an angular magnification of 10 as $10X$.

Simple microscope To view an object with naked eyes, the object must be placed between D and infinity. The maximum angle is subtended when it is placed at D.

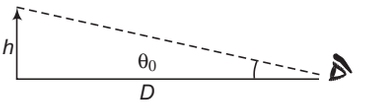


Fig. 24.70

Say this angle is θ_0. Then,

$$\theta_0 = \frac{h}{D}$$

This angle can be further increased, if a converging lens of short focal length is placed just in front of the eye. The lens used for this purpose is called **simple microscope** or a **magnifier**.

The object is placed at a distance u_0 from the lens (between pole and focus of lens). The virtual magnified image is formed as shown. This image subtends a visual angle (say) θ on the eye. Then,

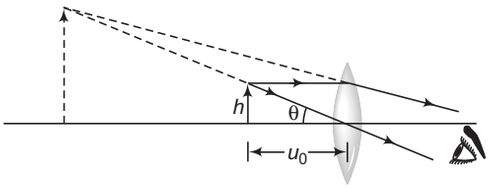


Fig. 24.71

$$\theta = \frac{h}{u_0}$$

From the definition of magnifying power for a microscope,

$$M = \frac{\theta}{\theta_0} = \frac{h/u_0}{h/D}$$

$\therefore$ $$M = \frac{D}{u_0}$$

For relaxed eye The final image should be at infinity. Thus, $u_0 = f$

$\therefore$ $$M_\infty = \frac{D}{f}$$

This is also called **magnifying power** for normal adjustment.We can see that $M_\infty > 1$ if $f < D$.

Magnifying power when final image is at *D* In the above case we saw that M is equal $\frac{D}{f}$. The magnification can be made large by chosing the focal length f small.

The magnifying power can be increased in an another way be moving the object still closer to the lens. Suppose,

the final virtual image is formed at a distance D. Then, from the equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, we have

$$\frac{1}{-D}+\frac{1}{u_0}=\frac{1}{f} \quad \text{or} \quad \frac{1}{u_0}=\frac{1}{D}+\frac{1}{f}$$

Substituting this value in the equation $M=\frac{D}{u_0}$, we have

$$M_D=1+\frac{D}{f}$$

Note Points

- That $M_D > M_\infty$, i.e. when final image is formed at 25 cm, angular magnification is increased but eye is most strained. On the other hand when final image is at infinity, angular magnification is slightly less but eye is relaxed. So, the choice is yours whether you want to see bigger size with strained eye or smaller size with relaxed eye.
- That M can be increased by decreasing f, but due to several other aberrations the image becomes too defective at large magnification with a simple microscope. Roughly speaking a magnification upto 4 is trouble free.

Compound microscope Figure shows a simplified version of a compound microscope. It consists of two converging lenses arranged coaxially. The one facing the object is called **objective** and the one close to eye is called eye piece. The objective has a smaller aperture and smaller focal length than those of the eye piece.

The separation between the objective and the eye piece (called the length of the microscope L) can be varied by appropriate screws fixed on the panel of microscope.

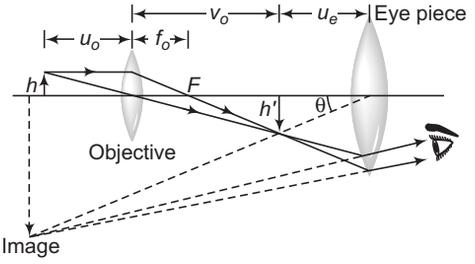


Fig. 24.72

The object is placed beyond first focus of objective, so that an inverted and real image (intermediate image) is formed by the objective. This intermediate image acts as an object for the eye piece and lies between first focus and pole of eye piece. The final magnified virtual image is formed by the eye piece. Let θ be the angle subtended by the final image on the eye, then,

$$\theta=\frac{h'}{u_e}$$

Here, h' is the height of the first image and u_e is its distance from the eye piece.

Further $$\theta_0=\frac{h}{D}$$

∴ Magnifying power of the compound microscope will be,

$$M=\frac{\theta}{\theta_0}=\frac{h'}{u_e}\times\frac{D}{h}=\left(\frac{h'}{h}\right)\left(\frac{D}{u_e}\right)$$

Here, $\frac{h'}{h}$ is the linear magnification by the objective. Thus,

$$\frac{h'}{h}=|m_o|=\frac{v_o}{u_o}$$

$$\therefore \quad M=\frac{v_o}{u_o}\left(\frac{D}{u_e}\right)$$

Length of the microscope will be,

$$L=v_o+u_e$$

For relaxed eye For relaxed eye final image should be at infinity. Or,

$$u_e=f_e$$

$$\therefore \quad M_\infty=\frac{v_o}{u_o}\frac{D}{f_e} \text{ and } L_\infty=v_o+f_e$$

Final image at *D* When the final image (by eye piece) is formed at D. Then by the formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have,

$$\frac{1}{-D}+\frac{1}{u_e}=\frac{1}{f_e}$$

$$\therefore \quad \frac{1}{u_e}=\frac{1}{D}+\frac{1}{f_e} \text{ or } u_e=\frac{Df_e}{D+f_e}$$

Thus, $$M_D=\frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$$

and $$L_D=v_o+\frac{Df_e}{D+f_e}$$

Telescopes A microscope is used to view the objects placed closed to it. To look at distant objects such as a star, a planet or a distant tree etc., we use telescopes. There are three types of telescopes in use:

(i) Astronomical telescope,
(ii) Terrestrial telescope
(iii) Galilean telescope

(i) Astronomical telescope Figure shows the construction and working of an astronomical telescope.

It consists of two converging lenses placed coaxially. The one facing the distant object is called the objective and has a large aperture and large focal length. The other is called the eye piece, as the eye is placed closed to it. The eye piece tube can slide within the objective tube, so that the

separation between the objective and the eye piece may be varied.

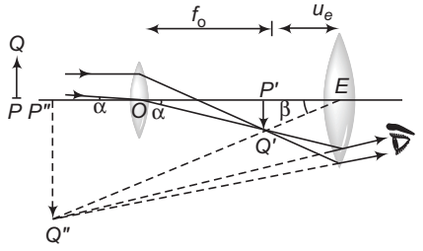


Fig. 24.73

Magnifying power Although a telescope can also be used to view the objects of few kilometers away but the magnifying power calculated below is for the case when object is at infinity. Rays coming from the object in that case will be almost parallel.

The image formed by objective will be at its second focus. This image called the **intermediate image** will act as the object for eye piece. This usually lies between pole and first focus of eye piece. So, that eye piece forms a virtual and magnified image of it.

$|\alpha|$ = angle subtended by object on objective

(or you can say at eye)

$$= \frac{P'Q'}{f_o}$$

$|\beta|$ = angle subtended by final image at eye piece

(or at eye)

$$= \frac{P'\theta'}{u_e}$$

From the definition of magnifying power

(for telescope),

$$M = \frac{|\beta|}{|\alpha|} = \frac{f_o}{u_e} \quad \text{or} \quad M = \frac{f_o}{u_e}$$

and length of telescope,

$$L = f_o + u_e$$

For relaxed eye For relaxed eye, intermediate image should lie at first focus of eye piece or

$$u_e = f_e$$

$$\therefore \qquad M_\infty = \frac{f_o}{f_e} \quad \text{and} \quad L_\infty = f_o + f_e$$

Final image at *D* When the final image is at D, then using the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for eye piece we have,

$$\frac{1}{-D} + \frac{1}{u_e} = \frac{1}{f_e}$$

$$\therefore \quad \frac{1}{u_e} = \frac{1}{D} + \frac{1}{f_e} \quad \text{or} \quad u_e = \frac{Df_e}{D + f_e}$$

Therefore, $M_D = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$ and $L_D = f_o + \frac{Df_e}{D + f_e}$

(ii) Terrestrial telescope In an astronomical telescope, the final image is inverted with respect to the object. To remove this difficulty, a convex lens of focal length f is included between the objective and the eye piece in such a way that the focal plane of the objective is a distance $2f$ away from this lens.

The role of the intermediate lens L is only to invert the image. The magnification produced by it is -1. The formulae of M does not change at all. They remain as it is, as were derived for astronomical telescope. The length of telescope will however increase by $4f$. Here, you should note that we are talking only about magnitude of M. Thus,

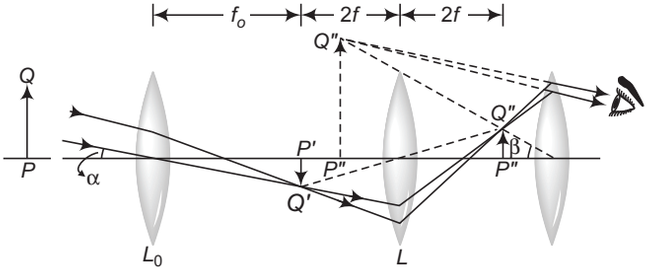


Fig. 24.74

$$M_\infty = \frac{f_o}{f_e} \quad \text{and} \quad M_D = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

$$L_\infty = f_o + 4f + f_e \quad \text{and} \quad L_D = f_o + 4f + \frac{Df_e}{D + f_e}$$

(iii) Galilean telescope Figure shows a simple model of Galilean telescope. A convergent lens is used as the objective and a divergent lens as the eye piece. The objective lens forms a real and inverted image $P'Q'$ but the divergent lens comes in between. This intermediate image acts as virtual object for eye piece. Final image $P''Q''$ is erect and magnified as shown in figure. The intermediate image is formed at second focus of objective.

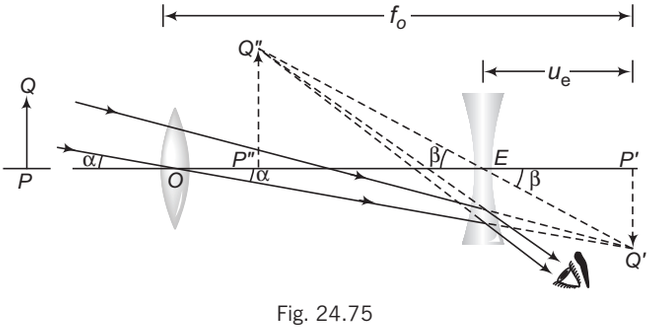


Fig. 24.75

Magnifying power From the figure, we can see that

$$|\alpha| = \frac{P'Q'}{f_o} \quad \text{and} \quad |\beta| = \frac{P'Q'}{u_e}$$

From the definition of magnifying power for telescope,

$$M = \frac{|\beta|}{|\alpha|} = \frac{f_o}{u_e}$$

and length of the telescope,

$$L = f_o - u_e$$

For relaxed eye For relaxed eye intermediate image should lie at first focus of eye piece. Or,

$$u_e = f_e$$

Hence, $M_\infty = \frac{f_o}{f_e}$ and $L_\infty = f_o - f_e$

Final image at *D* For the final image to be at a distance D from the eye piece, we have from the formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-D} - \frac{1}{u_e} = \frac{1}{-f_e}$$

$$\therefore \quad \frac{1}{u_e} = \frac{1}{f_e} - \frac{1}{D} \quad \text{or} \quad u_e = \frac{Df_e}{D - f_e}$$

Thus, $M_D = \frac{f_o}{f_e}\left(1 - \frac{f_e}{D}\right)$

and $L_D = f_o - \frac{f_e D}{D - f_e}$

Note Points

- In all above formulae of *M*, we are considering only the magnitude of *M*.
- For telescopes, formulae have been derived when the object is at infinity. For the object at some finite distance different formulae will have to be derived.
- Given below are formulae derived above of *M* and *L* in tabular form.

Table 24.2

Name of optical instruments	M	L	M_∞	M_D	L_∞	L_D
Simple Microscope	$\frac{D}{u_o}$	—	$\frac{D}{f}$	$1 + \frac{D}{f}$	—	—
Compound Microscope	$\frac{v_o}{u_o}\frac{D}{u_e}$	$v_o + u_e$	$\frac{v_o}{u_o}\frac{D}{f_e}$	$\frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$	$v_o + f_e$	$v_o + \frac{Df_e}{D + f_e}$
Astronomical Telescope	$\frac{f_o}{u_e}$	$f_o + u_e$	$\frac{f_o}{f_e}$	$\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$	$f_o + f_e$	$f_o + \frac{Df_e}{D + f_e}$
Terrestrial Telescope	— do —	$f_o + 4f + u_e$	— do —	— do —	$f_o + 4f + f_e$	$f_o + 4f + \frac{Df_e}{D + f_e}$
Galilean Telescope	$\frac{f_o}{u_e}$	$f_o - u_e$	$\frac{f_o}{f_e}$	$\frac{f_o}{f_e}\left(1 - \frac{f_e}{D}\right)$	$f_o - f_e$	$f_o - \frac{f_e D}{D - f_e}$

Resolving Power of a Microscope and a Telescope

Microscope The resolving power of a microscope is defined as the reciprocal of the distance between two objects which can be just resolved when seen through the microscope. It depends on the wavelength λ of the light, the refractive index μ of the medium between the object and the objective and the angle θ subtended by a radius of the objective on one of the objects.

$$R = \frac{1}{\Delta d} = \frac{2\mu \sin\theta}{\lambda}$$

To increase R, objective and object are immersed in oil.

Telescope The resolving power of a telescope is defined as the reciprocal of the angular separation between two distant objects which are just resolved by a telescope.

It is given by,

$$R = \frac{1}{\Delta\theta} = \frac{a}{1.22\lambda}$$

Here, a is the diameter of the objective. That is why, telescopes with larger objective aperture are used.

Example 24.17 *An object is seen through a simple microscope of focal length* 20 *cm. Find the angular magnification produced, if the image is formed at* 30 *cm from the lens.*

Sol. Given, $f = +20$ cm

and $v = -30$ cm

Using the formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have, $\frac{1}{-30} - \frac{1}{-u_o} = \frac{1}{20}$

$\therefore \quad u_o = 12$ cm

The angular magnification,

$$M = \frac{D}{u_o} = \frac{25}{12} = 2.08$$

Example 24.18 *A* 20 *D lens is used as a magnifier. Where should the object be placed to obtain maximum angular magnification? (Given* $D = 25$ *cm).*

Sol. Focal length of the lens,

$$f = \frac{1}{20}\text{ m} = \frac{100}{20}\text{ cm} = 5\text{ cm}$$

Maximum angular magnification is obtained when final image is formed at D. Hence,

$$\frac{1}{-25} - \frac{1}{-u_o} = \frac{1}{5}$$

$$\therefore \quad u_o = 4.17\text{ cm}$$

Example 24.19 *The separation between the objective and the eye piece of a compound microscope can be adjusted between* 9.8 *cm to* 11.8 *cm. If the focal length of the objective and the eye piece are* 1.0 *cm and* 6 *cm respectively, find the range of the magnifying power, if the image is always needed at* 24 *cm from the eye.*

Sol. For eye piece, it is given that

$$f_e = 6\text{ cm}, \quad v_e = -24\text{ cm}$$

$$\therefore \quad \frac{1}{-24} - \frac{1}{-u_e} = \frac{1}{6}$$

$$\therefore \quad u_e = 4.8\text{ cm}$$

When $L = 9.8$ cm, then $v_o = (9.8 - 4.8)\text{ cm} = 5.0$ cm

$$\therefore \quad \frac{1}{5.0} - \frac{1}{-u_o} = \frac{1}{1.0}$$

$$\therefore \quad u_o = 1.25\text{ cm}$$

or
$$M = \frac{v_o}{u_o}\frac{D}{u_e} = \left(\frac{5.0}{1.25}\right)\left(\frac{25.0}{4.8}\right) = 20.83$$

When $L = 11.8$ cm, then $v_o = (11.8 - 4.8)\text{ cm} = 7.0$ cm

$$\therefore \quad \frac{1}{7.0} - \frac{1}{-u_o} = \frac{1}{1.0}$$

or
$$u_o = 1.17\text{ cm}$$

$$\therefore \quad M = \left(\frac{7.0}{1.17}\right)\left(\frac{25.0}{4.8}\right) = 31.16$$

Therefore, range of magnifying power is from 20.83 to 31.16.

Example 24.20 *A compound microscope has a magnifying power of* 100 *when the image is formed at infinity. The objective has a focal length of* 0.5 *cm and the tube length is* 6.5 *cm. Find the focal length of the eye piece.*

Sol. When the final image is at infinity,

$$u_e = f_e = \text{tube length} - v_o$$

$$\therefore \quad f_e = 6.5 - v_o \qquad \ldots(i)$$

Since,
$$M_\infty = \frac{v_o}{u_o}\cdot\frac{D}{f_e}$$

$$\therefore \quad 100 = \frac{v_o}{u_o}\cdot\frac{25}{f_e}$$

or
$$\frac{v_o}{u_o f_e} = 4 \qquad \ldots(ii)$$

For the objective,

$$\frac{1}{v_o} - \frac{1}{-u_o} = \frac{1}{f_o} = \frac{1}{0.5}$$

$$\therefore \quad \frac{1}{v_o} + \frac{1}{u_o} = 2 \qquad \ldots(iii)$$

We have three unknowns v_o, u_o and f_e solving these three equations we get,

$$f_e = 2\text{ cm}$$

Example 24.21 *The eye piece of an astronomical telescope has a focal length of* 10 *cm. The telescope is focused for normal vision of distant objects when the tube length is* 1.0 *m. Find the focal length of the objective and the magnifying power of the telescope.*

Sol. Given, $f_e = 10$ cm, $L_\infty = 1.0\text{ m} = 100\text{ cm}$

Since, $L_\infty = f_o + f_e$

$$\therefore \quad f_o = 90.0\text{ cm}$$

Further,
$$M_\infty = \frac{f_o}{f_e} = \frac{90.0}{10.0} = 9.0$$

Example 24.22 *An astronomical telescope is to be designed to have a magnifying power of* 50 *in normal adjustment. If the length of the tube is* 102 *cm, find the powers of the objective and the eye piece.*

Sol. Given, $M_\infty = 50$

$$\therefore \quad \frac{f_o}{f_e} = 50 \qquad \ldots(i)$$

Further, $L_\infty = 102$ cm

$$\therefore \quad f_o + f_e = 102\text{ cm} \qquad \ldots(ii)$$

Solving Eqs. (i) and (ii), we have

$$f_o = 100\text{ cm} \quad \text{and} \quad f_e = 2\text{ cm}$$

$$\therefore \quad P_o = \frac{1}{1.0} = 1\text{ D} \quad \text{and} \quad P_e = \frac{1}{0.02} = 50\text{ D}$$

Example 24.23 *A Galilean telescope is* 27 *cm long when focused to form an image at infinity. If the objective has a focal length of* 30 *cm, what is the focal length of the eye piece?*

Sol. Given, $f_o = +30$ cm

Length of telescope is given 27 cm.

Therefore, $u_e = +3$ cm

For the final image at infinity, the intermediate image should lie at first focus of eye piece of the Galilean telescope.

$$\therefore \quad f_e = -3\text{ cm}$$

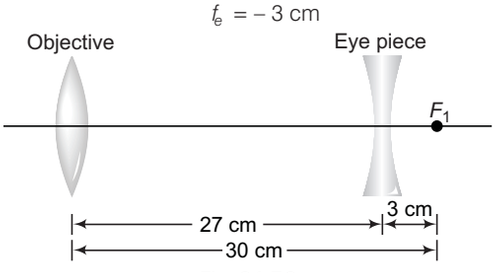
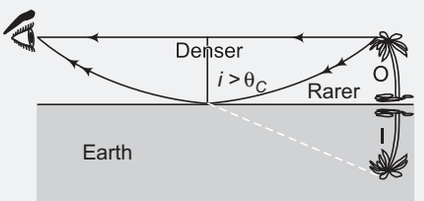


Fig. 24.76

Extra Knowledge Points

- Eye is most sensitive to yellow-green light ($\lambda = 5550$ Å).
- Photographic plate is most sensitive to blue and least to red.
- Persistance of eye is $\frac{1}{10}$s, i.e. if time interval between two successive light pulses is lesser than 0.1s, eye cannot distinguish them separately.
- Frequency of visible light is of the order of 10^{15} Hz.
- Colour of light is determined by its frequency and not the wavelength. During refraction of light frequency and colour of light do not change.
- During refraction, ray of light does not bend under following two conditions.
  (i) If light is incident normally on a boundary, i.e. $\angle i = 0$.
  (ii) If the refractive indices of two media are equal, i.e. $\mu_1 = \mu_2$.
- **Twinkling of stars** Due to fluctuations in refractive index of atmosphere the refraction of light (reaching to our eye from the star) becomes irregular and the light sometimes reaches the eye and sometimes it does not. This gives rise to twinkling of stars.
- **Oval shape of sun in the morning and evening**
  In the morning or evening, the sun is at the horizon. The refractive index decreases with height. Light reaching the earth's atmosphere from different parts of vertical diameter of the sun enters at different heights in the earth's atmosphere and so travels in media of different refractive indices at the same instant and hence, bends unequally. Due to this unequal bending of light from vertical diameter, the image of the sun gets distorted and it appears oval and larger. However, at noon when the sun is overhead, then due to normal incidence there will be no bending and the sun will appear circular.
- The sparkling of diamond is due to total internal reflection inside it.
- **Mirage** Mirage in deserts is caused by total internal reflection. Due to heating of the earth, the refractive index of air near the surface of the earth becomes lesser than above it. Light from distant objects reaches the surface of the earth with $i > \theta_c$, so that total internal reflection will take place and we see the image of an object along with the object as seen in figure, creating an illusion of water near the object.

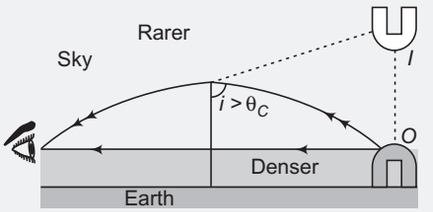


(A) Mirage

- **Duration of sun's visibility** In the absence of atmosphere the sun will be visible for its positions from *M* to *E* as shown in figure. However in presence of atmosphere, due to total internal reflection, the sun will become visible even when it is below the horizon.

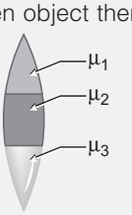


- **Looming** It is also due to total internal reflection. This phenomenon is observed in cold deserts and opposite to that of mirage.

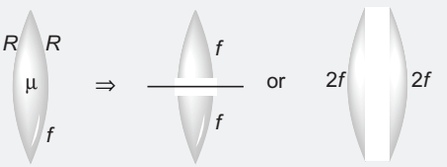


(B) Looming

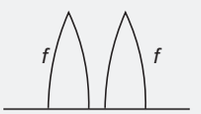
- A lens made of three different materials have three focal lengths. Thus, for a given object there are three images.



-

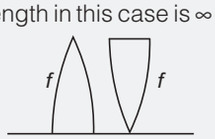


- The resultant focal length in this case is $\frac{f}{2}$.



In contact

- The resultant focal length in this case is ∞ or power is zero.



In contact

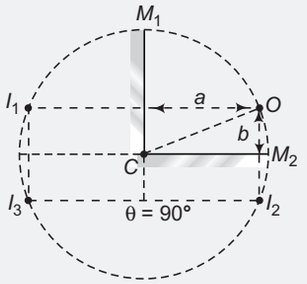
- **Exercise** Think the reasoning (*S*) of point number (*S*) 12, 13, 14 and 15.
- If an object is placed between two parallel mirrors ($\theta = 0°$) the number of images formed will be infinite but of decreasing intensity.
- **Number of images** If there are two plane mirrors inclined to each other at an angle θ, the number of images formed are determined as follows :
  (i) $n = (m - 1)$ for all positions of object, where $m = \frac{360}{\theta}$ is an even integer.

(ii) $n = m$, where $m = \frac{360}{\theta}$ is an odd integer and object is not on the bisector of mirrors.

(iii) $n = (m - 1)$, where $m = \frac{360}{\theta}$ is an odd integer and object is on the bisector of mirrors.

(iv) If $\frac{360}{\theta}$ is a fraction, the number of images formed will be equal to its integral part.

- The number of images formed by two mutually perpendicular ($\theta = 90°$) mirrors will be 3. All these three images will lie on a circle with centre at C, the point of intersection of mirrors M_1 and M_2 and whose radius is equal to distance between C and the object O.



Example Find total number of images formed by two mirrors inclined to each other at an angle $\theta = 60°$.

Sol. $\frac{360}{\theta} = \frac{360}{60} = 6 = m$

Since, m is an even integer, total number of images will be $m - 1$ or 5 for all positions of the object between the mirrors.

- In case of spherical mirrors, if object distance x_1 and image distance x_2 are measured from focus instead of pole, $u = (f + x_1)$ and $v = (f + x_2)$, the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ reduces to, } \frac{1}{f + x_2} + \frac{1}{f + x_1} = \frac{1}{f}$$

which on simplification gives, $x_1 x_2 = f^2$

This formula is called Newton's formula.

This formula applies to a lens also, but in that case x_1 is the object distance from first focus and x_2 the image distance from second focus.

- In case of a lens camera

(i) Time of exposure $\propto \frac{1}{(\text{Aperture})^2}$

(ii) The ratio of focal length to the aperture of lens is called f-number of the camera.

$$\therefore \quad f\text{-number} = \frac{\text{Focal length}}{\text{Aperture}}$$

If f-number of a camera is $f/11$, it implies that aperture is $\frac{1}{11}$ of its focal length. Movie cameras have very low f-number such as $f/1.5$.

- **Scattering of light** If the molecules of a medium after absorbing incoming radiations (light) emit them in all possible directions, the process is called scattering. In scattering, if the wavelength of radiation remains unchanged the scattering is called elastic otherwise inelastic.

- Rayleigh has shown, theoretically that in case of elastic scattering of light by molecules, the intensity of scattered light depends on both nature of molecules and wavelength of light. According to him,

Intensity of scattered light $\propto \frac{1}{\lambda^4}$.

Raman effect was based on inelastic scattering. For this CV Raman was awarded the Noble Prize in 1930.

Scattering helps us in understanding the following :

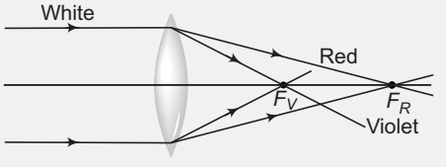
Why sky is blue When white light from the sun enters the earth's atmosphere scattering takes place. As scattering is proportional to $\frac{1}{\lambda^4}$, blue is scattered most. When we look at the sky we receive scattered light which is rich in blue and hence, the sky appears blue.

Why sun appears red during sunset and sunrise In the morning and evening when sun is at the horizon, due to oblique incidence light reaches the earth after traversing maximum path in the atmosphere and so suffers maximum scattering. Now, as scattering $\propto \frac{1}{\lambda^4}$, shorter wavelengths are scattered most leaving the longer one. As red light has longest wavelength in the visible region, it is scattered least. This is why sun appears red in the morning and evening. The same reason is why red light is used for danger signals.

- **Defects of images** Actual image formed by an optical system is usually imperfect. The defects of images are called aberrations. The defect may be due to light or optical system. If the defect is due to light it is called chromatic aberration, and if due to optical system, monochromatic aberration.

(i) **Chromatic aberration** The image of an object formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration. This defect arises due to the fact that focal length of a lens is different for different colours. For a lens,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



As μ is maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it.

The difference between f_R and f_V is a measure of longitudinal chromatic aberration. Thus,

$$\text{LCA} = f_R - f_V$$

This can also be written as,

$$f_R - f_V = -df$$

where, $df = f_V - f_R$

Differentiating the equation,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

We find, $\frac{-df}{f^2} = d\mu\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

or $\frac{-df}{f} = \frac{d\mu}{\mu - 1} = \omega$

$$\left[\text{where, } \omega = \frac{d\mu}{\mu - 1} = \text{dispersive power}\right]$$

Thus, $\text{LCA} = -df = f\omega$

For a single lens neither f nor ω can be zero. Thus, we cannot have a single lens free from chromatic aberration.

- Dispersive power is also written as,

$$\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1}, \text{ Here } \mu_Y = \frac{\mu_V + \mu_R}{2}$$

Condition of achromatism To get achromatism, we use a pair of two lenses in contact. For two thin lenses in contact we have,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \therefore \frac{-dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if,

$$dF = 0$$

$$\therefore \frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$$

$$\therefore \frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0$$

$$\therefore \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

This is the condition of achromatism. From the condition of achromatism, following conclusions can be drawn :

(a) As ω_1 and ω_2 are positive quantities, f_1 and f_2 should have opposite signs, i.e. if one lens is convex, the other must be concave.

(b) If $\omega_1 = \omega_2$, means both the lenses are of same material. Then,

$$\frac{1}{f_1} + \frac{1}{f_2} = 0 \text{ or } \frac{1}{F} = 0 \quad \text{or} \quad F = \infty$$

Thus, the combination behaves as a plane glass plate. So, we can conclude that both the lenses should be of different materials or, $\omega_1 \neq \omega_2$.

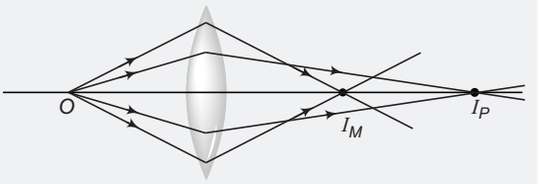
(c) Dispersive power of crown glass (ω_C) is less than that of flint glass (ω_F).

(d) If we want the combination to behave as a convergent lens, then convex lens should have lesser focal length or its dispersive power should be more.

Thus, convex lens should be made of flint glass and concave lens of crown. Thus, combination is converging, if convex is made of flint glass and concave of crown. Similarly, for the combination to behave as diverging lens, convex is made of crown glass and concave of flint glass.

(ii) **Monochromatic aberration** This is the defect in image due to optical system. Monochromatic aberration is of many types such as, spherical, coma, distortion, curvature and astigmatism. Here, we shall limit ourselves to spherical aberration only.

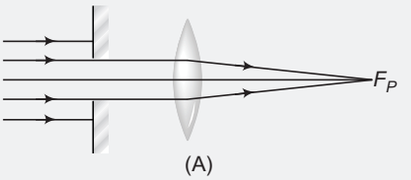
Spherical aberration Spherical aberration arises due to spherical nature of lens (or mirror).



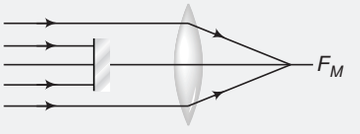
The paraxial rays (close to optic axis) get focused at I_P and marginal rays (away from the optic axis) are focused at I_M. Thus, image of a point object O is not a point.

The inability of the lens to form a point image of an axial point object is called spherical aberration. Spherical aberration can never be eliminated but can be minimised by the following methods :

(a) **By using stops** By using stops either paraxial or marginal rays are cut-off.



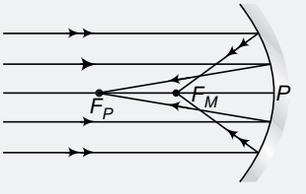
(A)



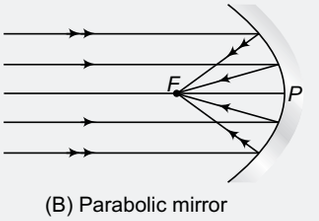
(B)

(b) **Using two thin lenses separated by a distance** Two thin lenses separated by a distance $d = f_2 - f_1$ has the minimum spherical aberration.

(c) **Using parabolic mirrors** If spherical mirror is replaced by parabolic mirror, spherical aberration is minimised.



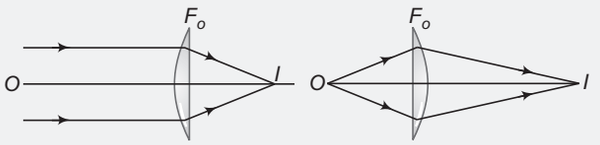
(A) Spherical mirror



(B) Parabolic mirror

(d) **Using lens of large focal length** It has been found that spherical aberration varies inversely as the cube of the focal length. So, if f is large, spherical aberration will be reduced.

(e) **Using plano-convex lens** In case of plano-convex lens spherical aberration is minimised, if its curved surface faces the incident or emergent ray whichever is more parallel.



(A) Telescope (B) Microscope

This is why in telescope the curved surface faces the object while in microscope curved surface is towards the image.

(f) **Using crossed lens** For a single lens with object at infinity, spherical aberration is found to be minimum, when R_1 and R_2 have the following ratio,

$$\frac{R_1}{R_2} = \frac{2\mu^2 - \mu - 4}{\mu(2\mu + 1)}$$

A lens which satisfies this condition is called a crossed lens.

Defects of vision Regarding eye following points are worth nothing :

(a) The human eye is most sensitive to yellow-green light ($\lambda = 5550$ Å).

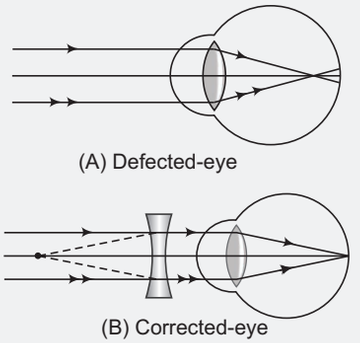
(b) The persistance of vision is $\frac{1}{10}$ s, i.e. if time interval between two consecutive light pulses is less than 0.1 s, eye cannot distinguish them separately.

(c) By the eye lens, real, inverted and diminished image is formed at retina.

(d) While testing your eye through reading chart, if doctor finds it to 6/12, it implies that you can read a letter from 6 m which the normal eye can read from 12 m. Thus, 6/6 means normal eye sight.

The common defects of vision are as follows :

(a) **Myopia or short-sightedness** Distant objects are not clearly visible in this defect. The image of distant object is formed before the retina.

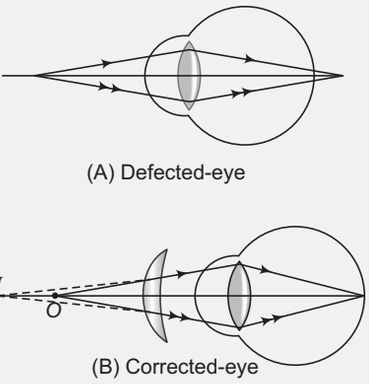


(A) Defected-eye

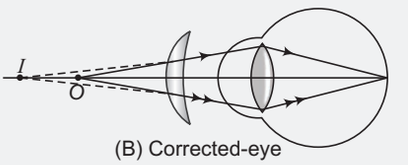
(B) Corrected-eye

The defect can be remedied by using a concave lens.

(b) **Hyperopia or far sightedness** The near objects are not clearly visible. Image of near object is formed behind the retina.



(A) Defected-eye



(B) Corrected-eye

This defect is remedied by using a convex lens.

(c) **Presbyopia** In it both near and far objects are not clearly visible. This is remedied either by using two separate lenses or by using single spectacle having bifocal lenses.

(d) **Astigmatism** In this defect eye cannot see objects in two orthogonal (perpendicular) directions clearly simultaneously. This defect is remedied by using cylindrical lens.

Chapter Summary with Formulae

▪ Reflection Laws

(a) $\angle i = \angle r$

(b) Incident ray, normal and reflected ray lie on same plane.

▪ Spherical Mirrors

(a) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

(b) Linear magnification, $m = -\frac{v}{u}$

(c) Image speed = $m^2 \times$ object speed

(d) Object velocity and image velocity always travel in opposite directions.

(e) $f = \frac{R}{2}$

▪ Refraction of Light

(a) In refraction, frequency of light remains unchanged. Speed and wavelength change. In a denser medium they decrease. In vacuum their values are maximum.

(b) Refractive index of a medium,

$$\mu = \frac{c}{v} = \frac{\lambda_0}{\lambda}$$

Here, c and λ_0 are the values in vacuum and v and λ are the values in medium.

(c) Refractive index of 1 with respect to 2,

$$_2\mu_1 = \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1} = \frac{\sin i_2}{\sin i_1}$$

(d) Snell's law

$$\mu \sin i = \text{constant}$$

or $$\mu_1 \sin i_1 = \mu_2 \sin i_2$$

$$\mu = \frac{\sin i}{\sin r} \quad \text{(When one medium is air)}$$

A ray of light follows that path along which time taken by the ray of light in reaching from one point to another point is least.

(e) Apparent depth for normal viewing

$$d_{\text{app.}} = \frac{d_{\text{actual}}}{\mu}$$

▪ Total Internal Reflection

(a) When a ray of light travels from a denser medium to a rarer medium with angle of incidence $i > \theta_c$, no refraction takes place and ray of light is 100% reflected in the denser medium.

(b) $\theta_c = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right)$

$= \sin^{-1}\left(\frac{1}{\mu}\right)$, (When rarer medium is air.)

(c) If μ increases, then θ_c decreases and changes of TIR increases in travelling from denser to rarer medium

▪ Refraction at Spherical Surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

For plane surface $R = \infty$, and this equation becomes,

$$v = \frac{\mu_2}{\mu_1} u$$

▪ Refraction by a Lens

(a) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

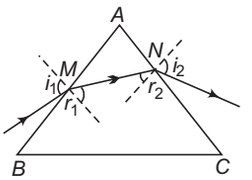
(b) $m = \frac{v}{u}$

(c) Power of a lens (in dioptre) = $\frac{1}{f \text{ (in meter)}}$

(d) For two or more than two thin lenses in contact we have,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{or} \quad P = P_1 + P_2$$

▪ Refraction through a Prism



(a) $r_1 + r_2 = A$

(b) $\delta = (i_1 + i_2) - A$

(c) $\delta \approx (\mu - 1)\, A$, if angles A and i_1 are small

(d) At minimum deviation, δ_m

$$r_1 = r_2 = \frac{A}{2} \quad \text{and} \quad i_1 = i_2 = \frac{A + \delta_m}{2}$$

(e) $\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

(f) At minimum deviation MN is parallel to BC, if $\angle B = \angle C$.

(g) If $A > 2\,\theta_c$, then total internal reflection will take place on face AC and it will not emerge out from the opposite face of the prism.

(h) $\mu = \mu_0 + \frac{A}{\lambda^2}$ (Cauchy's formula)

▪ The eye

(a) Focal length of eye lens is about 2.5 cm.

(b) Power of accommodation for normal eye D = 25 cm.

(c) In myopia or short-sightedness concave lens is used.

(d) In hyperopia or far-sightedness convex lens is used.

(e) In astigmatism, cylindrical lens is used.

▪ Optical Instruments

Name of optical instruments	M	L	M_∞	M_D	L_∞	L_D
Simple Microscope	$\frac{D}{u_o}$	—	$\frac{D}{f}$	$1 + \frac{D}{f}$	—	—
Compound Microscope	$\frac{v_o}{u_o}\frac{D}{u_e}$	$v_o + u_e$	$\frac{v_o}{u_o}\frac{D}{f_e}$	$\frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$	$v_o + f_e$	$v_o + \frac{Df_e}{D + f_e}$
Astronomical Telescope	$\frac{f_o}{u_e}$	$f_o + u_e$	$\frac{f_o}{f_e}$	$\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$	$f_o + f_e$	$f_o + \frac{Df_e}{D + f_e}$

Additional Examples

Example 1. *Suppose that the lower half of the concave mirror's reflecting surface is covered with an opaque (non-reflective) material. What effect will this have on the real image of an object placed in front of the mirror?*

Sol. Complete image will be formed. Only, intensity of image will be low, as this image is formed by less number of reflected rays.

Example 2. *If a mirror is immersed in a liquid. What will happen to its focal length? Will it increase, decrease or remain same?*

Sol. It will remain the same. Because focal length of a mirror is given by

$$f = \frac{R}{2}$$

Or f depends only on R. When the mirror is immersed in liquid, the radius of curvature R does not change. Therefore, focal length remains unchanged.

Example 3. *What type of mirror is used in vehicles for viewing the objects on rear side?*

Sol. Convex mirror is used as rear view mirror in vehicles. Because convex mirror always makes virtual and diminished image (of all real objects) and its field of view is large. It means, by keeping eye in one position we can see the images of most of the objects lying behind us.

Example 4. *Why is a concave mirror preferred to a plane mirror for shaving?*

Sol. When the face is placed between the pole and the focus of the concave mirror, a large and erect image of the face is seen in the mirror. In plane mirror, the image is of same size as the face, whenever the face may be.

Example 5. *How can we judge, without touching, whether a given mirror is plane, concave or convex?*

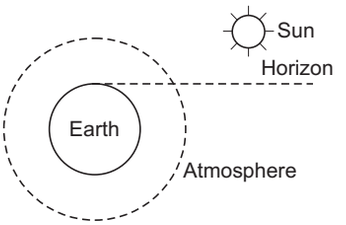
Sol. Let us hold the mirror close to our face and look into it. If the (virtual) image of our face is of the same size as the face, the mirror is plane; if enlarged, the mirror is concave; if diminished, the mirror is convex.

Example 6. *Why are mirrors used in searchlights parabolic and not concave spherical?*

Sol. In a searchlight, an intense, broad parallel beam of light, whose intensity remains undiminished with distance, is required. If a source of light is placed at the focus of a large aperture concave mirror, the reflected beam is not completely parallel due to spherical aberration, but somewhat divergent. This does not happen with parabolic mirrors.

Example 7. *Sun (or moon), when near the horizon, appears elliptical and not circular. Explain why?*

Sol. At sunrise or at sunset, the sun is just above the horizon. Its lower edge is nearer the horizon than its upper edge.



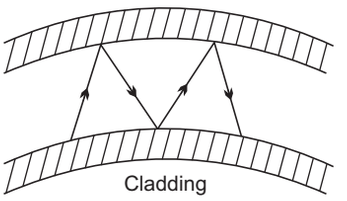
Therefore, light-rays from the lower edge are slightly more refracted in the atmosphere than those from the upper edge. In other words, the lower edge is apparently raised more than the upper edge. Thus, there is an apparent decrease in the vertical diameter of the sun (but not in the horizontal diameter) and so it appears elliptical.

Example 8. *Why do stars appear twinkling?*

Sol. Due to fluctuations in refractive index of atmosphere the refraction of light (reaching to our eye from the star) becomes irregular and the light sometimes reaches the eye and sometimes it does not. This gives rise to twinkling of stars.

Example 9. *What is the function of 'cladding' in a typical optical fibre?*

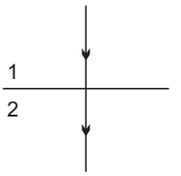
Sol. The refractive index of cladding material is lower than that of core material. Thus, it causes repeated total internal reflections of a light ray entering the optical fibre.



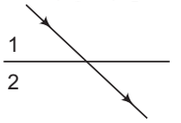
Example 10. *Under what conditions, a ray of light does not bend during refraction?*

Sol. Under following two conditions a ray of light does not bend during refraction.

(a) For normal incidence.
or , $\angle i = 0^\circ$



(b) If refractive index on both sides of the boundary is same. Or $\mu_1 = \mu_2$.



Example 11. *A thin converging lens is formed with one surface convex and the other plane. Does the position of image depend on whether the convex surface or the plane surface faces the object?*

Sol. All lens formulae have been derived under following two conditions.

(a) Thickness of lens should be negligible or it should be a thin lens.

(b) On both sides of the lens medium should be same (not necessarily air).

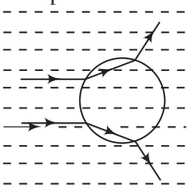
If above two conditions are met, then it hardly matters that object is placed on which side.

Example 12. *A single lens is mounted in a tube. A parallel beam enters the tube and emerges out of the tube as a divergent beam. Can you say with certainty that there is a diverging lens in the tube?*

Sol. No we can't say certainly that there is a diverging lens in the tube. We know that a lens changes its nature (convex behaves as concave and concave behaves as convex) if kept in a denser medium. So it is possible that there is a converging lens in the tube but the refractive index of the medium (in which the lens is kept) is more than the refractive index of the lens.

Example 13. *An air bubble is formed inside water. Does it act as a converging lens or a diverging lens ?*

Sol. It will behave like a diverging lens. Because refractive index of air (of which lens is made) is less than the refractive index of medium in which it is kept.



So, air bubble appears as convex lens but behaves like concave lens.

Example 14. *Can mirrors give rise to chromatic aberration ?*

Sol. No, mirrors don't show chromatic aberration. Law of reflection does not depend on refractive index and chromatic aberration is produced due to dependency of refractive index on wavelength.

Example 15. *Will the focal length of a lens for red light be more, same or less than that for blue light.*

Sol. Focal length of a lens for red light is more that that for blue light. It is because,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\mu_{\text{Red}} < \mu_{\text{Blue}}$$

$$\therefore \quad \left(\frac{1}{f}\right)_{\text{Red}} < \left(\frac{1}{f}\right)_{\text{Blue}}$$

or, $$f_{\text{Red}} > f_{\text{Blue}}.$$

Example 16. *An object is* 30.0 *from a spherical mirror, along the central axis. The absolute value of lateral magnification is* $\frac{1}{2}$. *The image produced is inverted. What is the focal length of the mirror?*

Sol. Image is inverted, so it is real u and v both are negative.

Magnification is $\frac{1}{2}$, therefore, $v = \frac{u}{2}$.

Given, $u = -30$ cm, $v = -15$ cm

Using the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we have, $$\frac{1}{f} = \frac{1}{-15} - \frac{1}{30} = -\frac{1}{10}$$

$\therefore$ $$f = -10 \text{ cm}$$

Example 17. *A concave mirror has a radius of curvature of* 24 *cm. How far is an object from the mirror, if an image is formed that is*

(a) virtual and 3.0 *times the size of the object,*

(b) real and 3.0 *times the size of the object and*

(c) real and 1/3 *the size of the object?*

Sol. Given $R = -24$ cm (concave mirror)

Hence, $f = \frac{R}{2} = -12$ cm

(a) Image is virtual and 3 times larger. Hence, u is negative and v is positive. Simultaneously, $|v| = 3|u|$. So let,

$$u = -x$$

then $$v = +3x$$

Substituting in the mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$,

we have $$\frac{1}{3x} - \frac{1}{x} = \frac{1}{-12}$$

$\therefore$ $$x = 8 \text{ cm}$$

Therefore, object distance is 8 cm.

(b) Image is real and three times larger. Hence, u and v both are negative and $|v| = 3|u|$. So let,

$$u = -x, \text{ then } v = -3x$$

Substituting in mirror formula, we have

$$\frac{1}{-3x} - \frac{1}{x} = -\frac{1}{12} \quad \text{or} \quad x = 16 \text{ cm}$$

$\therefore$ Object distance should be 16 cm.

(c) Image is real and $\frac{1}{3}$rd the size of object. Hence, both u and v are negative and $|v| = \frac{|u|}{3}$. So let,

$$u = -x \quad \text{then} \quad v = -\frac{x}{3}$$

Substituting in the mirror formula, we have

$$-\frac{3}{x} - \frac{1}{x} = -\frac{1}{12}$$

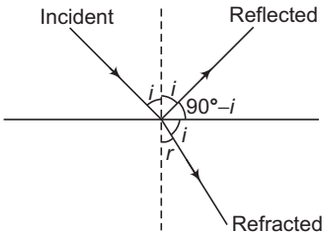
$\therefore$ $$x = 48 \text{ cm}$$

$\therefore$ Object distance should be 48 cm.

Example 18. *A ray of light falls on a glass plate of refractive index* $\mu = 1.5$. *What is the angle of incidence of the ray, if the angle between the reflected and refracted rays is* 90°*?*

Sol. In the figure, $r = 90° - i$

From Snell's law,

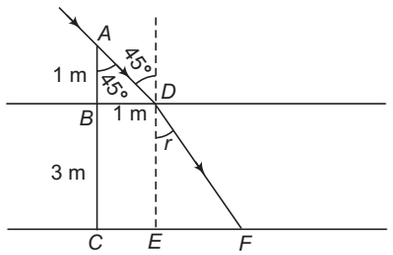


$$1.5 = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin (90° - i)} = \tan i$$

$$\therefore \quad i = \tan^{-1}(1.5) = 56.3°$$

Example 19. *A pile* 4 *m high driven into the bottom of a lake is* 1 *m above the water. Determine the length of the shadow of the pile on the bottom of the lake, if the sun rays make an angle of* 45° *with the water surface. The refractive index of water is* 4/3.

Sol. From Snell's law,



$$\frac{4}{3} = \frac{\sin 45°}{\sin r}$$

Solving this equation, we get

$$r = 32°$$

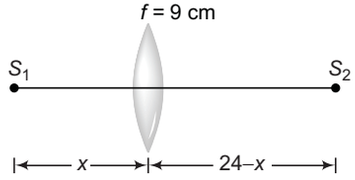
Further, $EF = (DE) \tan r$

$= (3) \tan 32° = 1.88$ m

$\therefore$ Total length of shadow $L = CF$

or $L = (1 + 1.88)$ m $= 2.88$ m

Example 20. *The distance between two point sources of light is* 24 *cm. Find out where would you place a converging lens of focal length* 9 *cm, so that the images of both the sources are formed at the same point.*



Sol. For $S_1 \quad \frac{1}{v_1} - \frac{1}{-x} = \frac{1}{9}$

$$\therefore \quad \frac{1}{v_1} = \frac{1}{9} - \frac{1}{x} \qquad \text{...(i)}$$

For $S_2 \quad \frac{1}{v_2} - \frac{1}{-(24 - x)} = \frac{1}{9}$

$$\therefore \quad \frac{1}{v_2} = \frac{1}{9} - \frac{1}{24 - x} \qquad \text{...(ii)}$$

Since, sign convention for S_1 and S_2 is just opposite. Hence,

$$v_1 = -v_2$$

or $$\frac{1}{v_1} = -\frac{1}{v_2}$$

$$\therefore \quad \frac{1}{9} - \frac{1}{x} = \frac{1}{24 - x} - \frac{1}{9}$$

$$x = 6$$

Solving this equation we get, $x = 6$ cm. Therefore, the lens should be kept at a distance of 6 cm from either of the object.

Example 21. *The angle of minimum deviation for a glass prism with* $\mu = \sqrt{3}$ *equals the refracting angle of the prism. What is the angle of the prism?*

Sol. Given, $A = \delta_m$

Using, $$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

we have, $$\sqrt{3} = \frac{\sin\left(\frac{A + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

or $$\sqrt{3} = \frac{\sin A}{\sin\left(\frac{A}{2}\right)}$$

$$= \frac{2 \sin \frac{A}{2} \cos \frac{A}{2}}{\sin\left(\frac{A}{2}\right)}$$

$$\therefore \quad \cos \frac{A}{2} = \frac{\sqrt{3}}{2}$$

$$\therefore \quad \frac{A}{2} = 30°$$

or $$A = 60°$$

Example 22. *A ray of light is incident at an angle of* 60° *on the face of a prism having refracting angle* 30°*. The ray emerging out of the prism makes an angle* 30° *with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges.*

Sol. Given, $i_1 = 60°$, $A = 30°$ and $\delta = 30°$.

From the relation,

$$\delta = (i_1 + i_2) - A$$

we have, $i_2 = 0$

This means that the emergent ray is perpendicular to the face through which it emerges.

Example 23. *The refracting angle of a glass prism is* 30°. *A ray is incident onto one of the faces perpendicular to it. Find the angle* δ *between the incident ray and the ray that leaves the prism. The refractive index of glass is* $\mu = 1.5$.

Sol. Given, $A = 30°, \mu = 1.5$ and $i_1 = 0°$

Since, $i_1 = 0°$, therefore, r_1 is also equal to 0°.

Further, since $r_1 + r_2 = A$

$\therefore$ $r_2 = A = 30°$

Using, $\mu = \dfrac{\sin i_2}{\sin r_2}$

we have, $1.5 = \dfrac{\sin i_2}{\sin 30°}$

or $\sin i_2 = 1.5 \sin 30°$

$= 1.5 \times \dfrac{1}{2} = 0.75$

$\therefore$ $i_2 = \sin^{-1}(0.75) = 48.6°$

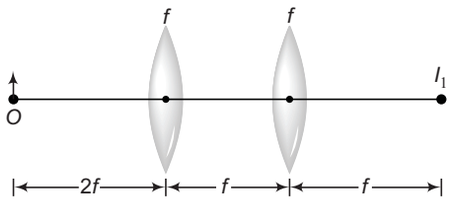
Now, the deviation, $\delta = (i_1 + i_2) - A$

$= (0 + 48.6) - 30$

or $\delta = 18.6°$

Example 24. *Two thin converging lenses are placed on a common axis, so that the centre of one of them coincides with the focus of the other. An object is placed at a distance twice the focal length from the left-hand lens. Where will its image be? What is the lateral magnification? The focal of each lens is f ?*

Sol.



The image formed by first lens will be at a distance $2f$ with lateral magnification $m_1 = -1$. For the second lens this image will behave as a virtual object. Using the lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$ we have,

$$\frac{1}{v} - \frac{1}{f} = \frac{1}{f}$$

$\therefore$ $v = \dfrac{f}{2}$

$$m_2 = \frac{v_2}{u_2} = \frac{(f/2)}{f} = \frac{1}{2}$$

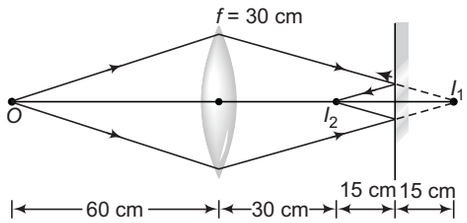
Therefore, final image is formed at a distance $\dfrac{f}{2}$ from the second lens with total lateral magnification,

$$m = m_1 \times m_2$$

$$= (-1) \times \left(\frac{1}{2}\right) = -\frac{1}{2}$$

Example 25. *A source of light is located at double focal length from a convergent lens. The focal length of the lens is* $f = 30$ *cm. At what distance from the lens should a flat mirror be placed, so that the rays reflected from the mirror are parallel after passing through the lens for the second time?*

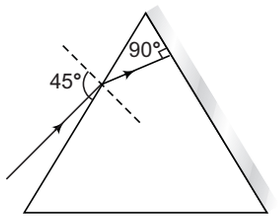
Sol.



Object is at a distance of $2f = 60$ cm from the lens. Image formed by lens I_1, should be at a distance 60 cm from the lens. Now I_2, the image formed by plane mirror should lie at focus or at a distance of 30 cm from the lens. Hence, the mirror should be placed at distance 45 cm from the lens as shown in figure.

Example 26. *One face of a prism with a refractive angle of* 30° *is coated with silver. A ray incident on another face at an angle of* 45° *is refracted and reflected from the silver coated face and retraces its path. What is the refractive index of the prism?*

Sol.



Given $A = 30°$, $i_1 = 45°$

and $r_2 = 0$

Since, $r_1 + r_2 = A$

$\therefore$ $r_1 = A = 30°$

Now, refractive index of the prism,

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 45°}{\sin 30°}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \sqrt{2}$$

Example 27. *Two equi-convex lenses of focal lengths* 30 *cm and* 70 *cm, made of material of refractive index* = 1.5*, are held in contact coaxially by a rubber band round their edges. A liquid of refractive index* 1.3 *is introduced in the space between the lenses filling it completely. Find the position of the image of a luminous point object placed on the axis of the combination lens at a distance of* 90 *cm from it.*

Sol. $|R_1| = |R_2| = f_1 = 30$ cm (As $\mu = 1.5$)

Similarly, $|R_3| = |R_4| = f_2 = 70$ cm

The focal length of the liquid lens (in air).

$$\frac{1}{f_3} = (\mu - 1)\left(\frac{1}{R_2} - \frac{1}{R_3}\right)$$

$$= (1.3 - 1)\left(\frac{1}{-30} - \frac{1}{+70}\right)$$

$$= -\frac{1}{70}$$



Further, equivalent focal length of the combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$= \frac{1}{30} + \frac{1}{70} - \frac{1}{70} = \frac{1}{30}$$

Using the lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$, we have

$$\frac{1}{v} - \frac{1}{-90} = \frac{1}{30}$$

$$\therefore \quad v = +45 \text{ cm}$$

Thus, image will be formed at a distance of 45 cm from the combination.

Example 28. *An astronomical telescope has an angular magnification of magnitude* 5 *for distant objects. The separation between the objective and eye piece is* 36 *cm and the final image is formed at infinity. Determine the focal length of objective and eye piece.*

Sol. For final image at infinity,

$$M_\infty = \frac{f_o}{f_e} \quad \text{and} \quad L_\infty = f_o + f_e$$

$$\therefore \quad 5 = \frac{f_o}{f_e} \qquad \text{...(i)}$$

and $$36 = f_o + f_e \qquad \text{...(ii)}$$

Solving these two equations, we have

$$f_o = 30 \text{ cm} \quad \text{and} \quad f_e = 6 \text{ cm}$$

Example 29. *A telescope has an objective of focal length* 50 *cm and an eye piece of focal length* 5 *cm. The least distance of distinct vision is* 25 *cm. The telescope is focused for distinct vision on a scale* 2 *m away from the objective. Calculate* (a) *magnification produced and* (b) *separation between objective and eye piece.*

Sol. Given, $f_o = 50$ cm and $f_e = 5$ cm

- Here, object is placed at finite distance from the objective. Hence, formulae derived for angular magnification *M* cannot be applied directly as they have been derived for the object to be at infinity. Here it will be difficult to find angular magnification. So, only linear magnification can be obtained.

For objective $\frac{1}{v_o} - \frac{1}{-200} = \frac{1}{50}$

$$\therefore \quad v_o = \frac{200}{3} \text{ cm}$$

$$m_o = \frac{v_o}{u_o}$$

$$= \frac{(200/3)}{-200} = -\frac{1}{3}$$

For eye piece $\frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5}$

$$\therefore \quad u_e = -\frac{25}{6} \text{ cm}$$

and $$m_e = \frac{v_e}{u_e}$$

$$= \frac{-25}{-(25/6)} = 6$$

(a) Magnification, $m = m_o \times m_e = -2$

(b) Separation between objective and eye piece,

$$L = v_o + |u_e| = \frac{200}{3} + \frac{25}{6}$$

$$= \frac{425}{6} = 70.83 \text{ cm}$$

NCERT Selected Questions

Q 1. A small candle 2.5 cm in size is placed 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image.

Sol. (a) $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$$

$$=\frac{1}{-18}-\frac{1}{(-27)}$$

$\therefore$ $v=-54$ cm

The negative sign shows that the image is formed in front of the mirror, i.e. on the side of the object itself. Thus, the screen must be placed at a distance of 54 cm in front of the mirror.

(b) $m=-\frac{v}{u}=\frac{I}{O}$

we get, $\frac{I}{2.5}=-\frac{(-54)}{(-27)}$

or $I=(2.5)\times(-2)$

$=-5$ cm

Thus, clearly the image is real, inverted and magnified.

Q 2. A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror?

Sol. Using the relation,

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v},$$

we get $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{15}-\left(\frac{1}{-12}\right)$

$\therefore$ $v=\frac{20}{3}$ cm

$=+6.67$ cm ≈ 6.7 cm

The positive sign shows that the image is formed behind the mirror.

Using the relation,

$$m=-\frac{v}{u},\text{we get}$$

$$m=-\frac{(20/3)}{-12}$$

$$=+\frac{5}{9}$$

Hence the image is virtual, erect and diminished.

As the needle is moved farther from the mirror, the image moves towards the focus (upto the focus only) and gets smaller and smaller in size.

Q 3. A tank is filled with water up to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank measured by a microscope is 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

Sol. **Case I** When tank is filled with water

$$\mu=\frac{\text{Real depth}}{\text{Apparent depth}}$$

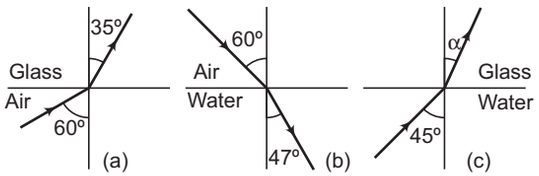
$$=\frac{12.5}{9.4}=1.33$$

Case II When the tank is filled with the liquid

apparent depth $=\frac{\text{Real depth}}{\mu}=\frac{12.5}{1.63}=7.67$ cm

$\therefore$ The distance through which the microscope has to be moved up $= 9.4-7.67=1.73$ cm.

Q 4. Figures (a) and (b) show refraction of an incident ray in air at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. (c)].



Sol. From Fig. (a)

$i=$ Ungle of incidence $=60°$

$r=$ Angle of refraction $=35°$

$\therefore$ RI of glass with respect to water

$$_a\mu_g=\frac{\sin i}{\sin r}=\frac{\sin 60°}{\sin 35°}$$

$$=\frac{0.8660}{0.5736}=1.51$$

From Fig. (b), here, $i=60°, r=47°$

$\therefore$ RI of water with respect to air

$$_a\mu_w=\frac{\sin i}{\sin r}=\frac{\sin 60°}{\sin 47°}$$

$$=\frac{0.8660}{0.7314}=1.18$$

$$_a\mu_g={}_a\mu_w\times{}_w\mu_g$$

or $_w\mu_g=\frac{{}_a\mu_g}{{}_a\mu_w}=\frac{1.51}{1.18}=1.28$

From Fig. (c), $i=$ Angle of incidence $=45°$

$\therefore$ Using the relation,

$$_w\mu_g = \frac{\sin i}{\sin r}$$

or $$1.28 = \frac{\sin 45^\circ}{\sin r}$$

or $$\sin r = \frac{\sin 45^\circ}{1.28} = \frac{1/\sqrt{2}}{1.28}$$

$$= \frac{0.707}{1.28} = 0.5525 = \sin 33.54'$$

$\therefore$ $$r = 33.54^\circ$$

Q 5. A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

Sol. Using the relation,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60+40}{2}\right)}{\sin\left(\frac{60}{2}\right)}$$

$$= \frac{\sin 50^\circ}{\sin 30^\circ} = \frac{0.7660}{0.50}$$

$$= 1.532 \approx 1.53$$

Let δ'_m be the new angle of minimum deviation of a parallel beam of light when the prism is placed in water $_a\mu_w = 1.33$, $_a\mu_g = 1.532$

$\therefore$ $$_w\mu_g = \frac{_a\mu_g}{_a\mu_w} = \frac{1.532}{1.33} = 1.152$$

$\therefore$ $$_w\mu_g = \frac{\sin\left(\frac{A+\delta'_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

or $$1.152 = \frac{\sin\left(\frac{60+\delta'_m}{2}\right)}{\sin\frac{60}{2}}$$

or $$\sin\left(30^\circ + \frac{\delta'_m}{2}\right) = 1.152 \times \frac{1}{2}$$

$$= 0.5759 = \sin 35.17^\circ$$

or $$30^\circ + \frac{\delta'_m}{2} = 35.17^\circ$$

or $$\frac{\delta'_m}{2} = 35.17^\circ - 30 = 5.17^\circ$$

$\therefore$ $$\delta'_m = 2 \times 5.17^\circ = 10.34^\circ$$

$$= 10^\circ 20'$$

Q 6. Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20 cm?

Sol. Using the relation,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \text{we get}$$

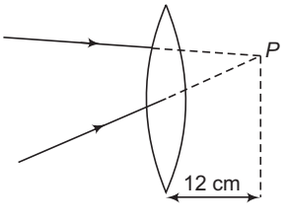
$$\frac{1}{20} = (1.55 - 1)\left[\frac{1}{R} - \left(\frac{1}{-R}\right)\right]$$

$$= 0.55\left[\frac{1}{R} + \frac{1}{R}\right] = 0.55 \times \frac{2}{R}$$

or $$R = 1.10 \times 20 = 22 \text{ cm}$$

Q 7. A beam of light converges to a point P. Now a lens is placed on the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm, (b) a concave lens of focal length 16 cm?

Sol. (a) $\therefore$ Using the formula,



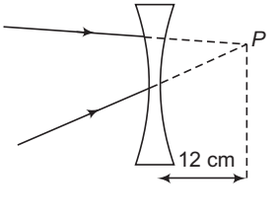
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}, \text{we get}$$

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{12} + \frac{1}{20}$$

$\therefore$ $$v = \frac{60}{8} = 7.5 \text{ cm}$$

i.e. the image is formed on the right of the lens and is real.

(b) For concave lens, $f = -16$ cm



$\therefore$ Using the relation,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}, \text{we get}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-16} + \frac{1}{12}$$

$\therefore$ $$v = +48 \text{ cm}$$

The image is formed at 48 cm to the right of the lens where the beam would converge and is real.

Q 8. An object of size 3 cm is palced 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

Sol. Using the relation

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}, \text{we get}$$

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{-21}+\frac{1}{(-14)}$$

$$\therefore \quad v=-8.4 \text{ cm}$$

$\therefore$ The image is virtual, erect and located at 8.4 cm from the lens on the same side as the object.

Also we know that

$$m=\frac{I}{O}=\frac{v}{u}$$

$$\therefore \quad I=\frac{v}{u}\times O$$

$$=\frac{-8.4}{-14}\times 3=1.8 \text{ cm}$$

i.e. the image is of diminished size.

If the object is moved away from the lens, the virtual image moves towards the focus of the lens (but never beyond focus).

Q 9. What is the focal length of a convex lens of focal length 30 cm in contact with a concave length of focal length 20 cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

Sol. $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$

$$=\frac{1}{30}+\frac{1}{(-20)}$$

$$\therefore \quad F=-60 \text{ cm}$$

As the focal length of the combination of two lenses is negative so the combination behaves as a diverging lens, i.e. as a concave lens.

Q 10. A small telescope has an objective lens of focal length 144 cm and an eye-piece of focal length 6 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eye-piece?

Sol. $M=$ Magnifying power of telescope

$L=$ Separation between the objective and the eyepiece

In normal adjustment (i.e. when the final image is formed at ∞)

$$M=-\frac{f_o}{f_e}=-\frac{144}{6}$$

$$=-24$$

$$\therefore \quad M=24$$

$$\therefore \quad L=f_o+f_e=144+6$$

$$=150 \text{ cm}$$

Q 11. (a) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1 cm is used, what is the angular magnification of the telescope?

(b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^6 m and the radius of lunar orbit is 3.8×10^8 cm.

Sol. (a) Angular magnification $=-\frac{f_o}{f_e}$

$$=-\frac{15}{10^{-2}}=-1500$$

(b) Let d be the diameter of the image of the moon formed by the objective lens.

$\therefore$ Angle subtended by the image $=\frac{d}{f_o}=\frac{d}{15}$ ...(i)

Also we know that angle subtended by the diameter of the moon

$$=\frac{\text{diameter of moon}}{\text{radius of lunar orbit}}$$

$$=\frac{3.48\times10^6}{3.8\times10^8} \quad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\frac{d}{15}=\frac{3.48\times10^6}{3.8\times10^8}$$

or $$d=\frac{3.48}{3.8}\times10^{-2}\times15$$

$$=13.73\times10^{-2} \text{ m}$$

$$=13.73 \text{ cm}$$

Q 12. A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

Sol. $d=\text{normal shift}=t-\frac{t}{u}=t\left(1-\frac{1}{\mu}\right)$

$$=15\left(1-\frac{1}{1.5}\right)=15\left(1-\frac{2}{3}\right)$$

$$=15\times\frac{1}{3}=5 \text{ cm}$$

i.e. the pin appears raised by 5 cm.

No, the answer does not depend upon the location of the slab for small angles of incidence.

Q 13. At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

Sol. Let C = critical angle for the material of the prism

$$\therefore \quad \sin C = \frac{1}{\mu} = \frac{1}{1.524} = 0.6562$$

$$\therefore \quad C = \sin^{-1}(0.6562) = 41°$$

We know that for a prism

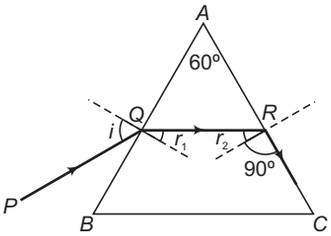
$$A = r_1 + r_2$$

Here $r_2 = C = 41°,\ A = 60°$

$\therefore \quad 60° = r_1 + 41°$ or $r_1 = 60° - 41° = 19°$

For the face AB,

$$r_1 = 19°, \mu = 1.524$$



$\therefore$ According to Snell's law,

$$\mu = \frac{\sin i}{\sin r_1} \quad \text{or} \quad \sin i = \mu \sin r_1 = 1.524 \sin 19°$$

$$= 1.524 \times 0.3256 = 0.4962$$

$$\therefore \quad i = \sin^{-1}(0.4962) = 29.75°$$

$$\therefore \quad i = 29.75° \approx 30°$$

Q 14. Does short sightedness (myopia) or long sightedness (hypermetropia) imply necessarily that the eye has partially lost its ability of accommodation? If not, what might cause these defects of vision?

Sol. No, short-sightedness (Myopia) or long sightedness (Hypermetropia) do not necessarily mean that eye has partially lost its ability of accommodation. Myopia arises when the eye can see nearby objects clearly but distant objects are not seen clearly due to the shift of the farthest point towards the eye and thus it becomes difficult to see beyond certain limit. This defect may be due to either the eye ball has become large or focal length of the eye lens has become too small. So the ray coming from infinity are focussed at a near point.

In long sightedness, one cannot see near by objects clearly. This defect may be due to either the eye ball has become short or the focal length of the lens has become too large.

Q 15. A myopic person has been using a spectacles of power 1 D for distant vision. During old age he also needs to use separate reading glass of power +2 D. Explain what may have happened?

Sol. Here, $P = -1$ D

$$\therefore \quad f = \frac{1}{P} = \frac{1}{-1} = -1 \text{ m} = -100 \text{ cm}$$

For normal vision, far point is at infinity

$$\therefore \quad u = -\infty$$

Using lens formula,

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad \text{...(i)}$$

we get $$-\frac{1}{-\infty} + \frac{1}{v} = -\frac{1}{100}$$

or $$\frac{1}{v} = -\frac{1}{100}$$

$$\therefore \quad v = -100 \text{ cm}$$

Thus, virtual image of the object at infinity is produced at 100 cm distance using spectacles.

To view objects at distance 25 cm to 100 cm, the person using ability of accommodation of eye which is partially lost in old age for which he needs another spectacles having power.

$$P = +2 \text{ D}$$

$$\therefore \quad f = \frac{1}{P} = \frac{1}{2} = 0.5 \text{ m} = 50 \text{ cm}$$

$$v = 25 \text{ cm}$$

Using Eq. (i), we get

$$-\frac{1}{u} + \frac{1}{25} = \frac{1}{50}$$

$$\therefore \quad u = 50 \text{ cm}$$

$\therefore$ His near point shifts to 50 cm.

Q 16. A person looking at another person wearing a shirt with a pattern comprising vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?

Sol. This defect is called **astigmatism.** The defect (called astigmatism) arises because the curvature of the cornea plus eye lens refracting system is not the same in different planes. [The eye lens is usually spherical, i.e. has the same curvature on different planes but the cornea is not spherical in case of an astigmatic eye.] In the present case, the curvature in the vertical plane is enough, so sharp images of vertical lines can be formed on the retina. But the curvature is insufficient in the horizontal plane, so horizontal lines appear blurred.

The defect can be corrected by using a cylindrical lens with its axis along the vertical. Clearly, parallel rays in the vertical plane will suffer no extra refraction, but those in the horizontal plane can get the required extra convergence due to refraction by the curved surface of the cylindrical lens if the curvature of the cylindrical surface is chosen appropriately.

Q 17. A small telescope has an objective lens of focal length 140 cm and an eye piece of focal length 5 cm. What is the magnifying power of the telescope for viewing distant objects when

(a) the telescope is in normal adjustment (i.e. when the final image is at infinity)?

(b) the final image is formed at the least distance of distinct vision (25 cm)?

Sol. (a) When the telescope is in normal adjustment, the magnifying power is given by

$$M = \frac{f_o}{|f_e|} = +\frac{140}{5} = 28$$

(b) When the final image is formed at the least distance of distinct vision, then M is given by,

$$M = \frac{f_o}{|f_e|}\left(1 + \frac{f_e}{D}\right)$$

$$= \frac{140}{5}\left(1 + \frac{5}{25}\right) = 33.6$$

***Q* 18.** Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of (a) reflected, and (b) refracted light? Refractive index of water is 1.33.

Sol. (a) For reflected light

(i) As the wavelength of the reflected light remains unchanged, so $\lambda' = \lambda = 589 \text{ nm} = 589 \times 10^{-9}$ m

(ii) As the reflection takes place in the same medium, so

v = speed of light

$= c = 3 \times 10^8 \text{ ms}^{-1}$

(iii) $\therefore$ Using the relation, $c = f\lambda$, we get

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{589 \times 10^{-9}}$$

$$= 5.093 \times 10^{14} \text{ Hz}$$

(b) For refracted light

$$\lambda_w = \frac{\lambda}{\mu_w}$$

$$= \frac{589 \times 10^{-9}}{1.33}$$

$$= 442.85 \times 10^{-9} \text{ m}$$

$$\approx 443 \text{ nm}$$

In moving from one medium to another medium, the frequency of light does not change so

$$f_w = f = 5.093 \times 10^{14} \text{ Hz}$$

$\therefore$ $$v_W = \frac{c}{\mu_w}$$

$$= \frac{3 \times 10^8}{1.33}$$

$$= 2.256 \times 10^8 \text{ ms}^{-1}$$

$$\approx 2.26 \times 10^8 \text{ ms}^{-1}$$

***Q* 19.** (a) The refractive index of glass is 1.3. What is the speed of light in glass? (speed of light in vacuum is 3×10^8 ms^{-1}).

(b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

Sol. (a) Using the relation,

$\mu = \frac{c}{v}$, we get

$$v_g = \frac{c}{\mu_g}$$

$$= \frac{3 \times 10^8}{1.5}$$

$$= 2 \times 10^8 \text{ ms}^{-1}$$

(b) No, the speed of light in glass is not independent of the colour of light. According to Cauchy's formula, the refractive index depends on the colour, i.e. wavelength of light as

$$\mu = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4} + \ldots$$

Thus, clearly the speed of light depends on the wavelength of colour of the light.

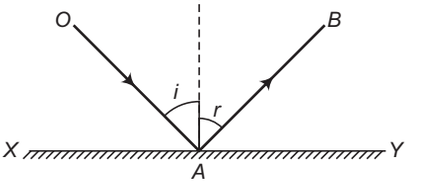
Also we know that $\lambda_v < \lambda_r$, i.e. wavelength of violet colour is lesser than the wavelength of red colour, thus clearly violet colour travels slower in glass than the red component.

***Q* 20.** Light of wavelength 5000 Å falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?

Sol. If f be the frequency of the incident light, then

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{5000 \times 10^{-10}} = 6 \times 10^{14} \text{ Hz}$$

The wavelength, μ and speed of the reflected light remains same. Frequency remains same in all media.



i.e. $f' = f = 6 \times 10^{14}$ Hz, $\lambda' = 5000$ Å

Here, XY is a plane reflecting surface.

OA and AB are the incident and reflected rays.

i = angle of incidence

r = angle of reflection

As $AB \perp OA$,

$\therefore$ $i + r = 90°$

Also, according to laws of reflection,

$r = i$

$\therefore$ $i + i = 90°$

$2i = 90°$

$$i = \frac{90°}{2}$$

$$= 45°$$

Objective Problems

[Level 1]

Reflection from Plane Mirror

1. A plane mirror reflects a pencil of light to form a real image. Then, the pencil of light incident on the mirror is
(a) parallel (b) convergent
(c) divergent (d) None of these

2. What should be the angle between two plane mirrors, so that whatever be the angle of incidence, the incident ray and the reflected ray from the two mirrors be parallel to each other?
(a) 60° (b) 90° (c) 120° (d) 45°

3. A man is 180 cm tall and his eyes are 10 cm below the top of his head. In order to see this entire height right from top to head, he uses a plane mirror kept at a distance of 1m from him. The minimum length of the plane mirror is
(a) 180 cm (b) 90 cm
(c) 85 cm (d) 170 cm

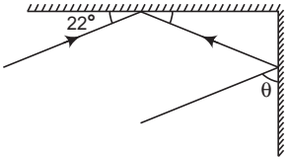
4. A ray of light is incident on a plane mirror at an angle of 30°. The deviation produced in the ray is
(a) 30° (b) 60°
(c) 90° (d) 120°

5. To get three images of a single object, one should have two plane mirrors at an angle of
(a) 30° (b) 60°
(c) 90° (d) 150°

6. Two mirrors are kept at 60° to each other and a body is placed at the middle. The total number of images formed is
(a) six (b) four
(c) five (d) three

7. To get three images of a single object, one should have two plane mirrors at an angle of
(a) 60° (b) 50°
(c) 30° (d) 90°

8. A ray of light is directed towards a corner reflector as shown. The incident ray makes an angle of 22° with one of the mirrors. At what angle θ does the ray emerge?



(a) 22° (b) 68°
(c) 44° (d) 46°

9. When a plane mirror is placed horizontally on level ground at a distance of 60 m from the foot of a tower, the top of the tower and its image in the mirror subtend an angle of 90° at the corner of the mirror nearer to the foot of tower. The height of the tower is
(a) 30 m (b) 60 m
(c) 90 m (d) 120 m

10. A watch shows the time as 3 : 25. What will be the time that appears when seen through a plane mirror?
(a) 8 : 35 (b) 9 : 35
(c) 7 : 35 (d) 8 : 25

Reflection from Spherical Mirror

11. A convex mirror of focal length f forms an image which is $\frac{1}{n}$ times the object. The distance of the object from the mirror is
(a) $(n-1)f$ (b) $\left[\frac{n-1}{n}\right]f$
(c) $\left[\frac{n+1}{n}\right]f$ (d) $(n+1)f$

12. The focal length of a concave mirror is 50 cm. Where an object be placed, so that its image is two times and inverted
(a) 75 cm (b) 60 cm
(c) 125 cm (d) 50 cm

13. An object of size 7.5 cm is placed in front of a convex mirror of radius of curvature 25 cm at a distance of 40 cm.The size of the image should be
(a) 2.3 cm (b) 1.78 cm
(c) 1 cm (d) 0.8 cm

14. The image formed by a convex mirror of focal length 30 cm is a quarter of the size of the object. The distance of the object from the mirror is
(a) 30 cm
(b) 90 cm
(c) 120 cm
(d) 60 cm

15. An object is placed 40 cm from a concave mirror of focal length 20 cm. The image formed is
(a) real, inverted and same in size
(b) real, inverted and smaller in size
(c) virtual, erect and larger in size
(d) virtual, erect and smaller in size

16. A concave mirror gives an image three times as large as the object placed at a distance of 20 cm from it. For the image to be real, the focal length should be
(a) 10 cm (b) 15 cm
(c) 20 cm (d) 30 cm

17. A concave mirror of focal length f (in air) is immersed in water ($\mu = 4/3$). The focal length of the mirror in water will be
(a) f (b) $\frac{4}{3}f$
(c) $\frac{3}{4}f$ (d) $\frac{7}{3}f$

18. A point object is placed at a distance of 30 cm from a convex mirror of focal length 30 cm. The image will form at
(a) infinity (b) pole
(c) focus (d) 15 cm behind the mirror

19. Convergence of concave mirror can be decreased by dipping in
(a) water (b) oil
(c) both (a) and (b) (d) None of these

20. A point object is placed at a distance of 10 cm and its real image is formed at a distance of 20 cm from a concave mirror. If the object is moved by 0.1 cm towards the mirror, the image will shift by about
(a) 0.4 cm away from the mirror
(b) 0.8 cm away from the mirror
(c) 0.4 cm towards the mirror
(d) 0.8 cm towards the mirror

21. An object is placed at a distance of 30 cm from a concave mirror and its real image is formed at a distance of 30 cm from the mirror. The focal length of the mirror is
(a) 15 cm (b) 45 cm
(c) 30 cm (d) 20 cm

22. A boy 2 m tall stands 40 cm in front of a mirror. He sees an erect image, 1 m high. The mirror is
(a) concave (b) convex
(c) plane (d) either convex or concave

23. An object is 20 cm away from a concave mirror with focal length 15 cm. If the object moves with a speed of 5 m/s along the axis, then the speed of the image will be
(a) 45 m/s (b) 27 m/s
(c) 9 m/s (d) 10 m/s

Refraction of Light

24. The refractive index of a certain glass is 1.5 for light whose wavelength in vacuum is 6000 Å. The wavelength of this light when it passes through glass is
(a) 4000 Å (b) 6000 Å
(c) 9000 Å (d) 15000 Å

25. A light wave has a frequency of 4×10^{14} Hz and a wavelength of 5×10^{-7} m in a medium. The refractive index of the medium is
(a) 1.5 (b) 1.33
(c) 1.25 (d) 1.75

26. A vessel of depth $2d$ cm is half filled with a liquid of refractive index μ_1 and the upper half with a liquid of refractive index μ_2. The apparent depth of the vessel seen perpendicularly from about is
(a) $d\left[\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right]$ (b) $d\left[\frac{1}{\mu_1}+\frac{1}{\mu_2}\right]$
(c) $2d\left[\frac{1}{\mu_1}+\frac{1}{\mu_2}\right]$ (d) $2d\left[\frac{1}{\mu_1\mu_2}\right]$

27. The refractive indices of glass and water with respect to air are 3/2 and 4/3 respectively.The refractive index of glass with respect to water will be
(a) 8/9 (b) 9/8
(c) 7/6 (d) None of these

28. If ${}_i\mu_j$ represents refractive index when a light ray goes from medium i to medium j, then the product ${}_2\mu_1 \times {}_3\mu_2 \times {}_4\mu_3$ is equal to
(a) ${}_3\mu_1$ (b) ${}_3\mu_2$
(c) $\frac{1}{{}_1\mu_4}$ (d) ${}_4\mu_2$

29. Electromagnetic radiation of frequency n, wavelength λ, travelling with velocity v in air, enters a glass slab of refractive index μ . The frequency, wavelength and velocity of light in the glass slab will be respectively
(a) $\frac{n}{\mu},\frac{\lambda}{\mu},\frac{v}{\mu}$ (b) $n,\frac{\lambda}{\mu},\frac{v}{\mu}$
(c) $n,\lambda,\frac{v}{\mu}$ (d) $\frac{n}{\mu},\frac{\lambda}{\mu},v$

30. A ray of light is incident on the surface of separation of a medium at an angle 45°and is refracted in the medium at an angle 30°. What will be the velocity of light in the medium?
(a) 1.96×10^8 m/s (b) 2.12×10^8 m/s
(c) 2.65×10^8 m/s (d) 1.24×10^8 m/s

31. Absolute refractive indices of glass and water are $\frac{3}{2}$ and $\frac{4}{3}$. The ratio of velocites of light in glass and water will be
(a) 4 : 3 (b) 9 : 8
(c) 8 : 9 (d) 3 : 4

32. An under water swimmer is at a depth of 12 m below the surface of water. A bird is at a height of 18 m from the surface of water, directly above his eyes. For the swimmer the bird appears to be at a distance from the surface of water equal to (refractive index of water is 4/3)
(a) 24 m (b) 12 m
(c) 18 m (d) 9 m

33. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of the virtual image from the surface of the sphere is
(a) 2 cm (b) 4 cm
(c) 6 cm (d) 12 cm

34. When light is refracted into a medium from vacuum
(a) its wavelength and frequency both increase
(b) its wavelength increases but frequency remains unchanged
(c) its wavelength decreases but frequency remains unchanged
(d) its wavelength and frequency both decrease

35. When light is refracted, which of the following does not change?
(a) Wavelength (b) Frequency
(c) Velocity (d) Amplitude

36. When a ray of light falls on a given plate at an angle of incidence 60°, the reflected and refracted rays are found to be normal to each other. The refractive index of the material of the plate is
(a) $\frac{\sqrt{3}}{2}$ (b) 1.5
(c) 1.732 (d) 2

37. A spot is placed on the bottom of a slab made of transparent material of refractive index 1.5. The spot is viewed vertically from the top when it seems to be raised by 2 cm. Then the height of the slab is
(a) 10 cm (b) 8 cm
(c) 6 cm (d) 4 cm

38. How does refractive index (μ) of a material vary with respect to wavelength (λ)? A and B are constants
(a) $\mu = A + \frac{B}{\lambda^2}$
(b) $\mu = A + B\lambda^2$
(c) $\mu = A + \frac{B}{\lambda}$
(d) $\mu = A + B\lambda$

39. An air bubble inside a glass slab ($\mu = 1.5$) appears 6 cm when viewed from one side and 4 cm when viewed from the opposite side. The thickness of the slab is
(a) 10 cm (b) 6.67 cm
(c) 15 cm (d) None of these

40. A piece of plane glass is placed on a word with letters of different colours. The letters which appear minimum raised are
(a) red (b) green
(c) yellow (d) violet

41. When a thin transparent plate of thickness t and refractive index μ is placed in the path of one of the two interfering waves of light, then the path difference changes by
(a) $(\mu + 1)t$ (b) $(\mu - 1)t$
(c) $(\mu + 1)/t$ (d) $(\mu - 1)/t$

42. A ray of light passes from vacuum into a medium of refractive index μ, the angle of incidence is found to be twice the angle of refraction. Then the angle of incidence is
(a) $\cos^{-1}(\mu/2)$ (b) $2\cos^{-1}(\mu/2)$
(c) $2\sin^{-1}\mu$ (d) $2\sin^{-1}(\mu/2)$

43. μ_1 and μ_2 are the refractive index of two mediums and v_1 and v_2 are the velocity of light in these two mediums respectively. Then, the relation connecting these quantities is
(a) $v_1 = v_2$ (b) $\mu_2 v_1 = \mu_1 v_2$
(c) $\mu_1^2 v_1 = \mu_2^2 v_2$ (d) $\mu_1 v_1 = \mu_2 v_2$

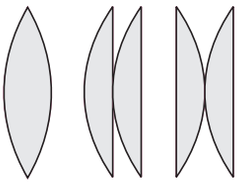
44. Three immiscible transparent liquids with refractive indices 3/2, 4/3 and 6/5 are arranged one on top of another. The depths of the liquids are 3 cm, 4 cm and 6 cm respectively. The apparent depth of the vessel is
(a) 10 cm (b) 9 cm
(c) 8 cm (d) 7 cm

Lens Problems

45. A convex lens of focal length 40 cm is in contact with a concave lens of focal length 25 cm. The power of combination is
(a) −1.5 D (b) −6.5 D
(c) +6.5 D (d) +1.5 D

46. Two lenses are placed in contact with each other and the focal length of combination is 80 cm. If the focal length of one is 20 cm, then the power of the other will be
(a) 1.66 D (b) 4.00 D
(c) –1.00 D (d) –3.75 D

47. Two similar plano-convex lenses are combined together in three different ways as shown in the adjoining figure. The ratio of the focal lengths in three cases will be

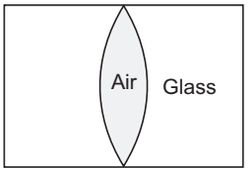


(a) 2 : 2 : 1 (b) 1 : 1 : 1
(c) 1 : 2 : 2 (d) 2 : 1 : 1

48. Two thin lenses, one of focal length +60 cm and the other of focal length −20 cm are put in contact. The combined focal length is
(a) +15 cm (b) −15 cm
(c) +30 cm (d) −30 cm

49. Distance of an object from a concave lens of focal length 20 cm is 40 cm. Then, linear magnification of the image
(a) = 1 (b) < 1 (c) < 1 (d) zero

50. In the figure, an air lens of radii of curvature 10 cm ($R_1 = R_2 = 10$ cm) is cut in a cylinder of glass $\left(\mu = \frac{3}{2}\right)$. The focal length and the nature of the lens is



(a) 15 cm, concave
(b) 15 cm, convex
(c) ∞, neither concave nor convex
(d) 0, concave

51. Two thin lenses of focal lengths f_1 and f_2 are in contact and coaxial. The combination is equivalent to a single lens of power

(a) $f_1 + f_2$ (b) $\frac{f_1 f_2}{f_1 + f_2}$
(c) $\frac{1}{2}(f_1 + f_2)$ (d) $\frac{f_1 + f_2}{f_1 f_2}$

52. A plano convex lens is made of glass of refractive index 1.5. The radius of curvature of its convex surface is R. Its focal length is

(a) $R/2$ (b) R
(c) $2R$ (d) $1.5R$

53. A concave lens of focal length f produces an image $\frac{1}{n}$ times, than that of the size of the object. The distance of the object from the lens is

(a) nf (b) $\frac{f}{n}$
(c) $(n+1)f$ (d) $(n-1)f$

54. We combined a convex lens of focal length f_1 and concave lens of focal length f_2 and their combined focal length was F. The combination of these lenses will behave like a concave lens, if

(a) $f_1 > f_2$
(b) $f_1 < f_2$
(c) $f_1 = f_2$
(d) $f_1 \leq f_2$

55. In a plano-convex lens the radius of curvature of the convex lens is 10 cm. If the plane side is polished, then the focal length will be (refractive index = 1.5)

(a) 20.5 cm (b) 10 cm
(c) 15.5 cm (d) 5 cm

56. In order to obtain a real image of magnification 2 using converging lens of focal length 20 cm, where should an object be placed

(a) 50 cm (b) 30 cm
(c) –50 cm (d) –30 cm

57. A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now the lens has been used to form the image of an object. At what distance from this lens an object be placed in order to have a real image of the size of the object?

(a) 20 cm (b) 30 cm
(c) 60 cm (d) 80 cm

58. A plano-convex lens is made of flint glass. Its focal length is

(a) inversely proportional to the wavelength of light
(b) longer for red than for blue
(c) longer for blue than for red
(d) the same for all colours

59. All of the following statements are correct except

(a) The magnification produced by a convex mirror is always less than one
(b) A virtual, erect, same sized image can be obtained by using a plane mirror
(c) A virtual, erect, magnified image can be formed using a concave mirror
(d) A real, inverted, same sized image can be formed using a convex mirror

60. Which of the following quantities increase when wavelength of light is increased? Consider only the magnitudes.

(a) The power of a converging lens
(b) The focal length of a converging lens
(c) The power of a diverging lens
(d) The focal length of a diverging lens

61. A biconvex lens has a focal length f. It is cut into two parts along a line perpendicular to principal axis. The focal length of each part will be

(a) $f/2$ (b) f (c) $(3/2)f$ (d) $2f$

62. The minimum distance between an object and its real image formed by a convex lens is

(a) $1.5f$ (b) $2f$ (c) $2.5f$ (d) $4f$

63. A plano-convex lens of curvature of 30 cm and refractive index 1.5 produces a real image of an object kept 90 cm from it. What is the magnification?

(a) 4 (b) 0.5 (c) 1.5 (d) 2

64. The real image which is exactly equal to the size of an object is to be obtained on a screen with the help of a convex lens of focal length 15 cm. For this, what must be in the distance between the object and screen?

(a) 15 cm (b) 30 cm (c) 45 cm (d) 60 cm

65. The magnification is more than unity when object is placed at a distance d from a convex lens. Its focal length is 20 cm. What is d ?

(a) Greater than 20 cm
(b) Less than 20 cm
(c) 40 cm
(d) Greater than 20 cm and less than 40 cm

66. The focal length of a converging lens is measured for violet, green and red colours. It is respectively f_v, f_g, f_r. We will find that
(a) $f_v = f_r$ (b) $f_v < f_r$ (c) $f_v > f_r$ (d) $f_g > f_r$

67. A double convex thin lens made of glass (refractive index $\mu = 1.5$) has both radii of curvature of magnitude 20 cm. Incident light rays parallel to the axis of the lens will converge at a distance L such that
(a) $L = 20$ cm (b) $L = 10$ cm
(c) $L = 40$ cm (d) $L = 20/3$ cm

68. A convex lens has a focal length of 20 cm. It is used to form an image of an object placed 15 cm from lens. The image is
(a) virtual, inverted and enlarged
(b) real, inverted and diminished
(c) real, inverted and enlarged
(d) virtual, erect and enlarged

69. At what distance from a convex lens of focal length 30 cm an object should be placed, so that the size of image be 1/4th of the object?
(a) 30 cm (b) 60 cm (c) 15 cm (d) 150 cm

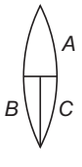
70. In a plano-convex lens the radius of curvature of convex surface is 10 cm and the focal length of the lens is 30 cm. The refractive index of the material of the lens will be
(a) 1.5 (b) 1.66 (c) 1.33 (d) 3

71. A convex lens forms an image of an object on a screen 30 cm from the lens. When the lens is moved 90 cm towards the object, the image is again formed on the screen. Then the focal length of the lens is
(a) 13 cm (b) 24 cm (c) 33 cm (d) 40 cm

72. An object has image thrice of its original size when kept at 8 cm and 16 cm from a convex lens. Focal length of the lens is
(a) less than 8 cm (b) 8 cm
(c) 16 cm (d) between 8 and 16 cm

73. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen, then
(a) half the image will disappear
(b) complete image will disappear
(c) intensity of image will increase
(d) intensity of image will decrease

74. A thin, symmetric double convex lens of power P is cut into three parts A, B and C as shown. The power of



(a) C is P (b) A is $2P$ (c) B is $\frac{P}{2}$ (d) B is $\frac{3}{2}P$

75. An object is placed at 10 cm from a lens and real image is formed with magnification of 0.5. Then, the lens is
(a) concave with focal length of 10/3 cm
(b) convex with focal length of 10/3 cm
(c) concave with focal length of 10 cm
(d) convex with focal length of 10 cm

76. A convex lens of refractive index 3/2 has a power of 2.5º. If it is placed in a liquid of refractive index 2, the new power of the lens is
(a) 2.5 D (b) – 2.5 D
(c) 1.25 D (d) – 1.25 D

77. A point source of light is placed at a distance of 2 f from a converging lens of focal length f. The intensity on the other side of the lens is maximum at a distance
(a) f
(b) between f and $2f$
(c) $2f$
(d) more than $2f$

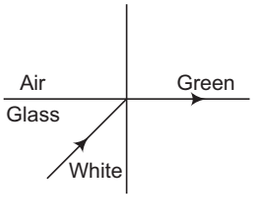
78. A point object O is placed on the principal axis of a convex lens of focal length $f = 20$ cm at a distance of 40 cm to the left of it. The diameter of the lens is 10 cm. An eye is placed 60 cm to right of the lens and a distance h below the principal axis. The maximum value of h to see the image is
(a) zero (b) 2.5 cm
(c) 5 cm (d) 10 cm

Total Internal Reflection

79. Critical angle of light passing from glass to air is minimum for
(a) red (b) green
(c) yellow (d) violet

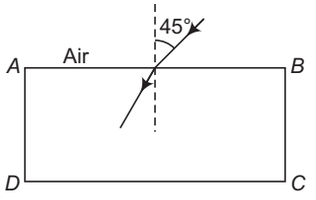
80. The wavelength of light in two liquids x and y is 3500 Å and 7000 Å respectively, then the critical angle of x relative to y will be
(a) 60° (b) 45°
(c) 30° (d) 15°

81. White light is incident on the interface of glass and air as shown in the figure. If green light is just totally internally reflected, then the emerging ray in air contains



(a) yellow, orange, red
(b) violet, indigo, blue
(c) all colours
(d) all colours except green

82. In the figure shown, for an angle of incidence 45°, at the top surface, what is the minimum refractive index needed for total internal reflection at vertical face *AD*?

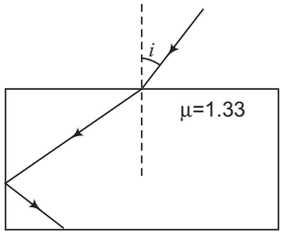


(a) $\frac{\sqrt{2}+1}{2}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\sqrt{\frac{1}{2}}$ (d) $\sqrt{2}$

83. The critical angle of a prism is 30°. The velocity of light in the medium is

(a) 1.5×10^8 m/s (b) 3×10^8 m/s
(c) 4.5×10^8 m/s (d) None of these

84. Given a slab with index $n=1.33$ and incident light striking the top horizontal face at angle i as shown in figure. The maximum value of i for which total internal reflection occurs is

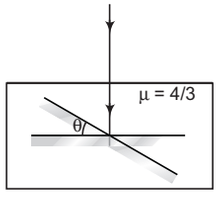


(a) $\sin^{-1}(\sqrt{0.77})$ (b) $\cos^{-1}(\sqrt{0.77})$
(c) $\sin^{-1}(0.77)$ (d) $\sin^{-1}(0.38)$

85. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45°. The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value of n from the following.

(a) 1.2 (b) 4/3 (c) 1.4 (d) 1.5

86. A plane mirror is placed horizontally inside water ($\mu = 4/3$). A ray falls normally on it. Then, mirror is rotated through an angle θ. The minimum value of θ for which ray does not come out of the water surface is



(a) $\pi/4$ (b) $\sin^{-1}\left(\frac{3}{4}\right)$
(c) $\frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)$ (d) $2\sin^{-1}\left(\frac{3}{4}\right)$

87. A glass slab has a critical angle of 30º when placed in air. What will be the critical angle when it is placed in liquid of refractive index 6/5?

(a) 45º (b) 37º
(c) 53º (d) 60º

88. A ray of light falls on a denser-rarer boundary from denser side. The critical angle is 45º. The maximum deviation the ray can undergo is

(a) 30º (b) 45º
(c) 90º (d) 120º

Prism Theory : Dispersion and Deviation

89. The dispersive powers of glasses of lenses used in an achromatic pair are in the ratio 5 : 3. If the focal length of the concave lens is 15 cm, then the nature and focal length of the other lens would be

(a) convex, 9 cm
(b) concave, 9 cm
(c) convex, 25 cm
(d) concave, 25 cm

90. When light rays are incident on a prism at an angle of 45°, the minimum deviation is obtained. If refractive index of the material of prism is $\sqrt{2}$, then the angle of prism will be

(a) 30° (b) 75°
(c) 90° (d) 60°

91. The refractive index of a prism for a monochromatic wave is $\sqrt{2}$ and its refracting angle is 60°. For minimum deviation, the angle of incidence will be

(a) 30° (b) 45°
(c) 60° (d) 75°

92. A ray of light is incident at an angle of 60° on one face of a prism of angle 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. The emergent ray is

(a) normal to the face through which it emerges
(b) inclined at 30°, to the face through which it emerges
(c) inclined at 60° to the face through which it emerges
(d) None of the above

93. In a thin prism of glass (refractive index 1.5), which of the following relations between the angle of minimum deviations δ_m and angle of prism r will be correct?

(a) $\delta_m = r$ (b) $\delta_m = 1.5r$
(c) $\delta_m = 2r$ (d) $\delta_m = \frac{r}{2}$

94. A ray of light passes through an equilateral glass prism in such a manner that the angle of incidence is equal to the angle of emergence and each of these angles is equal to 3/4 of the angle of the prism. The angle of deviation is

(a) 45° (b) 39°
(c) 20° (d) 30°

95. When light of wavelength λ is incident on an equilateral prism kept in its minimum deviation position, it is found that the angle of deviation equals the angle of the prism itself. The refractive index of the material of the prism for the wavelength λ is, then

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$

(c) 2 (d) $\sqrt{2}$

96. A thin prism P_1 with angle 6° and made from glass of refractive index 1.54 is combined with another thin prism P_2 of refractive index 1.72 to produce dispersion without deviation. The angle of prism P_2 will be

(a) 5°24′ (b) 4°30′

(c) 6° (d) 8°

97. If the refractive index of a material of equilateral prism is $\sqrt{3}$, then angle of minimum deviation of the prism is

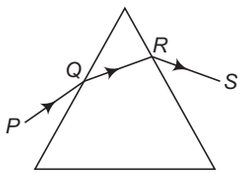
(a) 30° (b) 45° (c) 60° (d) 75°

98. Under minimum deviation condition in a prism, if a ray is incident at an angle 30°, the angle between the emergent ray and the second refracting surface of the prism is

(a) 0° (b) 30°

(c) 45° (d) 60°

99. A ray of light is incident on an equilateral glass prism placed on a horizontal table. For minimum deviation which of the following is true?



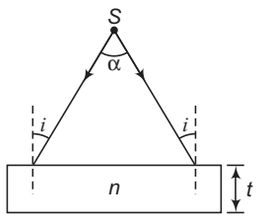
(a) PQ is horizontal

(b) QR is horizontal

(c) RS is horizontal

(d) Either PQ or RS is horizontal

100. A diverging beam of light from a point source S having divergence angle α, falls symmetrically on a glass slab as shown in figure. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and the refractive index n, then the divergence angle of the emergent beam is



(a) zero (b) α

(c) $\sin^{-1}\frac{1}{n}$ (d) $2\sin^{-1}\left(\frac{1}{n}\right)$

101. In the formation of a rainbow light from the sun on water droplets undergoes

(a) only dispersion

(b) only total internal reflection

(c) dispersion and total internal reflection

(d) None of the above

102. A narrow beam of white light goes through a slab having parallel faces

(a) the light never splits in different colours

(b) the emergent beam is white

(c) the light inside the slab is split into different colours

(d) the light inside the slab is white

103. A prism can have a maximum refracting angle of (C = critical angle for the material of the prism)

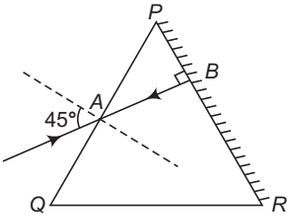
(a) 60° (b) C

(c) $2C$ (d) slightly less than 180°

104. A prism, having refractive index $\sqrt{2}$ and refracting angle 30°, has one of the refracting surfaces polished. A beam of light incident on the other refracting surface will trace its path, if the angle of incidence is

(a) 0° (b) 30° (c) 45° (d) 60°

105. The face PR of a prism PQR of angle 30° is silvered. A ray is incident on face PQ at an angle of 45° as shown in figure. The refracted ray undergoes reflection on face PR and retraces its path. The refractive index of the prism is

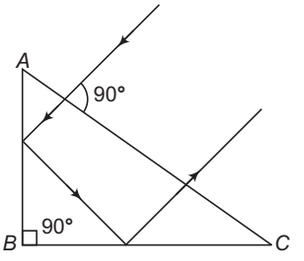


(a) $\sqrt{2}$ (b) $3/\sqrt{2}$ (c) 1.5 (d) 1.33

106. A ray is incident at an angle of incidence i on one surface of a prism of small angle A and emerges normally from opposite surface. If the refractive index of the material of prism is μ, the angle of incidence i is nearly equal to

(a) A/μ (b) $A/2\mu$ (c) μA (d) $\mu A/2$

107. A ray falls on a prism ABC ($AB = BC$) and travels as shown in figure. The minimum refractive index of the prism material should be



(a) 4/3 (b) $\sqrt{2}$ (c) 1.5 (d) $\sqrt{3}$

108. White light is passed through a prism …… colour shows maximum deviation.
(a) red (b) violet
(c) yellow (d) green

109. The angle of prism is 30°. The rays incident at 60° at one refracting face suffer a deviation at 30°. The angle of emergence is
(a) 0° (b) 30°
(c) 60° (d) 90°

110. The focal length of a thin convex lens for red and blue colours is 100.5 cm and 99.5 cm. The dispersive power of the lens is
(a) 0.01 (b) 0.02
(c) 1.005 (d) 0.995

111. When a ray of light is incident normally on one refracting surface of an equilateral prism, (refractive index of the material of the prism = 1.5)
(a) emerging ray is deviated by 30°
(b) emerging ray is deviated by 45°
(c) emerging ray just grazes the second refracting surface
(d) the ray undergoes total internal reflection at the second refracting surface

112. The angle of minimum deviation for a prism of refractive index 1.5 is equal to the angle of the prism. The angle of prism is …. (Given, $\cos 41° = 0.75$)
(a) 21° (b) 42°
(c) 60° (d) 82°

113. A ray of light passing through a prism of refracting angle 60° has to deviate by at least 30°. Then, refractive index of prism should be
(a) $\leq \sqrt{2}$ (b) $\geq \sqrt{2}$
(c) $\geq \sqrt{3}$ (d) $\leq \sqrt{3}$

114. A glass prism has a refractive angle of 90º and a refractive index of 1.5. A ray is incident at an angle of 30º. The ray emerges from an adjacent face at an angle of
(a) 60º (b) 30º
(c) 45º (d) the ray does not emerge

115. A combination is made of two lenses with focal lengths f_1 and f_2 and dispersive powers ω_1 and ω_2 respectively. The combination will be achromatic, if
(a) $\omega_1 = 2\omega_2$ and $f_1 = 2f_2$
(b) $2\omega_1 = \omega_2$ and $f_1 = 2f_2$
(c) $\omega_1 = 2\omega_2$ and $f_1 = -2f_2$
(d) $2\omega_1 = \omega_2$ and $2f_1 = f_2$

Photometry

116. If luminous efficiency of a lamp is 2 lumen/W and its luminous intensity is 42 cd, then power of the lamp is
(a) 62 W (b) 76 W
(c) 138 W (d) 264 W

117. A bulb of 100 W is hanging at a height of one meter above the centre of a circular table of diameter 4 m. If the intensity at a point on its rim is I_0, then the intensity at the centre of the table will be
(a) I_0 (b) $2\sqrt{5}I_0$
(c) $2I_0$ (d) $5\sqrt{5}I_0$

118. Lux is a unit of
(a) luminous intensity of a source
(b) illuminance on a surface
(c) transmission coefficient of a surface
(d) luminous efficiency of source of light

119. Total flux produced by a source of 1 cd is
(a) $\frac{1}{4\pi}$ (b) 8π
(c) 4π (d) $\frac{1}{8\pi}$

120. If the luminous intensity of a 100 W unidirectional bulb is 100 cd, then total luminous flux emitted from the bulb is
(a) 861 lumen (b) 986 lumen
(c) 1256 lumen (d) 1561 lumen

121. Lux is equal to
(a) 1 lumen/m^2 (b) 1 lumen/cm^2
(c) 1 cd/m^2 (d) 1 cd/cm^2

122. Five lumen/W is the luminous efficiency of a lamp and its luminous intensity is 35 cd. The power of the lamp is
(a) 80 W (b) 176 W
(c) 88 W (d) 36 W

123. A lamp rated at 100 cd hangs over the middle of a round table with diameter 3 m at a height of 2 m. It is replaced by a lamp of 25 cd and the height of the lamp is changed to 1 m so that the illumination at the centre of the table remains as before. The illumination at edge of the table becomes X times the original. Then, X is
(a) $\frac{1}{3}$ (b) $\frac{16}{27}$
(c) $\frac{1}{4}$ (d) $\frac{1}{9}$

124. A lamp is hanging 1m above the centre of circular table of diameter 1 m. The ratio of illuminances at the centre and the edge is
(a) $\frac{1}{2}$ (b) $\left(\frac{5}{4}\right)^{\frac{3}{2}}$ (c) $\frac{4}{3}$ (d) $\frac{4}{5}$

125. The intensity of direct sunlight on a surface normal to the rays is I_0. What is the intensity of direct sunlight on a surface, whose normal makes an angle of 60° with the rays of the sun?
(a) I_0 (b) $I_0\left(\frac{\sqrt{3}}{2}\right)$ (c) $\frac{I_0}{2}$ (d) $2I_0$

126. A lamp is hanging at a height of 4 m above a table. The lamp is lowered by 1 m. The percentage increase in illuminance will be
(a) 40% (b) 64%
(c) 78% (d) 92%

127. In a photometer two sources of light when placed at distances 20 cm and 30 cm respectively produce shadows of equal intensities. Their luminous intensities are in the ratio of
(a) $(2/3)^{1/2}$ (b) 2/3
(c) 4/9 (d) None of these

128. An electric bulb illuminates a plane surface. The intensity of illumination on the surface at a point 2 m away from the bulb is 5×10^{-4} phot (lumen/cm^2). The line joining the bulb to the point makes an angle of 60° with the normal to the surface. The intensity of the bulb in candela (candle power) is
(a) $40\sqrt{3}$ (b) 40
(c) 20 (d) 40×10^{-4}

Defects of Eye and Optical Instruments

129. For a normal eye, the least distance of distinct vision is
(a) 0.25 m (b) 0.50 m (c) 25 m (d) infinite

130. For the myopic eye, the defect is cured by
(a) convex lens (b) concave lens
(c) cylindrical lens (d) toric lens

131. Astigmatism (for a human eye) can be removed by using
(a) concave lens (b) convex lens
(c) cylindrical lens (d) prismatic lens

132. Image formed on the retina is
(a) real and inverted (b) virtual and erect
(c) real and erect (d) virtual and inverted

133. If there had been one eye of the man, then
(a) image of the object would have been inverted
(b) visible region would have decreased
(c) image would have not been seen three dimensional
(d) Both (b) and (c)

134. Image is formed for the short sighted person at
(a) retina (b) before retina
(c) behind the retina (d) image is not formed at all

135. The impact of an image on the retina remains for
(a) 0.1 s (b) 0.5 s
(c) 10 s (d) 15 s

136. When the power of eye lens increases, the defect of vision is produced. The defect is known as
(a) shortsightedness (b) longsightedness
(c) colourblindness (d) None of these

137. In human eye, the focusing is done by
(a) to and fro movement of eye lens
(b) to and fro movement of the retina
(c) change in the convexity of the lens surface
(d) change in the refractive index of the eye fluids

138. The exposure time of a camera lens at $\frac{f}{2.8}$ setting is $\frac{1}{200}$ s. The correct time of exposure at $\frac{f}{5.6}$ is
(a) 0.4 s (b) 0.02 s
(c) 0.002 s (d) 0.04 s

139. An object is placed at a distance u from a simple microscope of focal length f. The angular magnification obtained depends
(a) on f but not on u (b) on u but not on f
(c) on f as well as u (d) neither on f nor on u

140. The focal lengths of the objective and eye lens of a microscope are 1 cm and 5 cm respectively. If the magnifying power for the relaxed eye is 45, then the length of the tube is
(a) 30 cm (b) 25 cm
(c) 15 cm (d) 12 cm

141. Magnifying power of a simple microscope is (when final image is formed at $D = 25$ cm from eye)
(a) $\frac{D}{f}$ (b) $1+\frac{D}{f}$
(c) $1+\frac{f}{D}$ (d) $1-\frac{D}{f}$

142. The length of the compound microscope is 14 cm. The magnifying power for relaxed eye is 25. If the focal length of eye lens is 5 cm, then the object distance for objective lens will be
(a) 1.8 cm (b) 1.5 cm (c) 2.1 cm (d) 2.4 cm

143. If the focal length of objective and eye lens are 1.2 cm and 3 cm respectively and the object is put 1.25 cm away from the objective lens and the final image is formed at infinity. The magnifying power of the microscope is
(a) 150 (b) 200
(c) 250 (d) 400

144. The focal length of objective and eye lens of a microscope are 4 cm and 8 cm respectively. If the least distance of distinct vision is 24 cm and object distance is 4.5 cm from the objective lens, then the magnifying power of the microscope will be
(a) 18 (b) 32 (c) 64 (d) 20

145. The magnifying power of a microscope with an objective of 5 mm focal length is 400. The length of its tube is 20 cm. Then the focal length of the eye piece is
(a) 200 cm (b) 160 cm (c) 2.5 cm (d) 0.1 cm

146. The length of the tube of a microscope is 10 cm. The focal lengths of the objective and eye lenses are 0.5 cm and 1.0 cm. The magnifying power of the microscope is about
(a) 5 (b) 23 (c) 166 (d) 500

147. In a compound microscope, the intermediate image is
(a) virtual, erect and magnified
(b) real, erect and magnified
(c) real, inverted and magnified
(d) virtual, erect and reduced

148. If F_o and F_e are the focal lengths of the objective and eye piece respectively of a terrestrial telescope, then its magnifying power will be
(a) $F_o + F_e$ (b) $F_o \times F_e$
(c) F_o/F_e (d) $\frac{1}{2}(F_o + F_e)$

149. If the telescope is reversed i.e. seen from the objective side
(a) object will appear very small
(b) object will appear very large
(c) there will be no effect on the image formed by the telescope
(d) image will be slightly greater than the earlier one

150. The focal length of the objective of a terrestrial telescope is 80 cm and it is adjusted for parallel rays, then its magnifying power is 20. If the focal length of erecting lens is 20 cm, then full length of telescope will be
(a) 84 cm (b) 100 cm
(c) 124 cm (d) 164 cm

151. The aperture of a telescope is made large, because to
(a) increase the intensity of image
(b) decrease the intensity of image
(c) have greater magnification
(d) have lesser resolution

152. In Gallilean telescope, the final image formed is
(a) real, erect and enlarged
(b) virtual, erect and enlarged
(c) real, inverted and enlarged
(d) virtual, inverted and enlarged

153. In an astronomical telescope, the focal length of the objective lens is 100 cm and of eye piece is 2 cm. The magnifying power of the telescope for the normal eye is
(a) 50 (b) 10
(c) 100 (d) $\frac{1}{50}$

154. The focal lengths of the objective and eye lenses of a telescope are respectively 200 cm and 5 cm. The maximum magnifying power of the telescope will be
(a) -40 (b) -48
(c) -60 (d) -100

155. The number of lenses in a terrestrial telescope is
(a) two (b) three
(c) four (d) six

156. The focal lengths of the lenses of an astronomical telescope are 50 cm and 5 cm. The length of the telescope when the image is formed at the least distance of distinct vision is
(a) 45 cm (b) 55 cm (c) $\frac{275}{6}$ cm (d) $\frac{325}{6}$ cm

157. Correct exposure for a photographic print is 10 s at a distance of one metre from a point source of 20 cd. For an equal fogging of the print placed at a distance of 2 m from a 16 cd source, the necessary time for exposure is
(a) 100 s (b) 25 s
(c) 50 s (d) 75 s

158. A normal eye is not able to see objects closer than 25 cm because
(a) the focal length of the eye is 25 cm
(b) the distance of the retina from the eye lens is 25 cm
(c) the eye is not able to increase the focal length beyond a limit
(d) the eye is not able to decrease the focal length beyond a limit

159. When objects at different distances are seen by the eye, which of the following remain constant?
(a) The focal length of the eye lens
(b) The object distance from the eye lens
(c) The radii of curvature of the eye lens
(d) The image distance from the eye lens

160. The focal length of a normal eye lens is about
(a) 1 mm (b) 2 cm (c) 25 cm (d) 1 m

161. For a telescope, larger the diameter of the objective
(a) smaller is the resolving power
(b) greater is the magnifying power
(c) smaller is the magnifying power
(d) greater is the resolving power

162. If tube length of astronomical telescope is 105 cm and magnifying power is 20 for normal setting, calculate the focal length of objective
(a) 100 cm (b) 10 cm (c) 20 cm (d) 25 cm

163. Myopia is due to
(a) elongation of eye ball
(b) irregular change in focal length
(c) shortening of eye ball
(d) older age

164. The minimum magnifying power of telescope is M. If the focal length of its eye lens is halved, the magnifying power will become
(a) $M/2$ (b) $2M$ (c) $3M$ (d) $4M$

165. A telescope has a objective of focal length 50 cm and an eye piece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focussed for distinct vision on a scale 200 cm away. The separation between the objective and the eye piece is
(a) 75 cm (b) 60 cm
(c) 71 cm (d) 74 cm

166. When an astronomical telescope is focussed on a distant star, the distance of the eye piece from the objective is 60 cm. When focussed on a distant flag pole, the eye piece must be drawn out 10 cm. If the focal length of the eye piece is 5 cm, what will be the distance of the pole from the objective? Assume that the eye is focussed for infinity.
(a) 300 cm (b) 30 cm
(c) 320 cm (d) None of these

Miscellaneous Problems

167. When light wave suffers reflection at the interface from air to glass, the change in phase of the reflected wave is equal to
(a) zero (b) $\frac{\pi}{2}$
(c) π (d) 2π

168. A man has height 6 m. He observes image of 2 m height erect, then mirror used is
(a) concave (b) convex
(c) plane (d) None of these

169. The field of view is maximum for
(a) plane mirror
(b) concave mirror
(c) convex mirror
(d) cylindrical mirror

170. A virtual image larger than the object can be obtained by
(a) concave mirror (b) convex mirror
(c) plane mirror (d) concave lens

171. An astronaut in a spaceship see the outer space as
(a) white (b) black
(c) blue (d) red

172. The reason for shining of air bubble in water is
(a) diffraction of light
(b) dispersion of light
(c) scattering of light
(d) total internal reflection of light

173. Our eye is most sensitive for which of the following wavelength?
(a) 4500 Å
(b) 5500 Å
(c) 6500 Å
(d) equally sensitive for all wavelengths of visible spectrum

174. What will be the colour of sky as seen from the earth, if there were no atmosphere?
(a) Black (b) Blue
(c) White (d) Red

175. The splitting of white light into several colours on passing through a glass prism is due to
(a) refraction (b) reflection
(c) interference (d) diffraction

176. The sky would appear red instead of blue, if
(a) atmospheric particles scatter blue light more than the red light
(b) atmospheric particles scatter all colours equally
(c) atmospheric particles scatter red light more than the blue light
(d) the sun was much hotter

177. A school boy has a red badge with his name written in white. When seen through a red glass filter the badge appears
(a) black, the name appears red
(b) red, the name appears black
(c) red, the name appears red
(d) black, the name appears black

178. Which of the following can form a virtual, erect and magnified image?
(a) Plane mirror (b) Concave mirror
(c) Convex mirror (d) Concave lens

179. Match List I (Phenomenon) with List II (Principle) and select the correct answer using the codes given below the lists.

List I (Phenomenon)	List II (Principle)
I. Blue colour of a sky	(A) Total internal reflection
II. Glittering of diamond	(B) Dispersion of light
III. Formation of rainbow	(C) Scattering of light
IV. In the evening when the sun goes down below the horizon, it continues to remain visible for some time	(D) Refraction of light

Codes
(a) I-C, II-A, III-B, IV-D
(b) I-C, II-A, III-D, IV-B
(c) I-A, II-C, III-B, IV-D
(d) I-A, II-C, III-D, IV-B

180. A convex and a concave mirror of radii 10 cm each are placed facing each other and 15 cm apart. An object is placed exactly between them. If the reflection first takes place in concave and then in convex mirror, the position of the final image will be
(a) 7 cm behind concave mirror
(b) at the pole of the concave mirror
(c) at the pole of the convex mirror
(d) 6.7 cm in front of concave mirror

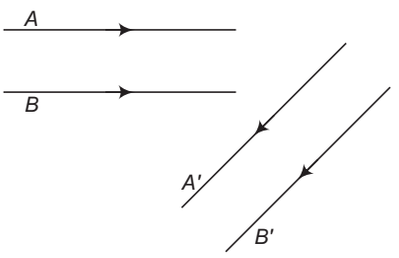
181. When the bodies placed in a dark room are exposed to X-rays, they appear
(a) yellow (b) red
(c) blue (d) invisible

182. The work of Prof. Raman for which he was awarded Nobel Prize was concerned with
(a) nuclear energy (b) scattering of light
(c) lasers (d) transistors

183. A double convex glass lens ($R_1 = R_2 = 10$ cm) having focal length equal to the focal length of a concave mirror. The radius of curvature of the concave mirror is ($\mu_g = 1.5$)
(a) 10 cm (b) 20 cm (c) 40 cm (d) 15 cm

184. Rainbow is observed when the sun is
(a) in front of the observer
(b) behind the observer
(c) vertically above the observer
(d) in any of these positions

185. Figure shows two rays A and B being reflected by a mirror and going as A' and B'. The mirror



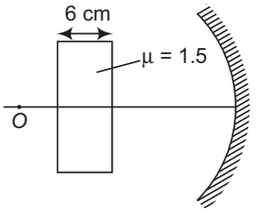
(a) is plane
(b) is convex
(c) is concave
(d) may be any spherical mirror

[Level 2]

Only One Correct Option

1. A ray incident at a point as an angle of incidence of 60° enters a glass sphere of refractive index $n = \sqrt{3}$ and is reflected and refracted at the further surface of the sphere. The angle between the reflected and refracted rays at this surface is
(a) 50° (b) 60°
(c) 90° (d) 40°

2. A slab of glass, of thickness 6 cm and refractive index $\mu = 1.5$ is placed in front of a concave mirror, as shown in the figure. If the radius of curvature of the mirror is 40 cm and the reflected image coincides with the object, then the distance of the object from the mirror is



(a) 30 cm (b) 22 cm
(c) 42 cm (d) 38 cm

3. An object is placed at a distance u from an equiconvex lens such that the distance between the object and its real image is minimum. The focal length of the lens is f. The value of u is
(a) ∞ (b) $1.5f$
(c) $2f$ (d) $4f$

4. If the distances of an object and its virtual image from the focus of a convex lens of focal length f are 1 cm each, then f is
(a) 4 cm
(b) $(\sqrt{2}+1)$ cm
(c) $2\sqrt{2}$ cm
(d) $(2+\sqrt{2})$ cm

5. When an object is at distance x and y from a lens, a real image and a virtual image is formed respectively having same magnification. The focal length of the lens is
(a) $\frac{x+y}{2}$
(b) $x - y$
(c) $\sqrt{xy}$
(d) $x + y$

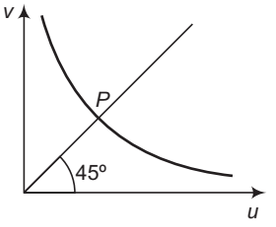
6. A convex lens of focal length 30 cm forms a real image three times larger than the object on a screen. Object and screen are moved until the image becomes twice the size of the object. If the shift of the object is 6 cm. The shift of screen is
(a) 28 cm
(b) 14 cm
(c) 18 cm
(d) 16 cm

7. In a plano convex lens of the radius of curvature of the convex lens is 10 cm. If the plane side is polished, then the focal length will be (Refractive index = 1.5)
(a) 20.5 cm
(b) 10 cm
(c) 15.5 cm
(d) 5 cm

8. A thin rod of length $f/3$ lies along the axis of a concave mirror of focal length f. One end of its magnified image touches an end of the rod. The length of the image is
(a) f
(b) $\frac{1}{2}f$
(c) $2f$
(d) $\frac{1}{4}f$

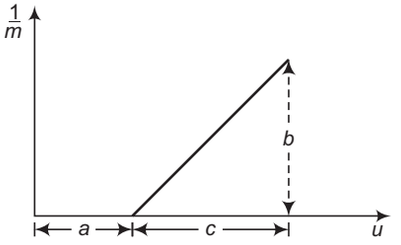
9. A concave mirror is placed at the bottom of an empty tank with face upwards and axis vertical. When sunlight falls normally on the mirror, it is focused at distance of 32 cm from the mirror. If the tank filled with water $\left(\mu = \frac{4}{3}\right)$ upto a height of 20 cm, then the sunlight will now get focused at
(a) 16 cm above water level
(b) 9 cm above water level
(c) 24 cm below water level
(d) 9 cm below water level

10. The graph shows part of variation of v with change in u for a concave mirror. Points plotted above the point P on the curve are for values of v



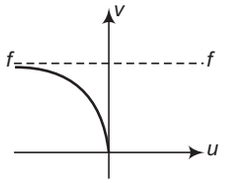
(a) smaller than f
(b) smaller than $2f$
(c) larger than $2f$
(d) larger than f but less than $2f$

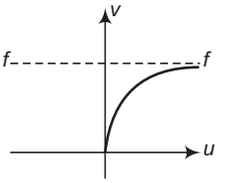
11. The graph shows how the inverse of magnification $\frac{1}{m}$ produced by a convex thin lens varies with object distance u. What was the focal length of the lens used?

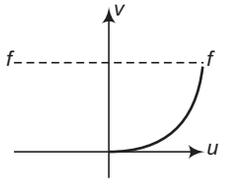


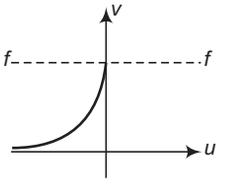
(a) $\frac{b}{c}$ (b) $\frac{b}{ca}$ (c) $\frac{bc}{a}$ (d) $\frac{c}{b}$

12. The graph between u and v for a convex mirror is

(a)


(b)


(c)


(d)


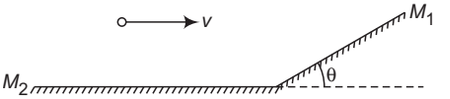
13. A concave lens of focal length 20 cm placed in contact with a plane mirror acts as a
(a) convex mirror of focal length 10 cm
(b) concave mirror of focal length 40 cm
(c) concave mirror of focal length 60 cm
(d) concave mirror of focal length 10 cm

14. A convex lens produces an image of a real object on a screen with a magnification of $\frac{1}{2}$. When the lens is moved 30 cm away from the object, the magnification of the image on the screen is 2. The focal length of the lens is
(a) 30 cm (b) 60 cm
(c) 20 cm (d) 15 cm

15. An infinitely long rod lies along the axis of a concave mirror of focal length f. The near end of the rod is at a distance $u > f$ from the mirror. Its image will have a length
(a) $\frac{f^2}{u-f}$ (b) $\frac{uf}{u-f}$
(c) $\frac{f^2}{u+f}$ (d) $\frac{uf}{u+f}$

16. A liquid is placed in a hollow prism of angle 60°. If angle of the minimum deviation is 30°, what is the refractive index of the liquid?
(a) $\sqrt{2}$ (b) 1.50
(c) $\frac{4}{3}$ (d) $\frac{5}{4}$

17. A point object is moving with a speed v before an arrangement of two mirrors as shown in figure.

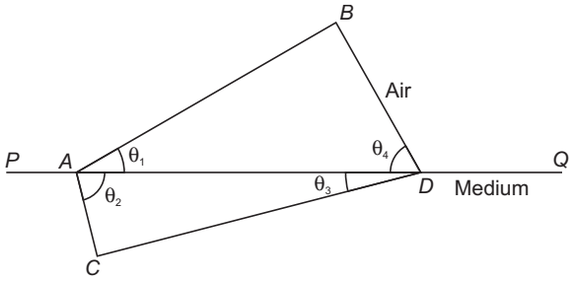


Find the velocity of image in morror M_1 with respect to image in mirror M_2.
(a) $2v \sin\theta$ (b) $v \sin\theta$
(c) $2v \cos\theta$ (d) $v \cos\theta$

18. How much water should be filled in a container of height 21 cm, so that it appears half filled to the observer when viewed from the top of the container ($\mu = 4/3$)?
(a) 8 cm (b) 10.5 cm
(c) 12 cm (d) 14 cm

19. Optic axis of a thin equi-convex lens is the x-axis. The coordinates of a point object and its image are (−40 cm, 1 cm) and (50 cm, −2 cm) respectively. Lens is located at
(a) $x = 20$ cm
(b) $x = -30$ cm
(c) $x = -10$ cm
(d) origin

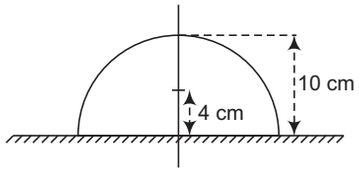
20. The adjoining figure represents a wavefront AB which passes from air to another transparent medium and produces a new wavefront CD after refraction.



The refractive index of the medium is (PQ is the boundary between air and the medium)

(a) $\frac{\cos\theta_1}{\cos\theta_3}$ (b) $\frac{\cos\theta_4}{\cos\theta_1}$ (c) $\frac{\sin\theta_1}{\sin\theta_3}$ (d) $\frac{\sin\theta_2}{\sin\theta_3}$

21. A hemispherical paper weight contains a small flower on its axis of symmetry at a distance of 4 cm from its flat surface. Where is the flower appears to an observer when he looks at it along the axis of symmetry from the top? (Index of refraction of glass = 1.5)



(a) 15 cm from top (b) 20 cm from top
(c) 5 cm from top (d) 25 cm from top

22. A thin plano convex lens acts like a concave mirror of radius of curvature 20 cm when its plane surface is silvered. The radius of curvature of the curved surface if index of refraction of its material is 1.5 will be

(a) 40 cm (b) 30 cm (c) 10 cm (d) 20 cm

23. A ray of light, travelling in a medium of refractive index μ, is incident at an angle i on a composite transparent plate consisting of three plates of refractive indices μ_1, μ_2 and μ_3. The ray emerges from the composite plate into a medium of refractive index μ_4, at angle x. Then,

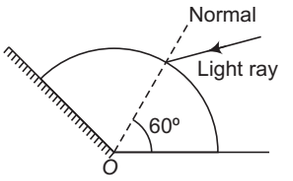
(a) $\sin x = \sin i$ (b) $\sin x = \frac{\mu}{\mu_4}\sin i$

(c) $\sin x = \frac{\mu_4}{\mu}\sin i$ (d) $\sin x = \frac{\mu_1}{\mu_2}\frac{\mu_3}{\mu_2}\frac{\mu}{\mu_4}\sin i$

24. The x-z plane separates two media A and B with refractive indices μ_1 and μ_2 respectively. A ray of light travels from A and B. Its directions in the two media are given by the unit vectors $\hat{\mathbf{r}}_A = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ and $\hat{\mathbf{r}}_B = \alpha\hat{\mathbf{i}} + \beta\hat{\mathbf{j}}$ respectively, where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors in the x and y directions. Then,

(a) $\mu_1 a = \mu_2\alpha$ (b) $\mu_1\alpha = \mu_2 a$
(c) $\mu_1 b = \mu_2\beta$ (d) None of these

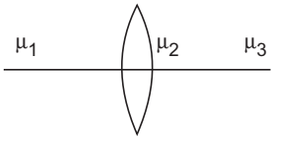
25. Consider the situation as shown in figure. The point O is the centre. The light ray forms an angle of 60° with the normal.



The normal makes an angle 60° with the horizontal and the mirror makes an angle 60° with the normal. The value of refractive index of that spherical portions so that light ray retraces its path is

(a) $\sqrt{2}$ (b) $\frac{2}{\sqrt{3}}$
(c) $\frac{3}{2}$ (d) $\sqrt{3}$

26. The figure shows an equiconvex lens. What should be the condition on the refractive indices so that the lens becoming diverging?



(a) $2\mu_3 > \mu_1 - \mu_2$
(b) $2\mu_2 < \mu_1 + \mu_3$
(c) $2\mu_2 > 2\mu_1 - \mu_3$
(d) None of the above

27. An object is kept at a distance of 16 cm from a thin lens and the image formed is real. If the object is kept at a distance of 6 cm from the same lens the image formed is virtual. If the size of the image formed are equal, the focal length of the lens will be

(a) 19 cm (b) 17 cm
(c) 21 cm (d) 11 cm

28. The apparent depth of water in cylindrical water tank of diameter $2R$ cm is reducing at the rate of x cm/min when water is being drained out at a constant rate. The amount of water drained in cc/min is (n_1 = refractive index of air, n_2 = refractive index of water)

(a) $\frac{x\pi R^2 n_1}{n_2}$ (b) $\frac{x\pi R^2 n_2}{n_1}$
(c) $\frac{2\pi R n_1}{n_2}$ (d) $\pi R^2 x$

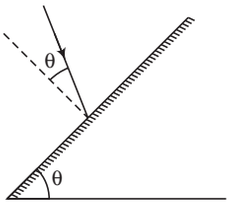
29. A ray of light strikes a horizontal plane mirror at an angle of 45°. A second plane mirror is attached at an angle θ with it. If ray after reflection from second mirror runs parallel to the first mirror, then θ is

(a) 45° (b) 60°
(c) 67.5° (d) 135°

30. A ray of light makes an angle of 10° with the horizontal above it and strikes a plane mirror which is inclined at an angle θ to the horizontal. The angle θ for which the reflected ray becomes vertical is
(a) 40° (b) 50°
(c) 80° (d) 100°

31. A plane mirror is placed along the y-axis such that x-axis is normal to the plane of the mirror. The reflecting surface of the mirror is towards negative x-axis. The mirror moves in positive x-direction with uniform speed of 5 m/s and a point object P is moving with constant speed 3 m/s in negative x-direction. The speed of image with respect to mirror is
(a) 8 m/s (b) 2 m/s
(c) 4 m/s (d) 16 m/s

32. A mirror is inclined at an angle of θ with the horizontal. If a ray of light is incident at an angle of incidence θ, then the angle makes by the reflected ray with makes horizontal is



(a) θ (b) 2θ
(c) $\frac{\theta}{2}$ (d) None of these

33. In a lake, a fish rising vertically to the surface of water uniformly at the rate of 3 m/s, observes a bird diving vertically towards the water at the rate of 9 m/s. The actual velocity of the dive of the bird is (Given, refractive index of water = 4/3)
(a) 3.6 m/s
(b) 4.5 m/s
(c) 6.0 m/s
(d) 12.0 m/s

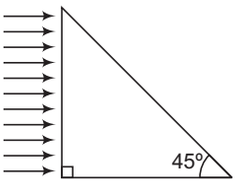
34. An equilateral prism deviates a ray through 45° for the two angles of incidence differing by 20°. The angle of incidence is
(a) 62.5°
(b) 42.5°
(c) Both are correct
(d) Both are wrong

35. A wavefront is represented by the plane $y = 3 - x$. The propagation of wave takes place at
(a) 45° with the +ve x-direction
(b) 30° with +ve x-direction
(c) 60° with the +ve x-direction
(d) No sufficient data

36. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror, what is the size of image?
(a) $\left(\frac{f}{u-f}\right) b$ (b) $\left(\frac{f}{u-f}\right)^2 b$
(c) $\left(\frac{f}{u-f}\right) b^2$ (d) $\left(\frac{f}{u-f}\right)$

37. A circular beam of light (diameter = d) falls on a plane surface of a liquid. The angle of incidence is 45° and refractive index of the liquid is μ. The diameter of the refracted beam is
(a) d (b) $(\mu - 1)\, d$
(c) $\frac{\sqrt{2\mu^2 - 1}}{\mu} d$ (d) $\frac{\sqrt{\mu^2 - 1}}{\mu} d$

38. A beam of light consisting of red, green and blue colours is incident on a right angled prism. The refractive indices of the material of prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will



(a) separates part of the red colour from the green and blue colours
(b) separates part of the blue colour from the red and green colours
(c) separate all the three colours from one another
(d) not separate even partially any colour from the other two colours.

39. A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is
(a) 7.5° (b) 5°
(c) 15° (d) 2.5°

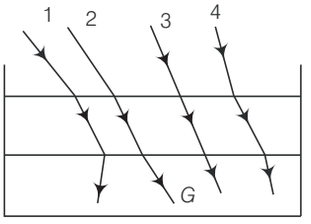
40. A short pulse of white light is incident from air to a glass slab at normal incidence. After travelling through the slab, the first colour to emerge is
(a) blue (b) green
(c) violet (d) red

41. An object approaches a convergent lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image
(a) moves away from the lens with an uniform speed 5 m/s
(b) moves away from the lens with an uniform acceleration
(c) moves away from the lens with a non-uniform acceleration
(d) moves towards the lens with a non-uniform acceleration

42. You are given four sources of light each one providing a light of a single colour-red, blue, green and yellow. Suppose the angle of refraction for a beam of yellow light corresponding to a particular angle of incidence at the interface of two media is 90°. Which of the following statements is correct if the source of yellow light is replaced with that of other lights without changing the angle of incidence?

(a) The beam of red light would undergo total internal reflection
(b) The beam of red light would bend towards normal while it gets refracted through the second medium
(c) The beam of blue light would undergo total internal reflection
(d) The beam of green light would bend away from the normal as it gets refracted through the second medium

43. The optical density of turpentine is higher than that of water while its mass density is lower. Figure shows a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in figure, the path shown is correct?



(a) 1 (b) 2
(c) 3 (d) 4

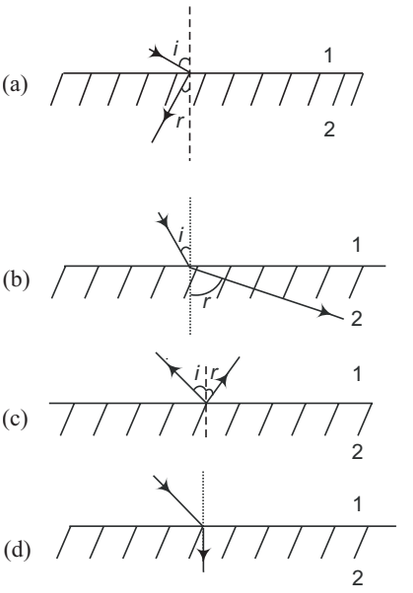
44. A car is moving with at a constant speed of 60 kmh^{-1} on a straight road. Looking at the rear view mirror, the driver finds that the car following him is at a distance of 100 m and is approaching with a speed of 5 kmh^{-1}.

In order to keep track of the car in the rear, the driver begins to glance alternatively at the rear and side mirror of his car after every 2 s till the other car overtakes. If the two cars were maintaining their speeds, which of the following statement (s) is/are correct?

(a) The speed of the car in the rear is 65 km h^{-1}
(b) In the side mirror, the car in the rear would appear to approach with a speed of 5 kmh^{-1} to the driver of the leading car
(c) In the rear view mirror, the speed of the approaching car would appear to decrease as the distance between the cars decreases
(d) In the side mirror, the speed of the approaching car would appear to increase as the distance between the cars decreases

45. There are certain material developed in laboratories which have a negative refractive index figure.

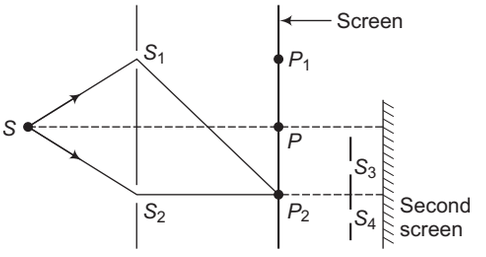
A ray incident from air (medium 1) into such a medium (medium 2) shall follow a path given by



46. In a Young's double-slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case,

(a) there shall be alternate interference patterns of red and blue
(b) there shall be an interference pattern for red distinct from that for blue
(c) there shall be no interference fringes
(d) there shall be an interference pattern for red mixing with one for blue

47. Figure shows a standard two slit arrangement with slits S_1, S_2, P_1, P_2 are the two minima points on either side of P (figure).



At P_2 on the screen, there is a hole and behind P_2 is a second 2-slit arrangement with slits S_3, S_4 and a second screen behind them.

(a) There would be no interference pattern on the second screen but it would be lighted
(b) The second screen would be totally dark
(c) There would be a single bright point on the second screen
(d) There would be a regular two slit pattern on the second screen

More than One Correct Options

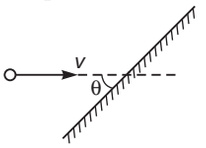
1. The image formed by a concave mirror is twice the size of the object. The focal length of the mirror is 20 cm. The distance of the object from the mirror is/are

(a) 10 cm (b) 30 cm
(c) 25 cm (d) 15 cm

2. Magnitude of focal length of a spherical mirror is f and magnitude of linear magnification is $\frac{1}{2}$. There

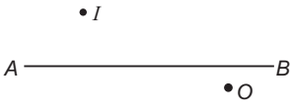
(a) if image is inverted it is a concave mirror
(b) if image is erect, it is a convex mirror
(c) object distance from the mirror may be $3f$
(d) object distance from the mirror may be f

3. A point object is moving towards a plane mirror as shown in figure.
Choose the correct options.



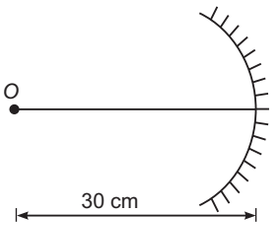
(a) Speed of image is also v
(b) Image velocity will also make an angle θ with mirror
(c) Relative velocity between object and image is $2v$
(d) Relative velocity between object and image is $2v \sin\theta$

4. AB is the principal axis of a spherical mirror. I is the point image corresponding to point object O. Choose the correct options.



(a) Mirror is lying to the right hand side of O
(b) Focus of mirror is lying to the right hand side of O
(c) Centre of curvature of mirror is lying to the right hand side of O
(d) Centre of curvature of mirror is lying between I and O

5. A point object is placed on the principal axis of a concave mirror of focal length −20 cm. At this instant object is given a velocity v towards the axis (event-1) or perpendicular to axis (event-2). Then, speed of image

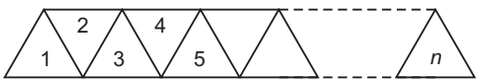


(a) In event-1 is $2v$ (b) In event-1 is $4v$
(c) In event-2 is $2v$ (d) In event-2 is $4v$

6. A point object is placed at equal distance $3f$ infront of a concave mirror, a convex mirror and a plane mirror separately (event-1). Now, the distance is decreased to $1.5f$ from all the three mirrors (event-2). Magnitude of focal length of convex mirror and concave mirror is f. Then choose the correct options.

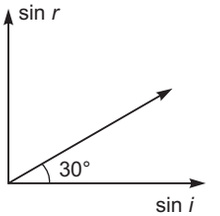
(a) Maximum distance of object in event-1 from the mirror is from plane mirror
(b) Minimum distance of object in event-1 from the mirror is from convex mirror
(c) Maximum distance of object in event-2 from the mirror is from concave mirror
(d) Minimum distance of object in event-2 from the mirror is from plane mirror

7. n number of identical equilateral prisms are kept in contact as shown in figure. If deviation through a single prism is δ. Then, (n, m are integers)



(a) if $n = 2m$, deviation through n prisms is zero
(b) if $n = 2m + 1$, deviation through system of n prisms is δ
(c) if $n = 2m$, deviation through system of n prisms is δ
(d) if $n = 2m + 1$, deviation through system of n prisms is zero

8. A ray of monochromatic light is incident on the plane surface of separation between two media x and y with angle of incidence i in the medium x and angle of refraction r in the medium y. The graph shows the relation between sin i and sin r.



(a) The speed of light in the medium y is $\sqrt{3}$ times than in medium x
(b) The speed of light in the medium y is $\frac{1}{\sqrt{3}}$ times than in medium x
(c) The total internal reflection can take place when the incidence is in x
(d) The total internal reflection can take place when the incidence is in y

9. Which of the following statements is/are true?

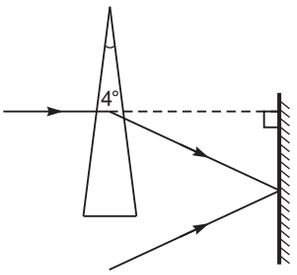
(a) In vacuum, the speed of red colour is more than that of violet color
(b) An object in front of a mirror is moved towards the pole of a spherical mirror from infinity, it is found that image also moves towards the pole. The mirror must be convex
(c) There exist two angles of incidence in a prism for which angles of deviation are same except minimum deviation
(d) A ray travels from a rarer medium to denser medium. There exist three angles of incidence for which the deviation is same

10. A lens of focal length f is placed in between an object and screen at a distance D. The lens forms two real images of object on the screen for two of its different positions, a distance x apart. The two real images have magnifications m_1 and m_2 respectively $(m_1 > m_2)$. Choose the correct statement(s).

(a) $m_1 m_2 = -1$ (b) $m_1 m_2 = 1$

(c) $f = \dfrac{D^2 - x^2}{4D}$ (d) $D \geq 4f$

11. A small angled prism of apex angle $A = 4°$ and refractive index $\mu = 1.5$ is placed in front of a vertical plane mirror as shown in figure. If the mirror is rotated through an angle θ, then the light ray becomes horizontal either after the mirror or after second time from the prism. The value of θ is



(a) 1° (b) 2°

(c) 4° (d) Not possible

12. Consider an extended object immersed in water contained in a plane trough. When seen from close to the edge of the trough the object looks distorted because

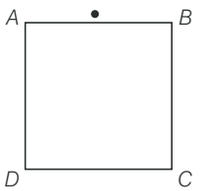
(a) the apparent depth of the points close to the edge are nearer the surface of the water compared to the points away from the edge

(b) the angle subtended by the image of the object at the eye is smaller than the actual angle subtended by the object in air

(c) some of the points of the object far away from the edge may not be visible because of total internal reflection

(d) water in a trough acts as a lens and magnifies the object

13. A rectangular block of glass $ABCD$ has a refractive index 1.6. A pin is placed midway on the face AB figure. When observed from the face AD, the pin shall



(a) appear to be near A

(b) appear to be near D

(c) appear to be at the centre of AD

(d) not be seen at all

14. A magnifying glass is used, as the object to be viewed can be brought closer to the eye than the normal near point. This results in

(a) a larger angle to be subtended by the object at the eye and hence, viewed in greater detail

(b) the formation of a virtual erect image

(c) increase in the field of view

(d) infinite magnification at the near point

15. An astronomical refractive telescope has an objective of focal length 20m and an eyepiece of focal length 2 cm.

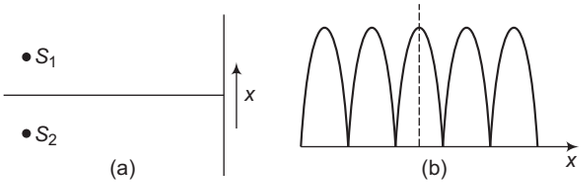
(a) The length of the telescope tube is 20.02 m

(b) The magnification is 1000

(c) The image formed is inverted

(d) An objective of a larger aperture will increase the brightness and reduce chromatic aberration of the image

16. Two sources S_1 and S_2 of intensity I_1 and I_2 are placed in front of a screen [Fig. (a)]. The pattern of intensity distribution seen in the central portion is given by Fig. (b).



In this case, which of the following statements are true?

(a) S_1 and S_2 have the same intensities

(b) S_1 and S_2 have a constant phase difference

(c) S_1 and S_2 have the same phase

(d) S_1 and S_2 have the same wavelength

17. For light diverging from a point source,

(a) the wavefront is spherical

(b) the intensity decreases in proportion to the distance squared

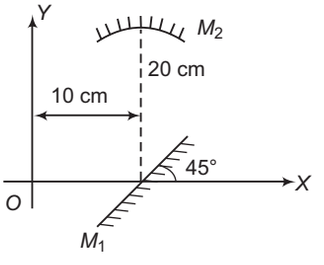
(c) the wavefront is parabolic

(d) the intensity at the wavefront does not depend on the distance

Comprehension Based Questions

Passage I (Q. 1 to 4)

A plane mirror (M_1) and a concave mirror (M_2) of focal length 10 cm are arranged as shown in figure. An object is kept at origin. Answer the following questions. (consider image formed by single reflection in all cases)



1. The co-ordinates of image formed by plane mirror are

(a) (– 20 cm, 0) (b) (10 cm, – 60 cm)

(c) (10 cm, – 10 cm) (d) (10 cm, 10 cm)

2. The co-ordinates of image formed by concave mirror are

(a) (10 cm, – 40 cm)

(b) (10 cm, – 60 cm)

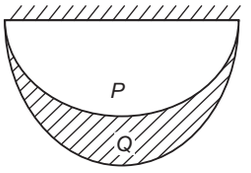
(c) (10 cm, 8 cm)

(d) None of the above

3. If concave mirror is replaced by convex mirror of same focal length, then coordinates of image formed by M_2 will be
(a) (10 cm, 12 cm) (b) (10 cm, 22 cm)
(c) (10 cm, 8 cm) (d) None of these

4. If concave mirror is replaced by another plane mirror parallel to x-axis, then co-ordinates of image formed by M_2 are
(a) (40 cm, 20 cm) (b) (20 cm, 40 cm)
(c) (– 20 cm, 20 cm) (d) None of these

Passage II (Q. 5 to 7)

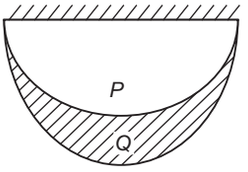


A plano-convex lens P and a concavo-convex lens Q are in contact as shown in figure. The refractive index of the material of the lens P and Q is 1.8 *and* 1.2 *respectively. The radius of curvature of the concave surface of the lens Q is double the radius of curvature of the convex surface. The convex surface of Q is silvered.*

5. An object is placed on the principal axis at a distance 10 cm from the plane surface. The image is formed at a distance 40 cm from the plane surface on the same side. The focal length of the system is
(a) – 8 cm (b) 8 cm (c) $-\frac{40}{3}$ cm (d) $\frac{40}{3}$ cm

6. The radius of curvature of common surface is
(a) 48 cm (b) 24 cm (c) 12 cm (d) 8 cm

7. If the plane surface of P is silvered as shown in figure, the system acts as



(a) convex mirror of focal length 24 cm
(b) concave mirror of focal length 8 cm
(c) concave mirror of focal length 24 cm
(d) convex mirror of focal length 8 cm

Assertion and Reason

Directions (Q. Nos. 1-20) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** In refraction from a plane surface if object is virtual its image will be real.
Reason Plane surface always makes opposite natured image. If object is real, image is virtual and *vice-versa*.

2. **Assertion** If a lens is immersed in a liquid its nature will change i.e. convex will behave concave and *vice-versa*.
Reason If both sides of a lens medium is same, then object can be placed on either side of the lens, image distance remains same.

3. **Assertion** Lens formula can be applied only for thin lenses.
Reason For thick lenses one cannot find image position.

4. **Assertion** If an object is placed at the centre of a glass sphere and it is observed from outside. Then, it will appear at centre itself.
Reason All rays starting from object fall normal on the surface. Hence they do not bend.

5. **Assertion** Shining of diamond is due to the phenomenon of total internal reflection.
Reason Refractive index of diamond is large. Hence critical angle is small.

6. **Assertion** Sun appears red during sunrise and sunset due to the phenomenon of refraction.
Reason In refraction of light frequency of light remains unchanged.

7. **Assertion** If an object is placed at a distance of 10 cm from a convex mirror of focal length 10 cm. Then, its image is formed at infinity.
Reason If an object is placed at the focus of a mirror its image is formed at infinity.

8. **Assertion** Although the surfaces of goggle lens are curved, it does not have any power.
Reason In case of goggles, both the curved surfaces have equal radii of curvature and have centre of curvature of the same side.

9. **Assertion** Image formed by a convex mirror can never be real.
Reason Convex mirror is diverging in nature.

10. **Assertion** A diverging lens (in air) cannot be made more diverging whatever be the medium we choose to completely immerse the lens.
Reason The minimum refractive index of any medium is 1.

11. Assertion When a wave travels from a denser to a rear medium its amplitude increases.

Reason In a rarer medium speed of wave is more.

12. Assertion In Snell's law $\mu = \frac{\sin i}{\sin r}$, angle i is always angle of incidence.

Reason $\mu = \frac{\sin i}{\sin r}$ is applicable one medium in air (or vacuum).

13. Assertion Reflected image always travels in opposite direction of object.

Reason Speed of point image formed by reflected rays is always equal to the speed of its point object.

14. Assertion A hollow lens behaves like a thin glass plate.

Reason Power of this lens becomes zero.

15. Assertion A ray of light travels from first medium to second medium. In first medium angle of ray with normal is $i_1 = 30°$ and in second medium $i_2 = 60°$. Then, second medium is rarer medium.

Reason Refractive index of first medium with respect to second medium is $\frac{\sin 30°}{\sin 60°}$.

16. Assertion Focal length of a lens depends on the wavelength of light used.

Reason The more the wavelengths, lesser is the focal length.

17. Assertion In front of a concave mirror a point object is placed between focus and centre of curvature. If a glass slab is placed between object and mirror, then image from mirror may become virtual.

Reason Glass slab always makes a virtual image of a real object.

18. Assertion Convex mirror will always make virtual image.

Reason Convex mirror is diverging in nature. Its power is negative.

19. Assertion If angle of incidence in case of a prism is gradually increased, then deviation produced by prism will first decrease then increase.

Reason At minimum deviation, $r_1 = \frac{A}{2}$.

20. Assertion A mirror has only one focus, while that of less has two focii.

Reason Both the focii of a mirror coincide at one point.

Match the Columns

1. Regarding the power of an optical instruments match the following two columns.

Column I	Column II
(A) Convex mirror	(p) Zero
(B) Concave mirror	(q) Infinite
(C) Plane mirror	(r) Positive
(D) Concave lens	(s) Negative

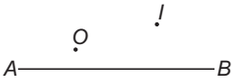
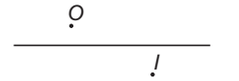
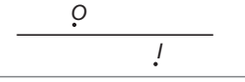
2. Focal length of a biconvex lens is f. Regarding the focal lengths match the following two columns.

Column I	Column II
(A) 	(p) f
(B) 	(q) $2f$
(C) 	(r) $\frac{f}{2}$
(D) 	(s) zero

3. Magnitudes of focal lengths of a convex lens, a concave lens, a convex mirror and a concave mirror are 20 cm each. An object is placed at a distance at 30 cm from pole/optical centre of each. Regarding the image match the following two columns.

Column I	Column II
(A) Convex mirror	(p) real
(D) Concave mirror	(q) virtual
(C) Convex lens	(r) magnified
(D) Concave lens	(s) diminished

4. Line AB in Column I represents the principal axis of a lens/mirror. Point O is a point object and I its point image. Now match the following two columns.

Column I	Column II
(A) 	(p) plane mirror
(B) 	(q) convex lens
(C) 	(r) concave mirror
(D) 	(s) concave lens

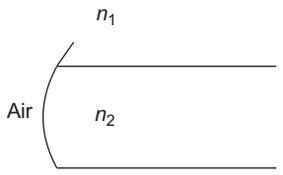
5. A thin convex lens has focal length f_1 and a concave lens has focal length f_2. They are kept in contact. Now match the following two columns.

Column I	Column II
(A) If $\lvert f_1 \rvert = \lvert f_2 \rvert$	(p) Power of system = 0
(B) If $\lvert f_1 \rvert > \lvert f_2 \rvert$	(q) Power of system = ∞
(C) If $\lvert f_1 \rvert < \lvert f_2 \rvert$	(r) Power of system is positive
	(s) Power of system is negative

Entrance Gallery

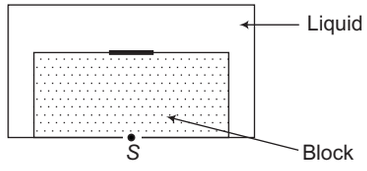
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1. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then, **[IIT Advanced]**



(a) $\lvert f_1 \rvert = 3R$ (b) $\lvert f_1 \rvert = 2.8R$
(c) $\lvert f_2 \rvert = 2R$ (d) $\lvert f_2 \rvert = 1.4R$

2. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is **[IIT Advanced]**

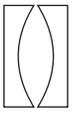


(a) 1.21 (b) 1.30
(c) 1.36 (d) 1.42

3. A thin convex lens made from crown glass ($\mu = 3/2$) has focal length f. When it is measured in two different liquids having refractive indices 4/3 and 5/3. It has the focal lengths f_1 and f_2, respectively. The correct relation between the focal length is **[JEE Main]**

(a) $f_1 = f_2 < f$
(b) $f_1 > f$ and f_2 becomes negative
(c) $f_2 > f$ and f_1 becomes negative
(d) f_1 and f_2 both become negative

4. Four combinations of two thin lenses are given in column I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in column I with their focal length in column II and select the correct answer using the codes given below the columns. **[JEE Advanced]**

Column I	Column II
P. 	1. $2r$
Q. 	2. $r/2$
R. 	3. $-r$
S. 	4. r

Codes

	P	Q	R	S
(a)	1	2	3	4
(b)	2	4	3	1
(c)	4	1	2	3
(d)	2	1	3	4

5. A green light is incident from the water to the air-water interface at the critical angle (θ). Select the correct statement. **[IIT Main]**

(a) The entire spectrum of visible light will come of the water at an angle of 90° to the normal
(b) The spectrum of visible light whose frequency is less than that of green light will come out of the air medium
(c) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium
(d) The entire spectrum of visible light will come out of the water of various angles to the normal

6. The apparent flattening of the sun at sunset and sunrise is due to **[Kerala CEE]**

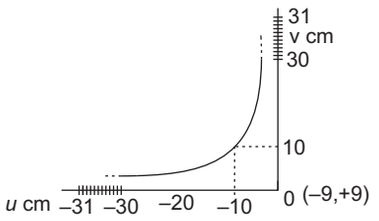
(a) refraction (b) diffraction
(c) total internal reflection (d) interference
(e) polarisation

7. Identify the mismatch in the following : **[Kerala CEE]**

(a) Myopia — Concave lens
(b) For rear view — Concave mirror
(c) Hypermetropia — Convex lens
(d) Astigmatism — Cylindrical lens
(e) Reflecting telescope — Convex mirror

8. A microscope is having objective focal lengths 1 cm and eye piece of focal length 6 cm. If tube length is 30 cm and image is formed at the least distance of distinct vision, what is the magnification produced by the microscope.
(take, $D = 25\,\text{cm}$) **[Karnataka CET]**
(a) 6 (b) 150
(c) 25 (d) 125

9. Graph of position of image *versus* position of point object from a convex lens is shown. Then, focal length of the lens is **[Karnataka CET]**



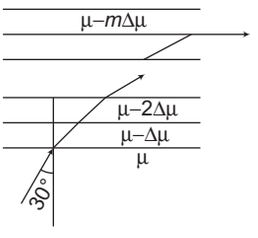
(a) 0.50 ± 0.05 cm
(b) 0.50 ± 0.10 cm
(c) 5.00 ± 0.05 cm
(d) 5.00 ± 0.10 cm

10. A focal length of a lens is 10 cm. What is power of a lens in dioptre? **[Karnataka CET]**
(a) 0.1 D (b) 10 D
(c) 15 D (d) 1 D

11. An object is placed at 20 cm in front of a concave mirror produces three times magnified real image. What is the focal length of the concave mirror? **[Karnataka CET]**
(a) 15 cm (b) 6.6 cm
(c) 10 cm (d) 7.5 cm

12. A glass slab consists of thin uniform layers of progressively decreasing Refractive Indices RI (see figure) such that the RI of any layer is $\mu - m\Delta\mu$. Here, μ and $\Delta\mu$ denote the RI of zeroth layer and the difference in RI between any two consecutive layers, respectively. The integer $m = 0, 1, 2, 3, \ldots$ denotes the numbers of the successive layers.

A ray of light from the zeroth layer enters the first layer at an angle of incidence of 30°. After undergoing the mth refraction, the ray emerges parallel to the interface. If $\mu = 1.5$ and $\Delta\mu = 0.015$, the value of m is **[WB JEE]**



(a) 20 (b) 30
(c) 40 (d) 50

13. A luminous object is separated from a screen by distance d. A convex lens is placed between the object and the screen such that it forms a distinct image on the screen. The maximum possible focal length of this convex lens is **[WB JEE]**
(a) $4d$ (b) $2d$ (c) $\frac{d}{2}$ (d) $\frac{d}{4}$

14. A concave mirror of focal length f produces a real image n times the size of the object. The distance of the object from the mirror is **[J&K CET]**
(a) $(n-1)f$ (b) $(n+1)f$
(c) $(n+1)f/n$ (d) $(n-1)f/n$

15. Which of the following relation is called mirror equation? **[J&K CET]**
(a) $u/v + f/u = 1/f$ (b) $1/v + 1/u = 1/f$
(c) $1/f + u = 1/v$ (d) $uf + vf = uv$

16. Light form a point source in air falls on a spherical glass surface ($n = 1.67$ and radius of curvature = 25 cm). The distance of the light source from the glass surface is 95 cm. At what position, the image is formed? **[J&K CET]**
(a) 75.45 cm
(b) 90.50 cm
(c) 105.25 cm
(d) 99.40 cm

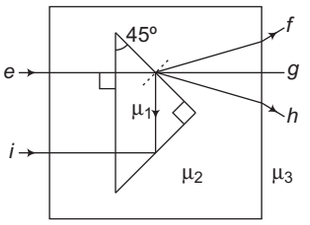
17. The angle of a prism is 42° and refractive index of its material is 3/2. Then, angle of minimum deviation for this prism is **[J&K CET]**
(a) 63° (b) 42°
(c) 28° (d) 21°

18. A compound microscope has a magnification of 30. The focal length of the eye-piece is 5 cm. If the final image is formed at the least distance of distinct vision (25 cm) the magnification produced by the objective is **[J&K CET]**
(a) 10 (b) 7.5
(c) 5 (d) 15

2013

19. A ray of light travelling in the direction $\frac{1}{2}(\hat{\mathbf{i}} + \sqrt{3}\hat{\mathbf{j}})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{\mathbf{i}} + \sqrt{3}\hat{\mathbf{j}})$. The angle of incidence is **[JEE Advanced]**
(a) 30° (b) 45°
(c) 60° (d) 75°

20. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure, A ray of light e enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths *ef, eg, eh* or *ei*. **[JEE Advanced]**



Match the paths in column I with conditions of refractive induces in column II and select the correct answer using the codes given below the columns.

Column I	Column II
P. $e \to f$	1. $\mu_1 > \sqrt{2}\mu_2$
Q. $e \to g$	2. $\mu_1 > \mu_1$ and $\mu_2 > \mu_3$
R. $e \to h$	3. $\mu_1 = \mu_2$
S. $e \to i$	4. $\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

Codes

	P	Q	R	S
(a)	2	3	1	4
(b)	1	2	4	3
(c)	4	1	2	3
(d)	2	3	4	1

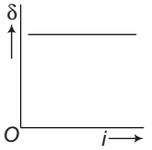
21. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and in one-third the size of the object. The wavelength of light inside the lens is $2/3$ times the wavelength in free space. The radius of the curved surface of the lens is **[JEE Advanced]**

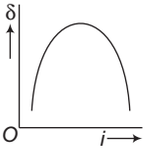
(a) 1 m (b) 2 m
(c) 3 m (d) 6 m

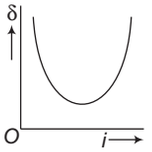
22. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 cm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is **[JEE Main]**

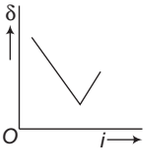
(a) 15 cm (b) 20 cm
(c) 30 cm (d) 10 cm

23. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by **[JEE Main]**

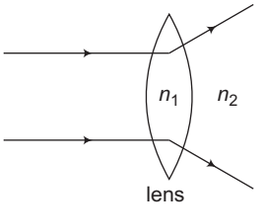
(a) 

(b) 

(c) 

(d) 

24. The relation between n_1 and n_2, if behaviour of light ray is as shown in figure is **[Karnataka CET]**



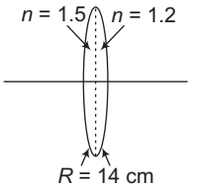
(a) $n_1 >> n_2$ (b) $n_2 > n_1$
(c) $n_1 > n_2$ (d) $n_1 = n_2$

25. A glass slab ($\mu = 1.5$) of thickness 6 cm is placed over a paper. What is the shift in the letters? **[OJEE]**

(a) 4 cm (b) 2 cm
(c) 1 cm (d) None of these

2012

26. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature $R = 14$ cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be **[IIT JEE]**



(a) – 280.0 cm (b) 40.0 cm
(c) 21.5 cm (d) 13.3 cm

27. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus on film? **[AIEEE]**

(a) 7.2 m (b) 2.4 m
(c) 3.2 m (d) 5.6 m

28. An object is placed at a distance 10 cm from a convex mirror of focal length 20 cm. Image distance is **[OJEE]**

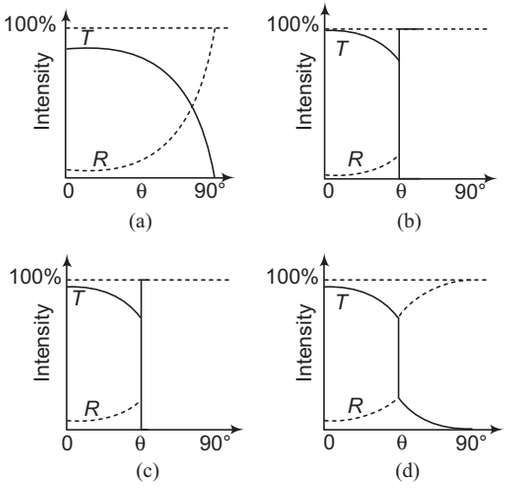
(a) 20/3 cm (b) 3/20 cm
(c) 4 cm (d) 4.5 cm

29. A beaker is of height 21 cm. A liquid of refractive index 4/3 is powered to the height h, so that the apparent height becomes halved. The value of h is **[OJEE]**

(a) 14 cm
(b) 16 cm
(c) 14/3 cm
(d) 18 cm

2011

30. A light ray travelling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is **[IIT JEE]**



31. A car is fitted with a convex side view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car is a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is **[AIEEE]**

(a) $\frac{1}{15}$ m/s (b) 10 m/s

(c) 15 m/s (d) $\frac{1}{10}$ m/s

32. Let the x-z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\mathbf{A} = 6\sqrt{3}\,\hat{\mathbf{i}} + 8\sqrt{3}\hat{\mathbf{j}} - 10\,\hat{\mathbf{k}}$ is incident on the plane of separation. The angle of refraction in medium 2 is **[AIEEE]**

(a) 45° (b) 60°

(c) 75° (d) 30°

33. A beaker contains water, up to a height h_1 and kerosene of height h_2 above water, so that the total height of (water + kerosene) is $(h_1 + h_2)$. Refractive index of water is μ_1 and that of kerosene is μ_2. The apparent shift in the position of the bottom of the beaker when viewed from above is **[AIEEE]**

(a) $\left(1-\frac{1}{\mu_1}\right)h_2 + \left(1-\frac{1}{\mu_2}\right)h_1$ (b) $\left(1+\frac{1}{\mu_1}\right)h_1 + \left(1+\frac{1}{\mu_2}\right)h_2$

(c) $\left(1-\frac{1}{\mu_1}\right)h_1 + \left(1-\frac{1}{\mu_2}\right)h_2$ (d) $\left(1+\frac{1}{\mu_1}\right)h_2 - \left(1+\frac{1}{\mu_2}\right)h_1$

34. When monochromatic red light is used instead of blue light in a convex lens, its focal length will **[AIEEE]**

(a) does not depend on colour of light

(b) increase

(c) decrease

(d) remain same

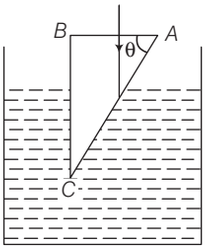
35. The focal length of the lens of refractive index ($\mu = 1.5$) in air is 10 cm. If air is replaced by water of $\mu = 4/3$, its focal length is **[Kerala CEE]**

(a) 20 cm (b) 30 cm

(c) 40 cm (d) 25 cm

(e) 35 cm

36. A glass prism of refractive index 1.5 is immersed in water $\left(\mu = \frac{4}{3}\right)$. Refer figure.



A light beam incident normally on the face AB is totally reflected to reach the face BC if **[Kerala CEE]**

(a) $2/3 < \sin\theta < 8/9$ (b) $\sin\theta \leq 2/3$

(c) $\cos\theta \geq 8/9$ (d) $\sin\theta > 8/9$

(e) $\cos\theta \leq 8/9$

37. Wavelength of given light waves in air and in a medium are 6000 Å and 4000 Å respectively. The critical angle is **[Karnataka CET]**

(a) $\sin^{-1}\left(\frac{3}{2}\right)$ (b) $\sin^{-1}\left(\frac{2}{3}\right)$

(c) $\tan^{-1}\left(\frac{3}{2}\right)$ (d) $\tan^{-1}\left(\frac{2}{3}\right)$

38. The time required for the light to pass through a glass slab (refractive index = 1.5) of thickness 4 mm is … ($c = 3\times10^8$ ms^{-1}, speed of light in free space). **[Karnataka CET, WB JEE]**

(a) 2×10^{-5}s (b) 2×10^{11} s

(c) 2×10^{-11} s (d) 10^{-11} s

39. A prism having refractive index 1.414 and refracting angle 30° has one of the refracting surfaces silvered. A beam of light incident on the other refracting surface will retrace its path, if the angle of incidence is **[Karnataka CET]**

(a) 45° (b) 60°

(c) 30° (d) 0°

40. A planoconvex lens has a maximum thickness of 6 cm. When placed on a horizontal table with the curved surface in contact with the table surface, the apparent depth of the bottom most point of the lens is found to be 4 cm. If the lens is inverted such that the plane face of the lens is in contact with the surface of the table, the apparent depth of the centre of the plane face is found to be (17/4) cm. The radius of curvature of the lens is [Karnataka CET]

(a) 34 cm (b) 128 cm
(c) 75 cm (d) 68 cm

41. Two thin lenses have a combined power of + 9 D. When they are separated by a distance of 20 cm, their equivalent power becomes $+\frac{27}{5}$ D. Their individual powers (in dioptre) are [Karnataka CET]

(a) 4, 5
(b) 3, 6
(c) 2, 7
(d) 1, 8

42. In fog, photographs of the objects taken with infrared radiations are more clear than those obtained during visible light because [Karnataka CET]

(a) *I-R* radiation has lesser wavelength than visible radiation
(b) scattering of *I-R* light is more than visible light
(c) the intensity of *I-R* light from the object is less
(d) scattering of *I-R* light is less than visible light

43. If the focal length of the eye piece of a telescope is double, its magnifying power m_1 will be [WB JEE]

(a) $2m$ (b) $3m$
(c) $\frac{m}{2}$ (d) $4m$

44. Two thin lenses of focal length 20 cm and 25 cm are placed in contact. The effective power of the combination is [WB JEE]

(a) 9 D (b) 2 D
(c) 3 D (d) 7 D

45. A convex lens of focal length 30 cm produces 5 times magnified real image of an object. What is the object distance? [WB JEE]

(a) 36 cm (b) 25 cm (c) 30 cm (d) 150 cm

46. A plano-concave lens is made of glass of refractive index 1.5 and the radius of curvature of its curved face is 100 cm. What is the power of the lens? [WB JEE]

(a) + 0.5 D (b) – 0.5 D
(c) – 2 D (d) + 2 D

47. If the aperture of a telescope is decreased the resolving power will [MHT CET]

(a) increases (b) decreases
(c) remain same (d) zero

2010

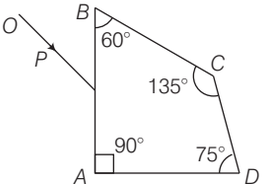
48. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 s. What is the speed of the object in kmh^{-1}? [IIT JEE]

(a) 3 (b) 4
(c) 5 (d) 6

49. A large glass slab $\left(\mu = \frac{5}{3}\right)$ of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R? [IIT JEE]

(a) 6 cm (b) 7 cm
(c) 8 cm (d) 9 cm

50. A ray OP of monochromatic light is incident on the face AB of prism $ABCD$ near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct? [IIT JEE]



(a) The ray gets totally internally reflected at face CD
(b) The ray comes out through face AD
(c) The angle between the incident ray and the emergent ray is 90°
(d) The angle between the incident ray and the emergent ray is 120°

51. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is [IIT JEE]

(a) virtual and at a distance of 16 cm from the mirror
(b) real and at a distance of 16 cm from the mirror
(c) virtual and at a distance of 20 cm from the mirror
(d) None of the above

52. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is [IIT JEE]

(a) 6
(b) 7
(c) 8
(d) 9

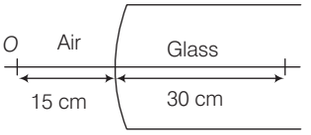
53. A square wire of side 1 cm is placed perpendicular to the principle axis of a concave mirror of focal length 15 cm at a distance of 20 cm. The area enclosed by the image of the wire is **[Kerala CEE]**

(a) 4 cm^2 (b) 6 cm^2
(c) 2 cm^2 (d) 8 cm^2
(e) 9 cm^2

54. A ray of light is incident on a plane mirror at an angle of 60°. The angle of deviation produced by the mirror is **[Karnataka CET]**

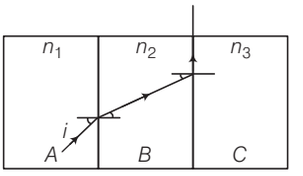
(a) 120° (b) 30°
(c) 60° (d) 90°

55. A point object O is placed in front of a glass rod having spherical end of radius of curvature 30 cm. The image would be formed at **[Karnataka CET]**



(a) 30 cm left
(b) infinity
(c) 1 cm to the right
(d) 18 cm to the left

56. A, B and C are the parallel sided transparent media of refractive indices n_1, n_2 and n_3 respectively. They are arranged as shown in the figure. A ray is incident at an angle i on the surface of separation of A and B which is as shown in the figure. After the refraction into the medium B, the ray grazes the surface of separation of the media B and C. Then, sin i equal to **[Karnataka CET]**



(a) $\frac{n_3}{n_1}$ (b) $\frac{n_1}{n_3}$ (c) $\frac{n_2}{n_3}$ (d) $\frac{n_1}{n_2}$

57. A boat has green light of wavelength $\lambda = 500$ nm on the mast. What wavelength would be measured and what colour would be observed for this light as seen by a diver submerged in water by the side of the boat?

Given, $n_w = \frac{4}{3}$. **[Karnataka CET]**

(a) Green of wavelength 376 nm
(b) Red of wavelength 665 nm
(c) Green of wavelength 500 nm
(d) Blue of wavelength 376 nm

58. Two beams of red and violet colours are made to pass separately through a prism of $A = 60°$. In the minimum deviation position, the angle of refraction inside the prism will be **[Karnataka CET]**

(a) greater for red colour
(b) equal but not 30° for both the colours
(c) greater for violet colour
(d) 30° for both the colours

Answers

Level 1

Objective Problems

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (b)	**9.** (b)	**10.** (a)
11. (a)	**12.** (a)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (d)	**19.** (a)	**20.** (a)
21. (a)	**22.** (b)	**23.** (a)	**24.** (a)	**25.** (a)	**26.** (b)	**27.** (b)	**28.** (c)	**29.** (b)	**30.** (b)
31. (c)	**32.** (a)	**33.** (c)	**34.** (c)	**35.** (b)	**36.** (c)	**37.** (c)	**38.** (a)	**39.** (c)	**40.** (a)
41. (b)	**42.** (b)	**43.** (d)	**44.** (a)	**45.** (a)	**46.** (d)	**47.** (b)	**48.** (d)	**49.** (b)	**50.** (a)
51. (d)	**52.** (c)	**53.** (d)	**54.** (a)	**55.** (b)	**56.** (d)	**57.** (a)	**58.** (b)	**59.** (d)	**60.** (b,d)
61. (d)	**62.** (d)	**63.** (d)	**64.** (d)	**65.** (b,d)	**66.** (b)	**67.** (a)	**68.** (d)	**69.** (d)	**70.** (c)
71. (b)	**72.** (d)	**73.** (d)	**74.** (c)	**75.** (b)	**76.** (d)	**77.** (c)	**78.** (b)	**79.** (d)	**80.** (c)
81. (a)	**82.** (b)	**83.** (a)	**84.** (a)	**85.** (d)	**86.** (c)	**87.** (b)	**88.** (c)	**89.** (a)	**90.** (d)
91. (b)	**92.** (a)	**93.** (a)	**94.** (d)	**95.** (a)	**96.** (b)	**97.** (c)	**98.** (d)	**99.** (b)	**100.** (b)
101. (c)	**102.** (c)	**103.** (c)	**104.** (c)	**105.** (a)	**106.** (c)	**107.** (b)	**108.** (b)	**109.** (a)	**110.** (a)
111. (d)	**112.** (d)	**113.** (b)	**114.** (d)	**115.** (c)	**116.** (d)	**117.** (d)	**118.** (b)	**119.** (c)	**120.** (c)
121. (a)	**122.** (c)	**123.** (a)	**124.** (b)	**125.** (c)	**126.** (c)	**127.** (c)	**128.** (b)	**129.** (a)	**130.** (b)
131. (c)	**132.** (a)	**133.** (d)	**134.** (b)	**135.** (a)	**136.** (a)	**137.** (c)	**138.** (b)	**139.** (a)	**140.** (c)
141. (b)	**142.** (a)	**143.** (b)	**144.** (b)	**145.** (c)	**146.** (d)	**147.** (c)	**148.** (c)	**149.** (a)	**150.** (d)
151. (a)	**152.** (b)	**153.** (a)	**154.** (b)	**155.** (b)	**156.** (d)	**157.** (c)	**158.** (d)	**159.** (d)	**160.** (b)
161. (d)	**162.** (a)	**163.** (a)	**164.** (b)	**165.** (c)	**166.** (d)	**167.** (c)	**168.** (b)	**169.** (c)	**170.** (a)
171. (b)	**172.** (d)	**173.** (b)	**174.** (a)	**175.** (a)	**176.** (c)	**177.** (a)	**178.** (a)	**179.** (a)	**180.** (c)
181. (d)	**182.** (b)	**183.** (b)	**184.** (b)	**185.** (a)					

Level 2

Only One Correct Option

1. (c)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (c)
11. (d)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (d)	**19.** (c)	**20.** (c)
21. (c)	**22.** (c)	**23.** (b)	**24.** (a)	**25.** (d)	**26.** (b)	**27.** (d)	**28.** (b)	**29.** (c)	**30.** (a)
31. (a)	**32.** (d)	**33.** (b)	**34.** (c)	**35.** (d)	**36.** (b)	**37.** (c)	**38.** (a)	**39.** (a)	**40.** (d)
41. (c)	**42.** (c)	**43.** (b)	**44.** (d)	**45.** (a)	**46.** (c)	**47.** (d)			

More than One Correct Options

1. (a,b)	**2.** (all)	**3.** (a,b,c)	**4.** (a,b,d)	**5.** (b,c)	**6.** (a,b,c)	**7.** (a,b)	**8.** (b,d)	**9.** (b,c)	**10.** (b,c,d)
11. (a,b)	**12.** (a,b,c)	**13.** (d)	**14.** (a,b)	**15.** (a,b,c)	**16.** (a,b,d)	**17.** (a,b)			

Comprehension Based Questions

1. (c)	**2.** (d)	**3.** (d)	**4.** (d)	**5.** (a)	**6.** (a)	**7.** (c)

Assertion and Reason

1. (a,b)	**2.** (d)	**3.** (c)	**4.** (a)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (a)	**9.** (d)	**10.** (a,b)
11. (a,b)	**12.** (d)	**13.** (d)	**14.** (a,b)	**15.** (c)	**16.** (c)	**17.** (c)	**18.** (d)	**19.** (b)	**20.** (a)

Match the Columns

1. (A→s; B→r; C→p; D→s)
2. (A→q; B→p; C→r; D→p)
3. (A→q,s; B→p,r; C→p,r; D→q,s)
4. (A→p; B→q,r; C→q,r; D→q,r)
5. (A→p; B→s; C→r)

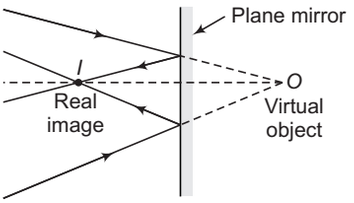
Entrance Gallery

1. (c)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (d)	**10.** (b)
11. (a)	**12.** (d)	**13.** (d)	**14.** (d)	**15.** (b)	**16.** (d)	**17.** (d)	**18.** (c)	**19.** (a)	**20.** (c)
21. (c)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (d)	**28.** (a)	**29.** (a)	**30.** (c)
31. (a)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (c)	**36.** (a)	**37.** (b)	**38.** (c)	**39.** (a)	**40.** (a)
41. (b)	**42.** (d)	**43.** (c)	**44.** (a)	**45.** (a)	**46.** (b)	**47.** (b)	**48.** (a)	**49.** (a)	**50.** (a,b,c)
51. (b)	**52.** (a)	**53.** (c)	**54.** (c)	**55.** (a)	**56.** (a)	**57.** (d)	**58.** (b)		

Solutions

Level 1 : Objective Problems

1. When convergent beam incident on a plane mirror, then mirror forms real image, for an virtual object.

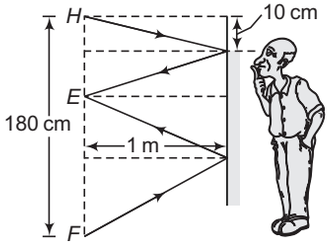


2. As incident ray and reflected ray are parallel to each other it means, deviation

$$\delta = 180° \quad \because \quad \delta = 360° - 2\theta$$

$$\therefore \qquad 180° = 360° - 2\theta \quad \Rightarrow \quad \theta = 90°$$

3. According to ray diagram shown in figure



$$\text{Length of mirror} = \frac{1}{2}(10 + 170) = 90 \text{ cm}$$

4. Deviation by a plane mirror

$$\delta = 180° - 2\theta$$
$$= 180° - 60° = 120°$$

5. As number of images $n = 3$

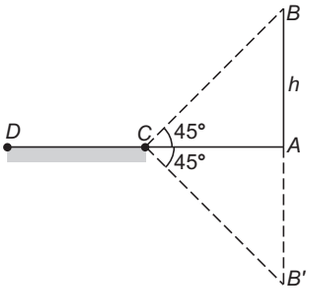
So, $\quad n = \dfrac{360}{\theta} - 1$

$$3 = \frac{360}{\theta} - 1 \quad \therefore \quad \theta = 90°$$

6. $m = \dfrac{360}{60°} = 6$

Since, m is an even integer, total number of images will be $m - 1$ or 5.

9. Given, $\quad AC = 60 \text{ m} \quad$ and $\quad \angle BCB' = 90°$



$$\therefore \qquad \angle BCA = \frac{\angle BCB'}{2} = 45°$$

$$AB = AC = 60 \text{ m}$$

11. Magnification, $\quad m = +\dfrac{1}{n} = -\dfrac{v}{u}$

$$\therefore \qquad v = -\frac{u}{n}$$

From mirror formula,

$$\frac{1}{f} = \frac{1}{\left(-\dfrac{u}{n}\right)} + \frac{1}{u}$$

$$\therefore \qquad u = -(n-1)f$$

12. In case of concave mirror, $m = -2$

Also, $\quad m = \dfrac{f}{f - u}$

$$\Rightarrow \qquad -2 = \frac{-50}{-50 - u}$$

$$\therefore \qquad u = -75 \text{ cm}$$

13. Magnification,

$$m = \frac{I}{O} = \frac{f}{f - u}$$

$$\therefore \qquad \frac{I}{7.5} = \frac{(R/2)}{(R/2) - u}$$

$$= \frac{(25/2)}{(25/2) - (-40)}$$

$$\therefore \qquad I = -1.78 \text{ cm}$$

14. Magnification, $m = \dfrac{f}{f-u}$

$\Rightarrow \quad \left(+\dfrac{1}{4}\right) = \dfrac{+30}{+30-u}$

$\Rightarrow \quad u = -90$ cm

15. Image formed is real, inverted and same in size because object is at the centre of curvature of the mirror.

16. Magnification,

$$m = \frac{f}{f-u} \quad \Rightarrow \quad -3 = \frac{f}{f-(-20)}$$

$\Rightarrow \quad f = -15$ cm

17. On immersing a mirror in water, focal length of the mirror remains unchanged.

18. By the mirror formula $\dfrac{1}{v} + \dfrac{1}{-30} = \dfrac{1}{30}$

$\therefore \quad v = +15$ cm

20. From the relation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we have, $\dfrac{-dv}{v^2} - \dfrac{du}{u^2} = 0$

$\therefore \quad dv = \dfrac{v^2}{u^2}(-du)$

$$= \left(\frac{20}{10}\right)^2 (0.1)$$

$$= 0.4 \text{ cm}$$

21. $\dfrac{1}{-30} + \dfrac{1}{-30} = \dfrac{1}{f}$

$\therefore \quad f = -15$ cm

22. Only convex mirror forms erect, virtual image of smaller size.

23. $\dfrac{1}{v} - \dfrac{1}{20} = \dfrac{1}{-15}$

$\therefore \quad v = -60$ cm

$$\text{Image speed} = \left(\frac{v^2}{u^2}\right)(\text{object speed})$$

$$= \left(\frac{60}{20}\right)^2 (5)$$

$$= 45 \text{ m/s}$$

24. $\lambda' = \dfrac{\lambda}{\mu} = \dfrac{6000}{1.5} = 4000$ Å

25. Refractive index,

$$\mu = \frac{c}{v} = \frac{c}{f\lambda}$$

$$= \frac{3\times10^8}{4\times10^{14}\times5\times10^{-7}}$$

$$= 1.5$$

26. Apparent depth $h' = \dfrac{d_1}{\mu_1} + \dfrac{d_2}{\mu_2}$

$$= \frac{d}{\mu_1} + \frac{d}{\mu_2}$$

$$= d\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)$$

27. Refractive index of glass with respect to water

$${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$$

28. ${}_2\mu_1 \times {}_3\mu_2 \times {}_4\mu_3 = \dfrac{\mu_1}{\mu_2} \times \dfrac{\mu_2}{\mu_3} \times \dfrac{\mu_3}{\mu_4}$

$$= \frac{\mu_1}{\mu_4} = {}_4\mu_1 = \frac{1}{{}_1\mu_4}$$

29. In going from one medium to another frequency remains same but wavelength and velocity decreases with the increase in refractive index, i.e. wavelength becomes $\lambda' = \lambda/\mu$ and velocity becomes,

$$v' = \frac{v}{\mu}$$

30. Refracticve index $\mu = \dfrac{c}{v} = \dfrac{\sin i}{\sin r} = \dfrac{\sin 45°}{\sin 30°}$

$\therefore \quad v = \dfrac{3\times10^8}{\sqrt{2}}$

$$= 2.12\times10^8 \text{ m/s}$$

31. As, $\mu = \dfrac{c}{v}$

$\Rightarrow \quad \nu \propto \dfrac{1}{\mu}$

$\therefore \quad \dfrac{v_1}{v_2} = \dfrac{\mu_2}{\mu_1}$

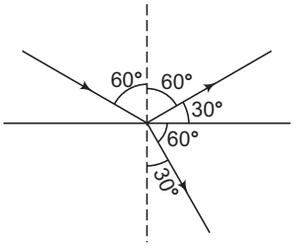
$\Rightarrow \quad \dfrac{v_g}{v_w} = \dfrac{\mu_w}{\mu_g} = \dfrac{4/3}{3/2} = \dfrac{8}{9}$

32. $h' = \mu h$

$$= \frac{4}{3}\times18 = 24 \text{ m}$$

33. All rays coming from object at centre are normal. So no bending will take place.

36.



$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 30°}$$

$$= \sqrt{3} = 1.732$$

37. $d_{\text{app}} = \dfrac{d}{\mu}$

or $(d-2) = \dfrac{d}{1.5}$

$\therefore \quad d = 6$ cm

38. This is from Cauchy's formula.

39. $d_{\text{actual}} = (d_{\text{app}})\mu$

From one side, $d_1 = 6\times1.5 = 9$

From other side, $d_2 = 4\times1.5 = 6$

$\therefore \quad d = d_1 + d_2 = 15$ cm

42. $\mu = \dfrac{\sin i}{\sin i/2} = \dfrac{2\sin i/2\cos i/2}{\sin i/2}$

$\dfrac{i}{2} = \cos^{-1}(\mu/2)$

$\therefore \quad i = 2\cos^{-1}\left(\dfrac{\mu}{2}\right)$

44. $d_{app} = \dfrac{d_1}{\mu_1} + \dfrac{d_2}{\mu_2} + \dfrac{d_3}{\mu_3}$

$= \dfrac{3}{3/2} + \dfrac{4}{4/3} + \dfrac{6}{6/5} = 2 + 3 + 5 = 10\,\text{cm}$

45. Focal length of the combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$\Rightarrow \quad \dfrac{1}{F} = \dfrac{1}{(+40)} + \dfrac{1}{(-25)}$

$\Rightarrow \quad F = -\dfrac{200}{3}\,\text{cm}$

As, $\quad \text{Power } P = \dfrac{100}{F} = \dfrac{100}{(-200/3)}$

$= -1.5\,\text{D}$

46. Focal length of the combination

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$\Rightarrow \quad \dfrac{1}{80} = \dfrac{1}{20} + \dfrac{1}{f_2}$

$\therefore \quad f_2 = -\dfrac{80}{3}\,\text{cm}$

$\therefore$ Power of second lens

$$P' = \frac{100}{f_2} = \frac{100}{(-80/3)} = -3.75\,\text{D}$$

47. In each case, there is a combination of two plano convex lenses placed close to each other. Focal length of combination $\dfrac{1}{F} = \dfrac{1}{f} + \dfrac{1}{f}$ is same in all cases, so ratio of focal length in three cases is 1 : 1 : 1.

48. Focal length of combination

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{60} + \frac{1}{(-20)}$$

$\Rightarrow \quad F = -30\,\text{cm}$

49. From a concave lens image is always smaller in size.

50. For air lens in glass, focal length of air lens,

$$\frac{1}{f} = ({}_g\mu_a - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{2}{3} - 1\right)\left[\frac{1}{10} - \left(-\frac{1}{10}\right)\right]$$

$\Rightarrow \quad f = -15\,\text{cm}$

So, nature of lens is concave type.

51. Focal length of combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$F = \frac{f_1 f_2}{f_1 + f_2}$$

Power of lens, $\quad P = \dfrac{1}{F} = \dfrac{f_1 + f_2}{f_1 f_2}$

52. Focal length of plano convex lens

$$f = \frac{R}{\mu - 1} = \frac{R}{(1.5 - 1)} = 2R$$

54. For combination of one convex and one concave lens

$$\frac{1}{F} = \frac{1}{f_1} - \frac{1}{f_2}$$

$\Rightarrow \quad F = \dfrac{f_1 f_2}{f_2 - f_1}$

If, $f_1 > f_2$, then F will be negative.

55. Apply, $\quad \dfrac{1}{f} = \dfrac{2(\mu_2/\mu_1)}{R_2} - \dfrac{2(\mu_2/\mu_1 - 1)}{R_1}$

56. For real image from converging lens,

$$m = -2$$

$\therefore \quad m = \dfrac{f}{f + u}$

$\Rightarrow \quad -2 = \dfrac{f}{u + f}$

$$= \frac{20}{u + 20}$$

$$u = -30\,\text{cm}$$

57. When plano-convex lens is silvered at curved surface, then it becomes concave mirror of focal length,

$$f = \frac{R}{2\mu} = \frac{30}{2 \times 1.5}$$

$$= 10\,\text{cm}$$

If we have to find real image of same size of the object, object must be placed at centre of curvature $u = (2f) = 2 \times 10 = 20\,\text{cm}$

60. With increase in wavelength, refractive index gets decreased.

61. $R_2 = \infty$ after cutting the lens.

63. $\dfrac{1}{v} - \dfrac{1}{-90} = \dfrac{1}{f}$

$$= (1.5 - 1)\left(\frac{1}{30}\right) = \frac{1}{60}$$

$\therefore \quad v = 180\,\text{cm}$

$\therefore \quad |m| = \left|\dfrac{v}{u}\right| = \dfrac{180}{90} = 2.0$

64. Keep the object at a distance $2f$ from the lens.

Hence, $d = 2f = 2 \times 15 = 30\,\text{cm}$

67. Rays will converge at focus.

68. When object lies between focus and pole of convex lens, image is virtual, erect and enlarged.

69. Image is real, because virtual image is enlarged.

$\therefore \quad \dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

or $\quad \dfrac{1}{x/4} + \dfrac{1}{x} = \dfrac{1}{30} \qquad (\because u = -x)$

$$\frac{5}{x} = \frac{1}{30}$$

$\therefore \quad x = 150\,\text{cm}$

70. $\dfrac{1}{30} = (\mu - 1)\left(\dfrac{1}{10}\right)$

or $\quad \mu = \dfrac{4}{3} = 1.33$

71. By the mirror formula $\frac{1}{30}-\frac{1}{-u}=\frac{1}{f}$

or $\frac{1}{30}+\frac{1}{u}=\frac{1}{f}$ ...(i)

Similarly, $\frac{1}{120}+\frac{1}{u-90}=\frac{1}{f}$ ...(ii)

Solving these two equations, we get

$$f = 24 \text{ cm}$$

72. If the object lies between first focus and optical centre, image is virtual and enlarged. When the object lies between F and $2F$ image is real and enlarged.

75. $\frac{1}{+5}-\frac{1}{-10}=\frac{1}{f}$ $\left(|v|=\frac{|u|}{2}\right)$

$\therefore$ $f=\frac{10}{3}$ cm

76. $2.5=\left(\frac{3}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\Rightarrow$ $5.0=\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$ ...(i)

$$P=\left(\frac{3/2}{2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$\Rightarrow$ $-4P=\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$ ...(ii)

From Eqs. (i) and (ii),

$\therefore$ $P=-1.25$ D

79. $\sin C=\frac{1}{\mu}$

As μ for violet colour is maximum, so $\sin C$ is minimum and hence critical angle C is minimum for violet colour.

80. Critical angle,

$$\sin C=\frac{n_2}{n_1}=\frac{\lambda_1}{\lambda_2} \Rightarrow \sin C=\frac{3500}{7000}=\frac{1}{2}$$

$\therefore$ $C = 30°$

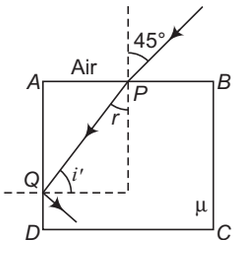
81. Critical angle,

$$\sin C=\frac{1}{\mu}$$

$\therefore$ $C=\sin^{-1}\left(\frac{1}{\mu}\right)$

and as μ decreases with increase in λ yellow, orange, red have higher wavelength than green, so μ will be less for these rays, and so critical angle for these rays will be high, hence if green is just totally internally reflected then yellow, orange and red rays will emerge out.

82. At point P,



$$\mu=\frac{\sin 45°}{\sin r}$$

$\Rightarrow$ $\sin r=\frac{1}{\mu\sqrt{2}}$ ...(i)

At point Q, for total internal reflection,

$$\sin i'=\frac{1}{\mu}$$

From figure, $i' = 90° - r$

$\therefore$ $\sin(90°-r)=\frac{1}{\mu}$

$\Rightarrow$ $\cos r=\frac{1}{\mu}$ ... (ii)

Now, $\cos r=\sqrt{1-\sin^2 r}=\sqrt{\left(1-\frac{1}{2\mu^2}\right)}$

$$=\sqrt{\left(\frac{2\mu^2-1}{2\mu^2}\right)}$$ ...(iii)

From Eqs. (ii) and (iii),

$$\frac{1}{\mu}=\sqrt{\frac{2\mu^2-1}{2\mu^2}} \Rightarrow \mu=\sqrt{\left(\frac{3}{2}\right)}$$

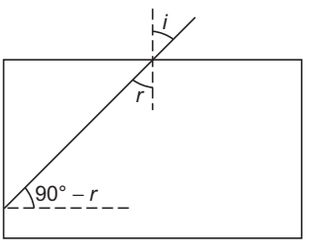
83. $\sin\theta_C=\frac{1}{\mu}$

$\therefore$ $\mu=\frac{1}{\sin\theta_C}=2$

$$v=\frac{c}{\mu}=\frac{3.0\times10^8}{2}$$

$$=1.5\times10^8 \text{ m/s}$$

84. For TIR to take place,



$$90°-r>\theta_C$$

or $r<90°-\theta_C$

From the relation,

$$n=\frac{\sin i}{\sin r} \Rightarrow \sin i=n\sin r$$

Since, for TIR to take place $r<90°-\theta_C$

$\therefore$ $i<\sin^{-1}\{n\sin r\}$

or $i<\sin^{-1}\{n\cos\theta_C\}$

or $i<\sin^{-1}\left\{n\sqrt{1-\frac{1}{n^2}}\right\}$

or $i<\sin^{-1}\left\{1.33\sqrt{1-\frac{1}{(1.33)^2}}\right\}$

or $i<\sin^{-1}\{\sqrt{0.77}\}$

For simplyfying the calculation, take $1.33=\frac{4}{3}$.

85. $\sin i > \sin\theta_C$

$\therefore \quad \sin 45° > \frac{1}{n}$

or $\quad \frac{1}{\sqrt{2}} > \frac{1}{n}$

$\therefore \quad n > \sqrt{2}$

or $\quad n > 1.414$

86. The reflected ray will rotate by angle 2θ. For TIR to take place at water-air boundary,

$$\sin 2\theta > \sin\theta_C$$

or $\quad \sin 2\theta > \frac{1}{\mu}$

$\therefore \quad \theta > \frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)$

87. $\mu_g = \frac{1}{\sin 30°} = 2$

Now, $\quad \sin\theta_C = \frac{6/5}{2} = \frac{3}{5}$

$\therefore \quad \theta_C = 37°$

88. The maximum deviation can be, when it is reflected back at

$$i = \theta_C = 45°$$

$\therefore \quad \delta = 180° - 2i = 90°$

89. From condition of achromatism

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

$\Rightarrow \quad \frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2}$

$$\frac{5}{3} = \frac{-(-15)}{f_2}$$

$$f_2 = 9 \text{ cm}$$

90. Refractive index of prism

$$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$$

$$i = 45° = \frac{A+\delta_m}{2}$$

So, $\quad \frac{\sin 45°}{\sin(A/2)} = \sqrt{2}$

So, $\quad \frac{1}{2} = \sin\frac{A}{2}$

$\Rightarrow \quad A = 60°$

91. Refractive index,

$$\mu = \frac{\sin i}{\sin A/2}$$

$$\sqrt{2} = \frac{\sin i}{\sin(60°/2)}$$

or $\quad \sqrt{2} \times \sin 30° = \sin i$

or $\quad \sqrt{2} \times \frac{1}{2} = \sin i$

$\Rightarrow \quad i = 45°$

92. Deviation of ray in prism is given by

$$\delta = i + e - A$$

So, $\quad e = \delta + A - i$

$$= 30° + 30° - 60°$$

$$= 0°$$

So, emergent ray will be perpendicular to face.

Or emergent ray will make an angle of 90° with the face through which it emerges.

93. Angle of minimum deviation in prism,

$$\delta_m = (\mu - 1)A$$

$$= (\mu - 1)(2r) \qquad (\because r = A/2)$$

$$= (1.5 - 1)(2r)$$

$$= 0.5 \times 2r = r$$

94. Given, $\quad i = e = \frac{3}{4}A$

$$= \frac{3}{4} \times 60 = 45°$$

In the position of minimum deviation,

$$2i = A + \delta_m$$

or $\quad \delta_m = 2i - A$

$$= 2 \times 45° - 60$$

$$= 30°$$

95. Refractive index for prism

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60°+60°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \sqrt{3}$$

96. To produce dispersion without deviation

$$\frac{A'}{A} = \frac{(\mu_y - 1)}{(\mu'_y - 1)}$$

$\Rightarrow \quad \frac{A'}{6} = -\frac{(1.54-1)}{(1.72-1)}$

$\Rightarrow \quad A' = -4.5°$

$$= 4°30'$$

97. Refractive index $\quad \mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\Rightarrow \quad \sqrt{3} = \frac{\sin\left(\frac{60°+\delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \sin\left(30° + \frac{\delta_m}{2}\right)$

$\therefore \quad \delta_m = 60°$

98. Under minimum deviation $i = e = 30°$, so angle between emergent ray and second refracting surface is $90° - 30° = 60°$.

99. In minimum deviation position, refracted ray inside an isoceles prism is parallel to the base of the prism.

100. Since, rays after passing through the glass slab just suffer lateral displacement. So, angle between emergent ray is also α.

103. For $A > 2C$, the ray of light does not emerge from the opposite fall of the prism.

104. Light should fall normally on the silvered face.

$$r_2 = 0$$

$$\therefore \quad r_1 = A = 30°$$

Now, $$\mu = \frac{\sin i_1}{\sin r_1}$$

or $$\sqrt{2} = \frac{\sin i_1}{\sin 30°}$$

This gives $$i_1 = 45°$$

105. $r_2 = 0,\ r_1 = A = 30°,\ i_1 = 45°$

$$\therefore \quad \mu = \frac{\sin i_1}{\sin r_1} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}$$

107. Angle i at both focus will be 45°. For TIR to take place,

$$i > \theta_C$$

or $$\sin i > \sin \theta_C$$

$$\therefore \quad \frac{1}{\sqrt{2}} > \frac{1}{\mu}$$

or $$\mu > \sqrt{2}$$

109. $\delta = (i_1 + i_2) - A$

or $$30° = (60° + i_2) - 30°$$

$$\therefore \quad i_2 = 0°$$

111. $i_1 = 0 = r_1$

Therefore, $$r_2 = A = 60°$$

$$\theta_C = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8°$$

$$r_2 > \theta_C$$

112. $$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)},$$

Substituting $A = \delta_m$ and $\mu = 1.5$, we get $A = 82°$.

113. Given, $A = 60°$ and $\delta_m = 30°$

$$\therefore \quad \mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

$$= \frac{\sin 45°}{\sin 30°} = \sqrt{2}$$

114. $\theta_C = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8°, \quad i_1 = 30°$

$$\therefore \quad \sin r_1 = \frac{\sin i_1}{\mu}$$

$$= \frac{\sin (30°)}{1.5},$$

$$r_1 = 19.5°$$

or $$r_2 = A - r_1 = 90 - 19.5$$

$$= 70.5°$$

Since, $$r_2 > \theta_C$$

The ray will not emerge from the prism.

115. Condition of achromatic combination is satisfied if

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

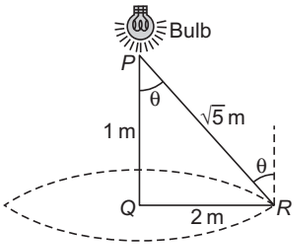
116. Power of lamp $= \frac{\text{Luminous flux}}{\text{Luminous efficiency}}$

$$= \frac{4\pi I}{\text{Luminous efficiency}}$$

$$= \frac{528}{2}$$

$$= 264 \text{ W}$$

117.



Illuminance at Q,

$$I_Q = \frac{L}{(1)^2} \quad \ldots(i)$$

Illuminance at R,

$$I_R = \frac{L\cos\theta}{(\sqrt{5})^2}$$

$$= \frac{L}{\sqrt{(5)^2}} \times \frac{1}{\sqrt{5}}$$

$$\therefore \quad I_R = \frac{L}{5\sqrt{5}} \quad \ldots(ii)$$

From Eqs. (i) and (ii),

$$I_Q = 5\sqrt{5} I_0$$

118. Unit of illuminance is lumen/m^2 and it is called lux.

119. Luminous intensity,

$$L = \frac{\phi}{4\pi}$$

$$\Rightarrow \quad 1 = \frac{\phi}{4\pi}$$

$$\therefore \quad \phi = 4\pi$$

120. Luminous intensity,

$$L = \frac{\phi}{4\pi}$$

$$\therefore \quad \phi = 4\pi L$$

$$= 4 \times 3.14 \times 100$$

$$= 1256 \text{ lumen}$$

121. The SI unit of illuminance is lumen/m^2 and is called lux.

122. Efficiency of light source is given by

$$\eta = \frac{\phi}{P} \quad \ldots(i)$$

Also, $$L = \frac{\phi}{4\pi} \quad \ldots(ii)$$

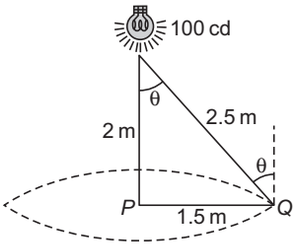
From Eqs. (i) and (ii),

$$P = \frac{4\pi L}{\eta}$$

$$= \frac{4\pi \times 35}{5}$$

$$= 88 \text{ W}$$

123. **For Ist case**



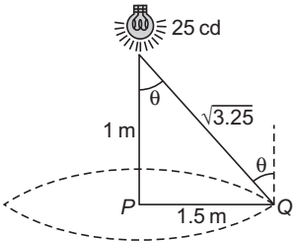
$$I_P = \frac{100}{2^2} = 25 \text{ cd}$$

and $$I_Q = \frac{100}{(2.5)^2}\cos\theta$$

$$= \frac{100}{(2.5)^2} \times \frac{2}{2.5}$$

$$= \frac{200}{(2.5)^3}$$

For IInd case



$$I_Q' = XI_Q$$

$$= \frac{25}{(3.25)^{3/2}}$$

$\therefore$ $$\frac{I_Q'}{I_Q} = \frac{25}{200} \times \frac{(2.5)^3}{(3.25)^{3/2}}$$

$\therefore$ $X = 1/3$

124. $$\frac{I_{\text{centre}}}{I_{\text{edge}}} = \frac{(r^2+h^2)^{3/2}}{h^3} = \frac{\left(1+\frac{1}{4}\right)^{3/2}}{1^3} = \left(\frac{5}{4}\right)^{3/2}$$

125. Intensity at an angle θ is given by

$$I_\theta = I_0 \cos\theta$$

$$= I_0 \cos 60° = \frac{I_0}{2}$$

126. **For Ist case**

$$I_1 = \frac{L}{r_1^2} = \frac{L}{16}$$

For IInd case

$$I_2 = \frac{L}{r_2^2} = \frac{L}{9}$$

$\therefore$ % increase in illuminance

$$= \frac{I_2 - I_1}{I_1} \times 100$$

$$= \left(\frac{16}{9} - 1\right) \times 100$$

$$= 78\%$$

127. $$\frac{L_1}{r_1^2} = \frac{L_2}{r_2^2}$$

$\therefore$ $$\frac{L_1}{L_2} = \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9}$$

128. $$I = \frac{L\cos\theta}{r^2}$$

$\therefore$ $$L = \frac{I \times r^2}{\cos\theta}$$

$$= \frac{(5\times10^{-4})(200)^2}{\cos 60°} = 40 \text{ cd}$$

133. If there had been one eye of the man, then visible region decreases, so the depth of image will not be seen.

135. The persistence of vision is (1/10 s) i.e. if time interval between two consecutive light pulses is lesser than 0.1 s,eye cannot distinguish them separately.

136. In short sightedness, the focal length of eyelens decreases and so the power of eyelens increases.

138. For camera lens,

Time of exposure $\propto$ (f-number)^2

$\Rightarrow$ $$\frac{t_2}{t_1} = \left(\frac{5.6}{2.8}\right)^2 = 4$$

$\therefore$ $$t_2 = 4t_1 = 4 \times \frac{1}{200}$$

$$= \frac{1}{50}\text{s} = 0.02\text{s}$$

140. Magnifying power

$$m_\infty = \frac{(L_\infty - f_o - f_e)\cdot D}{f_o f_e}$$

$\Rightarrow$ $$45 = \frac{(L_\infty - 1 - 5) \times 25}{1 \times 5}$$

$\Rightarrow$ $$L_\infty = 15 \text{ cm}$$

141. When final image is formed at $D = 25$ cm from eye.

In this situation, $v = -D$ from lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

we have $$\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f}$$

i.e. $$\frac{D}{u} = 1 + \frac{D}{f}$$

So, $$MP = \frac{D}{u} = \left(1 + \frac{D}{f}\right)$$

142. Length of compound microscope

$$L_\infty = v_o + f_e$$

$\Rightarrow$ $$14 = v_o + 5$$

$\therefore$ $$v_o = 9 \text{ cm}$$

Magnifying power of microscope for relaxed eye

$$M = \frac{v_o}{u_o} \cdot \frac{D}{f_e}$$

or $$25 = \frac{9}{u_o} \cdot \frac{25}{5}$$

or $$u_o = \frac{9}{5} = 1.8 \text{ cm}$$

143. When final image is formed at infinity, then magnifying power

$$M_\infty = -\frac{v_o}{u_o}\times\frac{D}{f_e}$$

From, $$\frac{1}{f_o}=\frac{1}{v_o}-\frac{1}{u_o}$$

$$\frac{1}{+1.2}=\frac{1}{v_o}-\frac{1}{(-1.25)}$$

$\Rightarrow$ $$v_o = 30\text{ cm}$$

$\therefore$ $$|M_\infty| = \frac{30}{1.25}\times\frac{25}{3}=200$$

144. For objective lens,

$$\frac{1}{f_o}=\frac{1}{v_o}-\frac{1}{u_o}$$

or $$\frac{1}{4}=\frac{1}{v_o}-\frac{1}{(-4.5)}$$

$\Rightarrow$ $$v_o = 36\text{ cm}$$

$\therefore$ $$|M| = \frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$$

$$=\frac{36}{4.5}\left(1+\frac{24}{8}\right)$$

$$=32$$

145. We assume that final image is formed at infinity and

$$M_\infty = \frac{(L_\infty - f_o - f_e)\cdot D}{f_o f_e}=\frac{LD}{f_o f_e}$$

or $$400=\frac{20\times 25}{0.5\times f_e}$$

$\therefore$ $$f_e = 2.5\text{ cm}$$

146. We assume that final image is formed at infinity and so

$$M_\infty = \frac{(L_\infty - f_o - f_e)D}{f_o f_e}=\frac{LD}{f_o f_e}$$

$\therefore$ $$M_\infty = \frac{10\times 25}{0.5\times 1}=500$$

149. If the telescope is reversed i.e. seen from the objective side, then object will appear very small, because in this case magnification becomes reciprocal of initial magnification.

150. For terrestrial telescope magnification,

$$M=\frac{f_o}{f_e}=\frac{80}{f_e}=20$$

$\Rightarrow$ $$f_e = 4\text{ cm}$$

Hence, length of terrestrial telescope

$$= f_o + f_e + 4f = 80 + 4 + 4\times 20$$

$$= 164\text{ cm}$$

151. In a telescope large aperture of objective helps in improving the brightness (image) by gathering more light from distant object.

152. If in an astronomical telescope, the convergent eye piece is replaced by a divergent lens which is placed in such a way that rays from objective are directed towards its focus, final image will be erect, enlarged and virtual, and it is called Gallilean telescope.

153. Magnification of astronomical telescope for normal eye is

$$m=-\frac{f_o}{f_e}=-\frac{100}{2}=-50$$

154. Magnifying power of astronomical telescope

$$m=-\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$$

$$=-\frac{200}{5}\left(1+\frac{5}{25}\right)$$

$$=-48$$

155. In terrestrial telescope there are three lenses, objective, eyepiece and third is erecting lens it is placed at $2f$ from intermediate image.

156. Length of the telescope when final image is formed at least distance of distinct vision is

$$L = f_o + u_e$$

$$= f_o + \frac{f_e D}{f_e + D}$$

$$= 50 + \frac{5\times 25}{5+25}$$

$$=\frac{325}{6}\text{ cm}$$

157. For equal fogging of print

$$I_2\times t_2 = I_1\times t_1$$

or $$\frac{L_2}{r_2^2}\times t_2 = \frac{L_1}{r_1^2}\times t_1 \quad\text{or}\quad \frac{16}{4}\times t_2 = \frac{20}{1}\times 10$$

$\therefore$ $$t_2 = 50\,\text{s}$$

162. $f_o + f_e = 105\,\text{cm}$ and $\frac{f_o}{f_e}=20$

165. For objective,

$$\frac{1}{v_o}-\frac{1}{-200}=\frac{1}{50}$$

$\therefore$ $$v_o=\frac{200}{3}\text{ cm}$$

For eye piece,

$$\frac{1}{-25}-\frac{1}{-u_e}=\frac{1}{5}$$

$\therefore$ $$u_e=\frac{25}{6}$$

Therefore, separation between objective and eye piece,

$$L = v_o + v_e = \frac{425}{6}\approx 71\text{ cm}$$

166. $f_o + f_e = 60\text{ cm}$ …(i)

Given, $$f_e = 5\text{ cm}$$

$\therefore$ $$f_o = 55\text{ cm}$$

In the second case,

$$v_o = (60+10) - f_e$$

$$= 65\text{ cm}$$

From the relation,

$$\frac{1}{v_o}-\frac{1}{u_o}=\frac{1}{f_o}$$

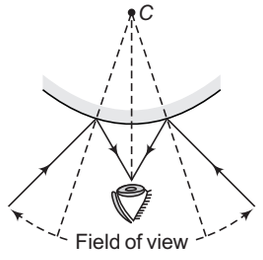
we have, $$\frac{1}{65}+\frac{1}{u_o}=\frac{1}{55}$$

$\therefore$ $$u_o = 357.5\text{ cm}$$

167. From a denser medium a phase difference of π occurs.

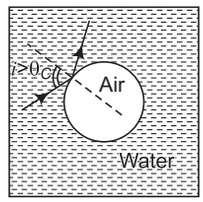
168. Erect, diminished image is formed by convex mirror.

169. In convex mirror, the field of view is increased as compared to plane mirror. This is why it is used as rear view mirror in vehicles.



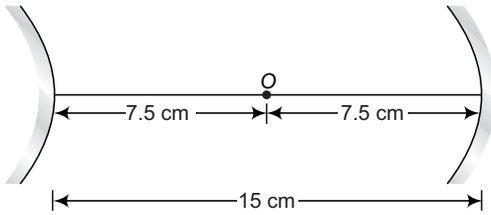
171. In a spaceship, as there is no scattering of light. So, space appears black.

172. An air bubble in water shines due to total internal reflection at its outer surface. Here light is propagating from denser medium (water) to rarer medium (air) and if $i > \theta_C$ total internal reflection takes place at the surface of the bubble.



174. Sky appears blue due to scattering. In absence of atmosphere no scattering will occur.

180. For concave mirror,



$$\frac{1}{v}-\frac{1}{7.5}=-\frac{1}{5} \quad \Rightarrow \quad \therefore \quad v=-15\ \text{cm}$$

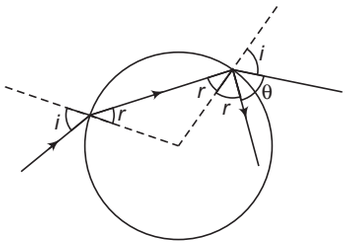
183. $\dfrac{1}{f}=(1.5-1)\left(\dfrac{1}{10}+\dfrac{1}{10}\right)$

$\therefore \quad f = 10\ \text{cm}$

$R = 2f = 20\ \text{cm}$

Level 2 : Only One Correct Option

1. $\sqrt{3}=\dfrac{\sin 60^\circ}{\sin r}$



$\therefore \quad r = 30^\circ$

The asked angle $\quad \theta = 180^\circ - i - r = 90^\circ$

2. Shift from the slab,

$$S=\left(1-\frac{1}{\mu}\right)$$

$$t = 2\ \text{cm}$$

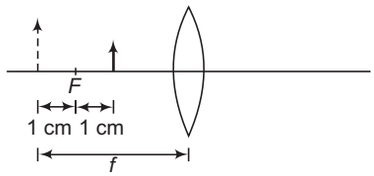
Let x be the distance between object and mirror. Then,

$$x - 2 = 40$$

or $\quad x = 42\ \text{cm}$

3. Object should be placed at distance $2f$.

4. $\dfrac{1}{-(f+1)}-\dfrac{1}{-(f-1)}=\dfrac{1}{f}$



Solving, we get

$$f = (\sqrt{2}+1)\ \text{cm}$$

5. For real image,

$$u = -x$$

$$v = +mx$$

$\therefore \quad \dfrac{1}{mx}-\dfrac{1}{-x}=\dfrac{1}{f}$ ...(i)

For virtual image,

$$u = -y$$

Then $\quad v = -my$

$\therefore \quad \dfrac{1}{-my}-\dfrac{1}{-y}=\dfrac{1}{f}$ ...(ii)

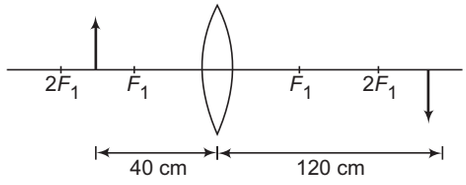
By solving Eqs. (i) and (ii), we get

$$f=\frac{x+y}{2}$$

6. $\dfrac{1}{3x}-\dfrac{1}{-x}=\dfrac{1}{+30}$

$\therefore \quad x = 40\ \text{cm}$

and $\quad 3x = 120\ \text{cm}$



To decrease the magnification object should be moved towards $2F_1$.

Hence, image will move towards $2F_2$. Let displacement is y. Then,

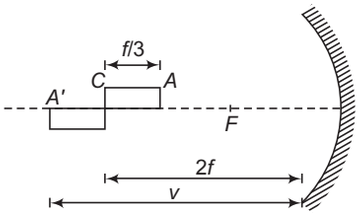
$$2(40+6)=(120-y)$$

$\therefore \quad y = 28\ \text{cm}$

7. Apply $\dfrac{1}{f}=\dfrac{2(\mu_2/\mu_1)}{R_2}-\dfrac{2(\mu_2/\mu_1-1)}{R_1}$

8. For end A of the rod,

$$u = 2f - \frac{f}{3} = \frac{5f}{3}$$



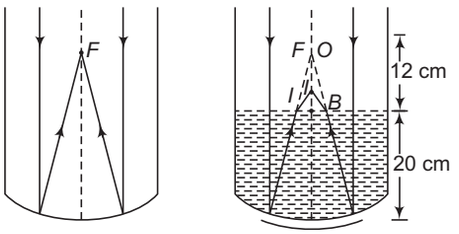
By using, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$$\frac{1}{-f} = \frac{1}{v} + \frac{1}{(-5f/3)}$$

$\therefore$ $v = -\frac{5}{2}f$

So, length of image $= \frac{5}{2}f - 2f = \frac{f}{2}$

9. In first case when sun is at infinity, i.e.



$$u = \infty$$

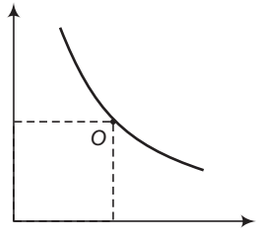
$$\frac{1}{f} = \frac{1}{-32} + \frac{1}{(-\infty)}$$

$\therefore$ $f = -32\,\text{cm}$

When water is filled in that tank upto a height of 20 cm, the image formed by the mirror will act as virtual object O for water surface.

$\therefore$ $BI = BO \times \frac{3}{4} = 12 \times \frac{3}{4} = 9\,\text{cm}$

10. At P, $u = v$ which is possible only when $u = 2f$



If we take another point Q just above P.

11. $\frac{1}{v} - \frac{1}{-u} = \frac{1}{+f}$

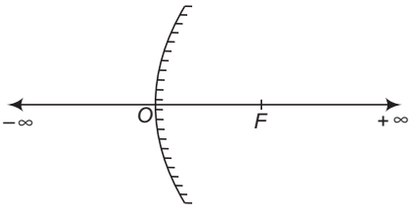
$\therefore$ $v = \frac{uf}{u-f}$

or $\frac{v}{u} = m = \frac{f}{u-f}$

$\therefore$ $\frac{1}{m} = \left(\frac{1}{f}\right)u - 1$

Comparing this with equation of straight line, $\frac{1}{f}$ is the slope of line, which is $\frac{b}{c}$.

12. u changes from 0 to $-\infty$. Then, v will change from 0 to $+f$.



13. $P = 2P_L + P_M$

or $-\frac{1}{F} = 2\left(-\frac{1}{0.2}\right) + 0$

or $F = 10\,\text{cm}$

Therefore, the combination behaves as a convex mirror of focal length 10 cm.

14. In the first case, let

$$u = -x$$

Then, $v = +\frac{x}{2}$

$\therefore$ $\frac{2}{x} + \frac{1}{x} = \frac{1}{f}$

or $\frac{3}{x} = \frac{1}{f}$ ...(i)

In the second case,

$$u = -(x - 30)$$

$$v = 2(x - 30)$$

$\therefore$ $\frac{1}{2(x-30)} + \frac{1}{x-30} = \frac{1}{f}$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$F = 20\,\text{cm}$$

15. From the relation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{-f}$$

$$\frac{1}{v} = \frac{1}{u} - \frac{1}{f}$$

or $v = \left(\frac{uf}{f-u}\right)$

Since, $u > f$, v is negative

or $|v| = \left(\frac{uf}{u-f}\right) > f$

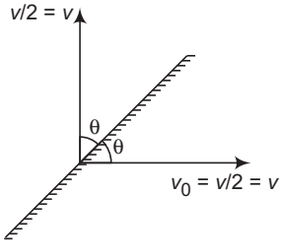
The end which is at infinity will have its image at focus.

$\therefore$ Length of image

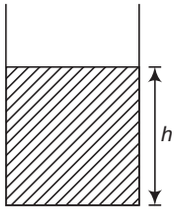
$$L = |v| - f = \frac{f^2}{u-f}$$

16. $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \sqrt{2}$

17. $v_r = \sqrt{v^2 + v^2 - 2v \cdot v \cdot \cos\theta} = 2v \sin\theta$



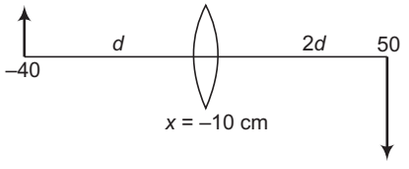
18. $\frac{h}{\mu} = \frac{21}{2} = 10.5$



$\therefore \quad h = 10.5 \times \frac{4}{3} = 14\,\text{cm}$

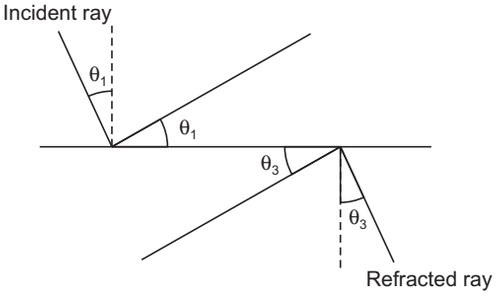
19. Magnitude magnification is 2

$3d = 90$



$\therefore \quad d = 30\,\text{cm}$

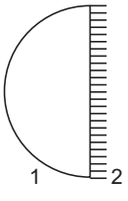
20. $\mu = \frac{\sin\theta_1}{\sin\theta_3}$



21. $\frac{1}{v} = \frac{1.5}{-6} = \frac{1-1.5}{-10}$

$\therefore \quad v = -5\,\text{cm}$

22. $\frac{1}{f} = \frac{2\mu}{R_2} - \frac{2(\mu-1)}{R_1}$



$\frac{1}{-10} = \frac{2 \times 1.5}{\infty} - \frac{2(1.5-1)}{R}$

$\therefore \quad R = 10\,\text{cm}$

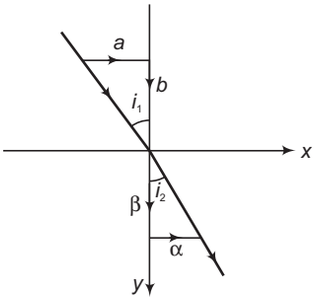
23. From Snell's law,

$\mu \sin i = \text{constant}$

$\mu_4 \sin x = \mu \sin i$

$\therefore \quad \sin x = \frac{\mu}{\mu_4} \sin i$

24. $\mu_1 \sin i_1 = \mu_2 \sin i_2$

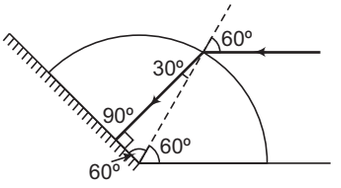


$\therefore \quad \frac{\mu_1 a}{\sqrt{a^2 + b^2}} = \frac{\mu_2 \alpha}{\sqrt{\alpha^2 + \beta^2}}$

$\sqrt{a^2 + b^2} = \sqrt{\alpha^2 + \beta^2} = 1$

$\therefore \quad \mu_1 a = \mu_2 \alpha$

25. $\mu = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$



26. $\frac{\mu_2}{v_1} - \frac{\mu_1}{\infty} = \frac{\mu_2 - \mu_1}{R}$ ...(i)

$\frac{\mu_3}{f} - \frac{\mu_2}{v_1} = \frac{\mu_3 - \mu_2}{-R}$ ...(ii)

From these two equations, we get

$\frac{\mu_3}{f} = \frac{\mu_2 - \mu_1 - \mu_3 + \mu_2}{R}$

Lens will become diverging if f is negative or $2\mu_2 < (\mu_1 + \mu_3)$.

27. Lens should be convex. Images should be magnified 16 cm length should lie between f and $2f$. Because corresponding to 6 cm ($< f$) length image should be magnified.

From the given options if $f = 11$ cm.

Then, $6\,\text{cm} < f$ and $11\,\text{cm} < 16\,\text{cm} < 22\,\text{cm}$

or $\quad f < 16\,\text{cm} < 2f$

28. $y_a = \left(\frac{n_1}{n_2}\right) y$

Here, $y = $ actual depth and

$y_a = $ apparent depth

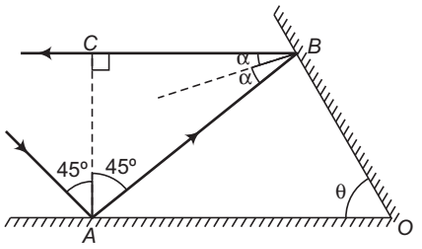
$\therefore \quad y = \left(\frac{n_2}{n_1}\right) \cdot y_a$

$$\left(-\frac{dy}{dt}\right)=\frac{n_2}{n_1}\left(-\frac{dy_a}{dt}\right)=\frac{n_2 x}{n_1}$$

$$\therefore \quad \frac{dy}{dt}=A\cdot\left(-\frac{dy}{dt}\right)$$

$$=\frac{\pi R^2 n_2 x}{n_1}$$

29. From the figure in ΔABC, $45°+90°+2\alpha=180°$



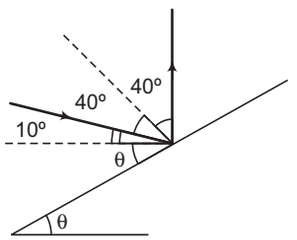
$$\Rightarrow \quad \alpha=22.5°$$

$$\angle OBA=90°-\alpha$$

In ΔOAB, $\quad \theta+45°+90°-\alpha=180°$

$$\Rightarrow \quad \theta=67.5°$$

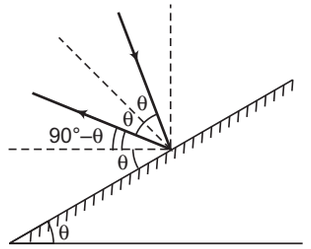
30. If $\theta=40°$, ray of light will become vertical after reflection.



31. $v_{CM}=8\,\text{m/s}$

$$\therefore \quad v_{IM}=8\,\text{m/s}$$

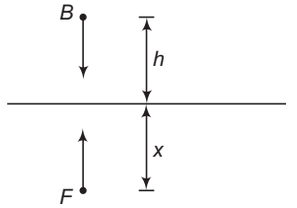
32. $90°-\theta+2\theta=90°$



$$\therefore \quad \theta=0°$$

Angle of reflected ray with horizontal $=90°-\theta=90°$

33. Fish is observer and bird is object



Apparent distance between F and B at some instant will be

$$y=(x+\mu h)$$

$$\left(-\frac{dy}{dt}\right)=\left(-\frac{dx}{dt}\right)+(\mu)\left(-\frac{dh}{dt}\right)$$

$$9=3+\frac{4}{3}\left(-\frac{dh}{dt}\right)$$

$$\therefore \quad -\frac{dh}{dt}=4.5\,\text{m/s}$$

34. $\delta=i_1+i_2-A$

$$45°=(i_1+i_2)-60°$$

$$\therefore \quad i_1+i_2=105° \qquad \ldots(i)$$

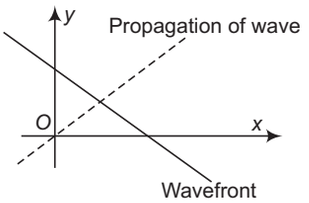
$$i_1-i_2=20° \qquad \ldots(ii)$$

Solving these two equations we get,

$$i_1=62.5°$$

and $\quad i_2=42.5°$

35.



36. Using mirror formula,

$$\frac{1}{v}+\frac{1}{-u}=\frac{1}{-f}$$

Multiplying with u,

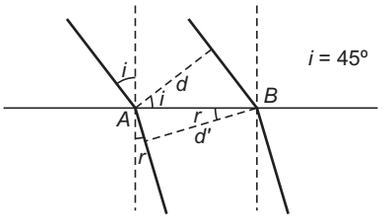
$$\frac{u}{v}=\frac{-u}{f}+1=\frac{f-u}{f}$$

$$\therefore \quad m=\frac{v}{u}=\left(\frac{f}{f-u}\right)$$

Now $\quad |dv|=m^2\,|du|$

Size of image $=\left(\frac{f}{f-u}\right)^2\cdot b$

37. $AB=\dfrac{d}{\cos i}=\dfrac{d'}{\cos r}$



$$\therefore \quad d'=\frac{\cos r}{\cos i}\,d \qquad \ldots(i)$$

Further, $\quad \sin r=\dfrac{\sin i}{\mu}=\dfrac{1}{\sqrt{2}\,\mu}$

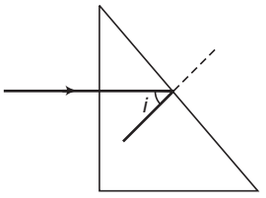
Hence $\quad \cos r=\sqrt{1-\dfrac{1}{2\mu^2}}$

$$=\frac{\sqrt{2\mu^2-1}}{\sqrt{2}\,\mu}$$

Substituting in Eq. (i), we have

$$d'=\frac{\sqrt{2\mu^2-1}}{\mu}\cdot d$$

38. $\angle i = 45°$



$$\sin i = \frac{1}{\sqrt{2}}$$

For only red colour $\angle i$, its critical angle, i.e. green and blue colours get total internally reflected.

39. Since, deviation $\delta = (\mu - 1)A = (1.5 - 1) \times 5° = 2.5°$

By geometry, angle of refraction by first surface is 5°.

But $\delta = \theta - r$, so, we have, $2.5° = \theta - 5°$ on solving $\theta = 7.5°$.

40. Since $v \propto \lambda$, the light of red colour is of highest wavelength and therefore of highest speed. Therefore, after travelling through the slab, the red colour emerge first.

41. When an object approaches a convergent lens from the left of the lens with a uniform speed of 5 m/s, the image away from the lens with a non-uniform acceleration.

42. According to VIBGYOR, among all given sources of light, the blue light have smallest wavelength.According to Cauchy relationship, smaller the wavelength higher the refractive index and consequently smaller the critical angle.

So, corresponding to blue colour, the critical angle is least which facilitates total internal reflection for the beam of blue light. The beam of green light would also undergo total internal reflection.

43. Here, light ray goes from (optically) rarer medium air to optically denser terpentine, then it bends towards the normal i.e. $i>r$ whereas when it goes from to optically denser medium terpentine to rarer medium water. then it bends away the normal i.e. $i<r$.

44. The speed of the image of the car would appear to increase as the distance between the cars decreases.

45. The negative refractive index metamaterials are those in which incident ray from air (Medium 1) to them refract or bend differently to that of positive refractive index medium.

46. For the interference pattern to be formed on the screen, the sources should be coherent and emits lights of same frequency and wavelength.

In a Young's double-slit experiment, when one of the holes is covered by a red filter and another by a blue filter. In this case due to filteration only red and blue lights are present. In Young's double-slit experiment monochromatic light is used for the formation of fringes on the screen. Hence, in this case there shall be no interference fringes.

47. According to question, there is a hole at point P_2. From Huygen's principle, wave will propagates from the sources S_1 and S_2. Each point on the screen will acts as secondary sources of wavelets.

Now, there is a hole at point P_2 (minima). The hole will act as a source of fresh light for the slits S_3 and S_4.

Therefore, there will be a regular two slit pattern on the second screen.

More than One Correct Options

1. For real image,

Let $u = -x$, then $v = -2x$

$$\therefore \quad \frac{1}{-2x} + \frac{1}{-x} = \frac{1}{-20}$$

Solving, we get $x = 30$ cm.

For virtual image,

Let $u = -x$, then $v = +2x$

$$\therefore \quad \frac{1}{2x} + \frac{1}{-x} = \frac{1}{-20}$$

or $$x = 10\,\text{cm}$$

2. Inverted and real image is formed by concave mirror.

Let $u = -x$, then $v = +x/2$

$$\therefore \quad \frac{1}{+x/2} + \frac{1}{-x} = \frac{1}{+f}$$

$$\therefore \quad x = 3f$$

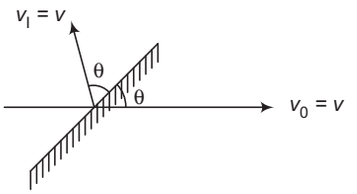
Erect and virtual image is formed by convex mirror.

Let, $$u = -x \quad \text{then} \quad v = +\frac{x}{2}$$

$$\frac{1}{-x/2} + \frac{1}{-x} = \frac{1}{+f}$$

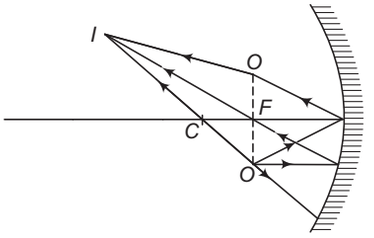
$$x = +f$$

3.



$$v_r = \sqrt{v^2 + v^2 - 2v \cdot v\cos 2\theta} = 2v\sin\theta$$

4. Ray diagram is as shown in figure.



5. $\frac{1}{v} + \frac{1}{-30} = \frac{1}{-20}$

Solving we get, $v = -60$ cm

$$m = \frac{v}{u} = -\frac{(-60)}{(-30)}$$

$$= -2$$

Speed of image (in event -1) is m^2 times and m times in event-2.

6. In event -1

For concave mirror,

$$\frac{1}{v} + \frac{1}{-3f} = \frac{1}{-f}$$

$$\therefore \quad v = -1.5f$$

For convex mirror,

$$\frac{1}{v}+\frac{1}{-3f}=\frac{1}{+f}$$

$$v=\frac{3}{4}f=0.75f$$

For plane mirror,

$$v=+3f$$

In event –2

For concave mirror,

$$\frac{1}{v}+\frac{1}{-1.5f}=\frac{1}{-f}$$

$$\therefore \quad v=-3f$$

For convex mirror,

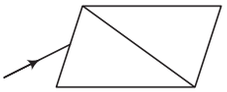
$$\frac{1}{v}+\frac{1}{-1.5f}=\frac{1}{+f}$$

$$\therefore \quad v=\frac{3f}{5}=0.6f$$

For plane mirror,

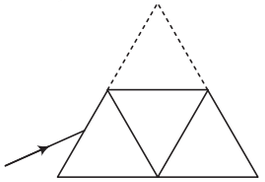
$$v=+1.5f$$

7. For $n=2m$, it is just like slab.



$\therefore$ Deviation $=0$

For $n=2m+1$, it is just like an identical prism of larger size.



8. $\frac{\sin i_1}{\sin i_2}=\frac{v_1}{v_2}$

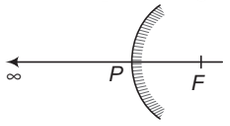
$$\therefore \quad \frac{\sin r}{\sin i}=\text{slope}=\frac{1}{\sqrt{3}}=\frac{v_y}{v_x}$$

$$\therefore \quad v_y=\frac{v_x}{\sqrt{3}}$$

Speed of light in medium-y is less. So, it is denser. T/R takes place when ray of travels from denser to rarer medium.

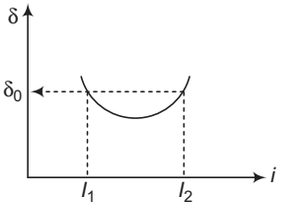
9. (a) If vacuum speed of light of all colours is same.

(b)



If object moves from ∞ to P then its virtual, erect and diminished image moves from F to P.

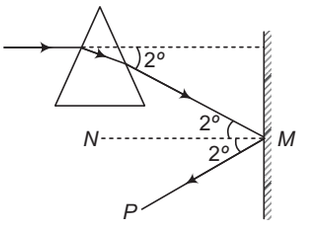
(c)



10. Displacement method of finding focal length of convex lens.

11. Deviation, $\delta=(\mu-1)A=(1.5-1)\,4°=2°$

(a)



To rotate ray MP by 2° (to make it parallel to MN) we will have to raotate the mirror only by1°.

(b) Without prism, ray of light was falling normal to plane mirror. So, ray of light was retracing its path prism has deviated it 2°. So if we rotate the mirror by 2°, ray of light further falls normal to plan mirror and retraces its path.

12. When immersed object is seen from close to the edge of the trough the object looks distorted because the apparent depth of the points close to the edge are nearer the surface of the water compared to the points away from the edge.

The angle subtended by the image of the object at the eye is smaller than the actual angle subtended by the object in air and some of the points of the object far away from the edge may not be visible because of total internal reflection.

13. For $\mu=1.6$, the critical angle, $\mu=1/\sin C$, we have $C=38.7°$, when viewed from AD, as long as angle of incidence on AD of the ray emanating from pin is greater than the critical angle , the light suffers from total internal reflection and cannot be seen through AD.

14. A magnifying glass is used, as the object to be viewed can be brought closer to the eye than the normal near point. This results in a larger angle to be subtended by the object at the eye and hence, viewed in greater detail. Morever, the formation of a virtual erect and enlarged image, takes place.

15. The length of the telescope tube is

$$f_o+f_e=20+(0.02)=20.02\,\text{m}$$

Also, $m=20/0.02=1000$

Also, the image formed is inverted .

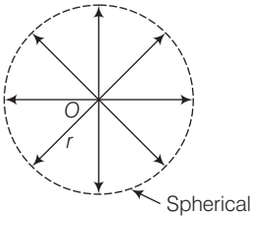
16. Consider the pattern of the intensity shown in the figure

(i) As intensities of all successive minima is zero, hence we can say that two sources S_1 and S_2 are having same intensities.

(ii) As width of the successive maxima (pulses) increases in continuous manner, we can say that the path difference (x) or phase difference varies in continuous manner.

(iii) We are using monochromatic light in YDSE to avoid overlapping and to have very clear pattern on the screen.

17. Consider the diagram in which light diverges from a point source (O).



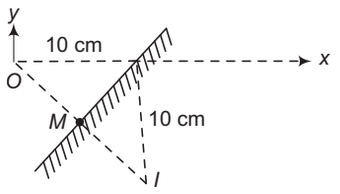
Due to the point source light propagates in all directions symmetrically and hence, wavefront will be spherical as shown in the diagram.

If power of the source is P, then intensity of the source will be

$$I = \frac{P}{4\pi r^2}$$

where, r is radius of the wavefront at any time.

Comprehension Based Questions

1. 

$$OM = MI$$

Coordinates of I are (10 cm, -10 cm)

2. Object is placed at centre at curvature of mirror. Hence, image is at the same point, real, inverted and of same size. Hence, coordinates are (20 cm, 0).

3. $\frac{1}{v} + \frac{1}{-20} = \frac{1}{+10}$

$$\therefore \quad v = +\frac{20}{3}\text{ cm}$$

$$m = -\frac{v}{u} = -\frac{(+20/3)}{(-20)} = +\frac{1}{3}$$

$$\therefore \quad I = 10\left(\frac{1}{3}\right) = \frac{10}{3}\text{ cm}$$

$$x = 10 - \frac{10}{3} = \frac{20}{3}\text{ cm}$$

and $$y = 20 + \frac{20}{3} = \frac{80}{3}\text{ cm}$$

4. Plane mirror forms image at equal distance on opposite side.

Hence, $x = 0, y = 40$ cm

5. Using the mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{-40} - \frac{1}{10}$$

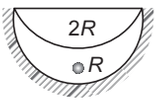
Solving we get, $f = -8$ cm

6. Focal length of lenses is

$$\frac{1}{F} = (1.8-1)\left(\frac{1}{\infty} - \frac{1}{-2R}\right) + (1.2-1)\left(\frac{1}{-2R} - \frac{1}{-R}\right)$$

$$F = 2R$$

Now, combined power of system,



$$P = 2P_L + P_m$$

or $$-\frac{1}{f} = 2P_L + P_m$$

or $$\frac{1}{8} = 2\left(\frac{1}{2R}\right) + \frac{2}{R}$$

or $$R = 24\text{ cm}.$$

$\therefore$ Radius of curvature of common surface

$$= 2R = 48\text{ cm}$$

7. Combined focal length of lens,

$$\frac{1}{f} = (1.2-1)\left(\frac{1}{+24} - \frac{1}{+48}\right) + (1.8-1)\left(\frac{1}{+48} - \frac{1}{\infty}\right)$$

$$= \frac{1}{48}$$

Now combined power of system

$$P = 2P_L + P_M$$

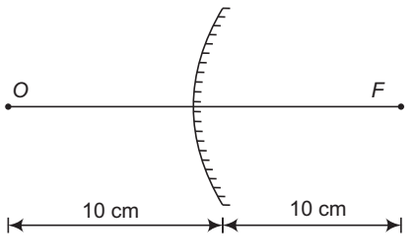
$$\therefore \quad -\frac{1}{F} = 2\left(\frac{1}{f}\right) + \infty = 2\left(\frac{1}{48}\right)$$

$$\therefore \quad F = -24\text{ cm}$$

Assertion and Reason

2. Nature of lens will change only if refractive index of liquid is more.

7. Object is not placed at focus.



8. $P = \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{R}\right) = 0$

9. For virtual object (u = positive)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

If $u < f$, v = + ve, i.e. image may be real.

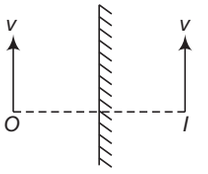
10. When a diverging lens is immersed in a liquid, its diverging power will either decrease or it will become converging.

11. $A_t = \left(\frac{2v_2}{v_1 + v_2}\right)A_i$

If $v_2 > v_1$, $A_t > A_i$

12. Angle i is the angle of ray of light with normal in air (or vacuum).

13. In the shown figure object and image are travelling in same direction.



14. $\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{R}\right) = 0$

$$\therefore \quad P_1 = 0$$

$$\frac{1}{f_2} = (\mu - 1)\left(\frac{1}{-R} - \frac{1}{-R}\right) = 0$$

$$\therefore \quad P_2 = 0$$

15. In rarer medium ray of light moves away from the normal. Further,

$$\mu_1 \sin 30^\circ = \mu_2 \sin 60^\circ$$

$$\therefore \quad \frac{\mu_1}{\mu_2} = {}_2\mu_1 = \frac{\sin 60^\circ}{\sin 30^\circ}$$

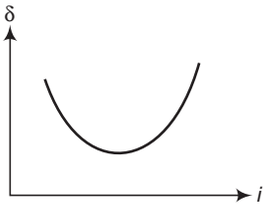
16. If λ is more, μ will be less as per Cauchy's formula $\left(\mu = A + \frac{B}{\lambda^2}\right)$.

Therefore, focal length will be more.

17. Glass slab will shift the object towards mirror. If it comes between pole and focus image will become virtual.

18. A convex mirror can also make a real image if object is virtual.

19. At minimum deviation,



$$r_1 = r_2 = \frac{A}{2}$$

Match the Columns

2. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

and $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

In case of biconvex lens,

$$|R_1| = |R_2|$$

3. Convex mirror and concave lens always make virtual and diminished images. In case of concave mirror object is placed between F and C. In case of convex lens object is placed between F and $2F$. In both the cases image is real and magnified.

4. If O and I are on the same side of AB, image is erect and virtual. If they are on opposite sides image is inverted and real.

5. $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ or $P = P_1 + P_2$

f_1 and P_1 are positive, f_2 and P_2 are negative.

Entrance Gallery

1. From lens maker's formula

$$\frac{1}{f_{\text{film}}} = (n_1 - 1)\left(\frac{1}{R} - \frac{1}{R}\right) \quad \Rightarrow \quad f_{\text{film}} = \infty \text{ (infinite)}$$

$\therefore$ There is no effect of presence of film.

From Air to Glass

Using the equation $\frac{n_2}{v} - \frac{1}{u} = \frac{n_2 - 1}{R}$ $\frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5 - 1}{R}$

$$\Rightarrow \quad v = 3R$$

$$\therefore \quad f_1 = 3R$$

From Glass to Air

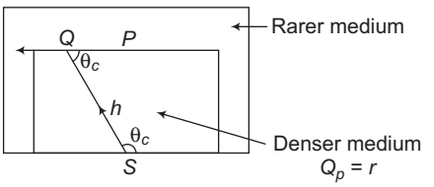
Again using the same equation

$$\frac{1}{v} - \frac{n_2}{u} = \frac{1 - n_2}{-R}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1.5}{\infty} = \frac{1 - 1.5}{-R}$$

$$\Rightarrow \quad v = 2R \quad \therefore f_2 = 2R$$

2. At point Q angle of incidence is critical angle θ_C, where



$$\sin\theta_C = \frac{\mu_i}{\mu_{\text{block}}}$$

In ΔPQS, $\sin\theta_C = \frac{r}{\sqrt{r^2 + h^2}}$

$$\therefore \quad \frac{\mu_l}{\mu_{\text{block}}} = \frac{r}{\sqrt{r^2 + h^2}}$$

$$\Rightarrow \quad \mu_l = \frac{r}{\sqrt{r^2 + h^2}} \times 2.72 = \frac{5.77}{11.54} \times 2.72 = 1.36$$

3. It is based on lens maker's formula and its magnification

i.e. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

According to lens maker's formula, when the lens in the air

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \frac{1}{2x} \quad \Rightarrow \quad f = 2x$$

Here, $\left(\frac{1}{x} = \frac{1}{R_1} - \frac{1}{R_2}\right)$

In case of liquid, where refractive index is $\frac{4}{3}$ and $\frac{5}{3}$, we get

Focal length in first liquid

$$\frac{1}{f_1} = \left(\frac{\mu_s}{\mu_{l_1}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f_1} = \left(\frac{\frac{3}{2}}{\frac{4}{3}} - 1\right)\frac{1}{x}$$

$\Rightarrow$ f_1 is positive.

$$\frac{1}{f_1} = \frac{1}{8x} = \frac{1}{4(2x)} = \frac{1}{4f} \quad \Rightarrow \quad f_1 = 4f$$

Focal length in second liquid

$$\frac{1}{f_2} = \left(\frac{\mu_s}{\mu_{l_2}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{\frac{3}{2}}{\frac{5}{3}} - 1\right)\frac{1}{x}$$

$\Rightarrow$ f_2 is negative.

4. The focal length of lens



(P) $\frac{1}{f}=\left(\frac{3}{2}-1\right)\left(\frac{1}{r}+\frac{1}{r}\right)=\frac{1}{r} \Rightarrow f=r$

$\Rightarrow \frac{1}{f_{eq}}=\frac{1}{f}+\frac{1}{f}=\frac{2}{r} \Rightarrow f_{eq}=\frac{r}{2}$



(Q) $\frac{1}{f}=\left(\frac{3}{2}-1\right)\left(\frac{1}{r}\right) \Rightarrow f=2r$

$\Rightarrow \frac{1}{f}+\frac{1}{f}=\frac{2}{f}=\frac{1}{r}$

$\Rightarrow f_{eq}=r$



(R) $\frac{1}{f}=\left(\frac{3}{2}-1\right)\left(-\frac{1}{r}\right)=-\frac{1}{2r}$

$\Rightarrow f=-2r$

$\Rightarrow \frac{1}{f_{eq}}=\frac{1}{f}+\frac{1}{f}=-\frac{2}{2r}$

$\Rightarrow f_{eq}=-r$



(S) $\frac{1}{f_{eq}}=\frac{1}{r}+\frac{1}{-2r}=\frac{1}{2r}$

$\Rightarrow f_{eq}=2r$



$(P) \rightarrow 2$;

$(Q) \rightarrow 4$;

$(R) \rightarrow 3$;

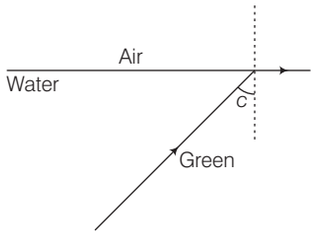
$(S) \rightarrow 1$

5. For total internal reflection of light take place, following conditions must be obeyed.

(*i*) The ray must travel from denser to rarer medium.

(*ii*) Angle of incidence (θ) must be greater than or equal to critical angle (C) i.e.

$$C=\sin^{-1}\left[\frac{\mu_{rarer}}{\mu_{denser}}\right]$$



Here, $\sin C=\frac{1}{n_{water}}$ and $n_{water}=a+\frac{b}{\lambda^2}$

If frequency is less and λ is greater and hence, RI $n_{(water)}$ is less and therefore, critical angle increases. So, they do not suffer reflection and come out at angle less than 90°.

6. The apparent flattering of the sun at sunset and sunrise is due to refraction, because the sun becomes visible a little before actual sunrise and remains visible a little after actual sunset, because of atmosphere refraction of light.

7. The mismatch is for rear view–Concave mirror.

8. By compound microscope,

$$m=\frac{L}{f_0}\left(1+\frac{D}{f_e}\right)$$

where, length of tube, $L=30$ cm, focal length of objective lens, $f_0=1$cm, focal length of eye piece, $f_e=6$cm, $D=25$cm)

$$=\frac{30}{1}\left(1+\frac{25}{6}\right)=30\times\frac{(6+25)}{6}$$

$$=5\times 31=155 \text{ cm} \cong 150$$

9. The focal length of the lens $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

$\Rightarrow f=5$ cm

$$f=\frac{uv}{u+v}$$

$$\Rightarrow \frac{\Delta f}{f}=\left|\frac{\Delta u}{u}\right|+\left|\frac{\Delta v}{v}\right|+\frac{|\Delta u|+|\Delta v|}{|u|+|v|}$$

$$\Delta f=0.15$$

The most appropriate answer is 5.00 ± 0.10 cm.

10. Given, $f=10$ cm,

The power of lens $P=\frac{1}{\text{focal length (m)}}=\frac{1}{f\text{ (m)}}$

or $P=\frac{100}{f(\text{cm})}=\frac{100}{10}=10$ D

11. We know that,

Linear magnification, $m=\frac{f}{f-u}$

Given, object displaced $u=-20$ cm

$m=-3$ ($\because$ all real image are inverted)

So, $-3=\frac{f}{f-(-20)}$

$\Rightarrow -3=\frac{f}{f+20}$

$\Rightarrow -3f-60=f$

$4f=-60$

$\Rightarrow f=-\frac{60}{4}=-15$ cm

Since, mirror is concave $f=-15$ cm

12. By Snell's law,

$\mu \sin i=$ constant

$\mu \sin i=(\mu-m\Delta\mu)\sin r$

where, $\mu=1.5, i=30°, r=90°, \Delta\mu=0.015$

$1.5 \sin 30°=(1.5-m\times 0.15 \sin 90°$

$\frac{3}{2}\times\frac{1}{2}=(1.5-m\times 0.015)\times 1$

$\frac{3}{4}=\frac{3}{2}-\frac{15m}{1000}$

$\Rightarrow 15m=\left(\frac{3}{2}-\frac{3}{4}\right)\times 1000$

$15m=\frac{6-3}{4}\times 1000$

$\Rightarrow m=\frac{3}{4}\times\frac{1000}{15}$

$m=\frac{3000}{60}$

$\Rightarrow m=50$

13. Focal length of a convex lens by displacement method

$$f=\frac{a^2-b^2}{4a}$$

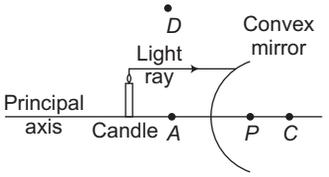
where, $a=$ distance between the image and object and $b=$ distance between two positions of lens.

Fact The distance between the two points should be greater than four times the focal length of the convex lens, i.e. $d > 4f$

So, the maximum possible focal length of this convex lens is $\frac{d}{4}$.

14. For mirror, $m = \frac{-v}{u} = \frac{h'}{h} = n$

For real image from concave mirror u and v both are negative,



So, $v = -nu$

$\Rightarrow$ From mirror equation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{-nu} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{u}\left(1 - \frac{1}{n}\right) = \frac{1}{f}$$

$$f\left(\frac{n-1}{n}\right) = u$$

15. Mirror equation can be represented as

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

16. From the formula the refraction at the spherical surface,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\Rightarrow \quad \frac{1.67}{v} - \frac{1}{(-95)} = \frac{1.67 - 1}{25}$$

$$\frac{1.67}{v} = \frac{0.67}{25} - \frac{1}{95}$$

$$= 0.0268 - 0.0105 = 0.0163$$

$$v = \frac{1.67}{0.0163} = 102.45 \approx 99.4$$

17. Angle of minimum deviation,

$$\delta_{\min} = (\mu - 1)A = \left(\frac{3}{2} - 1\right)42^\circ = \frac{1}{2} \times 42 = 21^\circ$$

18. Given, $m = 30$ and $f_e = 5$ cm

$$m = m_o \times m_e$$

Magnification of compound microscope

$$m = m_0 \times \left(1 + \frac{D}{f_e}\right) \quad \Rightarrow \quad 30 = m_0\left(1 + \frac{25}{5}\right)$$

$$\Rightarrow \quad m_o = 5$$

19. Component along the plane $= \frac{1}{2}$ and component perpendicular to the plane $= \frac{\sqrt{3}}{2}$

$$\therefore \quad \tan i = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad i = 30^\circ = \text{angle of incidence}$$

20. For $e \to i$, $45^\circ > \theta_c$

$$\sin 45^\circ > \sin \theta_c$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \quad \mu_1 > \sqrt{2}\,\mu_2$$

For $e \to f$, angle of refraction is lesser than angle of incidence, so $\mu_2 > \mu_1$ and then $\mu_2 > \mu_3$

For $e \to g, \mu_1 = \mu_2$

For $e \to h, \mu_2 < \mu_1 < \sqrt{2}\,\mu_2$ and $\mu_2 > \mu_3$

21. Refractive index $\mu = \frac{\lambda_{\text{air}}}{\lambda_{\text{medium}}} = \frac{1}{(2/3)} = \frac{3}{2}$

Further, $|m| = \frac{1}{3} = \left|\frac{v}{u}\right|$

$$\therefore \quad |v| = \frac{|u|}{3}$$

$$u = -24 \text{ m} \qquad \text{(Real object)}$$

$$\therefore \quad v = +8 \text{ m} \qquad \text{(Real image)}$$

Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = (\mu - 1)\left(\frac{1}{+R} - \frac{1}{\infty}\right)$

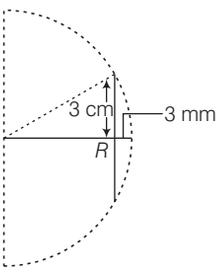
$$\therefore \quad \frac{1}{8} + \frac{1}{24} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R}\right)$$

$$\therefore \quad R = 3 \text{ m}$$

22. By Pythagoras theorem,

$$R^2 = (3)^2 + (R - 0.3)^2$$

$$\Rightarrow \quad R \approx 15\,\text{cm}$$



Refractive index of material of lens $\mu = \frac{c}{v}$

Here, c = speed of light in vacuum $= 3 \times 10^8$ m/s v = speed of light in material of lens $= 2 \times 10^8$ m/s

$$\mu = \frac{3 \times 10^8}{2 \times 10^8} = \frac{3}{2}$$

From lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

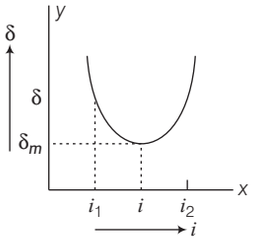
Here, $R_1 = R$ and $R_2 = \infty$ (For plane surface)

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$$

$$\Rightarrow \quad f = 30\,\text{cm}$$

23. We know that the angle of deviation depends upon the angle of incidence. If we determine experimentally, the angles of deviation corresponding to different angles of incidence and then plot i (on x-axis) and δ (on y-axis),

we get a curve as shown in the figure.



Clearly if angle of incidence is gradually increased, from a small value, the angle of deviation first decreases, becomes minimum for a particular angle of incidence and then begins to increase.

24. Given,

Refractive index of the lens $= n_1$

Refractive index of the second medium $= n_2$

Hence, parallel beam of light, after refraction through the lens, diverges only, if the refractive index of the second medium is greater than first medium of the lens.

Hence, $n_2 > n_1$.

25. If object in a denser medium is seen from a rarer medium, then image of object will appear at a lesser distance. The distance between object and its image, called as normal shift is given by

$$x = t\left[1 - \frac{1}{\mu}\right]$$

Here, $t = 6$ cm, and $\mu = 1.5$

$$\therefore \quad x = 6\left[1 - \frac{1}{1.5}\right] = 6\left[\frac{0.5}{1.5}\right] = 2 \text{ cm}$$

26. Using the lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

or

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f_1} + \frac{1}{f_2}$$

$$= \frac{1}{u} + (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_2 - 1)\left(\frac{1}{R_1'} - \frac{1}{R_2'}\right)$$

Substituting the values, we get

$$\frac{1}{v} = \frac{1}{-40} + (1.5 - 1)\left(\frac{1}{14} - \frac{1}{\infty}\right) + (1.2 - 1)\left(\frac{1}{\infty} - \frac{1}{-14}\right)$$

Solving this equation, we get

$$v = +40 \text{ cm}$$

27. Shift in image position due to glass plate,

$$S = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{1}{1.5}\right) \times 1 \text{ cm} = \frac{1}{3} \text{ cm}$$

For focal length of the lens,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{12} - \frac{1}{-240}$$

or

$$\frac{1}{f} = \frac{20 + 1}{240}$$

$$\Rightarrow \quad f = \frac{240}{21} \text{cm}$$

Now, to get back image on the film, lens has to form image at $\left(12 - \frac{1}{3}\right)\text{cm} = \frac{35}{3}$ cm such that the glass plate will shift the image on the film.

As,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{3}{35} - \frac{21}{240}$$

$$= \frac{48 \times 3 - 7 \times 21}{1680} = -\frac{1}{560}$$

$$\Rightarrow \quad u = -5.6 \text{ m}$$

28. For convex mirror, $u = -10$ cm, $f = 20$ cm

We have,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{20} = \frac{1}{v} + \frac{1}{-10} = \frac{1}{v} - \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{20} + \frac{1}{01} = \frac{1 + 2}{20} = \frac{3}{20}$$

$$\Rightarrow \quad v = \frac{20}{3} \text{cm}$$

29. Refractive index of liquid $\mu = 4/3$ and height $= h$

Apparent height $h' = \frac{h}{\mu} = \frac{h}{4/3}$

$$\frac{21}{2} = \frac{h \times 3}{4}$$

$$h = \frac{21 \times 4}{3 \times 2} = 14 \text{ m}$$

30. After critical angle reflection will be 100% and transmission is 0%. Options (b) and (c) satisfy this condition. But option (c) is the correct option. Because in option (b) transmission is given 100% at $\theta = 0°$, which is not true.

$\therefore$ Correct answer is (c).

31. From mirror formula, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\Rightarrow \quad -\frac{1}{u^2}\frac{du}{dt} - \frac{1}{v^2}\frac{dv}{dt} = 0$$

$$\Rightarrow \quad \frac{dv}{dt} = -\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$$

But

$$\frac{v}{u} = \frac{f}{u - f}$$

$$\therefore \quad \frac{dv}{dt} = -\left(\frac{f}{u - f}\right)^2\left(\frac{du}{dt}\right)$$

$$= \left(\frac{0.2}{-2.8 - 0.2}\right)^2 \times 15$$

$$= \frac{1}{15} \text{ms}^{-1}$$

32. As refractive index for $z > 0$ and $z \le 0$ is different X-Y plane should be boundary between two media.

Angle of incidence,

$$\cos i = \left|\frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}\right| = \frac{1}{2}$$

$$\therefore \quad i = 60°$$

From Snell's law

$$\frac{\sin i}{\sin r} = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\Rightarrow \quad r = 45°$$

33. Apparent shift $\Delta h = \left(1 - \frac{1}{\mu}\right) h$

$\therefore$ Apparent shift produced by water,

$$\Delta h_1 = \left(1 - \frac{1}{\mu_1}\right) h_1$$

and apparent shift produced by kerosene,

$$\Delta h_2 = \left(1 - \frac{1}{\mu_2}\right) h_2$$

The apparent shift in the position of the bottom of the beaker when viewed from the above and apparent shift produced by kerosene

$$\Delta h = \Delta h_1 + \Delta h_2 = \left(1 - \frac{1}{\mu_1}\right) h_1 + \left(1 - \frac{1}{\mu_2}\right) h_2$$

34. The focal length of the lens $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Also, by Cauchy's formula

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots$$

$$\lambda_{\text{blue}} < \lambda_{\text{red}} \quad \Rightarrow \quad \mu_{\text{blue}} > \mu_{\text{red}} \quad \Rightarrow \quad f_{\text{red}} > f_{\text{blue}}$$

Therefore, monochromatic red light is used instead of blue light in convex lens and its focal longth will increase.

35. $\frac{f_w}{f_a} = \frac{\mu - 1}{\frac{\mu_w}{\mu_a} - 1}$

$$\Rightarrow \qquad \frac{f_w}{10} = \frac{1.5 - 1}{1/8}$$

$$\Rightarrow \qquad f_w = \frac{0.5 \times 10}{1/8} = 40\,\text{cm}$$

36. Refractive index ${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{3/2}{4/3}\mu = \frac{9}{8}$

$$\sin C = \frac{1}{{}_w\mu_g} = \frac{8}{9}$$

$$\Rightarrow \qquad C = \sin^{-1}\left(\frac{8}{9}\right) \quad \Rightarrow \qquad \theta > \sin^{-1}\frac{8}{9}$$

37. We have, $\frac{n_2}{n_1} = \frac{1}{\sin C}$

$$\frac{1}{\sin C} = \frac{\lambda_1}{\lambda_2} = \frac{6000}{4000} = \frac{3}{2} \qquad \left[\because n = \frac{c}{\lambda}\right]$$

$$C = \sin^{-1}\left(\frac{2}{3}\right)$$

38. Refractive Index, $\frac{n_g}{n_a} = \frac{c_a}{c_g} \quad \Rightarrow \quad \frac{3}{2} = \frac{3 \times 10^8}{c_g}$

$$c_g = 2 \times 10^8$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{4 \times 10^{-3}}{2 \times 10^8} = 2 \times 10^{-11}\,\text{s}$$

39. $r_2 = 0$ ($\because$ No refraction is there at second surface)

$\therefore$ $$r_1 = A = 30^\circ$$

$$n = \frac{\sin i}{\sin r_1} = \frac{\sin i_1}{\sin 30^\circ} = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

$$\sin i_1 = \frac{1}{\sqrt{2}}$$

$$i_1 = 45^\circ$$

40. Refractive index $n = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{6}{4} = \frac{3}{2}$

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_1 \sim n_2}{R} \quad \Rightarrow \quad \frac{1.5}{6} - \frac{4}{17} = \frac{1.5 - 1}{R}$$

$$R = 34\,\text{cm}$$

41. Given, the combined power of two thin lens $P_1 + P_2 = 9$

If they are separated, the equivalent power

$$P = P_1 + P_2 - dP_1P_2$$

$$\frac{27}{9} = 9 - \frac{20}{100} \times P_1P_2$$

The above equation is correct for $P_1 = 3$ and $P_2 = 6$.

42. In fog, visible light is scattered more according to Rayleigh scattering, but scattering of infrared radiations is less due to high wavelengths, hence in fog, photographs of the objects taken with infrared radiations are more clear.

43. The magnifying power of a telescope, $m = -\frac{f_o}{f_e}$, if the focal length of eyepiece of telescope is double, then magnifying power, $m' = \frac{m}{2}$

44. Power of the lens $P = P_1 + P_2$

$$= \frac{1}{f_1} + \frac{1}{f_2} = \frac{100}{20} + \frac{100}{25} = 5 + 4 = 9\,\text{D}$$

45. According to the question

$$\nu = 5\mu \quad \frac{1}{5u} - \left(\frac{1}{-u}\right) = \frac{1}{30}$$

$$\frac{1}{5u} + \frac{1}{u} = \frac{1}{30} \quad \Rightarrow \quad \frac{5+1}{5u} = \frac{1}{30}$$

$$u = 36\,\text{cm}$$

46. Power of lens,

$$P = \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{1}\right) = 0.5(-1)$$

$$P = -0.5\,\text{D}$$

47. Resolving power of telescope $= \frac{a}{1.22\pi}$, where a is the aperture of a telescope.

So, if the aperture of telescope is decreased, the resolving power will decreases.

48. Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{+25/3} + \frac{1}{-u_1} = \frac{1}{+10}$$

or $$\frac{1}{u_1} = \frac{3}{25} - \frac{1}{10}$$

or $$u_1 = 50\,\text{m}$$

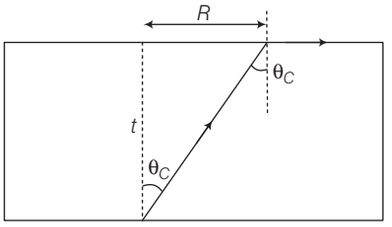
and $$\frac{1}{(+\,50/7)} + \frac{1}{-u_2} = \frac{1}{+10}$$

$\therefore$ $$\frac{1}{u_2} = \frac{7}{50} - \frac{1}{10} \quad \text{or} \quad u_2 = 25\,\text{m}$$

$$\text{Speed of object} = \frac{u_1 - u_2}{\text{time}} = \frac{25}{30}\,\text{ms}^{-1}$$

$$= 3\,\text{kmh}^{-1}$$

49. From the figure, we have $\frac{R}{t} = \tan\theta_C$ or $R = t(\tan\theta_C)$

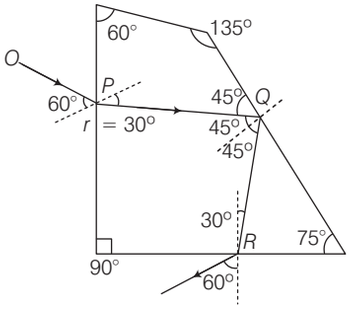


But, $\sin\theta_C = \frac{1}{\mu} = \frac{3}{5} \Rightarrow \therefore \tan\theta_C = \frac{3}{4}$

$$R = \frac{3}{4}t = \frac{3}{4}(8\text{ cm}) = 6\text{ cm}$$

Hence, the answer is 6.

50. Refractive index of the material $\mu = \frac{\sin i}{\sin r} \Rightarrow \sqrt{3} = \frac{\sin 60°}{\sin r}$



$\therefore \quad r = 30°$

$$\theta_C = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \text{ or } \sin\theta_C = \frac{1}{\sqrt{3}} = 0.577$$

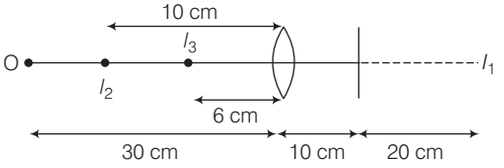
At point Q, angle of incidence inside the prism is $i = 45°$.

Since, $\sin i = \frac{1}{\sqrt{2}}$ is greater than $\sin\theta_C = \frac{1}{\sqrt{2}}$, ray gets totally internally reflected at face CD. Path of ray of light after point Q is shown in figure.

From the figure, we can see that angle between incident ray OP and emergent ray RS is 90°.

Therefore, correct options are (a), (b) and (c)

51. Object is placed at distance $2f$ from the lens. So, first image I will be formed at distance $2f$ on other side. This image I_1 will behave like a virtual object for mirror. The second image I_2 will be formed at distance 20 cm in front of the mirror, or at distance 10 cm to the left hand side of the lens.



Now, applying lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\therefore \quad \frac{1}{v} - \frac{1}{+10} = \frac{1}{+15} \Rightarrow v = 16\text{ cm}$

Therefore, the final image is at distance 16 cm from the mirror. But, this image will be real.

This is because ray of light is travelling from right to left.

52. From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{u}{v} - 1 = \frac{u}{f}$

or $\frac{u}{v} = \left(\frac{u+f}{f}\right) \Rightarrow m = \frac{v}{u} = \left(\frac{f}{u+f}\right)$

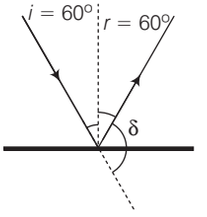
$$\frac{m_{25}}{m_{50}} = \frac{\left(\frac{20}{-25+20}\right)}{\left(\frac{20}{-50+20}\right)} = 6$$

53. Given, $f = -15\text{ cm}$, $u = -20\text{ cm}$

Magnification $m = \frac{f}{u-f} = \frac{-15}{-20+15} = 3\text{ cm}$

The area enclosed by the image of the wire $= m^2 = 9\text{ cm}^2$

54. Here, angle of incidence, $i = 60°$



$\therefore$ Angle of deviation,

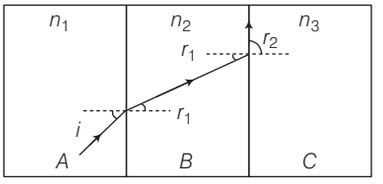
$$\delta = 180° - (i + r) = 180° - 2i \text{ (As } i = r)$$
$$= 180° - 2 \times 60° = 60°$$

55. By using formula, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \quad \frac{1.5}{v} - \frac{1}{(-15)} = \frac{(1.5-1)}{+30} \Rightarrow v = -30\text{ cm}$$

Negative sign shows that, image is obtained on the same side of object i.e. towards left.

56. Applying Snell's law between the surfaces A and B



$$n_1 \sin i = n_2 \sin r_1 \quad \ldots(i)$$

Again applying Snell's law between surfaces B and C

$$n_2 \sin r_1 = n_3 \sin r_2 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$n_1 \sin i = n_3 \sin r_2$$

Here, $r_2 = 90°$

$\therefore \quad n_1 \sin i = n_3 \Rightarrow \sin i = \frac{n_3}{n_1}$

57. When light travels from rarer to denser medium its wavelength reduces. So, wavelength in water $= \frac{\lambda_a}{\mu_w}$

$$= \frac{500}{(4/3)} = 375\text{ nm} \approx 376\text{ nm} \quad \text{(Blue colour)}$$

58. Here, angle of prism, $A = 60°$

For minimum deviation, $A = 2r$

or $r = \frac{A}{2} = \frac{60°}{2} = 30°$ for both colours.

25

Interference and Diffraction of Light

25.1 Introduction of Interference

In geometric optics, the propagation of light is described by rays that travel in straight lines and obey the laws of reflection and refraction. However, the rules of geometric optics are only valid when the wavelength of light is much smaller than the dimensions of the apertures or barriers involved. When this is not the case, phenomena arising from the wave nature of light are observed. Such phenomena fall under the general heading of **physical optics.** We have already discussed how superposed mechanical waves can interfere constructively or destructively, depending on their phase difference. Since, light is an electromagnetic wave, it must also exhibit **interference effects.** In this chapter, we'll look at interference phenomena in light. The colours seen in oil films and soap bubbles are a result of interference between light reflected from the front and back surfaces of a thin film of oil or soap solution. The first successful optical **interference** was carried out by **Thomas Young** in 1801.

Chapter Snapshot

- Introduction of Interference
- Energy Distribution in Interference
- Conditions for Interference
- Young's Double Slit Experiment
- Introduction to Diffraction
- Diffraction from Narrow Slits
- Diffraction of X-rays by Crystals

25.2 Energy Distribution in Interference

The term interference refers to any situation in which two or more waves overlap in space. When this occurs, the total wave at any point at any instant of time is governed by the principle of superposition. This principle states that, when two or more waves overlap, the resultant displacement at any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

If a single source of light is present in the medium, then the energy distribution is uniform. However, if there are two adjacent exactly similar sources, then the distribution no longer remains uniform. At some places, energy is maximum while at other places the energy is minimum, of course the total energy of the system remains the same.

This modification in energy distribution due to presence of two or more exactly similar sources is specific case of superposition of wave and known as **interference**.

If crest of a wave superposes over the crest of the other wave, then the resultant displacement becomes the sum of two amplitudes. It is double if two amplitudes are exactly equal. Naturally the intensity at such places becomes four times. ($I \propto A^2$). The same is true, if a wave trough coincides with the trough due to other wave. However, if a crest of a wave superposes over the trough of the other wave, then the resultant displacement is the difference in the amplitudes of the two waves and the intensity is minimum at such places.

The minimum energy is equal to zero, if the amplitude of two waves are exactly equal. Thus, the places of maxima have four times of the normal intensity while the places of minima have zero intensity. This condition of distribution of energy persists due to two waves having same amplitudes. However, in cases when the amplitudes due to two waves are not exactly equal, the interference occurs but intensity of minima is not zero.

25.3 Conditions for Interference

In order to obtain a sustained (permanent or stable) and observable interference pattern, the following conditions must be fulfilled.

(i) **Sources must be coherent** In order to produce a stable interference pattern, the individual waves must maintain a constant phase relationship with one another, i.e. the two interfering sources must emit waves having a constant phase difference between them. If the phase difference between two sources does not remain constant, then the places of maxima and minima shift. In case of mechanical waves, it is possible to keep a constant phase relationship between two different sources. But in case of light two different light sources can't be coherent, this is because of the way light is emitted.

In ordinary light sources, atoms gain excess energy by thermal agitation or by impact with accelerated break electrons.

An atom that is excited in such a way begins to radiate energy and continues until it has lost all the energy it can, typically in a time of the order of 10^{-8} s. The many atoms in a source ordinarily radiate in an unsynchronised in random phase relationship and the light that is emitted from two such sources has no definite phase relationship. Hence, to obtain a stable interference in light a single source is split into two coherent sources. Any random phase change in the source affects these secondary sources equally and does not change their relative phase. Laser light is much more coherent than ordinary light.

(ii) **Same frequency or wavelength** Phase relationship between two waves can be kept constant only when their frequencies are same. Thus, we can say that two coherent sources must have the same frequency.

(iii) **Equality of amplitudes** The amplitudes of two interfering waves should be equal or approximately equal. Otherwise the difference between the intensities of maximas and minimas will be too small and the contrast will be poor. Maximum contrast is however obtained when $A_1 = A_2$, because then minimum intensity will be zero.

Extra Knowledge Points

- Consider two coherent sources S_1 and S_2. Suppose, two waves emanating from these two sources superpose at point P. The phase difference between them at P is ϕ (which is constant). If the amplitude due to two individual sources at P is A_1 and A_2, then resultant amplitude at P, will be,



$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi} \quad \ldots(i)$$

Similarly, the resultant intensity at P is given by,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \quad \ldots(ii)$$

Here, I_1 and I_2 are the intensities due to independent sources. If the sources are incoherent then resultant intensity at P is given by,

$$I = I_1 + I_2 \quad \ldots(iii)$$

25.4 Young's Double Slit Experiment

One of the first to demonstrate the interference of light was Thomas Young in 1801.

Principle Monochromatic light (single wavelength) from a narrow vertical slit S falls on two other narrow slits S_1 and S_2 which are very close together and parallel to S. S_1 and S_2 act as two coherent sources (both being derived from S). If S, S_1 and S_2 all are very narrow, diffraction (bending of light at openings whose width is of the order of wavelength of light) causes the emerging beams to spread into the region beyond the slits. Superposition occurs in the shaded area, where the diffracted beams overlap. Alternate bright and dark equally spaced vertical bands (interference fringes) can be observed on a screen placed at some distance from the slits. If either of S_1 or S_2 is covered, the fringes disappear.

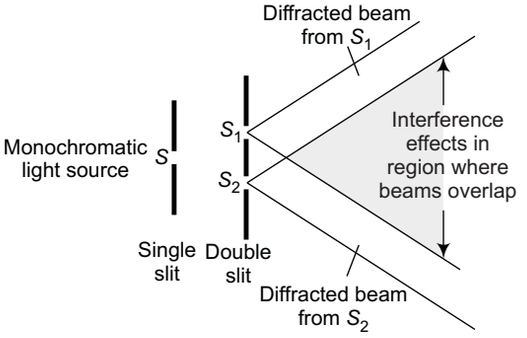


Fig. 25.1

Theory An expression for the separation of two bright (or dark) fringes (called fringe width w) can be obtained as under.

Figure shows the light waves from S_1 and S_2 meeting at an arbitrary point P on the screen. Since $D >> d$, the two light rays are approximately parallel with a path difference,

$$\Delta x = S_2P - S_1P \approx d \sin\theta \quad \ldots(i)$$

For **maximum** intensity at P,

$$\Delta x = n\lambda \quad (n = 0, \pm 1, \pm 2, \ldots)$$

or $$d \sin\theta = h\lambda \quad (n = 0, \pm 1, \pm 2, \ldots) \quad \ldots(ii)$$

The bright fringe corresponding to the integer n is called the nth order (or just nth) bright fringe. The bright fringe for $n = 0$, is known as the central fringe and its centre (point O) is called the central (or zero order) maximum. Higher order bright fringes are situated symmetrically about the central fringe.

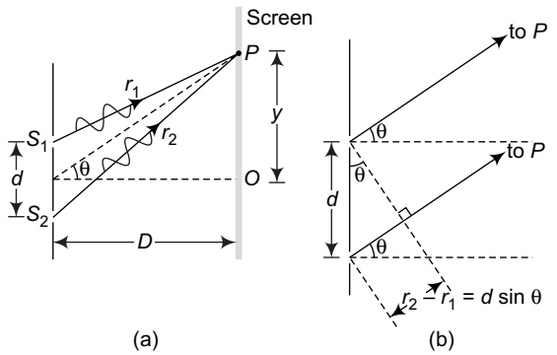


Fig. 25.2 (a) To reach P, the light waves from S_1 and S_2 must travel different distances. (b) The path difference between the two rays is $d \sin\theta$

For minimum intensity at P,

$$\Delta x = (2n-1)\frac{\lambda}{2} \quad (n = \pm 1, \pm 2, \ldots)$$

or $$d \sin\theta = (2n-1)\frac{\lambda}{2} \quad (n = \pm 1, \pm 2, \ldots) \quad \ldots(iii)$$

The first minima ($n = \pm 1$) are adjacent to the central maximum on either side. It is useful to obtain expressions for the positions of the bright and dark fringes measured vertically from O. In addition to our assumption that $D >> d$, we assume that $d >> \lambda$. These can be valid assumptions because in practice D is often of the order of 1 m, d is a fraction of a millimetre and λ is a fraction of a micrometre for visible light. Under these conditions θ is small, thus we can use the approximation.

$$\sin\theta \approx \tan\theta = \frac{y}{D}$$

Substituting in Eqs. (ii) and (iii), we get

$$y_{\text{bright}} = \frac{n\lambda D}{d} \quad (n = 0, \pm 1, \pm 2, \ldots) \quad \ldots(iv)$$

and $$y_{\text{dark}} = \frac{(2n-1)\lambda D}{2d} \quad (n = \pm 1, \pm 2, \ldots) \quad \ldots(v)$$

Fringe width (w) Distance between two adjacent bright (or dark) fringes is called the fringe width. It is denoted by w. Thus,

$$w = \frac{n\lambda D}{d} - \frac{(n-1)\lambda D}{d} = \frac{\lambda D}{d}$$

or $$w = \frac{\lambda D}{d} \quad \ldots(vi)$$

Alternatively we can show that distance between two successive dark fringes is also $\frac{\lambda D}{d}$.

Further Points

1. In YDSE (Young's Double Slit Experiment) we are basically interested in finding the resultant intensity at a general point P at a distance y from O on the screen or at an angle θ from C as shown. Rather we can say we will find intensity I as a function of y or θ.
2. Since, $d << D$, we can assume that intensity at P due to independent sources S_1 and S_2 are almost equal.

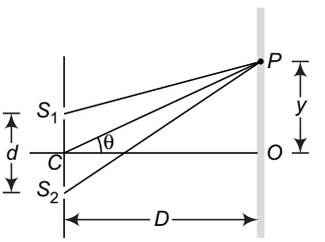


Fig. 25.3

or $$I_1 \approx I_2 = I_0 \text{ (say)} \quad \ldots(vii)$$

3. Path difference between S_2P and S_1P as a function of y or θ is given by

$$\Delta x = d \sin\theta \approx \frac{yd}{D} \quad \left(\text{as, } \sin\theta \approx \tan\theta = \frac{y}{D}\right) \quad \ldots(viii)$$

- $\Delta x = \frac{yd}{D}$ is applicable only when θ is small. However, $\Delta x = d \sin\theta$ can be applied to larger values of θ also provided $d << D$.

4. Once path difference as a function of y or θ is known, corresponding phase difference (we will denote it by ϕ not by $\Delta\phi$) can be obtained by the relation,

$$\phi = \frac{2\pi}{\lambda} \cdot \Delta x \qquad \ldots(ix)$$

5. For two coherent sources, the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

(See extra knowledge points)

Putting, $I_1 = I_2 = I_0$

We have, $I = I_0 + I_0 + 2\sqrt{I_0 \times I_0}\cos\phi$

Simplifying the above expression, we get

$$I = 4I_0 \cos^2\frac{\phi}{2} \qquad \ldots(x)$$

6. From Eq. (viii), we can see that intensity is maximum at points where,

$$\cos\frac{\phi}{2} = \pm 1 \quad \text{or} \quad \frac{\phi}{2} = n\pi \qquad (n = 0, \pm 1, \pm 2, \ldots)$$

or $\phi = 2n\pi$ or $\dfrac{2\pi}{\lambda}\Delta x = 2n\pi$

or $\Delta x = n\lambda$ or $d\sin\theta = n\lambda$

or $\dfrac{yd}{D} = n\lambda$ or $y = \dfrac{n\lambda D}{d}$

and this condition, we have already discussed earlier. Further, the maximum intensity is

$$I_{max} = 4I_0 \qquad \ldots(xi)$$

with this Eq. (viii), can also be written as,

$$I = I_{max}\cos^2\frac{\phi}{2} \qquad \ldots(xii)$$

Minimum intensity on the screen is found at points where,

$$\cos\frac{\phi}{2} = 0$$

or $\dfrac{\phi}{2} = \left(n - \dfrac{1}{2}\right)\pi \qquad (n = \pm 1, \pm 2, \pm 3, \ldots)$

or $\phi = (2n-1)\,\pi$

or $\dfrac{2\pi}{\lambda}\Delta x = (2n-1)\,\pi$

or $\Delta x = (2n-1)\dfrac{\lambda}{2}$

or $d\sin\theta = (2n-1)\dfrac{\lambda}{2}$

or $\dfrac{yd}{D} = (2n-1)\dfrac{\lambda}{2}$

or $y = \dfrac{(2n-1)\,\lambda D}{2d}$

The minimum intensity is zero

$$I_{min} = 0 \qquad \ldots(xiii)$$

A plot of light intensity *versus* $d\sin\theta$ is given in figure.

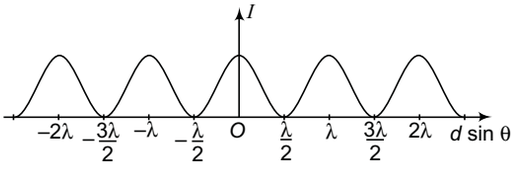


Fig. 25.4

The above points will remain continued but first let us take some simple examples in support of the theory discussed above.

Example 25.1 *In YDSE, the two slits are separated by* $0.1\,mm$ *and they are* $0.5\,m$ *from the screen. The wavelength of light used is* $5000\,Å$. *Find the distance between* $7th$ *maxima and* $11\,th$ *minima on the screen.*

Sol. Given, $d = 0.1\text{ mm} = 10^{-4}\text{ m}$, $D = 0.5\text{ m}$

and $\lambda = 5000\text{ Å} = 5.0 \times 10^{-7}\text{ m}$

$$\Delta y = (y_{11})_{dark} - (y_7)_{bright}$$

$$= \frac{(2\times 11 - 1)\,\lambda D}{2d} - \frac{7\lambda D}{d}$$

or $\Delta y = \dfrac{7\lambda D}{2d} = \dfrac{7\times 5.0\times 10^{-7}\times 0.5}{2\times 10^{-4}}$

$$= 8.75\times 10^{-3}\text{ m} = 8.75\text{ mm}$$

Example 25.2 *Maximum intensity in YDSE is* I_0. *Find the intensity at a point on the screen, where*

(a) the phase difference between the two interfering beams is $\dfrac{\pi}{3}$.

(b) the path difference between them is $\dfrac{\lambda}{4}$.

Sol. (a) From Eq. (xii), we have

$$I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

Here, I_{max} is I_0 (i.e. intensity due to independent sources is $I_0/4$). Therefore, at

$$\phi = \frac{\pi}{3} \quad \text{or} \quad \frac{\phi}{2} = \frac{\pi}{6}$$

$$I = I_0\cos^2\left(\frac{\pi}{6}\right) = \frac{3}{4}I_0$$

(b) Phase difference corresponding to the given path difference $\Delta x = \dfrac{\lambda}{4}$ is,

$$\phi = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)\left(\phi = \frac{2\pi}{\lambda}\Delta x\right) = \frac{\pi}{2}$$

or $\dfrac{\phi}{2} = \dfrac{\pi}{4} \Rightarrow \therefore\ I = I_0\cos^2\left(\dfrac{\pi}{4}\right) = \dfrac{I_0}{2}$

7. Fringe width (w) is the distance between two successive maximas or minimas. It is given by

$$w = \frac{\lambda D}{d} \quad \text{or} \quad w \propto \lambda$$

Two conclusions can be drawn from this relation:

(i) If YDSE apparatus is immersed in a liquid of refractive index μ, then wavelength of light and hence, fringe width decreases μ times.

(ii) If white light is used in place of a monochromatic light, then coloured fringes are obtained on the screen with red fringes of larger size than that of violet, because $\lambda_{\text{red}} > \lambda_{\text{violet}}$.

But note that centre is still white, because path difference there is zero for all colours. Hence, all the wavelengths interfere constructively. At other places light will interfere destructively for those wavelengths for whom path difference is $\lambda/2, 3\lambda/2, \ldots$ etc., and they will interfere constructively for the wavelengths for whom path difference is $\lambda, 2\lambda, \ldots$ etc.

8. Path difference produced by a slab Consider two light rays 1 and 2 moving in air parallel to each other. If a slab of refractive index μ and thickness t is inserted between the path of one of the rays then a path difference,

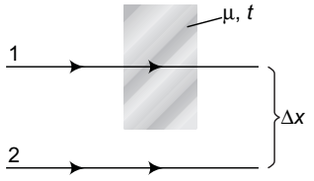


Fig. 25.5

$$\Delta x = (\mu - 1)\, t \qquad \ldots(\text{xiv})$$

is produced among them. This can be shown as under,

Speed of light in air $= c$

Speed of light in medium $= \frac{c}{\mu}$

Time taken by ray 1 to cross the slab,

$$t_1 = \frac{t}{c/\mu} = \frac{\mu t}{c}$$

and time taken by ray 2 to cross the same thickness t in air will be

$$t_2 = \frac{t}{c}$$

as $t_1 > t_2$

difference in time, $\Delta t = t_1 - t_2 = (\mu - 1)\frac{t}{c}$

During this time ray 2 will travel an extra distance, $\Delta x = (\Delta t)\, c = (\mu - 1)\, t$, which is same as Eq. (xiv).

Optical path length Now, we can show that a thickness t in a medium of refractive index μ is equivalent to a length μt in vacuum (or air). This is called optical path length. Thus,

$$\text{Optical path length} = \mu t$$

9. Shifting of fringes Suppose a glass slab of thickness t and refractive index μ is inserted onto the path of the ray emanating from source S_1, then the whole fringe pattern shifts upwards by a distance $\frac{(\mu - 1)\, tD}{d}$. This can be shown as under,

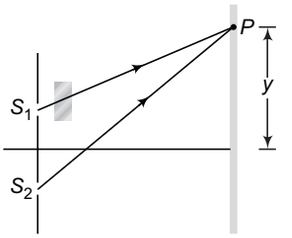


Fig. 25.6

Geometric path difference between S_2P and S_1P is

$$\Delta x_1 = S_2P - S_1P = \frac{yd}{D}$$

Path difference produced by the glass slab,

$$\Delta x_2 = (\mu - 1)\, t$$

Due to the glass slab path of ray 1 gets increased by Δx_2.

Therefore, net path difference between the two rays is

$$\Delta x = \Delta x_1 - \Delta x_2 \quad \text{or} \quad \Delta x = \frac{yd}{D} - (\mu - 1)\, t$$

For nth maxima on upper side,

$$\Delta x = n\lambda \quad \text{or} \quad \frac{yd}{D} - (\mu - 1)\, t = n\lambda$$

$$\therefore \qquad y = \frac{n\lambda D}{d} + \frac{(\mu - 1)\, tD}{d}$$

Earlier it was $\frac{n\lambda D}{d}$

$$\therefore \qquad \text{Shift} = \frac{(\mu - 1)t\, D}{d} \qquad \ldots(\text{xv})$$

Following three points are important with regard to Eq. (xv).

(a) Shift is independent of n, (the order of the fringe), i.e. shift of zero order maximum = shift of 7th order maximum or shift of 5th order maximum = shift of 9th order minimum and so on.

(b) Shift is independent of λ, i.e. if white light is used then, shift of red colour fringe = shift of violet colour fringe.

(c) Number of fringes shifted $= \frac{\text{Shift}}{\text{Fringe width}}$

$$= \frac{(\mu - 1)\, t\, D/d}{\lambda D/d} = \frac{(\mu - 1)t}{\lambda}$$

These numbers are inversely proportional to λ. This is because shift is same for all colours but fringe width of the colour having smaller value of λ is small, so more number of fringes will shift of this colour.

Example 25.3 *Bichromatic light is used in YDSE having wavelengths* $\lambda_1 = 400\,\text{nm}$ *and* $\lambda_2 = 700\,\text{nm}$. *Find minimum order of* λ_1 *which overlaps with* λ_2.

Sol. Let n_1 bright fringe of λ_1 overlaps with n_2 bright fringe of λ_2.

Then,

$$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

or
$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}$$
$$= \frac{700}{400} = \frac{7}{4}$$

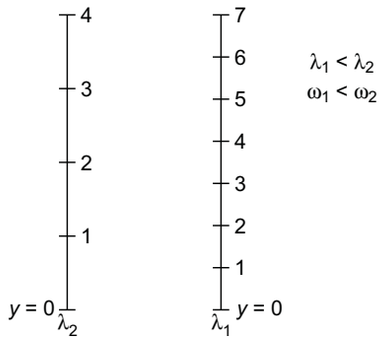


Fig. 25.7

The ratio $\frac{n_1}{n_2} = \frac{7}{4}$ implies that 7th bright fringe of λ_1 will overlap with 4th bright fringe of λ_2.

Similarly, 14th of λ_1 will overlap with 8th of λ_2 and so on.

So, the minimum order of λ_1 which overlaps with λ_2 is 7.

Example 25.4 *In YDSE find the thickness of a glass slab* $(\mu = 1.5)$ *which should be placed before the upper slit* S_1 *so that the central maximum now lies at a point, where* 5*th bright fringe was lying earlier (before inserting the slab). Wavelength of used light is* 5000 Å.

Sol. According to the question,

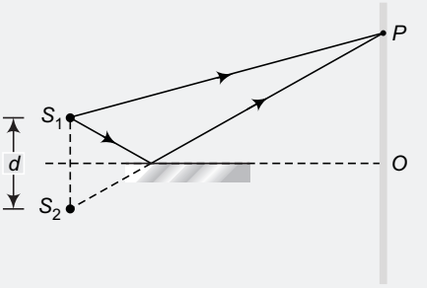
$$\text{Shift} = 5\ (\text{fringe width})$$

$$\therefore \quad \frac{(\mu - 1)tD}{d} = \frac{5\lambda D}{d}$$

$$\therefore \quad t = \frac{5\lambda}{\mu - 1}$$
$$= \frac{25000}{1.5 - 1}$$
$$= 50000\ \text{Å}$$

Extra Knowledge Points

- **Lloyd's mirror** A plane glass plate (acting as a mirror) is illuminated at almost grazing incidence by a light from a slit S_1. A virtual image S_2 of S_1 is formed closed to S_1 by reflection and these two act as coherent sources. The expression giving the fringe width is the same as for the double slit, but the fringe system differs in one important respect. In Lloyd's mirror, if the point P, e.g. is such that the path difference $S_2P - S_1P$ is a whole number of wavelengths, the fringe at P is dark not bright. This is due to 180° phase change which occurs when light is reflected from a denser medium. This is equivalent to adding an extra half wavelength to the path of the reflected wave. At grazing incidence a fringe is formed at O, where the geometrical path difference between the direct and reflected waves is zero and it follows that it will be dark rather than bright.



Thus, whenever there exists a phase difference of π between the two interfering beams of light, conditions of maximas and minimas are interchanged, i.e.

$$\Delta x = n\lambda \quad \text{(for minimum intensity)}$$

and $\Delta x = (2n - 1)\lambda/2$ (for maximum intensity)

- Resultant intensity due to two coherent sources is given by the equation,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$
$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

where, $\cos\phi = +1$

and $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

where, $\cos\phi = -1$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$$

If both the slits are of equal width

$$I_1 \approx I_2 = I_0 \quad \text{(say)}$$

and in that case, $I_{max} = 4I_0$ and $I_{min} = 0$

If the slits are of unequal width, then

$$I_1 \neq I_2 \qquad I_{min} \neq 0$$

According to some authors, amplitude of the light coming from a slit is proportional to the slit width, i.e. intensity is proportional to square of the slit width.

- **Interference in thin films** Interference effects are commonly observed in thin films, such as thin layers of oil on water or the thin surface of a soap bubble.

The varied colour's observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film.

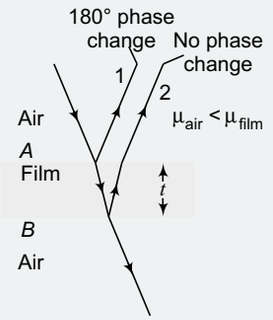
Consider a film of uniform thickness t and index of refraction μ, as shown in figure. Let us assume that the light rays travelling in air are nearly normal to the two surfaces of the film. To determine whether the reflected rays interfere constructively or destructively, we first note the following facts.

(i) The wavelength of light in a medium whose refractive index is μ is,

$$\lambda_\mu = \frac{\lambda}{\mu}$$

where, λ is the wavelength of light in vacuum (or air).

(iii) If a wave is reflected from a denser medium it undergoes a phase change of 180°. Let us apply these rules to the film shown in figure. The path difference between the two rays 1 and 2 is $2t$ while the phase difference between them is 180°. Hence, condition of constructive interference will be,



Interference in light reflected from a thin film is due to a combination of rays reflected from the upper and lower surfaces of the film

$$2t = (2n-1)\frac{\lambda_\mu}{2} \qquad (n = 1, 2, 3, \ldots)$$

or

$$2\mu t = \left(n - \frac{1}{2}\right)\lambda \quad \text{as} \quad \lambda_\mu = \frac{\lambda}{\mu}$$

Similarly, condition of **destructive** interference will be

$$2\mu t = n\lambda, \text{ where } n = 0, 1, 2, \ldots$$

Example 25.5 *Calculate the minimum thickness of a soap bubble film* ($\mu = 1.33$) *that results in constructive interference in the reflected light, if the film is illuminated with light whose wavelength in free space is* $\lambda = 600$ nm.

Sol. For constructive interference in case of soap film,

$$2\mu t = \left(n - \frac{1}{2}\right)\lambda \qquad (n = 1, 2, 3, \ldots)$$

For minimum thickness t, $n = 1$

or $$2\mu t = \frac{\lambda}{2} \quad \text{or} \quad t = \frac{\lambda}{4\mu} = \frac{600}{4 \times 1.33}$$

$$= 112.78 \text{ nm}$$

Example 25.6 *In solar cells a silicon solar cell* ($\mu = 3.5$) *is coated with a thin film of silicon monoxide SiO* ($\mu = 1.45$) *to minimize reflective losses from the surface. Determine the minimum thickness of SiO that produces the least reflection at a wavelength of* 550 *nm, near the centre of the visible spectrum.*

Sol. The reflected light is a minimum when rays 1 and 2 (shown in figure) meet the condition of destructive interference.

Both rays undergo a 180° phase change upon reflection. The net change in phase due to reflection is therefore zero, and the condition for a reflection minimum requires a path difference of $\lambda_\mu/2$. Hence,

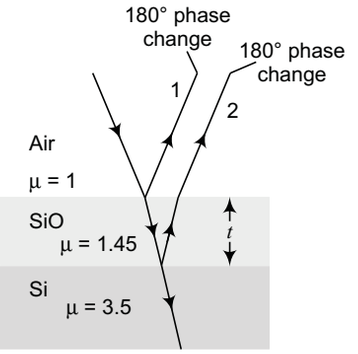


Fig. 25.8

$$2t = \frac{\lambda}{2\mu} \quad \text{or} \quad t = \frac{l}{4\mu} = \frac{550}{4\,(1.45)} = 94.8 \text{ nm}$$

Example 25.7 *Two slits are separated by* 0.32 *mm. A beam of* 500 *nm light strikes the slits producing an interference pattern. Determine the number of maxima observed in the angular range* $-30° < \theta < 30°$.

Sol. Fringe width, $w = \frac{\lambda D}{d}$ and $y = \frac{D}{\sqrt{3}}$

Therefore, number of fringe widths in a distance y is

$$n = \frac{y}{w} = \frac{d}{\sqrt{3}\lambda}$$

$$= \frac{0.32 \times 10^{-3}}{(\sqrt{3})\,(500 \times 10^{-9})}$$

$$= 369.5$$

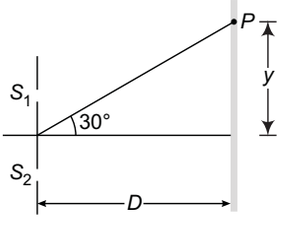


Fig. 25.9

Therefore, total number of maximas obtained in the angular range $-30° < \theta < 30°$ (including the central one) is

$$N = 2 \times 369 + 1 = 739$$

Example 25.8 *Distance between the slits shown in figure is $d = 20\,\lambda$, where λ is the wavelength of light used. Find the angle θ, where*

(a) central maxima (where, path difference is zero) is obtained.

(b) third order maxima is obtained.

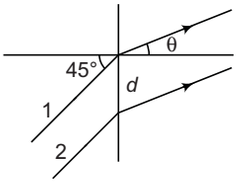


Fig. 25.10

Sol. Ray 1 has a longer path than that of ray 2 by a distance $d \sin 45°$, before reaching the slits. Afterwards ray 2 has a path longer than ray 1 by a distance $d \sin\theta$. The net path difference is therefore, $d \sin\theta - d \sin 45°$.

(a) Central maximum is obtained where, net path difference is zero, or $d \sin\theta - d \sin 45° = 0$ or $\theta = 45°$

(b) Third order maxima is obtained where, net path difference is $3\,\lambda$ or $d \sin\theta - d \sin 45° = 3\lambda$

$$\therefore \quad \sin\theta = \sin 45° + \frac{3\lambda}{d}$$

Putting $d = 20\,\lambda$

we have, $$\sin\theta = \sin 45° + \frac{3\lambda}{20\lambda}$$

or $$\theta \approx 59°$$

25.5 Introduction to Diffraction

When light waves pass through a small aperture, an interference pattern is observed rather than a sharp spot of light. This behaviour indicates that light, once it has passed through the aperture, spreads beyond the narrow path defined by the aperture into regions that would be in shadow if light travelled in straight lines. Other waves, such as sound waves and water waves, also have this property of spreading when passing through apertures or by sharp edges. This phenomenon, known as diffraction, can be described only with a wave model for light.

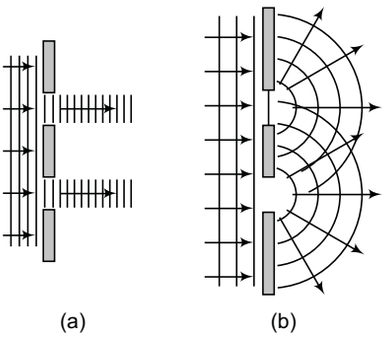


Fig. 25.11 (a) If light waves did not spread out after passing through the slits, no interference would occurs.
(b) The light waves from the two slits overlap as they spread out, filling what we

In general, diffraction occurs when waves pass through small openings, around obstacles or past sharp edges, as shown in figure. When an opaque object is placed between a point source of light and a screen, no sharp boundary exists on the screen between a shadowed region and an illuminated region. The illuminated region above the shadow of the object contains alternating light and dark fringes. Such a display is called a **diffraction pattern.**

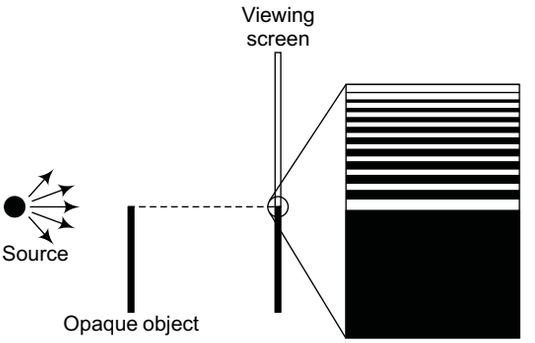


Fig. 25.12 Light from a small source passes by the edge of an opaque object. We might expect no light to appear on the screen below the position of the edge of the object. In reality, light bends around the top edge of the object and enter this region. Because of these effects, a diffraction pattern consisting of bright and dark fringes appears in the region above the edge of the object.

In this chapter, we restrict our attention to **Fraunhofer diffraction,** which occurs, e.g. when all the rays passing through a narrow slit are approximately parallel to one another. This can be achieved experimentally either by placing the screen far from the opening used to create the diffraction or by using a converging lens to focus the rays once they pass through the opening, as shown in Fig. 25.13.

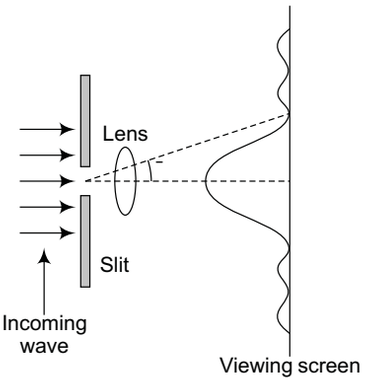


Fig. 25.13 Fraunhofer diffraction pattern of a single slit. The pattern consists of a central bright fringe flanked by much weaker maxima alternating with dark fringes.

25.6 Diffraction from Narrow Slits

Until now, we have assumed that slits are point sources of light. In this section, we abandon that assumption and see how the finite width of slits is the basis for understanding fraunhofer diffraction.

We can deduce some important features of this phenomenon by examining waves coming from various portions of the slit, as shown in Fig. 25.14. According to Huygens's principle, **each portion of the slit acts as a source of light waves**. Hence, light from one portion of the slit can interfere with light from another portion, and the resultant light intensity on a viewing screen depends on the direction θ.

To analyse the diffraction pattern, it is convenient to divide the slit into two halves, as shown in Fig. 25.14. Keeping in mind that all the waves are in phase as they leave the slit, consider rays 1 and 3. As these two rays travel toward a viewing screen far to the right of the Fig. 25.14, ray 1 travels farther than ray 3 by an amount equal to the path difference $(a/2)\sin\theta$, where a is the width of the slit. Similarly, the path difference between rays 2 and 4 is also $(a/2)\sin\theta$. If this path difference is exactly half a wavelength (corresponding to a phase difference of 180°), then the two waves cancel each other and destructive interference results. This is true for any two rays that originate at points separated by half the slit width, because the phase difference between two such points is 180°. Therefore, waves from the upper half of the slit interfere destructively with waves from the lower half when

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

or when

$$\sin\theta = \frac{\lambda}{a}$$

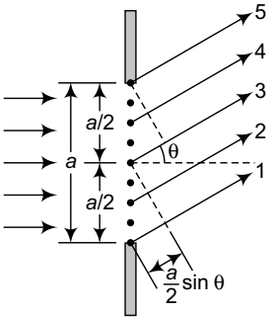


Fig. 25.14 Diffraction of light by a narrow slit of width a. Each portion of the slit acts as a point source of light waves. The path difference between rays 1 and 3 or between rays 2 and 4 is $(a/2)\sin\theta$ (drawing not to scale).

If we divide the slit into four equal parts and use similar reasoning, we find that the viewing screen is also dark when

$$\sin\theta = \frac{2\lambda}{a}$$

Likewise, we can divide the slit into six equal parts and show that darkness occurs on the screen when

$$\sin\theta = \frac{3\lambda}{a}$$

Therefore, the general condition for destructive interference is

$$\sin\theta = m\frac{\lambda}{a} \quad (m = \pm 1,\ \pm 2, \pm 3, \ldots) \qquad \ldots(i)$$

This equation gives the values of θ for which the diffraction pattern has zero light intensity i.e. when a dark fringe is formed. However, it tells us nothing about the variation in light intensity along the screen. The general features of the intensity distribution are shown in Fig. 25.15. A broad central bright fringe is observed, this fringe is flanked by much weaker bright fringes alternating with dark fringes. The various dark fringes occur at the values of θ that satisfy Eq. (i). Each bright fringe peak lies approximately halfway between its bordering dark fringe minima. Note that the central bright maximum is twice as wide as the secondary maxima.

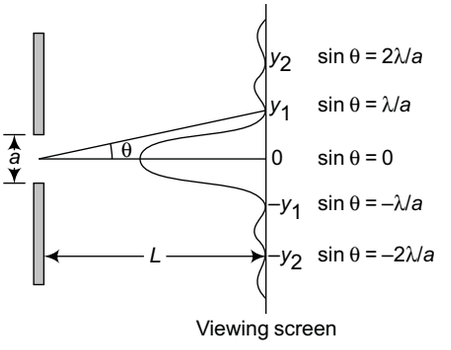


Fig. 25.15 Intensity distribution for a Fraunhofer diffraction pattern from a single slit of width a. The positions of two minima on each side of the central maximum are labelled.

Example 25.9 *Light of wavelength* 580 *nm is incident on a slit having a width of* 0.300 *mm. The viewing screen is* 2.00 *m from the slit. Find the positions of the first dark fringe and the width of the central bright fringe.*

Sol. The two dark fringes that flank the central bright fringe correspond to $m = \pm 1$ in Eq. (i), Hence, we find that

$$\sin\theta = \pm\frac{\lambda}{a} = \pm\frac{5.80\times10^{-7}\ \text{m}}{0.300\times10^{-3}\ \text{m}}$$
$$= \pm 1.93\times10^{-3}$$

From the triangle in Fig. 25.15 note that $\tan\theta = y_1/L$. Because θ is very small, we can use the approximation

$\sin\theta \approx \tan\theta$, thus $\sin\theta \approx y_1/L$. Therefore, the positions of the first minima measured from the central axis are given by

$$y_1 \approx L\sin\theta = \pm L\frac{\lambda}{a} = \pm\ 3.87\times 10^{-3}\ \text{m}$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe. Hence, the width of the central bright fringe is equal to $2|y_1| = 7.74\times 10^{-3}$ m $= 7.74$ mm. Note that this value is much greater than the width of the slit. However as the slit width is increased, the diffraction pattern narrows and corresponding to smaller values of θ. In fact, for large values of a, the various maxima and minima are so closely spaced that only a large central bright area resembling the geometric image of the slit is observed.

Exercise *Determine the width of the first-order* $(m=1)$ *bright fringe.*

Ans. 3.87 mm.

Intensity of Single Slit Diffraction Patterns

We can use phasors to determine the light intensity distribution for a single slit diffraction pattern. The proof is beyond our syllabus, we are just writing here the intensity at angle θ.

$$I = I_0\left[\frac{\sin\beta/2}{\beta/2}\right]^2 \Rightarrow \quad \text{Here,} \quad \frac{\beta}{2} = \frac{\pi\, a\sin\theta}{\lambda}$$

or
$$I = I_0\left[\frac{\sin(\pi\, a\sin\theta)/\lambda}{(\pi\, a\sin\theta)/\lambda}\right]^2 \qquad (\text{at angle}\ \theta)$$

where, I_0 is the intensity at $\theta = 0$ (the central maximum). From this result we see that minima occurs when,

$$\frac{\pi\, a\sin\theta}{\lambda} = m\pi \quad \text{or} \quad \sin\theta = m\frac{\lambda}{a}$$

$(m = \pm 1, \pm 2, \ldots)$

This is in agreement with Eq. (i).

- That $\sin\theta = 0$, corresponds to central maxima while $\frac{\pi\, a\sin\theta}{\lambda} = \pi$, corresponds to first minima.

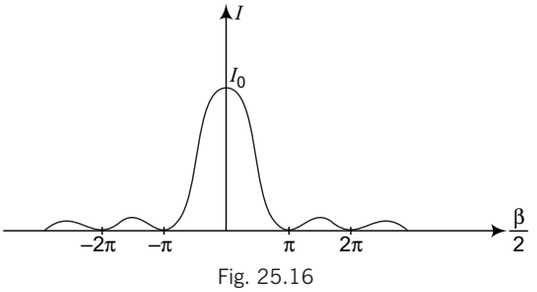


Fig. 25.16

Example 25.10 *Find the ratio of the intensities of the secondary maxima to the intensity of the central maximum for the single-slit Fraunhofer diffraction pattern.*

Sol. To a good approximation, the secondary maxima lie mid-way between the zero points. From Fig. 25.16, we see that this corresponds to $\beta/2$ values of $3\pi/2, 5\pi/2, 7\pi/2, \ldots$

$$\therefore \qquad \frac{I_1}{I_0} = \left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2 = \frac{1}{9\pi^2/4} = 0.045$$

$$\frac{I_2}{I_0} = \left[\frac{\sin(5\pi/2)}{5\pi/2}\right]^2 = \frac{1}{25\ \pi^2/4} = 0.016$$

i.e. the first secondary maxima (the ones adjacent to the central maximum) have an intensity of 4.5% that of the central maximum, and the next secondary maxima have an intensity of 1.6% that of the central maximum.

- Secondary maxima does not lie exactly between two minimas.

Exercise *Determine the intensity, relative to the central maximum, of the secondary maxima corresponding to* $m = \pm 3$.

Ans. 0.0083.

Diffraction Grating It consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, the direction of principal maxima is given by

$$d\sin\theta = n\lambda$$

where, d is the distance between two consecutive slits and is called grating element.

$n = 1, 2, 3, \ldots$ is the order of principal maximas.

Resolving Power of the Diffraction Grating

The diffraction grating is most useful for measuring wavelengths accurately. Like the prism, the diffraction grating can be used to disperse a spectrum into its wavelength components. Of the two devices, the grating is the more precise, if one wants to distinguish two closely spaced wavelengths.

For two nearly equal wavelengths λ_1 and λ_2 between which a diffraction grating can just barely distinguish, the **resolving power** R of the grating is defined as,

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda}$$

where, $\lambda = (\lambda_1 + \lambda_2)/2$ and $\Delta\lambda = \lambda_2 - \lambda_1$.

25.7 Diffraction of X-rays by Crystals

In principle, the wavelength of any electromagnetic wave can be determined if a grating of the proper spacing (of the order of λ) is available. X-rays, discovered by Wilhelm Roentgen (1845–1923) in 1895, are electromagnetic waves of very short wavelength (of the order of 0.1 nm). It becomes impossible to construct a grating having such a small spacing by cutting process.

However, the atomic spacing in a solid is known to be about 0.1 nm. In 1913, Max von Laue (1879–1960) suggested that the regular array of atoms in a crystal could act as a three-dimensional diffraction grating for X-rays.

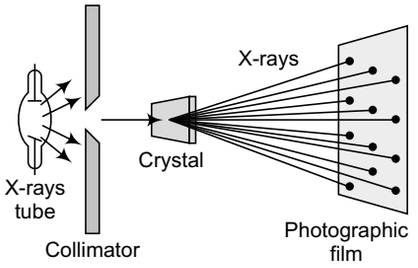


Fig. 25.17 Schematic diagram of the technique used to observe the diffraction of X-rays by a crystal. The array of spots formed on the film is called a Laue pattern.

Figure is one experimental arrangement for observing X-ray diffraction from a crystal. A collimated beam of X-rays is incident on a crystal. The diffracted beams are very intense in certain directions, corresponding to constructive interference from waves reflected from layers of atoms in the crystal. The diffracted beams can be detected by a photographic film, and they form an array of spots. One can deduce the crystalline structure by analyzing the positions and intensities of the various spots in the pattern.

The arrangement of atoms in a crystal of sodium chloride (NaCl) is shown in Fig. 25.18. Each unit cell (the geometric solid that repeats throughout the crystal) is a cube having an edge length a. Now, suppose that an incident X-ray beam makes an angle θ with one of the planes, as shown in figure. The beam can be reflected from both the upper plane and the lower one. However, the beam reflected from the lower plane travels farther than the beam reflected from the upper plane. The effective path difference is $2d \sin \theta$. The two beams reinforce each other (constructive interference) when this path difference equals some integer multiple of λ. The same is true for reflection from the entire family of parallel planes. Hence, the condition for constructive interference (maxima in the reflected beam) is

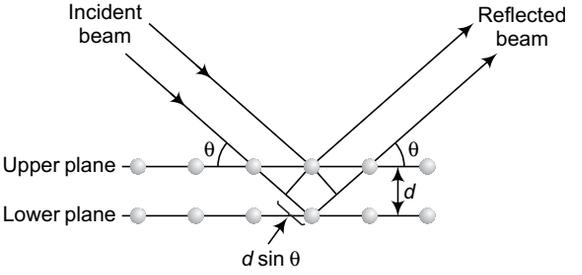


Fig. 25.18

$$2d \sin \theta = m\lambda \qquad (m = 1, 2, 3, \ldots)$$

This condition is known as **Bragg's law**, after WL Bragg (1890-1971), who first derived the relationship. If the wavelength and diffraction angle are measured, the above equation can be used to calculate the spacing between atomic planes.

Extra Knowledge Points

- **Types of diffraction** The diffraction phenomenon is divided into two types. **Fresnel diffraction** and **Fraunhofer diffraction.** In the first type either source or screen or both are at finite distance from the diffracting device (obstacle or aperture). In the second type both source and screen are effectively at infinite distance from the diffracting device. Fraunhofer diffraction is a particular limiting case of fresnel diffraction.
- **Difference between interference and diffraction** Both interference and diffraction are the results of superposition of waves, so they are often present simultaneously as in Young's double slit experiment. However interference is the result of superposition of waves from two different wavefronts while diffraction results due to superposition of wavelets from different points of the same wavefront.

Chapter Summary with Formulae

(i) $\frac{\sin i_1}{\sin i_2} = \frac{v_1}{v_2}$ is called law of refraction.

(ii) $\mu \sin i$ = constant or $\mu_1 \sin i_1 = \mu_2 \sin i_2$ is called Snell's law.

(iii) ${}_1\mu_2$ = Refractive index of medium-2 with respect to medium-1

$$= \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\sin i_1}{\sin i_2}$$

(iv) Young's double slit Experiment :

(a) It is an experiment of interference in light.

(b) For interference sources must be coherent.

(c) Interference is based on principle of superposition.

(d) Path difference,

$$\Delta x = d \sin\theta \approx \frac{yd}{D}$$

(e) Fringe width,

$$w = \frac{\lambda D}{d}$$

(f) $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$

(g) $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$

(h) If $A_1 = A_2 = A_0$
and $I_1 = I_2 = I_0$, then

$$A = 2A_0 \cos\frac{\phi}{2} \text{ and } I = 4\ I_0 \cos^2\frac{\phi}{2}$$

(i) In YDSE, $I_1 \approx I_2 = I_0$

(j) $I_{min} = 0$ at points where,

$$\phi = (2n - 1)\pi, \text{ where } n = \pm 1, \pm 2...$$

or $$\Delta x = (2n - 1)\ \lambda/2$$

or $$y = \frac{(2n - 1)\ \lambda D}{2d}$$

(k) $I_{max} = 4 I_0$ at points where, $\phi = 2n\pi$, where, $n = 0, \pm 1, \pm 2..$

or $$\Delta x = n\lambda \quad \text{or} \quad y = \frac{n\lambda D}{d}$$

(l) $\phi = \frac{2\pi}{\lambda} \cdot \Delta x$

(m) $$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$$

$$= \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1}\right)^2 = \left(\frac{A_1/A_2 + 1}{A_1/A_2 - 1}\right)^2 = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2$$

(v) **Diffraction**

(a) Spreading (or bending) of light, when passing through apertures or by sharp edges is called diffraction.

(b) In case of simple slit,

$$\sin\theta = m\frac{\lambda}{a}, \text{ where } m = \pm 1, \pm 2....$$

is the condition of minima.

(c) In fresnel type diffraction, either source or screen or both are at finite distance from the diffracting device. In Fraunhofer type diffraction source and screen both are effectively at infinite distance from the diffracting device.

Additional Examples

Example 1. *Why is the diffraction of sound waves more evident in daily life than that of light wave?*

Sol. Sound waves have wavelengths in the range of l5 m to 15 mm. Aperture or obstacle size are normally of this range that is why diffraction of sound waves is more evident in daily life than that of light wave. In case of light, wavelength is of the order of 10^{-7} m and normally, the size of aperture is much-much greater than this.

Example 2. *Can we conclude from the interference whether light is a transverse wave or a longitudinal wave?*

Sol. No. From interference we cannot conclude whether the light is transverse or longitudinal.

Example 3. *Why do not we have interference when two candles are placed closed to each other and the intensity is seen at a distant screen?*

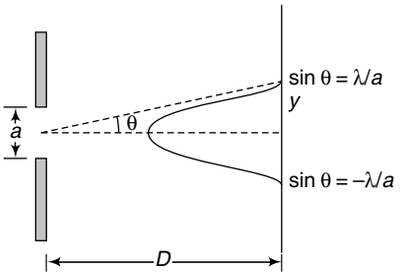
Sol. For interference to take place, sources must be coherent. Two different light sources are never coherent. Therefore, two candles cannot produce interference.

Example 4. *The wavelength of light in a medium is* $\lambda = \lambda_0/\mu$, *where* λ *is the wavelength in vacuum. A beam of red light* ($\lambda = 720\ nm$) *enters into water. The wavelength in water is* $\lambda = \lambda_0/\mu = 540\ nm$. *To a person under water does this light appear red?*

Sol. Yes. To a person inside water this light appears red. Because colour of light depends on frequency not on wavelength. In refraction from air to water frequency remains same. Therefore, colour will remain same.

Example 5. *In a single-slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band?*

Sol. In single slit, central maxima is as shown in figure. For small angle,



$$\sin\theta \approx \theta = \frac{y}{D} = \frac{\lambda}{a}$$

$$\therefore \qquad y = \frac{\lambda D}{a}$$

$\therefore$ Width of central maxima,

$$w = 2y = \frac{2\lambda D}{a}$$

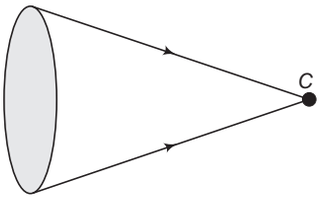
or $$w \propto \frac{1}{a}$$

If width of slit, a is made double then size of central diffraction band, w will remain half $\left(\text{as } w = \frac{1}{a}\right)$.

Intensity will however increase, because in central diffraction maxima light will fall on smaller area.

Example 6. *In what way is diffraction from each slit related to the interference pattern in a double-slit experiment?*

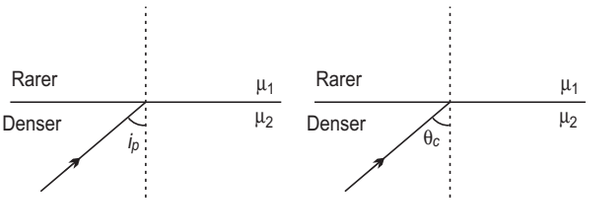
Sol. Path length of each wave diffracted from the edge of the circular obstacle is same.



Therefore, all waves interfere constructively at the centre of the shadow producing a bright spot.

Example 7. *Can reflection result in plane polarised light, if the light is incident on the interface from the side with higher refractive index.*

Sol. In the figure, $\mu_1 < \mu_2$.



Brewster's angle or polarising angle is given by

$$\tan i_p = \frac{\mu_1}{\mu_2} \qquad \text{...(i)}$$

Critical angle for total internal reflection is given by

$$\sin\theta_c = \frac{\mu_1}{\mu_2} \qquad \text{...(ii)}$$

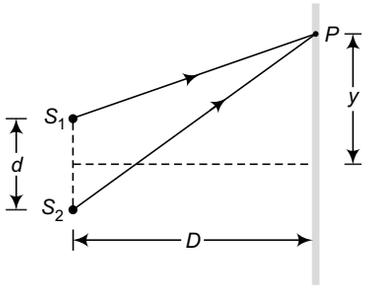
From Eqs. (i) and (ii), we can see that

$$i_p < \theta_c$$

Therefore, polarisation by reflection is possible in this case.

Example 8. *Young's double slit experiment is carried out using microwaves of wavelength* $\lambda = 3\,\text{cm}$. *Distance between the slits is* $d = 5\,\text{cm}$ *and the distance between the plane of slits and the screen is* $D = 100\,\text{cm}$.
(a) Find the number of maximas and
(b) Their positions on the screen.

Sol. (a) The maximum path difference that can be produced = distance between the sources or 5 cm.



Thus, in this case we can have only three maximas, one central maxima and two on its either side for a path difference of λ or 3 cm.

(b) For maximum intensity at P,

$$S_2P - S_1P = \lambda$$

$$\text{or } \sqrt{(y + d/2)^2 + D^2} - \sqrt{(y - d/2)^2 + D^2} = \lambda$$

Substituting, $d = 5$ cm,

$D = 100$ cm

and $\lambda = 3$ cm,

we get $y = \pm 75$ cm

Thus, the three maximas will be at

$y = 0$

and $y = \pm 75\,\text{cm}$

Example 9. *Interference fringes are produced by a double slit arrangement and a piece of plane parallel glass of refractive index* 1.5 *is interposed in one of the interfering beam. If the fringes are displaced through* 30 *fringe widths for light of wavelength* 6×10^{-5} *cm, find the thickness of the plate.*

Sol. Path difference due to the glass slab,

$$\Delta x = (\mu - 1)t$$

Thirty fringes are displaced due to the slab. Hence,

$$\Delta x = 30\lambda$$

$$\therefore \quad (\mu - 1)t = 30\lambda$$

$$\therefore \quad t = \frac{30\lambda}{\mu - 1}$$

$$= \frac{30 \times 6 \times 10^{-5}}{1.5 - 1}$$

$$= 3.6 \times 10^{-3}\ \text{cm}$$

Example 10. *Two coherent sources are* 0.3 *mm apart. They are* 0.9 *m away from the screen. The second dark fringe is at a distance of* 0.3 *cm from the centre. Find the distance of fourth bright fringe from the centre. Also find the wavelength of light used.*

Sol. Given, $d = 0.3 \times 10^{-3}$ m, $D = 0.9$ m

$$\frac{3\lambda D}{2d} = 0.3 \times 10^{-2}\ \text{cm}$$

(the distance of second dark fringe)

$$\therefore \quad \frac{\lambda D}{d} = (0.3 \times 10^{-2})\left(\frac{2}{3}\right)$$

$$= 0.2 \times 10^{-2}\ \text{m}$$

$$= 0.2\ \text{cm}$$

(i) Distance of fourth bright fringe from centre $= \dfrac{4\lambda D}{d}$

$$= 0.8\ \text{cm}$$

(ii) Wavelength of used light $\lambda = \left(\dfrac{d}{D}\right)(0.2 \times 10^{-2})$ m

$$= \left(\frac{0.3 \times 10^{-3}}{0.9}\right)(0.2 \times 10^{-2})$$

$$= 6.67 \times 10^{-7}\ \text{m}$$

Example 11. *In a Young's double slit set up the wavelength of light used is* 546 *nm. The distance of screen from slits is* 1*m. The slit separation is* 0.3 *mm.*
(a) Compare the intensity at a point P distant 10 *mm from the central fringe, where the intensity is* I_0.
(b) Find the number of bright fringes between P and the central fringe.

Sol. Given, $\lambda = 546\ \text{nm} = 5.46 \times 10^{-7}$ m

$D = 1.0$ m and $d = 0.3\ \text{mm} = 0.3 \times 10^{-3}$ m

(a) At a distance $y = 10\ \text{mm} = 10 \times 10^{-3}$ m from central fringe, the path difference will be,

$$\Delta x = \frac{y \cdot d}{D} = \frac{(10 \times 10^{-3})\,(0.3 \times 10^{-3})}{1.0}$$

$$= 3.0 \times 10^{-6}\ \text{m}$$

The corresponding phase difference between the two intertering beams will be

$$\phi = \frac{2\pi}{\lambda} \cdot \Delta x$$

$$= \left(\frac{2\pi}{5.46 \times 10^{-7}}\right)(3.0 \times 10^{-6})\ \text{rad}$$

$$= 1978°$$

$$\therefore \quad \frac{\phi}{2} = 989°$$

$$\therefore \quad I = I_0 \cos^2 \frac{\phi}{2}$$

$$= I_0 \cos^2 (989)$$

$$= 3.0 \times 10^{-4}\, I_0$$

(b) Fringe width, $w = \frac{\lambda D}{d} = \frac{(5.46 \times 10^{-7})\,(1.0)}{0.3 \times 10^{-3}}$ m

$= 1.82$ mm

Since, $\frac{y}{w} = \frac{10}{1.82} = 5.49$

Therefore, number of bright fringes between P and central fringe will be 5 (excluding the central fringe).

Example 12. *A parallel beam of white light falls on a thin film whose refractive index is equal to* 4/3. *The angle of incidence* $i = 53°$. *What must be the minimum film thickness, if the reflected light is to be coloured yellow* (λ *of yellow* $= 0.6\,\mu m$) *most intensively?* $(\tan 53° = 4/3)$

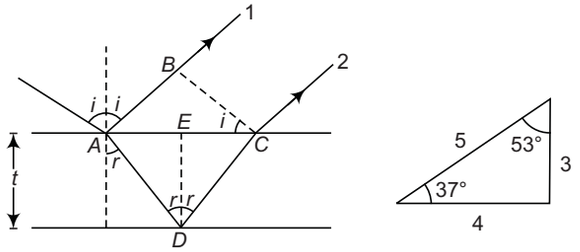
Sol. From the relation,

$$\mu = \frac{\sin i}{\sin r}$$

$$\frac{4}{3} = \frac{\sin 53°}{\sin r}$$

We have, $r \approx 37°$

Interference has to be taken place between rays 1 and 2. They have already a phase difference of π, because ray 1 is reflected from a denser medium.



They have a path difference of

$$\Delta x = 2\mu\,(AD) - AB$$

$$= 2\mu\left(\frac{t}{\cos r}\right) - 2t\,(\tan r)\,(\sin i)$$

$$= 2\left(\frac{4}{3}\right)\left(\frac{5}{3}\right)t - 2(t)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right)$$

$$= \frac{32}{15}t$$

For the two waves to interfere constructively,

$$\Delta x = (2n - 1)\frac{\lambda}{2} \qquad (n = 1, 2, 3, \ldots)$$

or $$\frac{32}{15}t = (2n - 1)\frac{\lambda}{2}$$

For minimum thickness, $n = 1$

or $$\frac{32}{15}t = \frac{\lambda}{2}$$

or $$t = \frac{15}{64}\lambda$$

$$= \frac{15 \times 0.6}{64}\,\mu m$$

$$= 0.14\,\mu m$$

Example 13. *In a double slit pattern* ($\lambda = 6000$ Å), *the first order and tenth order maxima fall at* 12.50mm *and* 14.75 mm *from a particular reference point. If* λ *is changed to* 5500 Å, *find the position of zero order and tenth order fringes, other arrangements remaining the same.*

Sol. Distance between 10 fringes is,

$$9w = (14.75 - 12.50)\text{ mm}$$

$$= 2.25\text{ mm}$$

$\therefore$ Fringe width, $w = 0.25$ mm

When the wavelength is changed from 6000 Å to 5500 Å, the new fringe width will become,

$$w' = \left(\frac{5500}{6000}\right)w = \left(\frac{5500}{6000}\right)(0.25)$$

As fringe width $\propto \lambda$.

$\therefore$ $w' = 0.23$ mm

The position of central (or zero order) maxima will remain unchanged. Earlier it was at a position,

$$y_0 = y_1 - w$$

$$= (12.50 - 0.25)$$

$$= 12.25\text{ mm}$$

The new position of tenth order maxima will be,

$$y_{10} = y_0 + 10w'$$

$$= (12.25) + (10)\,(0.23) = 14.55\text{ mm}$$

Example 14. *A beam of light of wavelength* 600 *nm from a distant source falls on a single slit* 1.0 *mm wide and the resulting diffraction pattern is observed on a screen* 2 *m away. What is the distance between the first dark fringe on either side of the central bright fringe?*

Sol. For the diffraction at a single slit, the position of minima is given by

$$d \sin\theta = n\lambda$$

For small value of θ,

$$\sin\theta \approx \theta = \frac{y}{D}$$

$\therefore$ $$d\left(\frac{y}{D}\right) = \lambda$$

or $$y = \frac{D}{d}\lambda$$

Substituting the values, we have

$$y = \frac{2 \times 6 \times 10^{-7}}{1 \times 10^{-3}}$$

$$= 1.2 \times 10^{-3}\text{ m}$$

$$= 1.2\text{ mm}$$

$\therefore$ Distance between first minima on either side of central maxima $= 2y = 2.4$ mm

Example 15. *A screen is placed* 50 *cm from a single slit, which is illuminated with* 6000 *Å light. If distance between the first and third minima in the diffraction pattern is* 3.0 *mm, what is the width of the slit?*

Sol. Position of first minima on a single slit diffraction pattern is given by

$$d \sin\theta = n\lambda$$

For small value of θ, $\sin\theta \approx \theta = \frac{y}{D}$

$\therefore$ $$\frac{y \cdot d}{D} = n\lambda$$

or $$y = \frac{n\lambda D}{d}$$

$\therefore$ Distance between third order minima and first order minima will be

$$\Delta y = y_3 - y_1 = \frac{(3-1)(\lambda D)}{d} = \frac{2\lambda D}{d}$$

Substituting the values, we have

$$\Delta y = \frac{(2)(6\times 10^{-7})(0.5)}{3\times 10^{-3}}$$
$$= 2\times 10^{-4} \text{ m}$$
$$= 0.2 \text{ mm}$$

Example 16. *In a single slit diffraction experiment first minimum for $\lambda_1 = 660$ nm coincides with first maxima for wavelength λ_2. Calculate λ_2.*

Sol. Position of minima in diffraction pattern is given by,

$$d \sin\theta = n\lambda$$

For first minima of λ_1, we have

$$d \sin\theta_1 = (1)\lambda_1$$

or $$\sin\theta_1 = \frac{\lambda_1}{d} \quad \text{...(i)}$$

The first maxima approximately lies between first and second maxima. For wavelength λ_2 its position will be

$$d \sin\theta_2 = \frac{3}{2}\lambda_2$$

$\therefore$ $$\sin\theta_2 = \frac{3\lambda_2}{2d} \quad \text{...(ii)}$$

The two will coincide if,

$$\theta_1 = \theta_2$$

or $$\sin\theta_1 = \sin\theta_2$$

$\therefore$ $$\frac{\lambda_1}{d} = \frac{3\lambda_2}{2d}$$

or $$\lambda_2 = \frac{2}{3}\lambda_1 = \frac{2}{3}\times 660 \text{ nm}$$
$$= 440 \text{ nm}$$

Example 17. *A parallel beam of monochromatic light of wavelength 450 nm passes through a slit of width 0.2 mm. Find the angular divergence in which most of the light is diffracted.*

Sol. Most of the light is diffracted between the two first order minima. These minima occur at angle given by

$$d \sin\theta = \pm n\lambda$$

$\therefore$ $$\sin\theta = \pm\frac{\lambda}{d}$$
$$= \pm\frac{450\times 10^{-9}}{0.2\times 10^{-3}}$$
$$= \pm 2.25\times 10^{-3} \text{ rad}$$

$\therefore$ The angular divergence $= 4.5\times 10^{-3}$ rad

NCERT Selected Questions

Q 1. What is the shape of the wavefront in each of the following cases?

(a) Light diverging from a point source.

(b) Light emerging out of a convex lens when a point source is placed at its focus.

(c) The portion of the wave front of light from a distance star intercepted by the earth.

Sol (a) When the light diverging from a point source, then the wavefront is of a converging spherical shape.

(b) When the point source is placed on the principal focus of the convex lens, then the rays of light emerging from the lens are parallel to each other, so the wavefront will be plane.

(c) In this case, the shape of the wavefront is almost plane because the source of light i.e. the star is far away from earth, thus a small area on the surface of a large sphere is nearly plane.

Q 2. In a Young's double slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.

Sol. For constructive interference,

$$y = n\lambda \frac{D}{d},$$

we get, $$\lambda = \frac{yd}{nD}$$

$$= \frac{1.2 \times 10^{-2} \times 0.28 \times 10^{-3}}{4 \times 1.4}$$

$$= 6000 \times 10^{-10} \text{ m}$$

$$= 6000 \text{ Å}$$

Q 3. In Young's double slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen, where path difference is λ, is K units. What is the intensity of light at a point where path difference is $\frac{\lambda}{3}$?

Sol. At path difference λ, maximum intensity is obtained.

$$\therefore \quad I_{max} = K \quad \Rightarrow \quad \Delta x = \frac{\lambda}{3}$$

$$\therefore \quad \phi = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{3} \quad \text{or} \quad \frac{\phi}{2} = \frac{\pi}{3} = 60°$$

Now, $$I = I_{max} \cos^2 \frac{\phi}{2}$$

$$= K \cos^2 \frac{\phi}{2} = K \cos^2 60° = \frac{K}{4}$$

Q 4. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double slit experiment.
Distance between two slits is 2 mm and between slits and screen is 1.2 m

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.

(b) What is the least distance from the central maximum, where the bright fringes due to both the wavelengths coincide?

Sol. (a) Using the relation,

$$x = \frac{n\lambda D}{d},$$

we get $$x = \frac{3 \times 6500 \times 10^{-10} \times 1.20}{2 \times 10^{-3}}$$

$$= 1.17 \times 10^{-3} \text{ m}$$

(b) Let n_1 be the least number of fringes of λ_1 (= 650 nm) which coincide with n_2 fringes of λ_2 (= 520 nm),

$$\frac{n_1 \lambda_1 D}{d} = \frac{n_2 \lambda_2 D}{d} \quad \text{or} \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{4}{5}$$

or $n_1 = 4$ and $n_2 = 5$

$\therefore$ $x' =$ least asked distance

$$= \frac{n_1 \lambda_1 D}{d} = 4 \times \frac{6500 \times 10^{-10} \times 1.20}{2 \times 10^{-3}}$$

$$= 1.56 \times 10^{-3} \text{ m}$$

$$= 1.56 \text{ mm}$$

Q 5. In a Young's double slit experiment the angular width of a fringe is found to be 0.2° on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe, if the entire experimental apparatus is immersed in water? Take refractive index of water to be $\frac{4}{3}$.

Sol. Angular fringe width is given by

$$\theta = \frac{\lambda}{d}$$

$$\therefore \quad \theta' = \frac{\lambda'}{d}$$

$$\therefore \quad \frac{\theta'}{\theta} = \frac{\lambda'}{\lambda} = \frac{\lambda/\mu}{\lambda} = \frac{1}{\mu}$$

$$\theta' = \frac{\theta}{\mu}$$

$$= \frac{0.2°}{4/3} = \frac{0.2° \times 3}{4}$$

$$= 0.15°$$

Objective Problems

[Level 1]

Interference

1. Two identical light sources S_1 and S_2 emit light of same wavelength λ. These light rays will exhibit interference, if
(a) their phase differences remain constant
(b) their phases are distributed randomly
(c) their light intensities remain constant
(d) their light intensities change randomly

2. If the amplitude ratio of two sources producing interference is 3 : 5, the ratio of intensities at maxima and minima is
(a) 25 : 16 (b) 5 : 3
(c) 16 : 1 (d) 25 : 9

3. For the sustained interference of light, the necessary condition is that the two sources should
(a) have constant phase difference
(b) be narrow
(c) be close to each other
(d) of same amplitude

4. Which of the following is conserved when light waves interfere?
(a) Intensity (b) Energy
(c) Amplitude (d) Momentum

5. If two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$ interfere at a point, the amplitude of the resulting wave will be about
(a) 7 (b) 6 (c) 5 (d) 3.5

6. Two coherent sources of intensities, I_1 and I_2 produce an interference pattern. The maximum intensity in the interference pattern will be
(a) $I_1 + I_2$ (b) $I_1^2 + I_2^2$
(c) $(I_1 + I_2)^2$ (d) $(\sqrt{I_1} + \sqrt{I_2})^2$

7. What is the path difference of destructive interference?
(a) $n\lambda$ (b) $n(\lambda + 1)$ (c) $\frac{(n+1)\lambda}{2}$ (d) $\frac{(2n+1)\lambda}{2}$

8. Which of the following is not an essential condition for interference?
(a) The two interfering waves must propagate in almost the same direction
(b) The waves must have the same period and wavelength
(c) The amplitudes of the two waves must be equal
(d) The two interfering beams of light must originate from the same source

9. Interference proves
(a) transverse nature of light (b) longitudinal nature of light
(c) wave nature of light (d) particle nature of light

10. When interference of light takes place
(a) energy is created in the region of maximum intensity
(b) energy is destroyed in the region of maximum intensity
(c) conservation of energy holds good and energy is redistributed
(d) conservation of energy does not hold good

11. Two waves of intensities I and $4I$ superpose, then the maximum and minimum intensities produced are
(a) $5I$ and $3I$ (b) $9I$ and I
(c) $9I$ and $3I$ (d) $5I$ and I

12. Two sources of sound of the same frequency produce sound intensities I and $4I$ at a point P when used individually. If they are used together such that the sounds from them reach P with a phase difference of $2\pi/3$, the intensity at P will be
(a) $2I$ (b) $3I$
(c) $4I$ (d) $5I$

13. What is the minimum thickness of a soap film needed for constructive interference in reflected light, if the light incident on the film is 750 nm? Assume that the index for the film is $\mu = 1.33$.
(a) 282 nm (b) 70.5 nm
(c) 141 nm (d) 387 nm

14. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then, the difference between the resultant intensities at A and B is
(a) $2I$ (b) $4I$
(c) $5I$ (d) $9I$

15. Two waves of same frequency and same amplitude from two monochromatic sources are allowed to superpose at a certain point. If in one case the phase difference is 0 and in other case it is $\pi/2$, then the ratio of the intensities in the two cases will be
(a) 1 : 1 (b) 2 : 1
(c) 4 : 1 (d) None of these

16. If the two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin(\omega t + \pi/3)$ interfere at a point, the amplitude of the resulting wave will be about
(a) 7 (b) 5 (c) 6 (d) 3.5

17. It is found that when waves from two identical coherent sources superpose at a certain point, then the resultant intensity is equal to the intensity of one wave only. This means that the phase difference between the two waves at that point is
(a) zero (b) $\pi/3$
(c) $2\pi/3$ (d) π

Young's Double Slit Experiment

18. Monochromatic green light of wavelength 5×10^{-7} m illuminates a pair of slits 1 mm apart. The separation of bright lines on the interference pattern formed on a screen 2 m away is
(a) 0.25 mm (b) 0.1 mm
(c) 1.0 mm (d) 0.01 mm

19. In Young's double slit experiment, the experiment, the ratio of intensities of bright and dark fringes is 9.This means that
(a) the intensities of individual sources are 5 and 4 units respectively
(b) the intensities of individual sources are 4 and 1 units respectively
(c) the ratio of their amplitudes is 3
(d) the ratio of their amplitudes is 2

20. In double slits experiment, for light of which colour the fringe width will be minimum
(a) violet (b) red
(c) green (d) yellow

21. A thin mica sheet of thickness 2×10^{-6} m and refractive index $(\mu = 1.5)$ is introduced in the path of the light from upper slit. The wavelength of the wave used is 5000 Å. The central bright maximum will shift
(a) 2 fringes upward
(b) 2 fringes downward
(c) 10 fringes upward
(d) None of the above

22. In Young's double slit experiment, the fringe width will remain same, if (D = distance between screen and plane of slits, d = separation between two slits and λ = wavelength of light used)
(a) both λ and D are doubled
(b) both d and D are doubled
(c) D is doubled but d is halved
(d) λ is doubled but d is halved

23. In Young's double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then, in the interference pattern
(a) the intensities of both the maxima and the minima increase
(b) the intensity of maxima increases and the minima has zero intensity
(c) the intensity of maxima decreases and that of the minima increases
(d) the intensity of maxima decreases and the minima has zero intensity

24. When a thin transparent plate of thickness t and refractive index μ is placed in the path of one of the two interfering waves of light, then the path difference between two waves is
(a) $(\mu + 1)t$ (b) $(\mu - 1)t$ (c) $\frac{(\mu + 1)}{t}$ (d) μt

25. In Young's double slit experiment, an interference pattern is obtained for $\lambda = 6000$ Å, coming from two coherent sources S_1 and S_2. At certain point P on the screen third dark fringe is formed. Then, the path difference $S_1P - S_2P$ in microns is
(a) 0.75 (b) 1.5
(c) 3.0 (d) 4.5

26. Intensity at centre in Young's double slit experiment is I_0. If one slit is covered then intensity at centre will be
(a) I_0 (b) $2I_0$ (c) $I_0/4$ (d) $I_0/2$

27. In Young's double slit experiment, when two light waves form third minimum intensity, they have
(a) phase difference of 3π (b) phase difference of $\frac{5\pi}{2}$
(c) path difference of 3λ (d) path difference of $\frac{5\lambda}{2}$

28. In Young's double slit experiment, the aperture screen distance is 2 m. The fringe width is 1 mm. Light of 600 nm is used. If a thin plate of glass $(\mu = 1.5)$ of thickness 0.06 mm is placed over one of the slits, then there will be a lateral displacement of the fringes by
(a) 0 cm (b) 5 cm
(c) 10 cm (d) 15 cm

29. If vacuum is created in the room, where Young's double slit experiment is performed. Then,
(a) no interference pattern is obtained
(b) fringe width will increase
(c) fringe width will decrease
(d) maximum intensity will decrease

30. Two slits are separated by a distance of 0.5 mm and illuminated with light of $\lambda = 6000$ Å. If the screen is placed 2.5 m from the slits. The distance of the third bright image from the centre will be
(a) 1.5 mm (b) 3 mm (c) 6 mm (d) 9 mm

31. Fringe width decreases with increase in
(a) λ (b) D
(c) d (d) None of these

32. Interference was observed in interference chamber when air is present. Now, the chamber is evacuated and if the same light is used, then for the same arrangement
(a) no interference pattern will be obtained
(b) exactly same interference pattern will be obtained with better contrast
(c) the fringe width is slightly decreased
(d) the fringe width is slightly increased

33. What happens to the fringe pattern, if in the path of one of the slits a glass plate which absorbs 50% energy is interposed?
(a) The bright fringes become bright and dark fringes become darker
(b) No fringes are observed
(c) The fringe width decreases
(d) None of the above

34. In Young's double slit experiment, the type of diffraction is
(a) Fresnel
(b) Fraunhofer
(c) Both (a) and (b)
(d) None of the above

35. Interference fringes were produced in Young's double slit experiment using light of wavelength 5000 Å. When a film of thickness 2.5×10^{-3} cm was placed in front of one of the slits, the fringe pattern shifted by a distance equal to 20 fringe widths. The refractive index of the material of the film is
(a) 1.25 (b) 1.35
(c) 1.4 (d) 1.5

36. The Young's double slit experiment is performed with blue and green light of wavelengths 4360 Å and 5460 Å respectively. If x is the distance of 4th maxima from the central one, then
(a) $x\,(\text{blue}) = x\,(\text{green})$ (b) $x\,(\text{blue}) > x\,(\text{green})$
(c) $x\,(\text{blue}) < x\,(\text{green})$ (d) $\dfrac{x\,(\text{blue})}{x(\text{green})} = \dfrac{5460}{4360}$

37. In Young's double slit experiment, wavelength $\lambda = 5000$ Å, the distance between the slits is 0.2 mm and the screen is at 200 cm from the slits. The central maximum is at $x = 0$. The third maximum (taking, the central maximum as zeroth maximum) will be at x equal to
(a) 1.67 cm (b) 1.5 cm
(c) 0.5 cm (d) 5.0 cm

38. The maximum intensity of fringes in Young's experiment is I. If one of the slit is closed, then the intensity at that place becomes I_0, which of the following relations is true?
(a) $I = I_0$
(b) $I = 2I_0$
(c) $I = 4I_0$
(d) There is no relation between I and I_0

39. In Young's double slit experiment, if the monochromatic source of light is replaced by white light, then one sees
(a) no interference fringe pattern
(b) coloured fringes
(c) black and white fringes
(d) white central fringe surrounded by few coloured fringes on either side

40. In Young's double slit experiment green light ($\lambda = 5461$ Å) is used and 60 fringes were seen in the field view. Now sodium light is used ($\lambda = 5890$ Å), then number of fringes observed are
(a) 40 (b) 60
(c) 50 (d) 55

41. In a Young's double slit experiment, the fringe width is β. If the entire arrangement is now placed inside a liquid of refractive index μ, the fringe width will become
(a) $\mu\beta$ (b) β/μ
(c) $\beta(\mu + 1)$ (d) $\beta(\mu - 1)$

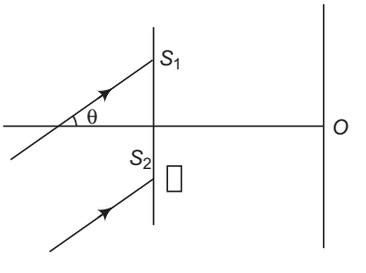
42. In Young's double slit experiment, the angular width of a fringe formed on a distant screen is 1°, the wavelength of light used is 6000 Å. The spacing between the slits is
(a) 6×10^{-7} m (b) 6.8×10^{-5} m
(c) 3.4×10^{-5} m (d) 3.4×10^{-4} m

43. In a Young's double slit experiment using red and blue lights of wavelengths 600 nm and 480 nm respectively, the value of n for which the nth red fringe coincides with $(n + 1)$th blue fringe is
(a) 5 (b) 4
(c) 3 (d) 2

44. Monochromatic light from a narrow slit illuminates two parallel narrow slits producing an interference pattern on a screen. The separation between the two slits is now doubled and the distance between the screen and the slits is reduced to half. The fringe width
(a) is doubled (b) becomes four times
(c) becomes one-fourth (d) remains the same

45. Two wavelengths of light of wavelength $\lambda_1 = 4500$ Å and $\lambda_2 = 6000$ Å are sent through a Young's double slit apparatus simultaneously, then
(a) no interference pattern will be formed
(b) the third order bright fringe of λ_1 will coincide with the fourth order bright fringe of λ_2
(c) the third order bright fringe of λ_2 will coincide with fourth order bright fringe of λ_1
(d) the fringes of wavelength λ_1 will be wider than the fringes of wavelength λ_2

46. A monochromatic beam of light falls on Young's double slit experiment apparatus at some angle (say θ) as shown in the figure. A thin sheet of glass is inserted in front of the lower slit S_2. The central bright fringe (path difference = 0) will be obtained



(a) at O
(b) above O
(c) below O
(d) anywhere depending on angle θ, thickness of plate t and refractive index of glass μ

47. Two slits separated by a distance of 1 mm, are illuminated with red light of wavelength 6.5×10^{-7} m. The interference fringes are observed on a screen placed 1 m from the slits. The distance between the third dark fringe and the fifth bright fringe is equal to
(a) 0.65 mm (b) 1.63 mm
(c) 3.25 mm (d) 4.88 mm

48. In Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen, when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by
(a) 12 (b) 18
(c) 24 (d) 30

Diffraction

49. A slit of width d is illuminated by white light. For red light ($\lambda = 6500$ Å), the first minima is obtained at $\theta = 30°$. Then, the value of d will be
(a) 3250 Å (b) 6.5×10^{-4} mm
(c) 1.24 microns (d) 2.6×10^{-4} cm

50. The light of wavelength 6328 Å is incident on a slit of width 0.2 mm perpendicularly, the angular fringe width will be
(a) 0.36° (b) 0.18°
(c) 0.72° (d) 0.09°

51. The bending of beam of light around corners of obstacles is called
(a) reflection
(b) diffraction
(c) refraction
(d) interference

52. Direction of the first secondary maximum in the Fraunhofer diffraction pattern at a single slit is given by (a is the width of the slit)
(a) $a \sin\theta = \frac{\lambda}{2}$ (b) $a \cos\theta = \frac{3\lambda}{2}$
(c) $a \sin\theta = \lambda$ (d) $a \sin\theta = \frac{3\lambda}{2}$

53. A light wave is incident normally over a single slit of width 24×10^{-5} cm. The angular position of second dark fringe from the central maxima is 30°. What is the wavelength of light ?
(a) 6000 Å (b) 5000 Å
(c) 3000 Å (d) 1500 Å

54. A parallel beam of monochromatic light of wavelength 5000 Å is incident normally on a single narrow slit of width 0.001 mm. The light is focused by a convex lens on a screen placed on the focal plane. The first minimum will be formed for the angle of diffraction equal to
(a) 0° (b) 15° (c) 30° (d) 60°

55. Diffraction of light was discovered by
(a) Young (b) Hertz
(c) Girmaldi (d) Malus

56. To observe diffraction, the size of an obstacle
(a) should be of the same order as wavelength
(b) should be much larger than the wavelength
(c) has no relation to wavelength
(d) should be exactly ($\lambda/2$)

57. Yellow light is used in a single slit diffraction experiment with slit width of 0.6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal
(a) that the central maximum is narrower
(b) more number of fringes
(c) less number of fringes
(d) no diffraction pattern

58. In Fresnel's class of diffraction, the
(a) obstacle screen distance is small
(b) the diffracted wavefront is considered as spherical
(c) no convex lens is used to focus the diffraction fringes on the screen
(d) All of the above

59. The X-rays cannot be diffracted by means of an ordinary grating due to
(a) large wavelength (b) high speed
(c) short wavelength (d) All of these

60. When the diffraction pattern from a certain slit illuminated with laser light ($\lambda = 6330$ Å) is projected on a screen 150 cm from the slit, the second minima on each side are separated by 8 cm. This tells us that
(a) the slit is approximately 0.005 cm wide
(b) the slit is approximately 0.05 cm wide
(c) a/λ is approximately 7.5 (a is the slit width)
(d) a/λ is approximately 750

61. A diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by blue light?
(a) No change
(b) Diffraction bands become narrower and crowded together
(c) Diffraction bands become broader and farther apart
(d) Bands disappear

62. Light of wavelength 6328 Å is incident normally on a slit having a width of 0.2 mm. The width of the central maximum measured from minimum to minimum of diffraction pattern on a screen 9 m away will be nearly
(a) 0.36° (b) 0.18°
(c) 0.72° (d) 0.09°

63. A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is
(a) π (b) zero (c) $\pi/2$ (d) 2π

64. In the diffraction pattern of single slit under bichromatic illumination, the first minimum with the wavelength λ_1 is found to be coincident with the third maximum at λ_2. So
(a) $3\lambda_1 = 0.3\lambda_2$ (b) $3\lambda_1 = \lambda_2$
(c) $\lambda_1 = 3.5\lambda_2$ (d) $0.3\lambda_1 = 3\lambda_2$

65. The first diffraction minima due to a single slit diffraction is at $\theta = 30°$ for a light of wavelength 5000 Å. The width of the slit is
(a) 5×10^{-5} cm (b) 10×10^{-5} cm
(c) 2.5×10^{-5} cm (d) 1.25×10^{5} cm

Miscellaneous Problems

66. In a wave, the path difference corresponding to a phase difference of ϕ is
(a) $\frac{\pi}{2\lambda}\phi$ (b) $\frac{\pi}{\lambda}\phi$ (c) $\frac{\lambda}{2\pi}\phi$ (d) $\frac{\lambda}{\pi}\phi$

67. The phase difference between incident wave and reflected wave is 180,° when light ray
(a) enters into glass from air
(b) enters into air from glass
(c) enters into glass from diamond
(d) enters into water from glass

68. The main difference in the phenomenon of interference and diffraction is that
(a) diffraction is due to interaction of light from the same wavefront, whereas interference is the interaction of waves from two isolated sources
(b) diffraction is due to interaction of light from same wavefront, whereas the interference is the interaction of two waves derived from the same source
(c) diffraction is due to interaction of waves derived from the same source, whereas the interference is the bending of light from the same wavefront
(d) diffraction is caused by the reflected waves from a source, whereas interference is caused due to refraction of waves from a source

69. Two sources are called coherent, if they produce waves
(a) of equal wavelength
(b) of equal velocity
(c) having same shape of wavefront
(d) having a constant phase difference

70. Huygen's wave theory of light cannot explain
(a) diffraction
(b) interference
(c) polarization
(d) photoelectric effect

71. When light waves suffer reflection at the interface between air and glass, the change of the phase of the reflected wave is equal to
(a) zero
(b) $\pi/2$
(c) π
(d) 2π

[Level 2]

Only One Correct Option

1. In the Young's double slit experiment, the intensities at two points P_1 and P_2 on the screen are respectively I_1 and I_2. If P_1 is located at the centre of a bright fringe and P_2 is located at a distance equal to a quarter of fringe width from P_1 then I_1/I_2 is
(a) 2
(b) 3
(c) 4
(d) None of the above

2. In Young's double slit experiment the 10th maximum of wavelength λ_1 is at a distance of y_1 from the central maximum. When the wavelength of the source is changed to λ_2, 5th maximum is at a distance of y_2 from its central maximum. The ratio $\left(\frac{y_1}{y_2}\right)$ is
(a) $\frac{2\lambda_1}{\lambda_2}$ (b) $\frac{2\lambda_2}{\lambda_1}$
(c) $\frac{\lambda_1}{2\lambda_2}$ (d) $\frac{\lambda_2}{2\lambda_1}$

3. Two light rays having the same wavelength λ in vacuum are in phase initially. Then, the first ray travels a path L_1 through a medium of refractive index n_1 while the second ray travels a path of length L_2 through a medium of refractive index n_2. The two waves are then combined to produce interference. The phase difference between the two waves is
(a) $\frac{2\pi}{\lambda}(L_2 - L_1)$ (b) $\frac{2\pi}{\lambda}(n_1L_1 - n_2L_2)$
(c) $\frac{2\pi}{\lambda}(n_2L_1 - n_1L_2)$ (d) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1} - \frac{L_2}{n_2}\right)$

4. In Young's double slit experiment, the two slits acts as coherent sources of equal amplitude A and wavelength λ. In another experiment with the same set up the two slits are sources of equal amplitude A and wavelength λ but are incoherent. The ratio of the intensity of light at the mid-point of the screen in the first case to that in the second case is

(a) 4 : 1 (b) 1 : 1
(c) 2 : 1 (d) 1 : 4

5. In the ideal double slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position, where the central maximum occurred previously remains unchanged. The minimum thickness of the glass plate is

(a) 2λ (b) $\frac{2\lambda}{3}$
(c) $\frac{\lambda}{3}$ (d) λ

6. In the standard Young's double slit experiment, the intensity on the screen at a point distant 1.25 fringe widths from the central maximum is (assuming slits to be identical)

(a) $\frac{1}{2} I_{max}$ (b) $\frac{1}{4} I_{max}$
(c) $\frac{1}{3} I_{max}$ (d) I_{max}

7. In a Young's double slit experiment, D equals to the distance of screen and d is the separation between the slits. The distance of the nearest point to the centre maximum, where the intensity is same as that due to a single slit, is equal to

(a) $\frac{D\lambda}{d}$ (b) $\frac{D\lambda}{2d}$
(c) $\frac{D\lambda}{3d}$ (d) $\frac{2D\lambda}{d}$

8. Two waves of same frequency and same amplitude from two monochromatic sources are allowed to superpose at a certain point. If in one case the phase difference is 0° and in other case it is $\pi/2$, then the ratio of the intensities in the two cases will be

(a) 1 : 1
(b) 2 : 1
(c) 4 : 1
(d) None of the above

9. An interference is observed due to two coherent sources separated by a distance 5λ along y-axis, where λ is the wavelength of light. A detector D is moved along the positive x-axis. The number of point on the x-axis excluding the points $x = 0$ and $x = \infty$ at which resultant intensity will be maximum, are

(a) 4 (b) 5
(c) ∞ (d) zero

10. In a Young's double slit experiment using unequal slit widths, the intensity at a point mid-way between a bright and dark fringes is $4I$. If one slit is covered by an opaque film, intensity at that point becomes $2I$. If the other is covered instead, the intensity at that point is

(a) $2I$ (b) $5I$
(c) $(5 + 2\sqrt{2})\, I$ (d) $(5 + 4\sqrt{2})\, I$

11. Microwaves of frequency 3×10^4 MHz and ultrasonic waves of wavelength 1 cm are passed through a slit of width 2 cm. Then

(a) diffraction will occur only in the microwaves
(b) diffraction will occur only in the ultrasonic waves
(c) diffraction will occur in both but the diffraction patterns will be different
(d) diffraction will occur in both and the diffraction patterns will be identical

12. In Young's double slit experiment, white light is used. The separation between the slits is b. The screen is at a distance $d (d >> b)$ from the slits. Some wavelengths are missing exactly in front of one slit. These wavelengths are

(a) $\lambda = \frac{b^2}{d}$ (b) $\lambda = \frac{2b^2}{d}$
(c) $\lambda = \frac{b^2}{3d}$ (d) $\lambda = \frac{2b^2}{3d}$

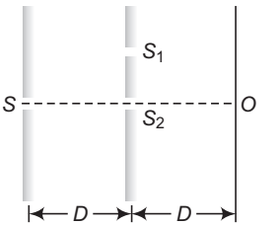
13. In Young's double slit experiment the y-coordinates of central maxima and 10th maxima are 2 cm and 5 cm respectively. When the Young's double slit experiment apparatus is immersed in a liquid of refractive index 1.5 the corresponding y-coordinates will be

(a) 2 cm, 7.5 cm (b) 3 cm, 6 cm
(c) 2 cm, 4 cm (d) 4/3 cm, 10/3 cm

14. In Young's double slit experiment how many maximas can be obtained on a screen (including the central maximum) on both sides of the central fringes if $\lambda = 2000$ Å and $d = 7000$ Å

(a) 12 (b) 7
(c) 18 (d) 4

15. Two ideal slits S_1 and S_2 are at a distance d apart and illuminated by light of wavelength λ passing through an ideal source slit S placed on the line through S_2 as shown. The distance between the planes of slits and the source slit is D.



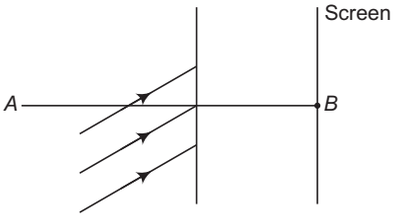
A screen is held at a distance D from the plane of the slits. The minimum value of d for which there is darkness at O is

(a) $\sqrt{\frac{3\lambda D}{2}}$ (b) $\sqrt{\lambda D}$
(c) $\sqrt{\frac{\lambda D}{2}}$ (d) $\sqrt{3\lambda D}$

16. Light of wavelength 589.3 nm is incident normally on two slits of separation 0.1 mm width 0.01 nm. The angular width of the central diffraction maximum at a distance of 1 m from the slits, is

(a) 0.68° (b) 0.34°
(c) 2.05° (d) None of these

17. A beam of light parallel to central line AB is incident on the plane of slits. The number of minima obtained on the large screen is n_1. Now, if the beam is tilted by some angle ($\neq 90°$) as shown in figure, then the number of minima obtained is n_2. Then



(a) $n_1 = n_2$ (b) $n_1 > n_2$
(c) $n_2 > n_1$ (d) n_2 will be zero

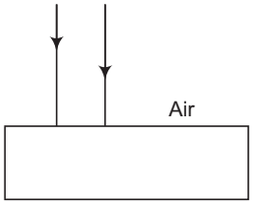
18. The intensity of each of the two slits in Young's double slit experiment is I_0. Calculate the minimum separation between the two points on the screen, where intensities are $2I_0$ and I_0. Given, the fringe width equal to α

(a) $\frac{\alpha}{4}$ (b) $\frac{\alpha}{3}$
(c) $\frac{\alpha}{12}$ (d) None of these

19. Young's double slit experiment is made in a liquid. The tenth bright fringe in liquid lies in screen, where 6th dark fringe lies in vacuum. The refractive index of the liquid is approximately

(a) 1.8 (b) 1.54
(c) 1.67 (d) 1.2

20. A parallel beam of light of intensity I is incident on a glass plate. 25% of light is reflected in any reflection by upper surface and 50% of light is reflected by any reflection from lower surface. Rest is refracted. The ratio of maximum to minimum intensity in interference region of reflected rays is



(a) $\left(\frac{\frac{1}{2}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$ (b) $\left(\frac{\frac{1}{4}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$ (c) $\frac{5}{8}$ (d) None of these

21. In a Young's experiment, one of the slits is covered with a transparent sheet of thickness 3.6×10^{-3} cm due to which position of central fringe shifts to a position originally occupied by 30th bright fringe. The refractive index of the sheet, if $\lambda = 6000$ Å is

(a) 1.5 (b) 1.2
(c) 1.4 (d) 1.6

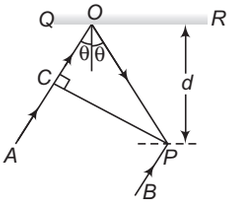
22. In an interference pattern, the position of zeroth order maxima is 4.8 mm from a certain point P on the screen. The fringe width is 0.2 mm. The position of second minima from point P is

(a) 5.1 mm (b) 5 mm
(c) 5.4 mm (d) 5.2 mm

23. One slit of a double slit experiment is covered by a thin glass plate of refractive index 1.4 and the other by a thin glass plate of the refractive index 1.7. The point on the screen, where the central maximum fall before the glass plate was inserted, is now occupied by what had been the fifth bright fringe was seen before. Assume the plate have the same thickness t and wavelength of light 480 nm. Then the value of t is

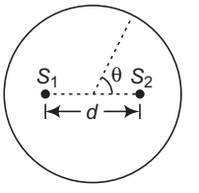
(a) 2.4 μm (b) 4.8 μm (c) 8 μm (d) 16 μm

24. In the adjacent diagram, CP represents a wavefront and AO and BP, the corresponding two rays. Find the condition of θ for constructive interference at P between the rays BP and reflected ray AOP



(a) $\cos\theta = 3\lambda/2d$ (b) $\cos\theta = \lambda/4d$
(c) $\sec\theta - \cos\theta = \lambda/d$ (d) $\sec\theta - \cos\theta = 4\lambda/d$

25. Two coherent sources separated by distance d are radiating in phase having wavelength λ. A detector moves in a big circle around the two sources in the plane of the two sources. The angular position of $n = 4$ interference maxima is given as



(a) $\sin^{-1}\frac{n\lambda}{d}$ (b) $\cos^{-1}\frac{4\lambda}{d}$
(c) $\tan^{-1}\frac{d}{4\lambda}$ (d) $\cos^{-1}\frac{\lambda}{4d}$

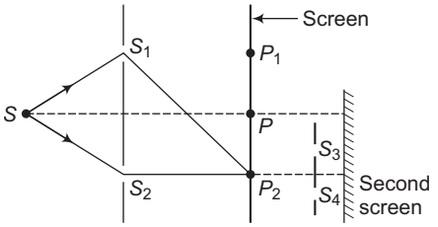
26. Two coherent light sources A and B are at a distance 3λ from each other (λ = wavelength). The distances from A on the x-axis at which the interference is constructive are



(a) 3λ (b) 4λ
(c) $5\lambda/4$ (d) 8.75λ

27. In a Young's double-slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case,
(a) there shall be alternate interference patterns of red and blue
(b) there shall be an interference pattern for red distinct from that for blue
(c) there shall be no interference fringes
(d) there shall be an interference pattern for red mixing with one for blue

28. Figure shows a standard two slit arrangement with slits S_1, S_2, P_1, P_2 are the two minima points on either side of P (figure).



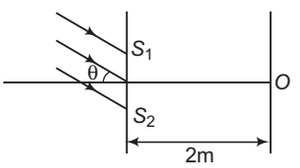
At P_2 on the screen, there is a hole and behind P_2 is a second 2-slit arrangement with slits S_3, S_4 and a second screen behind them.
(a) There would be no interference pattern on the second screen but it would be lighted
(b) The second screen would be totally dark
(c) There would be a single bright point on the second screen
(d) There would be a regular two slit pattern on the second screen

More than One Correct Options

1. A Young's double slit experiment is performed with white light, then
(a) the fringe next to the central will be red
(b) the central fringe will be white
(c) the fringe next to the central will be violet
(d) there will not be a completely dark fringe

2. If one of the slit of a standard Young's double slit experiment is covered by a thin parallel sided glass slab so that it transmits only one-half the light intensity of the other, then
(a) the fringe pattern will get shifted towards the covered slit
(b) the fringe pattern will get shifted away from the covered slit
(c) the bright fringes will be less bright and the dark ones will be more bright
(d) the fringe width will remain unchanged

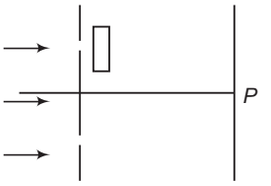
3. A parallel beam of light ($\lambda = 5000\,\text{Å}$) is incident at an angle $\theta = 30°$ with the normal to the slit plane in a Young's double slit experiment. The intensity due to each slit is I_0. Point O is equidistant from S_1 and S_2. The distance between slits is 1 mm.



(a) The intensity at O is $4I_0$
(b) The intensity at O is zero
(c) The intensity at a point on the screen 4 mm above O is $4I_0$
(d) The intensity at a point on the screen 4 mm above O is zero

4. In the phenomenon of interference
(a) sources must be coherent
(b) amplitudes must be same
(c) wavelengths must be same
(d) intensities may be different

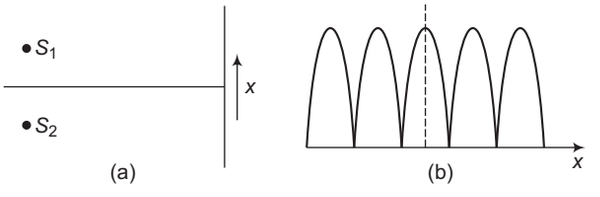
5. In Young's double slit experiment. set up shown in figure,



(a) zero order maximum will lie above point P
(b) first order maxima may lie above point P
(c) first order maxima may lie below point P
(d) zero order maxima may lie at point P

6. Bichromatic light of wavelengths $\lambda_1 = 5000\,\text{Å}$ and $\lambda_2 = 7000\,\text{Å}$ are used in Young's double slit experiment. Then,
(a) 14th order maxima of λ_1 will coincide with 10th order maxima of λ_2
(b) 21st order maxima of λ_2 will coincide with 15th order maxima of λ_1
(c) 11th order minima of λ_1 will coincide with 8th order minima of λ_2
(d) 3rd order minima of λ_1 will coincide with 4th order minima of λ_2

7. Two sources S_1 and S_2 of intensity I_1 and I_2 are placed in front of a screen [Fig. (a)]. The pattern of intensity distribution seen in the central portion is given by Fig. (b).



In this case, which of the following statements are true?
(a) S_1 and S_2 have the same intensities
(b) S_1 and S_2 have a constant phase difference
(c) S_1 and S_2 have the same phase
(d) S_1 and S_2 have the same wavelength

8. For light diverging from a point source,
(a) the wavefront is spherical
(b) the intensity decreases in proportion to the distance squared
(c) the wavefront is parabolic
(d) the intensity at the wavefront does not depend on the distance

Comprehension Based Questions

Passage (Q. 1 to 4)

A Young's double slit apparatus is immersed in a liquid of refractive index 1.33. *It has slit separation of* 1 *mm and interference pattern is observed on the screen at a distance* 1.33 *m from plane of slits. The wavelength in air is* 6300 *Å.*

1. Calculate the fringe width.
(a) 0.63 mm (b) 1.26 mm (c) 1.67 mm (d) 2.2 mm

2. Find the distance of seventh bright fringe from third bright fringe lying on the same side of central bright fringe .
(a) 2.52 mm (b) 4.41 mm
(c) 1.89 mm (d) 1.26 mm

3. One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to interchange the position of minima and maxima
(a) 2.57 mm (b) 1.57 mm
(c) 3.27 mm (d) 4.18 mm

4. One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the fringe width
(a) 0.63 mm (b) 1.26 mm
(c) 1.67 mm (d) 2.2 mm

Assertion and Reason

Directions (Q. Nos. 1-10) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** If a glass slab is placed in front of one of the slits, then fringe width will decrease.

Reason Glass slab will produce an additional path difference.

2. **Assertion** Interference shows wave nature of light.

Reason Photoelectric effect proves particle nature of light.

3. **Assertion** If wavelength is of the order of distance between the slits, then fringe size is large.

Reason Fringe width is given by

$$w = \frac{\lambda D}{d}$$

4. **Assertion** In Young's double slit experiment ratio $\frac{I_{max}}{I_{min}}$ is infinite.

Reason If width of any one of the slits is slightly increased, then this ratio will decrease.

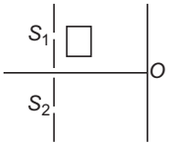
5. **Assertion** Stationary wave is an example of interference.

Reason Points of antinodes are points of constructive interference.

6. **Assertion** Two coherent sources transmit waves of equal intensity I_0. Resultant intensity at a point, where path difference is $\frac{\lambda}{3}$ is also I_0.

Reason In interference resultant intensity at any point is the average intensity of two individual intensities.

7. **Assertion** In the Young's double slit experiment. set up shown in figure, a glass slab is inserted in front of the slit S_1 as shown in figure. Then zero order maxima (where, path difference $= 0$) will lie above point *O*.



Reason Glass slab will produce an extra path difference in the path of the ray passing through S_1.

8. **Assertion** Fraunhofer diffraction occurs when all the rays passing through a narrow slit are approximately parallel to one another.

Reason Fraunhofer diffraction pattern can be achieved by placing the screen far from the slit.

9. **Assertion** In Young's double slit experiment, often both the phenomena interference and diffraction are present.

Reason Diffraction results due to superposition of wavelets from different points of the some wavefront.

10. **Assertion** In diffraction phenomenon different maximas have different intensities.

Reason In interference different maximas have same intensities.

Match the Columns

1. In Young's double slit experiment, match the following two columns.

Column I	Column II
(A) When width of one slit is slightly increased	(p) maximum intensity will increase
(B) When one slit is closed	(q) maximum intensity will decrease
(C) When both the sources are made incoherent	(r) maximum intensity will remain same
(D) When a glass slab is inserted in front of one of the slits	(s) fringe pattern will disappear

Assume absorption from glass slab to be negligible.

2. In normal Young's double slit experiment maximum intensity is $4I_0$. In Column I, y-coordinate is given corresponding to centre line. In Column II resultant intensities are given. Match the two columns.

Column I	Column II
(A) $y = \lambda D / d$	(p) $I = I_0$
(B) $y = \lambda D / 2d$	(q) $I = 2I_0$
(C) $y = \lambda D / 3d$	(r) $I = 4I_0$
(D) $y = \lambda D / 4d$	(s) I = zero

3. In normal Young's double slit experiment match the following two columns.

Column I	Column II
(A) In Young's double slit experiment apparatus is immersed in a liquid	(p) fringe width will increase
(B) When wavelength of light used in increased	(q) fringe width will decrease
(C) When distance between slits and screen (D) is increased	(r) fringe width will remain constant
(D) When distance between two slits (d) is increased	(s) fringe pattern will disappear

Entrance Gallery

2014

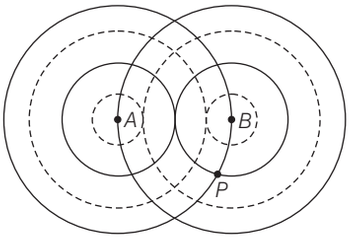
1. A light source, which emits two wavelengths $\lambda_1 = 400\,\text{nm}$ and $\lambda_2 = 600\,\text{nm}$, is used Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2 respectively, then **[JEE Advanced]**

(a) $\beta_2 > \beta_1$
(b) $m_1 > m_2$
(c) from the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1
(d) the angular separation of fringes of λ_1 is greater than λ_2

2. Two beams, A and B of plane polarised light with mutually perpendicular planes of polarisation are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then I_A / I_B equals. **[JEE Main]**

(a) 3 (b) $\frac{3}{2}$
(c) 1 (d) $\frac{1}{3}$

3. The diagram below shows, two sources A and B, vibrating in phase, in the same uniform medium and producing circular wave fronts. **[JEE Main]**



Crests-troughs

Which phenomenon occurs at point P?
(a) Destructive interference
(b) Constructive interference
(c) Reflection
(d) Refraction

4. Two coherent monochromatic beams of intensities I and $4I$ respectively are superposed. The maximum and minimum intensities in the resulting pattern are **[Karnataka CET]**

(a) $5I$ and $3I$ (b) $9I$ and $3I$
(c) $4I$ and I (d) $9I$ and I

5. A fringe width of a certain interference pattern is $\beta = 0.002$ cm. What is the distance of 5th dark fringe from centre? **[Karnataka CET]**

(a) 9×10^{-3} cm (b) 11×10^{-2} cm
(c) 1.1×10^{-2} cm (d) 3.28×10^{6} cm

6. Diameter of the objective of a telescope is 200 cm. What is the resolving power of telescope? (Take, wavelength of light = 500 Å) **[Karnataka CET]**

(a) 6.56×10^5 (b) 3.28×10^5
(c) 1×10^6 (d) 3.28×10^6

7. A polarised light of intensity I_0 is passed through another polariser whose pass axis makes an angle of 60° with the pass axis of the former. What is the intensity of emergent polarised light from second polariser? **[Karnataka CET]**

(a) $I = I_0$
(b) $I = I_0/6$
(c) $I = I_0/5$
(d) $I = I_0/4$

2013

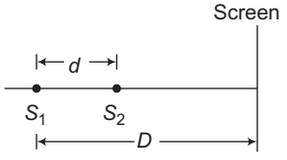
8. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is **[JEE Advanced]**

(a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$
(c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$

9. Using the expression $2d \sin\theta = \lambda$, one calculates the value of d by measuring the corresponding angles θ in the range 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As, θ increases from 0° **[JEE Advanced]**

(a) the absolute error in d remains constant
(b) the absolute error in d increases
(c) the fractional error in d remains constant
(d) the fractional error in d decreases

10. Two coherent point sources S_1 and S_2 are separated by a small distance d as shown in the figure. The fringes obtained on the screen will be **[JEE Main]**



(a) points (b) straight lines
(c) semi-circle (d) concentric circles

11. In a Young's double slit experiment, the slit separation is 1 mm and the screen is 1m from the slit. For a monochromatic light of wavelength 500 nm, the distance of 3rd minima from the central maxima is **[Karnataka CET]**

(a) 0.50 mm (b) 1.25 mm
(c) 1.50 mm (d) 1.75 mm

2012

12. Young's double slit experiment is carried out by using green, red and blue light, one colour at time. The fringe widths recorded are β_G, β_R and β_B respectively. Then, **[IIT JEE]**

(a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$
(c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$

13. In Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I, when they interfere at phase difference ϕ, is given by **[AIEEE]**

(a) $\frac{I_m}{9}(4 + 5\cos\phi)$ (b) $\frac{I_m}{3}\left(1 + 2\cos^2\frac{\phi}{2}\right)$
(c) $\frac{I_m}{5}\left(1 + 4\cos^2\frac{\phi}{2}\right)$ (d) $\frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$

14. For sustained interference fringes in double slit experiment, essential condition/s/is/are

I. sources must be coherent.

II. the intensities of the two sources must be equal.

Here, the correct option/s/is/are **[Karnataka CET]**

(a) Neither I nor II
(b) Both I and II
(c) Only I
(d) Only II

15. In Young's double slit experiment, fringes of width β are produced on a screen kept at a distance of 1 m from the slit. When the screen is moved away by 5×10^{-2} m, fringe width changes by 3×10^{-5} m. The separation between the slits is 1×10^{-3} m. The wavelength of the light used is … nm. **[Karnataka CET]**

(a) 400 (b) 500
(c) 600 (d) 700

16. In single slit experiment, the width of the slit is reduced. Then, the linear width of the principal maxima **[Karnataka CET]**

(a) decreases but becomes less bright
(b) increases but becomes less bright
(c) decreases but becomes more bright
(d) increases but becomes more bright

17. A parallel beam of light of wavelength 6000 Å gets diffracted by a single slit of width 0.3 mm. The angular position of the first minima of diffracted light is **[Karnataka CET]**

(a) 6×10^{-3} rad (b) 1.8×10^{-3} rad
(c) 3×10^{-3} rad (d) 2×10^{-3} rad

2011

18. At two points P and Q on screen in Young's double slit experiment. Waves from slits S_1 and S_2 have a path difference of 0 and $\frac{\lambda}{4}$ respectively. The ratio of intensities at P and Q will be **[AIEEE]**

(a) 3 : 2 (b) 2 : 1
(c) $\sqrt{2}$: 1 (d) 4 : 1

19. In a Young's double slit experiment, the two slits act as coherent sources of waves of equal amplitude A and wavelength λ. In another experiment with the same arrangement the two slits are made to act as incoherent sources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in the first case is I_1 and in the second case I_2, then the ratio $\frac{I_1}{I_2}$ is **[AIEEE]**

(a) 4 (b) 2
(c) 1 (d) 0.5

20. A beam of natural light falls on a system of 5 polaroids, which are arranged in succession such that the pass axis of each polaroid is turned through 60° with respect to the preceding one. The fraction of the incident light intensity that passes through the system is **[Kerala CEE]**

(a) $\frac{1}{64}$ (b) $\frac{1}{32}$
(c) $\frac{1}{256}$ (d) $\frac{1}{512}$
(e) $\frac{1}{128}$

21. The critical angle of a certain medium is $\sin^{-1}\left(\frac{3}{5}\right)$. The polarising angle of the medium is [Karnataka CET]

(a) $\tan^{-1}\left(\frac{4}{3}\right)$ (b) $\tan^{-1}\left(\frac{3}{4}\right)$

(c) $\tan^{-1}\left(\frac{5}{3}\right)$ (d) $\sin^{-1}\left(\frac{4}{5}\right)$

22. A narrow slit of width 2 mm is illuminated by monochromatic light of wavelength 500 nm. The distance between the first minima on either side on a screen at a distance of 1 m is [Kerala CEE]

(a) 5 mm (b) 0.5 mm
(c) 1 mm (d) 10 mm
(e) 2.5 mm

23. Consider the following statements in case of Young's double slit experiment. [Karnataka CET]

I. A slit S is necessary, if we use an ordinary extended source of light.

II. A slit S is not needed, if we use an ordinary but well collimated beam of light.

III. A slit S is not needed, if we use a specially coherent source of light.

Which of the above statements are correct?

(a) I and III (b) II and III
(c) I and II (d) I, II and III

24. The wavelength of the light in Young's double slit experiment is λ. The intensity at a point on the screen is I, where the path difference is $\frac{\lambda}{6}$. If I_0 denotes the maximum intensity, then the ratio of I and I_0 is [Karnataka CET]

(a) 0.866 (b) 0.707
(c) 0.807 (d) 0.75

25. What is the minimum thickness of a thin film required for constructive interference in the reflected light from it? Given, the refractive index of the film = 1.5, wavelength of the light incident on the film = 600 nm. [Karnataka CET]

(a) 100 nm (b) 300 nm
(c) 50 nm (d) 200 nm

26. Critical angle for certain medium is $\sin^{-1}(0.6)$. The polarising angle of that medium is [Karnataka CET]

(a) $\tan^{-1}[1.5]$ (b) $\sin^{-1}[0.8]$
(c) $\tan^{-1}[1.6667]$ (d) $\tan^{-1}[0.6667]$

27. In Young's double slit experiment the two slits are d distance apart. Interference pattern is observed on a screen at a distance D from the slits. A dark fringe is observed on the screen directly opposite to one of the slits. The wavelength of light is [WB JEE]

(a) $\frac{D^2}{2d}$ (b) $\frac{d^2}{2D}$

(c) $\frac{D^2}{d}$ (d) $\frac{d^2}{D}$

28. In a Young's double slit experiment, the fringe width is found to be 2 mm, when light of wavelength 6000 Å is used. Find the change in fringe width, if the whole apparatus is immersed in water of refractive index 1.33. [MHT CET]

(a) 0.5 mm (b) 1 mm
(c) 1.5 mm (d) 2 mm

29. In an interference experiment, phase difference for points where the intensity is minimum is $(n = 1, 2, 3, \ldots)$ [MHT CET]

(a) $n\pi$ (b) $(n + 1)\pi$
(c) $(2n - 1)\pi$ (d) zero

2010

30. In a given direction, the intensities of the scattered light by a scattering substance for two beams of light are in the ratio of 256 : 81. The ratio of the frequency of the first beam to the frequency of the second beam is [Karnataka CET]

(a) 64 : 127
(b) 1 : 2
(c) 64 : 27
(d) None of the abovc

31. In single slit diffraction pattern [MHT CET]

(a) central fringe has negligible width than others
(b) all fringes are of same width
(c) central fringes do not exist
(d) None of the above

Answers

Level 1

Objective Problems

1. (a)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (c)	**9.** (c)	**10.** (c)
11. (b)	**12.** (b)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (c)	**18.** (c)	**19.** (b,d)	**20.** (a)
21. (a)	**22.** (b)	**23.** (a)	**24.** (b)	**25.** (b)	**26.** (c)	**27.** (d)	**28.** (b)	**29.** (b)	**30.** (d)
31. (c)	**32.** (d)	**33.** (d)	**34.** (a)	**35.** (c)	**36.** (c)	**37.** (b)	**38.** (c)	**39.** (d)	**40.** (d)
41. (b)	**42.** (c)	**43.** (b)	**44.** (c)	**45.** (c)	**46.** (a)	**47.** (b)	**48.** (b)	**49.** (b)	**50.** (a)
51. (b)	**52.** (d)	**53.** (a)	**54.** (c)	**55.** (c)	**56.** (a)	**57.** (d)	**58.** (d)	**59.** (c)	**60.** (c)
61. (a)	**62.** (a)	**63.** (a)	**64.** (a)	**65.** (c)	**66.** (c)	**67.** (c)	**68.** (a)	**69.** (b)	**70.** (d)
71. (c)									

Level 2

Only One Correct Option

1. (a)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (a)
11. (d)	**12.** (a,c)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (c)	**19.** (a)	**20.** (b)
21. (a)	**22.** (a)	**23.** (d)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (c)	**28.** (d)		

More than One Correct Options

1. (b,c,d)	**2.** (a,c,d)	**3.** (a,c)	**4.** (a,c,d)	**5.** (a,b,c)	**6.** (a,c)	**7.** (a,b,d)	**8.** (a,b)

Comprehension Based Questions

1. (a)	**2.** (a)	**3.** (b)	**4.** (a)

Assertion and Reason

1. (d)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (a or b)	**8.** (b)	**9.** (b)	**10.** (b)

Match the Columns

1. (A→p, B→q,s or s, C→q, s or s, D→r) **2.** (A→r, B→s, C→p, D→q) **3.** (A→q, B→p, C→p, D→q)

Entrance Gallery

1. (c)	**2.** (d)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (b)	**9.** (b)	**10.** (d)
11. (b)	**12.** (d)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (d)	**17.** (d)	**18.** (b)	**19.** (b)	**20.** (d)
21. (c)	**22.** (a)	**23.** (a)	**24.** (d)	**25.** (a)	**26.** (c)	**27.** (d)	**28.** (c)	**29.** (c)	**30.** (d)
31. (d)									

Solutions

Level 1 : Objective Problems

1. For phenomenon of interference it is necessary that there should be a constant phase difference between two light rays.

2. In interference,

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

while $$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

So, $$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

$$= \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

$\therefore$ Given, $$\frac{A_1}{A_2} = \frac{3}{5}$$

$\therefore$ $$\frac{I_{max}}{I_{min}} = \frac{(3+5)^2}{(3-5)^2} = \frac{16}{1}$$

3. The sources which produce sustained, i.e. observable interference are called coherent sources. In case of interference as ($I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$), interference will be sustained, if the phase difference ϕ at a given point does not vary with time, so phase difference between two sources must be constant.

5. Here, phase difference, $\phi = \pi / 3$.

So, amplitude of resulting wave,

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$$

$$= \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \cos \pi/3} = 6$$

6. Resultant intensity for two interfering waves is given by

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi.$$

For maximum,

$$\cos\phi = \text{maximum} = 1$$

i.e. $$\phi = 0, \pm 2\pi, \pm 4\pi$$

or $$\phi = \pm 2\pi n, \quad n = 0, 1, 2, \ldots$$

$\therefore$ $$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= (\sqrt{I_1} + \sqrt{I_2})^2$$

7. For destructive interference,

$$\cos\phi = \text{min} = -1$$

i.e. $\phi = \pm\ \pi,\ \pm 3\pi,\ \pm 5\pi$

or $$\phi = \pm (2n+1)\pi$$

with $$n = 0, 1, 2, 3, \ldots$$

or $$\Delta x = \pm \frac{(2n+1)\lambda}{2}$$

11. $$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

and $$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

12. $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

13. We know that, $2\mu t = \frac{\lambda}{2}$

$\therefore$ $$t_{min} = \frac{\lambda}{4\mu} = 141 \text{ nm}$$

14. $\Delta I = (I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi_1) - (I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi_2)$

$= 2\sqrt{I_1 I_2}\,(\cos\phi_1 \sim \cos\phi_2)$

15. Where phase difference is 0, intensity is maximum.

where, $$\phi = \frac{\pi}{2} \quad \text{or} \quad \frac{\phi}{2} = \frac{\pi}{4},$$

The intensity will be $I = I_{max}\cos^2\frac{\phi}{2} = \frac{I_{max}}{2}$

$\therefore$ The desired ratio is 2 : 1.

16. Given, $$\phi = \frac{\pi}{3}$$

$\therefore$ $$A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}$$

$$= \sqrt{16 + 9 + 2(4)(3)\cos 60^\circ}$$

$$= \sqrt{37} \approx 6$$

17. We know that, $I = I + I + 2\sqrt{I\,I}\cos\phi$

$\therefore$ $$\cos\phi = -\frac{1}{2} \quad \text{or} \quad \phi = 120^\circ = \frac{2\pi}{3}$$

18. Separation of bright lines or fringe width,

$$\beta = \frac{D\lambda}{d} = \frac{5 \times 10^{-7} \times 2}{10^{-3}}$$

$$= 10^{-3} \text{ m} = 1.0 \text{ mm}$$

19. Given ,

$$\frac{I_{bright}}{I_{dark}} = \frac{I_{max}}{I_{min}} = 9$$

or $$\left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = 9$$

or $$\frac{a_1 + a_2}{a_1 - a_2} = 3$$

$\Rightarrow$ $$\frac{a_1}{a_2} = \frac{3+1}{3-1}$$

$\Rightarrow$ $$\frac{a_1}{a_2} = 2$$

So, $$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{4}{1}$$

20. Fringe width in double slit experiment is given by

$$\beta = \frac{D\lambda}{d}$$

it implies that $\beta \propto \lambda$,
so in given options, wavelength of violet light is λ_v = minimum.
Hence, fringe width of violet colour light is also minimum.

21. On introducing mica sheet, fringe shift $= \frac{\beta}{\lambda}(\mu - 1)t$

where, t is thickness of sheet.

So, $$\text{shift} = \frac{\beta}{(5000 \times 10^{-10})} \times (1.5) \times (2 \times 10^{-6})$$

$$= 2\beta$$

So, central bright maximum will shift 2 fringes upward.

22. Fringe width,

$$\beta = \frac{D\lambda}{d}$$

If both d and D are doubled, then

$$\beta' = \frac{(2D)\lambda}{2d} = \frac{D\lambda}{d} = \beta$$

23. In interference between waves of equal amplitudes a, the minimum intensity is zero and the maximum intensity is proportional to $4a^2$. (because slit width $\propto$ amplitude). For waves of unequal amplitudes a and $A(A > a)$, the minimum intensity is non-zero and the maximum intensity is proportional to $(a + A)^2$, which is greater than $4a^2$. So, the intensities of both the maxima and the minima increase.

25. For dark fringe at P,

$$S_1P - S_2P = \frac{5\lambda}{2} = 5 \times \frac{6000}{2}$$

$$= 15000\ \text{Å} = 1.5 \text{ micron.}$$

27. For minima, path difference should be an odd multiple of half wavelength.

or $$\Delta = (2n-1)\frac{\lambda}{2}$$

For third minima,

$$n = 3, \quad \Delta = (2 \times 3 - 1)\frac{\lambda}{2} = \frac{5\lambda}{2}$$

28. When thin plate of glass is introduced, then lateral displacement of fringes

$$= \frac{\beta}{\lambda}(\mu - 1)t$$

$$= \frac{1 \times 10^{-3}}{600 \times 10^{-9}}(1.5 - 1) \times 0.06 \times 10^{-3}$$

$$= \frac{1}{20}\text{ m}$$

$$= 5\text{ cm}$$

30. Distance of nth bright fringe from the centre,

$$y_n = \frac{nD\lambda}{d}$$

So, $$y_3 = \frac{3 \times 6000 \times 10^{-10} \times 2.5}{0.5 \times 10^{-3}}$$

$$= 9 \times 10^{-3}\text{ m} = 9\text{ mm}$$

31. $w = \dfrac{\lambda D}{d}$

32. Refractive index is slightly decreased.

33. In normal Young's double slit apparatus,

$$I_1 = I_2 = I_0$$

$\therefore$ $$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 = 4I_0$$

and $$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2 = 0$$

When one of the slit is covered by a glass plate.

$$I_1 < I_0,\ I_2 = I_0$$

$\therefore$ $$I_{\max} < 4I_0 \text{ and } I_{\min} > 0$$

35. $$\text{Shift} = \frac{(\mu - 1)tD}{d}$$

or $$20\left(\frac{\lambda D}{d}\right) = \frac{(\mu - 1)tD}{d}$$

$\therefore$ $$\mu = 1 + \frac{20\lambda}{t} = 1 + \frac{20\ (5.0 \times 10^{-7})}{2.5 \times 10^{-5}} = 1.4$$

36. $$\lambda_{\text{blue}} < \lambda_{\text{green}}$$

$\therefore$ $$w_{\text{blue}} < w_{\text{green}}$$

37. We know that, $x_3 = \dfrac{3\lambda D}{d}$

$$= \frac{3\ (5 \times 10^{-5})\ (200)}{0.02}\text{ cm} = 1.5\text{ cm}$$

40. Wavelength is increased, therefore, fringe width will increase. Number of fringes in the given region will decrease.

42. $$\theta\,(\text{in radian}) = \frac{\lambda}{d}$$

$\therefore$ $$d = \frac{\lambda}{\theta} = \frac{6 \times 10^{-7}}{\pi/180} = 3.4 \times 10^{-5}\text{ m}$$

43. Let nth red coincides with mth blue, then

$$\frac{n\lambda_R D}{d} = \frac{m\lambda_B D}{d}$$

$\therefore$ $$\frac{n}{m} = \frac{\lambda_B}{\lambda_R} = \frac{480}{600} = \frac{4}{5}$$

i.e. 4th red coincides with 5th blue. Next 8th red will coincide with 10th blue and so on. Thus,

$$n = 4 \quad \text{and} \quad m = n + 1 = 5$$

47. $$\Delta Y = \frac{5\lambda D}{d} - \frac{5\lambda D}{2d} = \frac{5\lambda D}{2d}$$

$$= \frac{5 \times 6.5 \times 10^{-7} \times 1}{2 \times 10^{-3}}\text{ m}$$

$$= 1.63\text{ mm}$$

49. For first minima,

$$d \sin\theta = \frac{\lambda}{2} \quad \text{or} \quad d = \frac{\lambda}{2\sin\theta}$$

50. Angular fringe width, $\theta = \dfrac{\lambda}{d}$.

51. Bending of beam of light around corners of obstacles/slit is called diffraction.

52. In single slit, for nth secondary maxima path difference,

$$a \sin\theta = (2n+1)\frac{\lambda}{2}$$

For first secondary maximum,

$$n = 1$$

$\therefore$ $$a \sin\theta = \frac{3\lambda}{2}$$

53. For nth dark fringe, in single slit

$$a \sin\theta = n\lambda$$

For second dark fringe,

$$a \sin\theta = 2\lambda$$

So, $24 \times 10^{-5} \times 10^{-2} \times \sin 30^\circ = 2\lambda$

$\therefore$ $$\lambda = 6 \times 10^{-7}\text{ m}$$

$$= 6000\text{ Å}$$

54. In single narrow slit, for first minima,

$$d \sin\theta = \lambda$$

$$\sin\theta = \frac{\lambda}{d}$$

$\therefore$ $$\theta = \sin^{-1}\left(\frac{5000 \times 10^{-10}}{0.001 \times 10^{-3}}\right) = 30^\circ$$

60. For minima, $a \sin\theta = n\lambda$ (a = slit width)

62. $d \sin\theta = \lambda$

$$\theta \approx \frac{\lambda}{d}$$

$$\therefore \quad 2\theta = \frac{2\lambda}{d} = \frac{2\times 6328\times 10^{-10}}{0.2\times 10^{-3}} \times \frac{180}{\pi} = 0.36^\circ$$

66. For 2π phase difference the path difference is λ.

So, for ϕ phase difference, path difference is $\frac{\lambda}{2\pi}\phi$.

67. When light reflects from denser surface, a phase change of π occurs.

Level 2 : Only One Correct Option

1. $y = \frac{w}{4} = \frac{\lambda D}{4d}$, $\Delta x = \frac{yd}{D} = \frac{\lambda}{4}$, $\phi = \frac{2\pi}{\lambda}\cdot\Delta x = \frac{\pi}{2}$

$$\frac{\phi}{2} = \frac{\pi}{4}$$

Now, $$I_2 = I_1 \cos^2\frac{\phi}{2}$$

or $$\frac{I_1}{I_2} = \frac{1}{\cos^2\phi/2} = 2$$

2. $y_1 = 10w_1 = \frac{10\lambda_1 D}{d}, y_2 = 5w_2 = \frac{5\lambda_2 D}{d}$

$$\therefore \quad \frac{y_1}{y_2} = 2\frac{\lambda_1}{\lambda_2}$$

3. The phase difference, $\Delta\phi = \frac{2\pi}{\lambda}\times\Delta x = \frac{2\pi}{\lambda}\times(n_1L_1 - n_2L_2)$.

5. Shift $= \frac{(\mu-1)tD}{d} = w = \frac{\lambda D}{d}$

$$\therefore \quad t = 2\lambda \qquad (\text{as } \mu = 1.5)$$

6. $$\Delta x = \frac{yd}{D} = \left(\frac{1.25\lambda D}{d}\right)\left(\frac{d}{D}\right)$$

$$= 1.25\,\lambda = \frac{5}{4}\lambda$$

$$\phi = \frac{2\pi}{\lambda}\cdot\Delta x = \frac{2\pi}{\lambda}\times\frac{5}{4}\lambda = \frac{5}{2}\pi$$

$$\frac{\phi}{2} = \frac{5}{4}\pi = 225^\circ$$

$$\therefore \quad I = I_{\max}\cos^2\frac{\phi}{2}$$

$$= \frac{1}{2}I_{\max}$$

7. $$I_0 = 4I_0\cos^2\frac{\theta}{2}$$

$$\phi = \frac{2\pi}{3} = \frac{2\pi}{\lambda}\cdot\Delta x = \frac{2\pi}{\lambda}\left(\frac{yd}{D}\right)$$

$$\therefore \quad y = \frac{\lambda D}{3d}$$

8. $A_1 = 2A$

$$\therefore \quad I_1 = 4I$$

$$A_2 = \sqrt{2}\,A$$

$$\therefore \quad I_2 = 2I$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{2}{1}$$

9. Possible path difference is λ, 2λ, 3λ and 4λ.

10. $y = \frac{w}{4} = \frac{\lambda D}{4d}$

$$\Delta x = \frac{yd}{D} = \frac{\lambda}{4}$$

$$\phi = \frac{2\pi}{\lambda}\cdot\Delta x = \frac{\pi}{2}$$

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$4I = 2I + I_2 + 0$$

$$\therefore \quad I_2 = 2I$$

11. Wavelength of microwaves,

$$\lambda = \frac{v}{f} = \frac{3\times 10^8}{3\times 10^{10}} = 10^{-2} \text{ m or 1 cm}$$

For diffraction pattern, width size should be of the order of width of slit.

12. Path difference between the rays reaching in front of slit S_1 is

$$S_1P - S_2P = \sqrt{(b^2 - d^2)} - d$$

For destructive interference at P

$$S_1P - S_2P = \frac{(2n-1)\lambda}{2}$$

i.e. $$\sqrt{(b^2 + d^2)} - d = \frac{(2n-1)\lambda}{2}$$

$$\Rightarrow \quad d\sqrt{\left(1 + \frac{b^2}{d^2}\right)} - d = \frac{(2n-1)\lambda}{2}$$

$$\Rightarrow \quad d\left(1 + \frac{b^2}{2d^2}\right) - d = \frac{(2n-1)\lambda}{2}$$

$$\Rightarrow \quad \frac{b^2}{2d} = \frac{(2n-1)\lambda}{2}$$

$$\therefore \quad \lambda = \frac{b^2}{(2n-1)\,d}$$

For $$n = 1, 2, \ldots,$$

$$\lambda = \frac{b^2}{d}, \frac{b^2}{3d} \ldots\ldots$$

13. Fringe width,

$$\beta = \frac{D\lambda}{d}$$

i.e. $$\beta \propto \lambda$$

So, wavelength λ and hence fringe width β decreases 1.5 times when immersed in liquid. The distance between central maxima and 10th maxima is 3 cm in vacuum. When immersed in liquid it will reduce to 2 cm. Position of central maxima will not change while 10th maxima will be obtained at $y = 4$ cm.

14. For maximum (brightness) intensity of the screen, path difference should be integer multiple of wavelength, i.e.

$$d = \sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{d} = \frac{n(2000)}{7000} = \frac{n}{3.5}$$

As, $$(\sin\theta)_{\max} = 1$$

So, $$n = 0, 1, 2, 3$$

Thus, only seven maximas can be obtained on both sides of the screen.

15. Path difference between the waves reaching at P,

$$\Delta = \Delta_1 + \Delta_2$$

where, Δ_1 = initial path difference

Δ_2 = path difference between the waves after emerging from slits.

Now, $\Delta_1 = SS_1 - SS_2$

$$= \sqrt{(D^2 + d^2)} - D$$

and $\Delta_2 = S_1O - S_2O$

$$= \sqrt{(D^2 + d^2)} - D$$

$$\therefore \quad \Delta = 2\{\sqrt{(D^2 + d^2)} - D\}$$

$$= 2\{(D^2 + d^2)^{1/2} - D\}$$

$$= 2\left\{\left(D + \frac{d^2}{2D}\right) - D\right\} \text{ from binomial expansion}$$

$$= \frac{d^2}{D}$$

For obtaining dark at O, Δ must be equals to $(2n-1)\frac{\lambda}{2}$,

i.e. $$\frac{d^2}{D} = (2n-1)\frac{\lambda}{2}$$

$$\therefore \quad d^2 = \frac{(2n-1)\lambda D}{2}$$

or $$d = \sqrt{\frac{(2n-1)\lambda D}{2}}$$

For minimum distance $n = 1$,

So, $$d = \sqrt{\left(\frac{\lambda D}{2}\right)}$$

16. $\beta = \frac{\lambda D}{d} = \frac{589.3 \times 10^{-9}}{0.1 \times 10^{-3}} D$

Hence, angular width

$$\theta = \frac{\beta}{D} = \frac{589.3 \times 10^{-9}}{0.1 \times 10^{-3}} \text{ rad}$$

$$= 589.3 \times 10^{-5} \times \frac{180}{\pi}$$

$$= 0.34°$$

17. Fringe size will not change.

18. Given, $2I_0 = 4I_0 \cos^2 \frac{\phi}{2}$

$$\phi = \frac{\pi}{2} = \frac{2\pi}{\lambda} \cdot \Delta x$$

$$= \frac{2\pi}{\lambda} \cdot \left(\frac{y_1 d}{D}\right)$$

$$\therefore \quad y_1 = \frac{\lambda D}{4d}$$

Further, $$I_0 = 4I_0 \cos^2 \frac{\phi}{2}$$

$$\phi = \frac{2\pi}{3} = \frac{2\pi}{\lambda} \Delta x = \frac{2\pi}{\lambda}\left(\frac{y_2 d}{D}\right)$$

$$\therefore \quad y_2 = \frac{\lambda D}{3d}$$

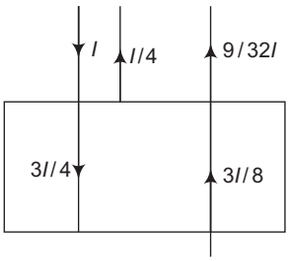
$$\Delta y = y_2 - y_1$$

$$= \frac{1}{12}\left(\frac{\lambda D}{d}\right) = \frac{1}{12}(\alpha)$$

19. $5.5\,\omega = 10\,\omega'$

$$\therefore \quad \frac{\omega}{\omega'} = \frac{10}{5.5} = 1.818 = \mu$$

20. $I_1 = \frac{I}{4}$ and $I_2 = \frac{9I}{32}$



$$\therefore \quad \frac{I_1}{I_2} = \frac{8}{9} \quad \text{or} \quad \sqrt{\frac{I_1}{I_2}} = \frac{2\sqrt{2}}{3}$$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1}/I_2 + 1}{\sqrt{I_1}/I_2 - 1}\right)^2$$

$$= \left[\frac{\frac{1}{4} + \sqrt{\frac{3}{8}}}{\frac{1}{2} - \sqrt{\frac{3}{8}}}\right]^2$$

21. Number of fringes shifted,

$$N = \frac{(\mu - 1)t}{\lambda}$$

$$\therefore \quad \mu = \frac{N\lambda}{t} + 1$$

22. Distance = $4.8 + 1.5w = 5.1$ mm

23. Five fringes have be shifted.

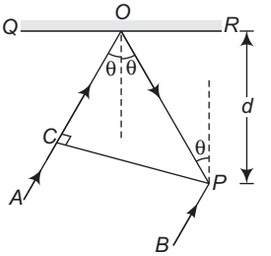
$$\because \quad 5 = \frac{(\mu_1 - 1)t - (\mu_2 - 1)t}{\lambda}$$

$$= \frac{(\mu_1 - \mu_2)t}{\lambda}$$

$$\therefore \quad t = \frac{5\lambda}{(\mu_1 - \mu_2)} = \frac{5 \times 480 \times 10^{-9}}{(1.7 - 1.4)}$$

$$= 16\,\mu\text{m}$$

24. Path difference between the two rays is given by



$$\Delta = CO + PO$$

$$\because \quad PR = d,$$

So, $$PO = d \sec\theta$$

and $$CO = PO\cos 2\theta = d \sec\theta \cos 2\theta$$

So, $$\Delta = (d \sec\theta + d \sec\theta \cos 2\theta)$$

Phase difference between two rays is $\phi = \pi$ (as one ray is reflected one and another is direct).

Now, for constructive interference, path difference should be even multiple of half wavelength

i.e. $\Delta = \lambda/2, \quad 3\lambda/2, \ldots$

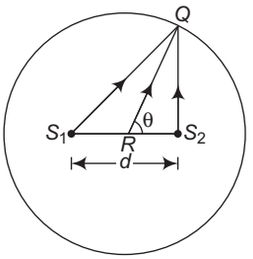
So, $d\sec\theta + d\sec\theta\cos 2\theta = \frac{\lambda}{2}$

or $d\sec\theta(1+\cos 2\theta) = \frac{\lambda}{2}$

or $d\sec\theta(2\cos^2\theta) = \frac{\lambda}{2}$

$\therefore \quad \cos\theta = \lambda/4d$

25. Path difference at a point Q on the circle is



$$\Delta y = d\cos\theta \qquad \ldots(i)$$

For maxima at Q path difference should be integer multiple of wavelength,

$$\Delta y = n\lambda \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$n\lambda = d\cos\theta$$

$\therefore \quad \theta = \cos^{-1}\left(\frac{n\lambda}{d}\right)$

For $n = 4$

$$\theta = \cos^{-1}\left(\frac{4\lambda}{d}\right)$$

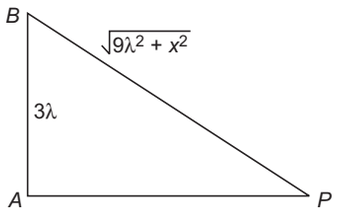
26. Say P be the point on x-axis, where constructive interference is obtained.

$$BP - AP = 2\lambda \text{ or } \lambda$$

$\therefore \quad \sqrt{9\lambda^2 + x^2} - x = 2\lambda$

$\therefore \quad 9\lambda^2 + x^2 = x^2 + 4\lambda^2 + 4x\lambda$

$$x = \frac{5\lambda}{4}$$



Further, $\sqrt{9\lambda^2 + x^2} - x = \lambda$

$\therefore \quad 9\lambda^2 + x^2 = x^2 + \lambda^2 + 2x\lambda$

$\therefore \quad x = 4\lambda$

27. For the interference pattern to be formed on the screen, the sources should be coherent and emits lights of same frequency and wavelength.

In a Young's double-slit experiment, when one of the holes is covered by a red filter and another by a blue filter. In this case due to filteration only red and blue lights are present. In YDSE monochromatic light is used for the formation of fringes on the screen. Hence, in this case there shall be no interference fringes.

28. According to question, there is a hole at point P_2. From Huygen's principle, wave will propagates from the sources S_1 and S_2. Each point on the screen will acts as secondary sources of wavelets.

Now, there is a hole at point P_2 (minima). The hole will act as a source of fresh light for the slits S_3 and S_4.

Therefore, there will be a regular two slit pattern on the second screen.

More than One Correct Options

1. λ_v is least. Therefore, ω_v is minimum (as $\omega \propto \lambda$). Hence, the fringe next to centre at will be violet.

At centre, $\Delta x = 0$ for all wavelengths. Hence, all wavelengths interfere constructively at centre. So, it is white.

2. (c) $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$,

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

when $I_1 = I_2 = I_0$

$I_{max} = 4I_0$ and $I_{min} = 0$

when $I_2 = \frac{I_0}{2}$,

then $I_{max} < 4I_0$ and $I_{min} > 0$.

3. $\Delta x_0 = d\sin\theta = (10^{-3})\left(\frac{1}{2}\right)$

$$= 5\times10^{-4}\,\text{m} = (10^3)\,\lambda$$

Since, Δx_0 is integer multiple of λ, it will produce maximum intensity or $4I_0$ at O.

$$w = \frac{\lambda D}{d} = \frac{(5\times10^{-7})(2)}{(10^{-3})}$$

$$= 10^{-3}\,\text{m} = 1\,\text{mm}$$

At 4 mm we will get 4th order maxima.

4. No solution is required.

5. Fringe pattern shifts in the direction of slab. But,

$$\text{shift} = \frac{(\mu-1)tD}{d}$$

So, actual shift will depend on the values of μ, t, D and d.

6. For overlapping of maximas

$$\frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

or $\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{7}{5}, \frac{14}{10}\ldots$

$\Rightarrow$ 14th order maxima of λ_1 will coincide with 10th order maxima of λ_2.

For overlapping of minimas

$$\frac{(2n_1-1)\lambda_1 D}{2d} = \frac{(2n_2-1)\lambda_2 D}{2d}$$

$\therefore \quad \frac{2n_1-1}{2n_2-1} = \frac{\lambda_2}{\lambda_1} = \frac{7}{5}$

(c) option with $n_1 = 11$ and $n_2 = 8$ gives this ratio.

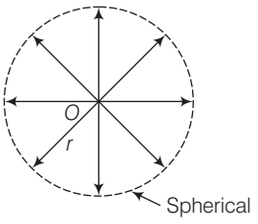
7. Consider the pattern of the intensity shown in the figure

(i) As intensities of all successive minima is zero, hence we can say that two sources S_1 and S_2 are having same intensities.

(ii) As width of the successive maxima (pulses) increases in continuous manner, we can say that the path difference (x) or phase difference varies in continuous manner.

(iii) We are using monochromatic light in YDSE to avoid overlapping and to have very clear pattern on the screen.

8. Consider the diagram in which light diverges from a point source (O).



Due to the point source light propagates in all directions symmetrically and hence, wavefront will be spherical as shown in the diagram.

If power of the source is P, then intensity of the source will be

$$I = \frac{P}{4\pi r^2}$$

where, r is radius of the wavefront at any time.

Comprehension Based Questions

1. $\lambda = \frac{\lambda_0}{\mu},\ w = \frac{\lambda D}{d} = \frac{\lambda_0 D}{\mu d}$

$$= \frac{(6300 \times 10^{-10})\ (1.33)}{1.33 \times 10^{-3}}$$

$$= 0.63 \times 10^{-3}\ \text{m}$$

$$= 0.63\ \text{mm}$$

2. $\Delta y = 7w - 3w = 4w = 4 \times 0.63\ \text{mm}$

$= 2.52\ \text{mm}$

3. $\Delta x = \left(\frac{\mu_2}{\mu_1} - 1\right) t = \frac{\lambda}{2}$

$$\therefore \quad t = \frac{\lambda}{2\left(\frac{\mu_2}{\mu_1} - 1\right)} = \frac{(6300 \times 10^{-10}/1.33)}{2\left(\frac{1.53}{1.33} - 1\right)}$$

$$= 1.575 \times 10^{-3}\ \text{m} = 1.575\ \text{mm}$$

4. Fringe width remains unchanged by the introduction of glass sheet.

Assertion and Reason

1. Fringe width does not change by the introduction of glass slab.

4. $I_{\max} > 4I_0$ and $I_{\min} = 0$

$$\therefore \quad \frac{I_{\max}}{I_{\min}} = \text{infinite}$$

If width of one slit is slightly increased $I_{\min} > 0$. Therefore, this ratio will be less than infinite.

6. $$\phi = \frac{2\pi}{\lambda} \cdot \Delta x$$

and $$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

7. Whole fringe pattern shifts towards that side, where slab is inserted.

Match the Columns

1. (a) $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$$I_1 = I_0$$

and $$I_2 > I_0$$

$$\therefore \quad I_{\max} > 4I_0$$

(b) When one slit is closed, fringe pattern will disappear.

$$I_{\max} = I_{\min} = I_0$$

(c) When both slits are made incoherent, again fringe pattern will disappear.

$$I_{\max} = I_{\min} = I_1 + I_2 = 2I_0$$

(d) Glass slab will only shift the fringe pattern.

2. $\Delta x = \frac{yd}{D},\ \phi = \frac{2\pi}{\lambda} \cdot \Delta x$

and $$I = 4I_0 \cos^2\frac{\phi}{2}$$

3. Fringe width, $w = \frac{\lambda D}{d}$

In a liquid, λ decreases μ times, therefore w also decrease μ times.

Entrance Gallery

1. We know that, fringe width $\beta = \frac{\lambda D}{d}$ or $\beta \propto \lambda$

$$\therefore \quad \lambda_2 > \lambda_1 \quad \Rightarrow \quad \beta_2 > \beta_1$$

Number of fringes in a given width, $m = \frac{Y}{\beta}$ or $m \propto \frac{1}{\beta}$

$$\Rightarrow \quad m_2 < m_1 \quad \Rightarrow \quad \beta_2 > \beta_1$$

Distance of 3rd maximum of λ_2 from central maximum

$$= \frac{3\lambda_2 D}{d} = \frac{1800\, D}{d}$$

Distance of 5th minimum of λ_1 from central maximum

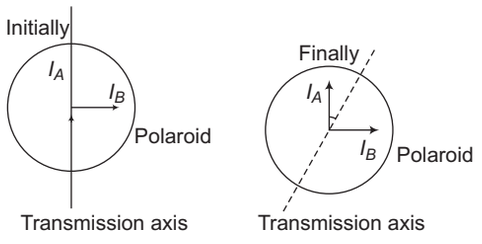
$$= \frac{9\lambda_1 D}{2d} = \frac{1800 D}{d}$$

So, 3rd maximum of λ_2 will overlap with 5th minimum of λ_1.

Angular separation (or angular fringe width) $= \frac{\lambda}{d} \propto \lambda$.

$\Rightarrow$ Angular separation for λ_1 will be lesser.

2. By law of Malus i.e. $I = I_0 \cos^2\theta$



Now, $I_{A'} = I_A \cos^2 30°,\ I_{B'} = I_B \cos^2 60°$

As, $I'_A = I'_B$

$I_A \cos^2 30° = I_B \cos^2 60°$

$\Rightarrow \quad I_A = \frac{3}{4} = I_B \frac{1}{4} \quad \Rightarrow \quad \frac{I_A}{I_B} = \frac{1}{3}$

3. According to Huygen's principle of secondary wavelets, wave interfering at point P are in same phase indicates P is point of constructive interference.

4. We know that,

the maximum intensities,

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 \qquad \ldots(i)$$

the minimum intensities,

$$I_{\min} = (\sqrt{I_1} + \sqrt{I_2})^2 \qquad \ldots(ii)$$

So, the ratio of the maximum and minimum of intensities is

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

$$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{4I} + \sqrt{I})^2}{(\sqrt{4I} - \sqrt{I})^2} = \left(\frac{3\sqrt{I}}{\sqrt{I}}\right)^2$$

$$= \frac{9}{1} = 9:1$$

5. The distance of 5th dark fringe from centre is given by

$$x_n = (2n+1)\frac{\lambda l}{2d}$$

As, $n = 4$ for 5th dark fringe.

So, $x_5 = \frac{9}{2}\frac{\lambda D}{d}$,

As, $\frac{\lambda D}{d} = \beta$

So, $x_5 = 9/2\beta = 9/2 \times 0.002 \quad \Rightarrow \quad x_5 = 9 \times 10^{-3}$ cm

6. Given, wavelength of light, $\lambda = 5000\ \text{Å} = 5000 \times 10^{-10}$

$= 5 \times 10^{-7}$ m

Diameter of telescope, $D = 200\,\text{cm} = 2$ m

Resolving power of a telescope $= \frac{1}{d\theta} = \frac{D}{1.22\,\lambda}$

where, $d\theta$ = Limit of resolution, λ = Wavelength of light used and D = Diameter of aperture of objective

So, resolving power $= \frac{2}{1.22 \times 5 \times 10^{-7}} = \frac{2 \times 10^7}{6.1} = \frac{20 \times 10^6}{6.1}$

$= 3.278 \times 10^6 = 3.28 \times 10^6$

7. By Malus law, $I = I_0 \cos^2\theta = I_0 \times \cos^2 60°$ (here, $\theta = 60°$)

$$I = I_0 \times \left(\frac{1}{2}\right)^2 = \frac{I_0}{4}$$

8. We know that, intensity $I = I_{\max} \cos^2 \frac{\phi}{2}$ $\ldots(i)$

Given, $I = \frac{I_{\max}}{2}$ $\ldots(ii)$

$\therefore$ From Eqs. (i) and (ii), we have

$$\phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

or path difference, $\Delta x = \left(\frac{\lambda}{2\pi}\right) \cdot \phi$

$\therefore \quad \Delta x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \cdots \left(\frac{2n+1}{4}\right)\lambda$

9. Given, $2d \sin\theta = \lambda$

So, $d = \frac{\lambda}{2\sin\theta}$

Differentiate, $\partial(d) = \frac{\lambda}{2}\partial(\text{cosec}\,\theta)$

$$\partial(d) = \frac{\lambda}{2}(-\,\text{cosec}\,\theta \cot\theta)\,\partial\theta$$

$$= \frac{-\lambda\cos\theta}{2\sin^2\theta}$$

As, θ = increases, $\frac{\lambda\cos\theta}{2\sin^2\theta}$ decreases

Alternate Solution

$$d = \frac{\lambda}{2\sin\theta}$$

$\ln d = \ln\lambda - \ln 2 - \ln\sin\theta$

$$\frac{\Delta(d)}{d} = 0 - 0 - \frac{1}{\sin\theta} \times \cos\theta\,(\Delta\theta)$$

Fractional error $|+(d)| = \cot\theta\,(\Delta\theta)$

Absolute error, $\Delta d = (d\cot\theta)\,\Delta\theta = \frac{d}{2\sin\theta} \times \frac{\cos\theta}{\sin\theta}$

$$\Delta d = \frac{\cos\theta}{2\sin^2\theta}$$

10. It will be concentric circle.

11. Given, $d = 1$ mm $= 1 \times 10^{-3}$ m, $D = 1$ m

$\lambda = 500$ mm $= 500 \times 10^{-9}$ m

Distance of nth minima from central maxima,

$$x_n = \frac{(2n-1)\lambda}{2}\frac{D}{d} = \frac{(2\times 3 - 1) \times 500 \times 10^{-9}}{2} \times \frac{1}{1 \times 10^{-3}}$$

$= 2.5 \times 500 \times 10^{-6} = 12.5 \times 10^{-4}$ m $= 1.25$ mm

12. We know that, fringe width, $\beta = \frac{\lambda D}{d}$

or $\beta \propto \lambda$

Now, $\lambda_R < \lambda_G > \lambda_B$

$\therefore \quad \beta_R > \beta_G > \beta_B$

13. Given, $a_1 = 2a_1 \quad \Rightarrow \quad I_1 = 4I_2 = 4I_0$

$\therefore \quad I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 = (3\sqrt{I_2})^2$

$= 9I_2 = 9I_0$

Now, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$= 4I_0 + I_0 + 2\sqrt{4I_0 \cdot I_0}\cos\phi$

$= 5I_0 + 4I_0\cos\phi = \frac{I_{\max}}{9}(5 + 4\cos\phi)$

$= \frac{I_{\max}}{9}[1 + 4(1 + \cos\phi)]$

$= \frac{I_{\max}}{9}(1 + 8\cos^2\phi/2)$

14. For sustained interference, sources must be coherent and the intensities of two sources must be nearly equal.

15. In Young's double slit experiment, we have

$$\beta = \frac{\lambda D}{d}$$

$$\Delta\beta = \frac{\lambda\Delta D}{d} \quad \Rightarrow \quad \lambda = \frac{\Delta\beta d}{\Delta D}$$

$$\lambda = \frac{3\times10^{-5}\times1\times10^{-3}}{5\times10^{-2}}$$

$$= 6\times10^{-7}\text{ m} \quad \text{or} \quad \lambda = 600\text{ nm}$$

16. If width of slit is reduced in a single slit diffraction, width of central maximum will increase $= \frac{2\lambda D}{a}$

But intensity of central maximum will decrease less number of waves will superpose at the centre.

17. We know that, $d\sin\theta = n\lambda$

$$0.3\times10^{-3}\times\theta = 6000\times10^{-10}$$

$$\theta = 2\times10^{-3}\text{ rad}$$

18. Let I_0 is intensity of light emitted from the source, then

Resultant intensity, $I = 4I_0\cos^2\frac{\phi}{2}$

$$I_1 = 4I_0\cos^2\frac{\phi}{2} = 4I_0$$

Now, $\Delta x = \frac{\lambda}{4}$

$$\phi = \frac{2\pi}{\lambda}\times\Delta x = \frac{2\pi}{\lambda}\times\frac{\lambda}{4}$$

$\Rightarrow$ $\phi = \frac{\pi}{2}$

and $I_2 = 4I_0\cos^2\frac{\pi}{4} = 2I_0$

$$I_1 : I_2 = 2:1$$

19. For coherent sources, $I_1 = 4I_0\cos^2\frac{\phi}{2} = 4I_0$

For incoherent sources, $I_2 = I_0 + I_0 = 2I_0$

$\therefore$ $\frac{I_1}{I_2} = 2$

20. Intensity, $I = \frac{I_0}{2}\cdot\frac{1}{(4)^4}$

$$= \frac{I_0}{512} = \frac{1}{512}$$

21. We know that, $\tan i = \frac{1}{\sin C}$

$$\cot i = \sin\left[\sin^{-1}\left(\frac{3}{5}\right)\right]$$

$\Rightarrow$ $\tan i = \frac{5}{3}$

$\Rightarrow$ $i = \tan^{-1}\left(\frac{5}{3}\right)$

22. We know that,

$$\text{Distance} = \frac{2\lambda}{b}\times d$$

$$= \frac{2\times0.5\times10^{-4}}{2}\times100$$

$$= 5\text{ mm}$$

23. A slit is necessary for ordinary extended source of light and slit is not needed for specially coherent source of light.

24. We know that, phase difference, $\phi = \frac{2\pi}{\lambda}\times$ path difference

$$\phi = \frac{2\pi}{\lambda}\times\frac{\lambda}{6} = \frac{\pi}{3} = 60°$$

Intensity, $I = I_0\cos^2\left(\frac{\phi}{2}\right)$

$$\frac{I}{I_0} = \cos^2(30°)$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 = 0.75$$

25. Condition for constructive interference is given by

$$2\mu t = [2n+1]\frac{\lambda}{2}$$

where, $n = 0, 1, 2, 3, \cdots$

For minimum thickness, $n = 0$

$$2\mu t = \frac{\lambda}{2} \quad \Rightarrow \quad t = \frac{\lambda}{4\mu}$$

$$= \frac{600\times10^{-9}}{4\times1.5}$$

$$= 100\text{ nm}$$

26. Critical angle, $C = \sin^{-1}(0.6)$

$$\sin(C) = 0.6$$

$\Rightarrow$ $\mu = \frac{1}{\sin C}$

$$= \frac{1}{0.6}$$

Polarising angle, $i_p = \tan^{-1}(\mu) = \tan^{-1}\left(\frac{1}{0.6}\right)$

$$= \tan^{-1}(1.6667)$$

27. Given, $Y = d$

As, we know that, from Young's double slit experiment,

$$y = \frac{\lambda D}{d}$$

$\Rightarrow$ $\frac{\lambda D}{d} = d$

$\Rightarrow$ $\lambda = \frac{d^2}{D}$

28. Given,

Fringe width, $\beta = 2\text{ mm} = 2\times10^{-3}$ m

$\lambda = 6000$ Å,

$\mu = 1.33, \beta' = ?$

$$\beta' = \frac{\beta}{\mu} = \frac{2}{1.33}$$

$$= 1.5\text{ mm}$$

29. The intensity I is minimum for phase difference,

$$\delta = (2n-1)\pi$$

where, $n = 1, 2, 3, \cdots$

30. According to Rayleigh scattering formula, intensity of scattered light, $I \propto \frac{1}{(\lambda)^4} \propto f^4$

$$\frac{f_1}{f_2} = \left[\frac{I_1}{I_2}\right]^{-1/4} = \left[\frac{256}{81}\right]^{-1/4}$$

$$= \frac{4}{3}$$

31. In single slit diffraction, the central fringe has maximum intensity and has the width double than other fringes.

26

Modern Physics

26.1 Dual Nature of Electromagnetic Waves

Classical physics treats particles and waves as separate components. The mechanics of particles and the optics of waves are traditionally independent disciplines, each with its own chain of principles based on their results. We regard electrons as particles, because they possess charge and mass and behave according to the laws of particle mechanics in such familiar devices as television picture tubes. We shall see, however, that it is just as correct to interpret a moving electron as a wave manifestation as it is to interpret it as a particle manifestation. We regard electromagnetic waves as waves, because under suitable circumstances they exhibit diffraction, interference and polarisation. Similarly, we shall see that under other circumstances they behave as a streams of particles. Rather we can say they have the **dual nature.**

The wave nature of light (a part of electromagnetic waves) was first demonstrated by Thomas Young, who observed the interference pattern of two coherent sources. The particle nature of light was first proposed by Albert Einstein in 1905 in his explanation of the photoelectric effect. A particle of light called a **photon**, has energy E that is related to the frequency f and wavelength λ of light wave by the Einstein equation,

$$E = hf = \frac{hc}{\lambda} \qquad \ldots(\text{i})$$

where, c is the speed of light (in vacuum) and h is Planck's constant.

$$h = 6.626 \times 10^{-34} \text{ J-s} = 4.136 \times 10^{-15} \text{ eV-s}$$

Since, energies are often given in electron volt $(1 \text{ eV} = 1.6 \times 10^{-19} \text{ J})$ and wavelengths are in Å, it is convenient to the combination hc in eV-Å. We have,

$$hc = 12375 \text{ eV-Å}$$

Hence, Eq. (i) in simpler form can be written as,

$$E \text{ (in eV)} = \frac{12375}{\lambda \text{ (in Å)}} \qquad \ldots(\text{ii})$$

The propagation of light is governed by its wave properties, whereas the exchange of energy between light with matter is governed by its particle properties. The wave-particle duality is a general property of nature. For example, electrons (and other so called particles) also propagate as waves and exchange energy as particles.

Chapter Snapshot

- Dual Nature of Electromagnetic Waves
- Electromagnetic Spectrum
- de-Broglie Wavelength of Matter Wave
- Early Atomic Structures
- The Bohr Hydrogen Atom
- Hydrogen Like Atoms
- Emission of Electrons
- Photoelectric Effect
- Nuclear Stability and Radioactivity
- The Radioactive Decay Law

26.2 Electromagnetic Spectrum

The basic source of electromagnetic wave is an accelerated charge. This produces the changing electric and magnetic fields which constitute an electromagnetic wave. An electromagnetic wave may have its wavelength varying from zero to infinity. Not all of them are known till date. Today, we are familiar with electromagnetic waves having wavelengths as small as 30 fm (1 fm = 10^{-15} m) to as large as 30 km. The boundaries separating different regions of spectrum are not sharply defined, with the exception of the visible part of the spectrum. The visible part of the electromagnetic spectrum covers from 4000 Å to 7000 Å. An approximate range of wavelengths is associated with each colour : violet (4000 Å-4500 Å), blue (4500 Å-5200 Å), green (5200 Å-5600 Å), yellow (5600 Å- 6000 Å), orange (6000 Å-6250 Å) and red (6250 Å-7000 Å).

Figure shows the spectrum of electromagnetic waves. The classification are based roughly on how the waves are produced and/or detected.

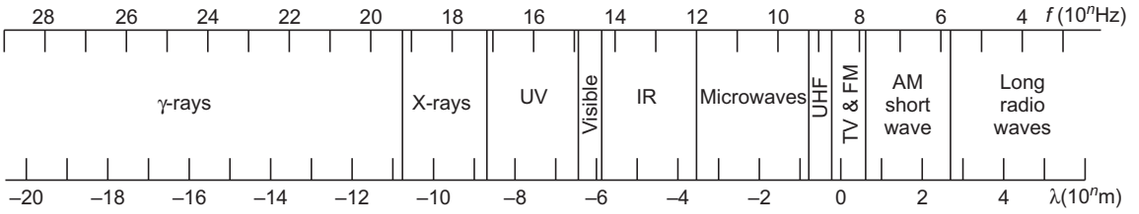


Fig. 26.1 The electromagnetic spectrum

γ-rays These were identified by P Villiard in 1900. These are usually produced within the nucleus of an atom and extremely energetic by atomic standards. They cover the range from 0.1 Å down or equivalently, from 10^{20} Hz up.

X-rays X-rays, discovered in 1895 by W Roentgen extend from 100 Å to 0.1 Å. These are produced by the rapid deceleration of electrons that bombard a heavy metal target. These are also produced by electronic transitions between the energy levels in an atom. X-rays are used to study the atomic structure of crystals or molecules such as DNA. Besides their diagnostic and therapeutic use in medicine they have become an important tool in studying the universe.

Ultraviolet radiation Ultraviolet (UV) rays were first discovered by JW Ritter in 1801. The ultraviolet region extends from 4000 Å to 100 Å. It plays a role in the production of vitamin-D in our skins. But prolonged doses of UV radiation can induce cancers in humans. Glass absorbs UV radiation and hence, can provide some protection against the sun's rays. If the ozone in our atmosphere did not absorb the UV below 3000 Å, there would be a large number of cell mutations, especially cancerous ones, in humans. For this reason, the depletion of the ozone in our atmosphere by chlorofluorocarbons (CFCs) is now a matter of international concern.

Visible light A lot of discussion has already been done on visible light in previous two chapters. As electrons undergo transitions between energy levels in an atom, light is produced at well defined wavelengths. Light covering a continuous range of wavelengths is produced by the random acceleration of electrons in hot bodies. Our sense of vision and the process of photosynthesis in plants have evolved within the range of those wavelengths of sunlight that our atmosphere does not absorb.

Infrared radiation The infrared region (IR) starts at 7000 Å and extends to about 1 mm. It was discovered in 1800 by M Herchel. It is associated with the vibration and rotation of molecules and is perceived by us as heat. Infrared radiation is used in the early detection of tumors.

Microwaves Microwaves cover wavelengths from 1 mm to about 15 cm. Microwaves upto about 30 GHz (1 cm) may be generated by the oscillations of electrons in a device called klystron. Microwave ovens are used in kitchens. Modern intercity communications such as phone conversations and TV programs are often carried *via* a cross country network of microwave antennas.

Radio and TV signals Radiowaves are generated when charges are accelerating through conducting wires. Their wavelengths lie in the range 10^{14} m to 10 cm. They are generated by *L-C* oscillators and are used in radio and television communication systems.

26.3 de-Broglie Wavelength of Matter Wave

The wave particle nature of electromagnetic waves discussed in article 26.1, led de-Broglie (pronounced de-Broy) to suggest that matter might also exhibit this duality and have wave properties. His ideas can be expressed quantitatively by first considering electromagnetic radiation. A photon of frequency f and wavelength λ has energy,

$$E = hf = \frac{hc}{\lambda}$$

By Einstein's energy mass relation, $E = mc^2$ the equivalent mass m of the photon is given by

$$m = \frac{E}{c^2} = \frac{hf}{c^2} = \frac{h}{\lambda c} \quad \ldots(i)$$

or $$\lambda = \frac{h}{mc} \quad \text{or} \quad \frac{h}{p} \quad \ldots(ii)$$

Here, p is the momentum of photon. By analogy de-Broglie suggested that a particle of mass m moving with speed v behaves in some ways like waves of wavelength λ given by,

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad \ldots(iii)$$

where, p is the momentum of the particle. Momentum is related to the kinetic energy by the equation,

$$p = \sqrt{2Km}$$

and a charge q when accelerated by a potential difference V gains a kinetic energy $K = qV$. Combining all these relations Eq. (iii) can be written as,

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$= \frac{h}{\sqrt{2Km}} = \frac{h}{\sqrt{2qVm}} \text{ (de-Broglie wavelength)} \quad \ldots(iv)$$

de-Broglie Wavelength for an Electron

If an electron (charge = e) is accelerated by a potential difference of V volts, it acquires a kinetic energy,

$$K = eV$$

Substituting the values of h, m and q in Eq. (iv), we get a simple formula for calculating de-Broglie wavelength of an electron. This is,

$$\lambda \text{ (in Å)} = \sqrt{\frac{150}{V \text{ (in volts)}}} \quad \ldots(v)$$

Example 26.1 *An electron is accelerated by a potential difference of* 25 V. *Find the de-Broglie wavelength associated with it.*

Sol. For an electron, de-Broglie wavelength is given by,

$$\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{25}} = \sqrt{6}$$

$$\approx 2.5 \text{ Å}$$

26.4 Early Atomic Structures

Every atom consists of a small nucleus of protons and neutrons with a number of electrons some distance away.

In the present article and in the next our chief concern will be the structure of the atom, since it is this structure that is responsible for nearly all the properties of matter. In nineteenth century many models were present by different scientists, but ultimately the first theory of the atom to meet with any success was put forward in 1913 by Neils Bohr. But before studying Bohr's model of atom let us have a look on other two models of the period one presented by JJ Thomson in 1898 and the other by Ernest Rutherford in 1911.

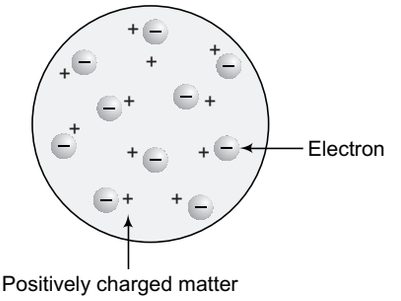


Fig. 26.2 The Thomson model of the atom. The Rutherford scattering

JJ Thomson suggested that atoms are just positively charged lumps of matter with electrons embedded in them like raisins in a fruit cake. Thomson's model called the 'plum pudding' model is illustrated in Fig. 26.2.

Thomson had played an important role in discovering the electron, his idea was taken seriously. But the real atom turned out to be quite different.

Rutherford's Nuclear Atom

The nuclear atom is the basis of the modern theory of atomic structure and was proposed by Rutherford in 1911. He, with his two assistants Geiger and Marsden did an experiment in which they directed a narrow beam of α-particles onto gold foil about 1 μm thick and found that while most of the particles passed straight through, some were scattered appreciably and a very few about 1 in 8000 suffered deflection of more than 90°.

To account for this very surprising result Rutherford suggested that : All the positive charge and nearly all the mass were concentrated in a very small volume or nucleus at the centre of the atom. The electrons were supposed to move in circular orbits round the nucleus (like planets round the sun). The electrostatic attraction between the two opposite charges being the required centripetal force for such motion.

The large angle scattering of α-particles would then be explained by the strong electrostatic repulsion from the nucleus.

Rutherford's model of the atom, although strongly supported by evidence for the nucleus, is inconsistent with classical physics. An electron moving in a circular orbit round a nucleus is accelerating and according to electromagnetic theory it should therefore, emit radiation continuously and thereby lose energy. If this happened the radius of the orbit would decrease and the electron would spiral into the nucleus in a fraction of second.

But atoms do not collapse. In 1913, an effort was made by Neils Bohr to overcome this paradox.

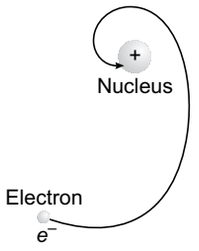


Fig. 26.3 An atomic electron should, classically, spiral rapidly into the nucleus as it radiates energy due to its acceleration.

26.5 The Bohr Hydrogen Atom

After Neils Bohr obtained his doctorate in 1911, he worked under Rutherford for a while. In 1913, he presented a model of the hydrogen atom, which has one electron. He postulated that an electron moves only in certain circular orbits, called stationary orbits. In stationary orbits electron does not emit radiation, contrary to the predictions of classical electromagnetic theory. According to Bohr, there is a definite energy associated with each stable orbit and an atom radiaties energy only when it makes a transition from one of these orbits to another. The energy is radiated in the form of a photon with energy and frequency given by

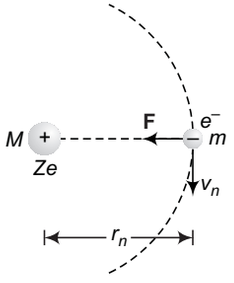


Fig. 26.4

$$\Delta E = hf = E_i - E_f \qquad \ldots(i)$$

Bohr found that the magnitude of the electron's angular momentum is quantised and this magnitude for the electron must be integral multiple of $\frac{h}{2\pi}$. The magnitude of the angular momentum is $L = mvr$ for a particle with mass m moving with speed v in a circle of radius r. So, according to Bohr's postulate,

$$mvr = \frac{nh}{2\pi} \qquad (n = 1, 2, 3 \ldots)$$

Each value of n corresponds to a permitted value of the orbit radius, which we will denote by r_n and the corresponding speed v_n. The value of n for each orbit is called **principal quantum number** for the orbit. Thus,

$$mv_n r_n = \frac{nh}{2\pi} \qquad \ldots(ii)$$

According to Newton's second law, a radially inward centripetal force of magnitude $F = \frac{mv^2}{r_n}$ is needed to the electron which is being provided by the electrical attraction between the positive proton and the negative electron.

Thus, $$\frac{mv_n^{\,2}}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^{\,2}} \qquad \ldots(iii)$$

Solving Eqs. (ii) and (iii), we get

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \qquad \begin{pmatrix} n\text{th orbit radius} \\ \text{in Bohr model} \end{pmatrix} \qquad \ldots(iv)$$

and $$v_n = \frac{e^2}{2\varepsilon_0 nh} \qquad \begin{pmatrix} n\text{th orbital speed} \\ \text{in Bohr model} \end{pmatrix} \qquad \ldots(v)$$

The smallest orbit radius corresponds to $n = 1$. We'll denote this minimum radius, called the **Bohr radius** as a_0. Thus,

$$a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$$

Substituting values of ε_0, h, π, m and e, we get

$$a_0 = 0.529 \times 10^{-10} \text{ m} = 0.529 \text{ Å} \qquad \ldots(vi)$$

Eq. (iv), in terms of a_0 can be written as,

$$r_n = n^2 a_0 \quad \text{or} \quad r_n \propto n^2 \qquad \ldots(vii)$$

Similarly, substituting values of e, ε_0 and h with $n = 1$ in Eq. (v), we get

$$v_1 = 2.19 \times 10^6 \text{ m/s} \approx \frac{c}{137} \qquad \ldots(viii)$$

This is the greatest possible speed of the electron in the hydrogen atom which is approximately equal to $c/137$ where, c is the speed of light in vacuum.

Eq. (v), in terms of v_1 can be written as,

$$v_n = \frac{v_1}{n} \quad \text{or} \quad v_n \propto \frac{1}{n} \qquad \ldots(ix)$$

Energy levels Kinetic and potential energies K_n and U_n in nth orbit are

$$K_n = \frac{1}{2} m v_n^2 = \frac{me^4}{8\varepsilon_0^{\,2} n^2 h^2}$$

and $$U_n = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = -\frac{me^4}{4\varepsilon_0^{\,2} n^2 h^2}$$

The total energy E_n is the sum of the kinetic and potential energies.

$$E_n = K_n + U_n = -\frac{me^4}{8\varepsilon_0^{\,2} n^2 h^2}$$

Substituting values of m, e, ε_0 and h with $n = 1$, we get the least energy of the atom in first orbit, which is -13.6 eV.

Hence, $$E_1 = -13.6 \text{ eV} \qquad \ldots(x)$$

and $$E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2} \text{ eV} \qquad \ldots(xi)$$

Substituting $n = 2, 3, 4 \ldots$, etc., we get energies of atom in different orbits.

$$E_2 = -3.40 \text{ eV}, \; E_3 = -1.51 \text{ eV}, \ldots E_\infty = 0$$

Ionisation energy of the hydrogen atom is the energy required to remove the electron completely. In ground state ($n = 1$) energy of atom is –13.6 eV and energy corresponding to $n = \infty$ is zero. Hence, energy required to remove the electron from ground state is 13.6 eV.

Emission Spectrum of Hydrogen Atom

Under normal conditions, the single electron in hydrogen atom stays in ground state ($n = 1$). It is excited to some higher energy state when it acquires some energy from external source. But it hardly stays therefore more than 10^{-8} s.

A photon corresponding to a particular spectrum line is emitted when an atom makes a transition from a state in an excited level to a state in a lower excited level or the ground level.

Let n_i and n_f be the initial the final energy state, then depending on the final energy state following series are observed in the emission spectrum of hydrogen atom.

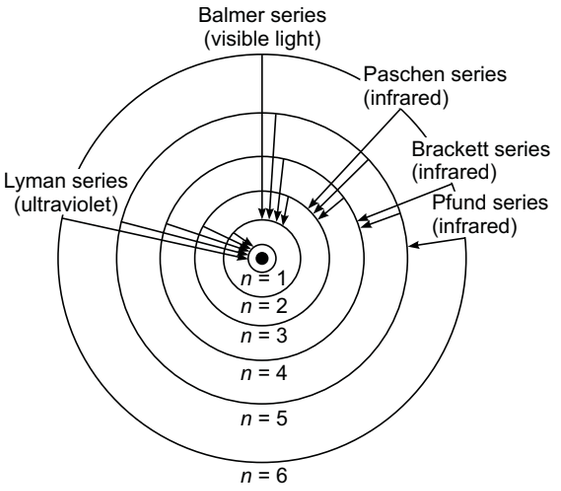


Fig. 26.5

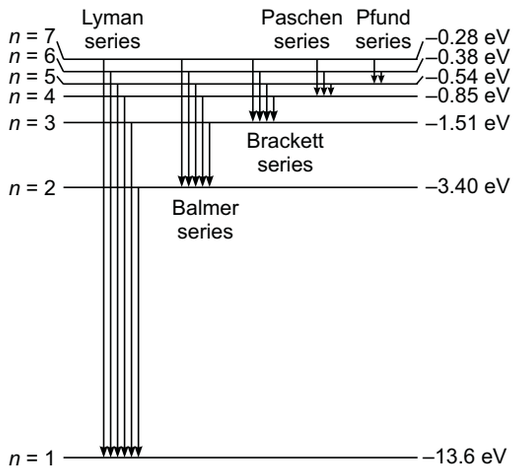


Fig. 26.6

For the Lyman series $n_f = 1$, for Balmer series $n_f = 2$ and so on. The relation of the various spectral series to the energy levels and to electron orbits is shown in figure.

Wavelength of Photon Emitted in De-excitation

According to Bohr when an atom makes a transition from higher energy level to a lower energy level, it emits a photon with energy equal to the energy difference between the initial and final levels. If E_i is the initial energy of the atom before such a transition, E_f is its final energy after the transition, and the photon's energy is $hf = \frac{hc}{\lambda}$, then conservation of energy gives,

$$hf = \frac{hc}{\lambda} = E_i - E_f \qquad \text{(energy of emitted photon)} \quad \ldots(xii)$$

By 1913, the spectrum of hydrogen had been studied intensively. The visible line with longest wavelength, or lowest frequency is in the red and is called H_α, the next line, in the blue-green is called H_β and so on.

In 1885, Johann Balmer, a swiss teacher found a formula that gives the wavelengths of these lines. This is now called the Balmer series. The Balmer's formula is

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad \ldots(xiii)$$

where, $n = 3, 4, 5 \ldots$, etc.

R = Rydberg constant $= 1.097 \times 10^7\ \text{m}^{-1}$

and λ is the wavelength of light/photon emitted during transition.

For $n = 3$, we obtain the wavelength of H_α-line.

Similarly, for $n = 4$, we obtain the wavelength of H_β- line. For $n = \infty$, the smallest wavelength (= 3646 Å) of this series is obtained. Using the relation, $E = \frac{hc}{\lambda}$ we can find the photon energies corresponding to the wavelength of the Balmar series. Multiplying Eq. (xiii) by hc, we find

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{Rhc}{2^2} - \frac{Rhc}{n^2}$$

$$= E_n - E_2$$

This formula suggests that

$$E_n = -\frac{Rhc}{n^2},\ n = 1, 2, 3 \ldots \qquad \ldots(xiv)$$

Comparing this with Eq. (xi), of the same article, we have

$$Rhc = 13.60\ \text{eV} \qquad \ldots(xv)$$

The wavelengths corresponding to other spectral series (Lyman, Paschen , etc.) can be represented by formulas similar to Balmer's formula.

Lyman series $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), n = 2, 3, 4, \ldots$

Paschen series $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), n = 4, 5, 6, \ldots$

Brackett series $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), n = 5, 6, 7, \ldots$

Pfund series $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), n = 6, 7, 8, \ldots$

The Lyman series is in the ultraviolet, and the Paschen, Brackett and Pfund series are in the infrared region.

Example 26.2 *Calculate* (*a*) *the wavelength and* (*b*) *the frequency of the* H_β*-line of the Balmer series for hydrogen.*

Sol. (a) H_β-line of Balmer series corresponds to the transition from $n = 4$ to $n = 2$ level. Using Eq. (xiii), the corresponding wavelength for H_β-line is,

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$= 0.2056 \times 10^7$$

$$\therefore \quad \lambda = 4.9 \times 10^{-7} \text{ m}$$

(b) $$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{4.9 \times 10^{-7}} = 6.12 \times 10^{14} \text{ Hz}$$

Example 26.3 *Find the largest and shortest wavelengths in the Lyman series for hydrogen. In what region of the electromagnetic spectrum does each series lie?*

Sol. The transition equation for Lyman series is given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \; n = 2, 3, \ldots$$

The largest wavelength is corresponding to $n = 2$

$$\therefore \quad \frac{1}{\lambda_{max}} = 1.097 \times 10^7\left(\frac{1}{1} - \frac{1}{4}\right)$$

$$= 0.823 \times 10^7 \text{ m}$$

$$\therefore \quad \lambda_{max} = 1.2154 \times 10^{-7} \text{ m} = 1215 \text{ Å}$$

The shortest wavelength corresponds to $n = \infty$

$$\therefore \quad \frac{1}{\lambda_{min}} = 1.097 \times 10^7\left(\frac{1}{1} - \frac{1}{\infty}\right)$$

or $$\lambda_{min} = 0.911 \times 10^{-7} \text{ m} = 911 \text{ Å}$$

Both of these wavelengths lie in ultraviolet (UV) region of electromagnetic spectrum.

26.6 Hydrogen Like Atoms

The Bohr model of hydrogen can be extended to hydrogen like atoms, i.e. one electron atoms, such as singly ionised helium (He^+), doubly ionised lithium (Li^{+2}) and so on. In such atoms, the nuclear charge is $+ze$, where z is the atomic number, equal to the number of protons in the nucleus.

The effect in the previous analysis is to replace e^2 everywhere by ze^2. Thus, the equations for, r_n, v_n and E_n are altered as under:

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m z e^2} = \frac{n^2}{z} a_0$$

or $$r_n \propto \frac{n^2}{z} \qquad \ldots(i)$$

where, $a_0 = 0.529$ Å (radius of first orbit of H)

$$v_n = \frac{ze^2}{2\varepsilon_0 nh} = \frac{z}{n} v_1$$

or $$v_n \propto \frac{z}{n} \qquad \ldots(ii)$$

where, $v_1 = 2.19 \times 10^6$ m/s

(speed of electron in first orbit of H)

$$E_n = -\frac{mz^2 e^4}{8\varepsilon_0^2 n^2 h^2}$$

$$= \frac{z^2}{n^2} E_1$$

or $$E_n \propto \frac{z^2}{n^2} \qquad \ldots(iii)$$

where, $E_1 = -13.60$ eV

(energy of atom in first orbit of H)

Fig. 26.7 compares the energy levels of H and He^+ which has $z = 2$. H and He^+ have many spectrum lines that have almost the same wavelengths.

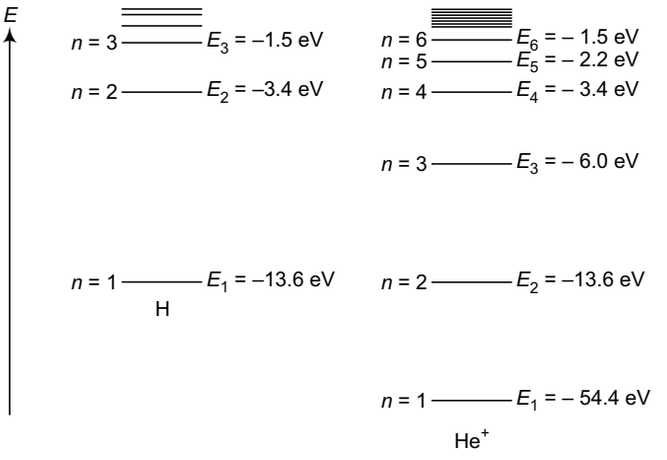


Fig. 26.7 Energy levels of H and He^+. Because of the additional factor z^2 in the energy expression, the energy of the He^+ ion with a given n is almost exactly four times that of the H atom with the same n. There are small differences (of the order of 0.05%), because the reduced masses are slightly different.

Extra Knowledge Points

- Total number of emission lines from some excited state n_1 to another energy state $n_2 (< n_1)$ is given by $\frac{(n_1 - n_2)(n_1 - n_2 + 1)}{2}$.

  For example total number of lines from $n_1 = n$ to $n_2 = 1$ are $\frac{n(n-1)}{2}$.
- As the principal quantum number n is increased in hydrogen and hydrogen like atoms, some quantities are decreased and some are increased. The table given below shows which quantities are increased and which are decreased.
- Table 26.1

Increased	Decreased
Radius	Speed
Potential energy	Kinetic energy
Total energy	Angular speed
Time period	
Angular momentum	

- Total energy of a closed system is always negative and the modulus of this is the binding energy of the system. For instance, suppose a system has a total energy of –100 J. It means that this system will separate, if 100 J of energy is supplied to this. Hence, binding energy of this system is 100 J. Thus, total energy of an open system is either zero or greater than zero.
- Kinetic energy of a particle cannot be negative, while the potential energy can be zero, positive or negative. It basically depends on the reference point where we have taken it zero. It is customary to take zero potential energy when the electron is at infinite distance from the nucleus. In some problem suppose we take zero potential energy in first orbit ($U_1 = 0$), then the modulus of actual potential energy in first orbit (when reference point was at infinity) is added in U and E in all energy states, while K remains unchanged.

Example 26.4 *Using the known values for hydrogen atom. Calculate*
(a) radius of third orbit for Li^{2+},
(b) speed of electron in fourth orbit for He^{+}.

Sol.

(a) $z = 3$ for Li^{+2}.

Further, we know that

$$r_n = \frac{n^2}{z} a_0$$

Substituting, $n = 3$, $z = 3$ and $a_0 = 0.529$ Å

we have r_3 for

$$Li^{+2} = \frac{(3)^2}{(3)} (0.529) \text{ Å} = 1.587 \text{ Å}$$

(b) $z = 2$ for He^{+}.

Also, we know that

$$v_n = \frac{z}{n} v_1$$

Substituting $n = 4$,

$z = 2$

and $v_1 = 2.19 \times 10^6$ m/s

we get, v_4 for

$$He^{+} = \left(\frac{2}{4}\right) (2.19 \times 10^6) \text{ m/s}$$

$$= 1.095 \times 10^6 \text{ m/s}$$

Example 26.5 *Find the kinetic energy, potential energy and total energy in first and second orbit of hydrogen atom, if potential energy in first orbit is taken to be zero.*

Sol. $E_1 = -13.60$ eV

$K_1 = -E_1 = 13.60$ eV

$U_1 = 2E_1 = -27.20$ eV

$E_2 = \frac{E_1}{(2)^2} = -3.40$ eV, $K_2 = 3.40$ eV

and $U_2 = -6.80$ eV

Now, $U_1 = 0$, i.e. potential energy has been increased by 27.20 eV. So, we will increase U and E in all energy states by 27.20 eV while kinetic energy will remain unchanged. Changed values in tabular form are as under.

Orbit	K (eV)	U (eV)	E (eV)
First	13.60	0	13.60
Second	3.40	20.40	23.80

Example 26.6 *A moving hydrogen atom makes a head on collision with a stationary hydrogen atom. Before collision both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state.*

Sol. Let K be the kinetic energy of the moving hydrogen atom and K', the kinetic energy of combined mass after collision.

From conservation of linear momentum,

$$p = p'$$

or $$\sqrt{2Km} = \sqrt{2K'(2m)}$$

or $$K = 2K' \quad \ldots(i)$$

From conservation of energy,

$$K = K' + \Delta E \quad \ldots(ii)$$

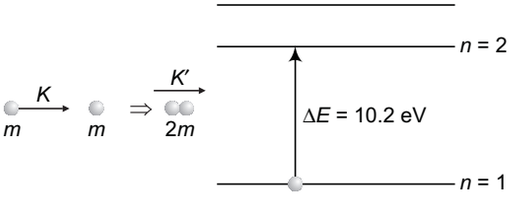


Fig. 26.8

Solving Eqs. (i) and (ii), we get

$$\Delta E = \frac{K}{2}$$

Now minimum value of ΔE for hydrogen atom is 10.2 eV.

or $\Delta E \geq 310.2$ eV

$\therefore$ $\frac{K}{2} \geq 310.2$

$\therefore$ $K \geq 20.4$ eV

Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV.

26.7 Emission of Electrons

At room temperature, the free electrons move randomly within the conductor, but they do not leave the surface of the conductor due to attraction of positive charges. Some external energy is required to emit electrons from a metal surface. Minimum energy is required to emit the electrons which are just on the surface of the conductor. This minimum energy is called the **work function** (denoted by W) of the conductor. Work function is the property of the metallic surface.

The energy required to liberate an electron from metal surface may arise from various sources such as heat, light, electric field, etc. Depending on the nature of source of energy, the following methods are possible.

(i) **Thermionic emission** The energy to the free electrons can be given by heating the metal. The electrons so emitted are known as **thermions.**

(ii) **Field emission** When a conductor is put under strong electric field the free electrons on it experience an electric force in the opposite direction of field. Beyond a certain limit electrons start coming out of the metal surface. Emission of electrons from a metal surface by this method is called the field emission.

(iii) **Secondary emission** Emission of electrons from a metal surface by the bombardment of high speed electrons or other particles is known as secondary emission.

(iv) **Photoelectric emission** Emission of free electrons from a metal surface by falling light (or any other electromagnetic wave which has an energy greater than the work function of the metal) is called photoelectric emission. The electrons so emitted are called **photoelectrons.**

26.8 Photoelectric Effect

When light of an appropriate frequency (or correspondingly of an appropriate wavelength) is incident on a metallic surface, electrons are liberated from the surface. This observation is known as **photoelectric effect.** Photoelectric effect was first observed in 1887 by Hertz. For photoemission to take place, energy of incident light photons should be greater than or equal to the work function of the metal.

or $E \geq W$ …(i)

$\therefore$ $hf \geq W$ or $f \geq \frac{W}{h}$

Here, $\frac{W}{h}$ is the minimum frequency required for the emission of electrons. This is known as threshold frequency f_0.

Thus, threshold frequency $f_0 = \frac{W}{h}$ …(ii)

Further Eq. (i) can be written as,

$$\frac{hc}{\lambda} \geq W$$

or $$\lambda \leq \frac{hc}{W}$$

where, $\frac{hc}{W}$ is the largest wavelength beyond which photoemission does not take place. This is called the threshold wavelength λ_0.

Thus, threshold wavelength $\lambda_0 = \frac{hc}{W}$ …(iii)

Hence, for the photoemission to take place either of the following conditions must be satisfied.

$$E \geq W \quad \text{or} \quad f \geq f_0 \quad \text{or} \quad \lambda \leq \lambda_0 \qquad \text{…(iv)}$$

Stopping Potential and Maximum Kinetic Energy of Photoelectrons

When the frequency f of the incident light is greater than the threshold frequency, some electrons are emitted from the metal with substantial initial speeds. Suppose E is the energy of light incident on a metal surface and W ($< E$) the work function of metal. As minimum energy is required to extract electrons from the surface, they will have the maximum kinetic energy which is $E - W$.

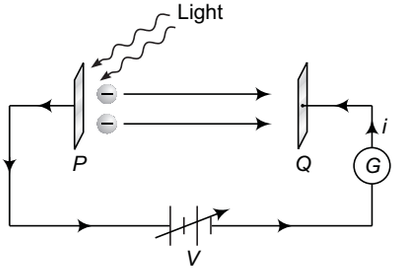


Fig. 26.9

Thus, $K_{max} = E - W$ …(v)

This value K_{max} can experimentally be found by keeping the metal plate P (from which electrons are emitting) at higher potential relative to an another plate Q

placed infront of P. Some electrons after emitting from plate P, reach the plate Q despite the fact that Q is at lower potential and it is repelling the electrons from reaching in itself. This is because the electrons emitted from plate P possess some kinetic energy and due to this energy they reach the plate Q and current i flows in the circuit in the direction shown in figure.

As the potential V is increased, the force of repulsion to the electrons gets increased and less number of electrons reach the plate Q and current in the circuit gets decreased. At a certain value V_0 electrons having maximum kinetic energy (K_{max}) also get stopped and current in the circuit becomes zero. This is called the **stopping potential**.

As an electron moves from P to Q, the potential decreases by V_0 and negative work $-eV_0$ is done on the (negatively charged) electron, the most energetic electron leaves plate P with kinetic energy $K_{max} = \frac{1}{2} mv_{max}^2$ and has zero kinetic energy at Q. Using the work energy theorem, we have

$$W_{ext} = -eV_0 = \Delta K = 0 - K_{max}$$

or $$K_{max} = \frac{1}{2} mv_{max}^2 = eV_0 \quad \ldots(vi)$$

Photoelectric Current

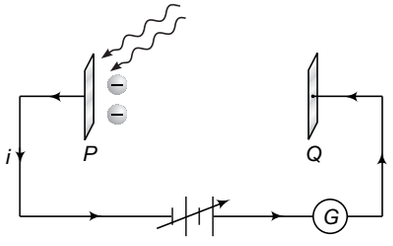


Fig. 26.10

Figure shows an apparatus used to study the variation of photocurrent i with the intensity and frequency of light falling on metal plate P. Photoelectrons are emitted from plate P which are being attracted by the positive plate Q and a photoelectric current i flows in the circuit, which can be measured by the galvanometer G.

Fig. (a) shows graphs of photocurrent as a function of potential difference V_{QP} for light of constant frequency and two different intensities. When V_{QP} is sufficiently large and positive the current becomes constant, showing that all the emitted electrons are being collected by the anode plate Q. The stopping potential difference $-V_0$ needed to reduce the current to zero is shown. If the intensity of light is increased, (or we can say the number of photons incident per unit area per unit time is increased) while its frequency is kept the same, the current becomes constant at a higher value, showing that more electrons are being emitted per unit time. But the stopping potential is found to be the same.

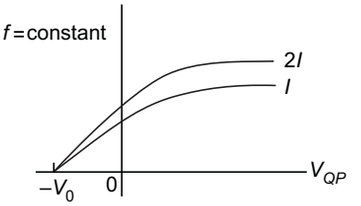


Fig. 26.11 (a) Photocurrent i as a function of the potential V_{QP} of the anode with respect to the cathode for a constant light frequency f, the stopping potential V_0 is independent of the light intensity I.

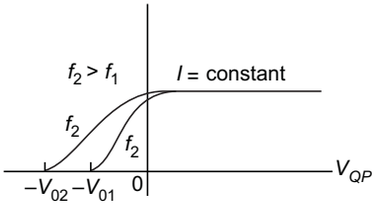


Fig. 26.11 (b) Photocurrent i as a function of the potential V_{QP} of an anode with respect to a cathode for two different light frequencies f_1 and f_2 with the same intensity. The stopping potential V_0 (and therefore, the maximum kinetic energy of the photoelectrons) increases linearly with frequency.

Fig. (b) shows current as a function of potential difference for two different frequencies, with the same intensity in each case. We see that when the frequency of the incident monochromatic light is increased, the stopping potential V_0 gets increased of course, V_0 turns out to be a linear function of the frequency f.

Graph between K_{max} and f

Let us plot a graph between maximum kinetic energy K_{max} of photoelectrons and frequency f of incident light.

The equation between K_{max} and f is,

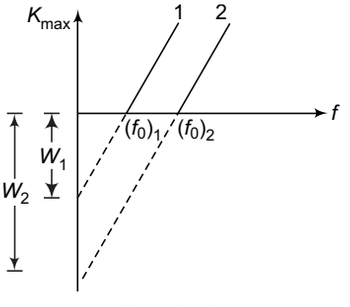


Fig. 26.12

$$K_{max} = hf - W$$

comparing it with $y = mx + c$, the graph between K_{max} and f is a straight line with positive slope and negative intercept.

From the graph we can note the following points.

(i) $K_{max} = 0$ at $f = f_0$

(ii) Slope of the straight line is h, a universal constant. i.e. if graph is plotted for two different metals 1 and 2, slope of both the lines is same.

(iii) The negative intercept of the line is W, the work function, which is characteristic of a metal, i.e. intercepts for two different metals will be different. Further,

$$W_2 > W_1$$

$$\therefore \quad (f_0)_2 > (f_0)_1 \quad \text{as} \quad W = hf_0$$

where, f_0 = threshold frequency.

Graph between V_0 and f

Let us now plot a graph between the stopping potential V_0 and the incident frequency f. The equation between them is,

$$eV_0 = hf - W$$

or

$$V_0 = \left(\frac{h}{e}\right) f - \left(\frac{W}{e}\right)$$

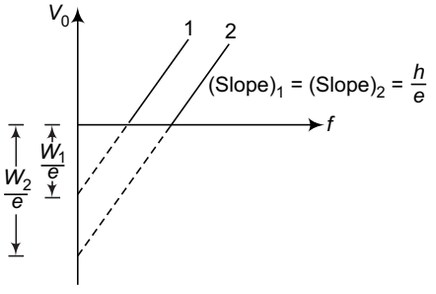


Fig. 26.13

Again comparing with $y = mx + c$, the graph between V_0 and f is a straight line with positive slope $\frac{h}{e}$ (a universal constant) and negative intercept $\frac{W}{e}$ (which depends on the metal). The corresponding graph is shown in figure.

Example 26.7 *The photoelectric work function of potassium is* 2.3 eV. *If light having a wavelength of* 2800 Å *falls on potassium, find*

(*a*) *the kinetic energy in electron volts of the most energetic electrons ejected.*

(*b*) *the stopping potential in volts.*

Sol. Given, $W = 2.3$ eV, $\lambda = 2800$ Å

$$\therefore \quad E \text{ (in eV)} = \frac{12375}{\lambda \text{ (in Å)}} = \frac{12375}{2800} = 34.4 \text{ eV}$$

(a) $K_{max} = E - W = (4.4 - 2.3)$ eV $= 2.1$ eV

(b) $K_{max} = eV_0$

$\therefore \quad 2.1 \text{ eV} = eV_0$

or $\quad V_0 = 2.1$ V

26.9 Nuclear Stability and Radioactivity

Among about 1500 known nuclides, less than 260 are stable. The others are unstable that decay to form other nuclides by emitting α and β-particles and γ-electromagnetic waves. This process is called radioactivity. It was discovered in 1896 by Henry Becquerel.

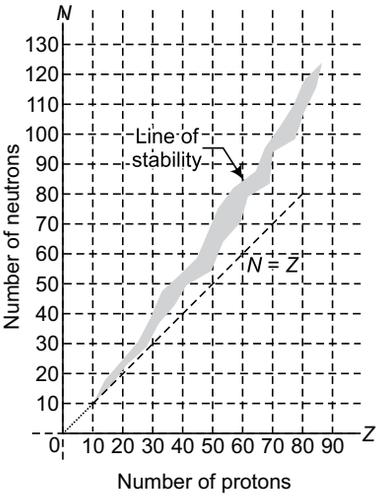


Fig. 26.14 The stable nuclides plotted on a graph or neutron number, *N*, *versus* proton number, *Z*. Note that for heavier nuclides, *N* is larger relative to *Z*. The stable nuclides group along a curve called the line of stability.

While the chemical properties of an atom are governed entirely by the number of protons in the nucleus (i.e. the proton number Z), the stability of an atom appears to depend on both the number of protons and the number of neutrons. For light nuclei, the greatest stability is achieved when the numbers of protons and neutrons are approximately equal ($N \approx Z$).

For heavier nuclei, instability caused by electrostatic repulsion between the protons is minimised when there are more neutrons than protons.

Figure shows a plot of N *versus* Z for the stable nuclei. For mass numbers upto about $A = 40$, we see that $N \approx Z$. 40Ca is the heaviest stable nucleus for which $N = Z$. For larger values of Z, the (short range) nuclear force is unable to hold the nucleus together against the (long range) electrical repulsion of the protons unless the number of neutrons exceeds the number of protons. At Bi ($Z = 83$, $A = 209$), the neutron excess is $N - Z = 43$. There are no stable nuclides with $Z > 83$.

The nuclide $^{209}_{83}$Bi is the heaviest stable nucleus.

Atoms are radioactive, if their nuclei are unstable and spontaneously (and randomly) emit various particles, the α, β and/or γ radiations. When naturally occuring nuclei are unstable, we call the phenomena **natural radioactivity.**

Other nuclei can be transformed into radioactive nuclei by various means, typically involving irradiation by neutrons, this is called **artificial radioactivity.**

A radioactive nucleus is called a **parent nucleus,** the nucleus resulting from its decay by particle emission is called **daughter nucleus.** Daughter nuclei also might be granddaughter nuclei, an so on. There are no son or grandson nuclei. For unstable nuclides and radioactivity following points can be made.

(i) Disintegrations tend to produce new nuclides near the stability line and continue until a stable nuclide is formed.

(ii) Radioactivity is a nuclear property, i.e. α, β and γ-emissions take place from the nucleus.

(iii) Nuclear processes involve huge amount of energy so the particle emission rate is independent of temperature and pressure. The rate depends solely on the concentration of the number of atoms of the radioactive substance.

(iv) A radioactive substance is either an α-emitter or a β-emitter. γ-rays emit with both.

Alpha Decay

An α-particle is a helium nucleus. Thus, a nucleus emitting an α-particle loses two protons and two neutrons. Therefore, the atomic number Z decreases by 2, the mass number A decreases by 4 and the neutron number N decreases by 2. The decay can be written as,

$$^{A}_{Z}X^{A} = {}^{A-4}_{Z-2}Y + {}^{4}_{2}\text{He}$$

where, X is the parent nucleus and Y is the daughter nucleus. For example, U^{238} and Ra^{226} are both α-emitter and α-decay according to,

$$^{238}_{92}\text{U} \longrightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$$

$$^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

As a general rule in any decay sum of mass numbers A and atomic numbers Z must be the same on both sides.

Note that a nuclide below the stability line in Fig. 26.14 disintegrates in such a way that its proton number decreases and its neutron to proton ratio increases. In heavy nuclides this can occur by α-emission.

If the original nucleus has a mass number A that is four times an integer, the daughter nucleus and all those in the chain will also have mass numbers equal to four times an integer. (Because in α-decay A decreases by four and in β-decay it remains the same). Similarly, if the mass number of the original nucleus is $4n+1$, where n is an integer, all the nuclei in the decay chain will have mass numbers given by $4n+1$ with n decreasing by 1 in each α-decay. We can see therefore, that there are four possible α-decay chains, depending on whether A equals $4n$, $4n+1$, $4n+2$ or $4n+3$ where, n is an integer.

Series $4n+1$ is now not found. Because its longest lived member (other than the stable end product Bi^{209}) is Np^{237} which has a half-life of only 2×10^{6} years. Because this is much less than the age of the earth so, this series has disappeared.

Figure shows the uranium $(4n+2)$ series.

The series branches at $Bi^{214,}$ which decays either by α-decay to Ti^{210} or β-decay to Po^{214}. The branches meet at the lead isotope Pb^{210}. Table 26.2 lists the four radioactive series.

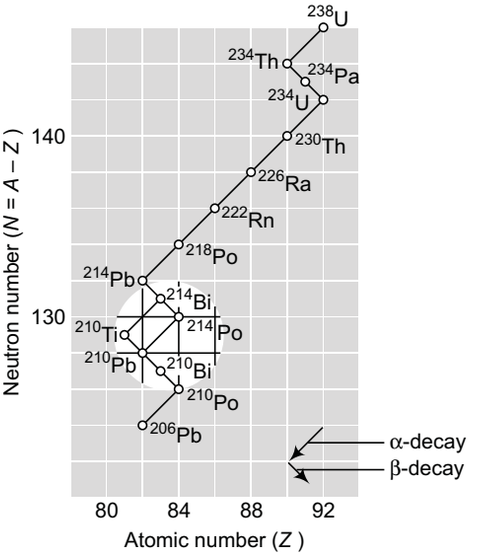


Fig. 26.15 The uranium decay series ($A = 4n + 2$). The decay of $^{214}_{83}$Bi may proceed either by α-emission and then β-emission or in the reverse order.

Table 26.2 Four Radioactive Series

Mass Numbers	Series	Parent	Half-Life, Years	Stable Product
$4n$	Thorium	$^{232}_{90}$Th	1.39×10^{10}	$^{208}_{82}$Pb
$4n+1$	Neptunium	$^{237}_{93}$Np	2.25×10^{6}	$^{209}_{83}$Bi
$4n+2$	Uranium	$^{238}_{92}$U	4.47×10^{9}	$^{206}_{82}$Pb
$4n+3$	Actinium	$^{235}_{92}$U	7.07×10^{8}	$^{207}_{82}$Pb

Beta Decay

β-decay can involve, the emission of either electrons or positrons. A positron is a form of antimatter, which has a charge equal to $+e$ and mass equal to that of an electron. The electrons or positrons emitted in β-decay do not exist inside the nucleus. They are only created at the time of emission, just as photons are created when an atom makes a transition from a higher to a lower energy state.

In β^--decay a neutron in the nucleus is transformed into a proton, an electron and an antineutrino.

$$n \longrightarrow P + e^- + \bar{\nu}$$

To conserve energy and momentum in the process, the emission of an antineutrino ($\bar{\nu}$) (alongwith proton and electron) was first suggested by W Pauli in 1930, but it was first observed experimentally in 1957.

Thus, a parent nucleus with atomic number Z and mass number A decays by β^--emission into a daughter with atomic number $Z+1$ and the same mass number A.

$$^{A}_{Z}X \xrightarrow{\beta^-} {}^{A}_{Z+1}Y$$

β^--decay occurs in nuclei that have too many neutrons. An example of β^--decay is the decay of carbon 14 into nitrogen,

$$^{14}_{6}\mathrm{C} \longrightarrow {}^{14}_{7}\mathrm{N} + e^- + \bar{\nu}$$

In β^+-decay, a proton changes into a neutron with the emission of a positron (and a neutrino)

$$P \longrightarrow n + e^+ + \nu$$

Positron (e^+) emission from a nucleus decreases the atomic number Z by 1 while keeping the same mass number A.

$$^{A}_{Z}X \xrightarrow{\beta^+} {}^{A}_{Z-1}Y$$

β^+- decay occurs in nuclei that have two few neutrons. A typical β^+- decay is,

$$^{13}_{7}\mathrm{N} \longrightarrow {}^{13}_{6}\mathrm{C} + e^+ + \nu$$

Electron capture Electron capture is competitive with positron emission, since both processes lead to the same nuclear transformation. This occurs when a parent nucleus captures one of its own orbital electrons and emits a neutrino.

$$^{A}_{Z}X + e^{-1} \longrightarrow {}^{A}_{Z-1}Y + \nu$$

In most cases, it is a K-shell electron that is captured, and for this reason the process is referred to as ***K*-capture.** One example is the capture of an electron by $_4\mathrm{Be}^7$.

$$^{7}_{4}\mathrm{Be} + e^- \longrightarrow {}^{7}_{3}\mathrm{Li} + \nu$$

Gamma Decay

Very often a nucleus that undergoes radioactive decay (α or β-decay) is left in an excited energy state (analogous to the excited states of the orbiting electrons, except that the energy levels associated with the nucleus have much larger energy differences than those involved with the atomic electrons). The typical half-life of an excited nuclear state is 10^{-10} s. The excited nucleus (X^*) then undergoes to a lower energy state, by emitting a high energy photon, called the γ-ray photon. The following sequence of events represents a typical situation in which γ-decay occurs.

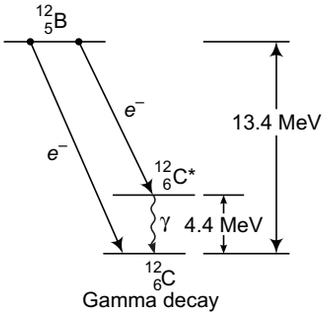


Fig. 26.16

$$^{12}_{5}\mathrm{B} \longrightarrow {}^{12}_{6}\mathrm{C}^* + e^- + \bar{\nu}$$
$$^{12}_{6}\mathrm{C}^* \longrightarrow {}^{12}_{6}\mathrm{C} + \gamma$$

Figure shows decay of B^{12} nucleus, which undergoes β-decay to either of two levels of C^{12}. It can either decay directly to the ground state of C^{12} by emitting a 13.4 MeV electron or undergo β-decay to an excited state of $^{12}_{6}\mathrm{C}^*$ followed by γ-decay to the ground state. The latter process results in the emission of a 9.0 MeV electron and a 4.4 MeV photon. The various pathways by which a radioactive nucleus can undergo decay are summarised in Table 26.3.

- In both α and β-decay, the Z value of a nucleus changes and the nucleus of one element becomes the nucleus of a different element. In γ-decay, the element does not change, the nucleus merely goes from an excited state to a less excited state.

Table 26.3. Various Decay Pathways

Alpha decay	$^{A}_{Z}X^{A} \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\mathrm{He}$
Beta decay (β^-)	$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}$
Beta decay (β^+)	$X \longrightarrow {}^{A}_{Z-1}Y + e^+ + \nu$
Electron capture	$^{A}_{Z}X + e^- \longrightarrow {}^{A}_{Z-1}Y + \nu$
Gamma decay	$^{A}_{Z}X^* \longrightarrow {}^{A}_{Z}X + \gamma$

26.10 The Radioactive Decay Law

Radioactive decay is a random process. Each decay is an independent event and one cannot tell when a particular nucleus will decay. When a particular nucleus decays, it is transformed into another nuclide, which may or may not be radioactive. When there is a very large number of nuclei in a sample, the rate of decay is proportional to the number of nuclei, N, that are present,

$$\left(-\frac{dN}{dt}\right) \propto N \quad \text{or} \quad \left(-\frac{dN}{dt}\right) = \lambda N$$

where, λ is called the **decay constant**. This equation may be expressed in the form $\frac{dN}{N} = -\lambda dt$ and integrated,

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

or $$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

where, N_0 is the initial number of parent nuclei at $t = 0$. The number that survive at time t is therefore,

$$N = N_0 e^{-\lambda t} \qquad \ldots(i)$$

This function is plotted in Fig. 26.17.

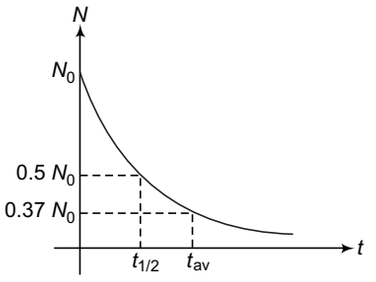


Fig. 26.17

Half-life The time required for the number of parent nuclei to fall to 50% is called half-life $t_{1/2}$ and may be related to λ as follows. Since,

$$0.5N_0 = N_0 e^{-\lambda t_{1/2}}$$

we have, $$\lambda t_{1/2} = \ln(2)$$

$$= 0.693$$

$$\therefore \quad t_{1/2} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda} \qquad \ldots(ii)$$

Mean-life The average or mean-life t_{av} is the reciprocal of the decay constant.

$$t_{av} = \frac{1}{\lambda} \qquad \ldots(iii)$$

The mean-life is analogous to the time constant in the exponential decrease in the charge on a capacitor in an *RC* circuit. After a time equal to the mean-lifetime, the number of radioactive nuclei and the decay rate have each decreased to 37% of their original values.

Activity of a Radioactive Substance

The decay rate R of a radioactive substance is the number of decays per second. And as we have seen above

$$-\frac{dN}{dt} \propto N \quad \text{or} \quad -\frac{dN}{dt} = \lambda N$$

Thus, $$R = -\frac{dN}{dt} \text{ or } R \propto N$$

or $$R = \lambda N \text{ or } R = \lambda N_0 e^{-\lambda t}$$

or $$R = R_0 e^{-\lambda t} \qquad \ldots(iv)$$

where, $R_0 = \lambda N_0$ is the activity of the radioactive substance at time $t = 0$. The activity *versus* time graph is shown in Fig. 26.17.

Thus, the number of nuclei and hence the activity of the radioactive substance decrease exponentially with time.

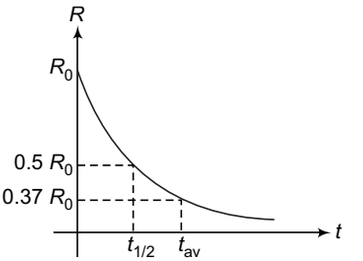


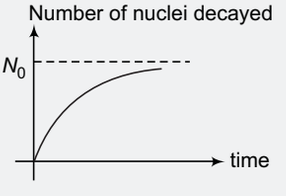
Fig. 26.18

Units of activity The SI unit for the decay rate is the Becqueral (Bq), but the Curie (Ci) and Rutherford (Rd) are often used in practice.

$1\ \text{Bq} = 1\ \text{decay/s}, 1\ \text{Ci} = 3.7 \times 10^{10}\ \text{Bq}$ and $1\ \text{Rd} = 10^6\ \text{Bq}$

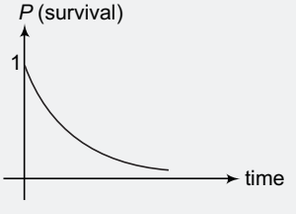
Extra Knowledge Points

- After n half-lives,
  (a) number of nuclei left $= N_0\left(\frac{1}{2}\right)^n$,
  (b) fraction of nuclei left $= \left(\frac{1}{2}\right)^n$ and
  (c) percentage of nuclei left $= 100\left(\frac{1}{2}\right)^2$
- Number of nuclei decayed after time t,
  $= N_0 - N = N_0 - N_0 e^{-\lambda t}$
  $= N_0(1 - e^{-\lambda t})$
  The corresponding graph is as shown in figure.

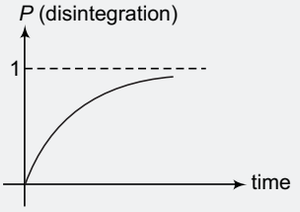


- Probability of a nucleus for survival of time t, The corresponding graph is shown in figure.

$$P(\text{survival}) = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$



- Probability of a nucleus to disintegrate in time t is,
  $$P\,(\text{disintegration}) = 1 - P\,(\text{survival}) = 1 - e^{-\lambda t}$$
  The corresponding graph is as shown



- half-life and mean-life are related to each other by the relation,
  $$t_{1/2} = 0.693\, t_{av} \quad \text{or} \quad t_{av} = 1.44\, t_{1/2}$$
- As we said in point number (2), number of nuclei decayed in time t are $N_0\,(1 - e^{-\lambda t})$. This expression involves power of e.
  So , to avoid it we can use,
  $$\Delta N = \lambda N \Delta t$$
  where, ΔN are the number of nuclei decayed in time Δt, at the instant when total number of nuclei are N. But this can be applied only when $\Delta t << t_{1/2}$.
- In same interval of time, equal percentage (or fraction) of nuclei are decayed (or left undecayed).

Example 26.8 *At time $t = 0$, number of nuclei of a radioactive substance are 100. At $t = 1$ s these numbers become 90. Find the number of nuclei at $t = 2$ s.*

Sol. In 1 s 90% of the nuclei have remained undecayed, so in another 1 s 90% of 90 i.e. 81 nuclei will remain undecayed.

Example 26.9 *At time $t = 0$, activity of a radioactive substance is 1600 Bq, at $t = 8$ s activity remains 100 Bq. Find the activity at $t = 2$ s.*

Sol. $R = R_0\left(\frac{1}{2}\right)^n$

Here, n is the number of half-lives.

Given, $R = \frac{R_0}{16} \Rightarrow \therefore \quad \frac{R_0}{16} = R_0\left(\frac{1}{2}\right)^n$

or $n = 4$

Four half-lives are equivalent to 8 s. Hence, 2 s is equal to one half-life. So in one half-life activity will remain half of 1600 Bq i.e. 800 Bq.

Example 26.10 *Prove mathematically that mean-life or average life of a radioactive substance is $t_{av} = 1/\lambda$.*

Sol. Let N be the number of atoms that exist at time t. Between t and $t + dt$, let dN atoms are decayed, then

$$\text{Mean or average life} = \frac{\int_{N_0}^{0} t\, dN}{\int_{N_0}^{0} dN}$$

Further, $-\frac{dN}{dt} = \lambda N \quad \text{or} \quad dN = -\lambda N dt$

$$\therefore \quad \text{Mean average life} = \frac{-\int_0^\infty t\ \lambda N dt}{-N_0}$$

But $N = N_0 e^{-\lambda t}$,

$$\text{Hence,} \quad \text{mean-life} = \frac{\int_0^\infty t \lambda N_0 e^{-\lambda t} dt}{N_0}$$

This integration is done by parts. The result is,

$$t_{av} = \frac{1}{\lambda}$$ **Hence proved.**

Example 26.11 *Uranium ores on the earth at the present time typically have a composition consisting of 99.3% of the isotope $_{92}U^{238}$ and 0.7% of the isotope $_{92}U^{235}$. The half-lives of these isotopes are 4.47×10^9 yr and 7.04×10^8 yr respectively. If these isotopes were equally abundant when the earth was formed, estimate the age of the earth.*

Sol. Let N_0 be number of atoms of each isotope at the time of formation of the earth ($t = 0$) and N_1 and N_2 the number of atoms at present ($t = t$). Then

$$N_1 = N_0 e^{-\lambda_1 t} \quad \ldots(i)$$

and $$N_2 = N_0 e^{-\lambda_2 t} \quad \ldots(ii)$$

$$\therefore \quad \frac{N_1}{N_2} = e^{(\lambda_2 - \lambda_1)t} \quad \ldots(iii)$$

Further, it is given that

$$\frac{N_1}{N_2} = \frac{99.3}{0.7} \quad \ldots(iv)$$

Equating Eqs. (iii) and (iv) and taking log both sides, we have

$$(\lambda_2 - \lambda_1)t = \ln\left(\frac{99.3}{0.7}\right)$$

$$\therefore \quad t = \left(\frac{1}{\lambda_2 - \lambda_1}\right)\ln\left(\frac{99.3}{0.7}\right)$$

Substituting the values, we have

$$t = 3\frac{1}{\frac{0.693}{7.04 \times 10^8} - \frac{0.693}{4.47 \times 10^9}}\ln\left(\frac{99.3}{0.7}\right) \text{ or } t = 5.97 \times 10^9 \text{ yr}$$

Comparison of the Properties of α-Particles, β-Particles and γ-Rays

S.No.	Property	α-particle	β-particle	γ-rays
1.	Nature	Helium nucleus	Fast moving electrons	Electromagnetic waves
2.	Charge	$+2e$	$-e$	Zero
3.	Rest mass	6.67×10^{-27} kg	9.1×10^{-31} kg	Zero
4.	Speed	1.4×10^7 to 2.2×10^7 m/s	1 – 99% of c	$c = 3 \times 10^8$ m/s
5.	Ionising power	10^4	10^2	1

Estimation of Earth's Age With the help of half-life we can estimate the age of mineral deposits, rocks and the earth. For example we know that uranium ($_{92}U^{238}$) is ultimately decayed into stable lead ($_{82}Pb^{206}$). Suppose in some rock of the earth, the quantities of uranium and lead are found to be in the ratio of 2 : 1. We can assume that initially when the rock came into existence, there was no lead in it and that all the lead now present in it is due to the decay of uranium. The half- life of uranium is 4.5×10^9 yr.

Using, $N = N_0\, e^{-\lambda t}$

we have, $\dfrac{2N_0}{3} = N_0\, e^{-\lambda t}$

or $\lambda t = \ln(3/2)$

$\therefore \quad \left(\dfrac{0.693}{t_{1/2}}\right) t = \ln(3/2)$

or $t = \dfrac{\ln(3/2)[4.5\times10^9]}{(0.6930)}$

$= 2.63\times10^9$ yr

Thus, we can say that the rock is about 2.63×10^9 yr old. Similarly, it has been found by analysis that the oldest rock on the earth is about 4×10^9 yr.

Carbon dating Cosmic rays coming from the outer space produce radioactive atoms by colliding with atoms present in the earth's upper atmosphere. One of the radioactive atoms so produced is an isotope of $_6C^{14}$, which is formed by collision of neutron ($_0n^1$) present in cosmic rays and nitrogen atom present in the atmosphere.

$$_7N^{14} + {}_0n^1 \longrightarrow {}_6C^{14} + {}_1H^1$$

This carbon isotope is oxidised to carbon dioxide ($^{14}CO_2$) which is mixed with ordinary carbon dioxide ($^{12}CO_2$) of the atmosphere. The ratio of ordinary carbon ($_6C^{12}$) and radioactive carbon ($_6C^{14}$) in the atmospheric carbon dioxide is 10^{12} : 1.

In nature, all living trees and plants take carbon dioxide from the atmosphere. So, in living plants also the ratio of $_6C^{12}$ and $_6C^{14}$ is 10^{12} :1. When a plant dies, it stops taking carbon dioxide from the atmosphere. Therefore in the dead plant, the quantity of ordinary carbon ($_6C^{12}$) remains unchanged, but the radioactive carbon ($_6C^{14}$) continues to decay according to following equation.

$$_6C^{14} \longrightarrow {}_7N^{14} + {}_{-1}\beta^0 + \bar{\gamma}$$

The half-life of this decay is 5568 years. The dead plant is ultimately converted into wood, coal, etc. If we measure the activity of some dead material, then the age of that material can be determined. From the relation $R = R_0\, e^{-\lambda t}$. We can find t, the age of dead plant. Here, R_0 is the activity when plant was dead and R is the activity when observation is taken.

Let us take an example in support of the theory.

Example 26.12 *A* 25.0 g *piece of charcoal is found in some ruins of an ancient city. The sample shows a* ^{14}C *activity of* 250 decay/min. *How long has the tree this charcoal came from been dead? Given, the half-life of* ^{14}C *is* 5730 *yr. The ratio of* ^{14}C *and* ^{12}C *in the living sample is* 1.3×10^{-12}.

Sol. The decay constant λ of ^{14}C will be

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{(5730)(365\times24\times60\times60)}$$

$$= 3.84\times10^{-12}\ s^{-1}$$

The number of ^{14}C nuclei can be calculated in two steps. First, the number of ^{12}C nuclei in 25.0 g of carbon is

$$N(^{12}C) = (6.02\times10^{23})\left(\frac{25}{12}\right) = 1.25\times10^{24}$$

The ratio of ^{14}C to ^{12}C in the live sample was 1.3×10^{-12}, we see that number of ^{14}C nuclei in 25.0 g before decay was,

$$N_0(^{14}C) = (1.3\times10^{-12})(1.25\times10^{24}) = 1.6\times10^{12}$$

Hence, the initial activity of the sample was

$$R_0 = \lambda N_0$$
$$= (3.84\times10^{-12})(1.6\times10^{12})$$
$$= 6.14 \text{ decay/s}$$
$$= 370 \text{ decay/min}$$

Activity at the time of observation is given as, 250 decay/min.

From the equation,

$$R = R_0\, e^{-\lambda t}$$

we have, $\lambda t = \ln\left(\dfrac{R_0}{R}\right)$ or $t = \dfrac{1}{\lambda}\ln\left(\dfrac{R_0}{R}\right)$

Substituting the values, we have

$$t = \frac{1}{3.84\times10^{-12}}\ln\left(\frac{370}{250}\right)$$
$$= 1.0\times10^{11}\ s$$
$$= 3200\ yr$$

26.11 Equivalence of Mass and Energy

In 1905, while developing his special theory of relativity, Einstein made the suggestion that energy and mass are equivalent. He predicted that if the energy of a body changes by an amount E, its mass changes by an amount m given by the equation,

$$E = mc^2$$

where, c is the speed of light. Everyday examples of energy gain are much too small to produce detectable changes of mass. But in nuclear physics this plays an important role. Mass appears as energy and the two can be regarded as equivalent. In nuclear physics mass is measured in **unified atomic mass units (u)**, 1 u being one-twelfth of the mass of carbon-12 atom and equals 1.66×10^{-27} kg. It can readily be shown using $E = mc^2$ that, 1 u mass has energy 931.5 MeV

Thus, $1\text{ u} \equiv 931.5\text{ MeV}$ or $c^2 = 931.5\text{ MeV/u}$

A unit of energy may therefore be considered to be a unit of mass. For example, the electron has a rest mass of about 0.5 MeV.

If the principle of conservation of energy is to hold for nuclear reactions it is clear that mass and energy must be regarded as equivalent. The implication of $E = mc^2$ is that any reaction producing an appreciable mass decrease is a possible source of energy.

Example 26.13 *Find the increase in mass of water when* 1.0 kg *of water absorbs* 4.2×10^3 J *of energy to produce a temperature rise of* 1 K.

Sol. $m = \dfrac{E}{c^2} = \dfrac{4.2\times 10^3}{(3.0\times 10^8)^2}\text{ kg} = 4.7\times 10^{-14}\text{ kg}$

Pair Production

When an energetic γ-ray photon falls on a heavy substance, it is absorbed by some nucleus of the substance and an electron and a positron are produced.

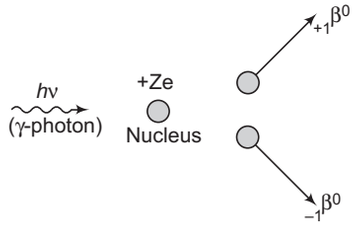


Fig.26.19

This phenomenon is called 'pair production' and may be represented by the equation

$$h\nu \longrightarrow {}_{+1}\beta^0 + {}_{-1}\beta^0$$

From the equation, $E = mc^2$, the least energy (of photon) required for pair production will be,

$$E = 2\,(9.1\times 10^{-31}\text{ kg})\,(3.0\times 10^8\text{ m/s})^2$$
$$= 1.64\times 10^{-13}\text{ J}$$
$$= \frac{1.64\times 10^{-13}}{1.6\times 10^{-13}}\text{ MeV} = 1.02\text{ MeV}$$

Hence, for pair production, it is essential that the energy of γ-photon must be at least 1.02 MeV.

Note Points

- In pair production, if the heavy nucleus is not present, then the energy and the momentum will not be conserved.
- If the energy of γ-photon is less than 1.02 MeV, it would cause photoelectric effect or Compton effect on striking the matter (no pair-production).
- If the energy of γ-photon is more than 1.02 MeV, then electron and positron are produced and the energy in excess of 1.02 MeV is obtained as the kinetic energy of these particles. This kinetic energy is not distributed equally between these two particles, but the positron gets more than half. This is because (positively charged) nucleus accelerates the escaping (positively charged) positron due to repulsion, but retards the electron due to attraction

Pair Annihilation The converse phenomenon of pair production is called pair annihilation. When an electron and a positron come very close to each other, they annihilate each other by combining together and two γ-photons are produced. This phenomenon can be represented by the following equation.

$$_{+1}\beta^0 + {}_{-1}\beta^0 \longrightarrow h\nu + h\nu$$

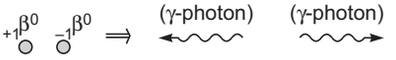


Fig. 26.20

- That two photons (not one) are produced in pair annihilation to conserve both the energy and the momentum.

26.12 Binding Energy and Nuclear Stability

The existence of a stable nucleus means that the nucleons (protons and neutrons) are in a bound state. Since, the protons in a nucleus experience strong electrical repulsion, there must exist a stronger attractive force that holds the nucleus together. The **nuclear force** is a short range interaction that extends only to about 2 fm. (In contrast, the electromagnetic interaction is a long range interaction). An important feature of the nuclear force is that it is essentially the same for all nucleons, independent of charge.

The **binding energy** (E_b) of a nucleus is the energy required to completely separate the nucleons. The origin of the binding energy may be understood with the help of mass-energy relation, $\Delta E = \Delta m c^2$, where Δm is the difference between the total mass of the separated nucleons and the mass of the stable nucleus. The mass of the stable nucleus is less than the sum of the mass of its nucleons. The binding energy of a nuclide ${}_zX^A$ is thus,

$$E_b = [\,z m_p + (A - z)\,m_N - m_X\,]\,c^2 \qquad \ldots(\text{i})$$

where, m_p = mass of proton, m_N = mass of neutron and m_X = mass of nucleus.

Note Points

- $\Delta m = [zm_p + (A - z)m_N - m_X]$ is called the mass defect. This much mass is lost during the formation of a nucleus. Energy $\Delta E = (\Delta m)c^2$ is liberated during the making of the nucleus. This is the energy due to which nucleons are bound together. So, to break the nucleus in its constituent nucleons this much energy has to be given to the nucleus.
- It is better to write Eq. (i) as under,

$$E_b = [zm_H + (A - z)m_N - m_A]c^2 \quad \ldots(ii)$$

where, m_H is the mass of H atom and m_A the atomic mass. By using the masses of H atoms rather than protons, masses of the electrons in the atom cancel out. We do this because it is atomic masses that are measured directly by mass spectrometer. A slight error is made by doing so but that is negligible.

- **Stability** Although nuclides with z values upto $z = 92$ (Uranium) occur naturally, not all of these are stable. The nuclide $^{209}_{83}$Bi is the heaviest stable nucleus. Even though uranium is not stable, however, its long lived isotope ^{238}U, has a half-life of some 4 billion year.

Binding energy per nucleon If the binding energy of a nucleus is divided by its mass number, the binding energy per nucleon is obtained. A plot of binding energy per nucleon E_b/A as a function of mass number A for various stable nuclei is shown in figure.

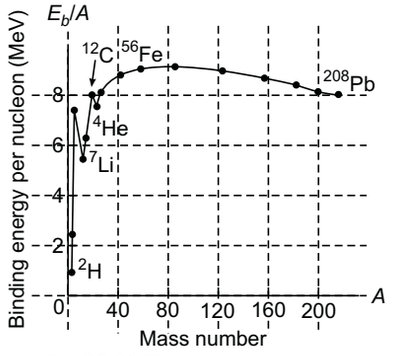


Fig. 26.21 The binding energy per nucleon, E_b/A, as a function of the mass

- That it is the binding energy per nucleon which is more important for stability of a nucleus rather than the total binding energy.

Following conclusions can be drawn from the above graph.

1. The greater the binding energy per nucleon the more stable is the nucleus. The curve reaches a maximum of about 8.75 MeV in the vicinity of $^{56}_{26}$Fe and then gradually falls to 7.6 MeV for $^{238}_{92}$U.
2. In a nuclear reaction energy is released, if total binding energy is increasing. Let us take an example. Suppose a nucleus X, which has total binding energy of 100 MeV converts into some another nucleus Y which has total binding energy 120 MeV. Then, in this process 20 MeV energy will be released. This is bacause 100 MeV energy has already been released during the formation of X while in case of Y it is 120 MeV. So, the remaining 20 MeV will be released now.
   Energy is released if ΣE_b is increasing.
3. ΣE_b in a nuclear process is increased, if binding energy per nucleon of the daughter products gets increased. Let us take an example. Consider a nucleus $X(A_X = 100)$ breaks in two lighter nuclei Y $(A_Y = 60)$ and $Z(A_Z = 40)$.
   $$X \rightarrow Y + Z$$
   Binding energy per nucleon of these three are say, 7 MeV, 7.5 MeV and 8.0 MeV. Then, total binding energy of X is $100 \times 7 = 700$ MeV and that of $Y + Z$ is $(60 \times 7.5) + (40 \times 8.0) = 770$ MeV. So, in this process 70 MeV energy will be released.
4. Binding energy per nucleon is increased, if two or more lighter nuclei combine to form a heavier nucleus. This process is called **nuclear fusion.**



Fig. 26.22

In **nuclear fission** a heavy nucleus splits into two or more lighter nuclei of almost equal mass.

In both the processes E_b/A is increasing. Thus, energy will be released.

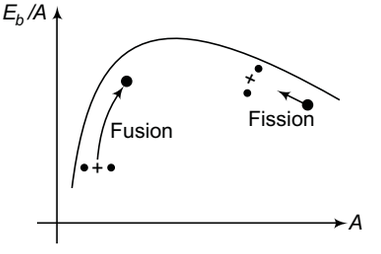


Fig. 26.23

26.13 Nuclear Fission
(Divide and Conquer)

As we saw in the above article nuclear fission occurs when a heavy nucleus such as ^{235} U splits into two lighter nuclei. In nuclear fission the combined mass of the daughter nuclei is less than the mass of the parent nucleus. The difference is called the mass defect. Fission is initiated when a heavy nucleus captures a thermal neutron (slow neutrons). Multiplying the mass defect by c^2 gives the numerical value of the released energy. Energy is released, because the binding energy per nucleon of the daughter nuclei is about 1 MeV greater than that of the parent nucleus.

The fission of ^{235}U by thermal neutrons can be represented by the equation,

$$^1_0n + {}^{235}_{92}\text{U} \longrightarrow {}^{236}_{92}\text{U}^* \longrightarrow X + Y + \text{neutrons}$$

where, $^{236}\mathrm{U}^{*}$ is an intermediate excited state that lasts only for 10^{-12} s before breaking into nuclei X and Y, which are called fission fragments. In any fission equation there are many combinations of X and Y that satisfy the requirements of conservation of energy and charge with uranium, for example, there are about 90 daughter nuclei that can be formed.

Fission also results in the production of several neutrons, typically two or three. On the average, about 2.5 neutrons are released per event. A typical fission reaction for uranium is

$$^{1}_{0}n + {}^{235}_{92}\mathrm{U} \longrightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,(^{1}_{0}n)$$

About 200 MeV is released in the fission of a heavy nucleus. The fission energy appears mostly as kinetic energy of the fission fragments (e.g. barium and krypton nuclei) which fly apart at great speed. The kinetic energy of the fission neutrons also makes a slight contribution. In addition one or both of the large fragments are highly radioactive and small amount of energy takes the form of beta and gamma radiation.

Chain reaction Shortly after nuclear fission was discovered, it was realised that, the fission neutrons can cause further fission of $^{235}\mathrm{U}$ and a chain reaction can be maintained.

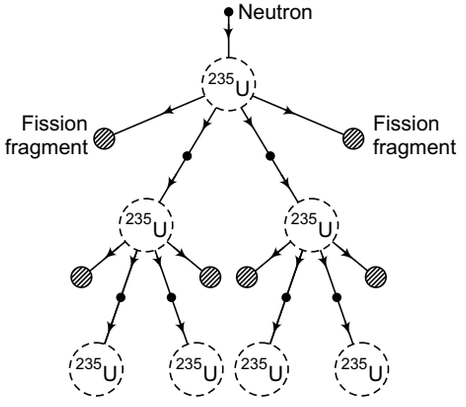


Fig. 26.24 A chain reaction

In practice, only a proportion of the fission neutrons is available for new fissions since, some are lost by escaping from the surface of the uranium before colliding with another nucleus. The ratio of neutrons escaping to those causing fission decreases as the size of the piece of uranium-235 increases and there is a **critical size** (about the size of a cricket ball) which must be attained before a chain reaction can start.

In the **atomic bomb** an increasing uncontrolled chain reaction occurs in a very short time when two pieces of uranium-235 are rapidly brought together to form a mass greater than the critical size.

Nuclear reactors In a nuclear reactor the chain reaction is steady and controlled so that on average only one neutron from each fission produces another fission. The reaction rate is adjusted by inserting neutron-absorbing rods of boron steel into the uranium-235.

Graphite core is used as a **moderator** to slow down the neutrons.

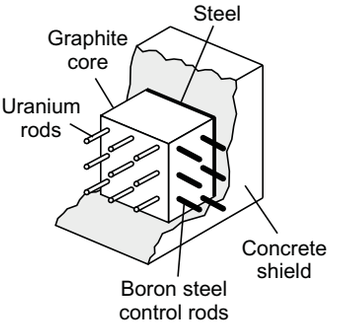


Fig. 26.25 Nuclear reactor

Natural uranium contains over 99% of $^{238}\mathrm{U}$ and less than 1% of $^{235}\mathrm{U}$. The former captures the medium speed fission neutrons without fissioning. It fissions with very fast neutrons. On the other hand $^{235}\mathrm{U}$ (and plutonium-239) fissions with slow neutrons and the job of moderator is to slow down the fission neutrons very quickly so that most escape capture by $^{238}\mathrm{U}$ and then cause the fission of $^{235}\mathrm{U}$.

A bombarding particle gives up most energy when it has an elastic collision with a particle of similar mass. For neutrons, hydrogen atoms would be most effective but they absorb the neutrons. But deuterium (in heavy water) and carbon (as graphite) are both suitable as moderator.To control the power level **control rods** are used. These rods are made of materials such as cadmium, that are very efficient in absorbing neutrons. The first nuclear reactor was built by Enrico Fermi and his team at the University of Chicago in 1942.

26.14 Nuclear Fussion

Binding energy for light nuclei ($A < 20$) is much smaller than the binding energy for heavier nuclei. This suggests a process that is the reverse of fission. When two light nuclei combine to form a heavier nucleus, the process is called nuclear fusion. The union of light nuclei into heavier nuclei also lead to a transfer of mass and a consequent liberation of energy. Such a reaction has been achieved in **hydrogen bomb** and it is believed to be the principal source of the sun's energy.

A reaction with heavy hydrogen or deuterium which yields 3.3 MeV per fusion is,

$$^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + {}^{1}_{0}n$$

By comparison with the 200 MeV per fission of ^{235}U this seems small, but per unit mass of material it is not. Fusion of two deuterium nuclei, i.e. deuterons, will only occur, if they overcome their mutual electrostatic repulsion. This may happen, if they collide at very high speed when, for Example, they are raised to a very high temperature (10^8-10^9 K). So, much high temperature is obtained by using an atomic (fission) bomb to trigger off fusion.

If a controlled fusion reaction can be achieved an almost unlimited supply of energy will become available from deuterium in the water of the oceans.

Example 26.14 *Neon-23 decays in the following way*

$$^{23}_{10}Ne \longrightarrow {}^{23}_{11}Na + {}^{0}_{-1}e + \bar{\nu}$$

Find the minimum and maximum kinetic energy that the beta particle ($^{0}_{-1}e$) *can have. The atomic masses of* ^{23}Na *and* ^{23}Na *are* 22.9945 u *and* 22.9898 u*, respectively.*

Sol. Here, atomic masses are given (not the nuclear masses), but still we can use them for calculating the mass defect, because mass of electrons get cancelled both sides. Thus,

Mass defect $\Delta m = (22.9945 - 22.9898) = 0.0047$ u

$\therefore$ $Q = (0.0047 \text{ u})\,(931.5 \text{ MeV/s})$

$= 4.4$ MeV

Hence, the energy of β-particles can range from 0 to 4.4 MeV.

26.15 X-Rays

Electromagnetic radiation with wavelengths from 0.1 Å to 100 Å falls into the category of X-rays. The boundaries of this category are not sharp. The shorter wavelength end overlaps γ-rays and the longer wavelength end overlaps ultraviolet rays. Photoelectric effect (will be discussed later) provides convincing evidence that photons of light can transfer energy to electrons. Is the inverse process also possible?

That is, can part or all of the kinetic energy of a moving electron be converted into a photon? Yes, it is possible. In 1895 **Wilhelm Roentgen** found that a highly penetrating radiation of unknown nature is produced when fast moving electrons strike a target of high atomic number and high melting point. These radiations were given a name X-rays as their nature was unknown (in mathematics an unknown quantity is normally designated by X). Later it was discovered that these are high energy photons (or electromagnetic waves)

Production of X-rays Figure shows a diagram of a X-ray tube, called the coolidge tube. A cathode (a plate connected to negative terminal of a battery), heated by a filament through which an electric current is passed, supplies electrons by thermionic emission.

The high potential difference V maintained between the cathode and a metallic target accelerate the electrons toward the latter. The face of the target is at an angle relative to the electron beam, and the X-rays that leave the target pass through the side of the tube. The tube is evacuated to permit the electrons to get to the target unimpeded.

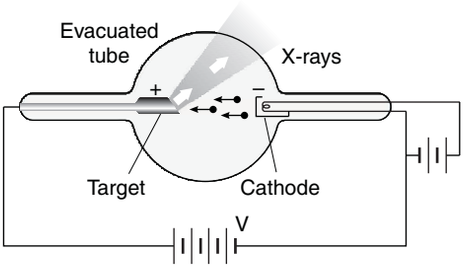


Fig. 26.26 An X-ray tube. The higher the accelerating voltage V, the faster the electrons and the shorter the wavelengths of the X-rays.

Continuous and characteristic X-rays X-rays so produced by the coolidge tube are of two types, continuous and characteristic. While the former depends only on the accelerating voltage V the later depends on the target used.

Continuous X-rays Electromagnetic theory predicts that an accelerated electric charge will radiate electromagnetic waves, and a rapidly moving electrons when suddenly brought to rest is certainly accelerated (of course negative). X-rays produced under these circumstances is given the German name **bremsstrahlung** (braking radiation). Energy loss due to bremsstrahlung is more important for electrons than for heavier particles because electrons are more violently accelerated when passing near nuclei in their paths. The continuous X-rays (or bremsstrahlung X-rays) produced at a given accelerating potential V vary in wavelength, but none has a wavelength shorter than a certain value λ_{min}. This minimum wavelength corresponds to the maximum energy of the X-rays which in turn is equal to the maximum kinetic energy qV or eV of the striking electrons. Thus,

$$\frac{hc}{\lambda_{min}} = eV \quad \text{or} \quad \lambda_{min} = \frac{hc}{eV}$$

After substituting values of h, c and e we obtain the following simple formula for λ_{min}.

$$\lambda_{min} \text{ (in Å)} = \frac{12375}{V \text{ (in volts)}} \qquad \ldots(i)$$

Increasing V decreases λ_{min}. This wavelength is also known as the cutoff wavelength or the threshold wavelength.

Characteristic X-rays The X-ray spectrum typically consists of a broad continuous band containing a series of sharp lines, as shown in Fig. 26.27.

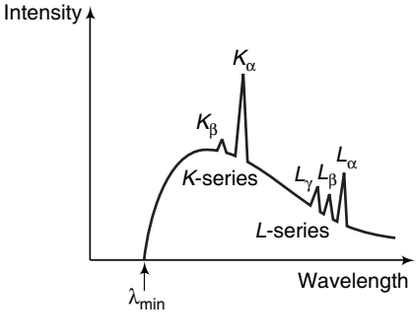


Fig. 26.27 X-ray spectrum

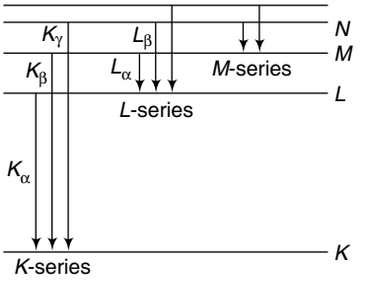


Fig. 26.28

As discussed above the continuous spectrum is the result of collisions between incoming electrons and atoms in the target. The kinetic energy lost by the electrons during the collisions emerges as the energy of the X-ray photons radiated from the target.

The sharp lines superimposed on the continuous spectrum are known as **characteristic X-rays,** because they are characteristic of the target material. They were discovered in 1908, but their origin remained unexplained until the details of atomic structure, particularly the shell structure of the atom, were discovered.

Characteristic X-ray emission occurs when a bombarding electron that collides with a target atom has sufficient energy to remove an inner shell electron from the atom. The vacancy created in the shell is filled when an electron from a higher level drops down into it. This transition is accompanied by the emission of a photon whose energy equals the difference in energy between the two levels.

Let us assume that the incoming electron his dislodged an atomic electron from the innermost shell the *K*-shell. If the vacancy is filled by an electron droping from the next higher shell, the *L*-shell the photon emitted has an energy corresponding to the K_α characteristic X-ray line. If the vacancy is filled by an electron dropping from the *M* shell, the K_β-line is produced. An L_α-line is produced as an electron drops from the *M*-shell to the *L*-shell, and an L_β-line is produced by a transition from the *N*-shell to the *L*-shell.

Moseley's Law for Characteristic Spectrum

Although multi-electron atoms cannot be analysed with the Bohr model, Henery GJ Moseley in 1914 made an effort towards this. Moseley measured the frequencies of characteristic X-rays from a large number of elements and plotted the square root of the frequency $\sqrt{f}$ against the atomic number *Z* of the element. He discovered that the plot is very close to a straight line. He plotted the square root of the frequency of the K_α line *versus* the atomic number *Z*.

As figure shows, Moseley's plot did not pass through the origin. Let us see why. It can be understood from Gauss's law. Consider an atom of atomic number *Z* in which one of the two electrons in the *K*-shell has been ejected. Imagine that we draw a Gaussian sphere just inside the most probable radius of the *L*-electrons. The effective charge inside the Gaussian surface is the positive nuclear charge and one negative charge due to the single *K*-electron. If we ignore the interactions between *L*-electrons, a single *L*-electron behaves as if it experiences an electric field due to a charge $(Z-1)\,e$ enclosed by the Gaussian surface.

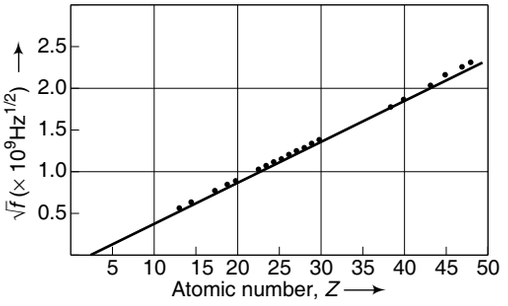


Fig. 26.29

Thus, Moseley's law of the frequency of K_α-line is

$$\sqrt{f_{K_\alpha}} = a\,(Z-1) \qquad \ldots(ii)$$

where, *a* is a constant that can be related to Bohr's theory.

The above law in general can be stated as under,

$$\sqrt{f} = a\,(Z-b) \qquad \ldots(iii)$$

For K_α-line, $\Delta E = hf = Rhc\,(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$

or $\sqrt{f} = \sqrt{\frac{3Rc}{4}}\,(Z-1)$ or $a = \sqrt{\frac{3Rc}{4}}$

and $b = 1$

After substituting values of *R* and *c*, we get

$$a = 4.98 \times 10^7\ (\text{Hz})^{1/2}$$

Eq. (iii) can also be written as,

$$f = a^2\,(Z-b)^2 \qquad \ldots(iv)$$

For K_α-line, $a^2 = \frac{3Rc}{4} = (2.48 \times 10^{15}\ \text{Hz})$ and $b = 1$

Hence, $f_{K_\alpha} = (2.48 \times 10^{15}\ \text{Hz})\,(Z-1)^2$

Example 26.14 *Find the cut-off wavelength for the continuous* X*-rays coming from an* X*-ray tube operating at* 40 kV.

Sol. Cut-off wavelength λ_{min} is given by,

$$\lambda_{min}\ (\text{in Å}) = \frac{12375}{V(\text{in volts})} = \frac{12375}{40 \times 10^3} = 0.31\ \text{Å}$$

Example 26.15 *Use Moseley's law with* $b = 1$ *to find the frequency of the* K_α X*-rays of* La $(Z = 57)$*, if the frequency of the* K_α X*-rays of* Cu $(Z = 29)$ *is known to be* 1.88×10^{18} Hz.

Sol. Using the equation,

$$\sqrt{f} = a\,(Z - b) \qquad (b = 1)$$

$$\frac{f_{La}}{f_{Cu}} = \left(\frac{Z_{La} - 1}{Z_{Cu} - 1}\right)^2 \quad \text{or} \quad f_{La} = f_{Cu}\left(\frac{Z_{La} - 1}{Z_{Cu} - 1}\right)^2$$

$$= 1.88 \times 10^{18}\left(\frac{57 - 1}{29 - 1}\right)^2 = 7.52 \times 10^{18}\ \text{Hz}$$

26.16 Some General Points

Classification of Nuclei

The nuclei have been divided in isotopes, isobars and isotones on the basis of number of protons (atomic number) or the total number of nucleons (mass number).

Isotopes The elements having the same number of protons but different number of neutrons are called isotopes. In other words, isotopes have same value of atomic number (Z) but different value of mass number (A). Almost every element has isotopes. Because of the same atomic number isotopes of an element have the same place in the periodic table.

The isotopes of some elements are given below.

Element	Its isotopes	Number of protons	Number of neutrons
Hydrogen	$_1H^1$	1	0
	$_1H^2$	1	1
	$_1H^3$	1	2
Oxygen	$_8O^{16}$	8	8
	$_8O^{17}$	8	9
	$_8O^{18}$	8	10
Chlorine	$_{17}Cl^{35}$	17	18
	$_{17}Cl^{37}$	17	20
Uranium	$_{92}U^{235}$	92	143
	$_{92}U^{238}$	92	146

In nature, the isotopes of chlorine ($_{17}Cl^{35}$ and $_{17}Cl^{37}$) are found in the ratio 75.4% and 24.6%. When chlorine is prepared in laboratory, its atomic mass is found to be,

$$M = (35 \times 0.754) + (37 \times 0.246) = 35.5$$

Since, the isotopes have the same atomic number, they have the same chemical properties. Their physical properties are different as they have different mass numbers. Two isotopes, thus cannot be separated by chemical method, but they can be separated from the physical methods.

Isobars The elements having the same mass number (A) but different atomic number (Z) are called isobars. They have different places in periodic table. Their chemical (as well as physical) properties are different.

$_1H^3$ and $_2He^3$, $_8O^{17}$ and $_9F^{17}$ are examples of isobars.

Isotones Elements having the equal number of neutrons $(A - Z)$ are called isotones.

$_3Li^7$ and $_4Be^8$, $_1H^3$ and $_2He^4$ are examples of isotones.

Nuclear Forces In nucleus the positively charged protons and the uncharged neutrons are held together in an extremely small space ($\approx 10^{-15}$ m) inspite of the strong electrostatic repulsion between the protons. Obviously there are some strong attractive forces operating within the nucleus between the nucleons. The nuclear forces are non-electric and non-gravitational forces. These forces are extremely short-range forces. They become operative only when the distance between two nucleons is a small multiple of 10^{-15} m. They do not exist when the distance is appreciably larger than 10^{-15} m and become repulsive when the distance is appreciably smaller than 10^{-15} m. Nuclear forces between protons and protons between neutrons and neutrons and between protons and neutrons are all essentially the same in magnitude. Thus, we can say that nuclear forces are charge independent.

Yukawa's Meson Theory of Nuclear Forces A Japanese scientist Yukawa in 1935 suggested that the nuclear forces are 'exchange forces'. Which are produced by the exchange of new particles called π-mesons between nucleons. These particles were later on actually discovered in cosmic radiation. There are three types of π-mesons, π^+, π^- and π^0. There is a continuous exchange of π-mesons between protons and neutrons due to which they continue to be converted into one another. When a π^+ meson jumps from a proton to a neutron, the proton is converted into a neutron and the neutron is converted into a proton.

$$p - \pi^+ \longrightarrow n \text{ and } n + \pi^+ \longrightarrow p$$

Conversely, when a π^- meson jumps from a neutron to a proton, the neutron is converted into a proton and the proton is converted into a neutron. Thus,

$$n - \pi^- \longrightarrow p \text{ and } p + \pi^- \longrightarrow n$$

The exchange of π^+ and π^- mesons between protons and neutrons is responsible for the origin of nuclear forces between them. Similarly, nuclear forces between two protons and between two neutrons are generated by a continuous exchange of π^0-mesons between them. Thus, the basis of nuclear forces is the exchange of mesons and hence these are called 'exchange forces'.

- According to modern concept, electrical and gravitational forces are also exchange forces. Electrical forces between two charged particles are generated by exchange of photons. A new particle is assumed to be responsible for the origin of gravitational forces between two bodies.

Size and Shape of the Nucleus The Rutherford scattering experiment established that mass of an atom is concentrated within a small positively charged region at the centre which is called the nucleus of the atom. The nuclear radius is given by

$$R = R_0 A^{1/3}$$

Here, A is the mass number of the particular nucleus and $R_0 = 1.3$ fm (fermi) $= 1.3 \times 10^{-15}$ m. This means that the nucleus radius is of the order of 10^{-15} m.

Here, $R_0 = 1.3$ fm is the distance of closest approach to the nucleus and is also known as nuclear unit radius.

Nuclear Density Let us consider the nucleus of an atom having the mass number A.

$$\text{Mass of nucleus} \approx A \times 1.67 \times 10^{-27} \text{ kg}$$

$$\text{Volume of the nucleus} = \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}\pi (R_0 A^{1/3})^3$$

$$= \frac{4}{3}\pi R_0^3 A$$

$$\therefore \quad 3 \text{ Density of the nucleus, } \rho = \frac{\text{Mass}}{\text{Volume}}$$

or
$$\rho = \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3} \times \pi \times (1.3 \times 10^{-15})^3 \times A}$$

$$= 1.8 \times 10^{17} \text{ kg/m}^3$$

Thus density of a nucleus is of the order of 10^{17} kg/m^3.

Magic Numbers

We know that the electrons in an atom are grouped in 'shells' and 'sub shells'. Atoms with 2, 10, 18, 36, 54 and 86 electrons have all of their shells completely filled. Such atoms are unusually stable and chemically inert. A similar situation exists with nuclei also. Nuclei having 2, 8, 20, 28, 50, 82 and 126 nucleons of the same kind (either protons or neutrons) are more stable than nuclei of neighbouring mass numbers. These numbers are called as 'magic numbers'.

Fluorescence and Phosphorescence

Fluorescence There are certain substances which on being illuminated by high frequency light (blue or ultraviolet) emit light of relatively low frequency. The emission occurs so long as the substance is being illuminated. This phenomenon is called fluorescence. Fluorescence has many applications in daily life. For example, the presence of invisible ultraviolet rays can be detected by their fluorescent effect.

Explanation Every atom has discrete energy levels associated with it. Normally, the electrons occupy the lowest energy states. When light of some appropriate energy falls on them, they absorb energy and jump to some higher energy state. They stay there only for 10^{-8} s. But they do not return directly to their ground state but are transferred step by step, emitting light of some lower frequency.

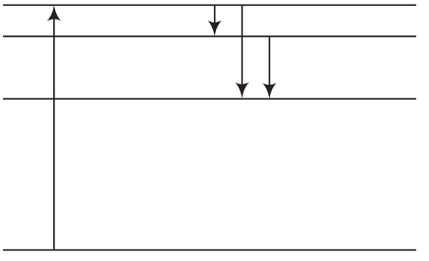


Fig. 26.30

Phosphorescence Fluorescent materials emit light only so long as light is incident on them. There are certain substances which continue emitting light for some time after the light incident on them is stopped. This phenomenon is called 'phosphorescence'. Phosphorescent substances are painted on watch hands, electric switch boards and sign boards. These substances absorb sunlight during day time and illuminate during the dark night by phosphorescence.

Explanation Phosphorescent materials have metastable energy states. In these states the electron can remain for a period longer than 10^{-8} s. Thus, while returning back to the lower energy levels the electron stays for some time in metastable energy states. These delayed transitions are responsible for phosphorescence.

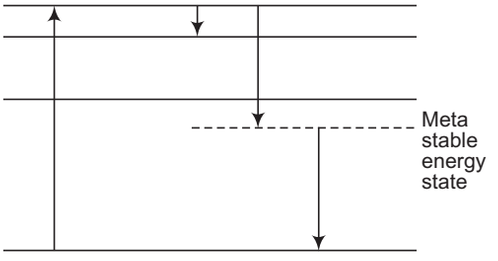


Fig. 26.31

Fundamental Particles

The particles which are not constituted by any other particles are called fundamental particles. A brief discussion of important fundamental particles is as follows.

(i) **Electron** It was discovered in 1897 by Thomson. Its charge is $-e$ and mass is 9.1×10^{-31} kg. Its symbol is e^- (or ${}_{-1}\beta^0$).

(ii) **Proton** It was discovered in 1919 by Rutherford in artificial nuclear disintegration. It has a positive charge $+e$ and its mass is 1836 times (1.673×10^{-27} kg) the mass of electron. In free state, the proton is a stable particle. Its symbol is p^+. It is also written as ${}_1H^1$.

(iii) **Neutron** It was discovered in 1932 by Chadwick. Electrically it is a neutral particle. Its mass is 1839 times (1.675×10^{-27} kg) the mass of electron. In free state the neutron is unstable (mean-life $\approx$ 17 min) but it constitutes a stable nucleus with the proton. Its symbol is n or ${}_0n^1$.

(iv) **Positron** It was discovered by Anderson in 1932. It is the antiparticle of electron, i.e. its charge is $+e$ and its mass is equal to that of the mass of electron. Its symbol is e^+ (or ${}_{+1}\beta^0$).

(v) **Antiproton** It is the antiparticle of proton. It was discovered in 1955. Its charge is $-e$ and its mass is equal to that of the mass of proton. Its symbol is p^-.

(vi) **Antineutron** It was discovered in 1956. It has no charge and its mass is equal to the mass of neutron. The only difference between neutron and antineutron is that if they spin in the same direction, their magnetic momenta will be in opposite directions. The symbol for antineutron is n.

(vii) **Neutrino and antineutrino** The existence of these particles was predicted in 1930 by Pauli while explaining the emission of β-particles from radioactive nuclei, but these particles were actually observed experimentally in 1956. Their rest mass and charge are both zero but they have energy and momentum. These are mutually antiparticles of each other. They have the symbol ν and $\bar{\nu}$.

(viii) **π-Mesons** The existence of π-mesons was predicted by Yukawa in 1935, but they were actually discovered in 1947 in cosmic rays. Nuclear forces are explained by the exchange of π-mesons between the nucleons. π-mesons are of three types, positive π-mesons (π^+), negative π-mesons (π^-) and neutral π-mesons (π^0). Charge on $\pi^\pm$ is $\pm e$. Whereas mass of $\pi^\pm$ is 274 times the mass of electron. π^0 has mass nearly 264 times the electronic mass.

(ix) **Mu-Mesons** These were discovered in 1936 by Anderson and Neddermeyer. These are found in abundance in the cosmic rays at the ground level. There are two types of mu-mesons. Positive mu-meson (μ^+) and negative mu-meson (μ^-). There is no neutral mu-meson. Both the mu-mesons have the same rest mass 207 times the rest mass of the electron.

(x) **Photon** These are bundles of electromagnetic energy and travel with the speed of light. Energy and momentum of a photon of frequency ν are $h\nu$ and $\frac{h\nu}{c}$ respectively.

Antiparticles For every fundamental particle there exists an identical fundamental particle just opposite in some property. For example electron and positron are identical in all respects, except that charges on them are opposite.

The following table shows various particles and their antiparticles. Some particles are their own antiparticles. For example π^0 and γ.

Name of Particle	Symbol	Antiparticle	Mass in comparison to mass of electron	Average life (in second) for the unstable particles
Electron	e^-	e^{+1}	1	Stable
Proton	p^+	p^-	1836	Stable
Neutron	n	$\bar{n}$	1839	1010
Neutrino	ν	$\bar{\nu}$	0	Stable
π-Mesons	π^+	π^-	274	2.6×10^{-8}
	π^0	π^0	264	0.9×10^{-16}
μ-Mesons	μ^-	μ^+	207	2.2×10^{-6}
Photon	γ	γ	0	Stable

Chapter Summary with Formulae

(i) **Photon**

(a) Energy of photon, $E = hf = \frac{hc}{\lambda}$

(b) Energy (in eV) $= \frac{12375}{\lambda\,(\text{in Å})}$

(c) Momentum of photon, $p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$

(ii) **Photoelectric Effect**

(a) Work function of a metal,

$$W = hf_0 = \frac{hc}{\lambda_0}$$

where, f_0 = threshold frequency and

λ_0 = threshold wavelength

(b) Energy of incident photons,

$$E = hf = \frac{hc}{\lambda}$$

where, f = frequency of incident light and

λ = wavelength of incident light

(c) Photoemission from a metal surface takes place, if and only if $E \geq W$ or $\lambda \leq \lambda_0$ or $f \geq f_0$

(d) Kinetic energy of emitted photons lies between zero and a maximum value. Here, maximum kinetic energy is corresponding to those photoelectrons which used minimum energy (= work function) for their emission.

(e) Maximum kinetic energy of photoelectrons is given by

$$K_{max} = \frac{1}{2} mv_{max}^2 = eV_0 = E - W$$

$$= hf - hf_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

where, V_0 is the stopping potential. This is the potential at which photoelectric current becomes zero or the photoelectrons having the maximum kinetic energy are also stopped.

(f) Stopping potential *versus* frequency of incident photon equation is

$$V_0 = \left(\frac{h}{e}\right) f - \left(\frac{W}{e}\right)$$

Therefore, V_0 *versus* f graph is a straight line with

slope = $+\frac{h}{e}$ = a universal constant and intercept $= -\frac{W}{e}$.

(g) Maximum kinetic energy of photoelectrons *versus* frequency of incident photon equation is $K_{max} = hf - W$

Therefore, K_{max} *versus* f graph is a straight line with :

slope = $+ h$ = a universal constant and intercept = $-W$

(h) Stopping potential depends upon the energy (or frequency) of incident photons.

(i) Saturation current depends upon the number of photons incident per unit area per unit time. If energy of photons is same, then we can also say that saturation current depends upon the intensity of incident light.

(iii) **Dual Nature**

(a) Like an electromagnetic wave which has dual nature (wave as well as particle), a particle also shows dual nature. It also behaves as wave.

(b) de-Broglie wavelength associated with a particle is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2Km}} = \frac{h}{\sqrt{2qVm}}$$

(c) de-Broglie wavelength of macroscopic bodies used in routine life is very small and cannot be detected.

(d) Interference, diffraction, polarisation etc, are wave phenomena. Photoelectric effect and compton effect are particle phenomena.

(e) Davisson and Germer experiment proved the wave nature of electron, as the photoelectric effect proved the particle nature of light (or electromagnetic waves in general).

Miscellaneous Points

(i) Electron was discovered by J J Thomson in 1897.

(ii) Plum pudding model of atom was proposed by J J Thomson in 1898.

(iii) J J Thomson's model was electrostatically unstable.

(iv) In α-particle scattering experiment impact parameter is the perpendicular distance of the initial velocity vector of the α-particle from the centre of the nucleus.

(v) **Rutherford's Atomic Model**

(a) According to this model, the entire positive charge and most of the mass of the atom are concentrated in the nucleus. The electrons revolve around the nucleus just as planets do around the sun. The nucleus size is about 10^{-14} m to 10^{-15} m and the atom size is around 10^{-10} m.

(b) $r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$ (c) $K = \frac{e^2}{8\pi\varepsilon_0 r}$

(d) $U = -\frac{e^2}{4\pi\varepsilon_0 r}$ (e) $E = -\frac{e^2}{8\pi\varepsilon_0 r}$

(f) According to the classical electromagnetic theory, the frequency of the electromagnetic waves emitted by the revolving electrons is equal to the frequency of revolution of electron.

(g) Electron is charged particle and it is accelerated due to its revolution in circle. Therefore, according to classical electromagnetic theory it should radiate energy in the form of electromagnetic waves. The energy of electron should therefore, continuously decrease.

So, following a spiral path the electron should eventually fall into the nucleus. In spiral path frequency of revolution should continuously decrease and it should emit a continuous spectrum. But actual spectrum observed was a line spectrum. So Rutherford's model could not explain these discrepancies and finally this model was not accepted fully.

(vi) **Atomic Spectra** Emission spectrum of solids, liquids and dense gases is continuous due to interaction of each atom (or molecule) with its neighbours. On the other hand emission spectrum of rarefied gases is discrete (or line spectrum). Because distance between atoms is large and they cannot interact with each other.

(vii) **Bohr's Atomic Model**

(a) This model is applicable for hydrogen and hydrogen like atoms (called hydrogenic atoms), which have a single electron. Although their atomic number may be different. For example

$H \rightarrow z = 1 \quad \Rightarrow \quad He^+ \rightarrow z = 2 \quad \Rightarrow \quad Li^{+2} \rightarrow z = 3$

In all above atoms number of electron is only one.

(b) Bohr's model is based on following three postulates.

First Postulate An electron in an atom could revolve in certain stable orbits without emitting radiant energy.

Second Postulate The electron revolves around the nucleus only in those orbits for which the angular momentum of the electron is an integral multiple of $h/2\pi$.

i.e. $L = mvr = n\left(\frac{h}{2\pi}\right)$ (where, $n = 1, 2, ...$)

Third Postulate Energy is radiated by an atom only when its electron makes a transition from higher energy state to lower energy state. The energy is radiated in the form of a photon with energy and frequency given by :

$$\Delta E = hf = E_i - E_f$$

(c) $r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m z e^2} = \frac{n^2}{z} a_0$ or $r_n \propto \frac{n^2}{z}$

where, $r_0 = 0.529$ Å = radius of first orbit of hydrogen atom

$$v_n = \frac{ze^2}{2\varepsilon_0 nh} = \frac{z}{n} v_1 \quad \text{or} \quad v_n \propto \frac{z}{n}$$

where, $v_1 = 2.19 \times 10^6$ m/s = speed of electron is first orbit of hydrogen atom

$$E_n = -\frac{mz^2e^4}{8\varepsilon_0^2 n^2 h^2} = \frac{z^2}{n^2} E_1 \quad \text{or} \quad E_n \propto \frac{z^2}{n^2}$$

where, $E_1 = -13.6$ eV = energy of atom in first orbit of hydrogen

(d) $E = -K = \frac{U}{2}$ and $K = \frac{|U|}{2}$

(e) $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ or $f = Rc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

(f)

Name of Series	n_1	n_2	Lies in
Lyman	1	2, 3, 4.... etc	Ultraviolet region
Balmer	2	3, 4, 5 ... etc	Visible region
Paschen	3	4, 5, 6 ... etc	Infrared region
Brackett	4	5, 6, 7 ... etc	Infrared region
Pfund	5	6, 7, 8 ... etc	Infrared region

(g) $Rhc = 13.6$ eV

(viii) **de-Broglie's Explanation of Bohr's Second Postulate Regarding Angular Momentum**

(a) Only those orbits are allowed in which standing waves (of matter wave) associated with revolving electrons can be formed.

(b) Circumference of nth allowed orbit

$$2\pi r = n\lambda \qquad (n = 1, 2, ...)$$

where, λ = wavelength of electron in that orbit.

(ix) 1 atomic mass unit = 1u = 1.660539×10^{-27} kg.

(x) In isotopes, atomic number Z (or number of protons) is same but mass number is different. They have identical chemical behaviour and are placed in the same location in the periodic table.

(xi) In isobars, mass number A (protons + neutrons) is same but atomic number is different.

(xii) In isotones, number of neutrons or $(A - Z)$ are same.

(xiii) **Size of Nucleus**

(a) $R = R_0 A^{1/3}$, where, $R_0 = 1.3$ fm $= 1.3 \times 10^{-15}$ m

(b) Density of a nucleus is of the order of 10^{17} kg/m^3. It is independent of A.

(xiv) **Radioactivity**

(a) half-life, $t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$

(b) Average life, $t_{av} = \frac{1}{\lambda}$

(c) $t_{av} = 1.44\ t_{1/2}$

(d) $t_{1/2} = (\ln 2)\ t_{av} = 0.693\ t_{av}$

(e) Number of nuclei left after time t

$$N = N_0\ e^{-\lambda t} = \left(\frac{1}{2}\right)^n N_0$$

where, N_0 = initial number of nuclei

and n = number of half-lives $= \frac{t}{t_{1/2}}$

(f) Number of nuclei decayed in time t

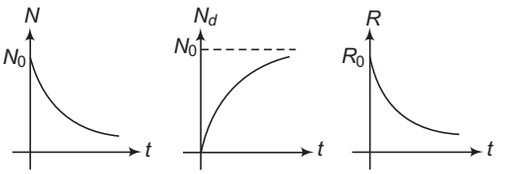
$$N_d = N_0\ (1 - e^{-\lambda t}) = N_0 - \left(\frac{1}{2}\right)^n N_0$$

(g) Activity of a radioactive substance

$$R = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

where, $R_0 = \lambda N_0$ = initial activity

(h)



(i) Units of Activity :

1 Curie = 1 Ci = 3.7×10^{10} dps

1 Rutherford = 1rd = 10^6 dps

1 Becquerel = 1 Bq = 1 dps

(xv) **Equivalence of Mass and Energy**

(a) $E = mc^2$ (b) 1 u = 931.5 MeV/c^2

(xvi) **Binding Energy and Nuclear Stability**

(a) Mass defect, $\Delta m = [Zm_p + (A - Z)\ m_n - m_{\text{nucleus}}]$

(b) Binding energy of a nucleus (in MeV)

$= [\ \Delta m$ (in atomic mass unit) $] \times 931.5 = E_b$

(c) Binding energy per nucleon $= \frac{E_b}{A}$

(d) For average mass numbers, the binding energy per nucleon is approximately 8 MeV.

(e) In a nuclear process, energy is released, if total binding energy increases. Or total binding energy of daughter nuclei is more than the total binding energy of parent nuclei

(f) Q- value = final kinetic energy – initial kinetic energy

= (sum of initial masses – sum of final masses) c^2

(g) Energy is released or total binding energy in a nuclear process increases, if binding energy per nucleon of daughter nuclei increases.

Or, the daughter nuclei lie towards the peak of the graph (between binding energy per nucleon and mass number). This happens in fission and fusion.

In fusion two or more lighter nuclei combine to form a heavy nucleus. In nuclear fission a heavy nucleus splits into two or more lighter nuclei.

Additional Examples

Example 1. *Photoemission from a metal surface is just taking place by green light. From the following select those radiations which can emit photoelectrons from the same metal.*

(a) Violet light

(b) Red light

(c) Ultraviolet radiation

(d) Infrared radiation

Sol. Energy of violet light and ultraviolet radiation is more than the energy of green light. Therefore, they can emit photoelectrons from the same metal.

Example 2. *An electron and a photon have the same momentum. Do they have the same wavelength or different ? Explain.*

Sol. Wavelength of a photon is given by

$$\lambda_1 = \frac{h}{p}$$

and wavelength of a particle (or electron) is also given by

$$\lambda_2 = \frac{h}{p}$$

Since, both have same momentum. Therefore, their wavelengths are also same.

Example 3. *In the above example do they have the same wavelength, if their energies are same.*

Sol. No. In this case they do not have the same wavelength. In case of a photon,

$$\lambda_1 = \frac{h}{p} = \frac{h}{E/c} = \frac{hc}{E}$$

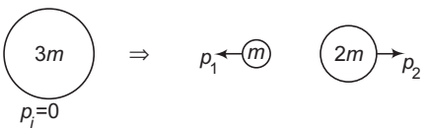
In case of an electron (a particle),

$$\lambda_2 = \frac{h}{\sqrt{2Km}}$$

We can see that $\lambda_1 \neq \lambda_2$, if $K = E$.

Example 4. *A bomb of mass* $3m$ *at rest explodes into two pieces of mass* m *and* $2m$*. What is the ratio of de-Broglie wavelengths associated with them after explosion.*

Sol. From conservation of momentum we can see that momenta of two pieces after explosion are equal and opposite.



Because,

$$p_i = p_f \quad \text{or} \quad 0 = p_1 + p_2$$

or $$p_1 = -p_2$$

or $$|p_1| = |p_2|$$

or $$p_1 = p_2$$

Now, from the expression of de-Broglie wavelength

$$\lambda = \frac{h}{p}$$

We can see that wavelengths are equal, if momenta are equal.

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = 1$$

Example 5. *Do all the electrons that absorb a photon come out as photoelectrons ?*

Sol. No, most of the electrons get scattered into the metal. Only few of them come out as photoelectrons.

Example 6. *Can a photon be deflected by an electric field ? By a magnetic field ?*

Sol. No. A photon cannot be deflected by either of the fields.

Example 7. *The threshold wavelength of a metal is* λ_0*. Light of wavelength slightly less than* λ_0 *is incident on an insulated plate made of this metal. It is found that photoelectrons are emitted for sometime and after that the emission stops. Explain.*

Sol. Photoelectrons after emission from the metal are collected over its surface. Now, these electrons repel the new electrons which wish to come out as photoelectrons. This is why photoemission is stopped after sometime.

Example 8. *Consider two different hydrogen atoms. The electron in each atom is in an excited state. Is it possible for the electrons to have different energies but the same orbital angular momentum according to the Bohr model ?*

Sol. No. This is not possible. Different energies means different value of quantum number n. So, their angular momenta will be different, as

$$L = mvr = n\left(\frac{h}{2\pi}\right)$$

If n are different, then L will be different.

Example 9. *If Bohr's quantisation postulate (angular momentum* $nh/2\pi$*) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do, we never speak its quantisation of orbits of planets around the sun?*

Sol. Angular momentum in planetory motion is very large (as compared to Planck's constant h) . For large value of angular momentum the value of n should also be very large $\left(\text{as } L = n\frac{h}{2\pi}\right)$. For such large values of n, the differences in the successive energies and angular momenta are so small that for practical purposes they may be considered continuous.

Example 10. *If neutrons exert only attractive force, why do not we have a nucleus containing neutrons only?*

Sol. Electrons need centripetal force for rotation. Neutrons (neutral particles) cannot provide this force. That is why protons (positively charged particles) are required for this purpose.

Example 11. *In β-decay, an electron (or a positron) is emitted by a nucleus. Does the remaining atom get oppositely charged.*

Sol. No. The left atom does not get oppositely charged. It exchanges the charge from atmosphere for getting stability. For example, in electron β-decay the remaining atom receives an electron from the atmosphere and it remains neutral.

Example 12. *In a typical fission reaction the nucleus is split into two middle weight nuclei of unequal masses. Which of two (heavier or lighter) has greater kinetic energy? Which one has greater linear momentum.*

Sol. From conservation of linear momentum both have the same momentum (in magnitude).

Further, the kinetic energy is given by

$$K = \frac{p^2}{2m} \quad \text{or} \quad K \propto \frac{1}{m} \qquad (\text{as } p \text{ is same})$$

Therefore, the lighter nucleus will have more kinetic energy.

Example 13. *Why heavy stable nuclei must contain more neutrons than protons ?*

Sol. In heavy nuclei, the number of protons is very large. Due to this, the electrostatic force, which is a long range force, also becomes very large. To contain the nucleons inside the nucleus, the only attractive force is nuclear force which is of course a short range force but almost same between any two pair of nucleons. This is why a nucleus will be stable, the number of neutrons should be more than the number of protons so as to provide sufficient attractive force.

Example 14. *"A fusion reaction is more energetic than a fission reaction" Comment.*

Sol. In a nuclear fusion reaction, the energy liberated per unit mass of the nuclei taking part in the reaction is many times larger than the energy liberated in a fission reaction.

Example 15. *A hydrogen like atom of atomic number z is in an excited state of quantum number* $2n$. *It can emit a maximum energy photon of* $204\,\text{eV}$. *If it makes a transition to quantum state n, a photon of energy* $40.8\,\text{eV}$ *is emitted. Find n, Z and the ground state energy* (*in* eV) *for this atom. Also, calculate the minimum energy* (*in* eV) *that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is* $13.6\ \text{eV}$.

Sol. Given, $E_{2n} - E_1 = 204\,\text{eV}$

$$\therefore \quad (13.6)Z^2\left(1 - \frac{1}{4n^2}\right) = 204 \qquad \text{...(i)}$$

$$E_{2n} - E_n = 40.8\ \text{eV}$$

$$\therefore \quad 13.6\,Z^2\left(\frac{1}{n^2} - \frac{1}{4n^2}\right) = 40.8 \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii),

we have, $n = 2$ and $Z = 4$

$$E_1 = (-13.6)\,Z^2\ \text{eV}$$
$$= (-13.6)\,(4)^2\ \text{eV}$$
$$= -217.6\ \text{eV}$$

During de-excitation, minimum energy emitted is,

$$E_{\min} = E_{2n} - E_{2n-1} = E_4 - E_3$$
$$= \frac{-217.6}{4^2} - \left(\frac{-217.6}{3^2}\right)$$
$$= 10.58\ \text{eV}$$

Example 16. *When a beam of* $10.6\ \text{eV}$ *photons of intensity* $2.0\ \text{W/m}^2$ *falls on a platinum surface of area* $1.0\times 10^{-4}\ \text{m}^2$ *and work function* $5.6\ \text{eV}$, 0.53% *of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies* (*in* eV). *Take,* $1\ \text{eV} = 1.6\times 10^{-19}\ \text{J}$.

Sol. Number of photoelectrons emitted per second

$$= \frac{(\text{Intensity})\,(\text{Area})}{(\text{Energy of each photon})} \times \frac{0.53}{100}$$
$$= \frac{(2.0)\,(1.0\times 10^{-4})}{(10.6\times 1.6\times 10^{-19})} \times \frac{0.53}{100}$$
$$= 6.25\times 10^{11}$$

Minimum kinetic energy of photoelectrons,

$$K_{\min} = 0$$

and maximum kinetic energy is,

$$K_{\max} = E - W$$
$$= (10.6 - 5.6)\ \text{eV}$$
$$= 5.0\ \text{eV}$$

Example 17. *A doubly ionised lithium atom is hydrogen like with atomic number* 3*. Find the wavelength of the radiation required to excite the electron in* Li^{2+} *from the first to the third Bohr orbit. The ionisation energy of the hydrogen atom is* 13.6 eV.

Sol. $E_n = -\frac{Z^2}{n^2}(13.6 \text{ eV})$

By putting $Z = 3$, we have

$$E_n = -\frac{122.4}{n^2} \text{ eV}$$

$$E_1 = -\frac{122.4}{(1)^2}$$

$$= -122.4 \text{ eV}$$

and $$E_3 = -\frac{122.4}{(3)^2} = -13.6 \text{ eV}$$

$$\therefore \quad \Delta E = E_3 - E_1 = 108.8 \text{ eV}$$

The corresponding wavelength is

$$\lambda = \frac{12375}{\Delta E \text{ (in eV)}} \text{ Å}$$

$$= \frac{12375}{108.8} \text{ Å}$$

$$= 113.74 \text{ Å}$$

Example 18. *The wavelength of the first line of Lyman series for hydrogen is identical to that of the second line of Balmer series for some hydrogen like ion X. Calculate energies of the first four levels, of X.*

Sol. Wavelength of the first line of Lyman series for hydrogen atom will be given by the equation

$$\frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4} \quad \text{...(i)}$$

The wavelength of second Balmer line for hydrogen like ion X is

$$\frac{1}{\lambda_2} = RZ^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3RZ^2}{16} \quad \text{...(ii)}$$

Given that $$\lambda_1 = \lambda_2 \quad \text{or} \quad \frac{1}{\lambda_1} = \frac{1}{\lambda_2}$$

i.e. $$\frac{3R}{4} = \frac{3RZ^2}{16}$$

$$\therefore \quad Z = 2$$

i.e. X ion is He^+. The energies of first four levels of X are,

$$E_1 = -(13.6)Z^2 = -54.4 \text{ eV}$$

$$E_2 = \frac{E_1}{(2)^2} = -13.6 \text{ eV}$$

$$E_3 = \frac{E_1}{(3)^2} = -6.04 \text{ eV}$$

and $$E_4 = \frac{E_1}{(4)^2} = -3.4 \text{ eV}$$

Example 19. *In the fusion reaction* ${}_1^2H + {}_1^2H \longrightarrow {}_2^3He + {}_0^1n$*, the masses of deuteron, helium and neutron expressed in* amu *are* 2.015, 3.017 *and* 1.009 *respectively. If* 1 *kg of deuterium undergoes complete fusion, find the amount of total energy released.*
1 *amu* $\equiv 931.5$ *MeV*$/c^2$.

Sol. $\Delta m = 2\,(2.015) - (3.017 + 1.009) = 0.004$ amu

$\therefore$ Energy released = (30.004×931.5) MeV = 3.726MeV

Energy released per deuteron

$$= \frac{3.726}{2} = 1.863 \text{ MeV}$$

Number of deuterons in 1 kg

$$= \frac{6.02 \times 10^{26}}{2} = 3.01 \times 10^{26}$$

$\therefore$ Energy released per kg of deuterium fusion

$$= (3.01 \times 10^{26} \times 1.863) = 5.6 \times 10^{26} \text{ MeV}$$

$$\approx 9.0 \times 10^{13} \text{ J}$$

Example 20. *In the chemical analysis of a rock the mass ratio of two radioactive isotopes is found to be* 100 : 1*. The mean-lives of the two isotopes are* 4×10^9 *yr and* 2×10^9 *yr respectively. If it is assumed that at the time of formation the atoms of both the isotopes were in equal proportion, calculate the age of the rock. Ratio of the atomic weights of the two isotopes is* 1.02 : 1.

Sol. At the time of observation ($t = t$),

$$\frac{m_1}{m_2} = \frac{100}{1} \quad \text{(given)}$$

Further it is given that $\frac{A_1}{A_2} = \frac{1.02}{1}$

Number of atoms $N = \frac{m}{A}$

$$\therefore \quad \frac{N_1}{N_2} = \frac{m_1}{m_2} \times \frac{A_2}{A_1} = \frac{100}{1.02} \quad \text{...(i)}$$

Let N_0 be the number of atoms of both the isotopes at the time of formation, then

$$\frac{N_1}{N_2} = \frac{N_0\, e^{-\lambda_1 t}}{N_0\, e^{-\lambda_2 t}} = e^{(\lambda_2 - \lambda_1)t} \quad \text{...(ii)}$$

Equating Eqs. (i) and (ii), we have

$$e^{(\lambda_2 - \lambda_1)t} = \frac{100}{1.02}$$

or $$(\lambda_2 - \lambda_1)t = \ln(100) - \ln(1.02)$$

$$\therefore \quad t = \frac{\ln(100) - \ln(1.02)}{\left(\frac{1}{2 \times 10^9} - \frac{1}{4 \times 10^9}\right)}$$

Substituting the values, we have

$$t = 1.834 \times 10^{10} \text{ yr}$$

Example 21. *The disintegration rate of a certain radioactive sample at any instant is* 4750 disintegrations per minute. 5 min *later the rate becomes* 2700 per minute. *Calculate*
(a) decay constant and
(b) half-life of the sample.

Sol. (a) From the relation

$$R = R_0\, e^{-\lambda t} \qquad \text{(Here, } R = \text{activity of sample)}$$

Substituting the values, we have

$$2700 = 4750\, e^{-5\lambda}$$

$$\therefore \quad 5\lambda = \ln\left(\frac{4750}{2700}\right)$$

$$= 0.56$$

$$\therefore \quad \lambda = 0.113\ \text{min}^{-1}$$

(b) Half-life of the sample,

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.113} = 6.132\ \text{min}$$

Example 22. *There is a stream of neutrons with a kinetic energy of* 0.0327*eV. If the half-life of neutrons be* 700 s, *what fraction of neutrons will decay before they travel a distance of* 10 *m? Mass of neutron equal to* 1.675×10^{-27}*kg.*

Sol. Kinetic energy of neutrons is given to be 0.0327 eV. From the relation, $K = \frac{1}{2}mv^2$, we can find the speed of the neutrons.

$$\therefore \quad K = \frac{1}{2}mv^2 \quad \text{or} \quad v = \sqrt{\frac{2K}{m}}$$

$$= \sqrt{\frac{2\times 0.0327\times 1.6\times 10^{-19}}{1.675\times 10^{-27}}}$$

$$= 2.5\times 10^{3}\ \text{m/s}$$

Time taken by the neutrons to travel a distance of 10 m with this speed,

$$t = \frac{10}{2.5\times 10^{3}}$$

$$= 4.0\times 10^{-3}\ \text{s}$$

Fraction of neutrons decayed in the given time interval,

$$= \frac{N_0\,(1 - e^{-\lambda t})}{N_0} \quad = (1 - e^{-\lambda t})$$

$$= 1 - e^{-\left(\frac{0.693}{700}\right)(4.0\times 10^{-3})}$$

$$= 3.96\times 10^{-6}$$

Example 23. *In a neutron induced fission of* $_{92}U^{235}$ *nucleus, usable energy of* 185 *MeV is released. If a* $_{92}U^{235}$ *reactor is continuously operating it at a power level of* 100 *MW how long will it take for* 1 *kg of uranium to be consumed in this reactor?*

Sol. Total number of nuclei in 1 kg of uranium,

n = (Number of moles) × (Avogadro's number)

$$= \left(\frac{1000}{235}\right)(6.02\times 10^{23})$$

$$= 2.56\times 10^{24}$$

By the fission of one nucleus, 185 MeV energy is released. Hence, total energy released by the fission of 1 kg of uranium is,

$$E = (2.56\times 10^{24})\,(185\times 1.6\times 10^{-13})\ \text{J}$$

$$= 7.58\times 10^{13}\ \text{J}$$

Power of the power plant is given 100 MW or 10^8 J/s. Therefore, total time upto which the power plant can be run by 1 kg uranium is,

$$t = \frac{7.58\times 10^{13}}{10^{8}}$$

$$= 7.58\times 10^{5}\ \text{s}$$

Example 24. *Determine the amount of* $_{84}Po^{210}$ *(polonium) necessary to provide a source of* α*-particles of* 5 *millicurie strength, if half-life of polonium is* 138 *days.*

(Given, $1\ curie = 3.7\times 10^{10}$ *disintegration/s)*

Sol. From the relation, $R = \lambda N$, we have

$$N = \frac{R}{\lambda} \quad \text{(Here, } R = \text{activity of sample)}$$

$$= \frac{(5\times 10^{-3})\,(3.7\times 10^{+10})}{(0.693/138\times 24\times 3600)}$$

$$= 3.18\times 10^{+15}$$

Number of moles required, $n = \dfrac{3.18\times 10^{15}}{6.02\times 10^{23}}$

$$= 5.28\times 10^{-9}$$

$\therefore$ Amount of polonium required for the desired activity,

$$m = (5.28\times 10^{-9})\ (210)\ \text{g}$$

$$= 1.11\times 10^{-6}\ \text{g}$$

Example 25. *A light source, emitting three wavelengths* 5000 *Å,* 6000 Å *and* 7000 *Å, has a power of* 10^{-3}*W on a beam of diameter* 2×10^{-3} *m. The power density is distributed equally amongst the three wavelengths. The beam shines normally on a metallic surface of area on* 10^{-4} m^2 *and having a work function of* 1.9 *eV. Assuming that each photon liberates an electron, calculate the charge emitted per second from the metal surface.*

Sol. Let E_1, E_2 and E_3 be the energies (in eV) of the photons of given wavelengths. Then,

$$E_1 \ = \ \frac{12375}{5000} = 2.475\ \text{eV}$$

$$E_2 = \frac{12375}{6000} = 2.0625 \text{ eV}$$

and $$E_3 = \frac{12375}{7000} = 1.767 \text{ eV}$$

As, E_1 and E_2 are greater than the work function W and E_3 is less than it, only first two wavelengths can emit the photoelectrons. Since, the power is equally distributed amongst all wavelengths. Let n_1 and n_2 be the number of photons incident per unit time, then,

$$n_1 = \frac{\left(\frac{10^{-3}}{3}\right)\left[\frac{10^{-4}}{\frac{\pi}{4}(2\times 10^{-3})^2}\right]}{2.475\times 1.6\times 10^{-19}} = 2.67\times 10^{16}$$

and $$n_2 = \frac{\left(\frac{10^{-3}}{3}\right)\left[\frac{10^{-4}}{\frac{\pi}{4}(2\times 10^{-3})^2}\right]}{2.0625\times 1.6\times 10^{-19}} = 3.21\times 10^{16}$$

Since, one photon can emit one electron. Hence, total charge emitted per second will be,

$$q = (n_1 + n_2)\, e$$
$$= (2.67 + 3.21)\times 10^{16} \times 1.6\times 10^{-19}$$
$$= 9.4\times 10^{-3} \text{ C/s}$$

Example 26. *Monochromatic radiation of wave length λ is incident on a hydrogen sample in ground state. Hydrogen atoms absorb a fraction of light and subsequently emit radiation of six different wavelengths. Find the value of λ.*

Sol. From nth energy state a total of $\frac{n(n-1)}{2}$ lines are emitted.

$$\therefore \quad \frac{n(n-1)}{2} = 6$$

or $$n = 4$$

Further, $$E_n = \frac{E_1}{n^2}$$

$$\therefore \quad E_4 = \frac{-13.6}{(4)^2} = -0.85 \text{ eV}$$

The energy needed to take a hydrogen atom from its ground state to $n = 4$ is,

$$\Delta E = E_4 - E_1$$
$$= -0.85 - (13.6) = 12.75 \text{ eV}$$

The corresponding wavelength of light,

$$\lambda = \frac{12375}{\Delta E} \text{ Å}$$
$$= \frac{12375}{12.75} = 970.6 \text{ Å}$$

NCERT Selected Questions

Q 1. The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the

(a) maximum kinetic energy of the emitted electrons?

(b) stopping potential, and

(c) maximum speed of the emitted photoelectrons?

Sol. (a) Using Einstein's photoelectric equation,

$$E_{max} = h\nu - W$$
$$= 6.62\times 6\times 10^{-20} - 2.14\times 1.6\times 10^{-19}$$
$$= 39.72\times 10^{-20} - 34.24\times 10^{-20}\ \text{J}$$
$$= 5.48\times 10^{-20}\ \text{J}$$
$$= \frac{0.548\times 10^{-19}}{1.6\times 10^{-19}}\ \text{eV} \qquad (\because 1\ \text{eV} = 1.6\times 10^{-19}\ \text{J})$$
$$= 0.342\ \text{eV}$$

(b) $eV_0 = E_{max}$

or $$V_0 = \frac{E_{max}}{e}$$
$$= \frac{0.342\times 1.6\times 10^{-19}\ \text{J}}{1.6\times 10^{-19}\ \text{C}}$$
$$= 0.342\ \text{JC}^{-1}$$
$$= 0.342\ \text{V}$$

(c) $\frac{1}{2} mv_{max}^2 = E_{max}$

$$\therefore \quad v_{max} = \sqrt{\frac{2E_{max}}{m}}$$
$$= \sqrt{\frac{2\times 5.48\times 10^{-20}}{9.1\times 10^{-31}}}$$
$$= 3.45\times 10^5\ \text{ms}^{-1}$$

Q 2. The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

Sol. $E_{max} = eV_0$, we get

$$E_{max} = 1.6\times 10^{-19}\times 1.5$$
$$= 2.4\times 10^{-19}\ \text{J}$$

Q 3. The energy flux of sunlight reaching the surface of the earth is $1.388\times 10^3\ \text{W/m}^2$. How many photons (nearly) per square metre are incident on the earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

Sol. Energy flux = Energy of photons from reaching the earth per second per unit area

$$\phi = 1.388\times 10^3\ \text{W/m}^2$$
$$= 1.388\times 10^3\ \text{Js}^{-1}\ \text{m}^{-2}$$

Let n = number of photons incident on the earth per sec per m^2 and E' = energy of each photon $= \frac{hc}{\lambda}$

If E be the energy of all photons reaching the surface of the earth per sec per unit area, then

$$E = nE' = \frac{nhc}{\lambda}$$

$$\therefore \quad \frac{nhc}{\lambda} = \phi$$

or $$n = \frac{\phi}{\left(\frac{hc}{\lambda}\right)}$$
$$= \frac{1.388\times 10^3}{hc}\times \lambda$$
$$= \frac{1.388\times 10^3\times 550\times 10^{-9}}{6.62\times 10^{-34}\times 3\times 10^8}$$
$$= 3.85\times 10^{21}$$

Q 4. In an experiment on photoelectric effect, the slope of the cut-off voltage *versus* frequency of incident light is found to be 4.12×10^{-15} Vs. Calculate the value of Planck's constant.

Sol. $eV_0 = h\nu - W$

$$\therefore \quad V_0 = \left(\frac{h}{e}\right)\nu - \frac{W}{e}$$

$$\therefore \quad \text{Slope} = \frac{h}{e}$$

or $$4.12\times 10^{-15} = \frac{h}{e}$$

$$\therefore \quad h = 4.12\times 10^{-15}\times e$$
$$= 4.12\times 10^{-15}\times 1.6\times 10^{-19}\ \text{Js}^{-1}$$
$$= 6.592\times 10^{-34}\ \text{Js}^{-1}$$

Q 5. A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm.

(a) What is the energy per photon associated with the sodium light?

(b) At what rate are the photons delivered to the sphere?

Sol. (a) Using the formula, $E = \frac{hc}{\lambda}$, we get

$$E = \frac{6.62\times 10^{-34}\times 3\times 10^8}{589\times 10^{-9}}$$
$$= 3.38\times 10^{-19}\ \text{J}$$
$$= \frac{3.38\times 10^{-19}}{1.6\times 10^{-19}} = 2.1\ \text{eV}$$

(b) Let n = rate at which the photons are delievered to the sphere.

$$n = \frac{\text{Energy radiated per second}}{\text{Energy of each photon}} = \frac{P}{E}$$

$$= \frac{100}{3.38 \times 10^{-19}}$$

$$= 2.96 \times 10^{20} \text{ photon / s}$$

Q 6. The threshold frequency for a certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.

Sol. Using the relation, $h\nu - h\nu_0 = eV_0$, we get

$$V_0 = h\left(\frac{\nu - \nu_0}{e}\right)$$

$$= 6.62 \times 10^{-34} \times \left(\frac{8.2 \times 10^{14} - 3.3 \times 10^{14}}{1.6 \times 10^{-19}}\right)$$

$$= 2.03 \text{ V}$$

Q 7. The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

Sol. Here, W = Work function of certain metal = 4.2 eV

$$= 4.2 \times 1.6 \times 10^{-19} \text{ J}$$

$$= 6.72 \times 10^{-19} \text{ J}$$

If E be the energy of the photon of incident light, then

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}} \text{ J}$$

$$= 6.0 \times 10^{-19} \text{ J}$$

As $E < W$, so photoelectric emission will not take place.

Q 8. Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^5 m / s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

Sol. Using Einstein's photoelectric equation,

$$h\nu - h\nu_0 = \frac{1}{2} m v_{\max}^2, \text{ we get}$$

$$h\nu_0 = h\nu - \frac{1}{2} m v_{\max}^2$$

$$\therefore \quad \nu_0 = \nu - \frac{m v_{\max}^2}{2h}$$

$$= 7.21 \times 10^{14} - \frac{9.1 \times 10^{-31} \times (6.0 \times 10^5)^2}{2 \times 6.62 \times 10^{-34}}$$

$$= 7.21 \times 10^{14} - 2.47 \times 10^{14}$$

$$= 4.74 \times 10^{14} \text{ Hz}$$

Q 9. Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the cathode is made.

Sol. $\therefore$ Energy of incident photon,

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{488 \times 10^{-9}}$$

$$= 4.08 \times 10^{-19} \text{ J}$$

Using formula, $eV_0 = E - W$, we get

$$W = E - eV_0$$

$$= 4.08 \times 10^{-19} \text{ J} - 1.6 \times 10^{-19} \times 0.38$$

$$= (4.08 - 0.608) \times 10^{-19} \text{ J}$$

$$= 3.472 \times 10^{-19} \text{ J}$$

$$= \frac{3.472 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 2.17 \text{ eV}$$

Q 10. What is the

(a) momentum

(b) speed, and

(c) de-Broglie wavelength of an electron with kinetic energy of 120 eV?

Sol. (a) Using the realtion, $E = \frac{p^2}{2m}$, we get

$$p = \sqrt{2mE}$$

$$= \sqrt{2 \times 9.1 \times 10^{-31} \times 120 \times 1.6 \times 10^{-19}}$$

$$= 5.91 \times 10^{-24} \text{ kg - ms}^{-1}$$

(b) $v = \frac{p}{m} = \frac{5.91 \times 10^{-24}}{9.1 \times 10^{-31}}$

$$= 6.5 \times 10^6 \text{ ms}^{-1}$$

(c) de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{6.62 \times 10^{-34}}{5.91 \times 10^{-24}}$$

$$= 1.12 \times 10^{-10} \text{ m}$$

Q 11. The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which

(a) an electron, and

(b) a neutron, would have the same de-Broglie wavelength.

Sol. (a) Let E_1 = KE of electron

Using the relation,

$$\lambda = \frac{h}{\sqrt{2m_e E_1}}, \text{ we get}$$

$$\lambda^2 = \frac{h^2}{2m_e E},$$

$$\therefore \quad E_1 = \frac{h^2}{2m_e\lambda^2}$$

$$= \frac{(6.62\times10^{-34})^2}{2\times9.1\times10^{-31}\times(589\times10^{-9})^2}$$

$$= 6.94\times10^{-25}\text{ J}$$

$$= \frac{6.94\times10^{-25}}{1.6\times10^{-19}}\text{ eV}$$

$$= 4.34\times10^{-6}\text{ eV}$$

(b) E_2 = KE of neutron

$$E_2 = \frac{h^2}{2m_n\lambda^2}$$

$$= \frac{(6.62\times10^{-34})^2}{2\times1.67\times10^{-27}\times(589\times10^{-9})^2}$$

$$= 3.782\times10^{-28}\text{ J}$$

$$= \frac{3.782\times10^{-28}}{1.6\times10^{-19}}\text{ eV}$$

$$= 2.36\times10^{-9}\text{ eV}$$

Q 12. What is the de-Broglie wavelength of
(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s.
(b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and
(c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

Sol. (a) Using formula, $\lambda = \frac{h}{mv}$, we get

$$\lambda_b = \frac{h}{m_b v_b}$$

$$= \frac{6.62\times10^{-34}}{0.040\times1000}$$

$$= 1.655\times10^{-33}\text{ m}$$

(b) Here, m = 0.060 kg

$$v = 1.0\text{ ms}^{-1}$$

$$\therefore \quad \lambda = \frac{6.62\times10^{-34}}{60\times10^{-3}\times1.0}$$

$$= 1.1\times10^{-32}\text{ m}$$

(c) Here, $m = 1.0\times10^{-9}$ kg

v = drift speed of dust particle

$= 2.2\text{ ms}^{-1}$

λ = de-Broglie wavelength

$$p = mv = 1.0\times10^{-9}\times2.2$$

$$= 2.2\times10^{-9}\text{ kg - ms}^{-1}$$

$$\therefore \quad \lambda = \frac{h}{p} = \frac{6.62\times10^{-34}}{2.2\times10^{-9}}$$

$$= 3.01\times10^{-25}\text{ m}$$

Q 13. An electron and a photon each have a wavelength of 1.00 nm. Find
(a) their momenta
(b) the energy of the photon, and
(c) the kinetic energy of electron.

Sol. (a) (i) Using the formula, $\lambda = \frac{h}{p}$, we get

Let p_e = momentum of electron

$$p_e = \frac{h}{\lambda_e} = \frac{6.63\times10^{-34}}{10^{-9}}$$

$$= 6.63\times10^{-25}\text{ kg - ms}^{-1}$$

(ii) Let p_p = momentum of photon

$$\therefore \quad p_p = \frac{h}{\lambda_p} = \frac{6.63\times10^{-34}}{10^{-9}}$$

$$= 6.63\times10^{-25}\text{ kg - ms}^{-1}$$

(b) Let E_p = energy of photon

Using the formula, $E = \frac{hc}{\lambda}$, we get

$$E_p = \frac{hc}{\lambda_p} = \frac{6.63\times10^{-34}\times3\times10^{8}}{10^{-9}}$$

$$= 19.89\times10^{-17}\text{ J}$$

$$= \frac{19.89\times10^{-17}}{1.6\times10^{-19}}\text{ eV}$$

$$= 12.43\times10^{2}\text{ eV}$$

$$= 1.243\times10^{3}\text{ eV}$$

(c) E = KE of electron

Using the formula, $\lambda = \frac{h}{\sqrt{2mE}}$

or $\lambda^2 = \frac{h^2}{2m_e mE}$, we get

$$E = \frac{h^2}{2m_e\, m\lambda^2}$$

$$= \frac{(6.63\times10^{-34})^2}{2\times9.1\times10^{-31}\times(10^{-9})^2}$$

$$= 2.415\times10^{-19}\text{ J}$$

$$= \frac{2.415\times10^{-19}}{1.6\times10^{-19}}\text{ eV} = 1.51\text{ eV}$$

Q 14. (a) For what kinetic energy of a neutron will the associated de-Broglie wavelength be 1.40×10^{-10} m?

(b) Also, find the de-Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of $\frac{3}{2}KT$ at 300 K.

Sol. (a) Using the formula, $\lambda = \frac{h}{\sqrt{2mE}}$, we get

$$\lambda^2 = \frac{h^2}{2mE}$$

or $$E = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.62\times 10^{-34})^2}{2\times 1.67\times 10^{-27}\times (1.40\times 10^{-10})^2}$$

$$= 6.69\times 10^{-21}\ \text{J}$$

$$= \frac{6.69\times 10^{-21}}{1.6\times 10^{-19}}$$

$$= 4.181\times 10^{-2}\ \text{eV}$$

(b) Here, T = absolute temperature = 300 K

k = Boltzmann's constant

$= 1.38\times 10^{-23}\ \text{JK}^{-1}$

E = Kinetic energy of neutron

$$= \frac{3}{2}kT$$

$$= \frac{3}{2}\times 1.38\times 10^{-23}\times 300$$

$$= 6.21\times 10^{-21}\ \text{J}$$

Using the relation,

$\lambda = \frac{h}{\sqrt{2mE}}$, we get

$$\lambda = \frac{6.62\times 10^{-34}}{\sqrt{2\times 1.67\times 10^{-27}\times 6.21\times 10^{-21}}}$$

$$= 1.455\times 10^{-10}\ \text{m}$$

$$= 1.455\ \text{Å}$$

Q 15. Show that the wavelength of electromagnetic radiation is equal to the de-Broglie wavelength of its quantum.

Sol. Let ν, λ' be the frequency and wavelength of the photon having momentum p

$$\therefore \quad p = \frac{E}{c} = \frac{h\nu}{c}$$

where, c = speed of photon

= speed of electromagnetic radiation

$\therefore$ de-Broglie wavelength λ' of the photon, i.e. quantum of electromagnetic wave is given by

$$\lambda' = \frac{h}{p} = \frac{h}{\left(\frac{h\nu}{c}\right)} = \frac{c}{\nu} \qquad \ldots(i)$$

Also let λ be the wavelength of electromagnetic radiation of frequency ν.

$$\therefore \quad \lambda = \frac{c}{\nu} \qquad \ldots(ii)$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\lambda = \lambda' = \frac{c}{\nu}$$

Q 16. What is the de-Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean-square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)

Sol. Let v = rms speed of N_2 molecule at 300 K.

or $$v = \sqrt{\frac{3kT}{m}}$$

or $$v = \sqrt{\frac{3\times 1.38\times 10^{-23}\times 300}{28.0152\times 1.67\times 10^{-27}}}$$

$$= 5.15\times 10^2\ \text{ms}^{-1}$$

Using the formula, $\lambda = \frac{h}{mv}$, we get

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.62\times 10^{-34}}{28.0152\times 1.67\times 10^{-27}\times 5.15\times 10^2}$$

$$= 2.75\times 10^{-11}\ \text{m}$$

Q 17. In an accelerator experiment on high energy collisions of electrons with positrons, a certain event is interpreted as anihilation of an electron-positron pair of total energy 10.2 BeV into two γ-rays of equal energy. What is the wavelength associated with each γ-ray? (1 BeV = 10^9 eV)

Sol. Here, energy of 2γ-rays = 10.2 BeV = 10.2×10^9 eV

$\therefore$ Energy of each γ-ray is given by

$$E = \frac{1}{2}(10.2\times 10^9\ \text{eV})$$

$$= \frac{1}{2}\times 10.2\times 10^9\times 1.6\times 10^{-19}\ \text{J}$$

$$= 8.16\times 10^{-10}\ \text{J}$$

Using the formula, $E = \frac{hc}{\lambda}$, we get

$$\lambda = \frac{hc}{E} = \frac{6.63\times 10^{-34}\times 3\times 10^8}{8.16\times 10^{-10}}$$

$$= 2.44\times 10^{-16}\ \text{m}$$

Q 18. Choose the correct alternative from clues given at end of the each statement :

(a) The size of the atom in Thomson's model is …… the atomic size in Rutherford's model. (much greater than/no different from/much less than)

(b) In the ground state of …… electrons are in stable equilibrium, while in …… electrons always experience a net force. (Thomson's model/Rutherford's model)

(c) A classical atom based on …… is assumed to collapse.

(Thomson's model/Rutherford's model)

(d) An atom has a nearly continuous mass distribution in a …… but has a highly non-uniform mass distribution in …… (Thomson's model/Rutherford's model)

(e) The positively charged part of the atom possesses most of the mass in …… (Rutherford's model/both the models)

Sol. (a) No different from

(b) Thomson's model, Rutherford's model

(c) Rutherford's model

(d) Thomson's model, Rutherford's model

(e) Both the models

Q 19. Suppose you are given a chance to repeat the α-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperature below 14 K). What results do you expect?

Sol. The nucleus of a hydrogen atom is a proton. The mass of it is 1.67×10^{-27} kg, whereas the mass of an incident α-particle is 6.64×10^{-27} kg. Because the scattering particle is more massive than the target nuclei (proton), the α-particle won't bounce back in even in a head-on collision. It is similar to football colliding with a tennis ball at rest. Thus, there would be no large angle scattering.

Q 20. What is the shortest wavelength present in the Paschen series of spectral lines?

Sol. For Paschen series, $n_1 = 3$, $n_2 = \infty$ for shortest wavelength. If λ_{min} be the minimum wavelength in this series of spectral lines, then

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) = \frac{R}{9}$$

or $$\lambda_{min} = \frac{9}{R}$$

$$= \frac{9}{1.097\times10^{7}} \text{ m}$$

$$= 8204.2\times10^{-10} \text{ m}$$

Q 21. A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of photon.

Sol. If $E = h\nu$ be the energy absorbed, then

$$h\nu = E_4 - E_1$$

Now $$E_4 = \frac{-13.6}{4^2}$$

$$= \frac{-13.6}{16} = -0.85 \text{ eV} \quad \ldots(i)$$

and $$E_1 = -13.6 \text{ eV} \quad \ldots(ii)$$

∴From Eqs. (i) and (ii), we get

$$h\nu = -0.85 - (-13.6)$$

$$= 12.75 \text{ eV}$$

$$= 12.75\times1.6\times10^{-19} \text{ J}$$

$$\nu = \frac{12.75\times1.6\times10^{-19}}{6.62\times10^{-34}}$$

$$= 3.08\times10^{15} \text{ Hz} \quad .$$

Using the relation

∴ $c = \nu\lambda$, we get

$$\lambda = \frac{c}{\nu} = \frac{3\times10^{8}}{3.08\times10^{15}}$$

$$= 974.02\times10^{-10} \text{ m}$$

$$\approx 974 \text{ Å}$$

Q 22. A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom make a transition from the upper level to the lower level?

Sol. Here, $\Delta E = 2.3 \text{ eV} = 2.3\times1.6\times10^{-19}$ J

$$h = 6.62\times10^{-34} \text{ Js}$$

Using the relation,

$$h\nu = \Delta E = E_2 - E_1$$

or $$\nu = \frac{\Delta E}{h}$$

$$= \frac{2.3\times1.6\times10^{-19}}{6.62\times10^{-34}}$$

$$= 0.5559\times10^{15} \text{ Hz}$$

$$\approx 5.6\times10^{14} \text{ Hz}$$

Q 23. The ground state energy of hydrogen atom is – 13.6 eV. What is the kinetic and potential energies of the electron in this state?

Sol. Kinetic energy = – Total energy

$$= 13.6 \text{ eV}$$

Potential energy = + 2 (Total energy)

$$= -27.2 \text{ eV}$$

Q 24. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Sol. Using the relation, $E = \frac{hc}{\lambda}$, we get

$$\lambda = \frac{hc}{E} = \frac{6.62\times10^{-34}\times3\times10^{8}}{12.5\times1.6\times10^{-19}}$$

$$= 0.993\times10^{-7} \text{ m}$$

$$= 993\times10^{-10} \text{ m}$$

$$= 993 \text{ Å}$$

This wavelength falls in the range of Lyman series (912 Å to 1216 Å), thus we conclude that Lyman series is emitted.

Q 25. The total energy of an electron in the first excited state of the hydrogen atom is about – 3.4. eV.

(a) What is the kinetic energy of the electron in this state?

(b) What is the potential energy of the electron in this state?

(c) Which of the answers above would change if the choice of the zero of potential energy is changed?

Sol. (a) $\therefore \text{KE} = -E = -(-3.4) = +3.4$ eV

(b) $\text{PE} = -2\,(\text{KE}) = -2 \times 3.4 = -6.8$ eV

(c) If zero of the potential energy is changed, KE does not change and continues to be + 3.4 eV as it is independent of the choice of the zero of potential energy.

But the potential energy and total energy of the state would change with the choice of the zero of the potential energy.

Q 26. Obtain the binding energy in MeV of a nitrogen nucleus ($^{14}_{7}\text{N}$), given $m(^{14}_{7}\text{N}) = 14.00307$ u.

Sol. $^{14}_{7}\text{N}$ is made up of 7 protons and 7 neutrons.

$\therefore$ Mass of nucleons

$= 7\,m_{\text{H}} + 7\,m_n$

$= 7 \times 1.00783\text{ u} + 7 \times 1.00867\text{ u}$

$= 7.05481 + 7.06069$

$= 14.1155$ u

Mass defect, Δm = mass of nucleons – mass of nucleus

$= 14.11550\text{ u} - 14.00307\text{ u}$

$= 0.11243$ u

$\therefore \quad \text{BE} = \Delta m \times 931$ MeV

$= 0.11243 \times 931$ MeV

$= 104.67$ MeV

Q 27. The half-life of $^{90}_{38}\text{Sr}$ is 28 yr. What is the disintegration rate of 15 mg of this isotope?

Sol. $T_{1/2} = 28\text{ yr} = 88.3 \times 10^7$ s

Number of atoms in 15 mg is given by

$$N = \frac{6.023 \times 10^{23}}{90} \times 15 \times 10^{-3}$$

$$= 1.004 \times 10^{20}$$

$$\lambda = \frac{0.693}{T_{1/2}}, \text{ we get}$$

$$\lambda = \frac{0.693}{88.3 \times 10^7}\ \text{s}^{-1}$$

Using the relation, $R = \lambda N$, we get

$$R = \frac{0.693}{88.3 \times 10^7} \times 1.004 \times 10^{20}$$

$$= 7.879 \times 10^{10}\ \text{disintegration/s}$$

Q 28. Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}\text{Au}$ and the silver isotope $^{107}_{47}\text{Ag}$.

Sol. $R = R_0\, A^{1/3}$

where, A is the mass number of the nucleus, R_0 is an empirical constant.

Let R_1, A_1 be the nuclear radius and mass number of $^{197}_{79}\text{Au}$ isotope.

Also, let R_2, A_2 be the nuclear radius and mass number of $^{107}_{47}\text{Ag}$ isotope.

$$R_1 = A_0\,(197)^{1/3}$$

and $$R_2 = A_0\,(107)^{1/3}$$

$\therefore$ $$\frac{R_1}{R_2} = \frac{(197)^{1/3}}{(107)^{1/3}}$$

$$= \left(\frac{197}{107}\right)^{1/3} = (1.84)^{1/3}$$

$$= 1.23$$

Q 29. The fission properties of $^{239}_{94}\text{Pu}$ are very similar to those of $^{235}_{92}\text{U}$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $^{239}_{94}\text{Pu}$ undergo fission?

Sol. Here, average energy released per fission of

$^{239}_{94}\text{Pu} = 180$ MeV

Amount of $^{239}_{94}\text{Pu} = 1\text{ kg} = 1000$ g

We know that 239 g of $^{239}_{94}\text{Pu}$ contains $= 6.023 \times 10^{23}$ fissionable nuclei.

Number of fissionable nuclei in 1 kg or 1,000 g of $^{239}_{94}\text{Pu}$

$$= \frac{6.023 \times 10^{23}}{239} \times 1{,}000$$

$$= 25.2 \times 10^{23}\ \text{Nuclei}$$

$\therefore$ Total energy released in fission of 1 kg of Pu

$= 180 \times 25.2 \times 10^{23}$ MeV

$= 4.536 \times 10^{36}$ MeV

Q 30. How long can an electric lamp of 100 W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$$^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}_{0}n^{1} + 3.27\text{ MeV}$$

Sol. From the given equation it is clear that 2 nuclei of $^{2}_{1}\text{H}$ (deuterium) combine to give 3.27 MeV of energy.

Now, 2 g of $^{2}_{1}\text{H}$ contains 6.023×10^{23} nuclei.

$\therefore$ Number of $^{2}_{1}\text{H}$ nuclei in 2 kg of deuterium

$$= \frac{6.023 \times 10^{23}}{2 \times 10^{-3}} \times 2$$

$$= 6.023 \times 10^{26}\ \text{nuclei}$$

Energy released in fusion of two nuclei of $^{2}_{1}\text{H} = 3.27$ MeV

$\therefore$ Energy released in fusion of 6.023×10^{26} nuclei of it is given by

$$E = \frac{3.27 \times 6.023}{2} \times 10^{26} \text{ MeV}$$

or $$E = \frac{3.27 \times 6.023}{2} \times 10^{26} \times 1.6 \times 10^{-13} \text{ J}$$

$$= 15.76 \times 10^{13} \text{ J}$$

Let t = time for which the electric lamp can glow due to this energy.

$\therefore$ Energy consumed by the lamp, $E' = Pt = 100\ t$

$\therefore$ According to law of conservation of energy

$$E' = E$$

or $$100t = 15.76 \times 10^{13}$$

or $$t = \frac{15.76 \times 10^{13}}{100} = 15.76 \times 10^{11} \text{ s}$$

$$= \frac{15.76 \times 10^{11}}{365 \times 24 \times 60 \times 60} = 4.99 \times 10^{4} \text{ yr}$$

Q 31. From the relation $R = R_0\ A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Sol. Mass of the nucleus of the atom

$$= A \text{ amu}$$

$$= A \times 1.66 \times 10^{-27} \text{ kg} \quad \ldots(i)$$

If V be the volume of the nucleus, then

$$V = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi\ (R_0\ A^{1/3})^3$$

$$= \frac{4}{3} \pi\ R_0^3\ A \quad \ldots(ii)$$

Using $$R_0 = 1.1 \times 10^{-15} \text{ m}$$

$$V = \frac{4}{3} \pi\ (1.1 \times 10^{-15})^3 \text{ A} \quad \ldots(iii)$$

If ρ be the density of the nucleus, then

$$\rho = \frac{\text{mass of nucleus}}{\text{Volume of nucleus}}$$

$$= \frac{A \times 1.66 \times 10^{-27}}{\frac{4}{3} \pi\ (1.1 \times 10^{-15})^3}$$

$$= 2.97 \times 10^{17} \text{ kg - m}^{-3} \quad \ldots(iv)$$

Thus from Eq. (iv), we see that ρ is independent of A, hence we conclude that ρ is nearly constant for all nuclei.

Objective Problems

[Level 1]

Properties of a Photon

1. The momentum of a photon is 3. 3×10^{-29} kg-m/s. Its frequency will be
(a) 3×10^{3} Hz (b) 6×10^{3} Hz
(c) 7.5×10^{12} Hz (d) 1.5×10^{13} Hz

2. The momentum of a photon is 2×10^{-16} g-cm/s. Its energy is
(a) 0.61×10^{-26} erg (b) 2×10^{-26} erg
(c) 6×10^{-6} erg (d) 6×10^{-8} erg

3. The rest mass of photon is
(a) 0 (b) ∞
(c) between 0 and ∞ (d) equal to that of an electron

4. The momentum of the photon of wavelength 5000Å will be
(a) 1.3×10^{-27} kg-m/s (b) 1.3×10^{-28} kg-m/s
(c) 4×10^{-29} kg-m/s (d) 4×10^{-18} kg-m/s

5. A photon in motion has a mass
(a) $c/h\nu$ (b) ν/h
(c) $h\nu$ (d) $h\nu/c^2$

6. The approximate wavelength of a photon of energy 2.48 eV
(a) 500 Å (b) 5000 Å
(c) 2000 Å (d) 1000 Å

7. The number of photons of wavelength 540 nm emitted per second by an electric bulb of power 100 W is (taking, $h = 6\times10^{-34}$ J-s)
(a) 100 (b) 1000
(c) 3×10^{20} (d) 3×10^{18}

8. If the energy of the photon is increased by a factor of 4, then its momentum
(a) does not change (b) decreases by a factor of 4
(c) increases by a factor of 4 (d) increases by a factor of 2

9. Which of one is correct?
(a) $E^2 = p^2c^2$ (b) $E^2 = p^2c$
(c) $E^2 = pc^2$ (d) $E^2 = p^2/c^2$

10. The energy of incident photons corresponding to maximum wavelength of visible light is
(a) 3.2 eV (b) 7 eV
(c) 1.76 eV (d) 1 eV

11. The momentum of a photon is 6.6×10^{-29} kg-ms^{-1}. The frequency of the radiation (in Hz) is
(a) 3×10^{13} (b) 3×10^{8}
(c) 3×10^{15} (d) 3×10^{6}

12. The number of photons of wavelength 340 nm emitted per second by an electric bulb of power 200 W is approximately (taking, $h = 6\times10^{-34}$ J-s)
(a) 4×10^{16} (b) 4×10^{10}
(c) 3.78×10^{20} (d) 3.78×10^{18}

13. Two photons having
(a) equal wavelengths and have equal linear momenta
(b) equal energies and have equal linear momenta
(c) equal frequencies and have equal linear momenta
(d) All of the above

14. Let p and E denote the linear momentum and energy of a photon. If the wavelength is decreased, then
(a) Both p and E increase
(b) p increases and E decreases
(c) p decreases and E increases
(d) Both p and E decrease

Bohr's Theory of Atomic Model

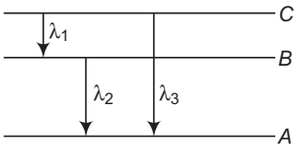
15. For the Bohr's first orbit of circumference $2\pi r$, the de-Broglie wavelength of revolving electron will be
(a) $2\pi r$ (b) πr (c) $\frac{1}{2\pi r}$ (d) $\frac{1}{4\pi r}$

16. In the nth orbit, the energy of an electron $E_n = -\frac{13.6}{n^2}$ eV for hydrogen atom. The energy required to take the electron from first orbit to second orbit will be
(a) 10.2 eV (b) 12.1 eV (c) 13.6 eV (d) 3.4 eV

17. Which one of the series of hydrogen spectrum is in the visible region?
(a) Lyman series (b) Balmer series
(c) Paschen series (d) Brackett series

18. Ionisation potential of hydrogen atom is 13.6 V. Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy 12.1 eV. The spectral lines emitted by hydrogen atoms according to Bohr's theory will be
(a) one (b) two
(c) three (d) four

19. Energy levels *A, B* and *C* of a certain atom corresponding to increasing values of energy i.e. $E_A < E_B < E_C$. If λ_1, λ_2 and λ_3 are the wavelengths of radiations corresponding to the transitions *C* to *B, B* to *A* and *C* to *A* respectively, which of the following statements is correct?



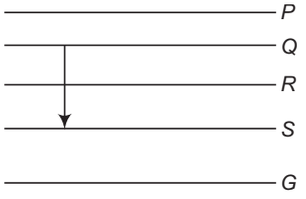
(a) $\lambda_3 = \lambda_1 + \lambda_2$ (b) $\lambda_3 = \dfrac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$

(c) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (d) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$

20. If the wavelength of the first line of the Balmer series of hydrogen atom is 6561 Å, the wavelength of the second line of the series should be

(a) 13122 Å (b) 3280 Å (c) 4860 Å (d) 2187 Å

21. Figure shows the energy levels *P, Q, R, S* and *G* of an atom where, *G* is the ground state. A red line in the emission spectrum of the atom can be obtained by an energy level change from *Q* to *S*. A blue line can be obtained by following energy level change



(a) *P* to *Q* (b) *Q* to *P* (c) *R* to *S* (d) *R* to *G*

22. An electron makes a transition from orbit $n = 4$ to the orbit $n = 2$ of a hydrogen atom. The wave number of the emitted radiations (*R* = Rydberg's constant) will be

(a) $\dfrac{16}{3R}$ (b) $\dfrac{2R}{16}$

(c) $\dfrac{3R}{16}$ (d) $\dfrac{4R}{16}$

23. The wavelength of radiation emitted is λ_0, when an electron jumps from the third to the second orbit of hydrogen atom. For the electron jump from the fourth to the second orbit of the hydrogen atom, the wavelength of radiation emitted will be

(a) $\dfrac{16}{25}\lambda_0$ (b) $\dfrac{20}{27}\lambda_0$ (c) $\dfrac{27}{20}\lambda_0$ (d) $\dfrac{25}{16}\lambda_0$

24. When an electron in hydrogen atom is excited from its fourth to fifth stationary orbit, the change in angular momentum of electron is (take, Planck's constant $h = 6.6 \times 10^{-34}$ J-s)

(a) 4.16×10^{-34} J-s (b) 3.32×10^{-34} J-s

(c) 1.05×10^{-34} J-s (d) 2.08×10^{-34} J-s

25. The concept of stationary orbits was proposed by

(a) Neil Bohr (b) J J Thomson

(c) Rutherford (d) I Newton

26. The ratio of the speed of the electrons in the ground state of hydrogen to the speed of light in vacuum is

(a) 1/2 (b) 2/137 (c) 1/137 (d) 1/237

27. The ionisation potential of H-atom is 13.6 V. When it is excited from ground state by monochromatic radiations of 970.6 Å, the number of emission lines will be (according to Bohr's theory)

(a) 10 (b) 3 (c) 6 (d) 4

28. Excitation energy of a hydrogen like ion in its first excitation state is 40.8 eV. Energy needed to remove the electron from the ion in ground state is

(a) 54.4 eV (b) 13.6 eV

(c) 40.8 eV (d) 27.2 eV

29. In Bohr's model, the atomic radius of the first orbit is r_0, then the radius of the third orbit is

(a) $r_0/9$ (b) r_0 (c) $9r_0$ (d) $3r_0$

30. The frequency of K_α, K_β and L_α, X-rays of material are γ_1, γ_2 and γ_3 respectively. Which of the following relations holds good?

(a) $\gamma_2 = \sqrt{\gamma_1\gamma_3}$ (b) $\gamma_3 = \sqrt{\gamma_1\gamma_2}$

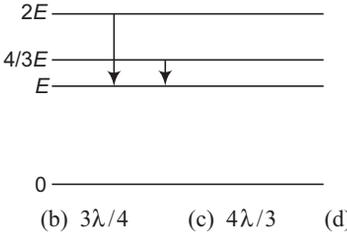
(c) $\gamma_2 = \dfrac{\gamma_1 + \gamma_3}{2}$ (d) $\gamma_2 = \gamma_1 + \gamma_3$

31. When the electron in hydrogen atom jumps from the second orbit to the first orbit, the wavelength of the radiation emitted is λ. When the electron jumps from the third to the first orbit, the wavelength of the radiation emitted is

(a) $\dfrac{9}{4}\lambda$ (b) $\dfrac{4}{9}\lambda$

(c) $\dfrac{27}{32}\lambda$ (d) $\dfrac{32}{27}\lambda$

32. The given diagram indicates, the energy levels of a certain atom. When the system moves from 2*E* level to *E*, a photon of wavelength λ is emitted. The wavelength of photon produced during its transition from 4*E*/3 level to *E* is



(a) $\lambda/3$ (b) $3\lambda/4$ (c) $4\lambda/3$ (d) 3λ

33. The ground state energy of hydrogen atom is – 13.6 eV. The kinetic energy of the electron in this state is

(a) 1.85 eV (b) 13.6 eV (c) 6.8 eV (d) 3.4 eV

34. Atomic hydrogen is excited from the ground state to the nth state. The number of lines in the emission spectrum will be

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$

(c) $\frac{(n-1)^2}{2}$ (d) $\frac{(n+1)^2}{2}$

35. The difference in angular momentum associated with the electron in the two successive orbits of hydrogen atom is

(a) $\frac{h}{\pi}$ (b) $\frac{h}{2\pi}$

(c) $\frac{h}{2}$ (d) $(n-1)\frac{h}{2\pi}$

36. The ratio of the largest to shortest wavelengths in Balmer series of hydrogen spectra is

(a) $\frac{25}{9}$ (b) $\frac{17}{6}$

(c) $\frac{9}{5}$ (d) $\frac{5}{4}$

37. In hydrogen atom, if the difference in the energy of the electron in $n = 2$ and $n = 3$ orbits is E, the ionisation energy of hydrogen atom is

(a) $13.2E$ (b) $7.2E$ (c) $5.6E$ (d) $3.2E$

38. Which of the following is not correct in Bohr model of hydrogen atom?

(a) The radius of nth orbit is proportional to n^2

(b) The total energy of electron in n th orbit is proportional to n

(c) The angular momentum of an electron in an orbit is an integral multiple of $h/2\pi$

(d) The magnitude of the potential energy of an electron in any orbit is greater than its kinetic energy

39. If ω is the speed of electron in the nth orbit hydrogen atom, then

(a) $\omega \propto n^{1/2}$ (b) $\omega \propto \frac{1}{n}$

(c) $\omega \propto \frac{1}{n^2}$ (d) $\omega \propto \frac{1}{n^3}$

40. An electron revolves round a nucleus of charge Ze. In order to excite the electron from the state $n = 2$ to $n = 3$, the energy required is 47.2 eV. Z is equal to

(a) 3 (b) 4

(c) 5 (d) 2

41. An electron with kinetic energy 5 eV is incident on a H-atom in its ground state. The collision

(a) must be elastic

(b) may be partially elastic

(c) must be completely inelastic

(d) may be completely inelastic

42. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionised Li atom is

(a) 1.51 (b) 13.6 (c) 40.8 (d) 122.4

43. The first excitation potential of hydrogen atom in the ground state is

(a) 13.6 V (b) 3.4 V

(c) 10.2 V (d) 1.89 V

44. If the wavelength of first member of Balmer series of hydrogen spectrum is 6563 Å, the wavelength of second member of Balmer series will be

(a) 1215 Å

(b) 4861 Å

(c) 6050 Å

(d) Data insufficient

45. Hydrogen atom emits blue light when it jumps from $n = 4$ level to the $n = 2$ level. Which colour of light would the atom emit when it changes from the $n = 5$ level to the $n = 2$ level?

(a) Red (b) Yellow

(c) Green (d) Violet

46. The second line in Paschen series is obtained when the electron makes transition from

(a) fourth orbit to third orbit (b) seventh orbit to third orbit

(c) sixth orbit to third orbit (d) fifth orbit to third orbit

47. The electron in a hydrogen atom makes transition from M-shell to L-shell. The ratio of magnitude of initial to final acceleration of the electron is

(a) 9 : 4 (b) 81 : 16

(c) 4 : 9 (d) 16 : 81

48. The orbital angular momentum of electron in the n_1th shell of element of atomic number Z_1 is L_1 and the same in the n_2th shell of element of atomic number Z_2 is L_2. If $L_2 > L_1$, then

(a) $n_2 > n_1$ (b) $Z_2 > Z_1$

(c) $n_2 Z_2 > n_1 Z_1$ (d) Both (a) and (b)

49. The velocity of an electron in the first orbit of H-atom is v. The velocity of an electron in the second orbit of He^+ is

(a) $2v$ (b) v

(c) $v/2$ (d) $v/4$

50. An electron moves in a circular orbit at a distance from a proton with kinetic energy E. To escape to infinity, the minimum energy which must be supplied to the electron is

(a) E (b) $2E$

(c) $0.5E$ (d) $E\sqrt{2}$

51. In a hypothetical Bohr hydrogen, the mass of the electron is doubled. The energy E_0 and radius r_0 of the first orbit will be (a_0 is the Bohr radius)

(a) $E_0 = -27.2$ eV, $r_0 = a_0/2$

(b) $E_0 = -27.2$ eV, $r_0 = a_0$

(c) $E_0 = -13.6$ eV, $r_0 = a_0/2$

(d) $E_0 = -13.6$ eV, $r_0 = a_0$

52. In which of the following transitions will the wavelength be minimum?

(a) $n = 5$ to $n = 4$ (b) $n = 4$ to $n = 3$

(c) $n = 3$ to $n = 2$ (d) $n = 2$ to $n = 1$

53. In which of the following systems will the radius of the first orbit ($n = 1$) be minimum?
(a) Hydrogen atom
(b) Deuterium atom
(c) Singly ionised helium
(d) Doubly ionised lithium

54. In which of the following systems will the wavelength corresponding to $n = 2$ to $n = 1$ be minimum?
(a) Hydrogen atom
(b) Deuterium atom
(c) Singly ionised helium
(d) Doubly ionised lithium

55. As one considers orbits with higher values of n in a hydrogen atom, the electric potential energy of the atom
(a) decreases (b) increases
(c) remains the same (d) does not increase

56. Which of the following parameters are the same for all hydrogen like atoms and ions in their ground states?
(a) Radius of the orbit
(b) Speed of the electrons
(c) Energy of the atom
(d) Orbital angular momentum of the electron

57. Ionisation energy of a hydrogen like ion A is greater than that of another hydrogen like ion B. Let r, u, E and L represent the radius of the orbit, speed of the electron, energy of the atom and orbital angular momentum of the electron respectively. In ground state
(a) $r_A > r_B$ (b) $u_A > u_B$
(c) $E_A > E_B$ (d) $L_A > L_B$

Radioactivity

58. A radioactive nucleus undergoes a series of decay according to the scheme

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic nubmer of A are 180 and 72 respectively, then what are these number for A_4 ?
(a) 172 and 69 (b) 174 and 70
(c) 176 and 69 (d) 176 and 70

59. An archaeologist analyses the wood in a prehistoric structure and finds that C^{14} (half-life = 5700 yr) to C^{12} is only one-third of that found in the cells buried plants. The age of the wood is about
(a) 5700 yr (b) 2850 yr
(c) 11400 yr (d) 22800 yr

60. Unit of radioactivity is Rutherford. Its value is
(a) 3.7×10^{10} disintegration/s
(b) 3.7×10^{6} disintegration/s
(c) 1.0×10^{10} disintegration/s
(d) 1.0×10^{6} disintegration/s

61. The counting rate observed from a radioactive source at $t = 0$ was 1600 count/s and at $t = 8$ s it was 100 count/s. The counting rate observed, as count/s at $t = 6$ s, will be
(a) 400 (b) 300
(c) 200 (d) 150

62. In the given nuclear reaction A, B, C, D, E represent

$${}_{92}U^{238} \xrightarrow{\alpha} {}_{B}Th^{A} \xrightarrow{\beta} {}_{D}Pa^{C} \xrightarrow{E} {}_{92}U^{234}$$

(a) $A = 234,\ B = 90, C = 234,\ D = 91, E = \beta$
(b) $A = 234,\ B = 90, C = 238, D = 94, E = \alpha$
(c) $A = 238,\ B = 93, C = 234,\ D = 91, E = \beta$
(d) $A = 234,\ B = 90, C = 234, D = 93, E = \alpha$

63. 1 curie is equal to
(a) 3×10^{10} disintegration/s
(b) 3.7×10^{7} disintegration/s
(c) 5×10^{7} disintegration/s
(d) 3.7×10^{10} disintegration/s

64. The half-life of a sample of a radioactive substance is 1 h. Wavelength If 8×10^{10} atoms are present at $t = 0$, then the number of atoms decayed in the duration $t = 2$ h to $t = 4$ h will be
(a) 2×10^{10} (b) 1.5×10^{10}
(c) zero (d) infinity

65. Threshold frequency for a metal is 10^{15} Hz. Light of wavelength, $\lambda = 4000$ Å falls on its surface. Which of the following statements is correct?
(a) No photoelectric emission takes place
(b) Photoelectron come out with zero speed
(c) Photoelectron come out with 10^3 m/s speed
(d) Photoelectron come out with 10^5 m/s speed

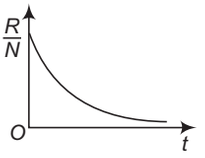
66. Half-life of a radioactive substance is 20 min. Difference between points of time when it is 33% disintegrated and 67% disintegrated is approximately
(a) 10 min (b) 20 min
(c) 30 min (d) 40 min

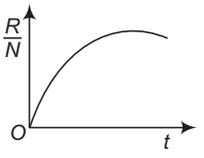
67. A and B are two radioactive substances whose half-lives are 1 and 2 years respectively. Initially 10 g of A and 1 g of B is taken. The time (approximate) after which they will have same quantity remaining is
(a) 6.62 yr (b) 5 yr
(c) 3.2 yr (d) 7 yr

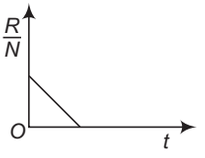
68. Half-life of a radioactive substance is 20 min. The time between 20% and 80% decay will be
(a) 20 min (b) 40 min
(c) 30 min (d) 25 min

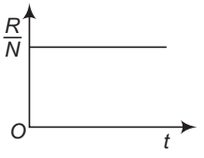
69. After 280 days, the activity of a radioactive sample is 6000 dps. The activity reduces to 3000 dps after another 140 days. The initial activity of the sample in dps is
(a) 6000 (b) 9000
(c) 3000 (d) 24000

70. A radioactive sample has N_0 active atoms at $t = 0$. If the rate of disintegration at any time is R and the number of atoms is N, then the ratio R/N varies with time as

(a) 

(b) 

(c) 

(d) 

71. If 10% of radioactive material decays in 5 days, then the amount of the original material left after 20 days is approximately

(a) 60% (b) 65%
(c) 70% (d) 75%

72. In a sample of radioactive material, what fraction of the initial number of active nuclei will remain undisintegrated after half of a half-life of the sample?

(a) $\sqrt{2} - 1$ (b) $1/\sqrt{2}$
(c) $1/2\sqrt{2}$ (d) 1/4

73. 90% of a radioactive sample is left undecayed after time t has elapsed. What percentage of the initial sample will decay in a total time $2t$?

(a) 20% (b) 19%
(c) 40% (d) 38%

74. In a radioactive sample the fraction of initial number of radioactive nuclei, which remains undecayed after n mean lives is

(a) $\frac{1}{e^n}$ (b) e^n
(c) $1 - \frac{1}{e^n}$ (d) $\left(\frac{1}{e-1}\right)^n$

75. A stationary radioactive nucleus of mass 210 units disintegrates into an α-particle of mass 4 units and residual nucleus of mass 206 units. If the kinetic energy of the α-particle is E, the kinetic energy of the residual nucleus is

(a) $\left(\frac{2}{105}\right)E$ (b) $\left(\frac{2}{103}\right)E$
(c) $\left(\frac{103}{105}\right)E$ (d) $\left(\frac{103}{2}\right)E$

76. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is

(a) 128 h (b) 24 h (c) 6 h (d) 12 h

77. An atom of mass number A and atomic number Z emits successively a γ-ray, a β-particle and an α-particle. The mass number and the atomic number of the end product are

(a) $A - 4, Z - 4$ (b) $A, Z + 1$
(c) $A - 4, Z + 2$ (d) $A - 4, Z - 1$

78. The dacay constant of a radioactive substance is λ. Its half-life and mean life, respectively are

(a) $\frac{1}{\lambda}$ and $\log_e 2$ (b) $\frac{\log_e 2}{\lambda}$ and $\frac{1}{\lambda}$
(c) $2 \log_e 2$ and $\frac{1}{\lambda}$ (d) $\frac{1}{\lambda}$ and $\frac{1}{\lambda}$

79. A β-particle is emitted by a radioactive nucleus at the time of conversion of a

(a) neutron into a proton (b) proton into a neutron
(c) nucleon into energy (d) positron into energy

80. Atomic mass number of an element is 232 and its atomic number is 90. The end product if this radioactive element is an isotope of lead ($^{208}_{82}$Pb). The number of α and β-particles emitted is

(a) $\alpha = 3$ and $\beta = 3$ (b) $\alpha = 6$ and $\beta = 4$
(c) $\alpha = 6$ and $\beta = 2$ (d) $\alpha = 1$ and $\beta = 6$

81. The decay constant of a radioactive element is defined as the reciprocal of the time interval after which the number of atoms of the radioactive element falls to nearly

(a) 50% of its original number
(b) 36.8% of its original number
(c) 63.2% of its original number
(d) 75% of its original number

82. At any instant, the ratio of the amount of radioactive substance is 2 : 1. If their half-lives be respectively 12h and 16 h, then after 2 days, what will be the ratio of the substances?

(a) 1 : 1 (b) 2 : 1
(c) 1 : 2 (d) 1 : 4

83. A radioactive isotope X with half-life 1.5×10^9 yr decays into a stable nucleus Y. A rock sample contains both elements X and Y in the ratio 1: 15. The age of the rock is

(a) 3.0×10^9 yr (b) 4.5×10^9 yr
(c) 6.0×10^9 yr (d) 9.0×10^9 yr

84. The decay constant of radioactive element is 1.5×10^{-9} /s. Its mean life (in second) will be

(a) 1.5×10^9 (b) 4.62×10^8
(c) 6.67×10^8 (d) 10.35×10^8

85. A radioactive material has mean-lives of 1620 yr and 520 yr for α and β-emission respectively. The material decays by simultaneous α and β-emission. The time in which 1/4th of the material remains intact is

(a) 4675 yr (b) 720 yr
(c) 545 yr (d) 324 yr

86. A radioactive nucleus emits a β-particle. The parent and daughter nuclei are
(a) isotopes (b) isotones
(c) isomers (d) isobars

87. In which sequence, the radioactive radiations are emitted in the following nuclear reactions?

$${}_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} \rightarrow {}_{Z-1}K^{A-4} \rightarrow {}_{Z-1}K^{A-4}$$

(a) β, α and γ (b) α, β and γ
(c) β, γ and α (d) γ, α and β

88. A radioactive substance A disintegrates into a radioactive substance B. Let n_A and n_B denote the number of nuclei of A and B surviving against radioactive decay at any moment and λ_A and λ_B be the respective disintegration constants. The rate of change in number of nuclei of the substance B is given by
(a) $\frac{dn_B}{dt} = \lambda_A n_A - \lambda_B n_B$ (b) $\frac{dn_B}{dt} = -\lambda_A n_A + \lambda_B n_B$
(c) $\frac{dn_B}{dt} = -\lambda_A n_A - \lambda_B n_B$ (d) $\frac{dn_B}{dt} = \lambda_A n_A + \lambda_B n_B$

89. An atom of mass number 15 and atomic number 7 captures an α-particle and then emits a proton. The mass number and atomic number of the resulting product will respectively be
(a) 14 and 2 (b) 15 and 3
(c) 16 and 4 (d) 18 and 8

90. Identify the correct statement given below.
(a) α-rays have the highest penetrating power
(b) β-rays have the highest penetrating power
(c) γ-rays have the highest penetrating power
(d) All three kinds of rays have the same penetrating power

91. A radioactive substance disintegrates 1/64 of initial value in 60 s. The half-life of this substance is
(a) 5 s (b) 10 s
(c) 30 s (d) 20 s

92. A radioactive nucleus is being produced at a constant rate α per second. Its decay constant is λ. If N_0 are the number of nuclei at time $t = 0$, then maximum number of nuclei possible are
(a) $\frac{\alpha}{\lambda}$ (b) $N_0 + \frac{\alpha}{\lambda}$
(c) N_0 (d) $\frac{\lambda}{\alpha} + N_0$

93. After three 'average life times' of a radioactive sample, the amount of substance remains x% of the original amount. Then, approximate value of x is
(a) 14 (b) 37 (c) 7 (d) 5

94. In one average life
(a) half the active nuclei decay
(b) less than half the active nuclei decay
(c) more than half the active nuclei decay
(d) all the nuclei decay

Photoelectric Effect

95. Threshold frequency for a metal is 10^{15} Hz. Light of $\lambda = 4000$ Å falls on its surface. Which of the following statements is correct?
(a) No photoelectric emission takes place
(b) Photoelectron come out with zero speed
(c) Photoelectron come out with 10^3 m/s speed
(d) Photoelectron come out with 10^5 m/s speed

96. Einstein got Nobel prize on which of the following works?
(a) Mass-energy relation
(b) Special theory of relativity
(c) Photoelectric equation
(d) Both (a) and (b)

97. The work function of a metal is 4.2 eV, its threshold wavelength will be
(a) 4000 Å (b) 3500 Å (c) 2955 Å (d) 2500 Å

98. The number of photoelectrons emitted per second from a metal surface increases when
(a) the energy of incident photons increases
(b) the frequency of incident light increases
(c) the wavelength of the incident light increases
(d) the intensity of the incident light increases

99. The work function of a metal is 1 eV. Light of wavelength 3000 Å is incident on this metal surface. The velocity of emitted photoelectrons will be
(a) 10 m/s (b) 1×10^3 m/s
(c) 1×10^4 m/s (d) 1×10^6 m/s

100. The work function of a metal is 1.6×10^{-19} J. When the metal surface is illuminated by the light of wavelength 6400 Å, then the maximum kinetic energy of emitted photoelectrons will be
(Given , Planck's constant $h = 6.4 \times 10^{-34}$ J-s)
(a) 14×10^{-19} J (b) 2.8×10^{-19} J
(c) 1.4×10^{-19} J (d) 1.4×10^{-19} eV

101. The work function for tungsten and sodium are 4.5 eV and 2.3 eV respectively. If the threshold wavelength λ for sodium is 5600 Å, the value of λ for tungsten is
(a) 5893 Å (b) 10683 Å
(c) 2862 Å (d) 528 Å

102. Photoelectric effect was successfully explained first by
(a) Planck (b) Hallwash
(c) Hertz (d) Einstein

103. In a photoelectric experiment for 4000 Å incident radiation, the potential difference to stop the ejection is 2 V. If the incident light is changed to 3000 Å, then the potential required to stop the ejection of electrons will be
(a) 2 V (b) less than 2 V
(c) zero (d) greater than 2 V

104. The maximum wavelength of radiation that can produce photoelectric effect in a certain metal is 200 nm. The maximum kinetic energy acquired by electron due to radiation of wavelength 100 nm will be
(a) 12.4 eV (b) 6.2 eV
(c) 100 eV (d) 200 eV

105. Energy conversion in a photoelectric cell takes place from
(a) chemical to electrical (b) magnetic to electrical
(c) optical to electrical (d) mechanical to electrical

106. The work function of aluminium is 4.2 eV. If two photons, each of energy 3.5 eV strike an electron of aluminum, then emission of electrons will be
(a) possible
(b) not possible
(c) depend upon the area of the surface
(d) data insufficient

107. When radiation is incident on photoelectron emitter, the stopping potential is found to be 9 V. If *e/m* for the electron is 1.8×10^{11} C kg^{-1}, the maximum velocity of the ejected electrons is
(a) 6×10^5 ms^{-1} (b) 8×10^5 ms^{-1}
(c) 1.8×10^6 ms^{-1} (d) 1.8×10^5 ms^{-1}

108. Two identical metal plates show photoelectric effect by a light of wavelength λ_A falling on plate A and λ_B on plate B ($\lambda_A = 2\lambda_B$). The maximum kinetic energy is
(a) $K_A = K_B$ (b) $K_A < K_B/2$
(c) $K_A = 2K_B$ (d) $K_A = K_B/2$

109. When radiation of wavelength λ is incident on a metallic surface, the stopping potential is 4.8 V. If the same surface is illuminated with radiation of double the wavelength, then the stopping potential becomes 1.6 V. Then, the threshold wavelength for the surface is
(a) 2λ (b) 4λ
(c) 6λ (d) 8λ

110. In a photoemissive cell with executing wavelength λ, the fastest electron has speed v. If the exciting wavelength is changed to $\frac{\lambda}{4}$, the speed of the fastest emitted electron will be
(a) $2v$ (b) $\frac{v}{2}$
(c) $< 2v$ (d) $> 2v$

111. Photoelectric effect supports quantum nature of light, because
(a) the energy of released electron is discrete
(b) the maximum kinetic energy of photoelectrons depends only on the frequency of light and not on its intensity
(c) even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately
(d) electric charge of the photoelectrons is quantised

112. If the work function of potassium is 2 eV, then its photoelectric threshold wavelength is
(a) 310 nm (b) 620 nm
(c) 6200 nm (d) 3100 nm

113. Threshold wavelength for a metal is 5200 Å. The photoelectron will be ejected, if it is irradiated by light from
(a) 50 W infrared lamp
(b) 1 W infrared lamp
(c) 50 W ultraviolet lamp
(d) 0.5 W infrared lamp

114. A photocell is illuminated by a point source of light, which is placed at a distance d from the cell. If the distance becomes $d/2$, then number of electrons emitted per second will
(a) remain the same (b) become four times
(c) become two times (d) become one-fourth

115. Light of frequency $4v_0$ is incident on the metal of threshold frequency v_0. The maximum kinetic energy of the emitted photoelectron is
(a) $3hv_0$ (b) $2hv_0$
(c) $\frac{3}{2}hv_0$ (d) $\frac{1}{2}hv_0$

116. In a photoelectric experiment, the stopping potential V_S is plotted against the frequency ν of incident light. The resulting curve is a straight line which makes an angle θ with the v-axis. The $\tan\theta$ will be equal to (ϕ = work function of surface)
(a) h/e (b) e/h
(c) $-\phi/e$ (d) eh/ϕ

117. The work functions of metals A and B are in the ratio 1 : 2. If light of frequencies f and $2f$ are incident on metal surfaces of A and B respectively, the ratio of the maximum kinetic energies of photoelectron emitted is (f is greater than threshold frequency of A, $2f$ is greater than threshold frequency of B)
(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

118. The surface of some material is radiated, by waves of $\lambda_1 = 3.5 \times 10^{-7}$ m and $\lambda_2 = 5.4 \times 10^{-7}$ m respectively. The ratio of the stopping potential in two cases is 2 : 1. The work function of the material is
(a) 0.86 eV (b) 1.345 eV
(c) 1.04 eV (d) 2.20 eV

119. The retarding potential necessary to stop the emission of photoelectron, when a target material of work function 1.24 eV is irradiated with light of wavelength 4.36×10^{-7} m is
(a) 0.36 V (b) 1.60 V
(c) 2.84 V (d) 4.08 V

120. Light of wavelength 3500 Å is incident on two metals *A* and *B, A* of work function 4.2 eV and *B* of work function 1.19 eV. The photoelectron will be emitted by
(a) metal *A*
(b) metal *B*
(c) Both *A* and *B*
(d) Neither metal *A* nor metal *B*

121. When stopping potential is applied in an experiment on photoelectric effect, no photocurrent is observed. This means that
(a) the emission of photoelectrons is stopped
(b) the photoelectrons are emitted but are reabsorbed by the emitter metal
(c) the photoelectrons are accumulated near the collector plate
(d) the photoelectrons are dispersed from the sides of the apparatus

122. If the frequency of light in a photoelectric experiment is doubled, the stopping potential will
(a) be doubled
(b) be halved
(c) become more than double
(d) become less than double

123. A non-monochromatic light is used in an experiment on photoelectric effect. The stopping potential is
(a) related to the mean wavelength
(b) related to the longest wavelength
(c) related to the shortest wavelength
(d) not related to the wavelength

124. When a monochromatic point source of light is at a distance *r* from a photoelectric cell, the cut-off voltage is *V* and the saturation current is *I*. If the same source is placed at a distance 3*r* away from the photoelectric cell, then
(a) the saturation current will not change
(b) the saturation current will change to *I*/9
(c) the stopping potential will not change
(d) the stopping potential will increase to 3*V*

Matter Waves

125. The idea of matter waves was given by
(a) Davisson and Germer (b) de-Brogile
(c) Einstein (d) Planck

126. The de-Broglie wavelength associated with the particle of mass *m* moving with velocity *v* is
(a) *h*/*mv* (b) *mv*/*h*
(c) *mh*/*v* (d) *m*/*hv*

127. A photon, an electron and a uranium nucleus all have the same wavelength. The one with the most energy
(a) is the photon
(b) is the electron
(c) is the uranium nucleus
(d) depends upon the wavelength and the properties of the particle

128. When the kinetic energy of an electron is increased, the wavelength of the associated wave will
(a) increase
(b) decrease
(c) not depend on the kinetic energy
(d) None of the above

129. If the de-Broglie wavelengths for a proton and for α-particle are equal, then the ratio of their velocities will be in the ratio
(a) 4 : 1 (b) 2 : 1
(c) 1 : 2 (d) 1 : 4

130. An electron of mass *m*, when accelerated through a potential difference *V* has de-Broglie wavelength λ. The de-Broglie wavelength associated with a proton of mass *M* accelerated through the same potential difference will be
(a) $\lambda\frac{m}{M}$ (b) $\lambda\sqrt{\frac{m}{M}}$ (c) $\lambda\frac{M}{m}$ (d) $\lambda\sqrt{\frac{M}{m}}$

131. What will be the ratio of de-Broglie wavelengths of proton and α-particle of same energy?
(a) 2 : 1 (b) 1 : 2 (c) 4 : 1 (d) 1 : 4

132. The de-Broglie wavelength of an electron having 80 eV of energy is nearly
(given, $1\text{ eV} = 1.6\times10^{-19}$ J, Mass of electron $= 9\times10^{-31}$ kg, Planck's constant $= 6.6\times10^{-34}$ J-s)
(a) 140 Å (b) 0.14 Å
(c) 14 Å (d) 1.4 Å

133. A particle of mass *M* at rest decays into two particles of masses m_1 and m_2, having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles, λ_1/λ_2 is
(a) m_1/m_2 (b) m_2/m_1
(c) 1 : 1 (d) $\sqrt{m_2}/\sqrt{m_1}$

134. Proton and α-particles have the same de-Broglie wavelength. What is the same for both of them?
(a) Charge (b) Energy
(c) Speed (d) Momentum

135. The de-Broglie wavelength of a body of mass *m* and kinetic energy *E* is given by
(a) $\frac{h}{\sqrt{2mE}}$ (b) $\frac{h}{2mE}$
(c) $\frac{\sqrt{2mE}}{h}$ (d) $\frac{h}{mE}$

136. Two electrons are moving with the same speed *v*. One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field. After some time, if the de-Broglie wavelengths of the two are λ_1 and λ_2, then
(a) $\lambda_1 = \lambda_2$ (b) $\lambda_1 > \lambda_2$
(c) $\lambda_1 < \lambda_2$ (d) $\lambda_1 > \lambda_2$ or $\lambda_1 < \lambda_2$

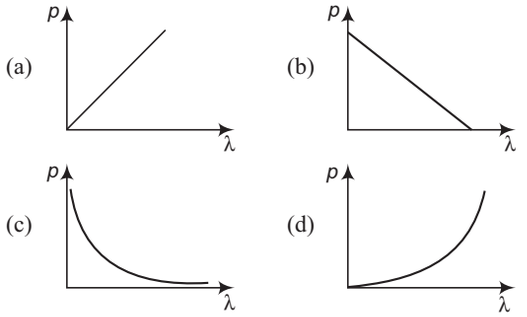
137. If λ_p and λ_e denote the de-Broglie wavelength of proton and electron after they are accelerated from rest through the same potential difference, then
(a) $\lambda_e = \lambda_p$ (b) $\lambda_e < \lambda_p$ (c) $\lambda_e > \lambda_p$ (d) $\lambda_e = \lambda_p/2$

138. An electron and proton are accelerated through the same potential difference. The ratio of their de-Broglie wavelength will be
(a) $(m_p/m_e)^{1/2}$ (b) m_e/m_p (c) m_p/m_e (d) 1

139. A proton accelerated through a potential difference of 100V, has de-Broglie wavelength λ_0. The de-Broglie wavelength of an α-particle, accelerated through 800 V is
(a) $\frac{\lambda_0}{\sqrt{2}}$ (b) $\frac{\lambda_0}{2}$ (c) $\frac{\lambda_0}{4}$ (d) $\frac{\lambda_0}{8}$

140. Matter waves
(a) are longitudinal waves
(b) are electromagnetic waves in nature
(c) always travel with the speed of light
(d) show diffraction

141. Which of the following graphs represents the variation of the particle momentum and the associated de-Broglie wavelength?



Nucleus and Nuclear Reactions

142. The masses of neutron and proton are 1.0087 amu and 1.0073 amu respectively. If two neutrons and two protons combine to form a helium nucleus (α-particles) of mass 4.0015 amu. The binding energy of the helium nucleus will be (1 amu = 931 MeV)
(a) 28.4 MeV (b) 20.8 MeV (c) 27.3 MeV (d) 14.2 MeV

143. Outside a nucleus
(a) neutron is stable
(b) proton and neutron both are stable
(c) neutron is unstable
(d) Neither neutron nor proton is stable

144. In an atomic bomb, the energy is released due to
(a) chain reaction of neutrons and $_{92}U^{235}$
(b) chain reaction of neutrons and $_{92}U^{238}$
(c) chain reaction of neutrons and $_{92}U^{240}$
(d) chain reaction of neutrons and $_{92}U^{236}$

145. If an electron and a positron annihiliate, then the energy released is
(a) 3.2×10^{-13} J (b) 1.6×10^{-13} J
(c) 4.8×10^{-13} J (d) 6.4×10^{-13} J

146. In the reaction $^2_1H + ^2_1H \rightarrow ^4_2He + ^1n_0$, if the binding energies of ^2_1H, ^3_1H and ^4_2He are respectively *a, b* and *c* (in MeV), then the energy (in MeV) released in this reaction is
(a) $c + a - b$ (b) $c - a - b$
(c) $a + b + c$ (d) $a + b - c$

147. A nuclear reaction along with the masses of the particle taking part in it is as follows;

A	+	B	$\rightarrow$	C	+	D	$+Q$ MeV
1.002 amu		1.004 amu		1.001 amu		1.003 amu	

The energy Q liberated in the reaction is
(a) 1.234 MeV
(b) 0.931 MeV
(c) 0.465 MeV
(d) 1.863 MeV

148. The principle of controlled chain reaction is used in
(a) atomic energy reactor (b) atom bomb
(c) the core of the sun (d) artificial radioactivity

149. For effective nuclear forces, the distance should be of the order of
(a) 10^{-10} m (b) 10^{-13} m
(c) 10^{-15} m (d) 10^{-20} m

150. The binding energy of the electron in the lowest orbit of the hydrogen atom is 13.6 eV. The energies required in eV to remove an electron from the three lowest orbits of the hydrogen atom are (in eV)
(a) 13.6, 6.8, 8.4
(b) 13.6, 10.2, 3.4
(c) 13.6, 27.2, 40.8
(d) 13.6, 3.4, 1.5

151. When the number of nucleons in nuclei increase, the binding energy per nucleon
(a) increases continuously with mass number
(b) decreases continuously with mass number
(c) remains constant with mass number
(d) first increases and then decreases with increase of mass number

152. What fissionable material was used in the bomb dropped at Nagasaki (Japan) in the year 1945?
(a) Uranium (b) Neptunium
(c) Berkelium (d) Plutonium

153. Fusion reactions take place at high temperature, because
(a) atoms are ionised at high temperature
(b) molecules break up at high temperature
(c) nuclei break up at high temperature
(d) repulsion between nuclei is large

154. A γ-ray photon creates an electron-positron pair. If the rest mass energy of an electron is 0.5 MeV and the total kinetic energy of the electron-positron pair is 0.78 MeV, then the energy of the γ-ray photon must be
(a) 0.78 MeV (b) 1.78 MeV
(c) 1.28 MeV (d) 0.25 MeV

155. The curve of binding energy per nucleon as a function of atomic mass number has a sharp peak, of helium nucleus. This implies that helium
(a) can easily be broken up
(b) is very stable
(c) can be caused as fissionable material
(d) is radioactive

156. Boron rods are used in nuclear reactors, because
(a) boron can absorb neutrons
(b) strength is given to the plant
(c) boron speeds up neutrons
(d) the reactors look good

157. 1 amu is equivalent to
(a) 1.6×10^{-12} J (b) 1.6×10^{-19} J
(c) 1.5×10^{-10} J (d) 1.5×10^{-19} J

158. Which of the following processes will result into fission reaction?
(a) ${}_{92}U^{235}$ is bombarded with fast moving neutrons
(b) ${}_{92}U^{235}$ is bombarded with thermal neutrons
(c) U^{238} is bombarded with slow moving neutrons
(d) U^{235} being unstable breaks into smaller fragments

159. The binding energies of the nuclei A and B are E_a and E_b respectively. Three atoms of the element B fuse to give one atom of element A and an energy Q is released. Then E_a, E_b and Q are related as
(a) $E_a - 3E_b = Q$ (b) $3E_b - E_a = Q$
(c) $E_a + 3E_b = Q$ (d) $E_b + 3E_a = Q$

160. The binding energy per nucleon for deuteron $[{}_1^2\text{H}]$ and helium $[{}_2^4\text{He}]$ are 1.1 MeV and 7 MeV respectively. The energy released when two deuterons fuse to form a helium nucleus is
(a) 32.4 MeV (b) 23.6 MeV
(c) 16.2 MeV (d) 11.8 MeV

161. Order of magnitude of density of uranium nucleus is
($m_p = 1.67\times10^{-27}$ kg)
(a) 10^{20} kg/m^3 (b) 10^{17} kg/m^3
(c) 10^{14} kg/m^3 (d) 10^{11} kg/m^3

162. The binding energy per nucleon is maximum in the case of
(a) ${}_2^4\text{He}$ (b) ${}_{26}^{56}\text{Fe}$
(c) ${}_{56}^{141}\text{Ba}$ (d) ${}_{92}^{235}\text{U}$

163. As compared to ${}^{12}\text{C}$-atom, ${}^{14}\text{C}$-atom has
(a) two extra protons and two extra electrons
(b) two extra protons but no extra electron
(c) two extra neutrons and no extra electron
(d) two extra neutrons and two extra electrons

164. Which of the following is a wrong description of binding energy in a nucleus?
(a) It is the energy required to break a nucleus into its constituent nucleons
(b) It is the energy made available when free nucleons combine to form a nucleus
(c) It is the sum of the rest mass energies of its nucleons minus the rest mass energy of the nucleus
(d) None of the above

165. An α-particle is bombarded on ${}^{14}\text{N}$. As a result, a ${}^{17}\text{O}$ nucleus is formed and a particle is emitted. This particle is a
(a) neutron (b) proton (c) electron (d) positron

166. As the mass number A increases, which of the following quantities related to a nucleus do not change?
(a) Mass (b) Volume
(c) Density (d) Binding energy

X-Rays

167. An X-ray tube is operated at 50 kV. The minimum wavelength produced is
(a) 0.5 Å (b) 0.75 Å (c) 0.25 Å (d) 1 Å

168. Which of the following wavelength falls in X-ray region?
(a) 10000 Å (b) 1000 Å (c) 1 Å (d) 10^{-2} Å

169. A metal block is exposed to beams of X-ray of different wavelengths. X-rays of which wavelength penetrate most?
(a) 2 Å (b) 4 Å (c) 6 Å (d) 8 Å

170. X-rays were discovered by
(a) Becquerel (b) Rontegen
(c) Marie Curie (d) Von Laue

171. Moseley's law relates the frequencies of line X-rays, which of the following characteristics of the target element is
(a) its density
(b) its atomic weight
(c) its atomic number
(d) interplaner spacing of the atomic planes

172. X-rays are in nature similar to
(a) β-rays (b) γ-rays
(c) de-Broglie waves (d) cathode rays

173. X-rays are not used for radar purpose, because
(a) they are not reflected by the target
(b) they are not electromagnetic waves
(c) they are completely absorbed by the air
(d) they sometimes damage the target

174. The patient is asked to drink $BaSO_4$ for examining the stomach by X-rays, because X-rays are
(a) reflected by heavy atoms
(b) refracted by heavy atoms
(c) less absorbed by heavy atoms
(d) more absorbed by heavy atoms

175. If V be the accelerating voltage, then the maximum frequency of continuous X-rays is given by
(a) $\frac{eh}{V}$ (b) $\frac{hV}{e}$
(c) $\frac{eV}{h}$ (d) $\frac{h}{eV}$

176. The minimum wavelength of X-rays produced by electrons accelerated by a potential difference of V volt is equal to
(a) $\frac{eV}{hc}$ (b) $\frac{eh}{cV}$
(c) $\frac{hc}{eV}$ (d) $\frac{cV}{eh}$

177. X-ray beam can be deflected by
(a) magnetic field (b) electric field
(c) Both (a) and (b) (d) None of these

178. In an X-rays tube, the intensity of the emitted X-rays beam is increased by
(a) increasing the filament current
(b) decreasing the filament current
(c) increasing the target potential
(d) decreasing the target potential

179. Bragg's law for X-rays is
(a) $d \sin\theta = 2n\lambda$ (b) $2d \sin\theta = n\lambda$
(c) $n \sin\theta = 2\lambda d$ (d) None of these

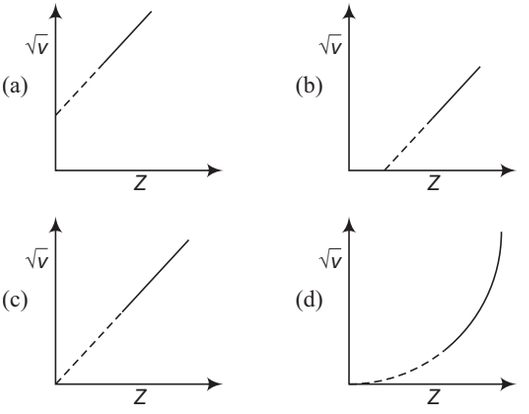
180. For production of characteristic K_β X-rays, the electron transition is
(a) $n = 2$ to $n = 1$ (b) $n = 3$ to $n = 2$
(c) $n = 3$ to $n = 1$ (d) $n = 4$ to $n = 2$

181. X-rays of $\lambda = 1$ Å have frequency
(a) 3×10^8 Hz (b) 3×10^{18} Hz
(c) 3×10^{10} Hz (d) 3×10^{15} Hz

182. In X-ray spectrum wavelength λ of K_α line depends on atomic number Z as
(a) $\lambda \propto Z^2$ (b) $\lambda \propto (Z-1)^2$
(c) $\lambda \propto \frac{1}{(Z-1)}$ (d) $\lambda \propto \frac{1}{(Z-1)^2}$

183. The wavelength of K_α-line for an element of atomic number 43 is λ. Then, the wavelength of K_α-line for an element of atomic number 29 is
(a) $\frac{43}{29}\lambda$ (b) $\frac{42}{28}\lambda$ (c) $\frac{9}{4}\lambda$ (d) $\frac{4}{9}\lambda$

184. The graph between the square root of the frequency of a specific line of characteristic spectrum of X-rays and the atomic number of the target will be



185. X-rays are produced by jumping of
(a) electrons from lower to higher energy orbit of atom
(b) electrons from higher to lower energy orbit of atom
(c) protons from lower to higher energy orbit of nucleus
(d) protons from higher to lower energy orbit of nucleus

186. The penetrating power of X-rays increases with the increase in its
(a) velocity
(b) intensity
(c) frequency
(d) wavelength

187. Molybdenum is used as a target element for production of X-rays, because it is
(a) light and can easily deflect electrons
(b) light and can absorb electrons
(c) a heavy element with a high melting point
(d) an element having high thermal conductivity

188. A X-ray tube operates at an accelerating potential of 20 kV. Which of the following wavelengths will be absent in the continuous spectrum of X-rays?
(a) 12 pm
(b) 75 pm
(c) 62 pm
(d) 95 pm

189. Hydrogen atom does not emit X-rays, because
(a) its energy levels are too close to each other
(b) its energy levels are too far apart
(c) it has a very small mass
(d) it has a single electron

190. The X-ray beam coming from an X-ray tube will be
(a) monochromatic
(b) having all wavelengths larger than a certain minimum wavelength
(c) having all wavelengths smaller than a certain maximum wavelength
(d) having all wavelengths between a minimum and a maximum wavelengths

191. Consider the following two statements *A* and *B* and identify the correct choice in the given answer.

(A) The characteristic X-ray spectrum depends on the nature of the material of the target.

(B) The short wavelength limit of continuous X-ray spectrum varies inversely on the potential difference applied to the X-ray tube.

(a) *A* is true and *B* is false (b) *A* is false and *B* is true
(c) Both *A* and *B* are true (d) Both *A* and *B* are false

192. For production of characteristic K_β X-rays, the electron transition is

(a) $n = 2$ to $n = 1$ (b) $n = 3$ to $n = 2$
(c) $n = 3$ to $n = 1$ (d) $n = 4$ to $n = 2$

193. The kinetic energy of electrons that strike the target is increased, then the cut-off wavelength of continuous X-ray spectrum

(a) increases (b) decreases
(c) no change (d) Cannot be said

194. The wavelength of K_α, X-rays for lead isotopes Pb^{208}, Pb^{206}, Pb^{204} are λ_1, λ_2 and λ_3 respectively. Then

(a) $\lambda_1 = \lambda_2 > \lambda_3$ (b) $\lambda_1 > \lambda_2 > \lambda_3$
(c) $\lambda_1 < \lambda_2 < \lambda_3$ (d) $\lambda_2 = \sqrt{\lambda_1 \lambda_3}$

195. The K_α and K_β lines of characteristic X-ray spectrum of molybdenum are 0.76 Å and 0.64 Å respectively. The wavelength of L_α line is

(a) 1.4 Å (b) 2.4 Å (c) 4.1 Å (d) 3.6 Å

196. Consider a photon of continuous X-ray coming from a coolidge tube. Its energy comes from

(a) the kinetic energy of the striking electron
(b) the kinetic energy of the free electrons of the target
(c) the kinetic energy of the ions of the target
(d) an atomic transition in the target

197. If the potential difference applied to the tube is doubled and the separation between the filament and the target is also doubled, the cut-off wavelength will

(a) remain unchanged
(b) be doubled
(c) be halved
(d) become four times the original

198. If the current in the circuit for heating the filament is increased, the cut off wavelength

(a) will increase
(b) will decrease
(c) will remain unchanged
(d) may increase or decrease

199. Moseley's law for characteristic X-rays is $\sqrt{v} = a(Z - b)$. In this

(a) Both *a* and *b* are independent of the material
(b) *a* is independent but *b* depends on the material
(c) *b* is independent but *a* depends on the material
(d) Both *a* and *b* depends on the material

Miscellaneous Problems

200. The mass of the electron varies with

(a) the size of the cathode ray tube
(b) the variation of *g*
(c) velocity
(d) size of the electron

201. When the speed of electrons increases, then the value of its specific charge

(a) increases
(b) decreases
(c) remains unchanged
(d) increases upto some velocity and then begins to decrease

202. The ratio of momenta of an electron and α-particle which are accelerated from rest by a potential difference of 100 V is

(a) 1 (b) $\sqrt{\frac{2m_e}{m_\alpha}}$ (c) $\sqrt{\frac{m_e}{m_\alpha}}$ (d) $\sqrt{\frac{m_e}{2m_\alpha}}$

203. The fact that electric charges are integral multiples of the fundamental electronic charge was proved experimentally by

(a) Planck (b) J J Thomson
(c) Einstein (d) Millikan

204. In Millikan oil drop experiment, a charged drop of mass 1.8×10^{-14} kg is stationary between its plates at distance 0.90 cm and pottential difference 2.0 kV. The number of electrons on the drop is

(a) 500 (b) 50
(c) 5 (d) 0

205. In Thomson's method of determining *e/m* of electrons

(a) electric and magnetic fields are parallel to electrons beam
(b) electric and magnetic fields are perpendicular to each other and perpendicular to electrons beam
(c) magnetic field is parallel to the electrons beam
(d) electric field is parallel to the electrons beam

206. The specific charge of an electron is

(a) 1.6×10^{-19} C
(b) 4.8×10^{-10} stat C
(c) 1.76×10^{11} C/kg
(d) 1.76×10^{-11} C/kg

207. Cathode rays are similar to visible light rays in that

(a) they both can be deflected by electric and magnetic fields
(b) they both have a definite magnitude of wavelength
(c) they both can ionise a gas through which they pass
(d) they both can expose a photographic plate

208. Electron volt is a unit of

(a) potential (b) charge
(c) power (d) energy

209. Which of the following have highest specific charge?

(a) Positron (b) Proton
(c) He (d) None of these

210. Dual nature of radiation is shown by
(a) diffraction and reflection
(b) refraction and diffraction
(c) photoelectric effect alone
(d) photoelectric effect and diffraction

211. The structure of solid crystals is investigated by using
(a) cosmic rays
(b) X-rays
(c) infrared radiations
(d) γ-rays

212. A photon and an electron have equal energy E. $\lambda_{\text{photon}}/\lambda_{\text{electron}}$ is proportional to
(a) $\sqrt{E}$ (b) $1/\sqrt{E}$
(c) $1/E$ (d) does not depend upon E

213. The particles which can be added to the nucleus of an atom without changing its chemical properties are called
(a) electrons (b) protons
(c) neutrons (d) None of these

214. Which of the following has the mass closest in value to that of the positron?
(a) Proton (b) Electron
(c) Photon (d) Neutrino

215. π-mesons can be
(a) π^+ or π^- (b) π^+ or π^0
(c) π^- or π^0 (d) π^+, π^- or π^0

216. The average kinetic energy of the thermal neutrons is of the order of
(a) 0.03 eV (b) 3 eV
(c) 3 keV (d) 3 MeV

217. Name of the India's first nuclear reactor is
(a) RAMBHA (b) MENAKA
(c) URVASI (d) APSARA

218. Which of the following is most unstable?
(a) Electrons (b) Protons
(c) Neutrons (d) α-particle

219. In which of the following, emission of electrons does not take place?
(a) Thermionic emission (b) X-ray emission
(c) Photoelectric emission (d) Secondary emission

220. Energy equivalent to 0.001 mg is
(a) 9×10^7 erg (b) 9×10^7 J
(c) 9×10^{10} J (d) 3×10^7 erg

221. Neutron decay in free space is given as follows,
$$_0n^1 \rightarrow {}_1\text{H}^1 + {}_{-1}e^0 + [\ ]$$
The parenthesis represents a
(a) neutrino (b) photon
(c) antineutrino (d) graviton

222. The antiparticle of electron is
(a) positron (b) α-particle (c) proton (d) β-particle

223. The radius of an atom is approximately equal to
(a) 10^{-8} cm (b) 10^{-4} cm
(c) 10^4 cm (d) 10^8 cm

224. The dimension of Planck's constant is
(a) $[ML^2T^{-1}]$ (b) $[ML^3T^{-1}]$
(c) $[MLT^{-1}]$ (d) $[M^0L^{-1}T^{-3}]$

225. The elliptical orbits of electron in the atom were proposed by
(a) J J Thomson (b) Bohr
(c) Sommerfeld (d) de-Broglie

226. The equation;
$$4\,{}_1\text{H}^1 \longrightarrow {}_2\text{He}^4 + 2e^+ + 26\ \text{MeV}$$
represents
(a) β-decay (b) γ-decay
(c) fusion (d) fission

227. Planck constant has the same dimensions as
(a) force × time (b) force × distance
(c) force × speed (d) force × distance × time

[Level 2]

Only One Correct Option

1. A ray of light of wavelength 5030 Å is incident on a totally reflecting surface. The momentum delivered by the ray is equal to
(a) 6.63×10^{-27} kg-m/s
(b) 2.63×10^{-27} kg-m/s
(c) 1.25×10^{-24} kg-m/s
(d) None of the above

2. What is the binding energy per nucleon of ${}_6\text{C}^{12}$ nucleus?
Given, mass of C^{12} $(m_c) = 12.000\,\text{u}$
Mass of proton $(m_p) = 1.0078\,\text{u}$,
Mass of neutron $(m_n) = 1.0087\,\text{u}$
and 1 amu = $931.4\,\dfrac{\text{MeV}}{\text{c}^2}$
(a) 5.26 MeV (b) 6.2 MeV (c) 4.65 MeV (d) 7.68 MeV

3. Energy of 24.6 eV is required to remove one of the electrons from a neutral helium atom. The energy (in eV) required to remove both the electrons from a neutral helium atom is

(a) 38.2 (b) 49.2
(c) 51.8 (d) 79.0

4. If the radius of first Bohr's orbit is *x,* then de-Broglie wavelength of electron in third orbit is nearly

(a) $2\pi x$ (b) $6\pi x$
(c) $9x$ (d) $\frac{x}{3}$

5. A star initially has 10^{40} deuterons. It produces energy *via* the processes ${}_1^2\mathrm{H} + {}_1^2\mathrm{H} \rightarrow {}_1^3\mathrm{H} + p$ and ${}_1^2\mathrm{H} + {}_1^3\mathrm{H} \rightarrow {}_2^4\mathrm{He} + n$ where, the masses of the nuclei are $m({}^2\mathrm{H}) = 2.014$ amu, $m(p) = 1.007$ amu, $m(n) = 1.008$ amu and $m({}^4\mathrm{He}) = 4.001$amu, if the average power radiated by the star is 10^{16} W. The deuteron supply of the star is exhausted in a time of the order of

(a) 10^{18} s (b) 10^{28} s
(c) 10^{12} s (d) 10^{16} s

6. Assuming that about 200 MeV energy is released per fission of ${}_{92}\mathrm{U}^{235}$ nuclei. What would be the mass of U^{235} consumed per day in the fission of reactor of power 1 MW approximately?

(a) 10 kg (b) 1 kg
(c) 1 g (d) 10 g

7. The ratio between acceleration of the electron in singly ionised helium atom and doubly ionised lithium atom (both in ground state) is

(a) $\frac{4}{9}$ (b) $\frac{27}{8}$
(c) $\frac{8}{27}$ (d) $\frac{9}{4}$

8. A H-atom moving with speed *v* makes a head on collision with a H-atom at rest. Both atoms are in ground state. The minimum value of velocity *v* for which one of the atom may excite is

(a) 6.25×10^4 m/s (b) 8×10^4 m/s
(c) 7.25×10^4 m/s (d) 13.6×10^4 m/s

$m_H = 1.67 \times 10^{-27}$ kg

9. In the reaction ${}_1^2\mathrm{H} + {}_2^3\mathrm{H} \rightarrow {}_2^4\mathrm{He} + {}_0^1 n$, if the binding energies per nucleon of ${}_1^2\mathrm{H}$, ${}_1^3\mathrm{H}$ and ${}_2^4\mathrm{He}$, are *x*, *y* and *z* respectively. Then, energy released in the process is

(a) $2x + 3y - 4z$
(b) $4z - 2x - 3y$
(c) $2x + 3y - 5z$
(d) None of these

10. The activity of a sample of radioactive material is A_1 at time t_1 and A_2 at time t_2 $(t_2 > t_1)$. Its mean life is *T*. Then

(a) $A_1 t_1 = A_2 t_2$ (b) $\frac{A_1 - A_2}{t_2 - t_1} = \text{constant}$
(c) $A_2 = A_1 e^{(t_1 - t_2)/T}$ (d) $A_2 = A_1 e^{(t_1/t_2)/T}$

11. A particular nucleus in a large population of identical radioactive nuclei did survive 5 half-lives of that isotope. Then, the probability that this surviving nucleus will survive the next half-life is

(a) $\frac{1}{32}$ (b) $\frac{1}{5}$
(c) $\frac{1}{2}$ (d) $\frac{1}{10}$

12. A stationary radioactive nucleus of mass 210 units disintegrates into an α-particle of mass 4 units and residual nucleus of mass 206 units. If the kinetic energy of the α-particle is *E,* the kinetic energy of the residual nucleus is

(a) $\left(\frac{2}{105}\right)E$ (b) $\left(\frac{2}{103}\right)E$
(c) $\left(\frac{103}{2}\right)E$ (d) $\left(\frac{105}{2}\right)E$

13. The magnetic field at the centre of a hydrogen atom due to the motion of the electron in the first Bohr orbit is *B.* The magnetic field at the centre due to the motion of the electron in the second Bohr orbit will be

(a) $\frac{B}{4}$ (b) $\frac{B}{8}$
(c) $\frac{B}{32}$ (d) $\frac{B}{64}$

14. An excited hydrogen atom emits a photon of wavelength λ in returning to the ground state. The quantum number *n* of the excited state is given by (*R* = Rydberg constant)

(a) $\sqrt{\lambda R(\lambda R - 1)}$ (b) $\sqrt{\frac{\lambda R}{(\lambda R - 1)}}$
(c) $\sqrt{\frac{(\lambda R - 1)}{\lambda R}}$ (d) $\sqrt{\frac{1}{\lambda R(\lambda R - 1)}}$

15. Magnetic moment of an electron in *n*th orbit of hydrogen atom is

(a) $\frac{neh}{\pi m}$ (b) $\frac{neh}{4\pi m}$
(c) $\frac{meh}{2\pi n}$ (d) $\frac{meh}{4\pi n}$

[*m* = mass of electron, *h* = Planck's constant]

16. The probability of survival of a radioactive nucleus for one mean life is

(a) $\frac{1}{e}$ (b) $1 - \frac{1}{e}$ (c) $\frac{\ln 2}{e}$ (d) $1 - \frac{\ln 2}{e}$

17. When the voltage applied to an X-ray tube is increased from $V_1 = 10\,\text{kV}$ to $V_2 = 20\,\text{kV}$, the wavelength difference between the K_α line and the short wavelength limit of the continuous X-ray spectrum increases by a factor 3. The atomic number of the element of which the tube anti-cathode is made will be

(a) 62 (b) 56
(c) 45 (d) 29

18. Light of wavelength 330 nm falling on a piece of metal ejects electrons with sufficient energy with requires voltage V_0 to prevent them from reaching a collector. In the same setup, light of wavelength 220 nm, ejects electrons which require twice the voltage V_0 to stop them in reaching a collector. The numerical value of voltage V_0 is

(a) $\frac{16}{15}$ V (b) $\frac{15}{16}$ V (c) $\frac{15}{8}$ V (d) $\frac{8}{15}$ V

19. Maximum KE of a photoelectron is E when the wavelength of incident light is λ. If energy becomes four times when wavelength is reduced to one-third, then work function of the metal is

(a) $\frac{3hc}{\lambda}$ (b) $\frac{hc}{3\lambda}$ (c) $\frac{hc}{\lambda}$ (d) $\frac{hc}{2\lambda}$

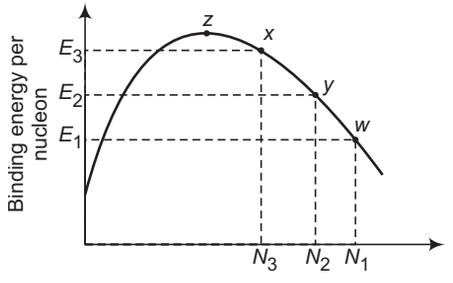
20. If the frequency of K_α X-ray emitted from the element with atomic number 31 is f, then the frequency of K_α X-ray emitted from the element with atomic number 51 would be

(a) $\frac{5f}{3}$ (b) $\frac{51f}{31}$
(c) $\frac{9f}{25}$ (d) $\frac{25f}{9}$

21. According to Moseley's law the ratio of the slope of graph between $\sqrt{f}$ and Z for K_β and K_α is

(a) $\sqrt{\frac{32}{27}}$ (b) $\sqrt{\frac{27}{32}}$ (c) $\sqrt{\frac{5}{36}}$ (d) $\sqrt{\frac{36}{5}}$

22. Consider the nuclear fission reaction $W \rightarrow X + Y$. What is the Q-value (energy released) of the reaction?



(a) $E_1N_1 - (E_2N_2 + E_3N_3)$
(b) $(E_2N_2 + E_3N_3 - E_1N_1)$
(c) $E_2N_2 + E_1N_1 - E_3N_3$
(d) $E_1N_1 + E_3N_3 - E_2N_2$

23. Two radioactive nuclei A and B have disintegration constants λ_A and λ_B and initially N_A and N_B number of nuclei of them are taken, then the time after which their undisintegrated nuclei are same is

(a) $\frac{\lambda_A\lambda_B}{(\lambda_A - \lambda_B)} \ln\left(\frac{N_B}{N_A}\right)$ (b) $\frac{1}{(\lambda_A + \lambda_B)} \ln\left(\frac{N_B}{N_A}\right)$
(c) $\frac{1}{(\lambda_B - \lambda_A)} \ln\left(\frac{N_B}{N_A}\right)$ (d) $\frac{1}{(\lambda_A - \lambda_B)} \ln\left(\frac{N_B}{N_A}\right)$

24. Light of wavelength 0.6 μm from a sodium lamp falls on a photocell and causes the emission of photoelectrons for which the stopping potential is 0.5 V. With light of wavelength 0.4 μm from a sodium lamp, stopping potential is 1.5 V. With this data, the value of h/e is

(a) 4×10^{-19} V/s (b) 0.25×10^{15} V/s
(c) 4×10^{-15} V/s (d) 4×10^{-8} V/s

25. In a hypothetical Bohr hydrogen, the mass of the electron is doubled. The energy E_0 and radius r_0 of the first orbit will be (a_0 is the Bohr's first orbit radius of hydrogen)

(a) $E_0 = -27.2$ eV, $r_0 = \frac{a_0}{2}$ (b) $E_0 = -27.2$ eV, $r_0 = a_0$
(c) $E_0 = -13.6$ eV, $r_0 = \frac{a_0}{2}$ (d) $E_0 = -13.6$ eV, $r_0 = a_0$

26. Imagine an atom made up of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atomic model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength of photon in the Balmer series has wavelength λ (given in terms of the Rydberg constant R for hydrogen atom) equal to

(a) $\frac{9}{(5R)}$ (b) $\frac{36}{(5R)}$
(c) $\frac{18}{(5R)}$ (d) $\frac{4}{(5R)}$

27. A photon has the same wavelength as the de-Broglie wavelength of electron. Given that c = speed of light and v = speed of electron. Which of the following relation is correct? [where, E_e = kinetic energy of electron, E_{ph} = energy of photon, p_e = momentum of electron and p_{ph} = momentum of photon]

(a) $\frac{E_e}{E_{ph}} = \frac{2c}{v}$ (b) $\frac{E_e}{E_{ph}} = \frac{v}{2c}$
(c) $\frac{p_e}{p_{ph}} = \frac{2c}{v}$ (d) $\frac{p_e}{p_{ph}} = \frac{c}{2v}$

28. The ratio of speed of the electron in the first Bohr orbit of hydrogen and the speed of light is equal to (where, e, h and c have their usual meanings)

(a) $\frac{2\pi hc}{e^2}$ (b) $\frac{e^2c}{2\pi h}$ (c) $\frac{e^2h}{2\pi c}$ (d) $\frac{e^2}{2\varepsilon_0 hc}$

29. In a hydrogen atom, the binding energy of the electron in the ground state is E_1. Then, the frequency of revolution of the electron in the nth orbit is

(a) $\frac{2E_1}{n^3h}$ (b) $\frac{2E_1n^3}{h}$

(c) $\sqrt{\frac{2mE_1}{n^3h}}$ (d) $\frac{E_1n^2}{h}$

30. de-Broglie wavelength of an electron in the nth Bohr orbit of hydrogen atom is λ_n and the angular momentum is J_n, then

(a) $J_n \propto \lambda_n$ (b) $\lambda_n \propto \frac{1}{J_n}$

(c) $\lambda_n \propto J_n^2$ (d) None of these

31. When photons of energy 4.25 eV strike the surface of a metal A, the ejected photoelectron have maximum kinetic energy T_A expressed in eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectron liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50)$ eV. If the de-Broglie wavelength of these photoelectron is $\lambda_B = 2\lambda_A$, then choose the wrong option.

(a) The work function of A is 2.25 eV

(b) The work function of B is 4.20 eV

(c) $T_A = 2.00$ eV

(d) $T_B = 2.75$ eV

32. The electron in a hydrogen atom makes a transition $n_1 \to n_2$, where n_1 and n_2 are the principal quantum numbers of two states. Assume the Bohr model to be valid. The time period of the electron in the initial state is eight times in the final state. The possible values of n_1 and n_2 are

(a) $n_1 = 4, n_2 = 2$ (b) $n_1 = 8, n_2 = 2$

(c) $n_1 = 8, n_2 = 1$ (d) $n_1 = 6, n_2 = 2$

33. A photon of energy E ejects a photoelectron from a metal surface whose work function is W_0. If electron having maximum kinetic energy enters into a uniform magnetic field of induction B in a direction perpendicular to the field and describes a circular path of radius r, then the radius r is given by, (in the usual notation)

(a) $\frac{\sqrt{2m(E - W_0)}}{eB}$ (b) $\sqrt{2m(E - W_0)\, eB}$

(c) $\frac{\sqrt{2m(E - W_0)}}{mB}$ (d) None of these

34. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time

(a) $\left(\frac{1}{10\lambda}\right)$ (b) $\frac{1}{(11\lambda)}$ (c) $\frac{11}{(10\lambda)}$ (d) $\frac{1}{(9\lambda)}$

35. A plane electromagnetic wave of frequency 30 MHz travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is $E = 6$ V/m along y-direction. Its magnetic field component B at this point would be

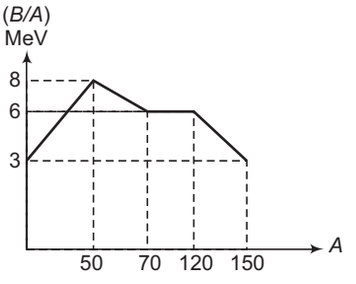
(a) 2×10^8 T along z-direction

(b) 6×10^8 T along x-direction

(c) 2×10^{-8} T along negative x-direction

(d) 2×10^{-8} T along z-direction

36. Assume that the nuclear binding energy per nucleons (B/A) *versus* mass number (A) is as shown in the figure. Consider a nucleus of $A = 110$. Fission of this nucleus results into 2 fragments.



Which of the following could possibly be the mass number of the resulting nuclei to release energy in fission?

(a) 55 and 55 (b) 70 and 40 (c) 100 and 10 (d) 90 and 20

37. A radioactive material decays by simultaneous emission of two particles with respective half-lives 1620 yr and 810 yr. The time, after which one-fourth of the material remains undecayed, is

(a) 3860 yr (b) 4240 yr (c) 2380 yr (d) 1080 yr

38. The recoil momentum of H-atom due to the transition of an electron from $n = 4$ state to $n = 1$ state is

(a) 13.6×10^{-19} kg-m/s (b) 6.8×10^{-27} kg-m/s

(c) 12.75×10^{-24} kg-m/s (d) None of these

39. The half-life of radium is 1620 yr and its atomic weight is 226 kg/K-mol. The number of atoms that will decay from its 1 g sample per second will be (Avogadro's number $N = 6.023 \times 10^{23}$ atom/mol)

(a) 3.61×10^{10} (b) 3.6×10^{12}

(c) 3.11×10^{15} (d) 31.1×10^{15}

40. A radioactive element A of decay constant λ_A decays into another radioactive element B of decay constant λ_B. Initially the number of active nuclei of A was N_0 and B was absent in the sample. The maximum number of active nuclei of B is found at $t = 2\frac{\ln 2}{\lambda_A}$. The maximum number of active nuclei of B is

(a) $\frac{N_0}{4}$ (b) $\frac{\lambda_A}{\lambda_B} N_0 e^{-\lambda_B t}$

(c) $\frac{\lambda_A}{\lambda_B}\frac{N_0}{4}$ (d) None of these

41. A bone containing 200 g carbon-14 has a β-decay rate of 375 decay/min. Calcualte the time that has elapsed since the death of the living one. Given the rate of decay for the living organism = 15 deca per min per gram of carbon and half-life of carbon-14 = 5730 yr

(a) 22920 yr (b) 11460 yr
(c) 17190 yr (d) None of these

42. If we assume only gravitational attraction between proton and electron in hydrogen atom and the Bohr's quantization rule to be followed, then the expression for the ground state energy of the atom will be (the mass of proton is M and that of electron is m.)

(a) $-\dfrac{G^2M^2m^2}{h^2}$ (b) $-\dfrac{2\pi^2G^2M^2m^2}{h^2}$
(c) $-\dfrac{2\pi^2GM^2m^3}{h^2}$ (d) None of these

43. Two identical samples (same material and same amount) P and Q of a radioactive substance having mean life T are observed to have activities A_P and A_Q respectively at the time of observation. If P is older than Q, then the difference in their age is

(a) $T \ln\left(\dfrac{A_P}{A_Q}\right)$ (b) $T \ln\left(\dfrac{A_Q}{A_P}\right)$
(c) $T\left(\dfrac{A_P}{A_Q}\right)$ (d) $T\left(\dfrac{A_P}{A_Q}\right)$

44. The acceleration of electron in the first orbit of hydrogen atom is

(a) $\dfrac{4\pi^2m}{h^3}$ (b) $\dfrac{h^2}{4\pi^2mr}$
(c) $\dfrac{h^2}{4\pi^2m^2r^3}$ (d) $\dfrac{m^2h^2}{4\pi^2r^3}$

45. n-alpha particles per second are emitted from N atoms of a radioactive element. Half-life of the radioactive element is

(a) $\dfrac{n}{N}$ s (b) $\dfrac{N}{n}$ s
(c) $\dfrac{0.693\,N}{n}$ s (d) $\dfrac{0.693\,n}{N}$ s

46. An α-particle accelerated through V volt is fired towards a nucleus. Its distance of closest approach is r. If a proton accelerated through the same potential is fired towards the same nucleus, the distance of closest approach of proton will be

(a) r (b) $2r$
(c) $\dfrac{r}{2}$ (d) $\dfrac{r}{4}$

47. A particle is dropped from a height H. The de-Broglie wavelength of the particle as a function of height is proportional to

(a) H (b) $H^{1/2}$ (c) H^0 (d) $H^{-1/2}$

48. The wavelength of a photon needed to remove a proton from a nucleus which is bound to the nucleus with 1 MeV energy is nearly

(a) 1.2 nm (b) 1.2×10^{-3} nm
(c) 1.2×10^{-6} nm (d) 1.2×10 nm

49. Consider a beam of electrons (each electron with energy E_0) incident on a metal surface kept in an evacuated chamber. Then,

(a) no electrons will be emitted as only photons can emit electrons
(b) electrons can be emitted but all with an energy, E_0
(c) electrons can be emitted with any energy, with a maximum of $E_0 - \phi$ (ϕ is the work function)
(d) electrons can be emitted with any energy, with a maximum of E_0

50. An electron (mass m) with an initial velocity $\mathbf{v} = v_0\hat{\mathbf{i}}\,(v_0 > 0)$ is in an electric field $\mathbf{E} = -E_0\hat{\mathbf{i}}$ (E_0 = constant > 0). Its de-Broglie wavelength at time t is given by

(a) $\dfrac{\lambda_0}{\left(1+\dfrac{eE_0}{m}\dfrac{t}{v_0}\right)}$ (b) $\lambda_0\left(1+\dfrac{eE_0t}{mv_0}\right)$
(c) λ_0 (d) $\lambda_0 t$

51. Taking the Bohr radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about

(a) 53 pm (b) 27 pm
(c) 18 pm (d) 13 pm

52. The binding energy of a H-atom, considering an electron moving around a fixed nuclei (proton), is

$$B = -\frac{me^4}{8n^2\varepsilon_0^2h^2} \qquad (m = \text{electron mass})$$

If one decides to work in a frame of reference where the electron is at rest, the proton would be moving around it. By similar arguments, the binding energy would be

$$B = -\frac{Me^4}{8n^2\varepsilon_0^2h^2} \quad (M = \text{proton mass})$$

This last expression is not correct, because

(a) n would not be integral
(b) Bohr-quantisation applies only two electron
(c) the frame in which the electron is at rest is not inertial
(d) the motion of the proton would not be in circular orbits, even approximately.

53. The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because

(a) of the electrons not being subject to a central force
(b) of the electrons colliding with each other
(c) of screening effects
(d) the force between the nucleus and an electron will no longer be given by Coulomb's law

54. O_2 molecule consists of two oxygen atoms. In the molecule, nuclear force between the nuclei of the two atoms
(a) is not important because nuclear forces are short-ranged
(b) is as important as electrostatic force for binding the two atoms
(c) cancels the repulsive electrostatic force between the nuclei
(d) is not important because oxygen nucleus have equal number of neutrons and protons

55. Suppose we consider a large number of containers each containing initially 10000 atoms of a radioactive material with a half life of 1 yr. After 1 yr,
(a) all the containers will have 5000 atoms of the material
(b) all the containers will contain the same number of atoms of the material but that number will only be approximately 5000
(c) the containers will in general have different numbers of the atoms of the material but their average will be close to 5000
(d) none of the containers can have more than 5000 atoms

56. When a nucleus in an atom undergoes a radioactive decay, the electronic energy levels of the atom
(a) do not change for any type of radioactivity
(b) change for α and β-radioactivity but not for γ-radioactivity
(c) change for α-radioactivity but not for others
(d) change for β-radioactivity but not for others

57. M_x and M_y denote the atomic masses of the parent and the daughter nuclei respectively in radioactive decay. The Q-value for a β^- decay is Q_1 and that for a β^+ decay is Q_2. If m_e denotes the mass of an electron, then which of the following statements is correct?
(a) $Q_1 = (M_x - M_y)c^2$ and $Q_2 = [M_x - M_y - 2m_e]c^2$
(b) $Q_1 = (M_x - M_y)c^2$ and $Q_2 = (M_x - M_y)c^2$
(c) $Q_1 = (M_x - M_y - 2m_e)c^2$ and $Q_2 = (M_x - M_y + 2c_e)c^2$
(d) $Q_1 = (M_x - M_y + 2m_e)c^2$ and $Q_2 = (M_x - m_y + 2m_e)c^2$

58. Heavy stable nuclei have more neutrons than protons. This is because of the fact that
(a) neutrons are heavier than protons
(b) electrostatic force between protons are repulsive
(c) neutrons decay into protons through beta decay
(d) nuclear forces between neutrons are weaker than that between protons

59. In a nuclear reactor , moderators slow down the neutrons which come out in a fission process. The moderator used have light nuclei. Heavy nuclei will not serve the purpose, because
(a) they will break up
(b) elastic collision of neutrons with heavy nuclei will not slow them down
(c) the net weight of the reactor would be unbearably high
(d) substances with heavy nuclei do not occur in liquid or gaseous state at room temperature

More than One Correct Options

1. If the potential difference of coolidge tube producing X-ray is increased, then choose the correct option (s).
(a) The interval between $\lambda_{K\alpha}$ and $\lambda_{K\beta}$ increases
(b) The interval between $\lambda_{K\alpha}$ and λ_0 increases
(c) The interval between $\lambda_{K\beta}$ and λ_0 increases
(d) λ_0 does not change
where, λ_0 is cut-off wavelength and λ_{K_α} and λ_{K_β} are wavelengths of K_α and K_β characteristic X-rays.

2. In Bohr model of the hydrogen atom, let R, v and E represent the radius of the orbit, speed of the electron and the total energy of the electron respectively. Which of the following quantities are directly proportional to the quantum number n ?
(a) vR (b) RE
(c) $\frac{v}{E}$ (d) $\frac{R}{E}$

3. The magnitude of angular momentum, orbital radius and time period of revolution of an electron in a hydrogen atom corresponding to the quantum number n are L, r and T respectively. Which of the following statements is/are correct?
(a) $\frac{rL}{T}$ is independent of n (b) $\frac{L}{T} \propto \frac{1}{n^2}$
(c) $\frac{T}{r} \propto n$ (d) $Lr \propto \frac{1}{n^3}$

4. In which of the following cases the heavier of the two particles has a smaller de-Broglie wavelength? The two particles
(a) move with the same speed
(b) move with the same linear momentum
(c) move with the same kinetic energy
(d) have the same change of potential energy in a conservative field

5. Hydrogen atom absorbs radiations of wavelength λ_0 and consequently emit radiations of 6 different wavelengths, of which two wavelengths are longer than λ_0. Choose the correct alternative(s).
(a) The final excited state of the atoms is $n = 4$
(b) The initial state of the atoms is $n = 2$
(c) The initial state of the atoms is $n = 3$
(d) There are three transitions belonging to Lyman series

6. In coolidge tube, if f and λ represent the frequency and wavelength of K_α line for a metal of atomic number Z, then identify the statement which represents a straight line
(a) $\sqrt{f}$ *versus* Z (b) $\frac{1}{\sqrt{\lambda}}$ *versus* Z
(c) f *versus* Z (d) λ *versus* Z

7. At $t = 0$, number of radioactive nuclei of a radioactive substance are x and its radioactivity is y. Half-life of radioactive substance is T. Then,

(a) $\frac{x}{y}$ is constant throughout

(b) $\frac{x}{y} > T$

(c) value of xy remains half after one half-life

(d) value of xy remains one fourth after one half-life

8. Choose the correct options.

(a) Isotopes have same number of atomic number

(b) Isobars have same atomic weight

(c) Isotones have same number of neutrons

(d) In neutral isotope atoms number of electrons are same

9. Choose the correct options.

(a) By γ-radiations atomic number is not changed

(b) By γ-radiations mass number is not changed

(c) By the emission of one α and two β-particles isotopes are produced

(d) By the emission of one α and four β-particles isobars are produced

10. Two radioactive substances have half lives T and $2T$. Initially they have equal number of nuclei. After time $t = 4T$, the ratio of their number of nuclei is x and the ratio of their activity is y. Then

(a) $x = 1/8$ (b) $x = 1/4$ (c) $y = 1/2$ (d) $y = 1/4$

11. Regarding the nuclear forces, choose the correct options.

(a) They are short range forces

(b) They are charge independent forces

(c) They are not electromagnetic forces

(d) They are exchange forces

12. Regarding a nucleus, choose the correct options.

(a) Density of a nucleus is directly proportional to mass number A

(b) Density of all the nuclei is almost constant, of the order of 10^{17} kg/m^3

(c) Nucleus radius is of the order of 10^{-15}m

(d) Nucleus radius $\propto A$

13. Relativistic corrections become necessary when the expression for the kinetic energy $\frac{1}{2}mv^2$, becomes comparable with mc^2, where m is the mass of the particle. At what de-Broglie wavelength, will relativistic corrections become important for an electron?

(a) $\lambda = 10$ nm (b) $\lambda = 10^{-1}$ nm

(c) $\lambda = 10^{-4}$ nm (d) $\lambda = 10^{-6}$ nm

14. Two particles A_1 and A_2 of masses m_1, m_2 $(m_1 > m_2)$ have the same de-Broglie wavelength. Then,

(a) their momenta are the same

(b) their energies are the same

(c) energy of A_1 is less than the energy of A_2

(d) energy of A_1 is more than the energy of A_2

15. The de-Broglie wavelength of a photon is twice, the de-Broglie wavelength of an electron. The speed of the electron is $v_e = \frac{c}{100}$. Then,

(a) $\frac{E_e}{E_p} = 10^{-4}$

(b) $\frac{E_e}{E_p} = 10^{-2}$

(c) $\frac{p_e}{m_e c} = 10^{-2}$

(d) $\frac{p_e}{m_e c} = 10^{-4}$

16. Photons absorbed in matter are converted to heat. A source emitting n photon/sec of frequency ν is used to convert 1 kg of ice at 0°C to water at 0°C. Then, the time T taken for the conversion

(a) decreases with increasing n, with ν fixed

(b) decreases with n fixed, ν increasing

(c) remains constant with n and ν changing such that $n\nu$ = constant

(d) increases when the product $n\nu$ increases

17. A particle moves in a closed orbit around the origin, due to a force which is directed towards the origin. The de-Broglie wavelength of the particle varies cyclically between two values λ_1, λ_2 with $\lambda_1 > \lambda_2$. Which of the following statement are true?

(a) The particle could be moving in a circular orbit with origin as centre

(b) The particle could be moving in an elliptic orbit with origin as its focus

(c) When the de-Broglie wavelength is λ_1, the particle is nearer the origin than when its value is λ_2

(d) When the de-Broglie wavelength is λ_2, the particle is nearer the origin than when its value is λ_1

18. An ionised H-molecule consists of an electron and two protons. The protons are separated by a small distance of the order of angstrom. In the ground state.

(a) the electron would not move in circular orbits

(b) the energy would be $(2)^4$ times that of a H-atom

(c) the electrons, orbit would go around the protons

(d) the molecule will soon decay in a proton and a H-atom

19. Consider aiming a beam of free electrons towards free protons. When they scatter, an electron and a proton cannot combine to produce a H-atom.

(a) Because of energy conservation

(b) Without simultaneously releasing energy in the form of radiation

(c) Because of momentum conservation

(d) Because of angular momentum conservation

20. The Bohr model for the spectra of a H-atom

(a) will not be applicable to hydrogen in the molecular from

(b) will not be applicable as it is for a He-atom

(c) is valid only at room temperature

(d) predicts continuous as well as discrete spectral lines

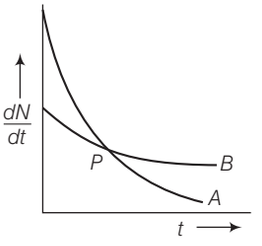
21. The Balmer series for the H-atom can be observed
(a) if we measure the frequencies of light emitted when an excited atom falls to the ground state
(b) if we measure the frequencies of light emitted due to transitions between excited states and the first excited state
(c) in any transition in a H-atom
(d) as a sequence of frequencies with the higher frequencies getting closely packed

22. Let $E_n = \frac{-1\,me^4}{8\varepsilon_0^2 n^2 h^2}$ be the energy of the nth level of H-atom. If all the H-atoms are in the ground state and radiation of frequency $\frac{(E_2 - E_1)}{h}$ falls on it,
(a) it will not be absorbed at all
(b) some of atoms will move to the first excited state
(c) all atoms will be excited to the $n = 2$ state
(d) no atoms will make a transition to the $n = 3$ state

23. Fusion processes, like combining two deuterons to form a He nucleus are impossible at ordinary temperatures and pressure.The reasons for this can be traced to the fact
(a) nuclear forces have short range
(b) nuclei are positively charged
(c) the original nuclei must be completely ionized before fusion can take place
(d) the original nuclei must first break up before combining with each other

24. Samples of two radioactive nuclides A and B are taken λ_A and λ_B are the disintegration constants of A and B respectively. In which of the following cases, the two samples can simultaneously have the same decay rate at any time?
(a) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A = \lambda_B$
(b) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A > \lambda_B$
(c) Initial rate of decay of B is twice the initial rate of decay of A and $\lambda_A > \lambda_B$
(d) Initial rate of decay of B is same as the rate of decay of A at $t = 2$h and $\lambda_B < \lambda_A$

25. The variation of decay rate of two radioactive samples A and B with time is shown in figure. Which of the following statements are true?



(a) Decay constant of A is greater than that of B, hence A always decays faster than B
(b) Decay constant of B is greater than that of A but its decay rate is always smaller than that of A
(c) Decay constant of A is greater than that of B but it does not always decay faster than B
(d) Decay constant of B is smaller than that of A but still its decay rate becomes equal to that of A at a later instant

Comprehension Based Questions

Passage I (Q. 1 to 3)

When a surface is irradiated with light of wavelength 4950 *Å, a photocurrent appears which vanishes, if a retarding potential greater than* 0.6 *V lt is applied across the photo tube. When a second source of light is used, it is found that the critical retarding potential is changed to* 1.1 *V.*

1. The work function of the emitting surface is
(a) 2.2 eV (b) 1.5 eV
(c) 1.9 eV (d) 1.1 eV

2. The wavelength of the second source is
(a) 6150 Å (b) 5150 Å
(c) 4125 Å (d) 4500 Å

3. If the photoelectrons (after emission from the source) are subjected to a magnetic field of 10 T, the two retarding potentials would
(a) uniformly increase
(b) uniformly decrease
(c) remain the same
(d) None of the above

Passage II (Q. 4 to 6)

In an experimental setup to study the photoelectric effect a point source of light of power 3.2×10^{-3} *W was taken. The source can emit mono energetic photons of energy* 5 *eV and is located at a distance of* 0.8 *m from the centre of a stationary metallic sphere of work function* 3.0 *eV. The radius of the sphere is* $r = 8 \times 10^{-3}$ *m. The efficiency of photoelectric emission is one for every* 10^6 *incident photons.*

Based on the information given above answer the questions given below.

(Assume that the sphere is isolated and photo electrons are instantly swept away after the emission).

4. The de-Broglie wavelength of the fastest moving photoelectron is
(a) 6.63 Å (b) 8.69 Å
(c) 2 Å (d) 5.26 Å

5. It was observed that after some time emission of photoelectrons from the sphere stopped. Charge on the sphere when the photon emission stops is
(a) $16\pi\varepsilon_0 r$ coulomb
(b) $8\pi\varepsilon_0 r$ coulomb
(c) $15\pi\varepsilon_0 r$ coulomb
(d) $20\pi\varepsilon_0 r$ coulomb

6. Time after which photo-electric emission stops is
(a) 100 s (b) 121 s
(c) 111 s (d) 141 s

Passage III (Q. 7 to 9)

The atomic masses of the hydrogen isotopes are

$$\text{Hydrogen } m_1H^1 = 1.007825 \text{ amu}$$

$$\text{Deuterium } m_1H^2 = 2.014102 \text{ amu}$$

$$\text{Tritium } m_1H^3 = 3.016049 \text{ amu}$$

7. The energy released in the reaction

$${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 \text{ is nearly}$$

(a) 1 MeV (b) 2 MeV (c) 4 MeV (d) 8 MeV

8. The number of fusion reactions required to generate 1 kWh is nearly

(a) 10^8 (b) 10^{18} (c) 10^{28} (d) 10^{38}

9. The mass of deuterium, ${}_1H^2$ that would be needed to generate 1 kWh

(a) 3.7 kg (b) 3.7 g
(c) 3.7×10^{-5} kg (d) 3.7×10^{-8} kg

Assertion and Reason

Directions (Q. Nos. 1-20) *These questions consist of two statements each linked as Assertion and Reason. While answering of these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** By increasing the accelerating voltage in Coolidge tube wavelength of characteristic X-rays does not change.

Reason Cut-off wavelength is inversely proportional to the acceleration voltage.

2. **Assertion** If distance of the point source is increased from the photoelectric plate, then stopping potential will remain unchanged.

Reason Saturation current will decrease.

3. **Assertion** If red light is just able to emit photoelectrons from a metal surface, then violet light cannot emit the photoelectrons from that metal.

Reason Energy of violet light photons is more than the red light photons.

4. **Assertion** If wavelength of incident photons is halved, then maximum kinetic energy of photoelectrons will become two times.

Reason Energy of photon $\propto 1/\lambda$.

5. **Assertion** Second orbit circumference of hydrogen atom is two times the de-Broglie wavelength of electrons in that orbit.

Reason de-Broglie wavelength of electron in ground state is minimum.

6. **Assertion** A particle at rest breaks into two particles of different masses. They fly off in different directions. Their de-Broglie wavelengths will be different.

Reason Their speed will be different.

7. **Assertion** In one half-life of a radioactive substance more number of nuclei are decayed in one average life.

Reason Average life = Half - life/ In (2)

8. **Assertion** Two different radioactive substances have initially same number of nuclei. Their decay constants are λ_1 and λ_2 ($< \lambda_1$). Then, initially first radioactive substance decays at faster rate.

Reason Half-life of first radioactive substance is less.

9. **Assertion** In any nuclear process energy is released, if total binding energy of daughter nuclei is more than the total binding energy of parent nuclei.

Reason Binding energy per nucleon is of the order of MeV.

10. **Assertion** Density of a nucleus is independent of the mass number.

Reason Radius of a nucleus $\propto A^3$. Where, A is the mass number.

11. **Assertion** Energy E_1 is required to remove first electron from helium atom and energy E_2 is to required to remove the second electron. Then, $E_1 < E_2$.

Reason Ionisation energy of single electron of He^+ is 54.4 eV.

12. **Assertion** Both in fusion and fission reactions total binding energy of daughter nuclei is more than the total binding energy of parent nuclei.

Reason In any nuclear reaction energy is released, if total binding energy increases.

13. **Assertion** Angular momentum of single electron is any orbit of hydrogen type atoms is independent of the atomic number of the element.

Reason In ground state angular momentum is minimum.

14. **Assertion** Energy of characteristic X-rays is more than the energy of continuous X-rays.

Reason Characteristic X-rays are produced due to transition of electrons from higher energy states to lower energy states.

15. Assertion Wavelength of characteristic X-rays is given by

$$\frac{1}{\lambda} \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

in the transition from n_2 to n_1. In the above relation proportionality constant does not depend upon the target material.

Reason Continuous X-rays are target independent.

16. Assertion Wavelength of characteristic X-rays is given by

$$\frac{1}{\lambda} \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

in transition from n_2 to n_1. In the above relation proportionality constant is series dependent. For different series (K-series, L-series, etc.) value of this constant will be different.

Reason For L-series value of this constant is less than the value for K-series.

17. Assertion If high pressure is applied on a radioactive, substance rate of radioactivity will increase.

Reason Radioactivity is a nuclear process.

18. Assertion At time $t = 0$, activity of a radioactive, substance is 10 units. At $t = 1$ s it remains 90 units. Then at $t = 2$ s it should remain 80 units.

Reason In equal interval of time percentage change is same in a radioactive substance.

19. Assertion If light continuously falls on a metal surface, then emission of electrons will stop after some time.

Reason We cannot extract all the electrons of a metal.

20. Assertion Ionization power of α-particles is maximum while penetration power of γ-rays is maximum.

Reason α-particles are heaviest particles, while γ-rays have maximum energy.

Match the Columns

1. Ground state energy of hydrogen atom is E_0. Match the following two columns.

Column I	Column II
(A) Electrostatic potential energy in ground state of hydrogen atom	(p) E_0
(B) Total energy is first excited state of He^+ ion	(q) $-E_0$
(C) Kinetic energy of electron in first excited state of He^+ ion	(r) $2E_0$
(D) Kinetic energy of electron in ground state of hydrogen atom	(s) $-2E_0$

2. In Column I physical quantities corresponding to hydrogen and hydrogen like atoms are given. In Column II, powers of principal quantum number n are given on which those physical quantities depend. Match the two columns.

Column I	Column II
(A) Angular velocity of circular motion of electron	(p) –4
(B) Centripetal acceleration in circular motion of electron	(q) 1
(C) Angular momentum of electron	(r) –3
(D) Moment of inertia of electron about centroidal axis	(s) 4

3. Wavelength of X-ray line is λ_0. Match the following two columns for same element.

Column I	Column II
(A) Wavelength of K_α line	(p) $\frac{128}{135}\lambda_0$
(B) Wavelength of K_γ line	(q) $\frac{288}{45}\lambda_0$
(C) Wavelength of L_α line	(r) $\frac{128}{27}\lambda_0$
(D) Wavelength of L_β line	(s) $\frac{32}{27}\lambda_0$

4. From a radioactive substance x numbers of α-particles and y-numbers of β-particles are emitted. As a result atomic number decreases by n and mass number by m. Then, match the following two columns.

Column I	Column II
(A) $x = 1$ and $y = 2$	(p) $n = 2, m = 0$
(B) $x = 2$ and $y = 1$	(q) $n = 0, m = 4$
(C) $x = 0$ and $y = 2$	(r) $n = 2, m = 4$
(D) $x = 1$ and $y = 0$	(s) None

5. Wavelength of X-ray photon is a momentum of X-ray photon is b.

Now, match the following two columns.

Column I	Column II
(A) Wavelength of γ-ray photon	(p) $> a$
(B) Momentum of γ-ray photon	(q) $< a$
(C) Wavelength of UV ray photon	(r) $> b$
(D) Momentum of UV ray photon	(s) $< b$

Entrance Gallery

2014

1. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2, respectively. If the ratio $u_1 : u_2 = 2:1$ and $hc = 1240$ eV nm, the work function of the metal is nearly

[JEE Advanced]

(a) 3.7 eV (b) 3.2 eV (c) 2.8 eV (d) 2.5 eV

2. The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to **[JEE Main]**

(a) 1.8 eV (b) 1.1 eV
(c) 0.8 eV (d) 1.6 eV

3. Hydrogen ($_1H^1$), deuterium ($_1H^2$), singly ionised helium ($_2He^4$)$^+$ and doubly ionised lithium ($_3Li^8$)$^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wavelengths of emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4, respectively for four elements, then approximately which one of the following is correct? **[JEE Main]**

(a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$
(d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

4. During the propagation of electromagnetic waves in a medium, **[JEE Main]**

(a) electric energy density is double of the magnetic energy density
(b) electric energy density is half of the magnetic energy density
(c) electric energy density is equal to the magnetic energy density
(d) Both electric and magnetic energy densities are zero

5. What is the de-Broglie wavelength of the electron accelerated through a potential difference of 100 V? **[Karnataka CET]**

(a) 12.27 Å (b) 1.227 Å
(c) 0.1227 Å (d) 0.001227 Å

6. The maximum kinetic energy of the photoelectrons depends only on **[Karnataka CET]**

(a) potential
(b) frequency
(c) incident angle
(d) pressure

7. The relation between half-life (T) and decay constant (λ) is **[Karnataka CET]**

(a) $\lambda T = 1$ (b) $\lambda T = \frac{1}{2}$
(c) $\lambda T = \log_e 2$ (d) $\lambda = \log, 2T$

8. Which of the following spectral series of hydrogen atom is lying in visible range of electromagnetic wave? **[Karnataka CET]**

(a) Paschen series (b) Pfund series
(c) Lyman series (d) Balmer series

9. What is the energy of the electron revolving in third orbit expressed in eV? **[Karnataka CET]**

(a) 1.51 eV (b) 3.4 eV
(c) 4.53 eV (d) 4 eV

10. The ionisation energy of hydrogen is 13.6 eV. The energy of the photon released when an electron jumps from the first excited state ($n = 2$) to the ground state of a hydrogen atom is **[WB JEE]**

(a) 3.4 eV (b) 4.53 eV
(c) 10.2 eV (d) 13.6 eV

11. Find the incorrect statement(s) about photoelectric effect. **[WB JEE]**

(a) There is no significant time delay between the absorption of a suitable radiation and the emission of electrons
(b) Einstein analysis gives a threshold frequency above which no electron can be emitted
(c) The maximum kinetic energy of the emitted photoelectrons is proportional to the frequency of incident radiation
(d) The maximum kinetic energy of electrons does not depend on the intensity of radiation

12. For the radioactive nuclei that undergo either α or β decay, which one of the following cannot occur? **[WBJEE]**

(a) Isobar of original nucleus is produced
(b) Isotope of original nucleus is produced
(c) Nuclei with higher atomic number that of the original nucleus is produced
(d) Nuclei with lower atomic number than that of the original nucleus is produced

13. Which of the following device is the application of photoelectric effect? **[J&K CET]**

(a) Light emitting diode
(b) Diode
(c) Photocell
(d) Transistor

14. A monochromatic light of frequency 3×10^{14} Hz, is produced by a laser, emits the power of 3×10^{-3} W. Find, how many number of photons are emitted per second? **[J&K CET]**

(a) 1.5×10^{16} (b) 2.5×10^{16}
(c) 4.5×10^{16} (d) 8.5×10^{16}

15. The ratio of radii of nuclei of two atoms of element of atomic mass numbers 27 and 64 is **[J&K CET]**

(a) 3 : 4 (b) 4 : 3
(c) 9 : 16 (d) 16 : 9

16. The value of Rydberg constant is **[J&K CET]**

(a) 1.997×10^7 m^{-1}
(b) 1.097×10^{-7} m^{-1}
(c) 1.097×10^7 m^{-1}
(d) 19.97×10^7 m^{-1}

17. The total energy of an electron in 4th orbit of hydrogen atom is **[J&K CET]**

(a) – 13.6 eV (b) – 3.4 eV
(c) – 1.51 eV (d) – 0.85 eV

2013

18. The work functions of silver and sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential *versus* frequency plot for silver to that of sodium is [JEE Advanced]

(a) 4 (b) 3
(c) 1 (d) 7

19. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de-Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is [JEE Advanced]

(a) 3 (b) 4
(c) 2 (d) 4.5

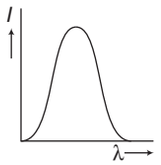
20. The correct statement is [JEE Advanced]

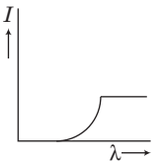
(a) The nucleus ${}^{6}_{3}\text{Li}$ can emit an α-particle
(b) The nucleus ${}^{210}_{84}\text{Po}$ can emit a proton
(c) Deuteron and α-particle can undergo complete fusion
(d) The nuclei ${}^{70}_{30}\text{Zn}$ and ${}^{82}_{34}\text{Se}$ can undergo complete fusion

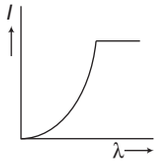
21. The kinetic energy (in keV) of the α- particle, when the nucleus ${}^{210}_{84}\text{Po}$ at rest undergoes α- decay, is [JEE Advanced]

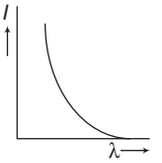
(a) 5316 KeV (b) 5422 KeV
(c) 5707 KeV (d) 5818 KeV

22. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current *I* of photocell varies as follows: [JEE Main]

(a) 

(b) 

(c) 

(d) 

23. In a hydrogen like atom, electron makes transition from an energy level with quantum number *n* to another with quantum number $(n-1)$. If $n >> 1$, the frequency of radiation emitted is proportional to [JEE Main]

(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$ (c) $\frac{1}{n^4}$ (d) $\frac{1}{n^3}$

24. The work function for aluminium is 4.125 eV. The cut-off wavelength for photoelectric effect for aluminium will be [Karnataka CET]

(a) 420 nm (b) 350 nm (c) 300 nm (d) 200 nm

25. If the ratio of radii of two spheres of same material is 1 : 4, then the ratio of their heat capacities is [Karnataka CET]

(a) $\frac{1}{64}$ (b) $\frac{1}{32}$ (c) $\frac{1}{16}$ (d) $\frac{1}{4}$

26. If *R* is the Rydberg's constant for hydrogen the wave number of the first line in the Lyman series will be [Karnataka CET]

(a) $\frac{R}{4}$ (b) $\frac{3R}{4}$ (c) $\frac{R}{2}$ (d) $2R$

27. A radioactive substance has an average life of 5 h. In a time of 5 h [Karnataka CET]

(a) half of the active nuclei decay
(b) less than half of the active nuclei decay
(c) more than half of the active nuclei decay
(d) all active nuclei decay

28. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately [OJEE]

(a) 540 nm (b) 400 nm
(c) 310 nm (d) 220 nm

29. Work function of a metal is 2.1 eV. Which of the waves of the following wavelengths will be able to emit photoelectrons from its surface? [OJEE]

(a) 4000 Å, 7500 Å (b) 5500 Å, 6000 Å
(c) 4000 Å, 6000 Å (d) None of these

30. What is de-Broglie wavelength of electron having energy 10 keV? [OJEE]

(a) 0.12 Å (b) 1.2 Å
(c) 12.2 Å (d) 2.12 Å

31. In a radioactive disintegration, the ratio of initial number of atoms to the number of atoms present at an instant of time equal to its mean life is [OJEE]

(a) $\frac{1}{e^2}$ (b) $\frac{1}{e}$
(c) e (d) e^2

32. In any fission process the ratio $= \frac{\text{mass of fission products}}{\text{mass of parent nucleus}}$ is [OJEE]

(a) less than 1
(b) greater than 1
(c) equal to 1
(d) depends on the mass of parent nucleus

33. In fog, photographs of the objects taken with infrared radiations are more clear than those obtained during visible light, because [OJEE]

(a) IR radiation has lesser wavelength than visible radiation
(b) scattering of IR light is more than visible light
(c) the intensity of IR light from the object is less
(d) scattering of IR light is less than visible light

34. When a certain metal surface is illuminated with light of frequency ν, the stopping potential for photoelectric current is V_0. When the same surface is illuminated by light of frequency $\frac{\nu}{2}$, the stopping potential is $\frac{V_0}{4}$. The threshold frequency for photoelectric emission is **[WB JEE]**

(a) $\frac{\nu}{6}$ (b) $\frac{\nu}{3}$
(c) $\frac{2\nu}{3}$ (d) $\frac{4\nu}{3}$

35. The de-Broglie wavelength of an electron is the same as that of a 50 keV X-ray photon. The ratio of the energy of the photon to the kinetic energy of the electron is (the energy equivalent of electron mass is 0.5 MeV) **[WB JEE]**

(a) 1 : 50 (b) 1 : 2 (c) 20 : 1 (d) 50 : 1

36. Which of the following is absorbed by the ozone layer? **[MHT CET]**

(a) γ - rays (b) Visible light
(c) Radio waves (d) Ultraviolet rays

2012

37. A proton is fired from very far away towards a nucleus with charge $Q = 120\,e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de-Broglie wavelength (in units of Fm) of the proton at its start is

[Take, the proton mass, $m_p = (5/3) \times 10^{-27}$ kg,

$$h/e = 4.2 \times 10^{-15} \text{ J-s/C},$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ m/F}, 1\text{ Fm} = 10^{-15} \text{ m}]$$

[IIT JEE]

(a) 7 Fm (b) 8 Fm (c) 10 Fm (d) 2 Fm

38. What is the maximum energy of the anti-neutrino? **[IIT JEE]**

(a) Zero
(b) Much less that 0.8×10^6 eV
(c) Nearly 0.8×10^6 eV
(d) Much larger than 0.8×10^6 eV

39. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then, the number of spectral lines in the emission spectra will be **[AIEEE]**

(a) 2 (b) 3 (c) 5 (d) 6

40. Assume that a neutron breaks into a proton and an electron. The energy released during this process is (Mass of neutron $= 1.6725 \times 10^{-27}$ kg, mass of electron $= 9 \times 10^{-31}$ kg,) **[AIEEE]**

(a) 0.73 MeV (b) 7.10 MeV
(c) 6.30 MeV (d) 5.4 MeV

41. An electromagnetic wave in vacuum has the electric and magnetic fields **E** and **B**, which are always perpendicular to each other. The direction of polarisation is given by **X** and that of wave propagation by **k.** Then, **[AIEEE]**

(a) **X** || **B** and **k** || **B** × **E**
(b) **X** || **E** and **k** || **E** × **B**
(c) **X** || **B** and **k** || **E** × **B**
(d) **X** || **E** and **k** || **B** × **E**

42. λ of a particle is 2000 Å and KE is 1 eV. If its KE is 1 MeV then, its λ is **[OJEE]**

(a) 3 Å (b) 200 Å
(c) 2 Å (d) 20 Å

43. A radioactive substance decays at the rate 5000 disintegration per minute. After 5 min, it disintegrates at 1250 disintegration per min. The decay constant is **[OJEE]**

(a) 0.4 ln 2 min^{-1} (b) 0.5 ln 3 min^{-1}
(c) 0.5 ln 2 min^{-2} (d) 0.2 ln 2 min^{-1}

44. $X + {}^{14}\text{N}_7 \longrightarrow {}^{17}\text{O}_8 + {}^{1}\text{P}_1$. What is the X? **[OJEE]**

(a) Helium (b) Deutron
(c) Neutrino (d) Positron

45. The de-Broglie wavelength of an electron moving with a velocity $c/2$ (c = velocity of light in vacuum) is equal to the wavelength of a photon. The ratio of the kinetic energies of electron and photon is **[WB JEE]**

(a) 1 : 4 (b) 1 : 2
(c) 1 : 1 (d) 2 : 1

46. Two elements A and B with atomic numbers Z_A and Z_B are used to produce characteristic, X-rays with frequencies ν_A and ν_B respectively. If $Z_A : Z_B = 1:2$, then $\nu_A : \nu_B$ will be **[WB JEE]**

(a) $1:\sqrt{2}$
(b) 1 : 8
(c) 4 : 1
(d) 1 : 4

2011

47. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly ionised helium atom is **[IIT JEE]**

(a) 1215 Å (b) 1640 Å
(c) 2430 Å (d) 4687 Å

48. Energy required for the electron excitation in Li^{2+} from the first to the third Bohr orbit is **[AIEEE]**

(a) 36.3 eV
(b) 108.8 eV
(c) 122.4 eV
(d) 12.1 eV

49. **Statement I** A nucleus having energy E_1 decays be β^- emission to daughter nucleus having energy E_2, but the β^- rays are emitted with a continuous energy spectrum having end point energy $E_1 - E_2$.

Statement II To conserve energy and momentum in β-decay at least three particles must take part in the transformation. **[AIEEE]**

(a) Statement I is correct, Statement II is incorrect
(b) Statement I is incorrect, Statement II is correct
(c) Statement I is incorrect, Statement II is incorrect, Statement II is the correct explanation of Statement I
(d) Statement I is correct, Statement II is incorrect, Statement II is not the correct explanation of Statement I

50. After absorbing a slowly moving neutron of mass m_N (momentum ~ 0) a nucleus of mass M breaks into two nuclei of masses m_1 and $5m_1$ $(6m_1 = M + m_N)$, respectively. If the de-Broglie wavelength of the nucleus with mass m_1 is λ, then the de-Broglie wavelength of the other nucleus will be **[AIEEE]**

(a) 25λ (b) 5λ (c) $\frac{\lambda}{5}$ (d) λ

51. The half-life of a radioactive substance is 20 min. The approximate time interval $(t_2 - t_1)$ between the time t_2 when 2/3 of it has decayed and time t_1 when 1/3 of it had decayed is **[AIEEE]**

(a) 14 min (b) 20 min (c) 28 min (d) 7 min

52. If the binding energy per nucleon of deutron is 1.115 MeV, its mass defect in atomic mass unit is **[Kerala CEE]**

(a) 0.0048 (b) 0.0024 (c) 0.0012 (d) 0.0006
(e) 2.230

53. The electric field of an electromagnetic wave travelling through vacuum is given by the equation $E = E_0 \sin(kx - \omega t)$. The quantity that is independent of wavelength is **[Kerala CEE]**

(a) $\frac{k}{\omega}$ (b) $k\omega$ (c) ω (d) k
(e) $k^2\omega$

54. The electric field of a plane electromagnetic wave varies with time of amplitude 2 Vm^{-1} propagating along Z-axis. The average energy density of the magnetic field is (in Jm^{-3}) **[Kerala CEE]**

(a) 13.29×10^{-12} (b) 8.85×10^{-12}
(c) 17.72×10^{-12} (d) 4.43×10^{-12}
(e) 2.22×10^{-12}

55. An electron of mass m_e and a proton of mass m_p are moving with the same speed. The ratio of their de-Broglie wavelength λ_e / λ_p is **[Karnataka CET]**

(a) 918 (b) $\frac{1}{1836}$ (c) 1836 (d) 1

56. If the binding energy per nucleon of deutron is 1.115 MeV, its mass defect in atomic mass unit is **[Kerala CEE]**

(a) 0.0048 (b) 0.0024 (c) 0.0012 (d) 0.0006
(e) 2.230

57. When an electron jumps from the orbit $n = 2$ to $n = 4$, then wavelength of the radiations absorbed will be (R is Rydberg's constant) **[Karnataka CET]**

(a) $\frac{3R}{16}$ (b) $\frac{5R}{16}$ (c) $\frac{16}{5R}$ (d) $\frac{16}{3R}$

58. The ratio of minimum wavelength of Lyman and Balmer series will be **[Karnataka CET]**

(a) 10 (b) 5
(c) 0.25 (d) 1.25

59. The fraction of the initial number of radioactive nuclei which remain undecayed after half of a half-life of the radioactive sample is **[Karnataka CET]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$
(c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{4}$

60. 1 curie represents **[Karnataka CET]**

(a) 1 disintegration per second
(b) 10^6 disintegration per second
(c) 3.7×10^{10} disintegration per second
(d) 3.7×10^7 disintegration per second

61. The photoelectric threshold wavelength for silver is λ_0. The energy of the electron ejected from the surface of silver by an incident wavelength λ $(\lambda < \lambda_0)$ will be **[Karnataka CET]**

(a) $hc\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)$ (b) $\frac{h}{c}\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)$
(c) $\frac{hc}{\lambda_0 - \lambda}$ (d) $hc(\lambda_0 - \lambda)$

62. In an inelastic collision, an electron excites a hydrogen atom from its ground state to a M-shell state. A second electron collides instantaneously with the excited hydrogen atom in the M-state and ionises it. At least how much energy the second electron transfers to the atom in the M-state? **[WB JEE]**

(a) + 3.4 eV
(b) + 1.51 eV
(c) – 3.4 eV
(d) – 1.51 eV

63. In the nuclear reaction

${}^{14}_{7}\mathrm{N} + X \rightarrow {}^{14}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H}$, the X will be **[WB JEE]**

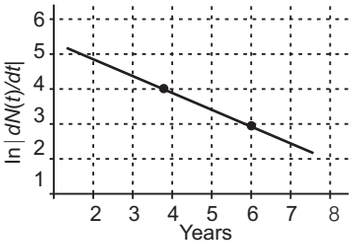
(a) ${}^{0}_{-1}e$ (b) ${}^{1}_{1}\mathrm{H}$
(c) ${}^{2}_{1}\mathrm{H}$ (d) ${}^{1}_{0}n$

2010

64. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de-Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is **[IIT JEE]**

(a) 3 (b) 4
(c) 2 (d) 4.5

65. To determine the half-life of a radioactive element, a student plots a graph of $\ln\left|\frac{dN(t)}{dt}\right|$ *versus* t. Where, $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 yr, the value of p is **[IIT JEE]**



(a) 8 (b) 7
(c) 4 (d) 8.5

66. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81\pi}{7}\times 10^5$ Vm^{-1}. When the field is switched off, the drop is observed to fall with terminal velocity 2×10^{-3} ms^{-1}. Given, $g = 9.8\,\text{ms}^{-2}$, viscosity of the air $= 1.8\times 10^{-5}$ N-s m^{-2} and the density of oil $= 900$ kgm^{-3}, the magnitude of q is **[AIEEE]**

(a) 1.6×10^{-19} C (b) 3.2×10^{-19} C
(c) 4.8×10^{-19} C (d) 8.0×10^{-19} C

67. **Assertion** (A) When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is $K_{\max}$. When the ultraviolet light is replaced by X-rays, both V_0 and $K_{\max}$ increase.

Reason (R) Photoelectrons are emitted with speeds ranging from zero to a maximum value, because of the range of frequencies present in the incident light. **[AIEEE]**

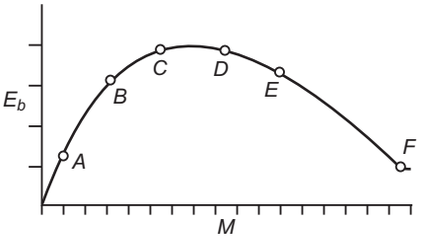
(a) If both A and R are correct and R is the correct explanation of A.
(b) If both A and R are correct but R is not the correct explanation of A
(c) If A is correct but R is incorrect
(d) If A is incorrect but R is correct.

68. If a source of power 4 kW produces 10^{20} photon/s the radiation belong to a part of the spectrum called **[AIEEE]**

(a) X-rays (b) ultraviolet rays
(c) microwaves (d) γ-rays

69. The above is a plot of binding energy per nucleon E_b, against the nuclear mass M, A, B, C, D, E and F correspond to different nuclei. Consider and four reactions

I. $A + B \to C + \varepsilon$ II. $C \to A + B + \varepsilon$
III. $D + E \to F + \varepsilon$ IV. $F \to D + E + \varepsilon$



where, ε is the energy released? In which reaction is ε positive? **[AIEEE]**

(a) I and IV (b) I and III
(c) II and IV (d) II and III

70. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be **[AIEEE]**

(a) $\frac{A-Z-8}{Z-4}$ (b) $\frac{A-Z-4}{Z-8}$
(c) $\frac{A-Z-12}{Z-4}$ (d) $\frac{A-Z-4}{Z-2}$

71. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then, **[AIEEE]**

(a) $E_2 = 2E_1$ (b) $E_1 > E_2$
(c) $E_2 > E_1$ (d) $E_1 = 2E_2$

72. The speed of daughter nuclei is **[AIEEE]**

(a) $c\frac{\Delta m}{M+\Delta m}$ (b) $c\sqrt{\frac{2\Delta m}{M}}$ (c) $c\sqrt{\frac{\Delta m}{M}}$ (d) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$

73. The de-Broglie wavelength of the electron in the ground state of the hydrogen atom is (radius of the first orbit of hydrogen atom = 0.53 Å) **[Karnataka CET]**

(a) 1.67Å (b) 3.33Å (c) 1.06Å (d) 0.53Å

74. According to Einstein's photoelectric equation, the graph of KE of the photoelectron emitted from the metal *versus* the frequency of the incident radiation gives a straight line graph, whose slope **[Karnataka CET]**

(a) depends on the intensity of the incident radiation
(b) depends on the nature of the metal and also on the intensity of incident radiation
(c) is same for all metals and independent of the intensity of the incident radiation
(d) depends on the nature of the metal

75. An electron is moving in an orbit of a hydrogen atom from which there can be a maximum of six transition. An electron is moving in an orbit of another hydrogen atom from which there can be a maximum of three transition. The ratio of the velocities of the electron in these two orbits is **[Karnataka CET]**

(a) $\frac{1}{2}$

(b) $\frac{2}{1}$

(c) $\frac{5}{4}$

(d) $\frac{3}{4}$

76. ν_1 is the frequency of the series limit of Lyman series, ν_2 is the frequency of the first line of Lyman series and ν_3 is the frequency of the series limit of the Balmer series. Then, **[Karnataka CET]**

(a) $\nu_1 - \nu_2 = \nu_3$

(b) $\nu_1 = \nu_2 - \nu_3$

(c) $\frac{1}{\nu_2} = \frac{1}{\nu_1} + \frac{1}{\nu_3}$

(d) $\frac{1}{\nu_1} = \frac{1}{\nu_2} + \frac{1}{\nu_3}$

77. The spectrum of an oil flame is an example for **[Karnataka CET]**

(a) line emission spectrum
(b) continuous emission spectrum
(c) line absorption spectrum
(d) band emission spectrum

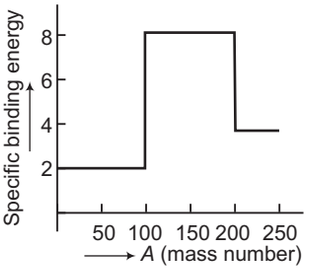
78. A radioactive sample S_1 having the activity A_1 has twice the number of nuclei as another sample S_2 of activity A_2. If $A_2 = 2A_1$, then the ratio of half-life of S_1 to the half-life of S_2 is **[Karnataka CET]**

(a) 4 (b) 2 (c) 0.25 (d) 0.75

79. When a neutron is disintegrated to give a β-particle,

(a) a neutrino alone is emitted **[Karnataka CET]**
(b) a proton and neutrino are emitted
(c) a proton alone is emitted
(d) a proton and an anti-neutrino are emitted

80. Assume the graph of specific binding energy *versus* mass number is as shown in the figure. Using this graph, select the correct choice from the following. **[Karnataka CET]**



(a) Fusion of two nuclei of mass number lying in the range of $100 < A < 200$ will release energy
(b) Fusion of two nuclei of mass number lying in the range of $51 < A < 100$ will release energy
(c) Fusion of two nuclei of mass number lying in the range of $1 < A < 50$ will release energy
(d) Fission of the nucleus of mass number lying in the rang of $100 < A < 200$ will release energy when broken into two fragments

81. Pick out the correct statement from the following. **[Karnataka CET]**

(a) Energy released per unit mass of the reactant is less in case of fusion reaction
(b) Packing fraction may be positive or may be negative
(c) Pu^{239} is not suitable for a fission reaction
(d) For stable nucleus, the specific binding energy is low

82. The speed of electromagnetic waves in vacuum depends upon the source of radiation. It **[Karnataka CET]**

(a) increases as we move from γ-rays to radio waves
(b) decreases as we move from γ-rays to radio waves
(c) is same for all of them
(d) None of the above

83. The de-Broglie wavelength of an electron in the ground state of the hydrogen atom is **[MHT CET]**

(a) πr^2 (b) $2\pi r$
(c) πr (d) $\sqrt{2\pi r}$

84. The orbital frequency of an electron in the hydrogen atom is proportional to **[MHT CET]**

(a) n^3 (b) n^{-3}
(c) n (d) n^0

85. The product of linear momentum and angular momentum of an electron of the hydrogen atom is proportional to n^x, where x is **[MHT CET]**

(a) 0 (b) 1
(c) –2 (d) 2

Answers

Level 1

Objective Problems

1. (d)	**2.** (c)	**3.** (a)	**4.** (a)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (a)	**10.** (c)
11. (a)	**12.** (c)	**13.** (d)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (c)
21. (d)	**22.** (c)	**23.** (b)	**24.** (c)	**25.** (a)	**26.** (c)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (d)
31. (c)	**32.** (d)	**33.** (b)	**34.** (b)	**35.** (b)	**36.** (c)	**37.** (b)	**38.** (b)	**39.** (d)	**40.** (c)
41. (a)	**42.** (d)	**43.** (c)	**44.** (b)	**45.** (d)	**46.** (d)	**47.** (d)	**48.** (a)	**49.** (b)	**50.** (a)
51. (a)	**52.** (d)	**53.** (d)	**54.** (d)	**55.** (b)	**56.** (d)	**57.** (b)	**58.** (a)	**59.** (c)	**60.** (d)
61. (c)	**62.** (a)	**63.** (d)	**64.** (b)	**65.** (a)	**66.** (b)	**67.** (a)	**68.** (b)	**69.** (d)	**70.** (d)
71. (b)	**72.** (b)	**73.** (b)	**74.** (a)	**75.** (b)	**76.** (d)	**77.** (d)	**78.** (b)	**79.** (a)	**80.** (b)
81. (b)	**82.** (a)	**83.** (c)	**84.** (c)	**85.** (c)	**86.** (d)	**87.** (a)	**88.** (a)	**89.** (d)	**90.** (c)
91. (b)	**92.** (a)	**93.** (d)	**94.** (c)	**95.** (a)	**96.** (c)	**97.** (c)	**98.** (d)	**99.** (d)	**100.** (d)
101. (c)	**102.** (d)	**103.** (d)	**104.** (b)	**105.** (c)	**106.** (b)	**107.** (c)	**108.** (b)	**109.** (b)	**110.** (d)
111. (b)	**112.** (b)	**113.** (c)	**114.** (b)	**115.** (a)	**116.** (a)	**117.** (b)	**118.** (c)	**119.** (b)	**120.** (b)
121. (b)	**122.** (c)	**123.** (c)	**124.** (b,c)	**125.** (b)	**126.** (a)	**127.** (a)	**128.** (b)	**129.** (a)	**130.** (d)
131. (a)	**132.** (d)	**133.** (c)	**134.** (d)	**135.** (a)	**136.** (d)	**137.** (c)	**138.** (a)	**139.** (d)	**140.** (d)
141. (c)	**142.** (a)	**143.** (c)	**144.** (a)	**145.** (b)	**146.** (b)	**147.** (d)	**148.** (a)	**149.** (c)	**150.** (d)
151. (d)	**152.** (d)	**153.** (d)	**154.** (b)	**155.** (b)	**156.** (a)	**157.** (c)	**158.** (b)	**159.** (a)	**160.** (b)
161. (b)	**162.** (b)	**163.** (c)	**164.** (d)	**165.** (b)	**166.** (c)	**167.** (c)	**168.** (c)	**169.** (a)	**170.** (b)
171. (a)	**172.** (b)	**173.** (a)	**174.** (d)	**175.** (c)	**176.** (c)	**177.** (d)	**178.** (a)	**179.** (b)	**180.** (c)
181. (b)	**182.** (d)	**183.** (a)	**184.** (b)	**185.** (b)	**186.** (c)	**187.** (c)	**188.** (a)	**189.** (a)	**190.** (b)
191. (c)	**192.** (c)	**193.** (b)	**194.** (d)	**195.** (c)	**196.** (a)	**197.** (c)	**198.** (c)	**199.** (a)	**200.** (c)
201. (b)	**202.** (d)	**203.** (d)	**204.** (c)	**205.** (b)	**206.** (c)	**207.** (d)	**208.** (d)	**209.** (a)	**210.** (d)
211. (b)	**212.** (b)	**213.** (c)	**214.** (b)	**215.** (d)	**216.** (a)	**217.** (d)	**218.** (c)	**219.** (b)	**220.** (b)
221. (c)	**222.** (a)	**223.** (a)	**224.** (a)	**225.** (c)	**226.** (c)	**227.** (d)			

Level 2

Only One Correct Option

1. (b)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (c)
11. (c)	**12.** (b)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (a)	**17.** (d)	**18.** (c)	**19.** (b)	**20.** (d)
21. (a)	**22.** (b)	**23.** (c)	**24.** (c)	**25.** (a)	**26.** (c)	**27.** (b)	**28.** (d)	**29.** (a)	**30.** (a)
31. (a)	**32.** (a)	**33.** (a)	**34.** (d)	**35.** (a)	**36.** (a,b)	**37.** (d)	**38.** (b)	**39.** (a)	**40.** (c)
41. (c)	**42.** (b)	**43.** (b)	**44.** (c)	**45.** (c)	**46.** (a)	**47.** (d)	**48.** (b)	**49.** (d)	**50.** (a)
51. (c)	**52.** (c)	**53.** (a)	**54.** (a)	**55.** (c)	**56.** (b)	**57.** (a)	**58.** (b)	**59.** (b)	

More than One Correct Options

1. (b,c)	**2.** (a,c)	**3.** (a,b,c)	**4.** (a,c)	**5.** (a,b,d)	**6.** (a,b)	**7.** (a,b,d)	**8.** (all)	**9.** (a,b,c)	**10.** (b,c)
11. (all)	**12.** (b,c)	**13.** (c,d)	**14.** (a,c)	**15.** (b,c)	**16.** (a,b,c)	**17.** (b,d)	**18.** (a,c)	**19.** (a,b)	**20.** (a,b)
21. (b,d)	**22.** (b,d)	**23.** (a,b)	**24.** (b,d)	**25.** (c,d)					

Comprehension Based Questions

1. (c)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (d)

Assertion and Reason

1. (b)	**2.** (b)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (c)
11. (b)	**12.** (a)	**13.** (b)	**14.** (d)	**15.** (d)	**16.** (b)	**17.** (d)	**18.** (d)	**19.** (c)	**20.** (a)

Match the Columns

1. (A→r; B→p; C→q; D→q) **2.** (A→r; B→p; C→q; D→s) **3.** (A→s; B→p; C→q; D→r)
4. (A→q; B→s; C→s; D→r) **5.** (A→q; B→r; C→p; D→s)

Entrance Gallery

1. (a)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (c)
11. (b)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (d)	**18.** (c)	**19.** (a)	**20.** (b)
21. (a)	**22.** (b)	**23.** (d)	**24.** (c)	**25.** (a)	**26.** (b)	**27.** (d)	**28.** (c)	**29.** (d)	**30.** (a)
31. (c)	**32.** (b)	**33.** (d)	**34.** (b)	**35.** (c)	**36.** (d)	**37.** (a)	**38.** (c)	**39.** (d)	**40.** (a)
41. (b)	**42.** (c)	**43.** (a)	**44.** (a)	**45.** (b)	**46.** (d)	**47.** (a)	**48.** (b)	**48.** (d)	**50.** (d)
51. (b)	**52.** (b)	**53.** (a)	**54.** (b)	**55.** (c)	**56.** (b)	**57.** (d)	**58.** (c)	**59.** (a)	**60.** (c)
61. (a)	**62.** (d)	**63.** (a)	**64.** (a)	**65.** (a)	**66.** (d)	**67.** (d)	**68.** (a)	**69.** (a)	**70.** (b)
71. (c)	**72.** (b)	**73.** (b)	**74.** (c)	**75.** (d)	**76.** (a)	**77.** (b)	**78.** (a)	**79.** (d)	**80.** (b)
81. (b)	**82.** (c)	**83.** (b)	**84.** (b)	**85.** (a)					

Solutions

Level 1 : Objective Problems

1. Momentum, $p = \frac{E}{c} = \frac{h\nu}{c}$

$\therefore \quad \nu = \frac{pc}{h} = \frac{3.3\times10^{-29}\times3\times10^{8}}{6.6\times10^{-34}}$

$= 1.5\times10^{13}$ Hz

2. Momentum, $p = \frac{E}{c}$

$\Rightarrow \quad E = pc = 2\times10^{-16}\times(3\times10^{10})$

$= 6\times10^{-6}$ erg

4. Momentum, $p = \frac{h}{\lambda} = \frac{6.6\times10^{-34}}{(5000\times10^{-10})}$

$= 1.3\times10^{-27}$ kg - m/s

5. As, $E = h\nu = mc^2 \quad \Rightarrow \quad \therefore \quad m = \frac{h\nu}{c^2}$

6. Energy (in eV) $= \frac{12375}{\lambda(\text{Å})} \Rightarrow \lambda = \frac{12375}{2.48} = 4989.8\ \text{Å} \approx 5000\ \text{Å}$

7. Power, $P = \frac{nhc}{\lambda t}$

$\therefore \quad 100 = \frac{n\times6\times10^{-34}\times3\times10^{8}}{540\times10^{-9}\times1}$

$\Rightarrow \quad n = 3\times10^{20}$

8. Momentum, $p = \frac{h}{\lambda}$ or $E = h\nu = \frac{hc}{\lambda} = pc$

So, if the energy of photon is increased by a factor of 4, then its momentum increases by a factor of 4.

9. Momentum, $p = \frac{E}{c}$

$\therefore \quad E = pc$

$\Rightarrow \quad E^2 = p^2c^2$

10. $\Delta E\,(\text{in eV}) = \frac{12375}{\lambda\,(\text{in Å})} = \frac{12375}{7000} = 1.76$ eV

11. $p = \frac{h\nu}{c}$

$\therefore \quad \nu = \frac{Pc}{h} = \frac{(6.6\times10^{-29})\ (3\times10^{8})}{(6.6\times10^{-34})}0$

$= 3\times10^{13}$ Hz

12. Power, $P = \frac{nhc}{\lambda t}$

$\therefore \quad 200 = \frac{n\times6\times10^{-34}\times3\times10^{8}}{340\times10^{-9}\times1}$

$\Rightarrow \quad n = \frac{200\times340\times10^{-9}}{6\times10^{-34}\times3\times10^{8}}$

$= 3.78\times10^{20}$

15. According to Bohr's first postulate

$mvr = \frac{nh}{2\pi}$

$\therefore \quad 2\pi r = n\left(\frac{h}{mv}\right) = n\lambda$

For $n = 1$, $\lambda = 2\pi r$

16. For $n = 2$, $E_2 = -\frac{13.6}{(2)^2} = -3.4$ eV

For $n = 1$, $E_1 = -13.6$ eV

$\therefore \quad E_{1\to2} = -3.4 - (-13.6)$

$= 10.2$ eV

18. Final energy of electron

$= -13.6 + 12.1 = -1.51$ eV

This energy corresponds to third level, i.e. $n = 3$. Hence, number of spectral lines emitted

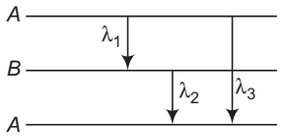
$= \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$

19. Let energy corresponding to state A, B and C be E_A, E_B and E_C.
So, from figure

$$(E_C - E_B) + (E_B - E_A) = (E_C - E_A)$$

or $$\frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = \frac{hc}{\lambda_3}$$

$\Rightarrow$ $$\lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$



20. Wavelength of spectral line in Balmer series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

For first line of Balmer series, $n = 3$

$\therefore$ $$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

For second line of Balmer series, $n = 4$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16}$$

$\therefore$ $$\frac{\lambda_2}{\lambda_1} = \frac{20}{27}$$

$\therefore$ $$\lambda_1 = \frac{20}{27} \times 6561 = 4860 \text{ Å}$$

21. If E is the energy radiated in transition, then from figure

$$E_{R \to G} > E_{Q \to S} > E_{R \to S} > E_{Q \to R} > E_{P \to Q}$$

For obtaining blue line energy radiated should be more.
Hence, correct option is (d).

22. Wave number is reciprocal of wavelength,

$\therefore$ $$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$= R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R}{16}$$

23. For $n = 3$ to $n = 2$ transition

$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

For $n = 4$ to $n = 2$ transition

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16}$$

$\therefore$ $$\frac{\lambda_2}{\lambda_1} = \frac{20}{27}$$

$\therefore$ $$\lambda_2 = \frac{20}{27}\lambda_1 = \frac{20}{27}\lambda_0$$

24. Change in angular momentum of electron

$$\Delta L = L_2 - L_1 = \frac{n_2 h}{2\pi} - \frac{n_1 h}{2\pi}$$

or $$\Delta L = \frac{h}{2\pi}(n_2 - n_1)$$

$$= \frac{6.6 \times 10^{-34}}{2 \times 3.14}(5 - 4) = 1.05 \times 10^{-34} \text{ J-s}$$

27. Wavelength,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{970.6 \times 10^{-10}} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

$\therefore$ $$n_2 = 4$$

So, number of emission lines

$$N = \frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$$

28. Excitation energy,

$$\Delta E = E_2 - E_1 = 13.6 Z^2 \left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$40.8 = 13.6 Z^2 \times \frac{3}{4}$$

$\therefore$ $$Z = 2$$

So, required energy to remove the electron from ground state

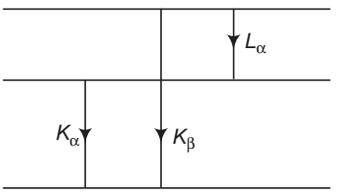
$$= + \frac{13.6 Z^2}{(1)^2} = 13.6 (Z)^2 = 54.4 \text{ eV}$$

29. $r \propto n^2$

30. $K_\beta = K_\alpha + L_\alpha$

$\therefore$ $$h\gamma_2 = h\gamma_1 + h\gamma_3$$

or $$\gamma_2 = \gamma_1 + \gamma_3$$



32. $\lambda \propto \frac{1}{\Delta E}$

$\therefore$ $$\frac{\lambda'}{\lambda} = \frac{(2E - E)}{(4E/3 - E)} = \frac{1}{1/3} = 3$$

$\therefore$ $$\lambda' = 3\lambda$$

36. $\lambda \propto \frac{1}{\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$ $\Rightarrow$ $\frac{\lambda_{\text{longest}}}{\lambda_{\text{shortest}}} = \frac{\left(\frac{1}{4} - \frac{1}{0}\right)}{\left(\frac{1}{4} - \frac{1}{9}\right)} = \frac{(1/4)}{(5/36)} = \frac{9}{5}$

37. $E_3 - E_2 = E$

or $$\frac{E_1}{9} - \frac{E_1}{4} = E \quad \text{or} \quad E_1 = -7.2\,E$$

$\therefore$ Ionisation energy of hydrogen atom is $7.2\,E$.

39. $L_n = \frac{nh}{2\pi}$

$$I\omega = \frac{nh}{2\pi} \qquad (mr^2)\omega = \frac{nh}{2\pi}$$

As $r \propto n^2$

$\therefore$ $$\omega \propto \frac{1}{n^3}$$

40. $Z^2 = \left(\frac{13.6}{4} - \frac{13.6}{9}\right) = 47.2$

$\therefore$ $$Z = 5$$

41. In hydrogen atom, $E_2 - E_1 = 10.2$ eV

Since, $5\,\text{eV} < 10.2\,\text{eV}$

The electron cannot excite the hydrogen atom. The collision must be therefore elastic.

42. $E \propto Z^2$

$\therefore$ Energy required $= (3)^2\,(13.6)\,\text{eV} = 122.4\,\text{eV}$

44. $\lambda \propto \dfrac{1}{\left(\dfrac{1}{n_f^{\,2}} - \dfrac{1}{n_i^{\,2}}\right)}$

$\therefore \quad \dfrac{\lambda}{6563} = \dfrac{(1/4 - 1/9)}{(1/4 - 1/16)} = \dfrac{(5/36)}{(12/64)}$

$\therefore \quad \lambda = 4861\,\text{Å}$

47. $a = \omega^2 r$

As $\omega \propto \dfrac{1}{n^3}$ (See the hint of Q-132)

and $r \propto n^2$

$$a \propto \frac{1}{n^4}$$

$$\frac{a_M}{a_L} = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$$

48. $L = \dfrac{nh}{2\pi}$, it does not depend on Z.

49. $v \propto \dfrac{Z}{n}$, since Z and n both have become twice, velocity of electron in second orbit of He^+ will be v.

50. If kinetic energy is E, then total energy will be $-E$, i.e. energy E must be supplied to the electron.

51. $E \propto m$ and $r \propto \dfrac{1}{m}$

i.e. Energy will become two times and radius will become half.

52. The wavelength will be minimum for which $\left(\dfrac{1}{n_f^{\,2}} - \dfrac{1}{n_i^{\,2}}\right)$ is maximum.

53. $r_n \propto \dfrac{n^2}{Z}$ or $r_n \propto \dfrac{1}{Z}$

54. Wavelength is minimum for which Z is maximum.

58. ${}_{72}A^{180} \xrightarrow{\alpha} {}_{70}A_1^{176} \xrightarrow{\beta} {}_{71}A_2^{176} \xrightarrow{\alpha} {}_{69}A_3^{172} \xrightarrow{\gamma} {}_{69}A_4^{172}$

59. $\dfrac{N_0}{N} = \dfrac{1}{4} = \left(\dfrac{1}{2}\right)^{t/5700}$

$\therefore \quad \left(\dfrac{1}{2}\right)^2 = \left(\dfrac{1}{2}\right)^{t/5700}$

$\therefore \quad \dfrac{t}{5700} = 2$

$\therefore \quad t = 11400\,\text{yr}$

61. $A = A_0\left(\dfrac{1}{2}\right)^{t/T} \Rightarrow \therefore \quad 100 = 1600\left(\dfrac{1}{2}\right)^{8/T}$

$\therefore \quad T = 2\,\text{s}$

at $t = 6\,\text{s}$, $A = 1600\left(\dfrac{1}{2}\right)^{6/2}$

$= 200\,\text{count/s}$

62. ${}_{92}U^{238} \xrightarrow{\alpha} {}_{90}Th^{234} \xrightarrow{\beta} {}_{91}Pa^{234} \xrightarrow{{}_{-1}\beta^0} {}_{92}U^{234}$

So, $A = 234, B = 90, C = 234, D = 91$ and $E = \beta$

64. As, $N = N_0\left(\dfrac{1}{2}\right)^{t/T}$

Number of atoms at $t = 2\,\text{h}$

$$N_1 = 8 \times 10^{10}\left(\frac{1}{2}\right)^{\frac{2}{1}} = 2 \times 10^{10}$$

Number of atoms at $t = 4\,\text{h}$

$$N_2 = 8 \times 10^{10}\left(\frac{1}{2}\right)^{\frac{4}{1}} = \frac{1}{2} \times 10^{10}$$

$\therefore$ Number of atoms decayed in given duration

$$= \left(2 - \frac{1}{2}\right) \times 10^{10} = 1.5 \times 10^{10}$$

65. Work function, $W = hf_0$

$$= 6.6 \times 10^{-34} \times 10^{15}\,\text{J}$$

$$= 6.6 \times 10^{-19}\,\text{J}$$

Energy incident of light, $E = \dfrac{hc}{\lambda}$

$$= \frac{6.6 \times \times 10^{-34} \times 3 \times 10^8}{4000 \times 10^{-10}}\,\text{J}$$

$$= 4.95 \times 10^{-19}\,\text{J}$$

Since, $E < W$. Therefore, no photoemnssion will take place.

66. Decay constant, $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{20} = 0.03465$

Now, time of decay,

$$t = \frac{2.303}{\lambda}\log\frac{N_0}{N}$$

$\Rightarrow \quad t_1 = \dfrac{2.303}{0.03465}\log\dfrac{100}{67} = 11.6\,\text{min}$

and $t_2 = \dfrac{2.303}{0.03465}\log\dfrac{100}{33} = 32\,\text{min}$

So, time difference between points of time

$$= t_1 - t_2 = 32 - 11.6 = 20.4\,\text{min} \approx 20\,\text{min}$$

67. As, $N = N_0\left(\dfrac{1}{2}\right)^{t/T}$

$\therefore \quad N_A = 10\left(\dfrac{1}{2}\right)^{t/1}$

$$N_B = 1\left(\frac{1}{2}\right)^{t/2}$$

Given, $N_A = N_B$

So, $10\left(\dfrac{1}{2}\right)^t = \left(\dfrac{1}{2}\right)^{t/2}$

So, $10 = \left(\dfrac{1}{2}\right)^{-t/2}$

or $10 = 2^{t/2}$

$\therefore \quad \log_{10} 10 = \dfrac{t}{2}\log_{10} 2$

$$1 = \frac{t}{2} \times 0.3010$$

$\therefore \quad t = 6.62\,\text{yr}$

68. Here, $T = 20$ min

As $N = N_0\left(\frac{1}{2}\right)^{t/T}$

So, for 20% decay $\frac{N}{N_0} = \frac{80}{100} = \left(\frac{1}{2}\right)^{t_1/20}$ ...(i)

For 80% decay $\frac{N}{N_0} = \frac{20}{100} = \left(\frac{1}{2}\right)^{t_2/20}$ ...(ii)

Dividing Eq. (ii) by Eq. (i), we get

$$\left(\frac{1}{2}\right)^{\left(\frac{t_2 - t_1}{20}\right)} = \frac{1}{4} \text{ or } \left(\frac{t_2 - t_1}{20}\right)\log\left(\frac{1}{2}\right) = \log\left(\frac{1}{4}\right)$$

$\therefore \quad t_2 - t_1 = 40$ min

69. Here, the activity of radioactive sample reduces to half in 140 days. Therefore, half-life of the sample is 140 days. Its two half-lives is 280 days, so before two half-lives, its activity was ($2^2 \times$ present activity).

$\therefore$ Initial activity $= 2^2 \times 6000 = 24000$ dps

70. $R = \lambda N \Rightarrow \therefore \frac{R}{N} = \lambda$

So, graph between $\frac{R}{N}$ and t, be a straight line parallel to the time axis.

71. $100 \xrightarrow{5\text{ days}} 90 \xrightarrow{5\text{ days}} 81 \xrightarrow{5\text{ days}} 72.9 \xrightarrow{5\text{ days}} 65.61\%$

In every five days only 90% is left undecayed.

72. $\frac{N}{N_0} = e^{-\lambda t} = e^{-\left(\frac{\ln 2}{t_{1/2}}\right)\left(\frac{t_{1/2}}{2}\right)} = e^{-\left(\frac{\ln 2}{2}\right)} = \frac{1}{\sqrt{2}}$

73. $100 \xrightarrow{t} 90\% \xrightarrow{t} 81\%$

81% is undecayed. Therefore, 19% is decayed.

74. $\frac{N}{N_0} = e^{-\lambda t} = e^{-\lambda\left(\frac{n}{\lambda}\right)} = \frac{1}{e^n}$

75. From conservation of linear momentum,

$$p_\alpha = p_{\text{nucleus}}$$

$$\sqrt{2E(4)} = \sqrt{2(E')(206)}$$

$\therefore \quad E' = \left(\frac{4}{206}\right)E = \left(\frac{2}{103}\right)E$

76. $1 = 64\left(\frac{1}{2}\right)^n$

$n = 6$ i.e. 6 half-lives

$\therefore \quad t = 6 \times 2 = 12$ h

82. $\frac{N_1}{N_2} = \frac{(2N_0)e^{-\frac{\ln(2)}{12}\times 48}}{(N_0)e^{-\left(\frac{\ln 2}{16}\right)\times 48}} = 1$

83. Initially whole was X

i.e. $N_0 = 1 + 15 = 16$ and $N = 1$

To convert 16 to 1, a tins of four half-lives is required.

85. $\lambda = (\lambda_1 + \lambda_2) = \frac{1}{1620} + \frac{1}{520} = 2.54 \times 10^{-3}$ yr^{-1}

$\therefore \quad t_{1/2} = \frac{\ln(2)}{\lambda} = 272.8$ yr

$\frac{1}{4}$th of the material remains intact after 2 half-lives.

89. ${}_7X^{15} + \alpha$ - particle $\longrightarrow {}_9Y^{19} - {}_1H^1 \longrightarrow {}_8Z^{18}$

91. $\frac{1}{64} = \left(\frac{1}{2}\right)^n$

$n = 6$

6 half-lives are equal to 60 s

$\therefore \quad 1$ half-life $= 10$ s

92. $x = \frac{N}{N_0} \times 100 = \left\{e^{-\lambda\left(\frac{3}{\lambda}\right)}\right\} \times 100 = 5\%$

95. Frequency of light of wavelength ($\lambda = 4000$ Å) is

$$\nu = \frac{c}{\lambda}$$

$\therefore \quad \nu = \frac{3 \times 10^8}{4000 \times 10^{-10}} = 0.75 \times 10^{15}$

This frequency is less than the given threshold frequency. Hence, no photoelectric emission takes place.

97. W_0 (eV) $= \frac{12375}{\lambda_0}$

$\therefore \quad \lambda_0 = \frac{12375}{4.2} = 2955$ Å

99. From Einstein's photoelectric equation

$$E = W_0 + K_{\max}$$

Now, $E = \frac{12375}{3000} = 4.125$ eV

$\therefore \quad K_{\max} = E - W_0$

$= 4.125$ eV $- 1$ eV

$= 3.125$ eV

$\therefore \quad \frac{1}{2}mv_{\max}^2 = 3.125 \times 1.6 \times 10^{-19}$ J

$\therefore \quad v_{\max} = \sqrt{\frac{2 \times 3.125 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$

$= 1 \times 10^6$ m/s

100. Maximum kinetic energy

$$K_{\max} = \frac{hc}{\lambda} - W_0$$

$$= \frac{6.4 \times 10^{-34} \times 3 \times 10^8}{6400 \times 10^{-10}} - 1.6 \times 10^{-19}$$

$$= 1.4 \times 10^{-19} \text{ J}$$

101. Work function, $W_0 = \frac{hc}{\lambda_0}$

$\therefore \quad \frac{(W_0)_T}{(W_0)_{Na}} = \frac{\lambda_{Na}}{\lambda_T}$

or $\lambda_T = \frac{\lambda_{Na} \times (W_0)_{Na}}{(W_0)_T} = \frac{5600 \times 2.3}{4.5}$

$= 2862$ Å

103. By decreasing the wavelength of incident light energy of incident light will increase.

104. Maximum kinetic energy,

$$K_{\max}(\text{in eV}) = \frac{hc}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$= 12375\left(\frac{1}{1000} - \frac{1}{2000}\right) = 6.2 \text{ eV}$$

106. For emission of electrons incident energy of each photon must be greater than work function (threshold energy).

107. $\frac{1}{2}mv_{\max}^2 = eV_0$

$\therefore \quad v_{\max} = \sqrt{\frac{2eV_0}{m}} = \sqrt{2\times1.8\times10^{11}\times9}$

$= 1.8\times10^6$ m/s

108. $\frac{hc}{\lambda} = W_0 + K_{\max}$

$\therefore \qquad \frac{hc}{\lambda_A} = W_0 + K_A \qquad ...(i)$

and $\qquad \frac{hc}{\lambda_B} = W_0 + K_B \qquad ...(ii)$

Subtracting Eq. (i) from Eq. (ii), we get

$$hc\left(\frac{1}{\lambda_B} - \frac{1}{\lambda_A}\right) = K_B - K_A$$

$$\Rightarrow \quad hc\left(\frac{1}{\lambda_B} - \frac{1}{2\lambda_B}\right) = K_B - K_A$$

$$\Rightarrow \quad \frac{hc}{2\lambda_B} = K_B - K_A \qquad ...(iii)$$

From Eqs. (ii) and (iii), we get

$$2K_B - 2K_A = W_0 + K_B$$

$$\Rightarrow \quad K_B - 2K_A = W_0$$

$$\Rightarrow \quad K_A = \frac{K_B}{2} - \frac{W_0}{2}$$

Which given, $\quad K_A < \frac{K_B}{2}$

109. From equation,

$$eV_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$4.8 = \frac{hc}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \qquad ...(i)$$

and $\qquad 1.6 = \frac{hc}{e}\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right) \qquad ...(ii)$

From Eqs. (i) and (ii), we get

$$\frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)}{\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right)} = \frac{4.8}{1.6} \quad \Rightarrow \lambda_0 = 4\lambda$$

110. $\frac{1}{2}mv_{\max}^2 = \frac{hc}{\lambda} - W$

$$v_{\max} = \sqrt{\frac{2}{m}\left(\frac{hc}{\lambda} - W\right)} = v$$

$$v'_{\max} = \sqrt{\frac{2}{m}\left(\frac{hc}{\lambda/4} - W\right)}$$

$$= 2\sqrt{\frac{2}{m}\left(\frac{hc}{\lambda} - W\right) + \frac{6}{m}W} > 2v$$

112. $\lambda\ (\text{in Å}) = \frac{12375}{2}$

113. The given wavelength lies in visible region.

114. Intensity of incident light $\propto \frac{1}{(\text{distance})^2}$

116. $eV_s = (h\nu) - \phi$

$\therefore \qquad V_s = \left(\frac{h}{e}\right)\nu - \frac{\phi}{e}$

$$\text{Slope} = \tan\theta = \frac{h}{e}$$

117. $K_1 = hf - Wz \quad \Rightarrow K_2 = 2hf - 2W$

$\therefore \qquad \frac{K_1}{K_2} = \frac{1}{2}$

118. $E_1 = \frac{12375}{3500}\text{eV} = 3.54\text{ eV}$

$$E_2 = \frac{12375}{5400}\text{eV} = 2.29\text{ eV}$$

Now, $\qquad \frac{2}{1} = \frac{3.54 - W}{2.29 - W}$

Solving this equation we get, $W = 1.04$ eV.

119. $E = \frac{12375}{\lambda\ (\text{in Å})} = \frac{12375}{4360} = 2.84\text{ eV}$

$\therefore \qquad K_{\max} = E - W = 1.6\text{ eV}$

or, the retarding potential is 1.6 V.

120. $E = \frac{12375}{3500}\text{eV} = 3.54\text{ eV}$

Since, $E > W_B$ but $E < W_A$, photoelectrons will be emitted by metal B.

122. $eV_0 = h\nu - W \quad$ or $\quad V_0 = \frac{h}{e}\nu - \frac{W}{e}$

When frequency is doubled,

$$V_0' = \frac{2h\nu}{e} - \frac{W}{e}$$

$$= 2\left(\frac{h\nu}{e} - \frac{W}{e}\right) + \frac{W}{e} = 2V_0 + \frac{W}{e}$$

or $\qquad V_0' > 2V_0$

124. When the distance is increased, frequency of incident light and hence the stopping potential does not change. But the intensity and hence, the saturation current decreases nine times.

125. Idea of matter waves was given by de-Broglie.

126. de-Broglie wavelength, $\lambda = \frac{h}{mv}$

127. All the three have same wavelength, therefore same momentum say p.

Energy of photon $E = pc$. Therefore, energy of photon is maximum.

128. Wavelength, $\qquad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$\therefore \qquad \lambda \propto \frac{1}{\sqrt{E}} \qquad$ (h and m are constants)

So, when the KE of an electron is increased, the wavelength of the associated wave will decrease.

129. As, $\quad \lambda = \frac{h}{p} = \frac{h}{mv}$

So, $\qquad \frac{h_1}{m_1v_1} = \frac{h_2}{m_2v_2} \quad \Rightarrow \quad \therefore \quad \frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{4}{1}$

130. As, $\lambda = \frac{h}{\sqrt{2mE}} \quad \Rightarrow \quad \lambda \propto \frac{1}{\sqrt{m}}$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}}$$

So, $$\frac{\lambda}{\lambda_2} = \sqrt{\frac{M}{m}}$$

$\therefore$ $$\lambda_2 = \lambda\sqrt{\left(\frac{m}{M}\right)}$$

131. As, $\lambda \propto \frac{1}{\sqrt{m}}$

$\Rightarrow$ $$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_p}} = \frac{2}{1}$$

132. $\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times80\times1.6\times10^{-19}}} = 1.4\ \text{Å}$

133. By law of conservation of momentum

$$0 = m_1v_1 + m_2v_2$$

$$m_1 v_1 = -m_2 v_2$$

Negative sign indicates that both the particles are moving in opposite direction.

Now de-Broglie wavelengths,

$$\lambda_1 = \frac{h}{m_1v_1}$$

and $$\lambda_2 = \frac{h}{m_2v_2}$$

$\therefore$ $$\frac{\lambda_1}{\lambda_2} = \frac{m_2v_2}{m_1v_1} = 1$$

134. $\lambda = \frac{h}{p}, \quad p_p = p_\alpha \quad \Rightarrow \quad \therefore \lambda_p = \lambda_\alpha$

136. The electrons may be accelerated or retarded.

137. $\lambda = \frac{h}{\sqrt{2qVm}}$ or $\lambda \propto \frac{1}{\sqrt{m}}$

$m_e < m_p \quad \Rightarrow \quad \therefore \lambda_e > \lambda_p$

138. $\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2qVm}} \propto \frac{1}{\sqrt{m}}$

$\therefore$ $$\frac{\lambda_e}{\lambda_p} = \left(\frac{m_p}{m_e}\right)^{1/2}$$

139. $\lambda = \frac{h}{\sqrt{2qVm}} \propto \frac{1}{\sqrt{qVm}}$

$\therefore$ $$\frac{\lambda_\alpha}{\lambda_p} = \frac{\sqrt{q_p\,V_p\,m_p}}{\sqrt{q_\alpha\,V_\alpha\,m_\alpha}} = \sqrt{\frac{1\times100\times1}{2\times800\times4}} = \frac{1}{8}$$

$\therefore$ $$\lambda_\infty = \frac{\lambda_p}{8} = \frac{\lambda_0}{8}$$

141. $\lambda = \frac{h}{p}$ or $\lambda \propto \frac{1}{p}$

i.e. λ -p graph is a rectangular hyberbola.

142. Binding energy of helium nucleus is given by

$$= (\Delta m)c^2$$

$$= [2(1.0087 + 1.0073) - 4.0015]\times(3\times10^8)^2$$

$$= 28.4\ \text{MeV}$$

144. To sustain a chain reaction in atom bomb one of the neutrons emitted in the fission of ^{235}U must be captured by another ^{235}U nucleus and cause it to fission.

145. Mass of electron = mass of positron = 9.1×10^{-31} kg

Energy released $E = (2m)c^2 = 2\times9.1\times10^{-31}\times(3\times10^8)^2$

$$= 1.6\times10^{-13}\ \text{J}$$

146. During fusion, BE of daughter nucleus is always greater than the total energy of the parent nuclei so energy released

$$= c - (a + b) = c - a - b$$

147. $Q = (1.002 + 1.004 - 1.001 - 1.003)\ (931.5)$ MeV

$= 1.863$ MeV

157. $E = mc^2 = (1.67\times10^{-27})\ (3.0\times10^8)^2$

$= 1.5\times10^{-10}$ J

159. Three atoms of $B \longrightarrow$ one atom of A

$\therefore$ $$Q = E_a - 3E_b$$

160. $2\ (_1\text{H}^2) \longrightarrow {}_2\text{He}^4$

$\therefore$ $$Q = 4(7) - 4(1.1) = 23.6\ \text{MeV}$$

167. Minimum wavelength produced

$$\lambda_{\min} = \frac{12375}{50\times10^3}\ \text{Å}$$

$$= 0.247 = 0.25\ \text{Å}$$

168. X-rays are electromagnetic waves of wavelength ranging from 0.1 to 100 Å.

169. Penetrating power is greater for lower wavelength.

170. X-rays were discovered by Rontegen.

171. According to Moseley's law.

$$\nu \propto (Z - b)^2 \quad \Rightarrow \quad \nu = a(Z - b)^2$$

Z = atomic number of element (a, b are constants)

172. Both X-rays and γ-rays are electromagnetic in nature.

173. When X-rays fall on a target, they are absorbed by the target, and they are not reflected by the target, so they are not used for radar purposes.

174. $BaSO_4$ is heavy element and it absorbs X-rays.

175. Energy, $E = eV = h\nu_{\max} \quad \Rightarrow \quad \therefore \quad \nu_{\max} = \frac{eV}{h}$

176. Energy, $E = eV = h\nu_{\max} = \frac{hc}{\lambda_{\min}}$

$\therefore$ $$\lambda_{\min} = \frac{hc}{eV}$$

177. X-rays are electromagnetic waves in nature and they are not deflected by electric and magnetic fields.

179. Bragg's law for diffraction of X-rays is

$$2d\sin\theta = n\lambda$$

181. Frequency, $\nu = \frac{c}{\lambda} = \frac{3\times10^8}{1\times10^{-10}}$

$$= 3\times10^{18}\ \text{Hz}$$

182. In X-ray spectrum wavelength λ of K_α-line is given by

$$\frac{1}{\lambda_{K_\alpha}} = \frac{3}{4}(Z - 1)^2$$

$\therefore$ $$\lambda_{K_\alpha} \propto \frac{1}{(Z-1)^2}$$

183. For K_α-line

$$\lambda_{K_\alpha} \propto \frac{1}{(Z-1)^2}$$

So, $$\frac{\lambda_2}{\lambda_1} = \left(\frac{Z_1 - 1}{Z_2 - 1}\right)^2$$

$\Rightarrow$ $$\frac{\lambda_2}{\lambda_1} = \left(\frac{43-1}{29-1}\right)^2 = \left(\frac{42}{28}\right)^2$$

$\Rightarrow$ $$\lambda_2 = \frac{9}{4}\lambda$$

184. According to Moseley's law

$$\nu \propto (Z-b)^2$$

or $$\sqrt{\nu} \propto (Z-b)$$

So, option (b) is correct.

188. $\lambda_{\min} = \dfrac{12375}{20\times 10^3}$ Å $= 61.875$ pm

194. Since, atomic numbers are same, $\lambda_1 = \lambda_2 = \lambda_3$

or $$\lambda_2 = \sqrt{\lambda_1\,\lambda_3}$$

195. $E_2 - E_1 = \dfrac{12375}{0.76}$ eV …(i)

$$E_3 - E_1 = \frac{12375}{0.64}\text{ eV} \quad \text{…(ii)}$$

$\therefore$ $$E_3 - E_2 = \frac{12375}{0.64} - \frac{12375}{0.76}$$

or $$\lambda_{K_\alpha} = \frac{12375}{E_3 - E_2} = 4.1\text{ Å}$$

200. According to Einstein's theory of relativity, if a body of mass m is moving with speed equivalent to speed of light, then mass of the particle changes according to formula.

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

201. When the speed of electron increases, then according to Einstan's equation mass of electron increases, hence the specific charge e/m decreases.

202. Momentum of particle is given by

$$p = mv$$

and velocity is given by, $v = \sqrt{\left(\dfrac{2qV}{m}\right)}$

$\therefore$ $$p = m\cdot\sqrt{\frac{2qV}{m}} = \sqrt{2qmV}$$

or $$p \propto \sqrt{qm}$$

$\Rightarrow$ $$\frac{p_e}{p_\alpha} = \sqrt{\frac{e\times m_e}{2e\times m_\alpha}} = \sqrt{\frac{m_e}{2m_\alpha}}$$

204. From Millikan's condition of equilibrium

$$qE = mg \;\Rightarrow\; q = \frac{mg}{E} \;\Rightarrow\; ne = \frac{mg}{E}$$

$\therefore$ $$n = \frac{mgd}{Ve} \qquad \left(\because E = \frac{V}{d}\right)$$

$\Rightarrow$ $$n = \frac{1.8\times 10^{-14}\times 10\times 0.9\times 10^{-2}}{2\times 10^3\times 1.6\times 10^{-19}}$$

$$= 5$$

206. Specific charge of electron

$$\frac{e}{m} = \frac{1.6\times 10^{-19}}{9.1\times 10^{-31}} = 1.76\times 10^{11}\text{ C/kg}$$

207. Light consists of photons and cathods rays consists of electrons. However, both effect the photographic plate.

208. Electron volt is a unit of energy

$$1\text{ eV} = 1.6\times 10^{-19}\text{ J}$$

210. Photoelectric effect shows particle nature while diffraction shows wave nature.

212. Wavelength of photon is given by

$$\lambda_{\text{photon}} = \frac{hc}{E}$$

and wavelength of proton is given by

$$\lambda_{\text{proton}} = \frac{h}{\sqrt{2mE}}$$

$\therefore$ $$\frac{\lambda_{\text{photon}}}{\lambda_{\text{electron}}} = c\sqrt{\left(\frac{2m}{E}\right)}$$

$\therefore$ $$\frac{\lambda_{\text{photon}}}{\lambda_{\text{electron}}} \propto \frac{1}{\sqrt{E}}$$

213. Neutrons can be added to the nucleus of an atom without changing its chemical properties.

214. Positron is antiparticle of electron, so electron has mass closest in value to that of the positron.

215. According to Yukawa, π-mesons may be positive, negative or neutral.

220. $E = mc^2 = (0.001\times 10^{-6})\,(3\times 10^8)^2 = 9\times 10^7$ J

221. The given equation is the equation of β-decay.

224. $E = h\nu$

or $$[h] = \left[\frac{\text{ML}^2\text{T}^{-2}}{\text{T}^{-1}}\right] = [\text{ML}^2\text{T}^{-1}]$$

Level 2 : Only One Correct Option

1. $\Delta p = 2\left(\dfrac{h}{\lambda}\right) = \dfrac{2\times 6.63\times 10^{-34}}{5030\times 10^{-10}} = 2.63\times 10^{-27}$ kg - m /s

2. Binding energy per nucleon

$$= \left(\frac{6\times 1.0078 + 6\times 1.0087 - 12}{12}\right)\times 931.4$$

$$= 7.68\text{ MeV}$$

3. After remaining one electron from helium atom it will become hydrogen like atom.

$\therefore$ $$E = 24.6 + (13.6)\,(2)^2$$

$$= 79\text{ eV}$$

4. $r_3 = (3)^2\cdot r_1 = 9x$

Now, $$3\lambda_3 = (2\pi r_3) = 18\pi x$$

$\therefore$ $$\lambda_3 = 6\pi x$$

5. Net process is

$${}_3\text{H}^2 \rightarrow {}_2\text{He}^4 + p + n$$

Mass defect,

$$\Delta m = (3\times 2.014) - 1.007 - 1.008 - 4.001$$

$$= 0.026\text{ amu}$$

Energy released by the fusion of three deuterons

$$= 0.026 \times 931.48 \text{ MeV}$$
$$= 0.026 \times 931.48 \times 1.6 \times 10^{-13} \text{ J}$$
$$= 3.87 \times 10^{-12} \text{ J}$$

$\therefore$ Total energy released

$$= \frac{10^{40}}{3} \times 3.87 \times 10^{-12} \text{ J}$$
$$= 1.29 \times 10^{28} \text{ J}$$

$$\text{Total time} = \frac{1.29 \times 10^{28}}{10^{16}} = 1.29 \times 10^{12} \text{ s}$$

6. $$\text{Mass} = \frac{1 \times 10^6 \times 24 \times 3600 \times 235}{200 \times 1.6 \times 10^{-13} \times 6.02 \times 10^{23}} \approx 1 \text{ g}$$

7. $a = \frac{v^2}{r}$ or $a \propto \frac{(Z^2)}{(1/Z)}$

$\therefore \quad a \propto Z^3$

For singly ionised helium atom, $Z = 2$
and doubly ionised lithium atom, $Z = 3$

8. Minimum 10.2 eV energy is required to excite a hydrogen atom. In perfectly inelastic collision maximum energy is lost.

$$\therefore \quad \frac{1}{2}mv^2 = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2$$
$$= 10.2 \times 1.6 \times 10^{-19}$$
$$\frac{1}{4}mv^2 = 1.632 \times 10^{-18}$$
$$\therefore \quad v = \sqrt{\frac{1.632 \times 4 \times 10^{-18}}{1.67 \times 10^{-27}}}$$
$$= 6.25 \times 10^4 \text{ m/s}$$

9. Energy released $= E_f - E_i$

10. $A_2 A_1 e^{-\lambda t} = A_1 e^{-\left(\frac{1}{t}\right)(t_2 - t_1)} = A_1 e^{\frac{(t_1 - t_2)}{T}}$

12. Momentum remains constant

$$K = \frac{p^2}{2m} \quad \text{or} \quad K \propto \frac{1}{m}$$
$$\therefore \quad \frac{K_n}{K_\alpha} = \frac{m_\alpha}{m_n} = \frac{4}{206} = \frac{2}{103}$$
$$\therefore \quad K_n = \left(\frac{2}{103}\right)K_\alpha = \left(\frac{2}{103}\right)E$$

13. $B = \frac{\mu_0 i}{2r} = \frac{\mu_0 (qf)}{2r}$

$$= \frac{\mu_0 (q)(v/2\pi r)}{r}$$
$$\therefore \quad B \propto \frac{v}{r^2} \propto \frac{(1/n)}{(n^4)}$$
$$\therefore \quad B \propto \frac{1}{n^5}$$

14. $\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right)$

$$\therefore \quad \frac{R}{n^2} = R - \frac{1}{\lambda} = \frac{\lambda R - 1}{\lambda}$$
$$\therefore \quad n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$

15. $\frac{M}{L} = \frac{q}{2m} = \frac{e}{2m}$

$$\therefore \quad M = \left(\frac{e}{2m}\right)L = \left(\frac{e}{2m}\right)\left(\frac{nh}{2\pi}\right) = \left(\frac{neh}{4\pi m}\right)$$

16. $P \text{ (survival)} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$

at $\quad t = \frac{1}{\lambda}, P \text{ (survival)} = \frac{1}{e}$

17. $\lambda_\alpha - \lambda'_m = 3(\lambda_\alpha - \lambda_m)$

$$\therefore \quad \frac{12375}{10.2(z-1)^2} - \frac{12375}{20 \times 10^3} = 3\left[\frac{12375}{10.2(z-1)^2} - \frac{12375}{10 \times 10^3}\right]$$

Solving we get, $z = 29$

18. Let W be the work function of metal. Then,

$$eV_0 = \frac{hc}{330 \times 10^{-9}} - W \quad \text{...(i)}$$
$$e(2V_0) = \frac{hc}{220 \times 10^{-9}} - W \quad \text{...(ii)}$$

Solving these two equations, we get

$$V_0 = \frac{10^9 \times h \times c}{110 \times e \times 6}$$
$$= \frac{10^9 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{110 \times 1.6 \times 10^{-19} \times 6}$$
$$= \frac{15}{8} \text{ V}$$

19. $E = \frac{hc}{\lambda} - W \quad \text{...(i)}$

$$4E = \frac{hc}{\lambda/3} - W \quad \text{...(ii)}$$

Solving these equations, we get $W = \frac{hc}{3\lambda}$

20. $f \propto (z-1)^2$ for K-series

$$\frac{f'}{f} = \frac{(51-1)^2}{(31-1)^2} = \frac{25}{9}$$
$$\therefore \quad f' = \frac{25}{9}f$$

21. $\sqrt{f} \propto (Z-1)$

$$\text{slope} \propto \sqrt{\frac{1}{n_1^2} - \frac{1}{n_2^2}}$$
$$\therefore \quad \frac{(\text{slope})\, K_\beta}{(\text{slope})\, K_\alpha} = \frac{\sqrt{1 - \frac{1}{9}}}{\sqrt{1 - \frac{1}{4}}}$$
$$= \sqrt{\frac{8}{9} \times \frac{4}{3}} = \sqrt{\frac{32}{27}}$$

22. Energy released = Final total binding energy – Initial total binding energy

23. $N_A e^{-\lambda_A t} = N_B e^{-\lambda_B t}$

$$\therefore \quad e^{(\lambda_B - \lambda_A)t} = \frac{N_B}{N_A}$$
$$\therefore \quad t = \frac{1}{\lambda_B - \lambda_A} \ln\left(\frac{N_B}{N_A}\right)$$

24. $eV = \frac{hc}{\lambda} - W$

$\therefore \quad V = \left(\frac{h}{e}\right)\cdot\frac{c}{\lambda} - \frac{W}{e}$

$V_1 = \left(\frac{h}{e}\right)\frac{c}{\lambda_1} - \frac{W}{e}$ ...(i)

$V_2 = \left(\frac{h}{e}\right)\frac{c}{\lambda_2} - \frac{W}{e}$ ...(ii)

Solving these two equations, we get

$$\frac{h}{e} = \frac{\lambda_1\lambda_2(V_1 - V_2)}{c(\lambda_2 - \lambda_1)}$$

$$= \frac{(0.6\times 0.4\times 10^{-12})\,(1.0)}{(3\times 10^{8})\,(0.2\times 10^{-6})}$$

$$= 4\times 10^{-15}\ \text{V-s}$$

25. $E \propto m, r \propto \frac{1}{m}$

26. $E \propto m \quad \Rightarrow \quad \therefore \quad \lambda \propto \frac{1}{m}$

In case of hydrogen atom,

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{9}\right) \text{ or } \lambda = \frac{36}{5R}$$

For this atom

$$\lambda' = \frac{\lambda}{2} = \frac{18}{5R}$$

27. $\lambda_{ph} = \lambda_e$

$\therefore \quad \frac{h}{P_{ph}} = \frac{h}{p_e}$

$\therefore \quad P_{ph} = p_e$

$\therefore \quad \frac{E_{ph}}{c} = \frac{2E_e}{v}$

$\therefore \quad \frac{E_e}{E_{ph}} = \frac{v}{2c}$

28. Speed of electron in first orbit of hydrogen atom is given by

$$v = \frac{e^2}{2\varepsilon_0 h}$$

$\therefore \quad \frac{v}{c} = \frac{e^2}{2\varepsilon_0 hc}$

29. $E_1 = \frac{me^4}{8\varepsilon_0^2 h^2}$

$$f_n = \frac{v_n}{2\pi r_n} = \frac{(e^2/2\varepsilon_0 nh)}{(2\pi)\,\varepsilon_0 n^2h^2/\pi me^2}$$

$$= \frac{me^4}{4\varepsilon_0^2 n^3h^3} = \frac{8E_1\varepsilon_0^2h^2}{4\varepsilon_0^2n^3h^3} = \frac{2E_1}{n^3h}$$

30. $\lambda = \frac{h}{p} = \frac{h}{mv}$ or $\lambda \propto \frac{1}{v}$

$\therefore \quad \lambda_n \propto n$

$J_n = n\frac{h}{2\pi}$

i.e. $J_n \propto n$

Hence, $\lambda_n \propto J_n$

31. $K_{max} = E - W$

$\therefore \quad T_A = 4.25 - W_A$ ...(i)

$T_B = (T_A - 1.50) = 4.70 - W_B$ ...(ii)

From these two equations, we have,

$W_B - W_A = 1.95$ eV ...(iii)

de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2Km}} \text{ or } \lambda \propto \frac{1}{\sqrt{K}}$$

$\therefore \quad \frac{\lambda_A}{\lambda_B} = \sqrt{\frac{K_A}{K_B}}$

$\Rightarrow \quad 2 = \sqrt{\frac{T_A}{T_A - 1.5}}$

$\Rightarrow \quad T_A = 2\,\text{eV}, W_A = 2.25\ \text{eV}, W_B = 4.20\ \text{eV}$

and $T_B = 0.5$ eV

32. $T = \frac{2\pi r}{v}$

$\therefore \quad T \propto \frac{r}{v}$

or $T \propto \frac{(n^2)}{(1/n)}$

or $T \propto n^3$

$T_i = 8T_f$

$\therefore \quad n_i = 2n_f$

33. $r = \frac{\sqrt{2Km}}{Bq} = \frac{\sqrt{2(E - W_0)\,m}}{eB}$

34. $\frac{1}{e} = \frac{N_0e^{-10\lambda t}}{N_0e^{-\lambda t}} = e^{-9\lambda t}$ or $t = \frac{1}{9\lambda}$

35. $\frac{E}{B} = c$

$\therefore \quad B = \frac{E}{c} = \frac{6}{3\times 10^8} = 2\times 10^{-8}$ T

Further, $c = E \times B$

36. To release energy in a nuclear process, binding energy per nucleon should increase.

37. $\lambda = \lambda_1 + \lambda_2$

or $\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$

or $\frac{1}{T} = \frac{1}{1620} + \frac{1}{810} = \frac{3}{1620}$

$T = \frac{1620}{3} = 540\,\text{yr}$

Suppose t is required time, then

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$ $\left[\text{Here, } \frac{N}{N_0} = \frac{1}{4}\right]$

$\frac{1}{4} = \left(\frac{1}{2}\right)^{t/T}$

or $\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/T}$

$\Rightarrow \quad \frac{t}{T} = 2$

or $t = 2\times T = 2\times 540 = 1080\,\text{yr}$

38. $\Delta E_{4-1} = \left(13.6 - \frac{13.6}{16}\right)\text{eV} = 12.75\text{ eV}$

Momentum of hydrogen atom = Momentum of photon

$$= \frac{E}{c}$$

$$= \frac{12.75 \times 1.6 \times 10^{-19}}{3.0 \times 10^{8}}$$

$$= 6.8 \times 10^{-27}\text{ kg-m/s}$$

39. $$\left(-\frac{dN}{dt}\right) = \lambda N \quad \text{...(i)}$$

$$\lambda = \frac{0.693}{T_{1/2}}$$

$$= \frac{0.693}{1620 \times 365 \times 24 \times 60 \times 60} \quad \text{...(ii)}$$

and $$N = \frac{6.023 \times 10^{23}}{226} \quad \text{...(iii)}$$

$\therefore$ $$\left(-\frac{dN}{dt}\right) = \frac{0.693 \times 6.023 \times 10^{23}}{1620 \times 365 \times 24 \times 60 \times 60 \times 226}$$

$$= 3.61 \times 10^{10}$$

40. N_B are maximum when

$$\lambda_B N_B = \lambda_A N_A$$

$\therefore$ $$N_B = \frac{\lambda_A}{\lambda_B} \cdot N_A$$

The given time is equivalent to two half lives of *A*. Hence,

$$N_A = \frac{N_0}{4}$$

$\therefore$ $$N_B = \frac{\lambda_A}{\lambda_B}\left(\frac{N_0}{4}\right)$$

41. $R_i = (15 \times 200) = 3000\text{ decay/min.}$

$$R = 375\text{ decay/min} = \frac{R_i}{8}$$

$\therefore$ $t = 3$ (half-lives)

$= 3 \times 5730$

$= 17190\text{ yr}$

42. $\frac{mv^2}{r} = \frac{GMm}{r^2}$

$$mvr = \frac{nh}{2\pi} = \frac{h}{2\pi} \quad \text{for} \quad n = 1$$

Solving these two equations, we get

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$= \frac{-2\pi^2 G^2 M^2 m^3}{h^2}$$

43. $\frac{A_P}{A_Q} = \frac{A_0 e^{-\lambda t_1}}{a_0 e^{-\lambda t_2}}$

$\therefore$ $$e^{\lambda(t_1 - t_2)} = \frac{A_Q}{A_P}$$

$\therefore$ $$\frac{t_1 - t_2}{T} = \ln\frac{A_Q}{A_P}$$

$\therefore$ $$(t_1 - t_2) = T \ln\left(\frac{A_Q}{A_P}\right)$$

44. $mvr = \frac{h}{2\pi}$ (in first orbit)

$\therefore$ $$v = \frac{h}{2\pi mr}$$

$$a = \frac{v^2}{r} = \frac{h^2}{4\pi^2 m^2 r^3}$$

45. $\left(-\frac{dN}{dt}\right) = \lambda N$

$\therefore$ $n = \lambda N$ or $\lambda = \frac{n}{N}$

or $t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693\,N}{n}$ sec.

46. Decrease in kinetic energy = Increase in potential energy

$\therefore$ $$qV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \cdot q}{r}$$

$\therefore$ $$r = \frac{Q}{(4\pi\varepsilon_0)V}$$

i.e. r is independent of q.

47. Velocity of a body falling from a height H is given by

$$v = \sqrt{2gH}$$

We know that de-broglie wavelength

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2gH}} \Rightarrow = \frac{h}{m\sqrt{2g}\sqrt{H}}$$

Here, $\frac{h}{m\sqrt{2g}}$ is a constant ϕ say 'K'.

So, $\lambda = K\frac{1}{\sqrt{H}} \Rightarrow \lambda \propto \frac{1}{\sqrt{H}} \Rightarrow \lambda \propto H^{-1/2}$

48. Given in the question,

Energy of a photon, $E = 1\text{ MeV} \Rightarrow = 10^6\text{ eV}$

Now $hc = 1240\text{ eVnm}$

Now, $E = \frac{hc}{\lambda}$

$\Rightarrow$ $$l = \frac{hc}{E} = \frac{1240\text{ eV nm}}{10^6\text{ eV}}$$

$$= 1.24 \times 10^{-3}\text{ nm}$$

49. When a beam of electrons of energy E_0 is incident on a metal surface kept in an evacuated chamber electrons can be emitted with maximum energy E_0 (due to elastic collision) and with any energy less than E_0, when part of incident energy of electron is used in liberating the electrons from the surface of metal.

50. Initial de-Broglie wavelength of electron,

$$\lambda_0 = \frac{h}{mv_0} \quad \text{...(i)}$$

Force on electron in electric field,

$$\mathbf{F} = -e\mathbf{E} = -e\,[-E_0\hat{\mathbf{i}}] = eE_0\hat{\mathbf{i}}$$

Acceleration of electron $\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{eE_0\hat{\mathbf{i}}}{m}$

Velocity of electron after time t,

$$v = v_0\hat{\mathbf{i}} + \left(\frac{eE_0\hat{\mathbf{i}}}{m}\right)t = \left(v_0 + \frac{eE_0}{m}t\right)\hat{\mathbf{i}}$$

$$= v_0\left(1 + \frac{eE_0}{mv_0}t\right)\hat{\mathbf{i}}$$

de-Broglie wavelength associated with electron at time t is

$$\lambda = \frac{h}{mv}$$

$$\Rightarrow \quad = \frac{h}{m\left[v_0\left(1+\frac{eE_0}{mv_0}t\right)\right]} = \frac{\lambda_0}{\left[1+\frac{eE_0}{mv_0}t\right]} \qquad \left[\because \lambda_0 = \frac{h}{mv_0}\right]$$

51. The atomic number of lithium is 3, therefore, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about $\frac{1}{3}$ times to that of Bohr radius. Therefore, the radius of lithium ion is near $\frac{53}{3} \approx 18$ pm.

52. When one decides to work in a frame of reference where the electron is at rest, the given expression is not true as it forms the non- inertial frame of reference.

53. The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because of the electrons not being subject to a central force.

54. In the molecules, nuclear force between the nuclei of the two atoms is not important because nuclear forces are short-ranged.

55. Radioactivity is a process due to which a radioactive material spontaneously decays. In half-life (t =1yr) of the material on the average half the number of atoms will decay.

Therefore, the containers will in general have different number of atoms of the material, but their average will be approx 5000.

56. α-β particle carries one unit of negative charge, an α-particle carries 2units of positive charge and γ (particle) carries no charge, therefore electronic energy levels of the atom charges for α and β decay, but not for γ-decay.

57. Let the nucleus is ${}_zX^A$. β^+ decay is represented as

$${}_zX^A \rightarrow {}_{z-1}y^A + {}_{+1}e^0 + \nu + Q_2$$

$$\therefore \quad Q_2 = [m_n({}_zX^A) - m_n({}_{z-1}y^A) - m_e]c^2$$

$$= [m_n({}_zX^A) + zm_e - m_n({}_{z-1}Y^A) - (z-1)m_e - 2m_e]c^2$$

$$= [m({}_zX^A) - m({}_{Z-1}Y^A) - 2m_e]c^2$$

$$= (M_x - M_y - 2m_e)c^2$$

β^- decay is represented as

$$= {}_zX^A \rightarrow {}_{z+1}A^Y + {}_{-1}e^0 + \bar{\nu} + \alpha_1$$

$$\alpha_1 = [m_n({}_zX^A) - m_n({}_{z+1}Y^A) - m_e]c^2$$

$$= [m_n({}_zX^A) + zm_e - m_n({}_{z+1}Y^A) - (z+1)me]c^2$$

$$= [m({}_zX^A) - m({}_{z-1}Y^A)]c^2$$

$$= (M_x - M_y)c^2$$

58. Stable heavy nuclei have more neutrons than protons. This is because electrostatic force between protons is repulsive , which may reduce stability.

59. According to the question, the moderator used have light nuclei (like proton). When protons undergo perfectly elastic collision with the neutron emitted their velocities are exchanged, *i.e.,* neutrons come to rest and protons move with the velocity of neutrons.

More than One Correct Options

1. λ_{K_α} and λ_{K_β} will remain unchanged. But λ_0 will decrease $\left(\text{as } \lambda_0 \propto \frac{1}{V}\right)$.

$\therefore \quad \Delta\lambda_1 = \lambda_{K_\alpha} - \lambda_0$ or $\Delta\lambda_2 = \lambda_{K_\beta} - \lambda_0$ will increase.

2. $R \propto n^2, v \propto \frac{1}{n}$ and $E \propto \frac{1}{n^2}$

3. $L \propto n, r \propto n^2$

and $\quad T = \frac{2\pi r}{v}$ or $T \propto \frac{r}{v} \propto \frac{n^2}{(1/n)}$

or $\quad T \propto n^3$

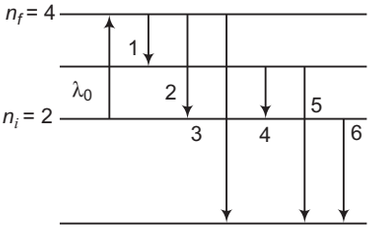
4. $\lambda = \frac{h}{P} = \frac{h}{mv} = \frac{h}{\sqrt{2Km}}$

$\lambda \propto \frac{1}{m}$ (if, v is same)

$\lambda \propto \frac{1}{\sqrt{m}}$ (if, K is same)

If change in potential energy is same, then change in kinetic energy is also same. But this does not mean that kinetic energy is same.

5. $\frac{n_f(n_f-1)}{2} = 6 \Rightarrow n_f = 4$



λ_1 and λ_4 are longer than λ_0.

λ_3, λ_5 and λ_6 belong to Lyman series.

6. $\sqrt{f} \propto (Z-b)$ …(i)

$\therefore$ If f *versus* Z is a straight line.

$$f = \frac{c}{\lambda}$$

Hence, $\frac{1}{\sqrt{\lambda}}$ *versus* Z is also a straight line.

7. $y = \lambda x = \frac{\ln 2}{T} \cdot x \Rightarrow \frac{x}{y} = \frac{1}{\lambda} = \text{constant}$

$\frac{x}{y} = \frac{T}{\ln 2}$ or $\frac{x}{y} > T$ (as, $\ln 2 = 0.693$)

Further,

$$xy = x(\lambda x) = \lambda x^2$$

After one half-life, x remains half. Hence, x^2 remains $\frac{1}{4}$th.

8. No solution is required.

9. By the emission of are α-particle, atomic number decreases by 2 and by the emission of two particles atomic number increases by 2. Hence, net atomic number remains unchanged.

10. At $t = 4T$

Number of half-lives of first $n_1 = 4$ and number of half-lives of second $n_2 = 2$

$$\frac{N_1}{N_2} = x = \frac{N_0\,(1/2)^4}{N_0\,(1/2)^2} = \frac{1}{4}$$

$$y = \frac{R_1}{R_2} = \frac{\lambda_1\,N_0\,(1/2)^4}{\lambda_2\,N_0\,(1/2)^2}$$

$$= \frac{\lambda_1}{4\lambda_2} = \frac{T_2}{4T_1} = \frac{2T}{4T} = \frac{1}{2}$$

11. No solution is required.

12. $R = R_0 A^{1/3}$ or $R \propto A^{1/3}$

13. de-Broglie wavelength

$$\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$$

Here, $h = 6.6 \times 10^{-34}$ Js

and for electron, $m = 9 \times 10^{-31}$ kg

Now consider each option one by one

(a) $\lambda_1 = 10\text{ nm} = 10 \times 10^{-9}\text{ m} = 10^{-8}\text{ m}$

$$\Rightarrow \quad v_1 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-8}}$$

$$\Rightarrow \quad = \frac{2.2}{3} \times 10^5 \approx 10^5\,\text{m/s}$$

(b) $\lambda_2 = 10^{-1}\text{ nm} = 10^{-1} \times 10^{-9}\text{ m} = 10^{-10}\text{ m}$

$$\Rightarrow\ v_2 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-10}} \approx 10^7\text{ m/s}$$

(c) $\lambda_3 = 10^{-4}\text{ nm} = 10^{-4} \times 10^{-9}\text{ m} = 10^{-13}\text{ m}$

$$\Rightarrow\ v_3 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-13}} \approx 10^{10}\text{ m/s}$$

(d) $\lambda_4 = 10^{-6}\text{ nm} = 10^{-6} \times 10^{-9}\text{ m} = 10^{-15}\text{ m}$

$$\Rightarrow\ v_4 = \frac{6.6 \times 10^{-34}}{9 \times 10^{-31} \times 10^{-15}} \approx 10^{12}\text{ m/s}$$

Thus, options (c) and (d) are correct as v_3 and v_4 is greater than 3×10^8 m/s.

14. de-Broglie wavelength $\lambda = \dfrac{h}{mv}$

where, $mv = p$ (moment)

$$\Rightarrow \quad \lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$$

Here, h is a constant.

So, $$p \propto \frac{1}{\lambda} \Rightarrow \frac{p_1}{p_2} = \frac{\lambda_2}{\lambda_1}$$

But $$(\lambda_1 = \lambda_2) = \lambda$$

Then, $$\frac{p_1}{p_2} = \frac{\lambda}{\lambda} = 1 \Rightarrow p_1 = p_2$$

Thus, their momenta is same.

Also, $$E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{mv^2 \times m}{m}$$

$$= \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{p^2}{m}$$

Here, p is constant $E \propto \dfrac{1}{m}$

$$\therefore \quad \frac{E_1}{E_2} = \frac{m_2}{m_1} < 1$$

$$\Rightarrow \quad E_1 < E_2$$

15. Suppose, Mass of electron $= m_e$, Mass of photon $= m_p$,

Velocity of electron $= v_e$ and velocity of photon $= v_p$

Thus, for electron, de-Broglie wavelength

$$\lambda_e = \frac{h}{m_e v_e}$$

$$= \frac{h}{m_e(c/100)} = \frac{100\,h}{m_e c} \quad \text{(given)} \qquad \ldots\text{(i)}$$

Kinetic energy, $E_e = \dfrac{1}{2}m_e v_e^2$

$$\Rightarrow \quad m_e v_e = \sqrt{2E_e m_e}$$

so, $$\lambda_e = \frac{h}{m_e v_e} = \frac{h}{\sqrt{2m_e E_e}}$$

$$\Rightarrow \quad E_e = \frac{h^2}{2\lambda_e^2 m_e} \qquad \ldots\text{(ii)}$$

For photon of wavelength λ_p, energy

$$E_p = \frac{hc}{\lambda_p} = \frac{hc}{2\lambda_e} \qquad [\because \lambda_p = 2\lambda_e]$$

$$\therefore \quad \frac{E_p}{E_e} = \frac{hc}{2\lambda_e} \times \frac{2\lambda_e^2 m_e}{h^2}$$

$$= \frac{\lambda_e m_e c}{h} = \frac{100h}{m_e c} \times \frac{m_e c}{h} = 100$$

So, $$\frac{E_e}{E_p} = \frac{1}{100} = 10^{-2}$$

For electron, $p_e = m_e v_e = m_e \times c/100$

So, $$\frac{p_e}{m_e c} = \frac{1}{100} = 10^{-2}$$

16. Energy spent to convert ice into water

$$= \text{mass} \times \text{ latent heat}$$

$$= mL = (1000\text{ g}) \times (80\text{ cal/g})$$

$$= 80000\text{ cal}$$

Energy of photons used $= nT \times E = nT \times h\nu$ $[\because E = h\nu]$

So, $nTh\nu = mL \Rightarrow T = \dfrac{mL}{nh\nu}$

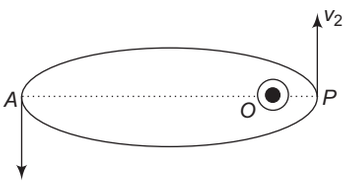
$\therefore$ $T \propto \dfrac{1}{n}$, when ν is constant.

$T \propto \dfrac{1}{\nu}$, when n is fixed.

$$\Rightarrow \quad T \propto \frac{1}{n\nu}.$$

Thus, T is constant, if $n\nu$ is constant.

17. The de-Broglie wavelength of the particle can be varying cyclically between two values λ_1 and λ_2, if particle is moving in an elliptical orbit with origin as its one focus.



Consider the figure,

Let v_1, v_2 be the speed of particle at A and B respectively and origin is at focus O. If λ_1, λ_2 are the de-Broglie wavelengths associated with particle while moving at A and B respectively. Then,

$$\lambda_1 = \frac{h}{mv_1} \quad \text{and} \quad \lambda_2 = \frac{h}{mv_2}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{v_2}{v_1} \quad \Rightarrow \quad \text{since} \quad \lambda_1 > \lambda_2$$

$$\therefore \quad v_2 > v_1$$

By law of conservation of angular momentum, the particle moves faster when it is closer to focus.

From figure, we note that origin O is closed to P than A.

18. The protons are separated by a small distance of the order of angstrom. In the ground state the electron would not move in circular orbits the electrons, orbit would go around the protons.

19. When beam of free electrons is aiming towards free protons. Then, they scatter but an electron and a proton cannot combine to produce a H-atom because of energy conservation and without simultaneously releasing energy in the form of radiation.

20. The Bohr model for the spectra of a H-atom will not be applicable to hydrogen in the molecular form. And also, it will not be applicable as it is for a He-atom.

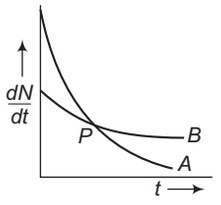
21. Balmer series for the H-atom can be observed if we measure the frequencies of light emitted due to transitions between higher excited states and the first excited state and as a sequence of frequencies with the higher frequencies getting closely packed.

22. When all the H-atoms are in the ground state and radiation of frequency $\frac{(E_2 - E_1)}{h}$ falls on it, some of atoms will move to the first excited state and no atoms will make a transition to the $n = 3$ state.

23. Fusion processes are impossible at ordinary temperatures and pressures.The reason is nuclei are positively charged and nuclear forces are short range strongest forces.

24. The two samples of the two radioactive nuclides A and B can simultaneously have the same decay rate at any time if initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A > \lambda_B$. Also, when initial rate of decay of B is same as rate of decay of A at $t = 2$ h and $\lambda_B < \lambda_A$.

25. From the given figure, it is clear that slope of curve A is greater than that of curve B.



So rate of decay is faster for A than that of B.

We know that $\left(\frac{dN}{dt}\right) \propto \lambda$, at any instant of time hence we can say that $\lambda_A > \lambda_B$. At point P shown in the diagram the two curve intersect.

Hence at point P, rate of decay for both A and B is the same.

Comprehension Based Questions

1. $E_1 = \frac{12375}{4950} = 2.5\,\text{eV}$

$K_1 =$ maximum kinetic energy $= 0.6\,\text{eV}$

$W = E_1 - K_2 = 1.9\,\text{eV}$

2. $K_2 = 1.1\,\text{eV}$

$$E_2 = W + K_2 = 3.0\ \text{eV}$$

$$\therefore \quad \lambda_2 = \frac{12375}{3.0} = 4125\ \text{Å}$$

3. Magnetic field cannot change the kinetic energy of charged particle.

4. $K_{\max} = E - W = 2\,\text{eV}$

$$\lambda = \sqrt{\frac{150}{\text{KE (in eV)}}}, \text{ for an electron}$$

$$= \sqrt{\frac{150}{2}} \approx 8.6\ \text{Å}$$

5. $K_{\max}$ is 2 eV. Hence, stopping potential is 2 V. Photoemission stops when potential of sphere becomes 2V.

$$\therefore \quad 2 = \frac{q}{4\pi\varepsilon_0 r} \quad \Rightarrow \quad \therefore \quad q = (8\pi\varepsilon_0 r)$$

6. $q = ne = 8\pi\varepsilon_0 r$

$$\therefore \quad n = \frac{8\pi\varepsilon_0 r}{e} = \frac{2 \times 8 \times 10^{-3}}{9 \times 10^9 \times 1.6 \times 10^{-19}} = 1.11 \times 10^7$$

So, this much number of electrons are required to be ejected form the sphere.

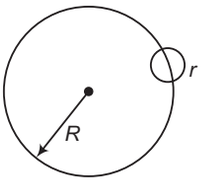
Number of photons emitted per second.

$$N_1 = \frac{\text{Power of source}}{\text{Energy of one photon}}$$

$$= \frac{3.2 \times 10^{-3}}{5 \times 1.6 \times 10^{-19}}$$

$$= 4 \times 10^{15}$$

Number of photon incident on sphere per second



$$N_2 = \left(\frac{N_1}{4\pi R^2}\right)(\pi r^2)$$

$$= \frac{N_1}{4R^2} r^2$$

$$= \frac{(4 \times 10^{15})\,(8 \times 10^{-3})^2}{4\,(0.8)^2} = 10^{11}$$

Number of photoelectron emitted per second,

$$N_3 = \frac{N_2}{10^6} = 10^5$$

Now, $(N_3)t = n$

$$\therefore \quad t = \frac{n}{N_3} = 111\ \text{s}$$

7. Energy released $= (\Delta m)\,(931.48)$ MeV

$$= [2 \times 2.01102 - 3.0160 - 1.007825] \times 931.5$$

$$= 4.03\ \text{MeV} \approx 4\ \text{MeV}$$

8. Let N number of fusion reactions are required, then

$$N \times 4 \times 1.6 \times 10^{-13} = 10^3 \times 3600$$

$$N = 5.625 \times 10^{18}$$

9. In one fusion reaction two ^2_1H nuclei are used. Hence, total number of ^2_1H nuclei are $2N$

or 1.125×10^{19}, Mass in kg $=\left(\dfrac{1.125\times10^{19}}{6.02\times10^{26}}\right)(2)$ kg

$=3.7\times10^{-8}$ kg

Assertion and Reason

4. $K=\dfrac{hc}{\lambda}-W$

$$K'=\frac{hc}{\lambda/2}-W=2\left(\frac{hc}{\lambda}-W\right)+W=2K+W$$

i.e. $K'>2K$

5. $mv_2r_2=2\left(\dfrac{h}{2\pi}\right)\ \Rightarrow\ \therefore\ 2\pi r_2=2\left(\dfrac{h}{mv_2}\right)=2\lambda_2$

Further, $\lambda=\dfrac{h}{p}$

Speed of momentum is maximum in ground state.

Hence, λ is minimum.

6. $\lambda=\dfrac{h}{p}$

Their momenta are same. Hence, their wavelengths are same.

7. Average life is more. Hence, more nuclei decay in one average life.

10. Radius $\propto A^{1/3}$.

11. E_{H} = 13.6 eV (ionisation energy)

$\therefore\quad E_{\text{He}^+}=13.6\,(z)^2=(13.6)\,(2)^2=54.4$ eV

13. $L=n\left(\dfrac{h}{2\pi}\right)$, i.e. $L\propto n$

15. Proportionality constant $\propto(z-b)^2$.

16. This proportionality constant $\propto(z-b)^2$.

For K-series : $b=1$

For L-series : $b=7.4$

17. Rate of a nuclear process cannot be altered by altering pressure or temperature. Because, any nuclear process involves huge amount of energy.

18. In 1 s only 90% remains.

$\therefore$ At $t=2$ s activity will remain 90% of 90, i.e. 81 units.

19. Already emitted electrons will repel the new electrons.

Match the Columns

1. $E\propto\dfrac{Z^2}{n^2}U=2E, K=-E$

2. (A) $\omega=\dfrac{v}{r}\propto\dfrac{(1/n)}{(n^2)}\propto n^{-3}$

(B) $a=r\omega^2\propto(n^2)\,(n^{-6})\propto n^{-4}$

(C) $L=n\dfrac{h}{2\pi}$ or $L\propto n$

(D) $I=mr^2$

or $I\propto r^2\propto(n^2)^2\propto n^4$

3. $\dfrac{1}{\lambda}\propto\left(\dfrac{1}{n_1^2}-\dfrac{1}{n_2^2}\right)$

in the transition from n_2 state to n_1 state.

4. By the emission of an α-particle, atomic number decreases by 2 and mass number by 4. But by the emission of a β-particle, atomic number increases by 1 while mass number remains unchanged.

5. Momentum is given by

$$p=\frac{h}{\lambda}\text{ or }p\propto\frac{1}{\lambda}$$

Entrance Gallery

1. Energy corresponding to 248 nm wavelength $=\dfrac{1240}{248}$ eV = 5 eV

Energy corresponding to 310 nm wavelength $=\dfrac{1240}{310}$ eV = 4 eV

$$\frac{\text{KE}_1}{\text{KE}_2}=\frac{u_1^2}{u_2^2}\ \Rightarrow\ \frac{4}{1}=\frac{5\text{ eV}-W}{4\text{ eV}-W}$$

$\Rightarrow\quad 16-4W=5-W\ \Rightarrow 11=3W$

$\Rightarrow\quad W=\dfrac{11}{3}=3.67\text{ eV}\cong3.7\text{ eV}$

2. The problem is based on frequency dependence of photoelectric emission. When incident light with certain frequency (greater than on the threshold frequency is focus on a metal surface), then some electrons are emitted from the metal with substantial initial speed.

When an electron moves in a circular path, then

$$r=\frac{mv}{eB}\ \Rightarrow\ \frac{r^2e^2B^2}{2}=\frac{m^2v^2}{2}$$

$$\text{KE}_{\max}=\frac{(mv)^2}{2m}\ \Rightarrow\ \frac{r^2e^2B^2}{2m}=(\text{KE})_{\max}$$

Work function of the metal, $W=h\nu-\text{KE}_{\max}$

$$1.89-\phi=\frac{r^2e^2B^2}{2m}-\frac{1}{2}\text{ eV}$$

$$=\frac{r^2eB^2}{2m}\text{ eV}$$

($h\nu$ = 1.89 eV, for the transition on from third to second orbit of H-atom)

$$=\frac{100\times10^{-6}\times1.6\times10^{-19}\times9\times10^{-8}}{2\times9.1\times10^{-31}}$$

$\therefore\quad \phi=1.89-\dfrac{1.6\times9}{2\times9.1}$

$=1.89-0.79$

$=1.1$ eV

3. For hydrogen, we get

$$\frac{1}{\lambda}=Rz^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)\ \Rightarrow\ \frac{1}{\lambda_1}=R(1)^2\left(\frac{3}{4}\right)$$

$$\frac{1}{\lambda_2}=R(1)^2\left(\frac{3}{4}\right)\ \Rightarrow\ \frac{1}{\lambda_3}=R(2)^2\left(\frac{3}{4}\right)$$

$$\frac{1}{\lambda_4}=R(3)^2\left(\frac{3}{4}\right)\ \Rightarrow\ \frac{1}{\lambda_1}=\frac{1}{4\lambda_3}=\frac{1}{9\lambda_4}=\frac{1}{\lambda_2}$$

$\Rightarrow\ \lambda_1=\lambda_2=4\lambda_3=9\lambda_4$

4. Both the energy densities are equal, i.e. energy is equally divided between electric and magnetic fields.

5. The de-Broglie wavelength of an electron is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}}\ \text{Å} \qquad (h = \text{Planck's constant})$$

where, m = mass of electron, e = electronic charge and V = potential difference with which electron is accelerated

$$\lambda = \frac{12.27}{\sqrt{100}}\ \text{Å} = \frac{12.27}{10}\ \text{Å} = 1.227\ \text{Å}$$

6. Above the threshold frequency, the maximum kinetic energy of the emitted photoelectrons depends on the frequency of the incident light, but is independent of the intensity of the incident light so long as the letter is not too high.

7. By radioactive decay law,
the time in which the final amount becomes half of its initial value is called half-time (T) of that radioactive element.

Relation between half-life and disintegration (decay) constant is given by

$$T = \frac{0.6931}{\lambda} \Rightarrow T = \frac{\log_e 2}{\lambda} \quad \Rightarrow \quad \lambda T = \log_e 2$$

8. When an atom comes down from some higher energy level to the second energy level ($n_1 = 2$ and $n_2 = 3,4,5\ldots$), then the lines of the spectrum are obtained in the visible part.
These lines were seen and studied for the first time by Balmer in 1885. The longest wavelength of this series (for $n = 3$) is 6563 Å and the shortest wavelength (for $n = \infty$) is 3646 Å. So, Balmer series is present in visible range of electromagnetic wave.

9. The energy of electron in any orbit is given by

$$E = \frac{-2\pi^2 me^4 Z^2 K^2}{n^2 h^2} = -\frac{13.6}{n^2}\ \text{eV}$$

$$= -\frac{13.6}{(3)^2} = -\frac{13.6}{9} = -1.51\ \text{eV}$$

($n = 3$ for 3rd Bohr's orbit)

10. Energy required to removed electron in the ($n = 2$) stage.

The energy of the photon are given by

$$= R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) = 13.6\ \text{eV}\left(\frac{1}{1} - \frac{1}{4}\right)$$

$$= 13.6\ \text{eV}\left(\frac{4-1}{4}\right) = 13.6\ \text{eV} \times \frac{3}{4} = \frac{40.8}{4}\ \text{eV} = 10.2\ \text{eV}$$

11. There is no significant time delay between the absorption of a suitable radiation and the emission of electrons and the maximum kinetic energy of electrons does not depend on the intensity of radiation.

The maximum kinetic energy of the emitted photoelectron is proportional to the frequency of incident radiation.

12. Isotope of the original nucleus is produced.

13. In photocell, concept of photoelectric effect is used.

14. Power = energy emitted per unit time = 3×10^{-3} J/s

Energy of one photon = $h\nu = 6.64 \times 10^{-34} \times 3 \times 10^{14}$

So, number of photons emitted per second

$$= \frac{3 \times 10^{-3}}{6.64 \times 10^{-34} \times 3 \times 10^{14}}$$

$$= \frac{1}{6.64} \times 10^{17}$$

$$= 0.15 \times 10^{17}$$

$$= 1.5 \times 10^{16}$$

15. Radius of a nucleus,

$$R = R_0 A^{\frac{1}{3}} \qquad [R_0 = \text{constant}]$$

So, $$\frac{R_1}{R_2} = \left(\frac{27}{64}\right)^{1/3} = \frac{3}{4}$$

16. Rydberg constant is given by $R = 1.097 \times 10^7\ \text{m}^{-1}$

17. For hydrogen atom, $E_n = \frac{-13.6}{n^2} = \frac{-13.6}{4^2}\ \text{eV} = -0.85\ \text{eV}$

18. Work function is given by

$$eV_0 = hf - W$$

$$\therefore \qquad V_0 = \left(\frac{h}{e}\right) f - \frac{W}{e}$$

V_0 versus f graph is a straight line with slope $= \frac{h}{e} = a$ universal constant.

Therefore, the ratio of two slopes should be 1.

19. Orbital angular momentum is given by

$$L = 3\left(\frac{h}{2\pi}\right) \Rightarrow \therefore \quad n = 3, \text{as } L = n\left(\frac{h}{2\pi}\right)$$

$$r_n \propto \frac{n^2}{Z} \Rightarrow r_3 = 4.5\, a_0 \Rightarrow \therefore \quad Z = 2$$

$$\frac{1}{\lambda_1} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 4R\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\Rightarrow \qquad \lambda_1 = \frac{9}{5R}$$

$$\frac{1}{\lambda_2} = RZ^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 4R\left(1 - \frac{1}{9}\right)$$

$$\Rightarrow \qquad \lambda_2 = \frac{9}{32R}$$

$$\frac{1}{\lambda_3} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 4R\left(1 - \frac{1}{4}\right)$$

$$\Rightarrow \qquad \lambda_3 = \frac{1}{3R}$$

20. (a) ${}_3\text{Li}^7 \rightarrow {}_2\text{He}^4 + {}_1\text{H}^3$

$$\Delta m = [\text{M}_{\text{Li}} - \text{M}_{\text{He}} - \text{M}_{{}_1\text{H}^3}]$$

$$\Delta m = [6.01513 - 4.002603 - 3.016050]$$

$$= -1.003523\ \text{u}$$

Δm is negative, so reaction is not possible.

(b) ${}_{84}\text{Po}^{210} \rightarrow {}_{83}\text{Bi}^{209} + {}_1\text{P}^1$

Δm is negative, so reaction is not possible.

(c) ${}_1\text{H}^2 \rightarrow {}_2\text{He}^4 + {}_3\text{Li}^6$

Δm is positive, so reaction is not possible.

(d) ${}_{30}\text{Zn}^{70} + {}_{34}\text{Se}^{82} \rightarrow {}_{64}\text{Gd}^{152}$

Δm is positive so, reaction is not possible.

21. ${}_{84}\text{Po}^{210} \rightarrow {}_2\text{He}^4 + {}_{82}\text{Pb}^{152}$

Mass defect, $\Delta m = (m_{\text{Po}} - m_{\text{He}} - m_{\text{Pb}}) = 0.005818$ u

$\therefore \quad Q = (\Delta m)(931.48)\ \text{MeV} = 5.4193\ \text{MeV} = 5419\ \text{MeV}$

$$\alpha \longleftarrow\!\bullet\longrightarrow \text{Pb}$$

From conservation of linear momentum,

$$\therefore \qquad p_{\text{Pb}} = p_\alpha$$

$$\sqrt{2m_{\text{Pb}} k_{\text{Pb}}} = \sqrt{2m_\alpha k_\alpha}$$

or $$\frac{k_\alpha}{k_{\text{Pb}}} = \frac{m_{\text{Pb}}}{m_\alpha} = \frac{206}{4}$$

$$\therefore \quad k_\alpha = \left(\frac{206}{206+4}\right)(k_{\text{total}})$$
$$= \left(\frac{206}{210}\right)(5419) = 5316 \text{ keV}$$

22. As λ is increased, there will be a value of λ above which photo-electron will cease to come out. So, photocurrent will be zero.

23. Energy is given by $\Delta E = h\nu$

$$\therefore \quad \nu = \frac{\Delta E}{h} = k\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{k2n}{n^2(n-1)^2} \approx \frac{2k}{n^3} \propto \frac{1}{n^3}$$

24. The work function for aluminium,

$$W_0 = 4.125 \text{ eV} = 4.125 \times 1.6 \times 10^{-19} = 6.6 \times 10^{-19} \text{ J}$$

The relation for work function is given by

$$W_0 = \frac{hc}{\lambda} \qquad \text{(where, } \lambda \text{ is the cut-off wavelength)}$$

$$6.6 \times 10^{-19} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{6.6 \times 10^{-19}}$$

$$= \frac{19.8 \times 10^{-26}}{6.6 \times 10^{-19}} = 3 \times 10^{-7} \text{ cm}$$

$$= 300 \times 10^{-9} \text{ m} = 300 \text{ nm}$$

25. Given,

radius of the first sphere, $R_1 = R$

radius of the second sphere, $R_2 = 4R$

The heat capacity is given by

$$\Delta Q = \frac{4}{3}\pi r^3 \rho c \propto R^3$$

Hence, $$\left(\frac{\Delta Q_1}{\Delta Q_2}\right) = \left(\frac{R_1}{R_2}\right) = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

26. The wave number in Lyman series is $= R\left(\frac{1}{1^2} - \frac{1}{(n)^2}\right)$

Hence, the wave number of first time when $n = 2$ is

$$= R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R^2 \times \frac{3}{4} = \frac{3R}{4}$$

27. Average life = 5 h, in one average life approximately 63% radioactive nuclei decay.

28. Threshold wavelength, $\lambda_0 = \frac{hc}{W_0} = \frac{12400}{4} = 3100$ Å

$$= 310 \text{ nm}$$

29. Threshold wavelength, $\lambda_0 = \frac{12375}{2.1}$

$$= 5892.8 \text{ Å}$$

30. The de-Broglie wavelength of a particle is given by

$$\lambda = \frac{h}{mv} \qquad \ldots(\text{i})$$

where, h is Planck's constant.

If kinetic energy of particle of mass m is v, then

$$K = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2K}{m}} \qquad \ldots(\text{ii})$$

Combining Eqs. (i) and (ii), we get

$$\lambda = \frac{h}{m\sqrt{\frac{2K}{m}}} = \frac{h}{\sqrt{2mK}} \qquad \ldots(\text{iii})$$

Given, $m = 9.1 \times 10^{-31}$ kg, $K = 10$ keV $= 10 \times 10^3 \times 1.6 \times 10^{-19}$ J

and $h = 6.6 \times 10^{-34}$ J-s

Substituting the above values in Eq. (iii), we get

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10 \times 10^3 \times 1.6 \times 10^{-19}}}$$

$$\Rightarrow \quad \lambda = 1.22 \times 10^{-11} \approx 0.12 \text{ Å}$$

31. Let the initial number of atoms at time, $t = 0$ be N_0.

Let N be the number of atoms at any instant t.

Mean life $\tau = \frac{1}{\lambda}$, where, λ is disintegration constant.

Given, $t = \tau$

According to radioactive disintegration law,

$$N = N_0 e^{-\lambda t} \quad \text{or} \quad N = N_0 e^{-\lambda \times \frac{1}{\lambda}} = \frac{N_0}{e} \quad \text{or} \quad \frac{N_0}{N} = e$$

32. In fission process, when a parent nucleus breaks into daughter produces, then some mass is lost in the form of energy. Thus, mass of fission products < mass of parent nucleus.

$$\Rightarrow \quad \frac{\text{Mass of fission products}}{\text{Mass of parent nucleus}} < 1$$

33. In fog, visible light is scattered more according to Rayleigh scattering, but scattering of infrared radiations is less due to high wavelengths, hence, in fog photographs of the objects taken with infrared radiations are more clear.

34. From Einstein's photoelectric equation,

***Case* I** $$h\nu = h\nu_0 + eV_0 \qquad \ldots(\text{i})$$

***Case* II** $$\frac{h\nu}{2} = h\nu_0 + \frac{eV_0}{4} \qquad \ldots(\text{ii})$$

Solving Eqs. (i) and (ii), we get the threshold frequency for photoelectric emission,

$$v_0 = \frac{\nu}{3}$$

35. According to the question,

$$\lambda_e = \lambda_p = \frac{hc}{E} = \frac{hc}{50 \times 10^3}$$

$$E_e = 0.5 \text{ MeV} = 50 \times 10^{-10} \text{ eV}$$

Again, $$\frac{E_p}{K_e} = \frac{2m_e C\lambda}{h} = \frac{2m_e C^2 h}{h \times 50 \times 10^3} = \frac{2m_e C^2}{50 \times 10^3}$$

$$= \frac{2 \times E_e C^2}{50 \times 10^3 \times C^2}$$

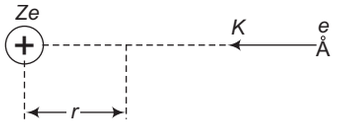
$$= \frac{2 \times 50 \times 10^4}{50 \times 10^3} \qquad \left(\because mc = \frac{Ee}{C^2}\right)$$

$$\frac{E_p}{K_e} = \frac{20}{1}$$

$$\therefore \quad E_p : K_e = 20 : 1$$

36. Ultraviolet rays are absorbed by the ozone layer.

37. r = closest distance = 10 Fm



From energy conservation, we have

$$K_i + U_i = K_f + U_f$$

or $K + 0 = 0 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r}$ or $K = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(120e)(e)}{r}$ ...(i)

The de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2Km}} = 7\times10^{-15}\text{ m} = 7\text{ Fm}$

38. Maximum kinetic energy of anti-neutrino is nearly (0.8×10^6) eV.

39. The emission spectrum, number of bright line is given by

$$\frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

40. According to given data, mass of neutron and proton are equal, which do not permit the breaking up of neutron and proton. But if we take standard mass of neuton as, 1.6750×10^{-27} kg, then Energy released = mass defect $\times c^2$

$$= (m_n - m_p - m_e)\times c^2$$

$$= \frac{(1.6750\times10^{-27} - 1.6725\times10^{-27} - 9\times10^{-31})}{1.66\times10^{-27}}\times 931.5\text{ MeV}$$

$$= \frac{16\times10^{-31}}{1.66\times10^{-27}}\times 931.5\text{ MeV} = \frac{1.6\times0.9315}{1.66}\text{ MeV} \approx 0.9\text{ MeV}$$

41. In electromagnetic wave, the direction of propagation of wave, electric field and magnetic field are mutually perpendicular, i.e. wave propagates perpendicular to **E** and **B** and along $\mathbf{E}\times\mathbf{B}$, while polarisation of wave takes place parallel to electric field vector.

42. Given, $\lambda_1 = 2000$ Å, $\text{KE}_1 = 1$ eV and $\text{KE}_2 = 1$ MeV $= 10^6$ eV

Wavelength of the particle,

$$\lambda = \frac{h}{\sqrt{2m\text{KE}}}$$

$$\therefore\quad \lambda_2 = \frac{\sqrt{\text{KE}_1}\times\lambda_1}{R\sqrt{\text{KE}_2}} = \frac{1\times2000}{10^3} = 2\text{ Å}$$

43. Decay constant, $\lambda = \frac{\log_e\frac{A_1}{A_2}}{t} = \frac{\log_e\frac{5000}{12500}}{5} = 0.4\ln 2\text{ min}^{-1}$

44. According to reaction,

$$14 + y = 17 + 1 \Rightarrow y = 4 \text{ and } 7 + x = 8 + 1$$

$$\Rightarrow\quad x = 9 - 7 = 2$$

X is ${}_2\text{He}^4$.

45. The de-Broglie wavelength, $\lambda = \frac{h}{mv}$

Here $\lambda_e = \frac{h}{m_e\frac{c}{2}}$ and $\lambda_p = \frac{h}{m_p c}$

Given, $\lambda_e = \lambda_p$

So, $\frac{h}{m_e\frac{c}{2}} = \frac{h}{m_p c} \Rightarrow \frac{\mu_\varepsilon}{\mu_\pi} = 2$

Ratio of KE $\frac{k_e}{k_p} = \frac{\frac{1}{2}m_e v_e^2}{\frac{1}{2}m_p v_p^2} = \frac{2mpc^2}{4mpc^2} = \frac{1}{2}$

46. Given, $Z_A : Z_B = 1:2$

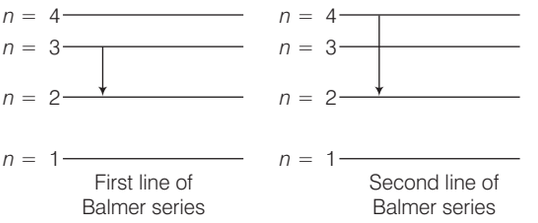
We know that, $v \propto Z^2$

Hence, $\frac{v_A}{v_B} = \frac{(1)^2}{(2)^2} \Rightarrow v_A : v_B = 1:4$

47. For hydrogen or hydrogen type atoms

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

In the transition from $n_i \to n_f$,



$$\therefore\quad \lambda \propto \frac{1}{\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)} \Rightarrow \therefore\ \frac{\lambda_2}{\lambda_1} = \frac{Z_1^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_1}{Z_2^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_2}$$

$$\Rightarrow\quad \lambda_2 = \frac{\lambda_1 Z_1^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_1}{Z_2^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)_2}$$

Substituting the values, we have

$$= \frac{(6561)(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{(2)^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right)} = 1215\text{ Å}$$

48. Change in energy, $\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\Delta E = 13.6(3)^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = 108.8\text{ eV}$$

49. In particle situation, at least three particles take place in transformation, so energy of β-particle + energy of third particle

$$= E_1 - E_2$$

Hence, energy of β-particle $\le E_1 - E_2$.

50. The de-Broglie wavelength is given by $\lambda = \frac{-h}{mv} = \frac{h}{p}$

where, p = momentum

By conservation of momentum, $p_1 + p_2 = 0$

$$p_1 = p_2 \Rightarrow \lambda_1 = \lambda_2 = \lambda$$

51. Given, $N_1 = N_0 - \frac{1}{3}N_0 = \frac{2}{3}N_0$

$$N_2 = N_0 - \frac{2}{3}N_0 = \frac{1}{3}N_0,\ \frac{N_1}{N_2} = \left(\frac{1}{2}\right)^n$$

$n = 1$

$\therefore\quad t_2 - t_1$ = one half-life = 20 min

52. Mass defect $= \frac{2 \times 1.115}{931} = 0.0024$ unit

53. Given, $E = E_0 \sin(kx - \omega t)$

Comparing this equation with standard equation, we get

Wavelength $= \frac{k}{\omega}$

54. Average energy density of magnetic field is U.

Average energy density of electric field $= \frac{1}{2} \times \frac{1}{2} \varepsilon_0 E^2$

$= \frac{1}{2} \times \frac{1}{2} \times 8.85 \times 10^{-12} \times (2)^2 = 8.85 \times 10^{-12}$ Jm^{-3}

55. Ratio of de-Broglie wavelength is given by

$$\frac{\lambda_e}{\lambda_p} = \frac{\frac{h}{m_e v}}{\frac{h}{m_p v}} = \frac{m_p}{m_e} = \frac{1.67 \times 10^{-27}}{9.1 \times 10^{-31}} = 0.18 \times 10^4 = 1836$$

56. Mass defect $= \frac{2 \times 1.115}{931} = 0.0024$ unit

57. Wavelength of radiation is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad \Rightarrow \quad n_1 = 2, n_2 = 4$$

$$\frac{1}{\lambda} = R\left[\frac{1}{4} - \frac{1}{16}\right] = R\left[\frac{4-1}{16}\right] = \frac{3R}{16} \quad \Rightarrow \lambda = \frac{16}{3R}$$

58. We know that, $\lambda \propto n^2$

$$\therefore \quad \frac{\lambda_{\text{Lyman}}}{\lambda_{\text{Balmer}}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25$$

59. We know that, $N = \frac{N_0}{2^n} = \frac{N_0}{2^{1/2}} = \frac{N_0}{\sqrt{2}}$ [given, $n = 1/2$]

60. $1\text{ Ci} = 3.7 \times 10^{10}$ dis/s

61. From the photoelectric equation,

$$E = W + \text{KE} \quad \Rightarrow \quad \text{KE} = E - W = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$$= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = hc\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)$$

62. We know that, $E_m = -\frac{13.6}{(n)^2}$

$$E_m = -\frac{13.6}{(3)^2} = -1.51$$

63. In the reaction, X stands for ${}^{0}_{-1}e$.

64. The de-Broglic wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2qVm}} \quad \text{or} \quad \lambda \propto \frac{1}{\sqrt{qm}}$$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{q_\alpha}{q_p} \cdot \frac{m_\alpha}{m_p}} = \sqrt{\frac{(2)(4)}{(1)(1)}} = 2.828$$

The nearest integer is 3.

65. $\left|\frac{dN}{dt}\right| = |\text{Activity of radioactive substance}|$

$= \lambda N = \lambda N_0 e^{-\lambda t}$ [$\because N = N_0 e^{-\lambda t}$]

Taking log on both sides

$$\ln\left|\frac{dN}{dt}\right| = \ln(\lambda N_0) - \lambda t$$

Hence, $\ln\left|\frac{dN}{dt}\right|$ *versus* t graph is a straight line with slope $-\lambda$.

From the graph we can see that, $\lambda = \frac{1}{2} = 0.5\text{ yr}^{-1}$

Now applying the equation,

$$N = N_0 e^{-\lambda t} = N_0 e^{-0.5 \times 4.16} = N_0 e^{-2.08} = 0.125\, N_0 = \frac{N_0}{8}$$

i.e. nuclei decreases by a factor of 8.

66. In steady state, $qE = mg$ …(i)

$$6\pi\eta r v = mg$$

We have, $\frac{4}{3}\pi r^3 \rho g = mg$ …(ii)

$$\therefore \quad r = \left(\frac{3mg}{4\pi\rho g}\right)^{1/3} \quad \text{…(iii)}$$

Substituting the value of r in Eq. (ii), we get

$$6\pi\eta v \left(\frac{3mg}{4\pi\rho g}\right)^{1/3} = mg \quad \text{or} \quad (6\pi\eta v)^3 \left(\frac{3mg}{4\pi\rho g}\right) = (mg)^3$$

Again substituting $mg = qE$, we get

$$(qE)^2 = \left(\frac{3}{4\pi\rho g}\right)(6\pi\eta v)^3 \quad \text{or} \quad qE = \left(\frac{3}{4\pi\rho g}\right)^{1/2} (6\pi\eta v)^{3/2}$$

$$\therefore \quad q = \frac{1}{E}\left(\frac{3}{4\pi\rho g}\right)^{1/2} (6\pi\eta v)^{3/2}$$

Substituting the values, we get

$$q = \frac{7}{81\pi \times 10^5} \sqrt{\frac{3}{4\pi \times 900 \times 9.8} \times 216\,\pi^3} \times \sqrt{(1.8 \times 10^{-5} \times 2 \times 10^{-3})^3}$$

$$= 8.0 \times 10^{-19}\text{ C}$$

67. Since the frequency of ultraviolet light is less than the frequency of X-rays, the energy of each incident photon will be more for X-rays, $\text{KE}_{\text{photoelectron}} = h\nu - \phi$

Stopping potential is to stop the fastest photoelectron,

$$V_0 = \frac{h\nu}{e} - \frac{\phi}{e}$$

So, $\text{KE}_{\max}$ and V_0 both increases.

But KE ranges from zero to $\text{KE}_{\max}$ because of loss of energy due to subsequent collisions before getting ejected and not due to range of frequencies in the incident light.

68. Given, Power of Source, $P = 4\text{ kW} = 4 \times 10^3$ W

Number of photon, $n = \frac{P}{h\nu} \quad \Rightarrow \quad \nu = \frac{P}{hn}$

$$\nu = \frac{4 \times 10^3}{10^{20} \times 6.023 \times 10^{-34}}$$

$$= 6.64 \times 10^{16}\text{ Hz}$$

The obtained frequency lies in the band of X-rays.

69. Energy ε is related only when lighter nuclei fuse to form a heavier nucleus such as in reaction (i),

$$A + B \rightarrow C + \varepsilon$$

Again, energy is released when a heavy nucleus splits into lighter nuclei as in (iv),

$$F \rightarrow D + E + \varepsilon$$

70. In positive beta decay, a proton is transformed into a neutron and a positron is emitted.

$$p^+ \longrightarrow n^0 + e^+$$

Number of neutrons initially was $A - Z$.
Number of neutrons after decay $(A - Z) - 3 \times 2$
(due to $\alpha -$ particles) $+ 2 \times 1$ (due to positive β-decay).
The number of protons will reduce by 8 [as 3×2]
(due to α – particles) + 2 (due to positive β-decay)].
Hence, atomic number reduces by 8.
So, the ratio of number of neutrons to that of protons

$$= \frac{A - Z - 4}{Z - 8}$$

71. After decay, the daughter nuclei will be more stable hence, binding energy per nucleon will be more than that of their parent nucleus.

72. Conserving the momentum, $\frac{M}{2} v_1 - \frac{M}{2} v_2 = 0$

$$v_1 = v_2 \quad \ldots(i)$$

$$\Delta mc^2 = \frac{1}{2} \cdot \frac{M}{2} v_1^2 + \frac{1}{2} \cdot \frac{M}{2} \cdot v_2^2 \quad \ldots(ii)$$

$$\Delta mc^2 = \frac{M}{2} v_1^2 \quad \Rightarrow \quad \frac{2\Delta mc^2}{M} = v_1^2$$

$$\Rightarrow \qquad v_1 = c\sqrt{\frac{2\Delta m}{M}}$$

73. According to Bohr's quantisation of angular momentum,

$$mvr = \frac{nh}{2\pi} \quad \text{or} \quad \frac{h}{mv} = \frac{2\pi r}{n} \quad \ldots(i)$$

The de-Broglie wavelength,

$$\lambda = \frac{h}{mv} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

Wavelength, $\lambda = \frac{2\pi r}{n} = \frac{2 \times \pi \times 0.53\ \text{Å}}{1} = 3.33\ \text{Å}$

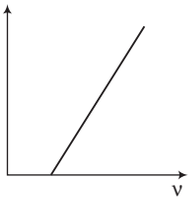
74. According to Einstein's photoelectric equation,

$$\text{KE}_{\max} = h\nu - \phi_0$$

Comparing with the equation of straight line,

$$y = mx + c$$

We get, slope of graph $= h$



75. Number of spectral lines obtained due to transition of electrons from nth orbit to lower orbit is given by

$$N = \frac{n(n-1)}{2}$$

***Case* I** $\qquad 6 = \frac{n_1 (n_1 - 1)}{2} \quad \Rightarrow \quad n_1 = 4$

***Case* II** $\qquad 3 = \frac{n_2 (n_2 - 1)}{2} \quad \Rightarrow \quad n_2 = 3$

Velocity of electron in hydrogen atom in nth orbit

$$v_n \propto \frac{1}{n} \quad \Rightarrow \quad \frac{v_n}{v_n'} = \frac{n_2}{n_1} \quad \Rightarrow \quad \frac{v_6}{v_3} = \frac{3}{4}$$

76. We know that, Frequency, $\nu = RC\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$$\nu_1 = Rc\left[1 - \frac{1}{\infty}\right] = Rc \quad \ldots(i)$$

$$\nu_2 = Rc\left[1 - \frac{1}{4}\right] = \frac{3}{4} Rc \quad \ldots(ii)$$

$$\nu_3 = Rc\left[\frac{1}{4} - \frac{1}{\infty}\right] = \frac{Rc}{4} \quad \ldots(iii)$$

On comparing Eqs. (i), (ii) and (iii), we get

$$\Rightarrow \qquad \nu_1 - \nu_2 = \nu_3$$

77. Since, spectrum of an oil flame consists of continuously varying wavelength in a definite wavelength range, it is an example for continuous emission spectrum.

78. Activity, $A = \lambda N = \frac{0.693}{T_{1/2}} N$

where, $T_{1/2}$ is the half-life of a radioactive sample.

$$\therefore \quad \frac{A_1}{A_2} = \frac{N_1}{T_1} \times \frac{T_2}{N_2} \quad \Rightarrow \quad \frac{T_1}{T_2} = \frac{A_2}{A_1} \times \frac{N_1}{N_2} = \frac{2A_1}{A_1} \times \frac{2N_2}{N_2} = \frac{4}{1}$$

79. A beta minus particle (β^-) is an electron. Emission of β^- involves transformation of a neutron into a proton, an electron and a third particle called anti-neutrino ($\bar{\nu}$).

$${}_0n^1 = {}_1p^1 + {}_{-1}\beta^0 + \bar{\nu}$$

80. When two nuclei of mass number lying in the range of $51 < A < 100$ combined, then a nucleus is formed in the range $100 < A < 150$ which has high value of specific binding energy. Thus, the fusion of two nuclei of mass number lying in range of $51 < A < 100$ will release energy.

81. Mass defect per nucleon is called packing fraction. Packing fraction $(f) = \frac{\Delta m}{A} = \frac{m - A}{A}$, where, m= mass of nucleus, A = mass number. Packing fraction measures the stability of a nucleus. Smaller the value of packing fraction, larger is the stability of nucleus. Packing fraction may be positive, may be negative or zero.

82. Speed of electromagnetic waves in vacuum

$$= \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \text{constant}$$

Therefore, speed of electromagnetic waves is same for all of them.

83. According to Bohr's theory, $mvr = \frac{nh}{2\pi} \Rightarrow 2\pi r = \frac{nh}{mv} = n\lambda$

For $\qquad n = 1, \lambda = 2\pi r$

84. We know that, Time period of electron,

$$T = \frac{4\varepsilon_0^2 n^3 h^3}{mZ^2 e^4} \Rightarrow T \propto n^3 \Rightarrow \therefore \frac{1}{\text{frequency}\,(f)} \propto n^3 \quad \text{or} \quad f \propto n^{-3}$$

85. Linear momentum $= mv = \frac{mcZ}{137n}$

Angular momentum $= \frac{nh}{2\pi}$

Given, linear momentum × angular momentum $\propto n^x$

$$\therefore \quad \frac{mcZ}{137n} \times \frac{nh}{2\pi} \propto n^x \quad \Rightarrow \quad n^0 \propto n^x \quad \Rightarrow \quad x = 0$$

27

Solids and Semiconductor Devices

27.1 Introduction

Solids can be classified in three types as per their electrical conductivity. (i) Conductors, (ii) Insulators and (iii) Semiconductors. In a conductor, large number of free electrons are present. They are always in *zig-zag* motion inside the conductor. In an insulator, all the electrons are tightly bound to the nucleus. If an electric field is applied inside a conductor, the free electrons experience force due to the field and acquire a drift speed. This results in an electric current. The conductivity of a conductor such as copper decreases as the temperature is increased. This is because as the temperature is increased, the random collisions of the free electrons with the particles in the conductor become more frequent. This results in a decrease in the drift speed and hence the conductivity decreases. In insulators almost zero current is obtained unless a very high electric field is applied. Semiconductors conduct electricity when an electric field is applied, but the conductivity is very small as compared to the usual metallic conductors. Silicon, germanium, carbon etc., are few examples of semiconductors.

Conductivity of silicon is about 10^{11} times smaller than that of copper and is about 10^{13} times larger than that of fused quartz.

Conductivity of a semiconductor increases as the temperature is increased.

Chapter Snapshot

- Introduction
- Energy Bands in Solids
- Intrinsic and Extrinsic Semiconductors
- *p* -*n* Junction Diode
- *p* -*n* Junction Diode as a Rectifier
- Junction Transistors
- Transistor as an Amplifier
- Digital Electronics

27.2 Energy Bands in Solids

To understand the energy bands in solids, let us consider the electronic configuration of sodium atom which has 11 electrons. The configuration is $(1s)^2$, $(2s)^2$, $(2p)^6$ and $(3s)^1$. The levels 1*s*, 2*s* and 2*p* are completely filled. The level 3*s* is half filled and the levels above 3*s* are empty. Consider a group of *N* sodium atoms all in ground state separated from each other by large distances such as in sodium vapour. There are total 11*N* electrons. Each atom has two energy states in 1*s* energy level. So, there are 2*N* identical energy states lebelled 1*s* and all them are filled from 2*N* electrons. Similarly, energy level 2*p* has 6*N* identical energy states which are also completely filled. In 3*s* energy levels *N* of the 2*N* states are filled by the electrons and the remaining *N* states are empty.

These ideas are shown in the table given below.

Table 27.1

Energy level	Total available energy states	Total occupied states
1*s*	2*N*	2*N*
2*s*	2*N*	2*N*
2*p*	6*N*	6*N*
3*s*	2*N*	*N*
3*p*	6*N*	0

In the above discussion, we have assumed that N sodium atoms are widely spread and hence the electrons of one atom do not interact with others. As a result, energy states of different states (e.g. 1*s*) are identical. When atoms are drawn closer to one another electron of one atom starts interacting with the electrons of the neighbouring atoms of the same energy states. For example, 1*s* electrons of one atom interact with 1*s* electrons of the other. Due to interaction of electrons the energy states are not identical, but a sort of energy band is formed.

These bands are shown in figure given below.

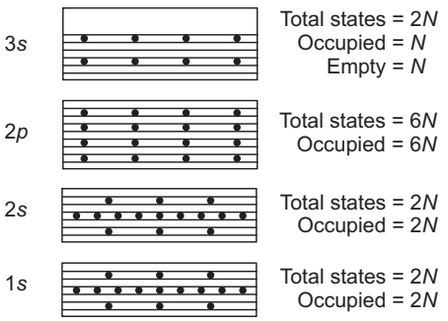


Fig. 27.1

The difference between the highest energy in a band and the lowest energy in the next higher band is called the **band gap** between the two gaps.

Thus, we can conclude that energy levels of an electron in a solid consists of bands of allowed states. There are regions of energy, called gaps, where no states are possible. In each allowed band, the energy levels are very closely spaced. Electrons occupy states which minimize the total energy, depending on the number of electrons and on the arrangement of the bands, a band may be fully occupied or partially occupied.

Now, electrical conductivity of conductors, insulators and semiconductors can be explained by these energy bands.

Conductors

The energy band structure of a conductor is shown in Fig. (a). The last occupied band of energy level (called conduction band) is only partially filled. In conductors, this band overlaps with completely filled valence band.

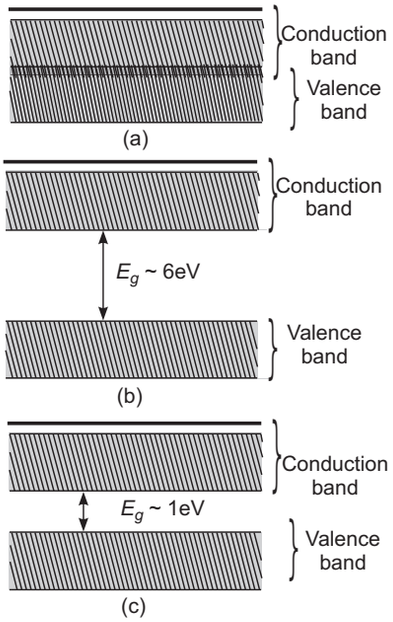


Fig. 27.2 Energy band diagram for a (a) metal, (b) insulator and (c) semiconductor : Note that one can have a metal either when the conduction band is partially filled or when the conduction and valence bands overlap in energy.

Insulators

The energy band structure of an insulator is shown in Fig. (b). The conduction band is separated from the valence band by a wide energy gap (e.g. 6 eV for diamond). But at any non-zero temperature, some electrons can be excited to the conduction band, which is now not fully empty.

Semiconductors

The energy band structure of a semiconductor is shown in Fig. (c). It is similar to that of an insulator but with a comparatively small energy gap. At absolute zero temperature, the conduction band of semiconductors is totally empty and all the energy states in the valence band are filled. The absence of electrons in the conduction band at absolute zero does not allow current to flow under the influence of an electric field. Therefore, they are insulators at low temperatures.

However at room temperatures, some valence electrons acquire thermal energy greater than the energy gap E_g and move to the conduction band where they are free to move under the influence of even a small electric field. Thus, a semiconductor originally an insulator at low temperatures becomes slightly conducting at room temperature. In unlike conductors, the resistance of semiconductors decreases with increasing temperature. We are generally concerned with only the highest valence band and the lowest conduction band. So, when we say valence band, it means the highest valence band. Similarly, when we say conduction band, it means the lowest conduction band.

27.3 Intrinsic and Extrinsic Semiconductors

As discussed above, in semiconductors the conduction band and the valence band are separated by a relatively small energy gap. For silicon, this gap is 1.1 eV and for germanium it is 0.7 eV.

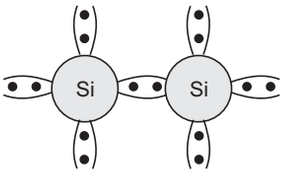


Fig. 27.3

Silicon has an atomic number 14 and electronic configuration $1s^2, 2s^2, 2p^6, 3s^2, 3p^2$.

The chemistry of silicon tells us that it has a valency 4. Each silicon atom makes covalent bonds with the four neighbouring silicon atoms. Semiconductors are divided into two groups on the basis of bonds between the neighbouring atoms.

Intrinsic Semiconductors

A pure (free from impurity) semiconductor which has a valency 4 is called an intrinsic semiconductor. Pure germanium, silicon or carbon in their natural state are intrinsic semiconductors. As discussed above, each atom makes four covalent bonds with their neighbouring atoms. At temperature close to zero, all valence electrons are tightly bound and so no free electrons are available to conduct electricity through the crystal. At room temperature, however a few of the covalent bonds are broken due to thermal agitation and thus some of the valence electrons become free. Thus, we can say that a valence electrons is shifted to conduction band leaving a **hole** (vacancy of electron) in valence band.

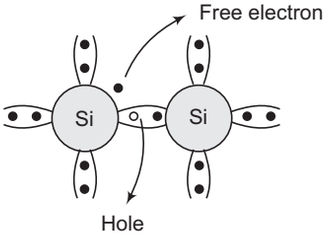


Fig. 27.4

In intrinsic semiconductors,

Number of holes = Number of free electrons

$$n_h = n_e$$

Extrinsic Semiconductors

The conductivity of an intrinsic semiconductor is very poor (unless the temperature is very high). At ordinary temperature, only one covalent bond breaks in 10^9 atoms of Ge. It means that only 1 atom in 10^9 atoms is available for conduction. Conductivity of an intrinsic (pure) semiconductor is significantly increased, if some pentavalent or trivalent impurity is mixed with it. Such impure semiconductors are called extrinsic or doped semiconductors. Extrinsic semiconductors are again of two types (i) *p*-type and (ii) *n*-type.

p-type Semiconductors

When a trivalent (e.g. boron, aluminium, gallium or indium) is added to a germanium or silicon crystal it replaces one of the Ge (or Si) atoms. Its three valence electrons form covalent bonds with neighbouring three Ge (or Si) atoms while the fourth valence electron of Ge (or Si) atom is not able to form the bond. Thus, there remains a hole (an empty space) on one side of the impurity atom.

The trivalent impurity atoms are called **acceptor atoms** because they create holes which accept electrons. Following points are worth noting regarding *p*-type semiconductors.

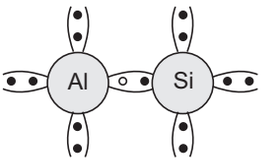


Fig. 27.5

(a) Holes are the majority charge carriers and electrons are minority charge carriers or number of holes are much greater than the number of electrons.

$$n_h >> n_e$$

(b) *p*-type semiconductor is electrically neutral.

(c) *p*-type semiconductor can be shown as

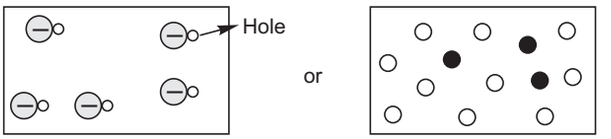


Fig. 27.6

n-type Semiconductors

When a pentavalent impurity atom (antimony, phosphorus or arsenic is added to a Ge (or Si) crystal it replaces a Ge (or Si) atom. Four of the five valence electrons of the impurity atom form covalent bonds with four neighbouring Ge (or Si) atoms and the fifth valence electron becomes free to move inside the crystal lattice. Thus, by doping pentavalent impurity number of free electrons increases.

The impurity (pentavalent) atoms are called **donor atoms** because they donate conduction electrons to the crystal. Following points are worth noting regarding *n*-type semiconductors:

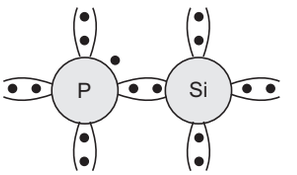


Fig. 27.7

(i) Electrons are the majority charge carriers and holes are minority charge carriers or number of electrons are much greater than the number of holes.

$$n_e >> n_h$$

(ii) *n*-type semiconductor is also electrically neutral.

(iii) *n*-type semiconductor can be shown as

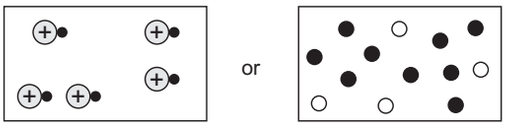


Fig. 27.8

Electrical Conduction through Semiconductors

When a battery is connected across a semiconductor (whether intrinsic or extrinsic) a potential difference is developed across its ends. Due to the potential difference, an electric field is produced inside the semiconductors. A current (although very small) starts flowing through the semiconductor. This current may be due to the motion of (i) free electrons and (ii) holes. Electrons move in opposite direction of electric field while holes move in the same direction.

The motion of holes towards right (in the figure) take place because electrons from right hand side come to fill this hole creating a new hole in their own position. Thus, we can say that holes are moving from left to right. Thus, current in a semiconductor can be written as,

$$i = i_e + i_h$$

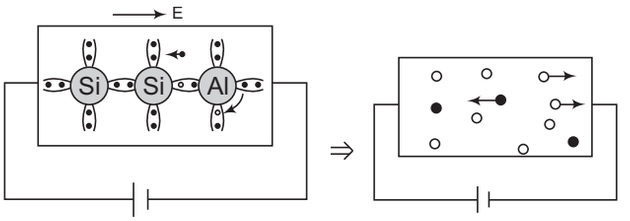


Fig. 27.9

But it should be noted that mobility of holes is less than the mobility of electrons.

Extra Knowledge Points

- Intrinsic semiconductors are pure tetravalent compounds, e.g. germanium, silicon and carbon.
- Due to thermal agitation some covalent bonds are broken. Bounded electrons become free electrons and equal number of holes are created. As the temperature is increased, more and more bonds are broken and conductivity of semiconductor is increased. Opposite to it conductivity decreases with decrease in temperature. At 0 K all intrinsic semiconductors act as insulators.
- In intrinsic semiconductors, number of free electrons and number of holes are same.
- When pentavalent or trivalent impurity is mixed with intrinsic semiconductor its conductivity is increased. Because by adding pentavalent compounds (antimony, phosphorus and arsenic) number of free electrons are increased and by adding trivalent (boron, aluminium, gallium and indium) compounds, number of holes are increased.
- *p*-type semiconductors are formed by doping trivalent compounds in intrinsic semiconductors. In *p*-type semiconductors, number of holes >> number of free electrons.
- *n*-type semiconductors are formed by doping pentavalent compounds in intrinsic semiconductors. In *n*-type semiconductors, number of free electrons >> number of holes.
- *p*-type or *n*-type semiconductors are electrically neutral.
- At 0 K electrical conductivity of extrinsic semiconductors is not zero.
- Current flow through a semiconductor is due to the motion of electron and hole both, i.e.

  $$i = i_e + i_h$$

  but mobility of electrons >> mobility of holes.
- Energy gap between conduction band (CB) and valence band (VB) is 6 eV for diamond, 1.1 eV for silicon and 0.7 eV for germanium.
- Trivalent atoms are called acceptor atoms and pentavalent are donor atoms.
- Holes are denoted by n_h and electrons by n_e .
- The resistivity ρ of a semiconductor is given by,

  $$\frac{1}{\rho} = e\,(n_e\,\mu_e + n_h\,\mu_h)$$

  where, n_e and n_h are electron and hole concentrations in the semiconductor and μ_e and μ_h are the electron and hole mobilities respectively.

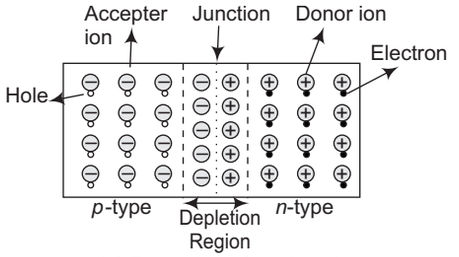
27.4 *p-n* Junction Diode

A *p*-type or *n*-type silicon crystal can be made by adding appropriate impurity as discussed above. These crystals are cut into thin slices called wafer. Semiconductor devices are usually made of these wafers.

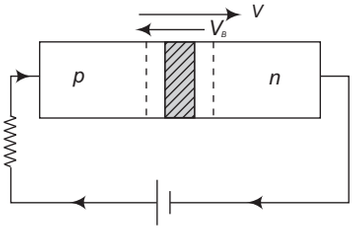
If on a wafer of *n*-type silicon, an aluminium film is placed and heated to a high temperature say 580°, aluminium diffuses into silicon.

In this way, a *p*-type semiconductor is formed on an *n*-type semiconductor. Such a formation of *p*-region on *n*-region is called the *p-n* junction. Another way to make a *p-n* junction is by diffusion of phosphorus into a *p*-type semiconductor. Such *p-n* junctions are used in a host of semiconductor devices of practical applications. The simplest of the semiconductor devices is a *p-n* junction diode.

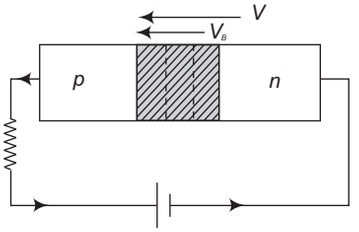
Biasing of a diode In a *p*-*n* junction diode, holes are majority carriers on *p*-side and electrons on *n*-side. Holes, thus diffuse to *n*-side and electrons to *p*-side.



(a) Formation of *p*-*n* junction



(b) Forward biased *p*-*n* junction



(c) Reverse biased *p*-*n* junction

Fig. 27.10

This diffusion causes an excess positive charge in the *n*-region and an excess negative charge in the *p*-region near the junction. This double layer of charge creates an electric field which exerts a force on the electrons and holes against their diffusion. This electric field becomes strong enough as diffusion proceeds to stop it. In the equilibrium position, there is a barrier, for charge motion with the *n*-side at a higher potential than the *p*-side.

The junction region has a very low density of either *p* or *n*-type carriers, because of inter diffusion. It is called **depletion region**. There is a barrier V_B associated with it, as described above. This is called **potential barrier**.

Now, suppose a DC voltage source is connected across the *p*-*n* junction. The polarity of this voltage can lead to an electric field across the *p*-*n* junction that is opposite to that present. The potential drop across the junction decreases and the diffusion of electrons and holes is thereby increased, resulting in a current in the circuit. This is called **forward biasing**. The depletion layer effectively becomes smaller. In the opposite case, called **reverse biasing** the barrier increases, the depletion region becomes larger, current of electrons and holes is greatly reduced.

Thus, the *p*-*n* junction allows a much larger current flow in forward biasing than in reverse biasing. This is crudely, the basis of the action of a *p*-*n* junction as a rectifier. The symbol of *p*-*n* junction diode is 

.

Diffusion Current and Drift Current

Because of concentration difference holes try to diffuse from the *p*-side to the *n*-side at the *p*-*n* junction. This diffusion give rise to a current from *p*-side to *n*-side called **diffusion current**.

Because of thermal collisions electron-hole pair are created at every part of a diode.However, if an electron-hole pair is created in the depletion region, the electron is pushed by the electric field towards the *n*-side and the hole towards the *p*-side. This gives rise to a current from *n*-side to *p*-side called the **drift current**.

Thus, $I_{df} \longrightarrow$ from *p*-side to *n*-side

$I_{dr} \longrightarrow$ from *n*-side to *p*-side

When diode is unbiased, $I_{df} = I_{dr}$ or $I_{\text{net}} = 0$

When diode is forward biased

$I_{df} > I_{dr}$ or I_{net} is from *p*-side to *n*-side.

When diode is reverse biased

$I_{dr} > I_{df}$ or I_{net} is from *n*-side to *p*-side.

Characteristic curve of a *p*-*n*-junction diode

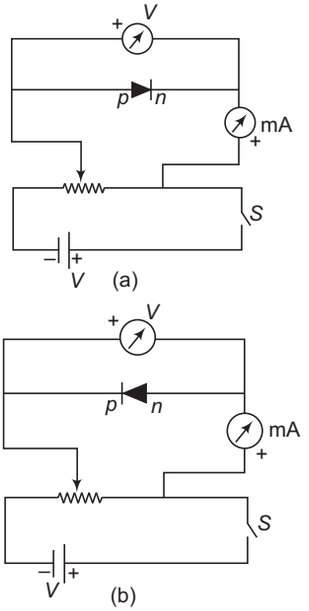


Fig. 27.11 (a) Circuit for obtaining the characteristics of a forward biased diode and
(b) Circuit for obtaining the characteristics of a reverse bias diode

When the diode is forward biased i.e. *p*-side is kept at higher potential, the current in the diode changes with the voltage applied across the diode. The current increases very slowly till the voltage across the diode crosses a certain value.

After this voltage, the diode current increases rapidly, even for very small increase in the diode voltage. This voltage is called the **threshold voltage or cut-off voltage**. The value of the cut in voltage is about 0.2 V for a germanium diode and 0.7 V for a silicon diode.

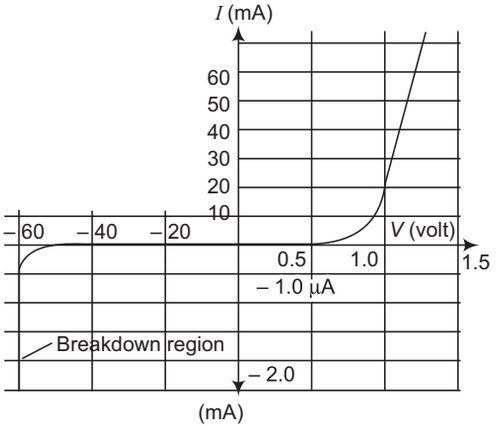


Fig. 27.12 Typical diode characteristics in the forward and reverse region (not to scale)

When the diode is reverse biased, a very small current (about a few micro amperes) produces in the circuit which remains nearly constant till a characteristic voltage called the breakdown voltage is reached. Then, the reverse current suddenly increases to a large valve. This phenomenon is called **avalanche breakdown**.

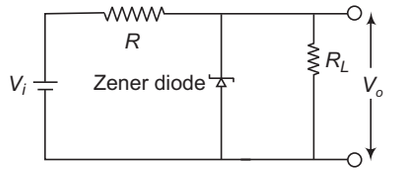


Fig. 27.13

Zener Diode A diode meant to operate in the breakdown region is called an **avalanche diode** or a **Zener diode**. Once the breakdown occurs, the potential difference across the diode does not increase even if the applied battery potential is increased. Such diodes are used to obtain constant voltage output.

Figure shows a Zener diode (in reverse biasing). Even if there is a small change in the input voltage V_i, the current through R_L remains almost the same. The current through the diode changes but the voltage across it remains the same.

- That even in reverse biasing current is not absolutely zero even if voltage is less than the breakdown voltage. Small current due to minority charge carriers on both sides flows in the circuit. Minority charge carriers are electrons on *p*-side and holes on *n*-side of the diode. If the reverse bias is made very high at breakdown voltage, the minority carriers acquire kinetic energy high enough to break the covalent bonds near the junction, thus liberating a huge of electron-hole pair.

27.5 *p-n* Junction Diode as a Rectifier

A rectifier is a device which converts an alternating current (or voltage) into a direct (or unidirectional) current (or voltage). A *p-n* junction diode can work as an excellent rectifier. It offers a low resistance for the current to flow when it is forward biased, but a very high resistance when reverse biased. Thus, it allows current through it only in one direction and acts as a rectifier. The junction diode can be used either as an half wave rectifier or as a full wave rectifier.

(i) ***p-n* junction diode as half wave rectifier** A simple rectifier circuit called the half-wave rectifier, using only one diode is shown in figure.

When the voltage at A is positive, the diode is forward biased and it conducts and when the voltage at A is negative, the diode is reverse biased and does not conduct. Since, the diode conducts only in the positive half cycles, the voltage between X and Y or across R_L will be DC but in pulses. When this is given to a circuit called filter (normally a capacitor), it will smoothen the pulses and will produce a rather steady DC voltage.

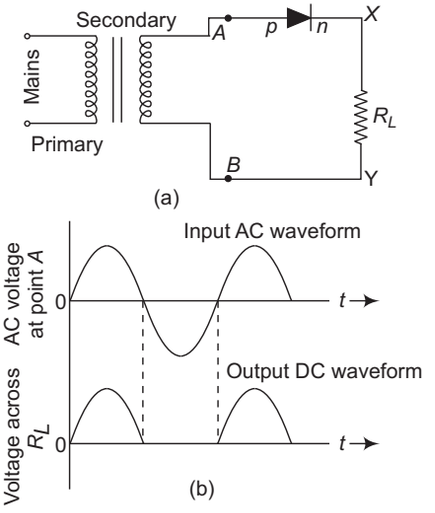


Fig. 27.14 (a) Half wave rectifier and (b) AC and DC voltage waveforms in a half wave rectifier

(ii) ***p-n* junction diode as full wave rectifier** Figure shows a circuit which is used in full wave rectification. Two diodes are used for this purpose. The secondary coil of the transformer is wound in two parts and the junction is called a centre-tap (CT). During one half cycle D_1 is forward biased and D_2 is reverse biased. Therefore, D_1 conducts but D_2 does not. Current flows from X to Y through load resistance R_L. During another half cycle D_2 is forward biased and D_1 is reverse

biased. Therefore, D_2 conducts and D_1 does not. In this half cycle also current through R_L flows from X to Y.

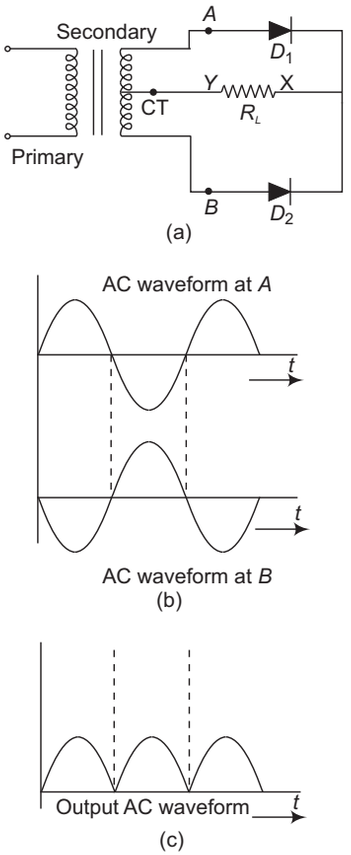


Fig. 27.15 (a) Full wave rectifier (b) AC voltage waveforms at points A and B and (c) Output DC wave forms of a full wave rectifier.

Thus, current through R_L in both the half cycles is in one direction, i.e. from X to Y.

Bridge rectifier Another full-wave rectifier called the bridge rectifier which uses four diodes is shown in figure.

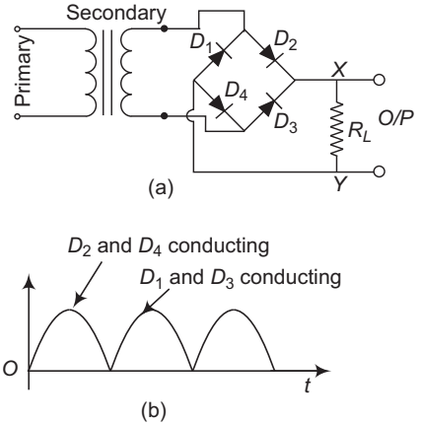


Fig. 27.16 (a) Bridge rectifier and (b) output waveforms for a bridge rectifier

For one half cycle diodes D_1 and D_3 are forward biased and D_2 and D_4 are reverse biased. So, D_1 and D_3 conduct but D_2 and D_4 do not. Current through R_L flows from X to Y. In another half cycle D_2 and D_4 are forward biased and D_1 and D_3 are reverse biased. So, in this half cycle D_2 and D_4 conduct but D_1 and D_3 do not. Current again flows from X to Y through R_L. Thus, we see that current through R_L always flows in one direction from X to Y.

Extra Knowledge Points

- Symbol of a p-n junction diode is 
- A p-n junction diode is said to be forward biased if p-side is on higher potential and n-side on lower potential. It is reverse biased if opposite is the case, i.e. p-side is on lower potential and n-side on higher potential.
- Even after rectification ripples are present in the output which can be removed upto great extent by a filter circuit. A filter circuit consists of a capacitor.

Example 27.1 *Find current passing through* $2\,\Omega$ *and* $4\,\Omega$ *resistance in the circuit shown in figure.*

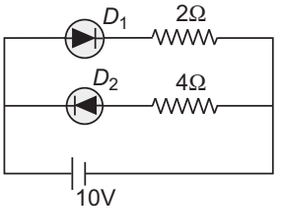


Fig. 27.17

Sol. In the given circuit diode D_1 is forward biased and D_2 is reverse biased. Hence, D_1 will conduct but D_2 does not. Therefore, current through $4\,\Omega$ resistance will be zero while through $2\,\Omega$ resistance will be, $\frac{10}{2} = 5$ A.

Example 27.2 *(a) Calculate the value of* V_o *and* i *if the Si and Ge diode start conducting at* $0.7\,V$ *and* $0.3\,V$ *respectively.*

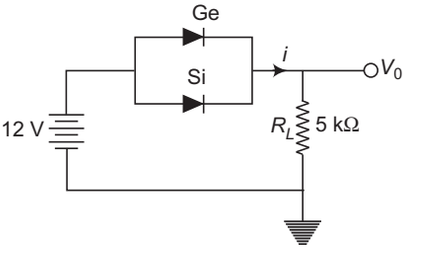


Fig. 27.18

(b) If the Ge diode connection is now reversed, what will be the new values of V_o *and* i *?*

Sol. (a) Ge diode will start conducting before the Si diode does so. The effective forward voltage across Ge diode is (12 – 0.3) V = 11.7 V. This will appear as the output voltage across the load, i.e.

$$V_o = 11.7 \text{ V}$$

The current through R_L,

$$i = \frac{11.7}{5 \times 10^3} \text{ A} = 2.34 \text{ mA}$$

(b) On reversing the connections of Ge diode, it will be reverse biased and conduct no current. Only Si diode will conduct. Therefore,

$$V_o = (12 - 0.7) \text{ V} = 11.3 \text{ V}$$

and $$i = \frac{11.3}{5 \times 10^3} \text{ A} = 2.26 \text{ mA}$$

27.6 Junction Transistors

A junction transistor is formed by sandwiching a thin wafer of one type of semiconductor between two layers of another type. The *n*-*p*-*n* transistor has a *p*-type wafer between two *n*-type layers. Similarly, the *p*-*n*-*p* transistor has a *n*-type wafer between two *p*-type layers.

p-*n*-*p* Transistor

Figure shows a *p*-*n*-*p* transistor, in which a thin layer of *n*-type semiconductor is sandwiched between two *p*-type semiconductors. The middle layer (called the base) is very thin (of the order of 1 μm) as compared to the widths of the two layers at the sides. Base is very lightly doped. One of the side layer (called emitter) is heavily doped and the other side layer (called collector) is moderately doped. Fig. (c) shows the symbol of *p*-*n*-*p* transistor.

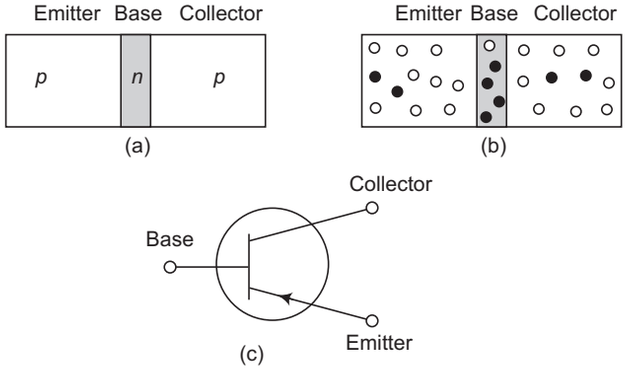


Fig. 27.19

n-*p*-*n* Transistor

In *n*-*p*-*n* transistor, *p*-type semiconductor is sandwiched between two *n*-type semiconductors. Symbol of *p*-*n*-*p* transistor is shown in Fig. (c).

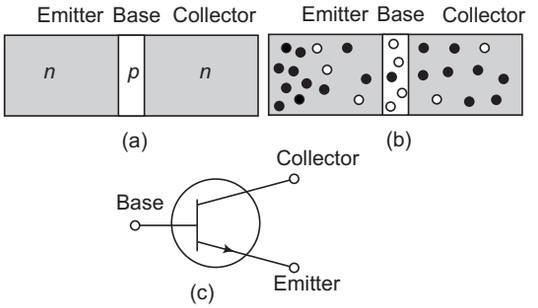


Fig. 27.20

More points about a transistor A transistor is basically a three terminal device. Terminals come out from the emitter, base and the collector for external connections. In normal operation of a transistor, the emitter-base junction is always forward biased and collector-base junction is reverse biased.

The arrow on the emitter-base line shows the direction of current between emitter and base. In an *n*-*p*-*n* transistor for example, there are a large number of conduction electrons in the emitter and a large number of holes in the base. If the junction is forward biased, the electrons will diffuse from emitter to the base and holes will diffuse from the base to the emitter. The direction of electric current at this junction is therefore from the base to the emitter. A transistor can be operated in three different modes.

(i) Common emitter (or grounded emitter)
(ii) Common collector (or grounded collector) and
(iii) Common base (or grounded base)

In common emitter mode, emitter is kept at zero potential. Similarly in common collector mode collector is at zero potential and so on.

Working of a *p*-*n*-*p* Transistor

Let us consider the working of a *p*-*n*-*p* transistor in common base mode. In emitter (*p* type) holes are in majority. Since, emitter-base is forward biased holes move toward base. Few of them combine with electrons in the base and rest go to the collector. Since, base-collector is reverse biased holes coming from base move toward the terminal of collector. They combine with equal number of electrons entering from collector terminal.

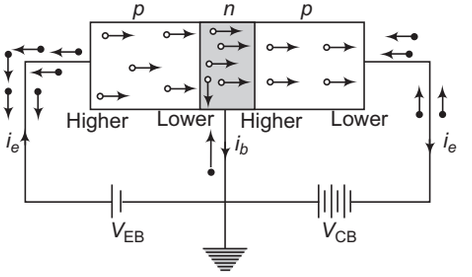


Fig. 27.21

Let us take an example with some numerical values.

Suppose 5 holes enter from emitter to base. This deficiency of 5 holes in emitter is compensated when 5 electrons emit from emitter and give rise to i_e. One out of five electrons which reach the base combine with one electron entering from base (the equivalent current is i_b). Rest four holes enter the collector and move towards its terminal. On the other hand 5 electrons which leave the emitter (as i_e) come to the base, emitter and collector junction. One electron of it goes to base and rest four to collector. This four electrons give rise to i_c (the collector current) and combine with the four holes coming from the base, and thus circuit is complete. From the figure we can see that,

$$i_e = i_c + i_b$$

- That i_b is only about 2% of i_e, or roughly around 2% of holes coming from emitter to base combine with the electrons. Rest 98% move to collector.

Working of *n-p-n* Transistor

A common base circuit of an *n-p-n* transistor is shown in figure. Majority charge carriers in the emitter (*n*-type) are electrons. Since, emitter-base circuit is forward biased. The electrons rush from emitter to base. Few of them leave the base terminal (comprising i_b) and rest move to collector. These electrons finally leave the collector terminal (give rise to i_c). Electrons coming from base and from collector meet at junction *O* and they jointly move to emitter, which gives rise to i_e.

Thus, here also we can see that

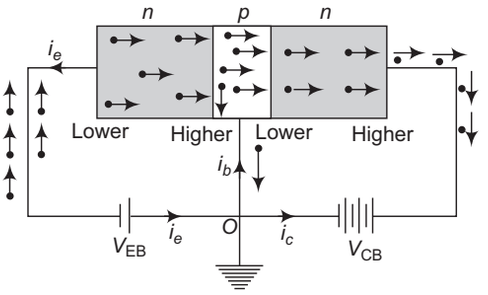


Fig. 27.22

$$i_e = i_b + i_c$$

- That although the working principle of *p-n-p* and *n-p-n* transistors are similar but the current carriers in *p-n-p* transistor are mainly holes whereas in *n-p-n* transistors the current carriers are mainly electrons. Mobility of electrons are however more than the mobility of holes, therefore *n-p-n* transistors are used in high frequency and computer circuits where the carriers are required to respond very quickly to signals.

α and β-parameters α and β-parameters of a transistor are defined as,

$$\alpha = i_c / i_e \quad \text{and} \quad \beta = i_c / i_b$$

As i_b is about 1 to 5% of i_e, α is about 0.95 to 0.99 and β is about 20 to 100. By simple mathematics we can prove that,

$$\beta = \frac{\alpha}{1-\alpha}$$

27.7 Transistor as an Amplifier

A transistor can be used for amplifying a weak signal.

When a transistor is to be operated as amplifier, three different basic circuit connections are possible. These are

(i) common base,

(ii) common emitter and

(iii) common collector circuits.

Whichever circuit configuration, the emitter is always forward biased while the collector is always reverse biased.

(a) **Common base amplifier using a *p-n-p* transistor** In common base amplifier, the input signal is applied across the emitter and the base, while the amplified output signal is taken across the collector and the base. This circuit provides a very low input resistance, a very high output resistance and a current gain of just less than 1. Still it provides a good voltage and power amplification. There is no phase difference between input and output signals.

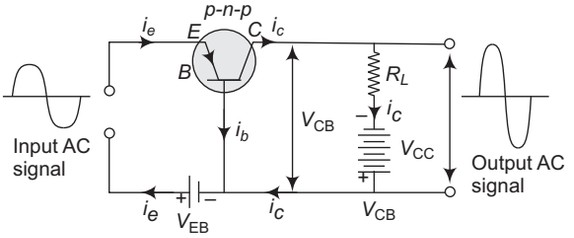


Fig. 27.23

The common base amplifier circuit using a *p-n-p* transistor is shown in figure. The emitter base input circuit is forward biased by a low voltage battery V_{EB}. The collector base output circuit is reversed biased by means of a high voltage battery V_{CC}. Since, the input circuit is forward biased, resistance of input circuit is small. Similarly, output circuit is reverse biased, hence resistance of output circuit is high.

The weak input AC voltage signal is superimposed on V_{EB} and the amplified output signal is obtained across collector-base circuit. In the figure we can see that,

$$V_{CB} = V_{CC} - i_c R_L$$

The input AC voltage signal changes net value of V_{EB}. Due to fluctuation in V_{EB}, the emitter current i_e also fluctuates which in turn fluctuates i_c. In accordance with the above equation there are fluctuations in V_{CB}, when the input signal is applied and an amplified output is obtained.

Current Gain, Voltage Gain and Power Gain

(i) **Current gain** Also called AC current gain (α_{AC}), is defined as the ratio of the change in the collector current to the change in the emitter current at constant collector-base voltage.

Thus, α_{AC} or simply $\alpha = \dfrac{\Delta i_c}{\Delta i_e}$ $(V_{CB} = \text{constant})$

As stated earlier also, α is slightly less than 1.

(ii) **Voltage gain** It is defined as the ratio of change in the output voltage to the change in the input voltage. It is denoted by A_V. Thus,

$$A_V = \frac{\Delta i_c \times R_{out}}{\Delta i_e \times R_{in}}$$

but $\dfrac{\Delta i_c}{\Delta i_e} = \alpha$, the current gain.

$\therefore$ $$A_V = \frac{\alpha R_{out}}{R_{in}}$$

Since, $R_{out} >> R_{in}$, A_V is quite high, although α is slightly less than 1.

(iii) **Power gain** It is defined as the change in the output power to the change in the input power. Since,

$$P = Vi$$

Therefore, power gain = Current gain × Voltage gain

or $$\text{power gain} = \alpha^2 \cdot \frac{R_{out}}{R_{in}}$$

Important Points in Common Base Amplifier

(i) The output voltage signal is in phase with the input voltage signal.

(ii) The common base amplifier is used to amplify high (radio) frequency signals and to match a very low source impedance (~20 Ω) to a high load impedance (~100 kΩ).

(b) **Common emitter amplifier using a *p-n-p* transistor** Figure shows a *p-n-p* transistor as an amplifier in common emitter mode. The emitter is common to both input and output circuits. The input (base-emitter) circuit is forward biased by a low voltage battery V_{BE}. The output (collector-emitter) circuit is reverse biased by means of a high voltage battery V_{CC}.

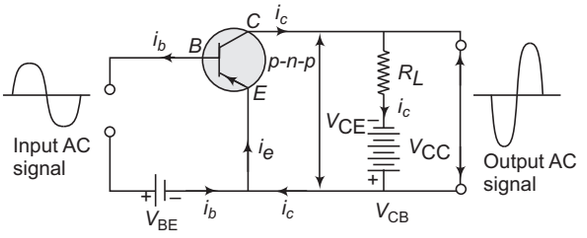


Fig. 27.24

Since, the base-emitter circuit is forward biased, input resistance is low. Similarly, collector-emitter circuit is reverse biased, therefore output resistance is high. The weak input AC signal is superimposed on V_{BE} and the amplified output signal is obtained across the collector-emitter circuit.

In the figure we can see that,

$$V_{CE} = V_{CC} - i_c R_L$$

When the input AC voltage signal is applied across the base-emitter circuit, it fluctuates V_{BE} and hence the emitter current i_e. This in turn changes the collector current i_c consequently V_{CE} varies in accordance with the above equation. This variation in V_{CE} appears as an amplified output.

Current Gain, Voltage Gain and Power Gain

(i) **Current gain** Also called AC current gain (β_{AC}), is defined as the ratio of the collector current to the base current at constant collector to emitter voltage.

β_{AC} or simply $\beta = \left(\dfrac{\Delta i_c}{\Delta i_b}\right)$ $(V_{CE} = \text{constant})$

(ii) **Voltage gain** It is defined as the ratio of the change in the output voltage to the change in the input voltage. It is denoted by A_V. Thus,

$$A_V = \frac{\Delta i_c \times R_{out}}{\Delta i_b \times R_{in}}$$

or $$A_V = \beta \left(\frac{R_{out}}{R_{in}}\right)$$

(iii) **Power gain** It is defined as the ratio of change in output power to the change in the input power. Since,

$$P = Vi$$

Therefore,

power gain = current gain × voltage gain

or $$\text{power gain} = \beta^2 \left(\frac{R_{out}}{R_{in}}\right)$$

Important Points in Common Emitter Amplifier

(i) The value of current gain β is from 15 to 50 which is much greater than α.

(ii) The voltage gain in common emitter amplifier is larger compared to that in common base amplifier.

(iii) The power gain in common emitter amplifier is extremely large compared to that in common base amplifier.

(iv) The output voltage signal is 180° out of phase with the input voltage signal in the common emitter amplifier.

Transconductance (g_m)

There is one more term called transconductance (g_m) in common emitter mode. It is defined as the ratio of the change in the collector current to the change in the base to emitter voltage at constant collector to emitter voltage. Thus,

$$g_m = \left(\frac{\Delta i_c}{\Delta V_{BE}}\right) \qquad (V_{CE} = \text{constant})$$

The unit of g_m is Ω^{-1} or siemen (S).

By simple calculation we can prove that,

$$g_m = \frac{\beta}{R_{in}}$$

Advantages of a Transistor over a Triode Valve

A transistor is similar to a triode valve in the sense that both have three elements. While the elements of a triode are, 'cathode', 'plate' and 'grid' the three elements of a transistor are 'emitter', 'collector' and 'base'. Emitter of a transistor can be compared with the cathode of the triode, the collector with the plate and the base with the grid.

Transistor has following advantages over a triode valve

(i) A transistor is small and cheap as compared to a triod valve. They can bear mechanical shocks.

(ii) A transistor has much longer life as compared to a triode valve.

(iii) Loss of power in a transistor is less as it operates at a much lower voltage.

(iv) In a transistor no heating current is required. So, unlike a triode valve, a transistor starts functioning immediately as soon as the switch is opened. In case of valves, they come in operation after some time of opening the switch (till cathode gets heated).

Drawbacks of a Transistor over a Triode Valve

Transistor have following drawbacks as compared to valves

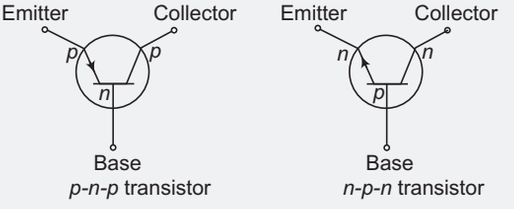
(i) Since, the transistors are made of semiconductors they are temperature sensitive. We cannot work on transistors at high temperatures.

(ii) In transistors noise level is high.

Keeping all the factors into consideration, transistors have replaced the valve from most of the modern electronic devices.

Extra Knowledge Points

- In the symbol of a transistor, arrow is made between base and emitter. The direction of arrow is from *p* to *n*. This arrow shows the direction of current in emitter terminal.



- In normal operation, the emitter base is forward biased and collector base is reverse biased.
- In the connecting wires, electrons are always responsible for current flow, but inside the transistor holes are mainly responsible for current flow in *p-n-p* transistor and electrons in case of *n-p-n* transistor.
- In $i_e = i_c + i_b$, i_c is slightly less than i_e while $i_b << i_e$ or $i_b << i_c$.

$$\alpha = i_c / i_e, \beta = i_c / i_b \text{ and } \beta = \frac{\alpha}{1-\alpha}.$$

  Here α is about 0.95 to 0.99 and β is about 20 to 100.
- In common base amplifier

  (i) Current gain $= \alpha = \dfrac{\Delta i_c}{\Delta i_e}$ $\qquad (V_{CB} = \text{constant})$

  Voltage gain $= \alpha \dfrac{R_{out}}{R_{in}}$ and Power gain $= \alpha^2 \dfrac{R_{out}}{R_{in}}$

  (ii) Input and output signals are in same phase.
- In common emitter amplifier

  (i) Current gain $= \beta = \dfrac{\Delta i_c}{\Delta i_b}$ $\qquad (V_{CE} = \text{constant})$

  Voltage gain $= \beta \dfrac{R_{out}}{R_{in}}$ and Power gain $= \beta^2 \dfrac{R_{out}}{R_{in}}$

  (ii) Input and output signals are 180° out of phase.

  (iii) Transconductance, $g_m = \dfrac{\Delta i_c}{\Delta V_{BE}}$ $\qquad (V_{CE} = \text{constant})$

Example 27.3 *The current gain of a transistor in a common base arrangement in* 0.98*. Find the change in collector current corresponding to a change of* 5.0 *mA in emitter current. What would be the change in base current ?*

Sol. Given, $\alpha = 0.98$ and $\Delta i_e = 5.0$ mA

From the definition of,

$$\alpha = \frac{\Delta i_c}{\Delta i_e}$$

Change in collector current,

$$\Delta i_c = (\alpha)(\Delta i_e) = (0.98)(5.0)\text{ mA} = 4.9\text{ mA}$$

Further, change in base current,

$$\Delta i_b = \Delta i_e - \Delta i_c = 0.1\text{ mA}$$

Example 27.4 *A transistor is connected in common-emitter configuration. The collector supply is* $8\,V$ *and the voltage drop across a resistor of* $800\,\Omega$ *in the collector circuit is* $8.5\,V$*. If the current gain factor* (α) *is* 0.96*, find the base current.*

Sol. $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.96}{1-0.96} = 24$

The collector current is,

$$i_c = \frac{\text{Voltage drop across collector resistor}}{\text{Resistance}}$$

$$= \frac{0.5}{800}\,\Omega = 0.625 \times 10^{-3}\text{ A}$$

From the definition of $\beta = \dfrac{i_c}{i_b}$

The base current $i_b = \dfrac{i_c}{\beta} = \dfrac{0.625 \times 10^{-3}}{24}$ A

$$= 26\,\mu\text{A}$$

Example 27.5 *In a common-emitter amplifier, the load resistance of the output circuit is* 500 *times the resistance of the input circuit. If* $\alpha = 0.98$*, then find the voltage gain and power gain.*

Sol. Given $\alpha = 0.98$ and $\dfrac{R_{out}}{R_{in}} = 500$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$$

(i) Voltage gain $= (\beta)\left(\dfrac{R_{out}}{R_{in}}\right) = (49)(500) = 24500$

(ii) Power gain $= (\beta^2)\left(\dfrac{R_{out}}{R_{in}}\right) = (49)^2\,(500) = 1200500$

27.8 Digital Electronics

(i) **Binary system** There are a number of questions which have only two answers Yes or No. A statement can be either True or False. A switch can be either ON or OFF. These values may be represented by two symbols 0 and 1. In a number system in which we have only two digits is called a binary system. (decimal system for example has ten digits).

In binary system, usually we write 1 for positive response (e.g. when a switch is ON) and 0 for negative respnonse (when a switch is OFF).

(ii) **Truth table** To understand the concept of truth table, let us take an example.

A bulb is connected to an AC source *via* two switches S_1 and S_2.

In binary system, we will write 0, if the switch (or bulb) is off and write 1 if it is on. Further let us write

A for state of switch S_1
B for state of switch S_2
and C for state of the bulb.

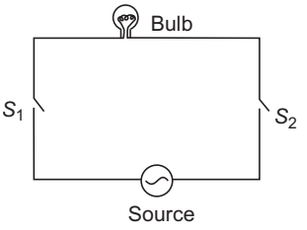


Fig. 27.27

Now, let us make a table (called truth table) which I think is self explanatory.

Table 27.2

Switch S_1	Switch S_2	Bulb	*A*	*B*	*C*
Off	On	Off	0	1	0
On	Off	Off	1	0	0
Off	Off	Off	0	0	0
On	On	On	1	1	1

Exercise *Make a truth table corresponding to the circuit as shown in figure.*

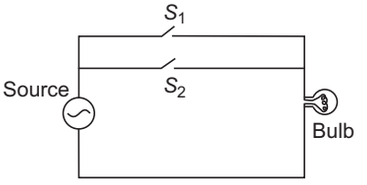


Fig. 27.28

Table 27.3

Switch S_1	Switch S_2	Bulb	*A*	*B*	*C*
On	Off	On	1	0	1
Off	On	On	0	1	1
Off	Off	Off	0	0	0
On	On	On	1	1	1

(iii) **Logical function** A variable (e.g. state of a switch or state of a bulb) which can assume only two values (0 and 1) is called a logical variable. A function of logical variables is called a logical function. **AND**, **OR** and **NOT** represents three basic operation on logical variables.

'AND' function Suppose C is a function of A and B, then it will be said an 'AND' function when C has value 1 when both A and B have value 1. Truth table corresponding to table 27.2 is an example of 'AND' function. The function is written as,

$$C = A \text{ and } B$$

AND function is also denoted as $C = A \cdot B$

'OR' function C, a function of A and B will be said an 'OR' function when C has value 1 when either of A or B has value 1. Truth table corresponding to table 27.3 is an example of 'OR' function. The function is written as,

$$C = A \text{ OR } B$$

OR function is also denoted as,

$$C = A + B$$

'NOT' function 'NOT' function is a function of a single variable.

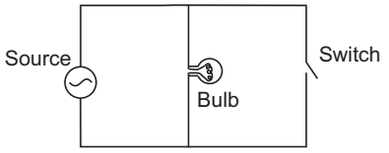


Fig. 27.29

A bulb is short circuited by a switch. If the switch is open, the current goes through the bulb and it is on. If the switch is closed the current goes through the switch and the bulb is off. The truth table corresponding to the above situation (or NOT function) is as under.

Table 27.4

Switch	Bulb	*A*	*B*
Open	On	0	1
Closed	Off	1	0

'NOT' function is denoted as,

$$B = \text{NOT } A \text{ or } B = \overline{A}$$

Example 27.6 *Write the truth table for the logical function $D = (A \text{ OR } B)$ AND B.*

Sol. A OR B is a logical function, say it is equal to X, i.e.

$$X = A \text{ OR } B$$

Now, $D = X$ AND B

The corresponding truth table is as under.

Table 27.5

A	*B*	$X = A$ **OR** B	$D = (A$ **OR** $B)$ **AND** B
1	0	1	0
0	1	1	1
0	0	0	0
1	1	1	1

That the given function can also be written as,

$$D = (A + B) \cdot B$$

(iv) **Logic gates** Logic gates are important building blocks in digital electronics. These are circuits with one or more inputs and one output. The basic gates are OR, AND, NOT, NAND, NOR and XOR. As we know, in digital electronics only two voltage levels are present. Conventionally these are 5V and 0V, referred to as 1 and 0 respectively or *vice-versa.* They are also referred to as high and low.

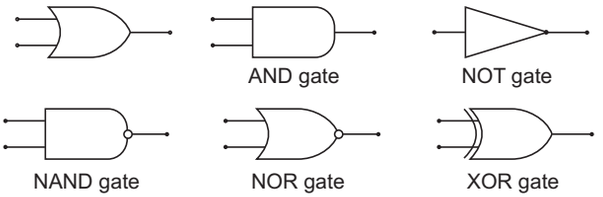


Fig. 27.30

Figure given above are the symbols of six basic gates.

OR gate The truth table of 'OR' gate is given below.

Table 27.6

A	*B*	*X*
0	0	0
0	1	1
1	0	1
1	1	1

The output X will be 1 (i.e. 5V) when the A input is 1, OR when the B input is 1, OR when both are 1. This is written as, $X = A + B$

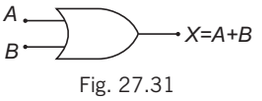


Fig. 27.31

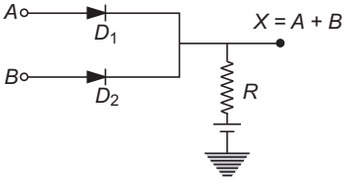


Fig. 27.32

Figure shows construction of an OR gate using two diodes.When either of point A or point B (or both) has potential +5V, diodes D_1 or D_2 (or both) are forward biased and the potential at X is the same as the common potential at A and B which is 5V.

AND gate The truth table of 'AND' gate is given below.

Table 27.7

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

The output Y will be 1 (i.e. 5V) when both the inputs A and B is 1. This is written as,

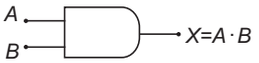


Fig. 27.33

$$Y = A \cdot B$$

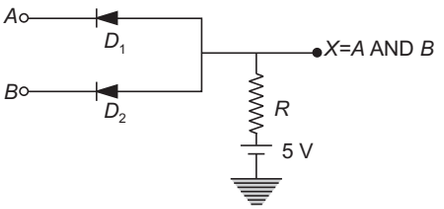


Fig. 27.34

Figure shows construction for an AND gate using two diodes D_1 and D_2.

When potentials at A and B both are zero, then both the diodes are forward biased and offer no resistance. The potential at X in this position is equal to the potential at A or B i.e. 0. Thus $X = 0$, when both A and B are zero. Now, suppose potential at A is zero but at B is 5V, then D_1 is forward biased. In this situation potential at X is also zero.

Thus, $X = 0$ when $A = 0$. Similarly, we can see that $X = 0$ when $B = 0$. Lastly when potentials at both A and B are 5V, so that both the diodes are unbiased and there will be no current through R and the potential at X will be equal to 5V. Thus, $X = 1$ when A and B both are 1.

NOT gate This has one input and one output. The output is the inverse of the input. When the input A is 1, the output X will be 0 and *vice-versa.* The truth table for 'NOT' gate is given below.

Table 27.8

A	X
0	1
1	0

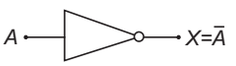


Fig. 27.35

A NOT gate cannot be constructed with diodes. Transistor is used for realisation of a NOT gate, but at this stage students do not require it. A NOT gate is written as $X = \bar{A}$.

NAND gate The function, $X =$ NOT (A and B) of two logical variables A and B is called NAND function. It is written as $X = A$ NAND B. It is also written as,

$$X = \overline{A \cdot B} \quad \text{or} \quad X = \overline{AB}$$

The truth table of a 'NAND' gate is given below

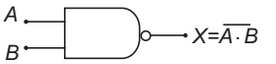


Fig. 27.36

Table 27.9

A	B	$A \cdot B$	$X = \overline{A \cdot B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

NOR gate The function $X =$ NOT (A OR B) is called a NOR function and is written as $X = A$ NOR B. It is also written as, $X = \overline{A + B}$.

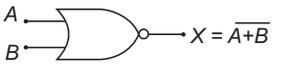


Fig. 27.37

The truth table for a NOR gate is given below.

Table 27.10

A	B	$A + B$	$X = \overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

XOR gate It is also called the exclusive OR function. It is a function of two logical variables A and B which evaluates to 1 if one of two variables is 0 and the other is 1. The function is zero, if both the variables are 0 or 1.

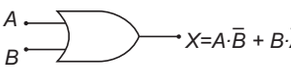


Fig. 27.38

$$A \text{ XOR } B = A \cdot \overline{B} + \overline{A} \cdot B$$

The truth table for a XOR gate is given below.

Table 27.11

A	B	$\bar{A}$	$\bar{B}$	$A \cdot \bar{B}$	$\bar{A} \cdot B$	$A = A \cdot \bar{B} + \bar{A} \cdot B$
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

Example 27.7 *Construct the truth table for the function X of A and B represented by figure shown here.*

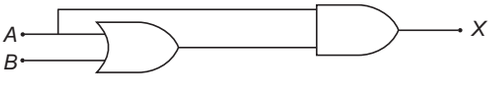


Fig. 27.39

Sol. The output X in terms of the input A and B can be written as, $X = A \cdot (A + B)$.

Let us make the truth table corresponding this function.

Table 27.12

A	*B*	*A* + *B*	$X = A \cdot (A + B)$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

(v) **Binary Mathematics** As we know in binary system only two digits 0 and 1 are used. For any calculation in binary system, natural numbers (1, 2, 3, etc.) are first converted into binary system. To understand binary system more clearly let us first take an example of decimal number system. Suppose we have a number 625.72. This number can be written as,

$$625.72 = 6\times 10^{2} + 2\times 10^{1} + 5\times 10^{0} + 7\times 10^{-1} + 2\times 10^{-2}$$

Thus, **the base of the decimal number system is 10.** The place values are found by raising the base 10 to the power of the place. Also, powers are numbered to the left of the decimal point starting with 0, and to the right of the decimal point starting with –1. In **binary system the base is 2**. The two binary digits 0 and 1 are called 'bits'. Thus, all binary numbers consist of a chain of the bits 0 and 1. Examples are 10, 101, 1101 etc. These are read as, one-zero, one-zero-one and one-one-zero-one etc.

In a binary number, the place value of each bit corresponds to some power of 2. For example, let us consider the binary number, 1101.011. This can be written as,

$$\begin{aligned}1101.011 &= 1\times 2^{3} + 1\times 2^{2} + 0\times 2^{1} + 1\times 2^{0} + 0\times 2^{-1} + 1\times 2^{-2} + 1\times 2^{-3}\\ &= 8 + 4 + 0 + 1 + 0 + 0.25 + 0.125\\ &= 13.375\end{aligned}$$

Binary numbers are used by all digital circuits, e.g. in computers, calculators etc.

Conversion of a decimal number into its equivalent binary number A decimal number can be converted into binary number by using divide by 2 rule. We go on dividing the given decimal number by 2, until the quotient is zero and write down the remainder after each division. These remainders taken in reverse order form the required binary number.

Let us convert 23 into its binary equivalent.

Table 27.13

Divide by 2	23	Remainder
Divide by 2	11	1
Divide by 2	5	1
Divide by 2	2	1
Divide by 2	1	0
	0	1

The remainders in reverse order are 10111. Hence, the binary equivalent of 23 is 10111. Symbolically we write,

$$(23)_{10} = (10111)_{3}$$

The conversion of a decimal fraction into binary fraction is performed by multiply-by-2 rule. Let us take an example, we convert 0.125 into its binary equivalent.

Table 27.14

Multiply by 2	Binary
$0.125\times 2 = 0.250$	0
$0.250\times 2 = 0.500$	0
$0.500\times 2 = 1.000$	1
$0.000\times 2 = 0.000$	0

The recovered [binary] taken from top to bottom form the required binary fraction.

The binary equivalent of 0.125 is therefore written 0.0010.

- If a decimal number contains both the integral and the fractional part, such as 23.125 then after determining the binary equivalent of each part separately, the two parts are combined. Thus, binary equivalent of 23.125 is 10111.0010.

Example 27.8 *Convert the decimal number* 10.625 *into its binary equivalent.*

Sol. Integral part is 10. It can be converted into its binary part by using divide by 2 rule.

Table 27.15

Divide by 2	10	Remainder
Divide by 2	5	0
Divide by 2	2	1
Divide by 2	1	0
	0	1

Thus, $(10)_{10} = (1010)_{2}$

Fractional part is 0.625. It can be converted into its binary part by using multiply-by-2 rule.

Table 27.16

Multiply by 2	Binary
$0.625\times 2 = 1.250$	1
$0.250\times 2 = 0.500$	0
$0.500\times 2 = 1.000$	1
$0.000\times 2 = 0.000$	0

Thus, $(0.625)_{10} = 0.1010$

$\therefore$ $(10.625)_{10} = (1010.1010)_{2}$

Example 27.9 *Convert the binary number* 101011.1101 *into equivalent decimal number.*

Sol. $101011.1101 = 1\times 2^5 + 0\times 2^4 + 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0 + 1\times 2^{-1} + 1\times 2^{-2} + 0\times 2^{-3} + 1\times 2^{-4}$

$= 32 + 0 + 8 + 0 + 2 + 1 + 0.5 + 0.25 + 0 + 0.0625$

$= 43.8125$

$\therefore (101011.1101)_2 = (43.8125)_{10}$

Exercise (1) *Convert the following binary numbers into their decimal equivalents*.

(*a*) 0.10110 (*b*) 1001.111

Ans. (*a*) 0.6875 (*b*) 9.875

(2) *Convert the following binary numbers into their decimal equivalents.*

(*a*) 11110 (*b*) 101101

Ans. (*a*) 30 (*b*) 45

(3) *Convert the following decimal numbers into their binary equivalents.*

(*a*) 25.5 (*b*) 0.65

Ans. (*a*) 11001.10 (*b*) 0.101001

Sum of binary numbers The sum of digits 0 and 1 with themselves and with each other are given by the following laws :

(i) $0+0=0$ (ii) $1+0=1$

(iii) $0+1=1$ (iv) $1+1=10$

Now, let us take few examples of different types. Through these examples we can easily understand the method of adding two binary numbers.

Example 27.10 *Add the binary numbers* 101 *and* 11.

Sol. We first write these numbers in columns as below.

```
  1 0 1
+   1 1
-------
1 0 0 0
```

Starting from right, in the first column we get 1 + 1 = 10. The 0 is written below this column and 1 is carried in the next column. In the second column, 1 + (0 + 1) = 1 + 1 = 10. From the value 0 is written below this column and 1 is carried in the third column. Now, in the third column 1 + 1 = 10 is obtained. Thus,

101 + 11 = 1000

Example 27.11 *Add the binary numbers* 101 *and* 110.

Sol. Rewriting the numbers in columns as below.

```
  1 0 1
+ 1 1 0
-------
1 0 1 1
```

The sum is in accordance with the four laws given in the beginning.

Thus, 101 + 110 = 1011

Example 27.12 *Add the binary numbers* 1101 *and* 1110.

Sol. Writing the numbers in columns we have,

```
  1 1 0 1
+ 1 1 1 0
---------
1 1 0 1 1
```

Here, in the last column we had 1 + (1 + 1) which we wrote 11. This is because,

$1 + (1 + 1) = 1 + 10 = 1\ 0$

$+\ 1$

$1\ 1$

Exercise Add the binary numbers 1011 and 1001.

Ans. 10100

For adding more than two binary numbers the above method is repeated again and again. For example if you want to add 101, 1011 and 1001. First add 101 and 1011 by the method discussed above, then add 1001 with the sum of 101 and 1011.

Exercise Add three binary numbers 111, 101 and 110.

Ans. 10010

Difference of binary numbers The four laws of difference (in binary system) are as below.

(i) $0-0=0$ (ii) $1-0=1$

(iii) $1-1=0$ (iv) $10-1=1$

Now, let us take an example in support of the above four laws.

Example 27.13 *Subtract the binary number* 100 *from the binary number* 101.

Sol. Let us write the given numbers in columns as below.

```
  1 0 1
- 1 0 0
-------
  0 0 1
```

Thus, 101 – 100 = 001 or simply 1.

Example 27.14 *Subtract the binary number* 1110 *from the binary number* 11011.

Sol. Rewriting the given numbers in columns as below.

```
  1 1 0 1 1
- 1 1 1 0
---------
    1 1 0 1
```

In this example, first and second column subtract in accordance with the four laws given in the starting. In the third column, for subtracting 1 from 0 we carry 1 from the fourth column. Thus, 10 – 1 = 1 is obtained. In the fourth column, for subtracting 1 from 0 (which remains 0 after giving 1 to the third column) we carry 1 from the fifth column. Thus 10 – 1 = 1 is obtained.

Thus, 11011 – 1110 = 1101

Chapter Summary with Formulae

(i) Intrinsic semiconductors are pure tetravalent compounds, e.g. germanium, silicon and carbon.

(ii) Due to thermal agitation some covalent bonds are broken. Bounded electrons become free electrons and equal number of holes are created. As the temperature is increased more and more bonds are broken and conductivity of semiconductor is increased. Opposite to it, conductivity decreases with decrease in temperature. At 0 K, all intrinsic semiconductors act as insulators.

(iii) In intrinsic semiconductors, number of free electrons and number of holes are same.

(iv) When pentavalent or trivalent impurity is mixed with intrinsic semiconductor its conductivity is increased. Because by adding pentavalent compounds (antimony, phosphorus and arsenic) number of free electrons are increased and by adding trivalent (boron, aluminium, gallium and indium) compounds, number of holes are increased.

(v) *p*-type semiconductors are formed by doping trivalent compounds in intrinsic semiconductors. In *p*-type semiconductors number of holes >> number of free electrons.

(vi) *n*-type semiconductors are formed by doping pentavalent compounds in intrinsic semiconductors. In *n*-type semiconductors number of free electrons >> number of holes.

(vii) *p*-type or *n*-type semiconductors are electrically neutral.

(viii) At 0 K, electrical conductivity of extrinsic semiconductors is not zero.

(ix) Current flow through a semiconductor is due to the motion of electron and hole both, i.e.

$$i = i_e + i_h$$

but mobility of electrons >> mobility of holes.

(x) Energy gap between Conduction Band (CB) and Valence Band (VB) is 6 eV for diamond, 1.1 eV for Si and 0.7 eV for Ge.

(xi) Trivalent atoms are called acceptor atoms and pentavalent are donor atoms.

(xii) Holes are denoted by n_h and electrons by n_e .

(xiii) The resistivity ρ of a semiconductor is given by,

$$\frac{1}{\rho} = e\,(n_e \mu_e + n_h \mu_h)$$

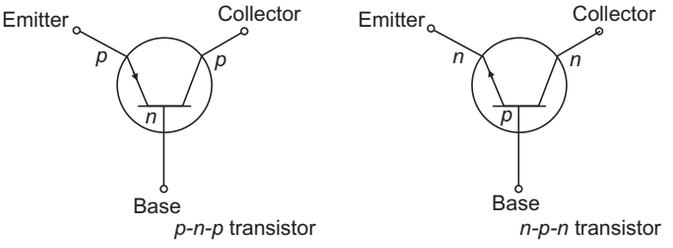
where, n_e and n_h are electron and hole concentrations in the semiconductor and μ_e and μ_h are the electron and hole mobilities respectively.

(xiv) Symbol of a *p*-*n* junction diode is 

(xv) A *p*-*n* junction diode is said to be forward biased, if *p*-side is on higher potential and *n*-side on lower potential. It is reverse biased, if opposite is the case, i.e. *p*-side is on lower potential and *n*-side on higher potential.

(xvi) Even after rectification ripples are present in the output which can be removed upto great extent by a filter circuit. A filter circuit consists of a capacitor.

(xvii) In the symbol of a transistor arrow is made between base and emitter. The direction of arrow is from *p* to *n*. This arrow shows the direction of current in emitter terminal.



(xviii) In normal operation, the emitter-base is forward biased and collector-base is reverse biased.

(xix) In the connecting wires, always electrons are responsible for current flow, but inside the transistor holes are mainly responsible for current flow in *p*-*n*-*p* transistor and electrons in case of *n*-*p*-*n* transistor.

(xx) In $i_e = i_c + i_b$, i_c is slightly less than i_e while $i_b << i_e$ or $i_b << i_c$.

(xxi) $\alpha = i_c / i_e$, $\beta = i_c / i_b$ and $\beta = \dfrac{\alpha}{1-\alpha}$

Here, α is about 0.95 to 0.99 and β is about 20 to 100.

(xxii) In common-base amplifier

(a) Current gain $= \alpha = \dfrac{\Delta i_c}{\Delta i_e}$ $(V_{CB} = \text{constant})$

Voltage gain $= \alpha \dfrac{R_{out}}{R_{in}}$ and Power gain $= \alpha^2 \dfrac{R_{out}}{R_{in}}$

(b) Input and output signals are in same phase.

(xxiii) In common-emitter amplifier

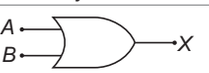
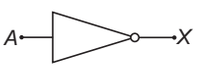
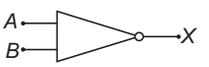
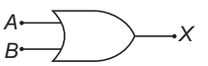
(a) Current gain $= \beta = \dfrac{\Delta i_c}{\Delta i_b}$ $(V_{CE} = \text{constant})$

Voltage gain $= \beta \dfrac{R_{out}}{R_{in}}$ and Power gain $= \beta^2 \dfrac{R_{out}}{R_{in}}$

(b) Input and output signals are 180° out of phase.

(c) Transconductance, $g_m = \dfrac{\Delta i_c}{\Delta V_{BE}}$ $(V_{CE} = \text{constant})$

(xxiv) Logic Gates

S.N.	Name of the Gate	Function	Symbol
1.	OR	$x = A + B$	
2.	AND	$X = A \cdot B$	
3.	NOT	$X = \overline{A}$	
4.	NAND	$X = \overline{A \cdot B}$	
5.	NOR	$X = \overline{A + B}$	
6.	XOR	$X = A \cdot \overline{B} + B \cdot \overline{A}$	

(xxv) Truth Tables

A	B	OR Gate $X = A + B$	AND Gate $x = A \cdot B$	NOT Gate $X = \overline{A}$	NAND Gate $X = \overline{A \cdot B}$	NOR Gate $X = \overline{A + B}$	$\overline{B}$	XOR Gate $X = A \cdot \overline{B} + B \cdot \overline{A}$
1	0	1	0	0	1	0	1	1
0	1	1	0	1	1	0	0	1
1	1	1	1	0	0	0	0	0
0	0	0	0	1	1	1	1	0

Additional Examples

Example 1. *Sn, C, Si and Ge are all group XIV elements. Yet, Sn is a conductor, C is an insulator while Si and Ge are semiconductors. Why ?*

Sol. It all depends on energy gap between valence band and conduction band. The energy gap for Sn is 0 eV, for carbon is 5.4 eV, for Si is 1.1 eV and for Ge is 0.7 eV.

Example 2. *Three photodiodes D_1, D_2 and D_3 are made of semiconductors having band gaps of* 2.5 *eV,* 2 *eV* and 3 *eV respectively. Which one will be able to detect light of wavelength* 6000 Å *?*

Sol. Energy of incident light

$$E \text{ (in eV)} = \frac{12375}{\lambda \text{ (in } \overset{\circ}{\text{A}})}$$

$$E = \frac{12375}{6000} \text{ eV}$$

or $$E = 2.06 \text{ eV}$$

For the incident radiation to be detected by the photodiode energy of incident radiation should be greater than the band gap. This is true only for D_2. Therefore, only D_2 will detect this radiation.

Example 3. *What is the range of energy gap* (E_g) *in insulators, semiconductors and conductors?*

Sol. For insulators, $E_g > 3$ eV, for semiconductors, $E_g = 0.2$ eV to 3eV while for conductors (or metals) $E_g = 0$.

Example 4. *n-type extrinsic semiconductor is negatively charged, while p-type extrinsic semiconductor is positively charged. Is this statement true or false?*

Sol. False. Intrinsic as well as extrinsic semiconductors are electrically neutral.

Example 5. *What is resistance of an intrinsic semiconductor at* 0 K*?*

Sol. At 0 K number of holes (or number of free electrons) in an intrinsic semiconductor becomes zero. Therefore, resistance of an intrinsic semiconductor becomes infinite at 0 K.

Example 6. *Consider an amplifier circuit using a transistor. The output power is several times greater than the input power. Where does the extra power come from?*

Sol. The extra power required for amplified output is obtained from the DC source.

Example 7. *A piece of copper and the other of germanium are cooled from the room temperature to* 80 *K. What will happen to their resistance?*

Sol. Copper is conductor and germanium is semiconductor. With decrease in temperature resistance of a conductor decreases and that of semiconductor increases. Therefore, resistance of copper will decrease and that of semiconductor will increase.

Example 8. *A transistor has three impurity regions, emitter, base and collector. Arrange them in order of increasing doping levels.*

Sol. The order of increasing doping levels is base > collector > emitter.

Example 9. *Name two gates which can be used repeatedly to produce all the basic or complicated gates.*

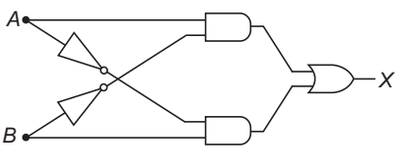
Sol. NAND and NOR gates can be used repeatedly to produce all the basic or complicated gates. This is why these gates are called digital building blocks.

Example 10. *In a transistor connected in common-emitter mode* $R_0 = 4\,k\Omega$, $R_i = 1\,k\Omega$, $i_c = 1\,mA$ *and* $i_b = 20\,\mu A$. *Find the voltage gain.*

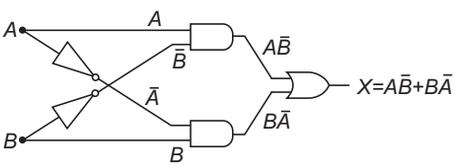
Sol. $\beta = \dfrac{i_c}{i_b} = \dfrac{1 \times 10^{-3}}{20 \times 10^{-6}} = 50$

$\therefore$ Voltage gain $A_V = \beta \left(\dfrac{R_0}{R_i}\right) = (50)\left(\dfrac{4}{1}\right) = 200$

Example 11. *Write the truth table for the circuit given in figure.*



Sol. The circuit can be redrawn as,



The corresponding truth table is shown below.

Table 27.17

A	B	$\bar{A}$	$\bar{B}$	$A\bar{B}$	$B\bar{A}$	$A\bar{B} + B\bar{A} = X$
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0

Example 12. *A n-p-n transistor is connected in common emitter configuration in which collector supply is* $8V$ *and the voltage drop across the load resistance of* $800\,\Omega$ *connected in the collector circuit is* $0.8V$*. If current amplification factor is,* 25*, determine collector emitter voltage and base current. If the internal resistance of the transistor is* $200\,\Omega$*, calculate the voltage gain and the power gain.*

Sol. The corresponding circuit is shown in figure.

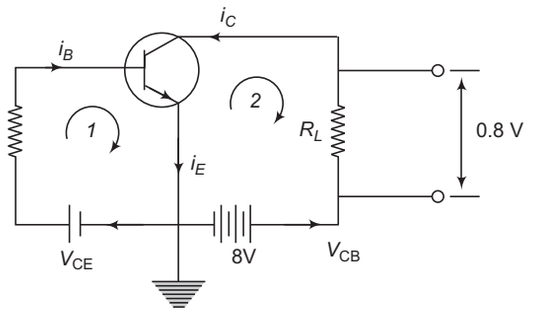
Voltage across $R_L = i_c R_L = 0.8$ V (given)

$$\therefore \quad i_c = \frac{0.8}{R_L} = \frac{0.8}{800}\text{ A} = 1\text{ mA}$$

Further it is given that,

$$\beta = 25 = \frac{i_c}{i_b} \quad \Rightarrow \quad \therefore \quad i_b = \frac{i_c}{25} = 40\,\mu\text{A}$$

Collector-Emitter Voltage (V_{CE})



Applying Kirchhoff's second law in loop 2, we have

$$V_{CE} = V_{CC} - i_c R_L = (8 - 0.8)\text{ V} = 7.2\text{ V}$$

Voltage gain (A_V)

Voltage gain, $A_V = \beta\left(\frac{R_{\text{out}}}{R_{\text{in}}}\right)$

or $A_V = 25\left(\frac{800}{200}\right) = 100$

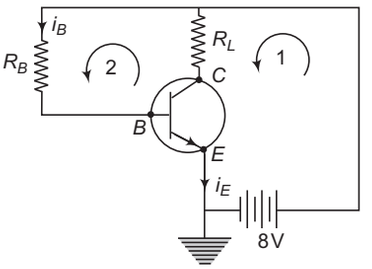
Power gain

$$\text{Power gain} = \beta^2\left(\frac{R_{\text{out}}}{R_{\text{in}}}\right) = (25)^2\left(\frac{800}{200}\right) = 2500$$

- Kirchhoff's laws can be applied in a transistor circuit in the similar manner as is done in normal circuits.

Example 13. *An n-p-n transistor in a common emitter mode is used as a simple voltage amplifier with a collector current of* 4 *mA. The positive terminal of a* $8V$ *battery is connected to the collector through a load resistance* R_L *and to the base through a resistance* R_B*. The collector-emitter voltage* $V_{CE} = 4V$*, the base-emitter voltage* $V_{BE} = 0.6\,V$ *and the current amplification factor* $\beta = 100$*. Calculate the values of* R_L *and* R_B*.*

Sol. Given, $i_c = 4$ mA



Applying Kirchhoff's second law in loop 1, we have

$$V_{CE} = 8 - i_c R_L$$

$$\therefore \quad R_L = \frac{8 - V_{CE}}{i_c} = \frac{8 - 4}{4 \times 10^{-3}} = 1000\ \Omega = 1\ \text{k}\Omega$$

Further, $\beta = \frac{i_c}{i_b}$

$$\therefore \quad i_b = \frac{i_c}{\beta} = \frac{4 \times 10^{-3}}{100}\text{ A} = 40\ \mu\text{A}$$

Now, $V_{BE} = 8 - i_b R_B$

$$\therefore \quad R_B = \frac{8 - V_{BE}}{i_b} = \frac{8 - 0.6}{40 \times 10^{-6}} = 1.85 \times 10^5\ \Omega$$

Example 14. *In an n-p-n transistor (in common - emitter mode)* 10^{10} *electrons enter the emitter in* 10^{-6} *s. Only* 2% *of the electrons are lost in the base. Calculate the base current and the current amplification factor. Charge on electron is* 1.6×10^{-19} *C.*

Sol. $i_e = \frac{q}{t} = \frac{10^{10} \times 1.6 \times 10^{-19}}{10^{-6}} = 1.6$ mA

Base current, $i_b = 2\%$ of $i_e = \left(\frac{2}{100}\right) \times 1.6$ mA

$= 0.032$ mA

$$\therefore \quad i_c = i_e - i_b = 1.568\text{ mA}$$

Therefore, current amplification factor

$$\beta = \frac{i_c}{i_b} = \frac{1.568}{0.032} = 49$$

Example 15. *A change of* 8.0 *in the emitter current brings a change of* 7.9 *mA in the collector current. How much change in the base current is required to have the same change* 7.9 *mA in the collector current. Find the values of* α *and* β*.*

Sol. Given, $\Delta i_e = 8.0$ mA

and $\Delta i_c = 7.9$ mA

$\therefore$ $\Delta i_b = \Delta i_e - \Delta i_c = 0.1$ mA

Further, $\alpha = \dfrac{\Delta i_c}{\Delta i_e} = \dfrac{7.9}{8.0} = 0.9875$

and $\beta = \dfrac{\Delta i_c}{\Delta i_b} = \dfrac{7.9}{0.1} = 79$

Example 16. *A transistor is used in common-emitter mode in an amplifier circuit. When a signal of* $20\,mV$ *is added to the base-emitter voltage, the base current changes by* $20\,\mu A$ *and the collector current changes by* $2\,mA$. *The load resistance is* $5\,k\Omega$. *Calculate*

(a) β,
(b) the input resistance R_i,
(c) the transconductance g_m *and*
(d) the voltage gain.

Sol. Given, $\Delta i_b = 20\,\mu A$ and $\Delta i_c = 2$ mA

(a) $\beta = \dfrac{\Delta i_c}{\Delta i_b} = \dfrac{2 \times 10^{-3}}{20 \times 10^{-6}} = 100$

(b) $\Delta V_i = (\Delta i_b) \times R_i$

$\therefore$ $R_i = \dfrac{\Delta V_i}{\Delta i_b} = \dfrac{20 \times 10^{-3}}{20 \times 10^{-6}}$

$= 1000\ \Omega = 1\ \text{k}\Omega$

(c) Transconductance $g_m = \dfrac{\Delta i_c}{\Delta V_i}$

$= \dfrac{2 \times 10^{-3}}{20 \times 10^{-3}} = 0.1\,\Omega^{-1}$

(d) Voltage gain, $A_V = \beta \left(\dfrac{R_{\text{out}}}{R_{\text{in}}} \right) = (100)\left(\dfrac{5}{1}\right) = 500$

Example 17. *Let* $X = A \cdot \overline{BC}$. *Evaluate X for*

(a) $A = 1, B = 0, C = 1$,
(b) $A = B = C = 1$ *and*
(c) $A = B = C = 0$.

Sol. (a) When, $A = 1$, $B = 0$ and $C = 1$

$BC = 0$

$\therefore$ $\overline{BC} = 1$

or $A\overline{BC} = 1$

(b) When, $A = B = C = 1$

Then, $BC = 1$

or $\overline{BC} = 0$

$\therefore$ $A\overline{BC} = 0$

(c) When, $A = B = C = 0$

Then, $BC = 0$

$\therefore$ $\overline{BC} = 1$

or $A\overline{BC} = 0$

NCERT Selected Questions

***Q* 1.** In an *n*-type silicon, which of the following statement is true?

(a) Electrons are majority carriers and trivalent atoms are the dopants.
(b) Electrons are minority carriers and pentavalent atoms are the dopants.
(c) Holes are minority carriers and pentavalent atoms are the dopants.
(d) Holes are majority carriers and trivalent atoms are the dopants.

Sol. (c) Holes are minority carriers and pentavalent atoms are the dopants.

***Q* 2.** Carbon, silicon and germanium have four valence electrons each. These are characterised by valence and conduction bands separated by energy band gap respectively equal to $(E_g)_C$, $(E_g)_{Si}$ and $(E_g)_{Ge}$. Which of the following statements is true?

(a) $(E_g)_{Si} < (E_g)_{Ge} < (E_g)_C$
(b) $(E_g)_C < (E_g)_{Ge} < (E_g)_{Si}$
(c) $(E_g)_C < (E_g)_{Si} < (E_g)_{Ge}$
(d) $(E_g)_{Ge} < (E_g)_{Si} < (E_g)_C$

Sol. (c) $(E_g)_C < (E_g)_{Si} < (E_g)_{Ge}$

***Q* 3.** In an unbiased *p-n* junction, holes diffuse from the *p*-region to *n*-region because

(a) free electrons in the *n*-region attract them
(b) they move across the junction by the potential difference
(c) hole concentration in *p*-region is more as compared to *n*-region
(d) All of the above

Sol. (c) Hole concentration in *p*-region is more as compared to *n*-region.

***Q* 4.** When a forward bias is applied to *p-n* junction, it

(a) raises the potential barrier
(b) reduces the majority carrier current to zero
(c) lowers the potential barrier
(d) None of the above

Sol. (c) Lowers the potential barrier.

Q 5. For transistor action, which of the following statements are correct?

(a) Base, emitter and collector regions should have similar size and doping concentrations

(b) The base region must be very thin and lightly doped

(c) The emitter junction is forward biased and collector junction is reverse biased

(d) Both the emitter junction as well as the collector junction are forward biased

Sol. (b) The base region must be very thin and lightly doped.

(c) The emitter junction is forward biased and collector junction is reverse biased.

Q 6. For a transistor amplifier, the voltage gain

(a) remains constant for all frequencies

(b) is high at high and low frequencies and constant in the middle frequency range

(c) is low at high and low frequencies and constant at mid frequencies

(d) None of the above

Sol. (c) Is low at high and low frequencies and constant at mid frequencies.

Q 7. In half wave rectification, what is the output frequency if the input frequency is 50 Hz. What is the output frequency of a full wave rectifier for the same input frequency.

Sol. A full-wave rectifier both the half cycles of the AC input, i.e. it conducts twice during a cycle.

So, frequency of AC output = 2 × frequency of AC input

$$= 2 \times 50 = 100 \text{ Hz.}$$

Q 8. For a common emitter transistor amplifier, the audio signal voltage across the collector resistance of 2 kΩ is 2 V. Suppose the current amplification factor of the transistor is 100, find the input signal voltage and base current, if the base resistance is 1 kΩ.

Sol. Here, β = current gain = 100

R_i = input (base) resistance = 1 kΩ

$= 1000\ \Omega$

R_o = output (collector) resistance

$= 2\ \text{k}\Omega = 2000\ \Omega$

V_o = output voltage = 2 V

$\therefore$ Resistance gain $= \dfrac{R_o}{R_i} = \dfrac{2000}{1000} = 2$

Using, the relation

Voltage gain = Current gain × Resistance gain

We get, voltage gain, $A_V = 100 \times 2 = 200$

Collector current, $I_c = \dfrac{V_o}{R_o}$

$$= \frac{2\text{V}}{2000\ \Omega} = \frac{1}{1000}\ \text{A}$$

$$= 10^{-3}\ \text{A} = 1\ \text{mA}$$

Using the relation,

$$\beta = \frac{I_c}{I_b}, \text{ we get}$$

$$I_b = \frac{I_c}{\beta} = \frac{10^{-3}}{100}$$

$$= 10^{-5}\ \text{A} = 10\ \mu\text{A}$$

Also using the relation,

$$A_V = \frac{V_o}{V_i}, \text{ we get}$$

$$V_i = \frac{V_o}{A_V}$$

$$= \frac{2}{200} = \frac{1}{100}\ \text{V}$$

$$= 0.01\ \text{V}$$

Q 9. Two amplifiers are connected one after the other in series (cascaded). The first amplifier has a voltage gain of 10 and the second has a voltage gain of 20. If the input signal is 0.01 V, calculate the output AC signal.

Sol. Let A_V be the voltage gain of the cascaded amplifier.

$\therefore$ $A_V = A_{V_1} \times A_{V_2}$

$= 10 \times 20 = 200$

$\therefore$ Using the relation,

$$A_V = \frac{V_o}{V_i}, \text{ we get}$$

$$V_o = A_V \times V_i$$

$$= 200 \times 0.01 = 2$$

Q 10. A *p-n* photodiode is fabricated from a semiconductor with band gap of 2.8 eV. Can it detect a wavelength of 6000 nm?

Sol. E_g = band gap of the semiconductor

$= 2.8$ eV

$= 2.8 \times 1.6 \times 10^{-19}\ \text{J} = 4.48 \times 10^{-19}\ \text{J}$

Wavelength of the radiation,

$\lambda = 6000$ nm

$= 6000 \times 10^{-9}$ m

If E be the energy corresponding to this radiation, then

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{6000 \times 10^{-9}}\ \text{J}$$

$$= 3.31 \times 10^{-20}\ \text{J}$$

Since, $E < E_g$, it cannot be detected.

Objective Problems

[Level 1]

1. The majority charge carriers in *p*-type semiconductor are
(a) electrons (b) protons
(c) holes (d) neutrons

2. A *p*-type semiconductor can be obtained by adding
(a) arsenic to pure silicon
(b) gallium to pure silicon
(c) antimony to pure germanium
(d) phosphorous to pure germanium

3. The valence of the impurity atom that is to be added to Ge crystal so as to make it a *n*-type semiconductor, is
(a) 6 (b) 5
(c) 4 (d) 3

4. When the electrical conductivity of a semiconductor is due to the breaking of its covalent bonds, then the semiconductor is said to be
(a) donar (b) acceptor
(c) intrinsic (d) extrinsic

5. A piece of copper and the other of germanium are cooled from the room temperature to 80 K, then which of the following would be a correct statement?
(a) Resistance of each increases
(b) Resistance of each decreases
(c) Resistance of copper increases while that of germanium decreases
(d) Resistance of copper decreases while that of germanium increases

6. In an insulator, the forbidden energy gap between the valence band and conduction band is of the order of
(a) 1 MeV (b) 0.1 MeV
(c) 1 eV (d) 5 eV

7. The energy band gap of Si is
(a) 0.70 eV
(b) 1.1 eV
(c) between 0.70 eV to 1.1 eV
(d) 5 eV

8. The impurity atoms which are mixed with pure silicon to make a *p*-type semiconductor are those of
(a) phosphorous (b) boron
(c) antimony (d) copper

9. In a semiconductor, the separation between conduction band and valence band is of the order of
(a) 100 eV
(b) 10 eV
(c) 1 eV
(d) zero

10. In a good conductor, the energy gap between the conduction band and the valence band is
(a) infinite
(b) wide
(c) narrow
(d) zero

11. *p*-type semiconductor is formed when
A. As impurity is mixed in Si
B. Al impurity is mixed in Si
C. B impurity is mixed in Ge
D. P impurity is mixed in Ge
(a) A and C
(b) A and D
(c) B and C
(d) B and D

12. At room temperature, a *p*-type semiconductor has
(a) large number of holes and few electrons
(b) large number of free electrons and few holes
(c) equal number of free electrons and holes
(d) no electrons or holes

13. In semiconductors at room temperature the
(a) valence band is partially empty and the conduction band is partially filled
(b) valence band is completely filled and the conduction band is partially filled
(c) valence band is completely filled
(d) conduction band is completely empty

14. A piece of semiconductor is connected in series in an electric circuit. On increasing the temperature, the current in the circuit will
(a) decrease
(b) remain unchanged
(c) increase
(d) stop flowing

15. Intrinsic semiconductor is electrically neutral. Extrinsic semiconductor having large number of current carriers would be
(a) positively charged
(b) negatively charged
(c) positively charged or negatively charged depending upon the type of impurity that has been added
(d) electrically neutral

16. If n_e and v_d be the number of electrons and drift velocity in a semiconductor. When the temperature is increased
(a) n_e increases and v_d decreases
(b) n_e decreases and v_d increases
(c) both n_e and v_d increases
(d) both n_e and v_d decreases

17. In a semiconducting material, the mobilities of electrons and holes are μ_e and μ_h respectively. Which of the following is true?
(a) $\mu_e > \mu_h$
(b) $\mu_e < \mu_h$
(c) $\mu_e = \mu_h$
(d) $\mu_e < 0; \mu_h > 0$

18. Serious draw back of the semiconductor device is that they
(a) cannot be used with high voltage
(b) pollute the environment
(c) are costly
(d) do not last for long time

19. The forbidden energy gap in the energy bands of germanium at room temperature is about
(a) 1.1 eV (b) 0.1 eV
(c) 0.67 eV (d) 6.7 eV

20. Which of the following statements is not true?
(a) The resistance of intrinsic semiconductor decreases with increase of temperature
(b) Doping pure Si with trivalent impurities give *p*-type semiconductors
(c) The majority carriers in *n*-type semiconductors are holes
(d) A *p*-*n* junction can act as a semiconductor diode

21. *n*-type semiconductor is formed
(a) when a Ge crystal is doped with an impurity containing 3 valence electrons
(b) from pure Ge
(c) from pure Si
(d) when a Ge crystal are doped with an impurity containing 5 valence of electrons

22. Electrical conductivity of intrinsic and *p*-type semiconductor increases with increase in
(a) pressure
(b) volume
(c) density
(d) temperature

23. Number of electrons in the valence shell of a semiconductor is
(a) 1 (b) 2
(c) 3 (d) 4

24. What enables Ge to behave as semiconductor even though all electrons in the valence band form covalent bonds? It is due to the small width of
(a) valence band
(b) conduction band
(c) forbidden energy gap
(d) None of the above

25. Temperature coefficient of resistance of semiconductor is
(a) zero
(b) constant
(c) positive
(d) negative

26. An *n*-type and a *p*-type silicon can be obtained by doping pure silicon with
(a) sodium and magnesium respectively
(b) phosphorus and boron respectively
(c) boron and phosphorus respectively
(d) indium and sodium respectively

27. In an insulator, the forbidden energy gap between the valence band and conduction band is of the order of
(a) 1 MeV
(b) 0.1 MeV
(c) 1 eV
(d) 5 eV

28. An *n*-type semiconductor is
(a) negatively charged
(b) positively charged
(c) neutral
(d) negatively or positively charged depending upon the amount of impurity

29. The valence band and conduction band of a solid, overlap at low temperature, the solid may be
(a) a metal
(b) a semiconductor
(c) an insulator
(d) None of the above

30. A Si specimen is made into *p*-type semiconductor by doping on an average one indium atom 6×10^7 silicon atoms. If the number density of atoms in Si be $6 \times 10^{28}/\text{m}^3$ what is the indium atom/cm^3?
(a) 10^{12} (b) 10^{15}
(c) 10^{18} (d) 10^{20}

31. E_g for silicon is 1.12 eV and that for germanium is 0.72 eV. Therefore, it can be concluded that
(a) more number of electron hole pairs will be generated in silicon than in germanium at room temperature
(b) less number of electron hole pairs will be generated in silicon than in germanium at room temperature
(c) equal number of electron hole pairs will be generated in both at lower temperatures
(d) equal number of electron hole pairs will be generated in both at higher temperatures

32. Under which of the following conditions does an avalanche breakdown in a semiconductor diode occur?
(a) When potential barrier is reduced to zero
(b) When reverse bias exceeds a certain value
(c) When forward bias exceeds a certain value
(d) When forward current exceeds a certain value

33. Let n_p and n_e be the numbers of holes and conduction electrons in an intrinsic semiconductor
(a) $n_p > n_e$
(b) $n_p = n_e$
(c) $n_p < n_e$
(d) $n_p \neq n_e$

34. In the forward bias arrangement of a *p-n*-junction diode, the
(a) *n*-end is connected to the positive terminal of the battery
(b) *p*-end is connected to the positive terminal of the battery
(c) direction of current is from *n*-end to *p*-end in the diode
(d) *p*-end is connected to the negative terminal of battery

35. The potential barrier for silicon diode is approximately
(a) 0.2 V (b) 0.6 V
(c) 1.1 V (d) 1.4 V

36. The reverse biasing in a *p-n* junction diode
(a) decreases the potential barrier
(b) increases the potential barrier
(c) increases the number of minority charge carriers
(d) increases the number of majority charge carriers

37. In the circuit given below, the value of the current is



(a) zero (b) 10^{-2} A
(c) 10^{2} A (d) 10^{-3} A

38. A *p-n* junction has a thickness of the order of
(a) 1 cm (b) 1 mm
(c) 10^{-6} m (d) 10^{-12} cm

39. In forward bias, the width of potential barrier in a *p-n* junction diode
(a) increases
(b) decreases
(c) remains constant
(d) first increases then decreases

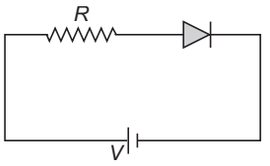
40. The dominant mechanisms for motion of charge carriers in forward and reverse biased silicon *p-n* junctions are
(a) drift in forward bias, diffusion in reverse bias
(b) diffusion in forward bias, drift in reverse bias
(c) diffusion in both forward and reverse bias
(d) drift in both forward and reverse bias

41. In *p-n* junction, avalanche current flows in circuit when biasing is
(a) forward (b) reverse
(c) zero (d) excess

42. When forward bias is applied to a *p-n* junction, then what happens to the potential barrier V_B, and the width of charge depleted region x?
(a) V_B increases, x decreases
(b) V_B decreases, x increases
(c) V_B increases, x increases
(d) V_B decreases, x decreases

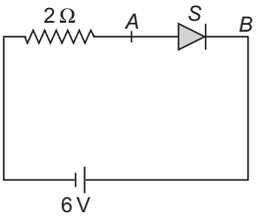
43. A potential barrier of 0.50 V exists across a *p-n* junction.If the depletion region is 5.0×10^{-7} m wide, the intensity of the electric field in this region is
(a) 1.0×10^{6} V/m (b) 1.0×10^{5} V/m
(c) 2.0×10^{5} V/m (d) 2.0×10^{6} V/m

44. For the given circuit of *p-n*-junction diode, which of the following statement is correct?



(a) In forward biasing, the voltage across R is V
(b) In forward biasing, the voltage across R is $2V$
(c) In reverse biasing, the voltage across R is V
(d) In reverse biasing, the voltage across R is $2V$

45. The diode shown in the circuit is a silicon diode. The potential difference between the points A and B will be

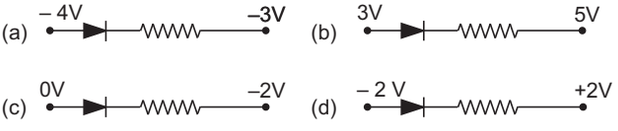


(a) 6 V (b) 0.6 V (c) 0.7 V (d) zero

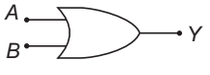
46. A diode having potential difference 0.5 V across its junction which does not depend on current, is connected in series with resistance of 20 Ω across source. If 0.1 A passes through resistance then what is the voltage of the source?
(a) 1.5 V (b) 2.0 V
(c) 2.5 V (d) 5 V

47. Potential barrier developed in a junction diode opposes
(a) minority carrier in both regions only
(b) majority carrier only
(c) electrons in *n*-region
(d) holes in *p*-region

48. The forward bias diode is



49. Below we give four entries for the truth table of two point input OR gate. Which two are wrong?

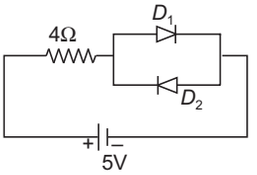


	A	B	Y
i	1	0	1
ii	1	1	0
iii	0	0	1
iv	0	1	1

(a) i, ii (b) ii, iii (c) iii, iv (d) iv, i

50. If no external voltage is applied across *p-n* junction, there would be
(a) no electric field across the junction
(b) an electric field pointing from *n*-type to *p*-type side across the junction
(c) an electric field pointing from *p*-type to *n*-type side across the junction
(d) a temporary electric field during formation of *p-n* junction that would subsequently disappear

51. In the network shown



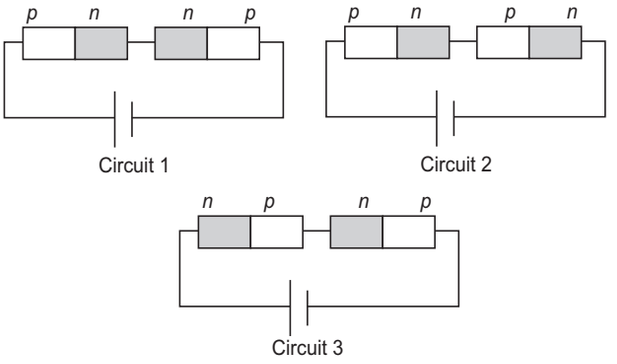
(a) the potential difference across D_2 is 5 V
(b) current through resistor equals 1.25 A
(c) current through diode D_1 is 1.25 A
(d) current through diode D_2 is 1.25 A

52. The width of depletion region in a *p-n* junction diode is 500 nm and an intense electric field of 6×10^5 Vm^{-1} is also found to exist. The height of potential barrier is
(a) 0.30 V (b) 0.40 V (c) 3 V (d) 4 V

53. When a diode is heavily doped
(a) the zener voltage will be low
(b) the avalanche voltage will be high
(c) the depletion region will be thin
(d) the leakage current will be low

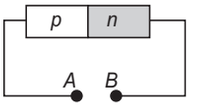
54. If the *p-n* junction diode is forward biased, then width of potential barrier in a *p-n* junction diode
(a) increase
(b) remains constant
(c) decrease
(d) first increase then remains constant

55. Two identical *p-n* junctions may be connected in series with a battery in three ways (figure). The potential differences across the two *p-n* junctions are equal in



(a) circuit 1 and circuit 2 (b) circuit 2 and circuit 3
(c) circuit 3 and circuit 1 (d) circuit 1 only

56. *p-n* junction conducts when *A* and *B* are connected to the



(a) positive and negative terminals of the battery respectively
(b) negative and positive terminals
(c) negative and positive terminals but the potential difference is large
(d) None of the above

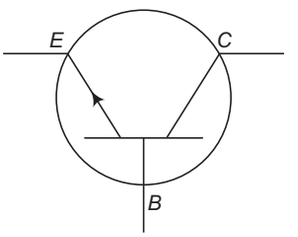
57. When *n-p-n* transistor is used as an amplifier,then
(a) electrons move from base to collector
(b) holes move from emitter to base
(c) electrons move from collector to base
(d) holes move from base to emitter

58. In the case of constants α and β of a transistor
(a) $\alpha = \beta$
(b) $\beta < 1, \alpha > 1$
(c) $\alpha\beta = 1$
(d) $\beta > 1, \alpha < 1$

59. Least doped region in a transistor is
(a) either emitter or collector
(b) base
(c) emitter
(d) collector

60. The transistors provide good power amplification when they are used in
(a) common collector configuration
(b) common emitter configuration
(c) common base configuration
(d) None of the above

61. The symbol given in figure represents



(a) *n-p-n* transistor
(b) *p-n-p* transistor
(c) forward biased *p-n* junction diode
(d) reverse biased *n-p* junction diode

62. The part of a transistor which is heavily doped to produce a large number of majority carriers, is
(a) base
(b) emitter
(c) collector
(d) None of the above

63. In a transistor
(a) both the emitter and the collector are equally doped
(b) base is more heavily doped than the collector
(c) collector is more heavily doped than the emitter
(d) the base is made very thin and is lightly doped

64. In a common base transistor circuit, the current gain is 0.98. On changing the emitter current by 5.00 mA, the change in collector current is
(a) 0.196 mA (b) 2.45 mA (c) 4.9 mA (d) 5.1 mA

65. To use a transistor as an amplifier
(a) both junctions are forward biased
(b) both junctions are reverse biased
(c) the emitter base junction is forward biased and the collector base junction is reverse biased
(d) no biasing voltages are required

66. In an *n-p-n* transistor, the collector current is 10 mA. If 90% of the electrons emitted reach the collector, then the emitter current will be about
(a) 9 mA (b) 11 mA (c) 1 mA (d) 0.1 mA

67. *n-p-n* transistor is more useful than a *p-n-p* transistor, because
(a) *n-p-n* transistor offers less resistance to the flow of current
(b) charge carriers in *n-p-n* move more easily than the charge carriers in *p-n-p*
(c) *p-n-p* transistor is very costly compared to an *n-p-n* transistor
(d) None of the above

68. The base of transistor is made very thin compared to emitter and collector. By doing this which one of the following is achieved?
(a) High emitter current (b) Low power gain
(c) Low emitter current (d) High power gain

69. The working of a triode valve and transistor are similar. A transistor is a current operated device while a triode valve is a
(a) current operated device
(b) voltage operated device
(c) charge operated device
(d) magnetically operated device

70. In a *p-n-p* transistor with normal bias
(a) only holes cross the collector junction
(b) only majority carriers cross the collector junction
(c) the collector junction has a low resistance
(d) the emitter base junction is forward biased and the collector base junction is reverse biased

71. A transistor has three impurity regions. All the three regions have different doping levels. In order of increasing doping level, the regions are
(a) emitter, base and collector
(b) collector, base and emitter
(c) base, emitter and collector
(d) base, collector and emitter

72. What is β value for transistor whose $\alpha = 0.98$?
(a) 29 (b) 38
(c) 49 (d) 56

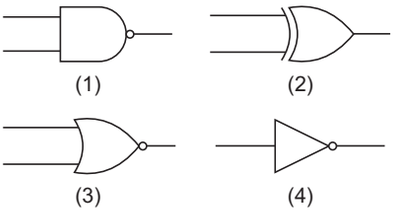
73. When the emitter current of a transistor is changed by 1 mA, its collector current changes by 0.99 mA. In the common base circuit current gain for the transistor is
(a) 0.099 (b) 1.01
(c) 1.001 (d) 0.990

74. In a transistor α is related to β by the relation
(a) $\beta = \dfrac{\alpha + 1}{\alpha}$ (b) $\beta = \dfrac{\alpha - 1}{\alpha}$
(c) $\beta = \dfrac{\alpha}{1 - \alpha}$ (d) $\beta = \dfrac{\alpha}{\alpha + 1}$

75. In case of *n-p-n* transistor the collector current is always less than the emitter current because
(a) collector side is reverse biased and emitter side forward biased
(b) a few electrons are lost in the base and only the remaining ones reach the collector
(c) collector side is forward biased and emitter side is reverse biased
(d) collector being reverse biased attracts less electrons

76. In a transistor the current amplification factor α is 0.9. The transistor is connected in common base configuration. The change in collector current when base current changes by 4 mA is
(a) 4 mA (b) 12 mA
(c) 24 mA (d) 36 mA

77. Given below are symbols for some logic gates.



The XOR gate and NOR gates respectively are
(a) 1 and 2 (b) 2 and 3
(c) 3 and 4 (d) 1 and 4

78. The given truth table is of

A	*X*
0	1
1	0

(a) OR gate (b) AND gate
(c) NOT gate (d) None of these

79. What will be the input of A and B for the Boolean expression $\overline{(A + B)} \cdot \overline{(A \cdot B)} = 1$?
(a) 0, 0 (b) 0, 1 (c) 1, 0 (d) 1, 1

80. The Boolean expression of NOR gate is
(a) $C = A + B$ (b) $C = \overline{A + B}$
(c) $C = A \cdot B$ (d) $C = \overline{A \cdot B}$

81. The given truth table corresponds to

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

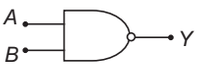
(a) NAND gate (b) XOR gate
(c) OR gate (d) NOR gate

82. The following truth table corresponds to the logic gate :

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

(a) NAND (b) OR
(c) AND (d) XOR

83. The given symbol represents



(a) NAND gate (b) OR gate
(c) AND gate (d) NOR gate

84. When the inputs of two input logic gate are 0 and 0, the output is 1. When the inputs are 1 and 0, the output is zero. The logic gate is of the type
(a) XOR (b) NAND
(c) NOR (d) OR

85. The electrical circuit used to get smooth DC output from a rectifier circuit is called
(a) oscillator (b) filter
(c) amplifier (d) logic gates

86. If a full wave rectifier circuit is operating in 50 Hz mains, the fundamental frequency in the ripple will be
(a) 50 Hz (b) 70.7 Hz
(c) 100 Hz (d) 25 Hz

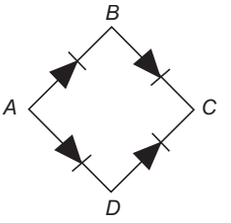
87. A *p*-type semiconductor has acceptor level 57 MeV above the valence band. The maximum wavelength of light required to create hole is
(a) 57 Å (b) 57×10^{-3} Å
(c) 217100 Å (d) 11.61×10^{-33} Å

88. In the Boolean algebra which of the following is not equal to A?
(a) $A \cdot A$ (b) $A + A$
(c) $\overline{A} \cdot A$ (d) $\overline{\overline{A} + \overline{A}}$

89. When the *p*-end of the *p*-*n* junction is connected to the negative terminal of the battery and the *n*-end to the positive terminal of the battery, then the *p*-*n* junction behaves like
(a) a conductor
(b) an insulator
(c) a super conductor
(d) a semiconductor

90. The Boolean expression of NOR gate is
(a) $C = \overline{A + B}$
(b) $C = \overline{A + B}$
(c) $C = \overline{A \cdot B}$
(d) $C = \overline{A \cdot B}$

91. In the diagram, the input is across the terminals A and C and the output is across B and D. Then, the output is



(a) zero
(b) same as the input
(c) full wave rectifier
(d) half wave rectifier

92. A semiconducting device is connected in a series circuit with a battery and a resistance. A current is found to pass through the circuit. If the polarity of the battery is reversed, the current drops to almost zero. The device may be
(a) extrinsic semiconductor
(b) intrinsic semiconductor
(c) *p*-*n* junction
(d) *p*-*n*-*p* transistor

93. With an AC input from 50 Hz power line, the ripple frequency is
(a) 50 Hz in the DC output of half wave as well as full wave rectifier
(b) 100 Hz in the DC output of half wave as well as full wave rectifier
(c) 50 Hz in the DC output of half wave and 100 Hz in DC output of full wave rectifier
(d) 100 Hz in the DC output of half wave and 50 Hz in the DC output of full wave rectifier

94. An electric field is applied to a semiconductor. Let the number of charge carriers be n and the average drift speed be v. If the temperature is increased
(a) both n and v will increase
(b) n will increase but v will decrease
(c) v will increase but n will decrease
(d) both n and v will decrease

95. The drift current in a *p-n* junction is
(a) from the *n*-side to the *p*-side
(b) from the *p*-side to the *n*-side
(c) from the *n*-side to the *p*-side, if the junction is forward biased and in the opposite direction if it is reverse biased
(d) from the *p*-side to the *n*-side, if the junction is forward biased and in the opposite direction if it is reverse biased

96. The diffusion current in a *p-n* junction is
(a) from the *n*-side to the *p*-side
(b) from the *p*-side to the *n*-side
(c) from the *n*-side to the *p*-side, if the junction is forward biased and in the opposite direction if it is reverse biased
(d) from the *p*-side to the *n*-side, if the junction is forward-biased and in the opposite direction if it is reverse-biased

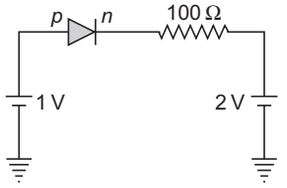
97. Diffusion current in a *p-n* junction is greater than the drift current in magnitude
(a) if the junction is forward biased
(b) if the junction is reverse biased
(c) if the junction is unbiased
(d) None of the above

[Level 2]

Only One Correct Option

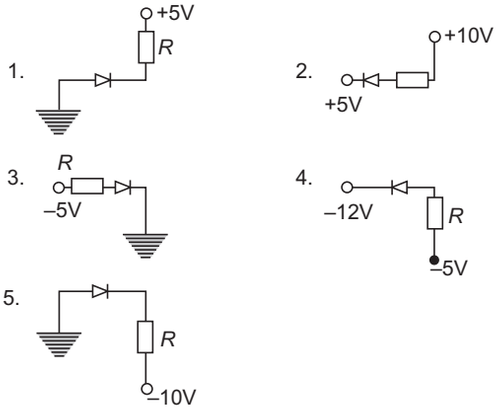
1. Mobilities of electrons and holes in a sample of intrinsic germanium at room temperature are $0.36\ m^2/Vs$ and $0.17\ m^2/Vs$. The electron and hole densities are each equal to $2.5\times 10^{19}\ m^3$. The electrical conductivity of germanium is
(a) 0.47 S/m (b) 5.18 S/m
(c) 2.12 S/m (d) 1.09 S/m

2. The current through an ideal *p-n* junction shown in the circuit diagram will be



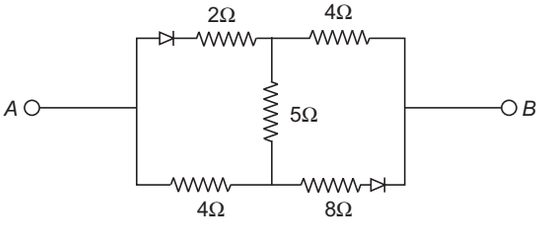
(a) zero (b) 1 mA
(c) 10 mA (d) 30 mA

3. In the given figure, which of the diodes are forward biased?



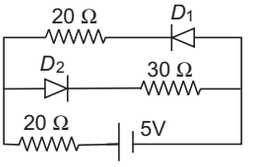
(a) 1, 2, 3
(b) 2, 4, 5
(c) 1, 3, 4
(d) 2, 3, 4

4. The equivalent resistance of the circuit across *AB* is given by



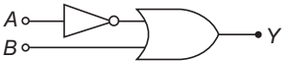
(a) 4 Ω (b) 13 Ω
(c) 4 Ω or 13 Ω (d) 4 Ω or zero

5. The current in the circuit will be



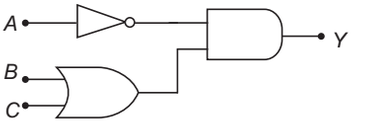
(a) 5/40 A (b) 5/50 A
(c) 5/10 A (d) 5/20 A

6. The Boolean expression for the circuit given in figure is



(a) $Y = A + \overline{B}$ (b) $Y = \overline{A + B}$
(c) $Y = \overline{A} + B$ (d) $Y = A + B$

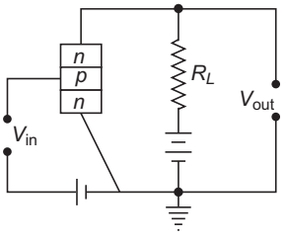
7. The Boolean expression for the circuit given in figure is



(a) $Y = \overline{A}\cdot B + C$ (b) $Y = \overline{A}\cdot(\overline{B} + \overline{C})$
(c) $Y = \overline{A}\cdot(B + \overline{C})$ (d) $Y = \overline{A}\cdot(B + C)$

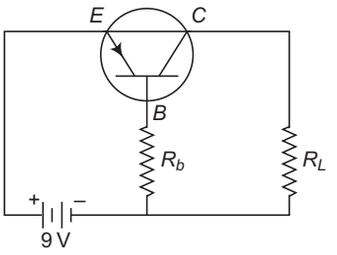
8. If the two ends of a *p-n* junction are joined by a wire
(a) there will not be a steady current in the circuit
(b) there will be a steady current from the *n*-side to the *p*-side
(c) there will a steady current from the *p*-side to the *n*-side
(d) there may or may not be a current depending upon the resistance of the connecting wire

9. An *n-p-n*-transistor circuit is arranged as shown in figure. It is



(a) a common base amplifier circuit
(b) a common emitter amplifier circuit
(c) a common collector amplifier circuit
(d) None of the above

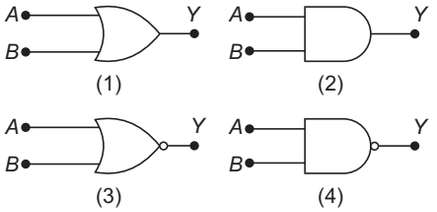
10. In a transistor circuit shown here the base current is $35\,\mu A$. The value of the resistor R_b is



(a) 123.5 kΩ
(b) 257 kΩ
(c) 380.05 kΩ
(d) None of the above

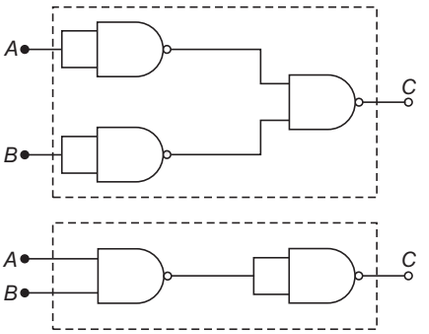
11. The transfer ratio β of a transistor is 50. The input resistance of the transistor when used in the common emitter configuration is 1 k Ω. The peak value of the collector AC current for a peak value of AC input voltage of 0.01 V is
(a) 100 μA
(b) 0.01μA
(c) 0.25 μA
(d) 500 μA

12. Given below are four logic gate symbols (figure). Those for OR, NOR and NAND are respectively



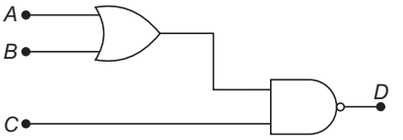
(a) 1, 4, 3
(b) 4, 1, 2
(c) 1, 3, 4
(d) 4, 2, 1

13. The combination of NAND gates shown here are equivalent to an



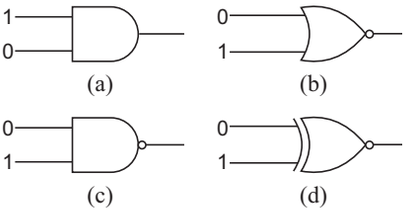
(a) OR gate and an AND gate respectively
(b) AND gate and a NOT gate respectively
(c) AND gate and an OR gate respectively
(d) OR gate and a NOT gate respectively

14. For the given combination of gates, if the logic states of inputs A, B and C are as follows. $A = B = C = 0$ and $A = B = 1, C = 0$, then the logic states of output D are

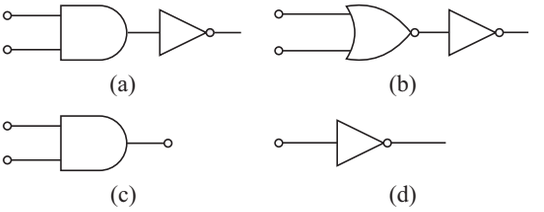


(a) 0, 0 (b) 0, 1 (c) 1,0 (d) 1, 1

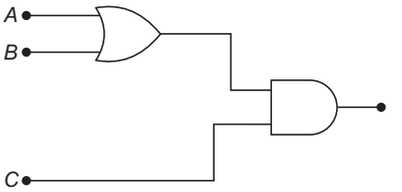
15. Which of the following gates will have an output of 1?



16. Which represents NAND gate?

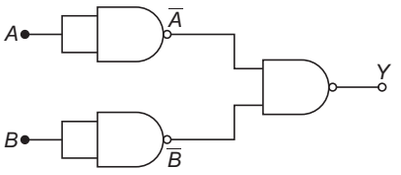


17. To get an output 1 from the circuit shown in the figure, the input must be



(a) $A = 0, B = 1, C = 0$ (b) $A = 1, B = 0, C = 0$
(c) $A = 1, B = 0, C = 1$ (d) $A = 1, B = 1, C = 0$

18. The combination of the gates shown in the figure below produces

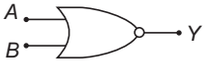


(a) NOR gate
(b) OR gate
(c) AND gate
(d) XOR gate

19. Given below are four logic gate symbols. Those for OR, NOR and NAND are respectively

(1) 

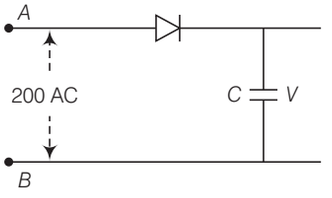
(3) 

(2) 

(4) 

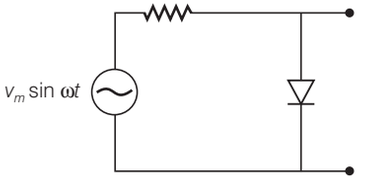
(a) 2, 4, 3
(b) 4, 1, 2
(c) 1, 3, 4
(d) 1, 2, 4

20. A 220 V AC supply is connected between points A and B (figure). What will be the potential difference V across the capacitor?



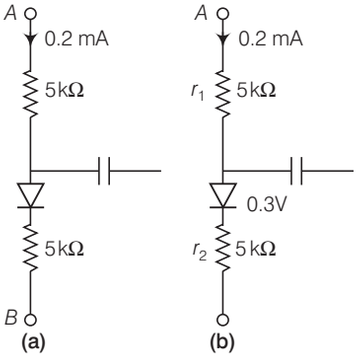
(a) 220V
(b) 110 V
(c) 0 V
(d) $220\sqrt{2}$V

21. The output of the given circuit in figure given below,



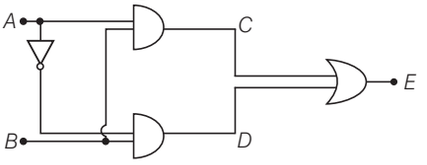
(a) would be zero at all times
(b) would be like a half wave rectifier with positive cycles in output
(c) would be like a half wave rectifier with negative cycles in output
(d) would be like that of a full wave rectifier

22. In the circuit shown in figure given below, if the diode forward voltage drop is 0.3 V, the voltage difference between A and B is



(a) 1.3 V
(b) 2.3 V
(c) 0
(d) 0.5 V

23. Truth table for the given circuit is



(a)

A	B	E
0	0	1
0	1	0
1	0	1
1	1	0

(b)

A	B	E
0	0	1
0	1	0
1	0	0
1	1	0

(c)

A	B	E
0	0	0
0	1	1
1	0	0
1	1	1

(d)

A	B	E
0	0	0
0	1	1
1	0	1
1	1	0

More than One Correct Options

1. When an electric field is applied across a semiconductor
(a) electrons move from lower energy level to higher energy level in the conduction band
(b) electrons move from higher energy level to lower energy level in the conduction band
(c) holes in the valence band move from higher energy level to lower energy level
(d) holes in the valence band move from lower energy level to higher energy level

2. Consider an *n-p-n* transitor with its base-emitter junction forward biased and collector base junction reverse biased. Which of the following statements are true?
(a) Electrons crossover from emitter to collector
(b) Holes move from base to collector
(c) Electrons move from emitter to base
(d) Electrons from emitter move out of base without going to the collector

3. In a *n-p-n* transistor circuit, the collector current is 10 mA. If 95 per cent of the electrons emitted reach the collector, which of the following statements are true?
(a) The emitter current will be 8 mA
(b) The emitter current will be 10.53 mA
(c) The base current will be 0.53 mA
(d) The base current will be 2 mA

4. In the depletion region of a diode
(a) there are no mobile charges
(b) equal number of holes and elections exist, making the region neutral
(c) recombination of holes and electrons has taken place
(d) immobile charged ions exist

5. What happens during regulation action of a Zener diode?
(a) The current and voltage across the Zener remains fixed
(b) The current through the series Resistance (R_s) changes
(c) The Zener resistance is constant
(d) The resistance offered by the Zener changes

6. The breakdown in a reverse biased *p-n* junction is more likely to occur due to
(a) large velocity of the minority charge carriers if the doping concentration is small
(b) large velocity of the minority charge carriers if the doping concentration is large
(c) strong electric field in a depletion region if the doping concentration is small
(d) strong electric field in the depletion region if the doping concentration is large

Entrance Gallery

2014

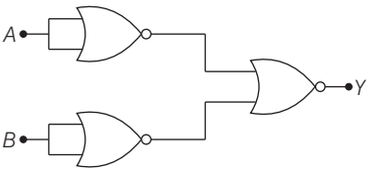
1. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage V is in volt and the temperature T is in kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA? **[JEE Main]**
(a) 0.2 mA (b) 0.02 mA
(c) 0.5 mA (d) 0.05 mA

2. The forward biased diode connection is **[JEE Main]**
(a) +2V —▷|— –2V
(b) –3V —▷|— –3V
(c) 2V —▷|— 4V
(d) –2V —▷|— +2V

3. In *n*-type semiconductor, electrons are majority charge carriers but it does not show any negative charge. The reason is **[Karnataka CET]**
(a) electrons are stationary
(b) electrons neutralise with holes
(c) mobility of electrons is extremely small
(d) atom is electrically neutral

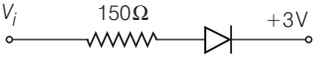
4. If α-current gain of a transistor is 0.98. What is the value of β-current gain of the transistor? **[Karnataka CET]**
(a) 0.49 (b) 49
(c) 4.9 (d) 5

5. For the given digital circuit, write the truth table and identify the logic gate it represents **[Karnataka CET]**



(a) OR gate (b) NOR gate
(c) NAND gate (d) AND gate

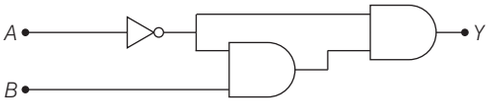
6. In the circuit shown assume the diode to be ideal. When V_i increases from 2V to 6 V, the change in the current is (in mA) **[WB JEE]**



(a) zero (b) 20
(c) 80/3 (d) 40

7. In a transistor output characteristics commonly used in common emitter configuration, the base current I_B, the collector current I_C and the collector-emitter voltage V_{CE} have values of the following orders of magnitude in the active region **[WB JEE]**
(a) I_B and I_C both are in μA and V_{CE} in volt
(b) I_B is in μA and I_C in mA and V_{CE} in volt
(c) I_B is in mA and I_C is in μA and V_{CE} in mV
(d) I_B is in mA and I_C is in mA and V_{CE} in mV

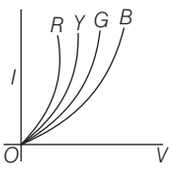
8. The output Y of the logic circuit given below is **[WB JEE]**

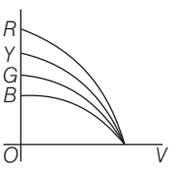


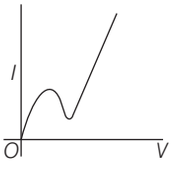
(a) $\overline{A} + B$ (b) $\overline{A}$
(c) $(\overline{\overline{A} + B}) \cdot \overline{A}$ (d) $(\overline{\overline{A} + B}) \cdot A$

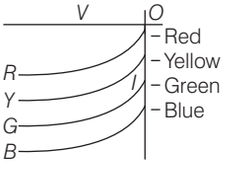
2013

9. The *I-V* characteristic of an LED is

(a) 

(b) 

(c) 

(d) 

10. A diode having potential difference 0.5 V across its junction which does not depend on current, is connected in series with resistance of 20Ω across source. If 0.1 A current passes through resistance then what is the voltage of the source? **[OJEE]**

(a) 1.5 V (b) 2.0 V (c) 2.5 V (d) 5 V

11. In a common-emitter amplifier, the load resistance of the output circuit is 1000 times the resistance of the input circuit. If $\alpha = 0.98$, then voltage gain is **[OJEE]**

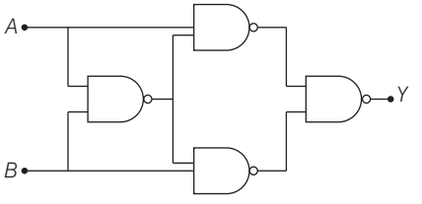
(a) 49×10^3 (b) 2.5×10^2 (c) 1.5×10^2 (d) 4.9

12. In a common emitter-amplifier, the input signal is applied across **[OJEE]**

(a) anywhere (b) emitter-collector
(c) collector-base (d) base-emitter

2012

13. Truth table for system of four NAND gate as shown in figure is **[AIEEE]**



(a)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(b)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(c)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(d)

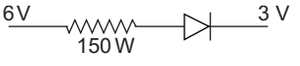
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

14. The given below truth table shows **[OJEE]**

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

(a) XOR gate (b) NAND gate
(c) OR gate (d) AND gate

15. As shown in the figure, the current in the part of circuit is



(a) 0.03 A (b) 0.02 A **[OJEE]**
(c) 0.04 A (d) 0.05 A

16. The decimal number equivalent to a binary number 1011001 is **[WB JEE]**

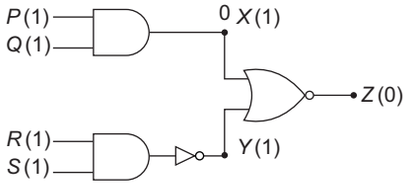
(a) 13 (b) 17 (c) 89 (d) 178

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17. The output of an OR gate is connected to both the inputs of a NAND gate. The combination will serve as a **[AIEEE]**

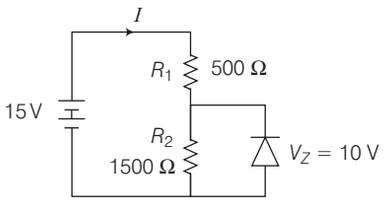
(a) OR gate (b) NOT gate
(c) NOR gate (d) AND gate

18. The circuit diagram shows a logic combination with the states of output *X*, *Y* and *Z* given for inputs *P, Q, R* and *S* all at state 1. When inputs *P* and *R* change to state 0 with inputs *Q* and *S* still at 1, the states of outputs *X, Y* and *Z* change to **[Kerala CEE]**



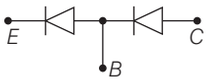
(a) 1, 0, 0 (b) 1, 1, 1 (c) 0, 1, 0
(d) 0, 0, 1 (e) 0, 1, 1

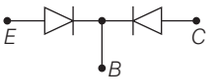
19. In the circuit given, the current through the zener diode is **[Kerala CEE]**

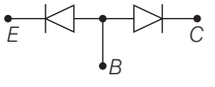


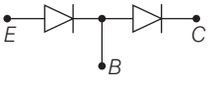
(a) 10 mA (b) 6.67 mA (c) 5 mA
(d) 3.33 mA (e) zero

20. An *n-p-n* transistor can be considered to be equivalent to two diodes, connected. Which of the following figures is the correct one? **[Karnataka CET]**

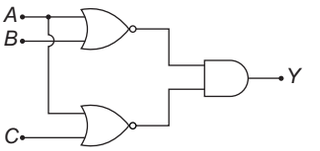
(a) 

(b) 

(c) 

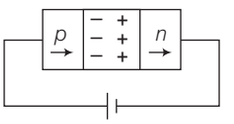
(d) 

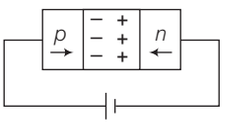
21. The output of given logic circuit is **[Karnataka CET]**

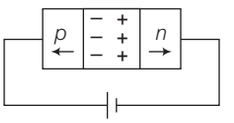


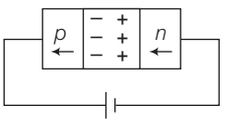
(a) $A + B + C$ (b) $(A + B) \cdot (A + C)$
(c) $A \cdot (B \cdot C)$ (d) $A \cdot (B + C)$

22. In the case of forward biasing of a *p-n* junction diode, which one of the following figures correctly depicts the direction of conventional current (indicated by an arrow mark)? **[Karnataka CET]**

(a) 

(b) 

(c) 

(d) 

23. Which type of gate the following truth table represents?

Inputs		Output
A	*B*	*Q*
0	0	1
0	1	1
1	0	1
1	1	0

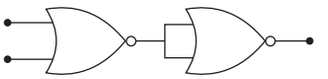
(a) NOT (b) AND [WB JEE]
(c) OR (d) NAND

24. In a *p*-*n* junction diode, are not connected to any circuit [Karnataka CET]
(a) the potential is the same everywhere
(b) the *p*-type side has a higher potential than the *n*-type side
(c) there is an electric field at the junction directed from the *n*-type side to *p*-type side
(d) there is an electric field at the junction directed from the *p*-type side to *n*-type side

25. Avalanche breakdown in a *p*-*n* junction diode is due to [Karnataka CET]
(a) sudden shift of Fermi level
(b) increase in the width of forbidden gap
(c) sudden increase of impurity concentration
(d) cumulative effect of increased electron collision and creation of added electron-hole pairs

26. The forbidden energy gap in Ge is 0.72 eV, given, $hc = 12400$ eV-Å. The maximum wavelength of radiation that will generate electron hole pair is [Karnataka CET]
(a) 172220 Å
(b) 172.2 Å
(c) 17222 Å
(d) 1722 Å

27. Identify the logic operation performed by the circuit given here. [Karnataka CET]



(a) OR (b) NOR
(c) NOT (d) NAND

28. LED is a *p*-*n* junction diode which is [MHT CET]
(a) forward biased
(b) either forward biased or reverse biased
(c) reverse biased
(d) neither forward biased nor reverse biased

29. How many NAND gates are required to form an AND gate? [MHT CET]
(a) 1 (b) 2
(c) 3 (d) 4

Answers

Level 1

Objective Problems

1. (c)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (d)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (d)
11. (c)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (a)	**17.** (a)	**18.** (a)	**19.** (c)	**20.** (c)
21. (d)	**22.** (d)	**23.** (d)	**24.** (c)	**25.** (d)	**26.** (b)	**27.** (d)	**28.** (c)	**29.** (a)	**30.** (b)
31. (b)	**32.** (b)	**33.** (b)	**34.** (b)	**35.** (b)	**36.** (b)	**37.** (b)	**38.** (c)	**39.** (b)	**40.** (b)
41. (b)	**42.** (d)	**43.** (a)	**44.** (a)	**45.** (a)	**46.** (c)	**47.** (b)	**48.** (c)	**49.** (b)	**50.** (b)
51. (b,c)	**52.** (a)	**53.** (c)	**54.** (c)	**55.** (b)	**56.** (a,c)	**57.** (a)	**58.** (d)	**59.** (b)	**60.** (a)
61. (a)	**62.** (b)	**63.** (d)	**64.** (c)	**65.** (c)	**66.** (b)	**67.** (b)	**68.** (d)	**69.** (b)	**70.** (d)
71. (d)	**72.** (c)	**73.** (d)	**74.** (c)	**75.** (b)	**76.** (d)	**77.** (b)	**78.** (c)	**79.** (a)	**80.** (b)
81. (a)	**82.** (b)	**83.** (a)	**84.** (c)	**85.** (b)	**86.** (c)	**87.** (c)	**88.** (d)	**89.** (b)	**90.** (b)
91. (a)	**92.** (c)	**93.** (c)	**94.** (b)	**95.** (a)	**96.** (b)	**97.** (a)			

Level 2

Only One Correct Option

1. (c)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (b)
11. (d)	**12.** (c)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (a)	**17.** (c)	**18.** (b)	**19.** (d)	**20.** (d)
21. (c)	**22.** (b)	**23.** (c)							

More than One Correct Options

1. (a,c)	**2.** (a,c)	**3.** (b,c)	**4.** (a,b,d)	**5.** (b,d)	**6.** (a,c,d)

Entrance Gallery

1. (a)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (c)
11. (a)	**12.** (d)	**13.** (a)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (c)

Solutions

Level 1 : Objective Problems

1. The majority charge carriers in p-type semiconductor are holes, while electrons are minority charge carriers.

2. A p-type semiconductor can be made by replacing a silicon atom in the crystal lattice with a gallium (a trivalent impurity).

3. Pentavalent impurity atom is to be added to Ge crystal so as to make a n-type semiconductor.

4. When the electrical conductivity of a semiconductor is due to the breaking of its covalent bonds, then the semiconductor is said to be intrinsic.

5. Resistance of conductor (Cu) decreases with decrease in temperature while that of semiconductor (Ge) increases with decrease in temperature.

6. A solid with completely filled valence band is an insulator, if the energy gap between the valence band and the empty conduction band is larger than about 5 eV.

7. The energy band gap of Si is 1.1 eV.

8. To make a p-type semiconductor trivalent impurity is added, boron is trivalent.

9. In semiconductors, forbidden energy gap is of the order of 1 eV.

10. In a good conductor, the conduction band and valence band overlap each other.

11. For making p-type semiconductor, we add trivalent impurity. So, B impurity is mixed in Ge and Al impurity is mixed in Si.

12. At room temperature, a p-type semiconductor has large number of holes and few electrons, as holes are majority charge carriers.

13. In semiconductors, at a room temperature, the valence band is partially empty and the conduction band is partially filled.

14. On increasing the temperature, the current in the circuit will increase because with rise in temperature, resistance of semiconductor decreases, hence overall resistance of the circuit increases, which in turn increases the current in the circuit.

15. Extrinsic semiconductors are also electrically neutral.

16. As, $i = neAv_d$

So, $v_d = \dfrac{i}{neA}$

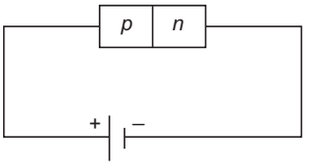
So, when temperature is increased, n increases and v_d decreases.

17. Mobility of electrons are higher than that of holes because electrons need less energy to move.

25. Resistance of semiconductor decreases with increase in temperature.

30. In $1\,cm^3$ of Si, there will be total $\dfrac{6\times10^{28}}{10^6} = 6\times10^{22}$ atoms per 6×10^7 silicon atoms there is one indium atom. Therefore, total number of indium atoms will be, $\dfrac{6\times10^{22}}{6\times10^7} = 10^{15}$.

34. In the forward bias arrangement of a p-n junction diode, the p-end is connected to the positive terminal of the battery.



36. In reverse biasing of p-n junction diode, the applied reverse voltage establishes an electric field which acts in the same direction as the electric field in the potential barrier. Therefore, the height of potential barrier is increased.

37. Diode is in forward biased condition. So, current flow

$$i = \frac{(4-1)}{300} = 10^{-2}\ \text{A}$$

38. The thickness of p-n junction has a order of 10^{-6} m.

39. In forward bias, the applied forward potential is opposite to potential barrier of depletion region, therefore potential barrier is decreased.

40. In forward biasing, the diffusion current increases and drift current remains constant, so net current is due to diffusion. In reverse biasing diffusion is not possible, so net current (very small) is due to the drift.

41. At a specific reverse voltage in p-n junction a huge current flows in reverse direction known as avalanche current.

42. In forward biasing, both potential barrier V_B and the width of charge depleted region x decreases.

43. Intensity of electric field

$$E = \frac{V}{d} = \frac{0.5}{5\times10^{-7}} = 10^6\ \text{V/m}$$

44. In case of forward biasing, resistance of p-n junction diode is zero, So, whole voltage appears across the resistance.

45. In the given circuit, diode is in reverse biasing, so it acts as open circuit. Hence, potential difference between A and B is 6 V.

46. Voltage of the source

$$V' = V_D + iR = 0.5 + 0.1\times20 = 2.5\ \text{V}$$

48. In forward biasing of a diode p-side is at higher potential.

51. D_1 is forward biased and D_2 is reverse biased.

Therefore, current through the resistance and D_1 will be equal and which is equal to $\dfrac{5}{4} = 1.25\ \text{A}$

52. $V = Ed = (6\times10^5)\,(500\times10^{-9}) = 0.30\ \text{V}$

57. When n-p-n transistor is used as an amplifier, then majority charge carriers electrons of n-type emitter move from emitter to base and then from base to colloctor.

58. α is current gain in case of common base amplifier while β is current gain in case of common emitter amplifier. α is always less than 1 and β is nearly 50.

59. In a transistor, emitter is heavily doped, base is least doped while collector is moderately doped.

64. $\alpha = \dfrac{\Delta i_c}{\Delta i_e}$

$\therefore \quad \Delta i_C = \alpha\, \Delta i_e = (0.98)\ (5.00) = 4.9$ mA

66. $90\% = 10$ mA

$\therefore \quad 10\% = \dfrac{10}{90} \times 100 = 11.11$

or emitter current is 11.11 mA.

72. $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.98}{1-0.98} = 49$

73. $\alpha = \dfrac{\Delta i_c}{\Delta i_e} = \dfrac{0.990}{1} = 0.99$

78. In the truth table, output $X = \overline{A}$. So, this truth table is of NOT gate.

79. The given Boolean expression can be written as

$$\begin{aligned} Y &= (\overline{A+B}) \cdot (\overline{AB}) \\ &= (\overline{A} \cdot \overline{B}) \cdot (\overline{A} + \overline{B}) \\ &= (\overline{A} \cdot \overline{A}) \cdot \overline{B} + \overline{A}(\overline{B} \cdot \overline{B}) \\ &= \overline{A} \cdot \overline{B} + \overline{A} \cdot \overline{B} \\ &= \overline{A} \cdot \overline{B} \end{aligned}$$

So, the truth table is

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

80. A NOR gate is combination of NOT and OR gate. The Boolean expression of NOR gate is

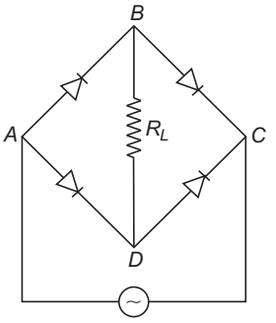
$$C = \overline{A+B}$$

85. From rectifier, we get pulsating DC, which consists of AC ripples, to remove these unwanted ripples we use filter circuits.

86. For full wave rectifier, the fundamental frequency in ripple is twice that of input frequency.

87. $\lambda\,(\text{in Å}) = \dfrac{12375}{E\,(\text{in eV})} = \dfrac{12375}{57 \times 10^{-3}}$ Å ≈ 217100 Å

91. During the first half cycle when $V_a > V_c$, all the four diodes are forward biased. Hence, no current will flow through R_L. During second half cycle when $V_c > V_a$, all the four diodes are reverse biased. Again, no current will flow through R_L.



93. $\alpha = \dfrac{\Delta i_c}{\Delta i_e}$

$$= \frac{\Delta i_c}{\Delta i_c + \Delta i_b}$$

$$0.01\, \Delta i_c = 0.9\, \Delta i_b$$

or $$\Delta i_c = 9 \Delta i_b$$

Level 2 : Only One Correct Option

1. The electrical conductivity of Ge,

$$\begin{aligned} \sigma &= \frac{1}{\rho} = e\,(\mu_e n_e + \mu_h n_n) \\ &= (1.6 \times 10^{-19})\,[0.36 + 0.17]\,(2.5 \times 10^{19}) \\ &= 2.12\ \text{S/m} \end{aligned}$$

2. The diode is in reverse biasing, so no current flows through it.

4. If $V_A > V_B$, both the junction diodes are forward biased and the given circuit diagram becomes a balanced Wheatstone bridge. The equivalent resistance in this case becomes 4 Ω.
If $V_A < V_B$, the diodes are reverse biased. In that case 4 Ω, 5 Ω and 4 Ω are in series.

5. D_1 is reverse biased. Therefore, no current will flow through 20 Ω resistance.

10. $V = i_b R_b$

$\therefore \quad R_b = \dfrac{V}{i_b} = \dfrac{9}{35 \times 10^{-6}}$

$= 257$ kΩ

11. $\beta = \dfrac{\Delta i_c}{\Delta i_b} = 50,$

$$R_i = 1\ \text{k}\Omega = 10^3\ \Omega$$

$$\Delta i_{\text{in}} = \Delta i_b = \frac{\Delta V_i}{R_i}$$

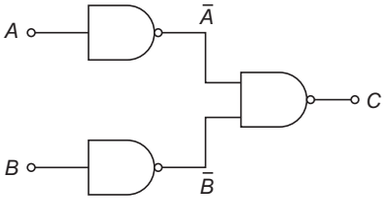
$$= \frac{0.01}{10^3} = 10^{-5}\ \text{A}$$

$\therefore \quad \Delta i_c = \beta\, \Delta i_b$

$= 50 \times 10^{-5}$ A

$= 500\,\mu$A

13. For upper part

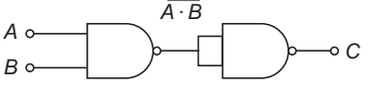


Here, $C = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A} + \overline{B}}$

$= A + B$ (From De-Morgan's theorem)

Here, output C is equivalent to OR gate.

For lower part



$$C = \overline{\overline{AB} \cdot \overline{AB}}$$
$$= \overline{\overline{AB}} + \overline{\overline{AB}}$$
$$= AB + AB = AB$$

In this case, output C is equivalent to AND gate.

14. The output D for the given combination,

$$D = \overline{(A + B) \cdot C} = (\overline{A + B}) + \overline{C}$$

If $A = B = C = 0$, then\

$$D = (\overline{0 + 0}) + \overline{0} = \overline{0} + \overline{0} = 1 + 1 = 1$$

If $A = B = 1, C = 0$, then,

$$D = (\overline{1 + 1}) + \overline{0} = \overline{1} + \overline{0} = 0 + 1 = 1$$

15. For option (c), it is a NAND gate,

$\therefore$ Its output $= \overline{0.1} = \overline{0} = 1$

16. In figure (a), first gate is AND gate and second gate is NOT gate. So, it is NAND gate.

17. The Boolean expression for the given combination is output

$$Y = (A + B) \cdot C$$

The truth table is

A	B	C	$Y = (A + B)C$
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
0	1	1	1
1	0	1	1
1	1	1	1

Hence, $A = 1, B = 0$ and $C = 1$

18. The boolean expression of given combination of gates is written as

$$Y = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$$

So, it produces OR gate.

20. As p-n junction conducts during positive half cycle only, the diode connected here will work is positive half cycle. Potential difference across C = peak voltage of the given AC voltage $= V_0 = V_{\text{rms}}\sqrt{2} = 220\sqrt{2}$ V.

21. Due to forward biased during positive half cycle of input AC voltage, the resistance of p-n junction is low. The current in the circuit is maximum. In this situation, a maximum potential difference will appear across resistance connected in series of circuit. This result into zero output voltage across p-n junction.

Due to reverse biase during negative half cycle of AC voltage, the p-n junction is reverse biased. The resistance of p-n junction becomes high which will be more than resistance in series. That is why, there will be voltage across p-n junction with negative cycle in output.

22. Consider the fig. (b) given here, suppose the potential difference between A and B. $r_1 = 5\,\text{k}\Omega$ and $r_2 = 5\,\text{k}\Omega$ are resistance in series connection.

Then, $V - 0.3 = [(r_1 + r_2)10^3] \times (0.2 \times 10^{-3})]$ $[\because V = ir]$

$$= [(5 + 5)10^3] \times (0.2 \times 10^{-3})$$
$$= 10 \times 10^3 \times 0.2 \times 10^{-3} = 2$$

$\Rightarrow$ $V = 2 + 0.3 = 2.3\,\text{V}$

23. Here, $C = A.B$ and $D = \overline{A}.B$

$$E = C + D = (A.B) + (\overline{A}\,.\,B)$$

Explanation The truth table of this arrangement of gates can be given by

A	B	$\overline{A}$	$C = A.B$	$d = \overline{A}.B$	$E = (C + D)$
0	0	1	0	0	0
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	0	1

More than One Correct Options

1. When electric field is applied across a semiconductor, the electrons in the conduction band get accelerated and acquire energy. They move from lower energy level to higher energy level.

While the holes in valence band move from higher energy level to lower energy level, where they will be having more energy.

2. Here emitter-base junction is forward biased *i.e.,* the positive pole of emitter base battery is connected to base and its negative pole to emitter. Also, the collector base junction is reverse biased, *i.e.,* the positive pole of the collector base battery is connected to collector and negative pole to base.

Thus, electron move from emmiter to base and crossover from emitter to collector.

3. Here, $I_c = 10$ mA,

Also, $I_c = \dfrac{95}{100} I_e$

$\Rightarrow$ $I_e = \dfrac{10 \times 100}{95} = 10.53$ mA

Also, $I_b = I_e - I_c = 10.53 - 10 = 0.53$ mA

4. The space-charge regions on both the sides of p-n junction which has immobile ions and entirely lacking of any charge carriers will form a region called depletion region of a diode. The number of ionized acceptors on the p-side equals the number of ionized donors on the n-side.

5. During regulation action of a Zener diode, the current through the R_s changes and resistance offered by the Zener changes. The current through the Zener changes but the voltage across the Zener remains constant.

6. In reverse biasing, the minority charge carriers will be accelerated due to reverse biasing, which on striking with atoms cause ionization resulting secondary electrons and thus more number of charge carriers.

When doping concentration is large, there will be large number of ions in the depletion region, which will give rise to a strong electric field.

Entrance Gallery

1. Given, $I = (e^{1000\,V/T} - 1)$ mA, $dV = \pm 0.01$ V

$$T = 300\,\text{K}, I = 5\,\text{mA}$$

So, $I = e^{1000\,V/T} - 1$

$\Rightarrow$ $I + 1 = e^{1000\,V/T}$

Taking log on both sides, we get

$$\log(I + 1) = \frac{1000\,\text{V}}{T}$$

On differentiation, $\frac{dI}{I+1}=\frac{1000}{T}dV$

$$dI=\frac{1000}{T}\times(I+1)\,dV$$

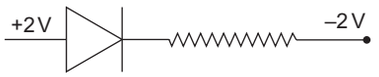
$$\Rightarrow \quad =\frac{1000}{300}\times(5+1)\times 0.01$$

$$=0.2\text{ mA}$$

So, error in the value of current is 0.2 mA.

2. For forward bias, *p*-side must be a higher potential than *n*-side.

So, it is forward biased.



3. The *n*-type semiconductor, region has (negative) electrons as majority charge carriers and an equal number of fixed positively charged donor ions. Again, the material as a whole is neutral. That is a reason, atom is electrically neutral.

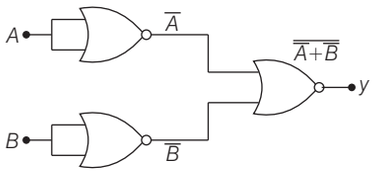
4. By constants (or parameters) of a transistor where, α and β are the AC current gains of a transistor in the common-base configuration and in the common-emitter configuration, respectively.

$$\beta=\frac{\alpha}{1-\alpha}=\frac{0.98}{1-0.98}$$

where, $\alpha=0.98, \beta=?$

$$=\frac{0.98}{0.02}=\frac{98}{2}=49$$

5. According to the logical relationship

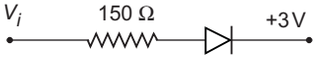


According to the logical relationship

$$\overline{\overline{A}+\overline{B}}+\overline{\overline{A\cdot B}}=A\cdot B$$

So, it is AND gate.

6. $I_{\text{initial}}=0$,

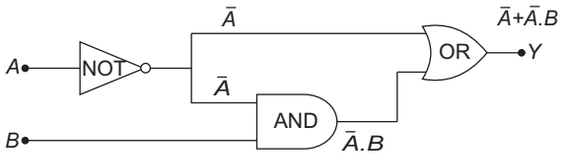


$$I_{\text{final}}=\frac{3}{150}=0.2\text{ A}$$

So, change in $I=0.2\text{ A}=20\text{ mA}$

7. I_B is in $\mu_{\text{amp.}}$, I_C is in *m* amp and V_{CE} is in volt.

8.

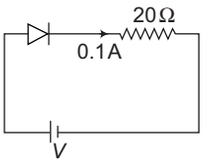


By absorption law,

$$Y=\overline{A}+\overline{A}\cdot B \quad \text{or} \quad Y=(\overline{A}\cdot B)+\overline{A}=\overline{A}\cdot(B+1)=\overline{A}\cdot 1=\overline{A}$$

9. For same value of current higher value of voltage is required for higher frequency.

10. The circuit diagram of the given situation is shown. Since, it is a closed mesh, the voltage is given by

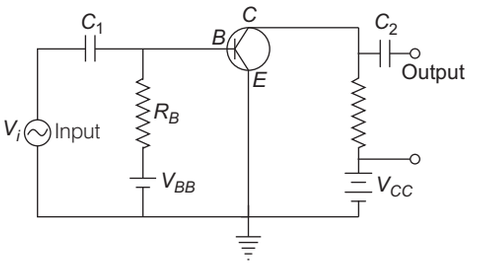


Given, $V'=0.5, I=0.1\text{ A}$ and $R=20\,\Omega$

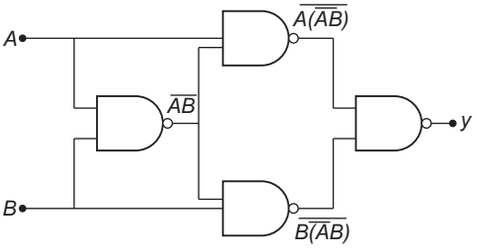
$\therefore \quad V=0.5+0.1\times 20=2.5\text{ V}$

11. We know that, $\beta=\frac{R_2}{R_1}=49\times 1000=49\times 10^3$

12. In CE amplifier, the input signal is applied across base-emitter junction as shown in the figure.



13. Output, $Y=\overline{(A\cdot(\overline{A\cdot B}))\cdot(B\cdot(\overline{A\cdot B}))}$



$$=\overline{(\overline{A}+A\cdot B)\cdot(\overline{B}+A\cdot B)}$$

$$=(\overline{\overline{A}+A\cdot B})+(\overline{\overline{B}+A\cdot B})$$

$$=A\cdot(\overline{A\cdot B})+B\cdot(\overline{A\cdot B})$$

$$=A\cdot(\overline{A}+\overline{B})+B\cdot(\overline{A}+\overline{B})$$

$$=A\cdot\overline{B}+B\cdot\overline{A}$$

A	B	$\overline{A}$	$\overline{B}$	$A\cdot\overline{B}$	$B\cdot\overline{A}$	Y
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

It is the truth table for XOR gate.

14. Since output *Y* is given by

$$Y=\overline{A}\cdot B+A\cdot\overline{B}$$

So, the truth table will be of XOR gate.

15. From the Kirchhoff's law,

$$6-i\times 150-3=0$$

$$\Rightarrow \quad i=\frac{3}{150}=\frac{1}{50}$$

$$=0.02\text{ A}$$

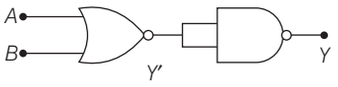
16. Given, binary number = 1011001

Its equivalent decimal number

$$= 2^0 + 0 + 0 + 2^3 + 2^4 + 0 + 2^6$$

$$= 1 + 8 + 16 + 64 = 89$$

17.



and $Y = \overline{Y'} = \overline{A + B}$

i.e. out of a NOR gate.

18. In that states, outputs *X, Y* and changes to 0, 1, 0.

19. Given circuit, $V_{r_2} = 10\,\text{V}$

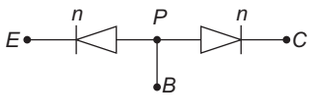
and $I_{r_2} = 6.67\,\text{mA}$

$$I_{r_1} = \frac{5}{500} = 10\,\text{mA}$$

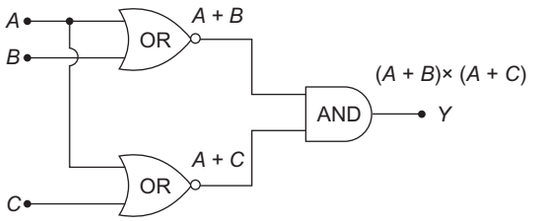
$\therefore$ $I_2 = 10 - 6.67$

$= 3.33\,\text{mA}$

20.

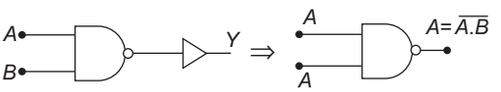


21.



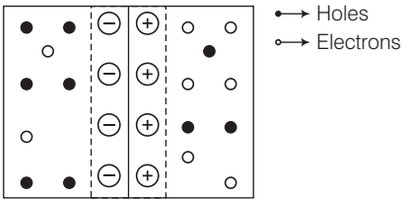
22. In *p*-region, direction of conventional current is same as flow of holes. In *n*-region, direction of conventional current is opposite to direction of flow electrons.

23.



It is a NAND gate.

24. At junction, a potential barrier/depletion layer is formed with *n*-side at higher potential and *p*-side at lower potential. Therefore, there is an electric field at the junction directed from the *n*-side to *p*-side.



25. At high reverse voltage, due to high electric field, the minority charge carriers, while crossing the junction acquire very high velocities. There by, collision breaks down the covalent bonds generating more carriers. A chain reaction is established giving rise to high current. This mechanism is called avalanche breakdown.

26. Energy gap, $E_g = \dfrac{hc}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{hc}{E_g} = \frac{12400}{0.72}$$

$$= 17222\,\text{Å}$$

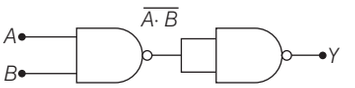
27. The given gate circuit is a combination of two NOR gates.

$\therefore$ $y = \overline{\overline{A + B}} = A + B$

It is boolean expression of OR gate.

28. When a junction diode is forward biased, energy is released at the junction due to recombination of electrons and holes. In the junction diode made of gallium arsenide or indium phosphide the energy is released in visible region. Such a junction diode is called a light emitting diode or LED.

29. Two NAND gates are required as follow



$$Y = \overline{\overline{AB} \cdot \overline{AB}} = AB$$

28

Basics of Communications

28.1 Introduction

Communication refers to the transfer of information or message from one point to another point. In modern communication systems, the information is first converted into electrical signals and then sent electronically. This has the advantage of speed, reliability and possibility of communicating over long distances. We are using these every day such as telephones, TV and radio transmission, satellite communication etc. Historically, long distance communication started with the advent of telegraphy in early nineteenth century. The milestone in trans-atlantic radio transmission in 1901 is credited to **Marconi.** However the concept of radio transmission was first demonstrated by Indian physicist **JC Bose**. Satellite communication started in 1962 with the launching of **Telstar** satellite. The first geostationary satellite **Early Bird** was launched in 1965. Around 1970, optical fibre communication entered in USA, Europe and Japan.

The basic units of any communication systems are shown below.

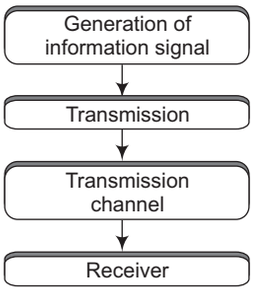


Fig. 28.1 Block diagram of communication system

We can identify different units when we orally communicate to our friend.

Our vocal chord, tongue, etc., transmit the sound waves. Air through which sound travels is the transmission channel and ear is the receiver.

In any communication system information (a physical quantity) is first converted into an electrical signal by a device called **transducer**. Most of the speech or information signal cannot be directly transmitted to long distances. For this an intermediate step of **modulation** is necessary in which the information signal is loaded or superimposed on a high frequency wave which acts as a carrier wave.

Chapter Snapshot

- Introduction
- Types of Communication Systems
- Modulation
- Pulse Modulation
- Digital Communication
- Data and Document Transmission : Fax and Modem
- Communication Channels

28.2 Types of Communication Systems

The communication systems can be classified according to the nature of information or mode of transmission as given below.

1. **According to the nature of information source**
   (a) Speech transmission (as in radio)
   (b) Picture transmission (as in TV)
   (c) Facsimile transmission (FAX)
   (d) Data transmission
2. **According to the type of modulation** Most of the information are of low frequency which cannot be transmitted to long distances. This low frequency information is superimposed on a high frequency wave and then it is transmitted to long distances. This process is called modulation. The transmitted carrier signal could be a **continuous wave** or **a pulse**.
   For continuous carrier waves, the types of modulation are :
   (a) Amplitude Modulation (AM)
   (b) Frequency Modulation (FM)
   (c) Phase Modulation (PM)
   For pulse carrier waves, the various modes of modulation are,
   (a) Pulse Amplitude Modulation (PAM)
   (b) Pulse Duration Modulation (PDM)
   (c) Pulse Position Modulation (PPM)
3. **According to mode of transmission**
   (a) Analog communication
   (b) Digital communication
4. **According to transmission channel**
   (a) **Line communication** It can have two wire transmission line, coaxial cable and optical fibre cable.
   (b) **Space communication** Now let us have a look on some important points regarding them.
      (i) In an **analog signal** current or voltage value varies continuously with time, e.g. a sinusoidal analog voltage signal. Such sinusoidal continuously varying signals may have infinite instantaneous values within the range of a maximum value (V_{max}) and a minimum value (V_{min}). A decimal number system with base 10 is used to deal with analog signals.
      (ii) In a **digital communication system**, signals are designed for two-state operation. The output voltage has only two states either low or high represented by 0 and 1. It is a discontinuous signal. A binary number system with base 2 is used to deal with digital signals. 0 of digital signal refers to open and 1 to close circuit.
      (iii) In **amplitude modulation** amplitude of high frequency carrier wave is changed in accordance with the intensity of the signal.
      (iv) In frequency modulation frequency of carrier wave is changed in accordance with the intensity of the signal.

28.3 Modulation

In this section, we will discuss in detail about modulation. What is it ? What is the need of modulation or how the modulation is done etc.

No signal in general is a single frequency but it spreads over a range of frequencies called the signal bandwidth. Suppose, we wish to transmit an electronic signal in the audio-frequency (20 Hz-20 kHz) range over a long distance. Can we do it ? No it cannot, because of the following problems :

1. **Size of antenna** For transmitting a signal we need an antenna. This antenna should have a size comparable to the wavelength of the signal. For an electromagnetic wave of frequency 20 kHz, the wavelength is 15 km. Obviously such a long antenna is not possible and hence direct transmission of such signal is not practical.
2. **Effective power radiated by an antenna** Power radiated by an antenna $\propto \frac{1}{(\lambda)^2}$. Therefore, power radiated by large wavelength would be small. For good transmission we require high power and hence need of high frequency transmission is required.
3. **Mixing up of signal from different transmitters** To avoid intermixing of different signals a band of frequencies is allotted to different users. For telephones, band of frequency allotted is 300 Hz to 3.4 kHz, suppose you are transmitting at around 1 MHz and you are allowing 10 kHz band width to each user. So different users or transmitters can transact at frequencies 1 ± 0.005 MHz, 1.01 ± 0.005M Hz, 0.99 ± 0.005 MHz etc. Therefore many channels get allowed, if the transmission frequency is high. This is achieved by a process modulation.

Modulation types Different types of modulation depend upon the specific characteristic of the carrier wave which is being varied in accordance with the message signal.

We know that a sinusoidal carrier wave can be expressed as $E = E_0 \sin(\omega t + \phi)$

The three distinct characteristics are :

Amplitude (E_0), angular frequency (ω) and phase angle (ϕ). Either of these three characteristics can be varied in accordance with the signal. The three types of modulation are, amplitude modulation, frequency modulation and phase modulation.

Similarly, the characteristics of a pulse are :

Pulse amplitude, pulse duration or pulse width and pulse position (time of rise or fall of the pulse amplitude).

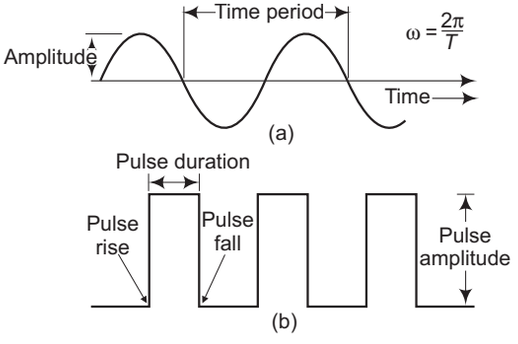
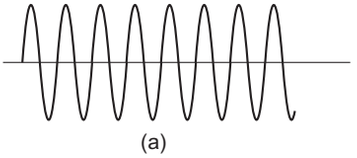


Fig. 28.2

Hence, different types of pulse modulation are, Pulse Amplitude Modulation (PAM), Pulse Duration Modulation (PDM) or Pulse Width Modulation (PWM) and Pulse Position Modulation (PPM).

Continuous wave or sinusoidal wave modulation As mentioned earlier also, there are three types of wave modulation :

(i) **Amplitude Modulation** (AM) In this type of modulation, the amplitude of the carrier signal varies in accordance with the information signal. The high frequency carrier wave (Fig. 28.3a) is superimposed on low frequency information signal (Fig. 28.3b). As a result in the amplitude modulated carrier wave amplitude no longer remains constant, but its envelope has similar sinusoidal variation as that of the low frequency or modulating signal. The carrier wave frequency ranges from 0.5 to 2 MHz. AM signals are noisy, because electrical noise signals significantly affect this.



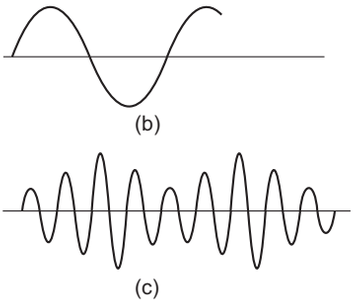


Fig. 28.3

(ii) **Frequency Modulation** (FM) In this type amplitude of carrier wave is fixed while its frequency is changing. Frequency of the carrier signal varies in accordance with the modulating signal. FM gives better quality transmission. It is preferred for transmission of music. The allotted frequencies for FM radio is 88 to 108 MHz. For VHF TV it is 47 to 230 MHz and for UHF TV it is 470 to 960 MHz.

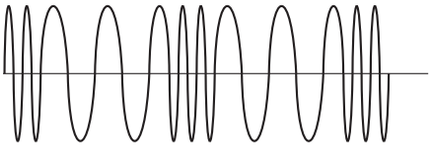


Fig. 28.4

(iii) **Phase Modulation** (PM) The phase angle ϕ of the carrier signal varies in accordance with the modulating voltage.

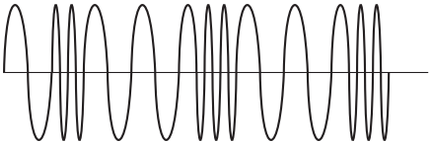


Fig. 28.5

In the figure, it seems as if frequency is changing. This is because phase term is an angle term like ω in the equation of sinusoidal wave.

28.4 Pulse Modulation

Here, the carrier wave is in the form of pulses. The common pulse modulation systems are as under.

Pulse Amplitude Modulation (PAM)

The amplitude of the pulse varies in accordance with the modulating signal as shown in figure. The pulse amplitude is increasing or decreasing as the modulating sinusoidal voltage is changing.

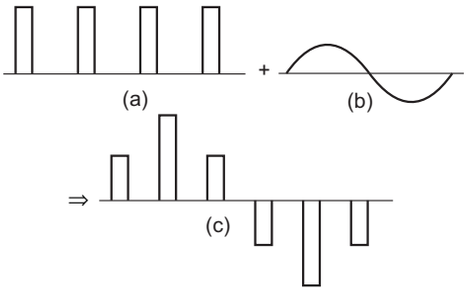


Fig. 28.6

Pulse Duration Modulation (PDM) or Pulse Width Modulation (PWM)

The pulse duration varies in accordance with the modulating signal as shown in figure. The pulse duration or the width of the original unmodulated signal is constant. However in PDM pulse duration keep on changing alternately.

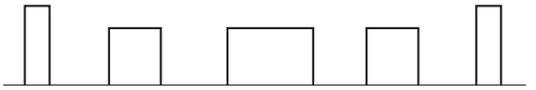


Fig. 28.7

Pulse Position Modulation (PPM)

The pulse position changes with the modulating signal as shown in the figure. The dashed lines in this figure show the original position of the pulse. In PPM these positions have shifted in time.

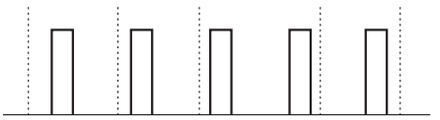


Fig. 28.8

28.5 Digital Communication

In digital communication, the modulating signals are discrete. However in analog communication the signals are continuous. There are a number of encoding steps in the digital communication. Which makes its circuit complicated. Digital communication is more error and noise free communication.

A schematic diagram of a digital communication system is shown in figure. The source encoder converts the information into binary code. Sometimes a simple encoding of the source signal is not compatible with channel through which the signal is to be carried. In such a situation an additional encoding called the channel encoding is carried out. In the final step, before transmission, the channel codes modulate a continuous waveform.

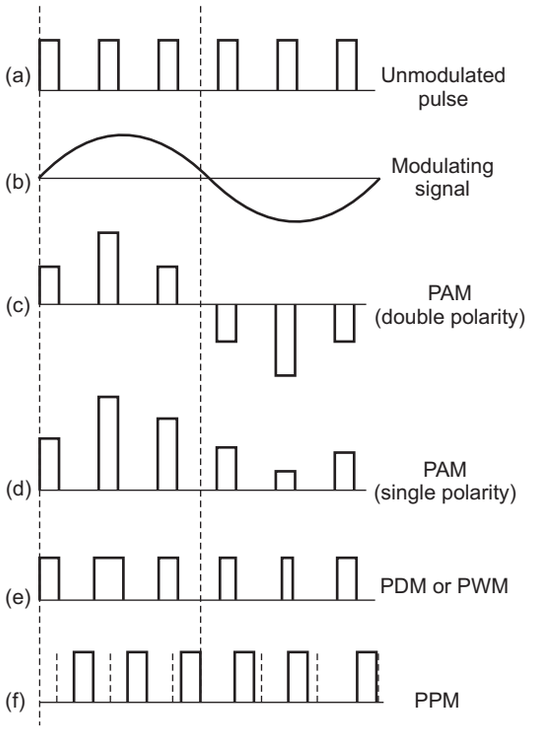


Fig. 28.9

The common modulation techniques employed for the digital data are :

(i) Amplitude Shift Keying (ASK)
(ii) Frequency Shift Keying (FSK)
(iii) Phase Shift Keying (PSK)

The digital data is 0 and 1. Obviously it has only discrete values and can be said to be **quantised** in the sense that it is either 0 or 1. Most of the information signals are analog. This analog signal is first quantised into digital signal. During the quantisation, the noise gets eliminated which makes the digital communication more reliable than the analog.

28.6 Data and Document Transmission Fax and Modem

The term data is applied to a representation of facts, concepts or instructions suitable for communication, interpretation or processing by human beings or by automatic means. Data in most cases consists of pulse type of signals. Even as analog signal is converted into a series of codes pulses. With the advent of computer, data transmission from one machine to another has become very important. Modems are used to interface two digital sources/receivers.

The term modem is contraction of the term modulater and demodulater. In the transmitting mode the modem accepts digital data and converts it to analog signals for use in modulating a carrier signal. At the receiving end the modem demodulates the carrier. Thus, the modems are placed at both ends of the communication circuits.

Many times we wish to transmit a document or a photograph. This is done by **Facsimile** (commonly called **Fax**) transmission. It means exact reproduction. The document to be transmitted is first scanned by a light source and the optical signals are converted into electrical signals by a photo-detector. These electrical signals are coded and transmission by any suitable communication method.

28.7 Communication Channels

Physical medium through which signals propagate between transmitting and receiving station is called the communication channel. There are basically two types of communications.

(i) Space communication
(ii) Line communication

Space Communication

Consider two friends playing with a ball in a closed room. One friend throws the ball (transmitter) and the other receives the ball (receiver). There are three ways in which the ball can be sent to the receiver.

(i) By rolling it along the ground
(ii) Throwing directly and
(iii) Throwing towards roof and then reflected towards the receiver. Similarly, there are three ways of transmitting an information from one place to the other using physical space around the earth.
  (a) Along the ground (ground waves)
  (b) Directly in a straight line through intervening topographic space (space wave, or tropospheric wave or surface wave) and
  (c) Upwards in sky followed by reflection from the ionosphere (sky wave).

The three modes are discussed below.

Ground Wave or Surface Wave Propagation

Information can be transmitted through this mode when the transmitting and receiving antenna are close to the surface of the earth.

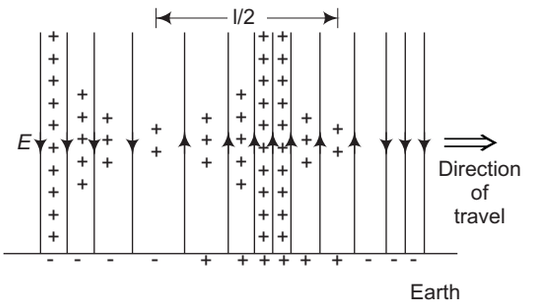


Fig. 28.10 Vertically polarised wave travelling over the surface of the earth. The solid lines represent the electric field (E) of the electromagnetic wave.

The radiowaves which progress along the surface of the earth are called ground waves or surface waves. These waves are vertically polarised in order to prevent short circuiting of the electric component. The electrical field due to the wave induce charges in the earth's surface as shown in figure. As the wave travels, the induced charges in the earth also travel along it.

This constitutes a current in the earth's surface. As the ground wave passes over the surface of the earth, it is weakened as a result of energy absorbed by the earth. Due to these losses the ground waves are not suited for very long range communication. Further these losses are higher for high frequency. Hence, ground wave propagation can be sustained only at low frequencies (500 kHz to 1500 kHz).

Space Wave Propagation or Tropospheric Wave Propagation

Television signal (80 MHz to 200 MHz) waves neither follow the curvature of the earth nor get reflected by ionosphere. Surface wave or sky wave cannot be employed in television communication. Television signals can be reflected from geostationary satellite or tall receiver antennas.

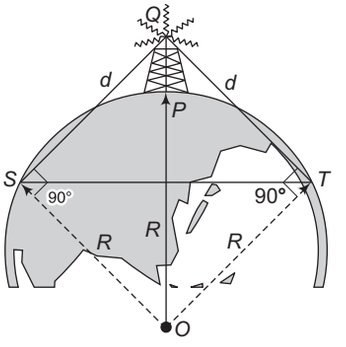


Fig. 28.11

Height of transmitting antenna The transmitted waves, travelling in a straight line, directly reach the receiver end and are then picked up by the receiving antenna as shown in figure. Due to finite curvature of the earth, such waves cannot be seen beyond the tangent points *S* and *T*.

Suppose h is the height of antenna PQ. Let R be the radius of the earth.

Further, let $QT = QS = d, \quad PQ = h, \quad OQ = R + h$

From the right angled triangle OQT,

$$OQ^2 = OT^2 + QT^2$$

$$\therefore \quad (R+h)^2 = R^2 + d^2$$

$$\therefore \quad d^2 = h^2 + 2Rh$$

Since, $R >> h, \; h^2 + 2Rh \approx 2Rh$

$$\therefore \quad d \approx \sqrt{2Rh}$$

- That this distance is of the order of 40 km. Area covered for TV transmission = $\pi d^2 = 2\pi Rh$.
  Further population covered = Population density × area covered.

Example 28.1 *A TV tower has a height of* 60 *m. What is the maximum distance and area upto which TV transmission can be received? Take radius of the earth as* 6.4×10^6 *m.*

Sol. (i) $d = \sqrt{2Rh}$

$= \sqrt{2 \times 6.4 \times 10^6 \times 60}$ m $= 27.7$ km

(ii) Area covered = $\pi d^2 = 2\pi Rh$

$(2 \times 3.14 \times 6.4 \times 10^6 \times 60)$ m^2 = 2411 km^2

Sky Wave Propagation or Ionospheric Propagation

If one wishes to send signals at far away stations, then either repeater transmitting stations are necessary or height of the antenna is to be increased. However much before the advent of satellites, radio broadcast covered long distances by the reflection of signals from the ionosphere. This mode of transmission is called ionospheric propagation or sky wave propagation.

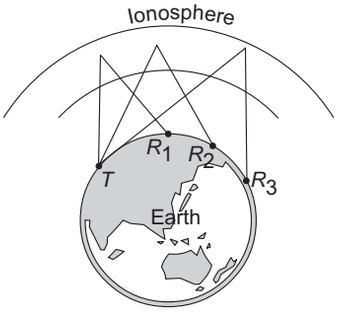


Fig. 28.12

$T \rightarrow$ Transmitter, $R \rightarrow$ Receiver

The ionosphere extends from a height of 80 km to 300 km. The refractive index of ionosphere is less than its free space value. That is, it behaves as a rare medium. As we go deep into the ionosphere, the refractive index keeps on decreasing. The bending of beam (away from the normal) will continue till it reaches critical angle after which it will be reflected back. The different points on the earth receive signals reflected from different depts of the ionosphere. There is a critical frequency f_c (5 to 100 MHz) : beyond which the waves cross the ionosphere and do not return back to the earth.

Satellite communication Long distance communication beyond 10 to 20 MHz was not possible before 1960, because all the three modes of communication discussed above failed (ground waves due to conduction losses, space wave due to limited line of sight and sky wave due to the penetration of the ionosphere by the high frequencies beyond f_c). Satellite communication made this possible.

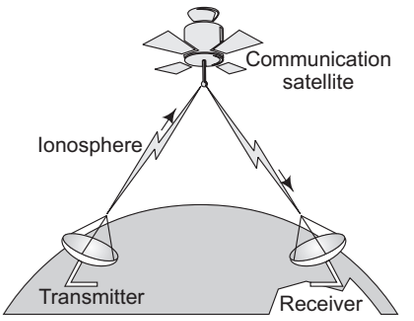


Fig. 28.13 Principle of satellite communication

The basic principle of satellite communication is shown in figure. A communication satellite is a spacecraft placed in an orbit around the earth. The frequencies used in satellite communication lie in UHF/microwave regions. These waves can cross the ionosphere and reach the satellite.

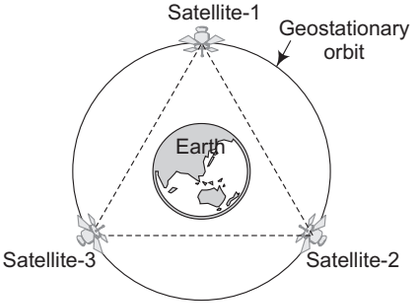


Fig. 28.14

For steady, reliable transmission and reception it is preferred that satellite should be geostationary. A geostationary satellite is one that appears to be stationary relative to the earth. It has a circular orbit lying in the

equatorial plane of the earth at an approximate height of 36000 km. Its time period is 24 h.

If we use three geostationary satellites placed at the vertices of an equilateral triangle as shown in figure the entire the earth can be covered by the communication network.

In addition to geostationary equatorial orbits, there are two more orbits which are being used for communication. These are

(a) **Polar circular orbit** This orbit passes over or very close to the poles. It is approximately at a height of 1000 km from the earth.

(b) **Highly inclined elliptical orbits**

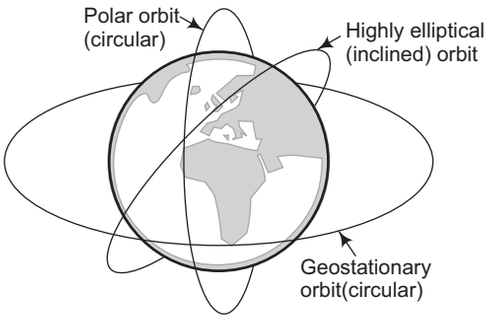


Fig. 28.15 A schematic diagram of various satellite orbits used in satellite communication

Remote sensing Remote sensing is an application of satellite communication. It is the art of obtaining information about an object or area acquired by a sensor that is not in direct contact with the target of investigation. Any photography is a kind of remote sensing. If we want to cover large areas for which information is required, we have to take photographs from larger distances. This is called aerial photography. Town and country planning can also be done by remote sensing.

A satellite equipped with appropriate sensors is used for remote sensing. Taking photograph of any object relies on the reflected wave from the object. We use visible light in normal photography. In principle, waves of any wavelength in the electromagnetic spectrum can be used for this purpose by using suitable sensors.

Some applications of remote sensing include meteorology (development of weather systems and weather forecasting), climatology (monitoring climate changes) and oceanography etc.

Line Communication

In space communication, there is no physical point to point contact between the receiver and the transmitter.

The point to point communication mode is line communication through transmission lines or wires (as in earlier telephone and telegraph lines) is the simplest and oldest mode of line communication.

There are mainly three types of line communication channels.

(i) Two wire transmission line

(ii) Coaxial cables

(iii) Optical fibres

Two Wire Transmission Line

You might have seen wire of telephone lines along the road side. The signal slowing through the wires create their respective electric (E) and magnetic fields (B). Each portion of the transmission line can be considered as a small inductor, resistor and capacitor as shown in figure (c). Such inductors, resistors and capacitors are distributed throughout the transmission line has a characteristic impedance.

Maximum power is transferred to the load when the impedance of the detecting device at the receiver is matched to the characteristic impedance. You might have seen telephone line man matching the load when the reception in your telephone is not good. In a two wire transmission line a high frequencies suffer from the disadvantage of being prone to electromagnetic interference and also become sources of radiation themselves. The later can be reduced, if we use a twisted wire as shown.

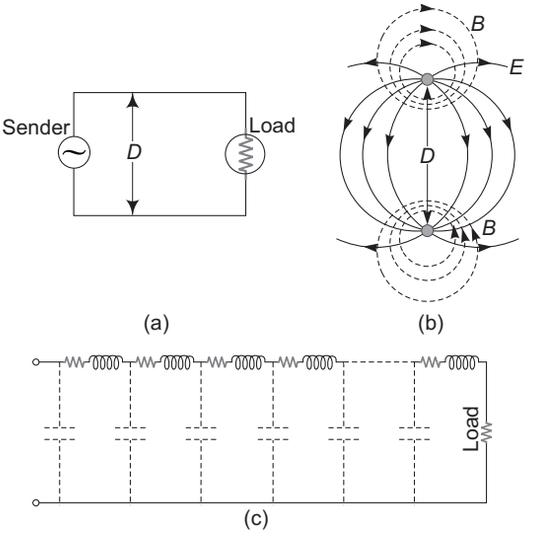


Fig. 28.16 Description of a two wire wave transmission line : (a) a typical line; (b) electric (solid lines) and magnetic (dashed lines) fields due to the signal in the transmission line; and
(c) equivalent circuit of transmission line geometry shown in (a). The impedances are distributed throughout the line.

Coaxial Cable

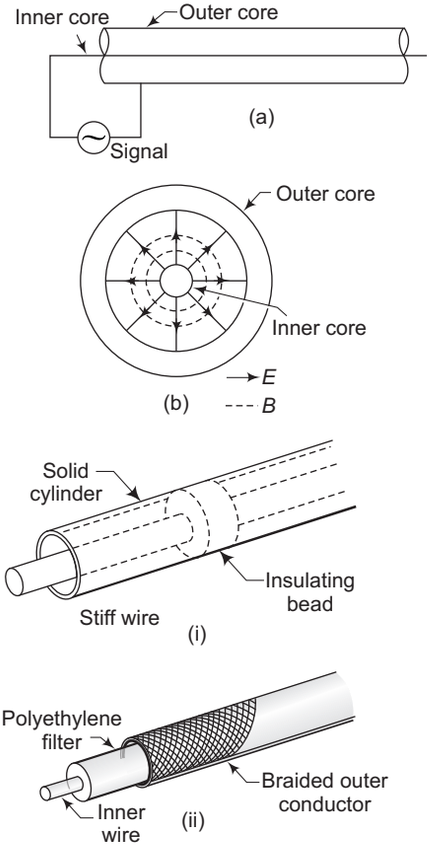


Fig. 28.17

The drawbacks of two wire transmission line (electromagnetic interference and radiation) can be avoided by use of shielded coaxial cables. This consists of a hollow outer cylindrical conductor which surrounds a single inner conductor kept separated from each other by an insulator. The outer conductor acts as a shield and minimises interference. The inner conductor is made up of copper wire and the outer conductor can be either a solid or a braided mesh of fine wires. The inner conductor is held at the centre by a solid dielectric material acting as a spacer. This dielectric medium plays an important role. These are good insulators only for DC or low frequency AC.

As the frequency increases, there is significant dielectric loss and can sometimes become quite high after a certain frequency. So, it puts an upper limit (approximately 20 MHz) upto which a coaxial cable can be effectively used. The outer conductor is normally connected to ground and thus it provides an electrical shield to the signals carried by the central conductor.

Optical Fibres

Optical communication is a better choice for large number of channels and higher bandwidth transmission. With the recent availability of optical fibres of small diameters ($\approx 10^{-5}$ m), the optical communication technology is developing very rapidly. There are following advantages of optical communication over the conventional two wire or cable electronic communication systems.

(i) In optical fibres losses are less.

(ii) Secret information like banking, defence etc. is more secure, because optical signal is confined to the inside of fibre and cannot be tempered easily.

(iii) It can be used for high frequencies ($\approx 10^{14}$ Hz).

Total Internal Reflection (TIR) is the principle behind optical fibres. Light can be transmitted along it with almost no loss, because of TIR. Important applications of fibre optics are in telecommunications and medicine.

The optical fibre and fibre cables An optical fibre consists typically of a transparent core fibre of glass of refractive index n_1 surrounded by a transparent glass sheath or cladding of slightly lower index n_2 with both enclosed in an opaque protective jacket. Figure shows a cross-section of an option fibre.

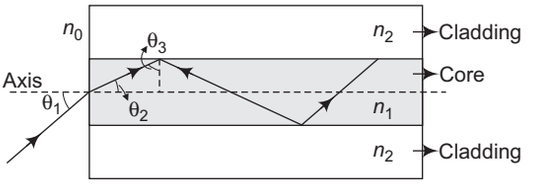


Fig. 28.18

A ray entering the core from an external medium of refractive index n_0 at an angle θ_1 will make an angle θ_2 with respect to axis inside the core.

From Snell's law,

$$n_0 \sin \theta_1 = n_1 \sin \theta_2$$

$$\therefore \quad \sin \theta_2 = \left(\frac{n_0}{n_1}\right) \sin \theta_1$$

The ray is now incident on the core-cladding boundary at an angle θ_3.

If $\theta_3 > \theta_c$, the critical angle the ray will be totally internally reflected and continue to propagate inside the core.

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$\sin\theta_1 = \left(\frac{n_1}{n_0}\right)\sin\theta_2$$

$$= \left(\frac{n_1}{n_0}\right)\sin(90° - \theta_c)$$

$$= \left(\frac{n_1}{n_0}\right)\cos\theta_c$$

$$= \left(\frac{n_1}{n_0}\right)\sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$$\therefore \quad \sin\theta_1 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

For air as the external medium, $n_0 = 1$.

Hence, $$\sin\theta_1 = \sqrt{n_1^2 - n_2^2}$$

Following points are worthnoting regarding an optical fibre.

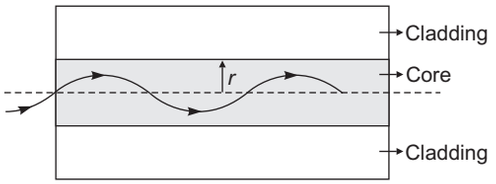


Fig. 28.19

(i) By gradually decreasing, the index of refraction n_1 of the optical fibre core as a function of radius r the ray path becomes a smooth undulating curve which does not reach the core cladding boundary.

(ii) Speed of light changes when light changes medium. There are three classification of fibres.

(a) In a **multimode step index fibre** the core diameter is large. There are many paths through which light can travel. The light by travelling the straight path through the centre reaches the end before the other rays, which follows a *zig-zag* path. Time lag between different light rays to exit the fibre is called the modal dispersion.

(b) In a **multimode graded index fibre** the light at the fibre core travels more slowly than the light nearer the surface on account of higher index of refraction of core. Thus modal dispersion is reduced.

(c) In a **single mode step index fibre** a light ray can travel only through one path, therefore modal dispersion is zero.

(iii) The refractive index of the fibre is determined by its composition. Usually n_1 and n_2 are 1.48 and 1.46 respectively.

Chapter Summary with Formulae

(i) Any information is first converted into electrical signals (in the form of voltage or current) and then sent electronically.

(ii) There are two basic models of communication: point to point and broadcast.

(iii) Three essential elements of any communication system are : transmitter, medium/channel and receiver.

(iv) Every message signal occupies a range of frequencies. This is called bandwidth of signals. Similarly, different types of transmission media (like optical fibres, ground wave or sky wave etc.) offer different bandwidths.

(v) Two important forms of communication system are : analog and digital. In analog system, the information to be transmitted is in continuous waveform while for digital it has only two discrete levels.

(vi) **Communication channels**

(a) There are two types of communication channels : space communication and line communication.

(b) Line communication include optical fibres, coaxial cables or two wire transmission line.

(c) There are three modes of space communication

Ground wave or surface wave propagation near the surface of the earth, electromagnetic waves propagate as surface waves. Surface wave propagation is useful up to a few MHz frequencies.

Space wave propagation antennas are used for this purpose.

If height of antenna is h, then the range is given by

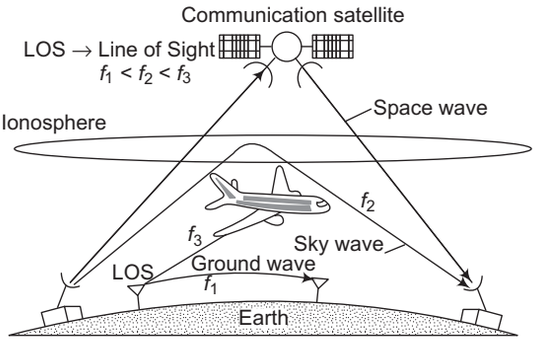
$$d = \sqrt{2Rh}$$

It height of receiving antenna is also given, then the maximum line of sight distance d_M is given by

$$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

where, R = radius of the earth, h_T = height of transmitting antenna and h_R = height of receiving antenna.

Sky wave propagation or ionospheric propagation long distance communication is achieved through reflection of electromagnetic waves by ionosphere. Sky wave propagation takes place to frequency of about 30 MHz.



(vii) **Modulation**

(a) Low frequencies cannot be transmitted to long distances. Therefore, they are superimposed on a high frequency carrier wave by a process known as modulation.

(b) Pulse modulation can be classified as : Pulse Amplitude Modulation (PAM), Pulse Duration Modulation (PDM) or Pulse Width Modulation (PWM) and Pulse Position Modulation (PPM).

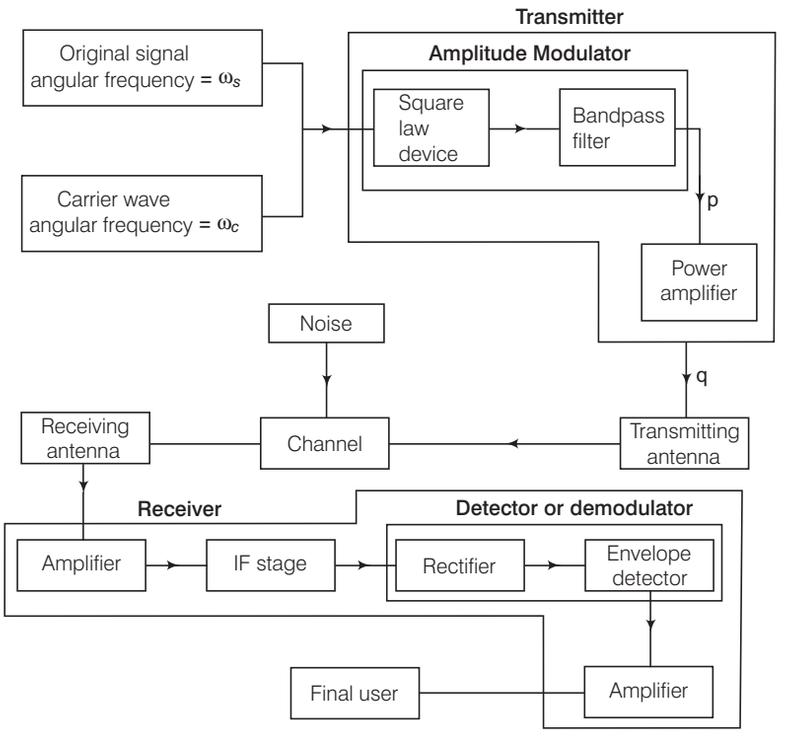
(c) Wave modulation can be classified as : Amplitude Modulation (AM), Frequency Modulation (FM) and Phase Modulation (PM).

(d) Angular frequency of original signal (to be transmitted) is ω_s and of carrier wave is $f\omega_c$.

(e) Signal p (shown in below block diagram) contains frequencies ω_c, $\omega_c - \omega_s$ and $\omega_c + \omega_s$.

(f) Signal q (shown in below block diagram) contains the original message signal frequency ω_s.

(g) Complete block diagram of amplitude modulated wave (from starting to end) is shown in following figure.



(viii) Thus on a whole following steps are followed in any communication system.

Step 1 → Information is converted into electrical signals.

Step 2 → Modulation of electrical signals (with carrier wave).

Step 3 → Modulated signal is sent to transmitter.

Step 4 → From the transmitter, signals are sent to communication channels.

Step 5 → Signals are received by receiver.

Step 6 → Signals are demodulated and original electrical signals are separated from modulated wave.

Step 7 → Demodulated electrical signal are further converted into actual information.

Additional Examples

Example 1. *Name the device fitted in the satellite which receives signals from the earth station and transmits them in different directions after amplification.*

Sol. Transponder.

Example 2. *An electromagnetic wave of frequency* 28 *MHz passes through the lower atmosphere of the earth and gets incident on the ionosphere. Shall the ionosphere reflects these waves?*

Sol. Yes. The ionosphere reflects back electromagnetic waves of frequency less than 30 MHz.

Example 3. *Which waves constitute amplitude-modulated band?*

Sol. Electromagnetic waves of frequency less than 30 MHz constitute amplitude-modulated band.

Example 4. *Give the frequency ranges of the following*

(i) High frequency band (HF)

(ii) Very high frequency band (VHF)

(iii) Ultra high frequency band

(iv) Super high frequency band.

Sol. (i) 3 MHz to 30 MHz (ii) 30 MHz to 300 MHz
(iii) 300 MHz to 3000 MHz (iv) 3000 MHz to 30000 MHz.

Example 5. *State the two functions performed by a modem.*

Sol. (i) Modulation (ii) Demodulation.

Example 6. *Why is the transmission of signals using ground waves restricted upto a frequency of* 1500 *kHz ?*

Sol. This is because at frequencies higher than 1500 kHz, there is an increase in the absorption of signal by the ground.

Example 7. *How does the effective power radiated by an antenna vary with wavelength?*

Sol. Power radiated by an antenna $\propto \left(\frac{1}{\lambda}\right)^2$.

Example 8. *Why is it necessary to use satellites for long distance TV transmission?*

Sol. Television signals are not properly reflected by the ionosphere. So, reflection is affected by satellites.

Example 9. *Why long distance radio broadcasts use shortwave bands?*

Sol. This is because ionosphere reflects waves in these bands.

Example 10. *What is a channel bandwidth ?*

Sol. Channel bandwidth is the range of frequencies that a system can transmit with efficient fidelity.

Example 11. *Give any one difference between FAX and e-mail systems of communication.*

Sol. Electronic reproduction of a document at a distant place is known as FAX. In e-mail system, message can be created, processed and stored. Such facilities are not there in Fax system.

Example 12. *Why ground wave propagation is not suitable for high frequency?*

Sol. At high frequency, the absorption of the signal by the ground is appreciable. So, ground wave propagation is not suitable for high frequency.

Example 13. *What is the purpose of modulating a signal in transmission?*

Sol. A low frequency signal cannot be transmitted to long distances, because of many practical difficulties. On the other hand, effective transmission is possible at high frequencies. So, modulation is always done in communication systems.

Example 14. *What is a transducer?*

Sol. A device which converts energy in one form to another is called a transducer.

Example 15. *Why do we need a higher band width for transmission of music compared to that for commercial telephone communication?*

Sol. As compared to speech signals in telephone communication, the music signals are more complex and correspond to higher frequency range.

Example 16. *From which layer of the atmosphere, radiowaves are reflected back?*

Sol. The electromagnetic waves of radio frequencies are reflected by ionosphere.

Example 17. *Why sky waves are not used in the transmission of television signals?*

Sol. The television signals have frequencies in 100-200 MHz range. As the ionosphere cannot reflect radio waves of frequency greater than 40 MHz back to the earth, the sky waves cannot be used in the transmission of TV signals.

Example 18. *Why are short waves used in long distance broadcasts?*

Sol. The short waves (wavelength less than 200 m or frequencies greater than 100 kHz) are absorbed by the earth due to their high frequency but are effectively reflected by layer in ionosphere. After reflection from the ionosphere, the

short waves reach the surface of the earth back only at a large distance from the transmitter. For this reason, short waves are used in long distance transmission.

Example 19. *Define the term critical frequency in relation to sky wave propagation of electromagnetic waves.*

Sol. The highest value of the frequency of radio waves, which on being radiated towards the ionosphere at some angle are reflected back to the earth is called critical frequency.

Example 20. *What mode of communication is employed for transmission of TV signals?*

Sol. Space wave communication.

Example 21. *How many AM broadcast stations can be accommodated in a* 100 *kHz bandwidth, if the highest frequency modulating a carrier is* 5 *kHz?*

Sol. Any station being modulated by a 5 kHz signal will produce an upper side frequency 5 kHz above its carrier and a lower side frequency 5 kHz below its carrier, thereby requiring a bandwidth of 10 kHz. Thus,

Number of stations accommodated

$$= \frac{\text{Total bandwidth}}{\text{Bandwidth per station}}$$

$$= \frac{100}{10} = 10$$

Example 22. *How many* 500 *kHz waves can be on a* 10 *km transmission line simultaneously?*

Sol. Let λ be the wavelength of 500 kHz signal. Then,

$$\lambda = \frac{c}{f}$$

$$= \frac{3.0 \times 10^8}{5.0 \times 10^4} \text{ m}$$

$$= 600 \text{ m}$$

The number of cycle on the line can be found from,

$$n = \frac{d}{\lambda} = \frac{10 \times 10^3}{600}$$

$$= 16.67$$

Example 23. *A two wire transmission line has a capacitance of* 20 *pF/m and a characteristic impedance of* 50 Ω.

(a) What is the inductance per metre of this cable?

(b) Determine the impedance of an infinitely long section of such cable.

Sol. (a) The characteristic impedance,

$$Z = \sqrt{L/C}$$

$$\therefore \quad L = (Z^2)(C) = (50)^2 \ (20 \times 10^{-12}) \text{ H}$$

$$= 0.05 \mu\text{H}$$

(b) The characteristic impedance of a transmission line is the impedance that an infinite length of line would present to a power supply at the input end of the line. Thus,

$$Z_\infty = Z_0 = 50 \ \Omega$$

Example 24. *TV transmission tower at a particular station has a height of* 160 *m.*

(a) What is the coverage range?

(b) How much population is covered by transmission, if the average population density around the tower is 1200 *per km*^{2}*?*

(c) By how much the height of tower be increased to double its coverage range. Given, radius of the earth = 6400 *km?*

Sol. (a) Coverage range $d = \sqrt{2Rh}$

$$= \sqrt{2 \times 6400 \times 10^3 \times 160} \text{ m}$$

$$= 45.254 \text{ km}$$

(b) Population covered

$$= (\text{population density}) \times (\text{area covered})$$

$$= (1200) \times (\pi d^2) = (2400 \ \pi Rh)$$

$$= 2400 \times 3.14 \times 6.4 \times 10^3 \times 0.16$$

$$= 77.17 \text{ lac}$$

(c) Coverage range $\propto \sqrt{h}$

Therefore, coverage range can be doubled by making height of the tower four times or 640 m. So, height of the tower should be increased by 480 m.

Example 25. *The core of an optical fibre is made of glass with refractive index equal to* 1.55 *and clad has refractive index* 1.55*. Calculate :*

(a) the critical angle for total internal reflection.

(b) maximum acceptance angle at the air-core interface.

Sol. (a) $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

Here, n_1 = refractive index of core = 1.55

and n_2 = refractive index of clad = 1.51

$$\therefore \quad \theta_c = \sin^{-1}\left(\frac{1.51}{1.55}\right) = 55.8°$$

(b) Maximum acceptance angle at the air-core interface

$$= \sin^{-1}(\sqrt{n_1^2 - n_2^2}) = \sin^{-1}\left[\sqrt{(1.55)^2 - (1.51)^2}\right]$$

$$= 20.4°$$

NCERT Selected Questions

Q 1. Which of the following frequencies will be suitable for beyond the horizon communication using sky waves?

(a) 10 kHz (b) 10 MHz

(c) 1 GHz (d) 1000 GHz

Sol. Option (b) is correct, because 10 kHz cannot be radiated due to large antenna size, 1 GHz and 1000 GHz will be lentrated.

Q 2. Frequencies in the UHF range normally propagate by means of

(a) ground waves (b) sky waves

(c) surface waves (d) space waves

Sol. Option (d) is correct, because the frequency in UHF range normally propagate by means of space waves.

Q 3. Digital signals

(i) do not provide a continuous set of values.

(ii) represent values as discrete steps.

(iii) can utilize binary system and

(iv) can utilize decimal as well as binary systems.

(a) (i) and (ii)

(b) (ii) and (iii)

(c) (i), (ii) and (iii) but not

(d) All of (i), (ii), (iii) and (iv)

Sol. Option (d) is correct, because in digital signal all cases are these.

Q 4. Is it necessary for a transmitting antenna to be at the same height as that of the receiving antenna for line of sight communication? A TV transmitting antenna is 81m tall. How much service area can it cover, if the receiving antenna is at the ground level?

Sol. Height of antenna = $h = 81\,\text{m}$

No, it is not necessary for line of sight communication, the two antenna may not be at the same height.

$$\text{Area} = \pi d^2,\ R = \text{radius of the earth}$$

$$= 6.4 \times 10^6\,\text{m}$$

$$d = \sqrt{2hR}$$

$$\text{So, area} = \pi \times 2hR = \frac{22}{7} \times 2 \times 81 \times 6.4 \times 10^6$$

$$= 3258.5 \times 10^6\,\text{m}^2$$

$$\text{Area} = 3258.5\ \text{km}^2$$

Q 5. A carrier wave of peak voltage 12 V is used to transmit a message signal. What should be the peak voltage of the modulating signal in order to have a modulation index of 7.5%?

Sol. Peak voltage $V_0 = 12\,\text{V}$

μ = modulation index = 75%

$$\mu = \frac{\text{Peak voltage of modulating signal}}{\text{Peak voltage}}$$

$$\text{Peak voltage of modulation signal} = \mu \times \text{peak voltage}$$

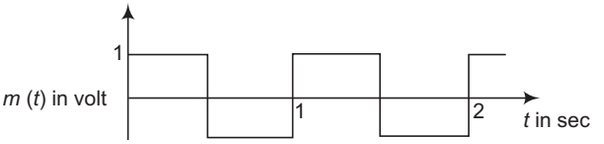
$$= \frac{75}{100} \times 12 = 9\,\text{V}$$

Q 6. A modulation signal is a square wave, as shown in figure.

The carrier wave is given by $c(t) = 2\sin(8\pi t)$ volt.

(a) Sketch the amplitude modulated waveform.

(b) What is the modulation index?



Sol. $c(t) = 2\sin(8\pi t)$ …(i)

(a) According to diagram $A_m = 1\text{V}$, $A_c = 2\,\text{V}$ by Eq. (i)

$T_m = 1\,\text{s}$ (from diagram)

$$\omega_m = \frac{2\pi}{T_m} = \frac{2\pi}{1} = 2\pi\ \text{rad/s} \quad \text{…(ii)}$$

From Eq. (i), $c(t) = 2\sin 8\pi t$

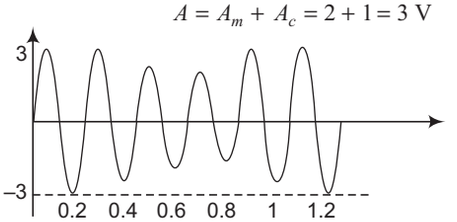
$c(t) = A_c \sin\omega_c t$

So, $\omega_c = 8\pi$

From Eq. (ii) $\omega_m = 2\pi$

So, $\omega_c = 4\omega_m$

$A = A_m + A_c = 2 + 1 = 3\ \text{V}$



(b) Modulation index $\mu = \frac{A_m}{A_c} = \frac{1}{2} = 0.5$

Q 7. For an amplitude modulated wave, the maximum amplitude is found to be 10 V, while the minimum amplitude is found to be 2V. Determine the modulation index μ.

What would be the value μ, if the minimum amplitude is zero volt?

Sol. $A_{\max} = 10\,\text{V}$

$A_{\min} = 2\,\text{V}$

Let A_c and A_m be the amplitudes of carrier wave and signal wave.

$$A_{max} = A_c + A_m = 10 \quad \ldots(i)$$
$$A_{min} = A_c - A_m = 2 \quad \ldots(ii)$$

From Eq. (i) and Eq. (ii), we get

$$2A_c = 12$$
$$A_c = 6\,\text{V},\ A_m = 10 - 6 = 4\,\text{V}$$

Modulation index $\mu = \frac{A_m}{A_c} = \frac{4}{6} = \frac{2}{3}$

When $A_{min} = 0$,

$$\left.\begin{aligned} A_c + A_m &= 10 \\ A_c - A_m &= 10 \end{aligned}\right\}$$

By solving $2A_m = 10 \Rightarrow A_m = 5 \Rightarrow A_c = 5$

$$\mu = \frac{A_m}{A_c} = \frac{5}{5} = 1$$

Q 8. Due to economic reasons, only the upper sideband of an AM wave is transmitted, but at the receiving station,there is a facility for generating the carrier. Show that, if a device is available which can multiply two signals, then it is a possible to recover the modulating signal at the receiver station.

Sol. Let ω_c be the angular frequency of carrier waves and ω_m be the signal waves.

Let the signal received at the receiving station be

$$e = E_1 \cos(\omega_c + \omega_m)\, t$$

Let the instantaneous voltage of carrier wave

$e_c = E_c \cos\omega_c t$ is available at receiving station

Now, $e \times e_c = E_1 E_c \cos\omega_c t \cos(\omega_c + \omega_m)\, t$

$$e \times e_c = \frac{E_1 E_c}{2} \cdot 2\cos\omega_c t \cdot \cos(\omega_c + \omega_m)\, t$$

$$= \frac{E_1 E_c}{2}[\cos(\omega_c + \omega_c + \omega_m)\, t + \cos(\omega_c + \omega_m - \omega_c)\, t]$$

Using formula $2\cos A \cos B = \cos(A + B) + \cos(A - B)$

$$= \frac{E_1 E_c}{2} = [\cos(2\omega_c + \omega_m)t + \cos\omega_m t]$$

Now at the receiving and at the signal passes through filter it will pass the high frequency $(2\omega_c + \omega_m)$ but obstruct the frequency ω_m. So, we can record the modulating signal $\frac{E_1 E_c}{2}\cos\omega_m t$.

Objective Problems

[Level 1]

1. Major parts of a communication system are
(a) transmitter and receiver
(b) receiver and communication channel
(c) transmitter and communication channel
(d) transmitter, receiver and communication channel

2. Communication channel consists of
(a) transmission line (b) optical fibre
(c) free space (d) All of these

3. Audio frequency range is from
(a) 20 Hz to 20 kHz
(b) 200 Hz to 2000 Hz
(c) 2 kHz to 20 kHz
(d) 20 kHz to 200 kHz

4. Basic components of a transmitter are
(a) message signal generator and antenna
(b) modulator and antenna
(c) signal generator and modulator
(d) message signal generator, modulator and antenna

5. Modulation is the phenomenon of
(a) superimposing the audio frequency signal over a carrier wave
(b) separating the audio frequency signal from the carrier wave
(c) separating carrier wave from the modulated wave
(d) any of (a), (b) or (c)

6. Range of frequencies allotted for commercial FM radio broadcast is
(a) 88 to 108 MHz (b) 88 to 108 kHz
(c) 8 to 88 MHz (d) 8 to 88 kHz

7. In an amplitude modulated wave, for audio frequency of 500 cps, the appropriate carrier frequency will be
(a) 50 c/s (b) 100 c/s
(c) 500 c/s (d) 50000 c/s

8. The ground wave propagation is suitable for radiowaves of frequency
(a) upto 2 MHz (b) from 2 MHz to 20 MHz
(c) from 2 MHz to 30 MHz (d) None of these

9. The sky wave propagation is suitable for radiowaves of frequency
(a) upto 2 MHz
(b) from 2 MHz to 20 MHz
(c) from 2 MHz to 30 MHz
(d) None of the above

10. The radiowaves of frequency 30 MHz to 300 MHz belong to
(a) high frequency band
(b) very high frequency band
(c) ultra high frequency band
(d) super high frequency band

11. The electromagnetic waves of frequency range 100 to 300 GHz are used in
(a) sky wave propagation
(b) TV communication
(c) microwave communication
(d) ground wave communication

12. The space wave propagation is utilized in
(a) television communication
(b) radar communication
(c) microwave communication
(d) All of the above

13. The electromagnetic waves of frequency 80 MHz to 200 MHz
(a) can be reflected by troposphere
(b) can be reflected by ionosphere
(c) can be reflected by mesosphere
(d) cannot be reflected by any layer of the earths atmosphere

14. The reception of television signals is possible by using
(a) sky wave propagation
(b) communication geostationary satellite
(c) polar satellite
(d) tall antenna which may directly intercept the signals coming from transmitter antenna

15. In satellite communication, the communication satellite
(a) acts as a reflector for a beam of modulated microwave from transmitter sent directly towards it
(b) acts as a reflector for signal reaching there, without any change in frequency
(c) receives the coming modulated microwave signal, amplifies it and returns it to the earth at a different frequency
(d) None of the above

16. The Indian remote sensing satellite is
(a) Aryabhatt (b) INSAT 2 B
(c) Sputnik-I (d) IRS IB

17. The air of the earth's atmosphere responsible for absorbing a large portion of ultraviolet radiations by the sun is
(a) mesosphere (b) troposphere
(c) ozone layer (d) ionosphere

18. Radiowaves of constant amplitude can be generated with
(a) filter (b) rectifier
(c) FET (d) oscillator

19. Optical fibre communication is generally preferred over general communication system, because
(a) it is more efficient
(b) it has signal security
(c) it cannot be jammed as easily as radiowaves
(d) All of the above

20. A laser is a coherent source, because it contains
(a) many wavelengths
(b) uncoordinated wave of particular wavelength
(c) coordinated waves of many wavelength
(d) coordinated waves of particular wavelength

21. Quality of transmission depends upon
(a) nature of medium (b) nature of signal
(c) Both (a) and (b) (d) Neither (a) nor (b)

22. Coaxial cables can carry
(a) digital signals
(b) analog signals
(c) Both digital and analog signals
(d) Neither digital nor analog signals

23. The most commonly used two wire lines are
(a) parallel wire lines (b) twisted pair wire lines
(c) co-axial wire lines (d) All of these

24. A TV tower has a height of 150 m. The area of the region covered by the TV broadcast is (radius of the earth $= 6.4 \times 10^6$ m)
(a) $9.6\pi \times 10^8$ m^2 (b) $19.2\pi \times 10^8$ m^2
(c) $19.2\pi \times 10^8$ m^2 (d) $1.92\pi \times 10^3$ km^2

25. When microwave signals follow the curvature of the earth, this is known as
(a) ionospheric reflection (b) window
(c) Faraday affect (d) ducting

26. In a 8 bit byte, the number of code combinations are
(a) 256 (b) 16
(c) 4 (d) 64

27. The carrier frequency generated by a tank circuit containing 1 nF capacitor and 10 μH inductor is
(a) 1592 Hz (b) 1592 kHz
(c) 159.2 Hz (d) 15920 kHz

28. What does the modem do?
(a) Modulation
(b) Demodulation
(c) Both modulation and demodulation
(d) Neither modulation nor demodulation

29. The modulating frequency is 5 kHz and the carrier frequency is 100 kHz. What is the band width of AM transmission?
(a) 105 kHz (b) 95 kHz
(c) 2.5 kHz (d) 10 kHz

30. In which of the following cases lies the broadcast band of FM?
(a) UHF (b) VHF
(c) SHF (d) HF

31. Which of the following is digital modulation technique?
(a) PDM (b) PAM
(c) PCM (d) PPM

32. What causes fading of the signal strength?
(a) Polarisation
(b) Diffraction
(c) Interference
(d) Refraction

33. Which of the following can be used to generate radiowaves?
(a) Rectifier (b) Modulator
(c) Amplifier (d) Oscillator

34. What is the radiowaves signal called, if it propagates from one place to another along the surface of the earth?
(a) Sky wave (b) Ground wave
(c) Carrier wave (d) Modulated wave

35. What are the waves used for long distance transmission called?
(a) Long waves (b) Carrier waves
(c) Short waves (d) Pulse waves

36. Which of the following electromagnetic waves are useful for telecommunication?
(a) Ultraviolet (b) Microwaves
(c) Infra red (d) Electrical waves

37. How many telephone conversations can be carried out by optical fibre?
(a) 10000 (b) 2000
(c) 1000 (d) 20000

38. What determines the absorption of radiowaves by the atmosphere?
(a) Distance of receiver (b) Polarisation
(c) Interference (d) Frequency

39. Which of the following is the disadvantage of FM over AM?
(a) Larger band width requirement
(b) Larger noise
(c) Higher modulation power
(d) Low efficiency

40. TV and radar system make use of
(a) VHF (b) HF
(c) UHF (d) EHF

41. Why is it difficult to transmit audio signals directly?
(a) A very high antenna is needed for their propagation
(b) Audio signals have a very high frequency
(c) Audio signals can't propagate independently
(d) Audio signals can't propagate through air

42. What is the range of UHF band?
(a) 30 to 400 MHz (b) 30 to 300 MHz
(c) 20 to 200 MHz (d) 20 to 300 MHz

43. Modern communication systems use
(a) analog circuits
(b) digital circuits
(c) combination of analog and digital circuits
(d) None of the above

44. Degree of modulation
(a) can take any value (b) should be less than 100%
(c) should exceed 100% (d) None of these

45. If the maximum and minimum voltage of an AM wave are V_{max} and V_{min} respectively, then modulation factor
(a) $m = \frac{V_{max}}{V_{max} + V_{min}}$ (b) $m = \frac{V_{min}}{V_{max} + V_{min}}$
(c) $m = \frac{V_{max} + V_{min}}{V_{max} - V_{min}}$ (d) $m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$

46. The AM wave is equivalent to the summation of
(a) two sinusoidal waves (b) three sinusoidal waves
(c) four sinusoidal waves (d) None of these

47. The process of recovering the audio signal from the modulated wave is known as
(a) amplification (b) rectification
(c) modulation (d) demodulation

48. What is the frequency above which radiation of electrical energy is practical?
(a) 0.2 kHz (b) 2 kHz (c) 20 kHz (d) 20 Hz

49. What is the need for doing modulation?
(a) To increase the intensity of audio signal
(b) To decrease the intensity of audio signal
(c) To transmit audio signal to large distances
(d) None of the above

50. What type of modulation is employed in India for radio transmission?
(a) Pulse modulation (b) Frequency modulation
(c) Amplitude modulation (d) None of these

51. What is the band width in amplitude modulation?
(a) Equal to audio signal frequency
(b) Two times the audio signal frequency
(c) Half the signal frequency
(d) None of the above

52. The diameter of an optical fibre is
(a) 10^{-3} cm (b) 10^{-4} cm
(c) 10^{-2} cm (d) 10^{-5} cm

53. In case of coaxial cables, the DC voltages applied at each station must be
(a) 1250 V (b) 1300 V
(c) 1350 V (d) 1400 V

54. The maximum frequency at which coaxial cables can be applied is
(a) 300 Mc/s (b) 3000 Mc/s
(c) 2500 Mc/s (d) 5000 Mc/s

55. Intelsat satellite is used for
(a) radio communication
(b) intercontinental communication
(c) radar communication
(d) None of the above

56. In which of the regions of the earth's atmosphere temperature decreases with height?
(a) Ionosphere (b) Stratosphere
(c) Troposphere (d) Mesosphere

57. The process of superimposing signal frequency (i.e. audio wave) on the carrier wave is known as
(a) transmission (b) reception
(c) modulation (d) detection

58. The sound waves after being converted into electrical waves are not transmitted as such, because
(a) they travel with the speed of sound
(b) the frequency is not constant
(c) they are heavily absorbed by the atmosphere
(d) the height of antenna has to be increased several times

59. An antenna is
(a) inductive
(b) capacitive
(c) resistive above its resonance frequency
(d) None of the above

60. The TV transmission tower in Delhi has a height of 240 m. The distance up to which the broadcast can be received (taking the radius of the earth to be 6.4×10^6 m) is
(a) 100 km (b) 60 km
(c) 55 km (d) 50 km

61. The power in a two the wire transmission line travels
(a) inside the conductors
(b) outside the conductors
(c) both inside and outside the conductors
(d) None of the above

62. In an amplitude modulated wave for audio frequency of 500 cycle/s, the appropriate carrier frequency will be?
(a) 50 cycle/s
(b) 100 cycle/s
(c) 500 cycle/s
(d) 50000 cycle/s

63. Which of the following statements is/are correct?
(a) Ground wave propagation is used for medium wave propagation
(b) Ground wave propagation is used for television broadcasting
(c) The ground wave propagation is suitable for low medium frequency upto 1.5 MHz
(d) The ground wave propagation is suitable for wavelengths greater than 200 m

64. Optical fibres are the best medium of transmission over copper wires, because
(a) the loss of data in optical fibers is very less
(b) optical fiber is cheap and easy to construct
(c) the optical fibers are made of glass and they are automatically isolated from the current
(d) the optical fiber can take large number of telephonic messages at a time

65. In amplitude modulation
(a) amplitude of carrier wave is varied about a mean value
(b) angular frequency ω_c is kept constant
(c) angular frequency ω_c is also varied
(d) amplitude of carrier wave is kept constant

66. In frequency modulation
(a) phase angle of the carrier wave is varied
(b) amplitude of the carrier wave is kept constant
(c) amplitude of carrier wave may also be varied
(d) phase angle of carrier wave is kept constant

67. FM broadcast is preferred over AM broadcast, because
(a) it is less noisy
(b) reproduction is poor
(c) it is more noisy
(d) reproduction is of much better quality

68. Demodulation is the
(a) process of decoupling of modulating wave from the carrier wave
(b) process of coupling modulated wave with modulating wave
(c) process of separating carrier wave from the modulated wave
(d) process of combining the message signal with the carrier wave

69. The process of superimposing signal frequency (i.e. audio wave) on the carrier wave is known as
(a) transmission (b) reception
(c) modulation (d) detection

70. The characteristic impedance of a coaxial cable is of the order of
(a) 50 Ω (b) 200 Ω
(c) 270 Ω (d) None of these

71. If μ_1 and μ_2 are the refractive indices of the materials of core and cladding of an optical fibre, then the loss of light due to its leakage can be minimised by having
(a) $\mu_1 > \mu_2$ (b) $\mu_1 < \mu_2$
(c) $\mu_1 = \mu_2$ (d) None of these

72. The maximum distance upto which TV transmission from a TV tower of height h can be received is proportional to
(a) $h^{1/2}$ (b) h
(c) $h^{3/2}$ (d) h^2

73. What type of modulation is employed in India for radio transmission?
(a) Amplitude modulation (b) Frequency modulation
(c) Pulse modulation (d) None of these

74. When the modulating frequency is doubled, the modulation index is halved and the modulating voltage remains constant, the modulation system is
(a) amplitude modulation
(b) phase modulation
(c) frequency modulation
(d) All of the above

75. The waves used in telecommunication are
(a) IR (b) UV
(c) Microwave (d) Cosmic rays

[Level 2]

Only One Correct Option

1. What should be the maximum acceptance angle at the air-core interface of an optical fibre, if n_1 and n_2 are the refractive indices of the core and the cladding, respectively?

(a) $\sin^{-1}(n_2/n_1)$ (b) $\sin^{-1}\sqrt{n_1^2 - n_2^2}$

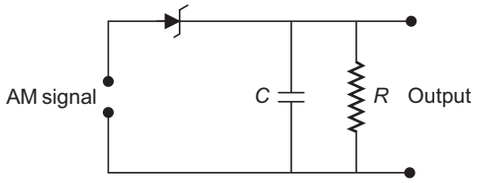
(c) $\left[\tan^{-1}\frac{n_2}{n_1}\right]$ (d) $\left[\tan^{-1}\frac{n_1}{n_2}\right]$

2. A leaf which contains only green pigments, is illuminated by a laser light of wavelength 0.6328 μm. It would appear to be

(a) brown (b) black

(c) red (d) green

3. Given below is a circuit diagram of an AM demodulator. For good demodulation of AM signal of carrier frequency f, the value of RC should be



(a) $RC = \frac{1}{f}$ (b) $RC < \frac{1}{f}$

(c) $RC \geq \frac{1}{f}$ (d) $RC >> \frac{1}{f}$

4. What maximum frequency can be reflected from ionosphere ?

(a) 5 MHz (b) 6 GHz

(c) 5 kHz (d) 500 MHz

5. Frequencies higher than 10 MHz were found not being reflected by the ionosphere on a particular day at a place. The maximum electron density of the ionosphere on the day was near to

(a) 1.5×10^{10} m^{-3}

(b) 1.24×10^{12} m^{-3}

(c) 3×10^{12} m^{-3}

(d) None of the above

6. Electromagnetic waves with frequencies greater than the critical frequency of ionosphere cannot be used for communication using sky wave propagation, because

(a) the refractive index of the ionosphere becomes very high for $f > f_c$

(b) the refractive index of the ionosphere becomes very low for $f > f_c$

(c) the refractive index of the ionosphere becomes very high for $f < f_c$

(d) None of the above

Assertion and Reason

Directions (Q. Nos. 1-12) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*

(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

(e) If both Assertion and Reason are false.

1. **Assertion** Optical fibres are free from electromagnetic disturbances.

Reason Optical fibres are electrically insulated.

2. **Assertion** Optical communication system is more economical than other systems of communications.

Reason The information carrying capacity of a communication system is directly proportional to its band width.

3. **Assertion** The electrical conductivity of the earth's atmosphere increase with altitude

Reason The high energy particles (i.e. γ-rays and cosmic rays) coming from outer space while entering the earth's atmosphere cause ionization of the atoms of the gasses present in the atmosphere.

4. **Assertion** The receiver reconstructs the original message or data after its propagation through communication channel.

Reason The reconstruction is necessary only in some cases.

5. **Assertion** Fax is a modulating and demodulating device.

Reason It is necessary for exact reproduction of a document.

6. **Assertion** The electromagnetic waves of longer wavelengths can travel longer distances on the earth's surface than those of shorter wavelengths.

Reason Velocity of wave propagation is same for all wavelengths.

7. **Assertion** Modulator or encoder is an essential component of a transmitter.

Reason The message signal has to be changed into a form suitable for transmission.

8. **Assertion** Ground wave transmission becomes weaker with increase in frequency.

Reason The ground waves can bend round the corners of the object on the earth.

9. **Assertion** Diode lasers are used as optical sources in optical communication.

Reason Diode lasers consume less energy.

10. **Assertion** In high latitude one sees colourful certains of light hanging down from high altitudes.

Reason The high energy charged particles from the sun are deflected to polar regions by the magnetic field of the earth.

11. **Assertion** Short wave bands are used for transmission of radio waves to a large distance.

Reason Short waves are reflected by ionosphere.

12. **Assertion** The electromagnetic waves of shorter wavelength can travel longer distances on the earth's surface than those of longer wavelengths.

Reason Shorter the wavelength, the larger is the velocity of wave propagation.

Entrance Gallery

2014

1. The waves used for Line of Sight (LOS) communication are **[J&K CET]**

(a) ground waves
(b) space waves
(c) sound waves
(d) sky waves

2012

2. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (radius of the earth = 6.4×10^6 m) is **[AIEEE]**

(a) 80 km (b) 16 km (c) 40 km (d) 64 km

2011

3. Which of the following four alternatives is not correct? We need modulation **[AIEEE]**

(a) to increase the selectivity
(b) to reduce the time lag between transmission and reception of the information signal
(c) to reduce the size of antenna
(d) to reduce the fractional bandwidth, that is the ratio of the signal bandwidth to the centre frequency

4. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement II The state of ionosphere varies from hour to hour, day to day and season to season. **[AIEEE]**

(a) Statement I is correct, Statement II is correct, Statement II is the correct explanation of Statement I
(b) Statement I is correct, Statement II is correct, Statement II is not the correct explanation of Statement I
(c) Statement I is incorrect Statement II is correct
(d) Statement I is correct Statement II is incorrect

5. If $E_c = 20 \sin 10^5 \pi t$ and $E_m = 10 \sin 400 \pi t$ are carrier and modulating signals respectively, the modulation index is **[Kerala CEE]**

(a) 56 % (b) 30 %
(c) 50 % (d) 48 %
(e) 60 %

6. 1000 kHz carrier wave is amplitude modulated by the signal frequency 200 Hz -4000 Hz. The channel width of this case is **[Kerala CEE]**

(a) 8 kHz (b) 4 kHz
(c) 7.6 kHz (d) 3.8 kHz
(e) 400 kHz

7. The distance of coverage of a transmitting antenna is 12.8 km. Then, the height of the antenna is (given that radius of the earth = 6400 km) **[Kerala CEE]**

(a) 6.4 m (b) 12.8 m (c) 3.2 m
(d) 16 m (e) 25.6 m

8. Which one of the following is incorrect statement in the transmission of electromagnetic waves? **[Kerala CEE]**

(a) Ground wave propagation is for high frequency transmission
(b) Sky wave propagation is facilitated by ionospheric layers
(c) Space wave is of high frequency and is suitable for line of sight communication
(d) Space wave is used for satellite communication
(e) Very high frequency waves cannot be reflected by the ionospheric layers

9. Fainted stars are called **[Karnataka CET]**

(a) dwarfs
(b) sixth magnitude stars
(c) second magnitude
(d) zero magnitude stars

10. Flash spectrum confirms a/an **[Karnataka CET]**

(a) magnetic storm (b) earthquake
(c) lunar eclipse (d) total solar eclipse

Answers

Level 1

Objective Problems

1. (d)	**2.** (d)	**3.** (a)	**4.** (d)	**5.** (a)	**6.** (a)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (b)
11. (c)	**12.** (d)	**13.** (d)	**14.** (b,d)	**15.** (c)	**16.** (d)	**17.** (c)	**18.** (d)	**19.** (d)	**20.** (d)
21. (c)	**22.** (c)	**23.** (d)	**24.** (d)	**25.** (d)	**26.** (a)	**27.** (b)	**28.** (c)	**29.** (d)	**30.** (b)
31. (c)	**32.** (c)	**33.** (d)	**34.** (b)	**35.** (c)	**36.** (b)	**37.** (b)	**38.** (d)	**39.** (a)	**40.** (c)
41. (a)	**42.** (b)	**43.** (b)	**44.** (c)	**45.** (d)	**46.** (a)	**47.** (d)	**48.** (c)	**49.** (c)	**50.** (c)
51. (b)	**52.** (b)	**53.** (c)	**54.** (b)	**55.** (b)	**56.** (c)	**57.** (c)	**58.** (d)	**59.** (a)	**60.** (c)
61. (b)	**62.** (d)	**63.** (all)	**64.** (a,c,d)	**65.** (a,b)	**66.** (a,b)	**67.** (a,d)	**68.** (a,c)	**69.** (c)	**70.** (c)
71. (a)	**72.** (a)	**73.** (a)	**74.** (c)	**75.** (c)					

Level 2

Only One Correct Option

1. (b)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (a)

Assertion and Reason

1. (a)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (d)	**7.** (a)	**8.** (b)	**9.** (b)	**10.** (a)
11. (a)	**12.** (c)								

Entrance Gallery

1. (b)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (b)	**10.** (d)

Solutions

Level 1 : Objective Problems

24. $A = 2\pi Rh = (2)(\pi)(6.4\times10^6)(150)\ \text{m}^2$

$= 1.92\pi\times10^9\ \text{m}^2 = 1.92\pi\times10^3\ \text{km}^2$

27. $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{10\times10^{-6}\times1.0\times10^{-9}}} = 1592\ \text{kHz}$

60. $d = \sqrt{2Rh} = \sqrt{2\times6.4\times10^6\times240}\ \text{m} = 55\ \text{km}$

62. Carrier frequency is always greater than modulating frequency (i.e. audio frequency), so option (d) is appropriate carrier frequency.

69. The process of superimposing signal frequency (i.e. audio wave) on carrier frequency is known as modulation.

70. The characteristic impedance of a coaxial cable is of the order of 270 Ω.

71. Refractive index of core is always greater than refractive index of cladding, to minimise the loss of light.

72. The maximum distance upto which TV transmission from TV tower of height h can be received is

$$d = \sqrt{2hR}$$

where, R is radius of the earth

$$\therefore \quad d \propto h^{1/2}$$

73. In India for radio transmission, Amplitude Modulation (AM) is used.

74. Here, modulating frequency is varying, so the modulation system is frequency modulation.

75. In telecommunication, microwaves are used.

Level 2 : Only One Correct Option

1. Acceptance angle is the maximum angle that a light ray can have relative to the axis of the fibre and propagate down the fibre. It is given by

$$\theta = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

3. For good demodulation of AM signal the value of RC (which is a time-constant) is chosen such that

$$\frac{1}{f} << RC \quad \text{or} \quad RC >> \frac{1}{f}$$

5. $$f^2 = \frac{80.6N}{\cos^2 i}$$

where, i is angle of incidence and N is electron density.

For the wave not reflected from ionosphere, $i = 0$.

$$\therefore \quad f^2 = \frac{80.6N}{\cos^2 0^\circ} = 80.6\,N \quad \text{or} \quad N = \frac{f^2}{80.6} = \frac{(10\times10^6)^2}{80.6}$$

$$= \frac{100}{80.6}\times10^{12}$$

$$= 1.24\times10^{12}\ /\text{m}^{-3}$$

Assertion and Reason

9. Diode lasers are used as optical sources in optical communication to generate analog signals or digital pulses for transmission or digital pulses for transmission through optical fibres. The advantage of diode lasers are their small size and low power input.

10. In high altitude one sees colourful certains of light hanging down from high altitudes, because the high energy charged particles from the sun are deflected to polar regions by the magnetic field of the earth, so both assertion and reason are true and reason is correct explanation of assertion.

11. Range of wavelength from 30 km to 30 km is known as short wave. These waves are used for radio transmission and for general communication purpose to a longer distance from ionosphere. Ionosphere is the outermost region of atmosphere extending from height of 80 km to 400 km approximately, above the surface of the earth. Therefore, both assertion and reason are true and reason is the correct explanation of assertion.

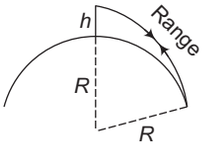
12. Electromagnetic waves of shorter wavelength do not suffer much diffraction from the obstacles of the earth's atmosphere so they can travel a long distance.

Also, shorter the wavelength, shorter is the velocity of wave propagation.

Entrance Gallery

1. Space wave is used for line of sight communication.

2. Range of radar on the earth surface (optical distance, for space wave, i.e. line of view)



$$\text{Range} = \sqrt{(R+h)^2 - R^2} = \sqrt{2Rh + h^2}$$

$$= \sqrt{2\times6400\times\frac{1}{2} + \left(\frac{1}{2}\right)^2} = 80\ \text{km}$$

3. Modulation does not change time lag between transmission and reception.

4. **Sky wave** refers to the propagation of radiowaves reflected or refracted back toward the earth from the ionosphere, an electrically charged layer of the upper atmosphere. Since, it is not limited by the curvature of the earth, sky wave propagation can be used to communicate beyond the horizon, at intercontinental distances. It is mostly used in the shortwaves frequency bands.

5. From the given equation,

$$E_m = 10\ \text{V}, E_c = 20\ \text{V} \quad \Rightarrow \quad \therefore \quad m = \frac{E_m}{E_c}\times100 = 50\%$$

6. Bandwidth $= 2\times f_m = 2\times4000 = 8000\ \text{Hz} = 8\ \text{kHz}$

7. Given, $d = 12.8\ \text{km}$, $R = 6400\ \text{km}$

We have $d = \sqrt{2hR} \quad \Rightarrow \quad h = \dfrac{d^2}{2R} = \dfrac{(12.8)^2}{2\times6400} = 12.8\ \text{m}$

8. **Ground wave propagation** is a method of radio frequency propagation that uses the area between the surface of the earth and the ionosphere for transmission. The ground wave can propagate a considerable distance over the earth's surface particularly in the low frequency and medium frequency portion of the radio spectrum.

9. Fainted stars are called sixth magnitude stars as they are visible to the naked eye. It depends on lot of stuff, like how dark the sky is how good your eyesight is and so on.

10. Flash spectrum confirms a total solar eclipse.

29

Electron Tubes

29.1 Thermionic Emission

In a metal, electrons in the outermost orbits are so loosely bound that they can be easily dislodged from the orbits and move freely within the metal. These electrons are called the **free** or **conduction electrons**. Although the free electrons can move within the metal, yet they cannot escape the metal surface due to the attraction of positive ions. The electron can escape from the metal surface, if it some how gains sufficient energy to overcome this attraction. The minimum amount of energy which must be given to an electron so that it may escape from the metallic surface is called the **work function** of that surface. The energy can be given to the electrons in the form of heat.

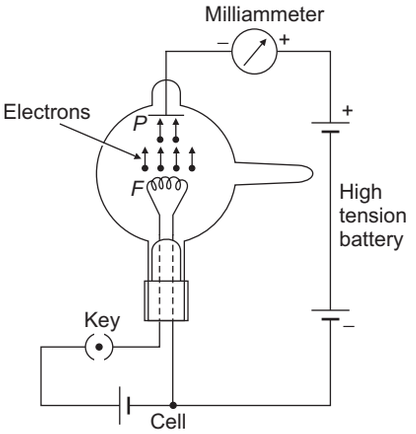


Fig. 29.1

If we heat a piece of metal to a high temperature, then electrons come out of the metal. These electrons are called **thermions**. The phenomenon of coming out of electrons by heating a metal surface is called **thermionic emission**. It was first observed in 1885 by **Edison.** So, it is also called **Edison effect**.

Thermionic Current

If we heat a filament *F* by a cell as shown in Fig. 29.1 electrons are emitted from it. These electrons can be attracted by a plate *P* kept at higher temperature with respect to the filament *F* by using a high tension battery. When the electrons emitted from *F* reach the

Chapter Snapshot

- Thermionic Emission
- Diode Valve
- Diode as Rectifier
- Triode Valve
- Triode Characteristics
- Uses of Triode
- Triode as an Amplifier

plate P circuit is completed and a current starts flowing in the external circuit. This current called the **thermionic current** can be measured by the milliammeter joined in series. If we go on increasing the positive potential of plate P, the thermionic current rises to a maximum value. This maximum current is called **saturation current**. Thermionic current reaches a saturation value, because all the electrons emitted from the filament reaches the plate. The value of the saturation current depends upon the temperature of the filament and increases with rise in temperature. **Richardson** stated that the relation between the saturation current i per unit area of filament and the temperature T of the filament can be expressed by the equation,

$$i = AT^2 \, e^{-b/T}$$

Here, A and b are constants for the given filaments.

The increase in saturation current (i) with rise in temperature (T) according to Richardson's equation is shown in Fig. 29.2.

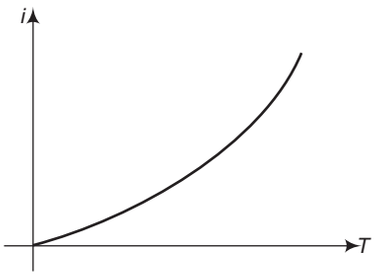


Fig. 29.2

Thermionic Emitters (Cathodes) The surface that acts as an electron emitter is called the cathode. The electron emission from a cathode depends on the temperature of the cathode and work function of the material of the cathode. The most commonly used cathodes are :

(i) **Tungsten** They have high melting point. The operating temperature is 2500 K and the work function is 4.52 eV. They are used in high voltage vacuum tubes.

(ii) **Thoriated tungsten** A layer of thorium is deposited on tungsten base. They have relatively low work function of 2.63 eV and work at 1900 K. They are not used at high voltages, because they are damaged by positive ion bombardment.

(iii) **Oxide coated cathode** They are the most efficient cathodes. They are prepared by coating a layer of mixture of barium and strontium oxide on a nickel alloy base. They have a very low work function of 1 eV and work at 1000 K. These are extensively used in small vacuum tubes, where plate potential does not exceed 1000 V.

- Cathodes are heated electrically, either directly or indirectly.

29.2 Diode Valve

The diode valve was invented by **Fleming** in 1904. It is a device in which electric current flows in high vacuum.

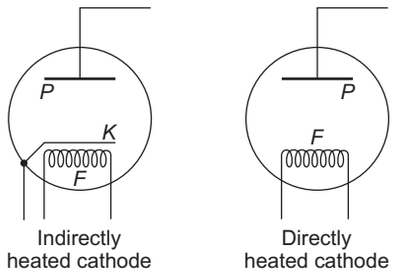


Fig. 29.3

A diode valve has a cathode, from which electrons are emitted by heating it either directly or indirectly. These electrons are collected by an anode. Both the anode and cathode are sealed in an evacuated glass bulb. The cathode can be denoted by (K) and the anode by plate (P).

In directly heated cathode a pure tungsten wire or a thorium coated tungsten wire is used as a cathode. This wire is directly connected to a heating battery.

In indirectly heated cathode a hollow nickel tube coated with barium oxide is used as the cathode. This tube surrounds a heating coil. When current is passed through this coil nickel tube gets heated indirectly and electrons are emitted. Symbols of directly and indirectly heated cathodes are shown in figure. In directly heated cathode filament (F) itself is cathode (K).

Working of Diode The circuit diagram of a diode is as shown in figure. Cathode is heated by passing current through the filament. After being heated cathode emits electrons. If the plate potential is kept positive with respect to the cathode, the electrons emitted by cathode are attracted by the plate. The circuit is completed and a current i_p flows from plate P to cathode C. On increasing the plate potential the plate current increases and finally attains a steady value. This steady current is called saturation current. If the plate P is given negative potential relative to the cathode, the electrons emitting from the cathode will be repelled and will not reach the plate. So, no current will flow in the valve.

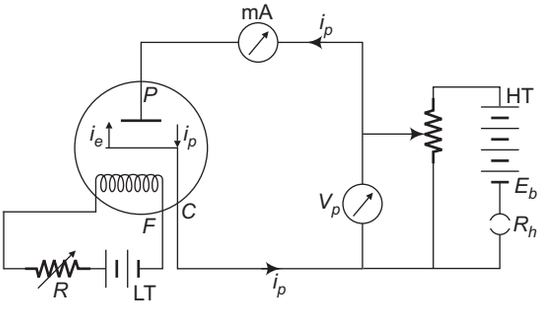


Fig. 29.4

(Although with zero, or even a small negative potential on the plate, a small current flows in the valve. It is so because a few of the electrons emerging from the filament have enough kinetic energy to reach the plate in spite of repulsion). Thus, diode allows current flow only in one direction, i.e. only when the plate is positive with respect to cathode. This is why it is called valve, and since it has two electrodes, it is called **diode valve**.

Diode Characteristics A graph representing the variation of plate current i_p with plate voltage V_p is known as characteristic curve of a diode. As the plate potential increases, the plate current rises to a maximum value. This maximum value of current (called the saturation current) corresponds to the plate potential when all the electrons emitted by the filament reach the plate. In the region AB, before the current reaches its saturation value, the relation between i_p and V_p may be represented by the equation,

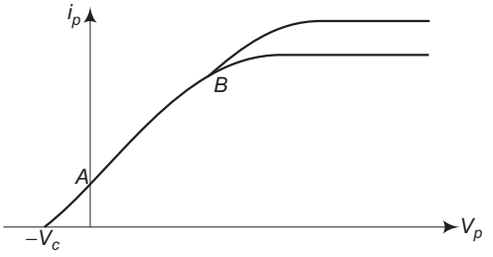


Fig. 29.5

$$i_p = kV_p^{3/2}$$

Here, k is a constant whose value depends upon the geometry of the diode valve. This equation is called **Child's equation**.

On applying negative potential to the plate, the plate current becomes zero at a particular negative potential. In this condition, no electrons reach the plate.This (negative) value of the plate potential is called the **cut off voltage**. It determines the maximum kinetic energy of the emitted electrons. Thus,

$$K_{\max} = \frac{1}{2} mv_{\max}^2 = e|V_c|$$

Note Points

- The current potential graph for a diode is non-linear i.e. the diode does not obey Ohm's law. Hence, the resistance of the diode valve is non-ohmic.
- The saturation current can be increased by increasing number of electrons emitting from the cathode per second. These numbers can be increased by increasing temperature of the filament.

Dynamic Plate Resistance Suppose the diode is operated at a plate voltage V_p and the plate current is i_p. Now, the plate voltage V_p is changed by a small amount ΔV_p. Consequently the current will change by a small amount Δi_p. The dynamic plate resistance r_p is then defined as,

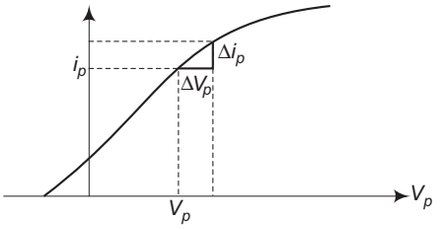


Fig. 29.6

$$r_p = \frac{\Delta V_p}{\Delta i_p}$$

- That value of r_p is not constant (unlike the resistance of a ohmic conductor) for a diode valve but it depends on the operating conditions (V_p, i_p) on characteristic curve.

Example 29.1 *When the plate voltage applied to a diode valve is changed from* 30 *V to* 31 *V, the plate current increases from* 40 *mA to* 45 *mA. Find the dynamic plate resistance at this operating condition.*

Sol. $\Delta V_p = (31 - 30)\,\text{V} = 1\,\text{V}$

$\Delta i_p = (45 - 40) \times 10^{-3}\ \text{A} = 5 \times 10^{-3}\ \text{A}$

$\therefore \quad r_p = \dfrac{\Delta V_p}{\Delta i_p} = \dfrac{1}{5 \times 10^{-3}}\ \Omega = 200\ \Omega$

29.3 Diode as Rectifier

When an alternating source of current is applied across a resistance, direction of current passing through the resistance keep on changing. By using a diode in the circuit we can have unidirectional current through the resistance. The conversion of AC into DC is called rectification. Diode(s) used for this purpose is (are) called **rectifier**. Rectifiers are of two types :

(i) **Half Wave Rectifier** A single diode acts as half wave rectifier. The required circuit containing an AC source, a diode and a resistor is shown in figure.

The AC signal to be rectified is connected in the primary P of a step up transformer. The terminal A of the secondary coil is connected to the plate P of the diode valve while the cathode K of the valve is connected to the terminal B of the secondary through the load R_L. The DC output is obtained across the load.

Working When the point A is at a higher potential then that at B, the anode is also at higher potential then that of the cathode. The diode allows current to pass through. Thus, the AC source sends a current which goes through the diode, through the resistor

and then back to the source. The current in the resistor is from a to b. When the potential at A becomes less than that at B, the diode does not allow a current. There is no current in the resistance in this case. Thus, it is clear that the current in the load flows for each positive half cycle of AC input signal but no current for the negative half cycle of AC input. Thus, only half of the AC is rectified, hence it is called **half wave rectifier**. We see that current in the resistor is always from a to b.

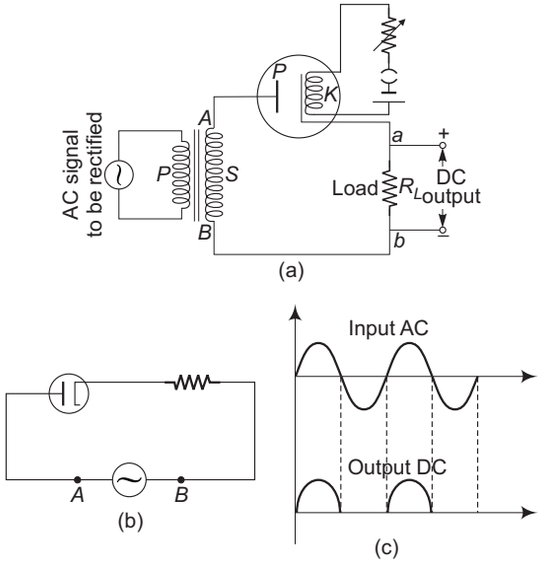


Fig. 29.7

In the output, besides DC some AC component is also obtained. These AC components cause fluctuations in the output and are known as ripples. These fluctuations can be removed by using filter circuits.

(ii) **Full Wave Rectifier** A full wave rectifier uses two diodes and gives a continuous supply. The circuit diagram for full wave rectifier is shown in figure. The AC input is connected to the primary P of the step up transformer. The terminals of the secondary are connected to the anodes of two diodes D_1 and D_2. The two cathodes are connected to one end of a resistor. The other end of the resistor is connected to the centre C of the secondary of the transformer.

Working During positive half of the cycle A has higher potential with respect to C and B has lower potential with respect to C. The diode D_1 conducts but D_2 does not. So, a current passes through D_1 and then through the resistor back to the secondary. The current in the resistor is from a to b. In the next half cycle, the potential of B is higher than C and that of C is higher than A. The diode D_2 conducts but D_1 does not. The current passes through D_2 and then through the resistor back to the secondary. Again the current in the resistor is from a to b. So, the current in the resistor is from a to b in both the half cycles. The input and output waves are as shown in figure.

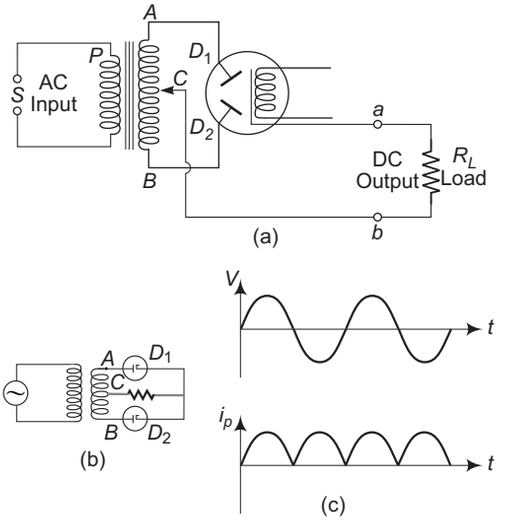


Fig. 29.8

29.4 Triode Valve

Triode valve was invented by **Lee De Forest** in 1907. It is similar in construction to a diode valve except that a wire grid is inserted between the cathode and the anode. Thus, the triode valve consists of three electrodes. (i) cathode, (ii) plate and (iii) grid enclosed in an evacuated glass tube.

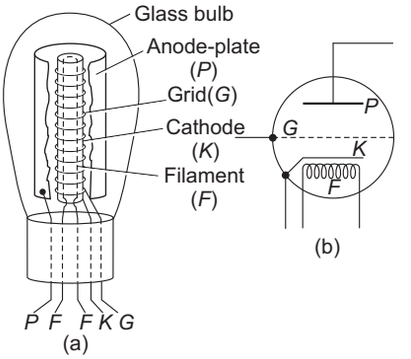


Fig. 29.9

When the cathode is heated, it emits thermions. These thermions pass through the holes in the grid and reach the anode. If the grid is kept at zero potential the triode works like a diode. The grid controls the current in the plate circuit. When it is given a positive potential relative to the cathode, it helps the electrons in reaching the plate. Contrary to this when the grid is given a negative potential relative to the cathode, it opposes the electrons moving towards the plate. Thus the plate current can be controlled by means of grid.

Hence, the grid is also called the **controlling electrode**.

Since, grid is nearer the cathode than the plate, therefore a small change in grid potential can bring about a large

change in the plate current. In absence of grid, the current is roughly given by $i_p = kV_p^{3/2}$. When the grid is added, the current is given by,

$$i_p = k\,(V_p + \mu V_g)^{3/2}$$

Here, k is a constant and μ is amplification factor of the given valve. It may be of the order of 10. The grid voltage V_g has the same effect as a plate voltage μV_g.

Further, $i_p = 0$ at $V_g = -\dfrac{V_p}{\mu}$. This value of V_g at which plate current becomes zero is called **cut off grid potential**.

29.5 Triode Characteristics

Curves showing relation between V_p, V_g and i_p are called characteristic curves of a triode valve. Two curves are mainly important. One is $i_p - V_p$ curves at constant V_g and another is $i_p - V_g$ curve at constant V_p.

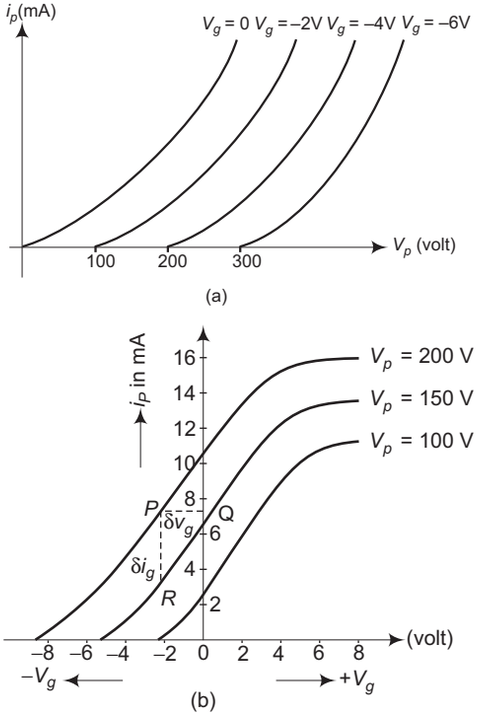


Fig. 29.10

The first curve is called **anode characteristic curve** and the second is called **mutual characteristic curve.** Plate characteristic curves of a triode at $V_g = 0\text{V}, -2\text{V}, -4\text{V}\ldots$ are shown in Fig. (a).

Fig. (b) shows the mutual characteristic curves. Mutual characteristic curves have the following important features:

(i) Higher the plate potential, higher is the cut-off grid voltage.

(ii) Higher the plate potential, larger is the plate current for any given grid potential.

(iii) The central portion of each curve is linear. In this part of curve equal change in grid potential causes equal change in plate current (for each curve).

Characteristic Constants There are three constants of a triode valve :

(i) Amplification factor

(ii) Internal resistance

(iii) Mutual conductance.

Now let us discuss them separately.

Amplification Factor (μ) If the grid voltage is increased, the plate current i_p also increases. The current can be brought back to i_p by decreasing the plate voltage V_p. Amplification factor is defined as the ratio of a small change in plate potential to a change in grid potential in the opposite direction so that the plate current remains the same. Thus,

$$\mu = \left(-\frac{\Delta V_p}{\Delta V_g}\right)_{\Delta i_p = 0}$$

The amplification factor can be understood in a different way as under.

The plate current can be increased either by increasing plate potential or grid potential. Since, the grid is nearer to the cathode a small change in grid potential produces the same change in the plate current as would do a much larger change in plate potential.

The ratio of ΔV_p (while V_g is constant) to ΔV_g (while V_p is constant) for a definite increase in the plate current is called the **amplification factor** of the triode valve. Thus,

$$\mu = \frac{\Delta V_p}{\Delta V_g} \quad \text{(for same amount of increase in } i_p\text{)}$$

That value of μ is always greater than 1. It has no unit.

Internal Resistance (r_p) If the grid voltage is constant and the plate voltage is changed, the plate current changes.

The ratio of the change in plate potential (ΔV_p) to the change in plate current (Δi_p), when the grid potential remains constant is defined as the **internal resistance** or the **plate resistance** r_p of the triode. Thus,

$$r_p = \frac{\Delta V_p}{\Delta i_p} \quad \text{(while } V_g \text{ is constant)}$$

Unit of r_p is ohm, but it is of the order of kilo ohm (kΩ).

Mutual Conductance (g_m) If the plate voltage V_p is kept constant and the grid voltage V_g is changed, the plate current changes. The ratio of change in plate current (Δi_p) to the corresponding change in grid potential (ΔV_g), when plate potential remains constant is defined as the **mutual conductance** of the triode valve.

Thus, $$g_m = \frac{\Delta i_p}{\Delta V_g} \qquad \text{(while } V_p \text{ is constant)}$$

Unit of g_m is ohm^{-1} or mho.

Relation between Three Constants The relation between three constants can be derived by simple mathematics as,

$$\mu = \frac{\Delta V_p}{\Delta V_g} = \frac{\Delta V_p}{\Delta i_p} \times \frac{\Delta i_p}{\Delta V_g} = r_p \times g_m$$

$$\therefore \quad \mu = r_p \times g_m$$

Example 29.2 *The mutual conductance of a triode valve is* 5×10^{-3} *ohm*$^{-1}$. *Find the change in plate current, if the grid voltage is changed from* –3.0 *V to* –5.0 *V.*

Sol. $g_m = \left(\frac{\Delta i_p}{\Delta V_g}\right)_{\Delta V_p = 0}$

$$\therefore \quad \Delta i_p = g_m \, \Delta V_g$$
$$= (5 \times 10^{-3})\,(-5.0 + 3.0)\,\text{A} = -10^{-2}\ \text{A}$$

Example 29.3 *The plate resistance and the amplification factor of a triode are* 10 *k*Ω *and* 20. *The tube is operated at plate voltage* 250 *V and grid voltage* –7.5 *V. The plate current is* 10 *mA.*

(a) To what value should the grid voltage be changed so as to increase the plate current to 15 *mA?*

(b) To what value should the plate voltage be changed to take the plate current back to 10 *mA?*

Sol. Given, $r_p = 10 \times 10^3\ \Omega$ and $\mu = 20$

$$\therefore \quad g_m = \frac{\mu}{r_p} = \frac{20}{10 \times 10^3}$$
$$= 2.0 \times 10^{-3}\ \text{ohm}^{-1}$$

(a) $g_m = \frac{\Delta i_p}{\Delta V_g}$

$$\therefore \quad \Delta V_g = \frac{\Delta i_p}{g_m}$$
$$= \frac{(15 - 10) \times 10^{-3}}{2.0 \times 10^{-3}} = 2.5\ \text{V}$$

∴ New grid potential should be –5.0 V

(b) $\mu = \left(-\frac{\Delta V_p}{\Delta V_g}\right)$

$$\therefore \quad (-\Delta V_p) = \mu\ \Delta V_g = (20)\,(2.5)\ \text{V} = 50\ \text{V}$$

29.6 Uses of Triode

Triode can be used as

(i) **An Oscillator** A triode as an oscillator can be used to generate electromagnetic waves of constant amplitude, which can be used in broadcasting.

(ii) **An Amplifier** Amplification means building up of an AC signal.

(iii) **A Modulator** The function of modulator is to superimpose sound waves over radio waves for their broadcasting.

(iv) **A Demodulator** A demodulator separates the audio waves from the modulated waves and send into a loudspeaker.

29.7 Triode as an Amplifier

A triode can be used to convert weak AC signals into strong AC signals. In this case the triode acts as an amplifier.

The amplification by a triode is based on the fact that a small change in grid voltage causes a large change in plate current.

The arrangement for using triode as an amplifier is shown in the figure.

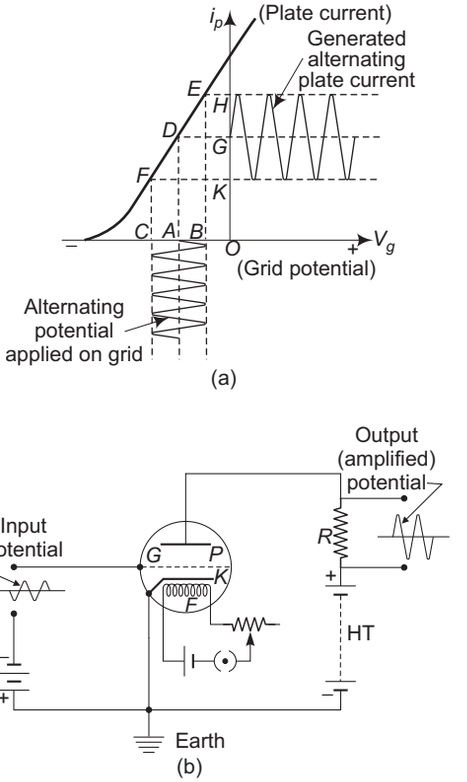


Fig. 29.11

A load resistance R is introduced in the plate circuit. A DC battery of low voltage (LT) gives a constant negative potential to the grid. The value of V_g (with no AC input signal) and V_p are so adjusted that the operating point lies approximately at the mid point of the straight part of the mutual characteristic curve. On the grid potential, the input AC signal (to be amplified) is superimposed. Due to this net grid potential oscillates alternately. These variations in grid potential cause corresponding variations in the plate current. This in turn sets

up alternating voltage across R. The output voltage is also alternating but with a greater amplitude. The additional energy required for the amplified voltage is drawn from the HT battery.

Amplifiers are used for reproduction of sound. The sound waves are converted into fluctuating electric currents, but these currents are very weak. These currents are amplified by an amplifier and then fed into a loudspeaker, which converts them again into (loud) sound.

Voltage Amplification (A) The ratio of output voltage to the input voltage of an amplifier is called the **voltage amplification**.

$$A = \frac{\Delta V_o}{\Delta V_i}$$

Here, $\Delta V_i = \Delta V_g =$ change in input potential

and $\Delta V_g = \Delta V_p / \mu$

$\therefore$ $\Delta V_i = \Delta V_p / \mu$

Also, $\Delta V_o = R.\Delta i_p =$ change in output signal

where $\Delta i_p = \dfrac{\Delta V_p}{R + r_p}$ (Here, $r_p =$ plate resistance)

$$\therefore \quad A = \frac{R\left(\dfrac{\Delta V_p}{R + r_p}\right)}{(\Delta V_p / \mu)} \quad \text{or} \quad A = \frac{\mu}{1 + \dfrac{r_p}{R}}$$

From this expression, we can see that value of A increases as R increases.

$$A_{\max} = \mu \quad \text{as} \quad R \to \infty$$

Thus, A cannot be greater than μ.

Note Points

- Normally one amplifier is not sufficient for the required level of amplification. For this a series of amplifiers of individual voltage amplifications A_1, A_2 ... etc., are used. The net voltage amplification in this case is given by
  $$A = A_1 \times A_2 \times \ldots$$
- Difference between a step up transformer and a triode amplifier is that the transformer does not amplify the power. If voltage is increased, then current is decreased in the same proportion. While in case of an amplifier, not only voltage is amplified but power is also amplified at the cost of DC power. This is the reason we use triode amplifier not a step-up transformer.
- The phase difference between input and output is π.

Additional Examples

Example 1. *The slope of the anode current grid potential curve of a triode is* 2 *mA/V and the slope of the anode current-anode potential curve is* 0.25 *mA/V. Calculate the voltage amplification that this triode will give across an anode resistance of* 12 *k*Ω.

Sol. Given $\dfrac{di_p}{dV_g} = (2 \times 10^{-3})\,\dfrac{\text{A}}{\text{V}}$

and $\dfrac{di_p}{dV_p} = (0.25 \times 10^{-3})\,\dfrac{\text{A}}{\text{V}}$

$\therefore$ $r_p = \dfrac{dV_p}{di_p}$

$$= \frac{1}{0.25 \times 10^{-3}} = 4\ \text{k}\Omega$$

$$\mu = \frac{dV_p}{dV_g} = \frac{(di_p/dV_g)}{(di_p/dV_p)}$$

$$= \frac{2 \times 10^3}{0.25 \times 10^{-3}} = 8$$

Therefore, voltage amplification

$$A = \frac{\mu}{1 + \dfrac{r_p}{R}} = \frac{8}{1 + \dfrac{4}{12}} = 6$$

Example 2. *The following readings were obtained from the linear portions of the static characteristics of a vacuum triode.*

V_p *(in volt)*	i_p *(in mA)*	V_g *(in volt)*
150	12	−1.5
150	5	−3
100	7.5	−1.5

Find

(a) Plate resistance,

(b) Mutual conductance and

(c) Amplification factor.

Sol. (a) $r_p = \dfrac{\Delta V_p}{\Delta i_p}$

$$= \frac{(100 - 150)\ \text{V}}{(7.5 - 12) \times 10^{-3}\ \text{A}}$$

(when V_g is constant at -1.5 V)

$$= \frac{-50}{-4.5} \times 10^3\ \Omega = 11.1\ \text{k}\Omega$$

(b) $g_m = \dfrac{\Delta i_p}{\Delta V_g} = \dfrac{(5 - 12) \times 10^{-3}\ \text{A}}{(-3 + 1.5)\ \text{V}} = 4.67\ \text{mW}^{-1}$

(c) $\mu = r_p \times g_m = 51.8$

Example 3. *In a triode valve, for grid voltage $V_g = -1.2V$, the plate current i_p and plate voltage V_p are given by the relation $i_p = -5.0 + 0.1V_p$, where i_p in mA and V_p in volt. When the grid voltage is changed to $-3.2V$ and plate voltage is kept as $150V$, a plate current of 5 mA is observed. Calculate the valve constants and voltage amplification for $20\,k\Omega$ load in plate circuit.*

Sol. Given, $i_p = -5.0 + 0.1\,V_p$ at $V_g = -1.2$ V

Differentiating this equation we have,

$$\frac{di_p}{dV_p} = 0.1\,\frac{\text{mA}}{\text{V}} = 10^{-4}\ \Omega^{-1}$$

or $$r_p = \frac{dV_p}{di_p} = 10^4\ \Omega$$

Putting $V_p = 150$ V in the given equation we have,

$i_p = 10$ mA (at $V_g = -1.2$ V)

Again it is given that $i_p = 5$ mA,

when $V_g = -3.2$ V at $V_p = 150$ V.

$$\therefore \quad g_m = \left(\frac{di_p}{dV_g}\right)_{V_p\ =\ \text{constant}}$$

$$= \frac{(5-10)\times 10^{-3}}{(-3.2+1.2)}$$

or $$g_m = 2.5\times 10^{-3}\ \Omega^{-1}$$

Amplification factor,

$$\mu = g_m \times r_p = 25$$

Voltage amplification,

$$A = \frac{\mu}{1+\dfrac{r_p}{R}} = \frac{25}{1+\left(\dfrac{10^4}{2\times 10^4}\right)} = 16.67$$

Example 4. *A triode valve has amplification factor 30 and dynamic plate resistance $15\ k\Omega$. This is used as an amplifier with a load of $30\ k\Omega$. Find the voltage amplifications A of the triode.*

Sol. The voltage amplification A is given by,

$$A = \frac{\mu}{1+\dfrac{r_p}{R}} = \frac{30}{1+\dfrac{15}{30}}$$

$$= 20$$

Example 5. *A triode has mutual conductance $= 2.0\ m\Omega^{-1}$ and plate resistance $= 20\,k\Omega$. It is desired to amplify a signal by a factor 30. What load resistance should be added in the circuit?*

Sol. Given, $g_m = 2\times 10^{-3}\ \text{W}^{-1}$

$$r_p = 20\times 10^3\ \Omega$$

$$\therefore \quad \mu = r_p \times g_m = 40$$

Further, $$A = \frac{\mu}{1+\dfrac{r_p}{R}}$$

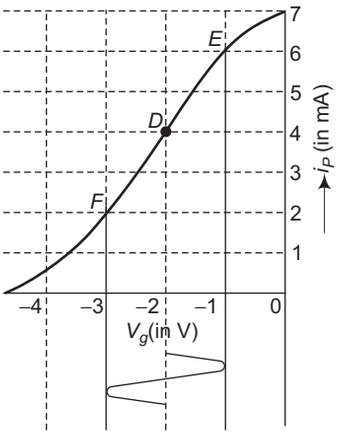
or $$30 = \frac{40}{1+\dfrac{20\times 10^3}{R}}$$

Solving this equation, we get

$$R = 60\times 10^3\ \Omega$$

or $$R = 60\ \text{k}\Omega$$

Example 6. *The mutual characteristic curve of a triode valve is shown in the figure. The load resistance in the plate circuit of the triode is $25\ k\Omega$ and its grid potential is -2 V. On applying an alternating potential of amplitude 1 V, find*



(a) the amplitude of alternating part of the plate current.

(b) the value of voltage-amplification.

Sol. (a) Constant applied grid potential V_g is -2 V. Amplitude of alternating part of the plate-current is 1 V. Therefore, V_g varies from -1 V to -3 V. Accordingly plate current i_p varies between 2 mA and 6 mA. The central value of i_p is 4 mA.

Hence, the amplitude of the alternating part of the plate current is 2 mA.

(b) The amplitude of output voltage

= (load resistance) × (amplitude of alternating of plate current)

$$= (25\times 10^3)\times(2\times 10^{-3}) = 50$$

$$\therefore \text{ Voltage amplification} = \frac{\text{output voltage}}{\text{input voltage}} = \frac{50}{1} = 50$$

Example 7. *The plate current in a triode can be written as*

$$i_p = k\left(V_g + \frac{V_p}{\mu}\right)^{3/2}$$

Prove that the mutual conductance is proportional to the cube root of the plate current.

Sol. From the definition of mutual conductance,

$$g_m = \left(\frac{di_p}{dV_g}\right)_{V_p \text{ = constant}}$$

Differentiating the given equation, w.r.t. V_g, keeping V_p to be constant we have,

$$g_m = \frac{3}{2}k\left(V_g + \frac{V_p}{\mu}\right)^{1/2} \quad ...(i)$$

From the given equation,

$$\left(V_g + \frac{V_p}{\mu}\right)^{\frac{1}{2}} = \left(\frac{i_p}{k}\right)^{\frac{1}{3}}$$

Substituting this in Eq. (i), we can see that

$$g_m \propto (i_p)^{\frac{1}{3}}$$

Example 8. *Show that the dynamic plate resistance of a diode is* $\frac{2V}{3i}$*, where V and i are the plate voltage and the plate current respectively. Assume Langmuir-Child equation to hold.*

Sol. The Langmuir-Child equation is,

$$i = kV^{3/2} \quad (k = \text{constant}) \quad ...(i)$$

$$\therefore \quad \frac{di}{dV} = \frac{3}{2}kV^{1/2} \quad ...(ii)$$

Dividing Eq. (i) by Eq. (ii), we have

$$i \cdot \frac{dV}{di} = \frac{2}{3}V$$

$\therefore$ Dynamic plate resistance $R = \frac{dV}{di} = \frac{2V}{3i}$

Example 9. *The internal resistance of a triode valve is* $1.0 \times 10^4\ \Omega$ *and the amplification factor is* 15*. What is the mutual conductance of the valve ? How much change in grid potential (at constant-potential) is required to increase the plate current from* 6 *mA to* 9 *mA ?*

Sol. Given, $r_p = 10^4\ \Omega$

and $\mu = 15$

Mutual conductance $= g_m = \frac{\mu}{r_p} = \frac{15}{10^4}$ mho

or $g_m = 1.5 \times 10^{-3}\ \Omega^{-1}$

Further, $g_m = \left(\frac{\Delta i_p}{\Delta V_g}\right)_{V_p \text{ = constant}}$

$\therefore \quad \Delta V_g = \frac{\Delta i_p}{g_m}$

$$= \frac{(9.0 - 6.0) \times 10^{-3}}{1.5 \times 10^{-3}}$$

$$= 2.0 \text{ V}$$

Example 10. *Calculate* μ *and* r_p *for a triode valve from the following observations.*

V_p (volt)	V_g (volt)	i_p (mA)
200	–1.0	5
200	–1.5	2
250	–1.0	9.8
250	–1.8	5
250	–2.3	2

Sol. $\mu = \left(\frac{\Delta V_p}{\Delta V_g}\right)$

For same amount of change in plate current

$$= \left[\frac{250 - 200}{-1.0 + 1.8}\right]$$

For change in current from 5mA to 9.8 mA

$$= \frac{50}{0.8} = 62.5$$

$$r_p = \left(\frac{\Delta V_p}{\Delta i_p}\right)_{V_g \text{ = constant}}$$

$$= \left[\frac{250 - 200}{(9.8 - 5.0) \times 10^{-3}}\right]$$

when V_g is constant at -1.0 V $= \frac{50}{4.8 \times 10^{-3}}\ \Omega$

$$= 10.4 \times 10^3\ \Omega = 10.4\ \text{k}\Omega$$

Objective Problems

[Level 1]

1. Thermionic emission from a heated filament varies with its temperature T as
(a) T^{-1} (b) T
(c) T^2 (d) $T^{3/2}$

2. The grid voltage of any triode valve is changed from –1 V to –3 V and the mutual conductance is 3×10^{-4} mho. The change in plate circuit current will be
(a) 0.8 mA (b) 0.6 mA
(c) 0.4 mA (d) 1 mA

3. In a triode, $g_m = 2 \times 10^{-3}\ \Omega^{-1}$, $\mu = 42$, load resistance, $R = 50\,\text{k}\Omega$. The voltage amplification obtained from this triode will be
(a) 30.42 (b) 29.57
(c) 28.18 (d) 27.15

4. If $R_p = 7\,\text{k}\Omega, g_m = 2.5$ milli-mho, then on increasing plate voltage by 50 V, how much the grid voltage is changed so that plate current remains the same
(a) –2.86 V (b) – 4 V
(c) +4 V (d) +2 V

5. In a triode amplifier, $\mu = 25, r_p = 40$ kΩ and load resistance $R_L = 10\,\text{k}\Omega$. If the input signal voltage is 0.5 V, then output signal voltage will be
(a) 1.25 V (b) 5 V
(c) 2.5 V (d) 10 V

6. The amplification factor of a triode is 20. If the grid potential is reduced by 0.2 V, then to keep the plate current constant its plate voltage is to be increased by
(a) 10 V (b) 4 V
(c) 40 V (d) 100 V

7. For a triode $r_p = 10\,\text{k}\Omega$ and $g_m = 3$ milli-mho. If the load resistance is double of plate resistance, then the value of voltage gain will be
(a) 10 (b) 20
(c) 15 (d) 30

8. For a given triode $\mu = 20$. The load resistance is 1.5 times the anode resistance. The maximum gain will be
(a) 16 (b) 12
(c) 10 (d) None of these

9. The amplification factor of a triode valve is 15. If the grid voltage is changed by 0.3 V, the change in plate voltage in order to keep the plate current constant (in volt) is
(a) 0.02 (b) 0.002 (c) 4.5 (d) 5.0

10. In the circuit of a triode valve, there is no change in the plate current, when the plate potential is increased from 200 V to 220 V and the grid potential is decreased from –0.5 V to –1.3 V. The amplification factor of this valve is
(a) 15 (b) 20
(c) 25 (d) 35

11. The unit of mutual conductance of a triode valve is
(a) siemen
(b) ohm
(c) ohm metre
(d) joule coulomb^{-1}

12. With a change of load resistance of a triode, used as an amplifier, from 50 kΩ to 100 kΩ, its voltage amplification changes from 25 to 30. Plate resistance of the triode is
(a) 25 kΩ (b) 75 kΩ
(c) 7.5 kΩ (d) 2.5 kΩ

13. Correct relation for triode is
(a) $\mu = g_m \times r_p$ (b) $\mu = \dfrac{g_m}{r_p}$
(c) $\mu = 2g_m \times r_p$ (d) None of these

14. Plate resistance of two triode valves is 2 kΩ and 4 kΩ, amplification factor of each of the valves is 40. The ratio of voltage amplification, when used with 4 kΩ load resistance, will be
(a) 10 (b) $\dfrac{4}{3}$
(c) $\dfrac{3}{4}$ (d) $\dfrac{16}{3}$

15. In a triode valve, the amplification factor is 20 and mutual conductance is 10^{-3} mho. The plate resistance is
(a) $2 \times 10^3\ \Omega$ (b) $4 \times 10^3\ \Omega$
(c) $2 \times 10^4\ \Omega$ (d) $2 \times 10^4\ \Omega$

16. The amplification factor of a triode is 50. If the grid potential is decreased by 0.20 V, what increase in plate potential will keep the plate current unchanged?
(a) 5 V (b) 10 V
(c) 0.2 V (d) 50 V

17. The correct relation for a triode is
(a) $g_m = \left.\dfrac{\Delta I_p}{\Delta V_p}\right|_{V_g = \text{constant}}$ (b) $g_m = \left.\dfrac{\Delta I_p}{\Delta V_g}\right|_{V_p = \text{constant}}$
(c) Both 'a' and 'b' (d) None of these

18. Select the correct statement.
(a) In a full wave rectifier, two diodes work alternately
(b) In a full wave rectifier, two diodes work simultaneously
(c) The efficiency of full wave and half wave rectifiers is same
(d) The full wave rectifier is bi-directional

19. Plate current in a triode valve will be maximum when
(a) grid voltage will be made – ve and plate voltage + ve
(b) grid voltage will be made + ve and plate voltage – ve
(c) grid voltage = 0 and plate voltage – ve
(d) grid voltage and plate voltage + ve

20. Plate voltage of a triode is increased from 200 V to 225 V. To maintain the plate current, change in grid voltage from 5.75 V to 5.0 V is needed. The amplification factor is
(a) 40 (b) 45 (c) 33.3 (d) 25

21. When triode is used as an amplifier, the phase difference between input voltage and output voltage is
(a) π (b) $\pi/2$ (c) $3\pi/2$ (d) $\pi/4$

22. An AC signal of 50 Hz frequency is input of a full wave rectifier using two diodes. The output frequency after full wave rectification is
(a) zero (b) 50 Hz
(c) 100 Hz (d) 200 Hz

23. In diode when there is saturation current, then the plate resistance r_p
(a) is zero
(b) is infinite
(c) is some finite quantity
(d) cannot say as data is insufficient

24. A diode can be used for
(a) modulation (b) amplification
(c) rectification (d) All of these

25. On bringing the grid nearer to the plate in triode valve, its amplification factor
(a) becomes zero (b) increases
(c) decreases (d) remains unchanged

26. The amplification factor of a triode valve depends upon
(a) temperature of the plate
(b) the plate voltage
(c) temperature of the cathode
(d) relative positions of cathode, grid and the plate

27. In a vacuum tube diode, the plate current (I) and the plate voltage (V) in the space charge limited region are related as
(a) $I \propto V^{3/2}$ (b) $I \propto V^{2/3}$ (c) $I \propto V$ (d) $I \propto V^2$

28. The ratio of the slope of the mutual characteristic and the anode characteristic for a triode valve is
(a) mutual conductance (b) plate resistance
(c) plate conductance (d) amplification factor

29. What is the limitation to the use of thoriated tungsten emitter?
(a) Deactivation, because of negative ion bombardment
(b) The temperature of the operation of the tube
(c) Deactivation, because of positive ion bombardment
(d) Cannot be used at low voltages

30. Why do we prefer indirectly heated cathode to the directly heated cathode?
(a) Equality of potential throughout the cathode
(b) Continuous emission of electrons
(c) Availability of filament material
(d) Due to some other reasons

31. If I_1 and I_2 be the currents in a diode under space-charge limited condition for plate voltages of 400 V and 200 V respectively, then the ratio I_1/I_2 is
(a) $\sqrt{2}$ (b) 2
(c) $2\sqrt{2}$ (d) 4

32. The use of the grid in a triode valve is
(a) to help in the emission of more electrons from cathode
(b) to control plate to cathode current
(c) to reduce the space charge
(d) None of the above

33. A diode can convert
(a) AC into DC but not DC into AC
(b) DC into AC but not AC into DC
(c) AC into DC and DC into AC
(d) Neither AC into DC nor DC into AC

34. In a diode, the plate current I_p will be zero when the potential is
(a) zero (b) slightly positive
(c) slightly negative (d) largely positive

35. The slopes of anode and mutual characteristics for a triode valve are 0.02 mA/V and 1.0 mA/V respectively. The amplification factor of the valve is
(a) 0.5 (b) 5 (c) 50 (d) 500

36. The diode is called a valve, because it allows the electric current to flow
(a) from cathode to plate only (b) from plate to cathode only
(c) in both directions (d) it works as two valves

37. A triode has a mutual conductance of 2×10^{-3} mho and an amplification factor of 50. The anode is connected through a resistance of 25×10^3 Ω to a 250 V supply. The voltage gain of this amplifier is
(a) 50 (b) 25 (c) 100 (d) 12.5

38. The material used for making thermionic cathode must have
(a) low work function and low melting point
(b) low work function and high melting point
(c) high work function and high melting point
(d) high work function and low melting point

39. With 448 mW power used, the number of electrons reaching the anode per second are 1.4×10^{16}, then anode voltage is
(a) 150 V (b) 200 V
(c) 15×48 V (d) (448/14) V

40. In a triode valve, the plate resistance is 10 kΩ and anode load resistance is 30 kΩ. If the amplification factor is 36, then voltage gain is
(a) 9 (b) 36 (c) 27 (d) 108

41. The plate current in a triode is initially 10 mA under certain conditions. When the plate voltage is increased by 10 V with a constant grid voltage, the plate current becomes 11 mA. After restoring to the original plate voltage, the grid voltage has to be increased by 1 V to bring the plate current again to 11 mA. The amplification factor of the triode is
(a) 10 (b) 11 (c) 100 (d) 110

42. The amplification factor of a triode is 20. If the grid voltage is decreased by 1 V, by how much should the plate voltage be increased so that the plate current remains constant?
(a) 10 V (b) 1/10 V (c) 1/20 V (d) 20 V

43. When a triode valve is operated at $V_p = 250$ V and $V_g = -10$ V, the plate current is 4.5 mA. If V_p is decreased to 200 V and V_g is increased to – 7.5 V, the plate current remains unaltered. The amplification factor of the triode is
(a) 10 (b) 15 (c) 20 (d) 40

44. In a vacuum triode the maximum kinetic energy of the electrons emitted by the cathode is 6.4×10^{-19} J. What is the cut-off voltage in volts which when properly applied to the grid just stops the flow of electrons from cathode to plate?
(a) 4 V (b) 6 V (c) 2 V (d) 1 V

45. A triode has cathode, grid and plate potentials of 0, – 2 and 90 V respectively. As the electrons leave the cathode with a kinetic energy of 3 eV, what is the kinetic energy of this electron when it reaches the grid?
(a) 3 eV (b) 2 eV
(c) 5 eV (d) 4 eV

46. When plate resistance is equal to load resistance, voltage gain is
(a) μ (b) μ/2 (c) μ/4 (d) μ/3

47. A load resistance of 50 kΩ is used in a triode amplifying stage. The valve has amplification factor 16 and plate resistance 30 kΩ. If a signal of amplitude 2.5 V is applied between the grid and the cathode, then the amplitude of the output signal will be
(a) 10 V (b) 15 V
(c) 30 V (d) 50 V

48. Three amplifiers, each having voltage amplification 10, are connected in series. The resultant amplification would be
(a) 10 (b) 30 (c) 1000 (d) 10/3

49. Two triodes *A* and *B* have the same amplification factor of 40. Their plate resistances are 4 kΩ and 8 kΩ respectively. If the amplifier circuit is made using any one of them and a load resistance of 8 kΩ, the ratio of the voltage gain obtained from them will be in the ratio
(a) 4/3 (b) 3/2 (c) 1 (d) 3

50. Under space-charge limited operation of a diode, the plate current is 10 mA at a plate potential of 100 V. If the plate potential is changed to 400 V, the plate current will be
(a) 20 mA (b) 40 mA
(c) $40\sqrt{2}$ mA (d) 80 mA

51. The plate current in a triode will become zero, if the negative voltage applied on the grid is
(a) V_p/μ (b) V_p
(c) $V_p \times \mu$ (d) V_p/r_p

52. A triode has a mutual conductance of 3 mA/V and anode slope resistance of 14 kΩ. The valve is used as a voltage amplifier with a load resistance of 20 kΩ. The amplifier gain is
(a) 4.5 (b) 24.7 (c) 18.8 (d) 36.4

53. The amplification factor of a triode valve is 15. If the grid voltage is changed by + 0.3 V, the change in plate voltage required to keep the plate current constant is
(a) + 4.5 V (b) – 4.5 V (c) + 45 V (d) – 45 V

54. In an experiment, the saturation in the plate-current in a diode is observed at 240 V. But a student still wants to increase the plate-current. It can be done, if
(a) the plate voltage is increased further
(b) the plate voltage is decreased
(c) the filament current is decreased
(d) the filament current is increased

55. The thermionic emission resembles
(a) conduction (b) convection
(c) evaporation (d) radiation

56. The cathode of a valve is coated with barium-strontium oxide, because
(a) it prevents the cathode from vaporization
(b) the cathode can be heated to a very high temperature
(c) the thermionic work-function of the cathode increases
(d) the thermionic work-function decreases

57. Which of the following has least variation with plate and grid voltages?
(a) μ
(b) r_p
(c) g_m
(d) Each one varies in the same manner

[Level 2]

Only One Correct Option

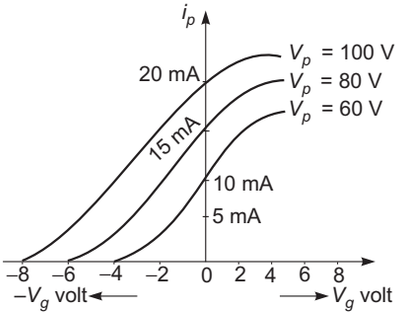
1. In a triode valve circuit, when the voltage is increased from 200 V to 220 V and grid potential is decreased from −0.5 V to −1.3 V, there is no change in plate current. The amplification factor of the triode is
(a) 120 (b) 35
(c) 25 (d) 15

2. If plate resistance of a triode is $3.3\times10^{3}\ \Omega$ and its mutual conductance is 3×10^{-3} mho, then the amplification factor is nearly equal to
(a) 10^{6} (b) 10^{3}
(c) 10^{1} (d) 10^{9}

3. In a diode, when there is a saturation current, the plate resistance will be
(a) data insufficient
(b) zero
(c) some finite quantity
(d) infinite quantity

4. A triode valve has an amplification factor of 20 and its plate is given a potential of 300 V. The grid voltage required to reduce the plate current to zero, is
(a) 25 V (b) 15 V
(c) 12 V (d) 10 V

5. The variation of anode current in a triode valve corresponding to a change in grid potential at three different values of the plate potential is shown in the given figure. The mutual conductance of triode is



(a) 5×10^{-3} mho
(b) 2.5×10^{-3} mho
(c) 7.5×10^{-3} mho
(d) 9.5×10^{-3} mho

6. In a triode valve, the current in the plate circuit is controlled by
(a) ammeter
(b) grid
(c) cathode
(d) anode

7. The thermions are
(a) positrons
(b) photons
(c) electrons
(d) protons

8. The amplification factor of a triode valve is 15. If the grid voltage is changed by 0.3 V, the change in plate voltage in order to keep the plate current constant (in volt) is
(a) 50 V
(b) 0.002 V
(c) 4.5 V
(d) 0.02 V

9. The voltage amplification of a triode is 30 when a load resistance of 50 kΩ is connected to it and it becomes 34 on attaching a load resistance of 85 kΩ, the mutual conductance is
(a) 3.1×10^{-3} mho
(b) 1.1×10^{-3} mho
(c) 4.1×10^{-3} mho
(d) 2.1×10^{-3} mho

10. When a vacuum diode is operated in the saturation region in order to obtain higher anode current, then
(a) temperature of the cathode should be increased
(b) separation of the electrodes should be increased
(c) anode voltage should be increased
(d) temperature of the cathode should be decreased

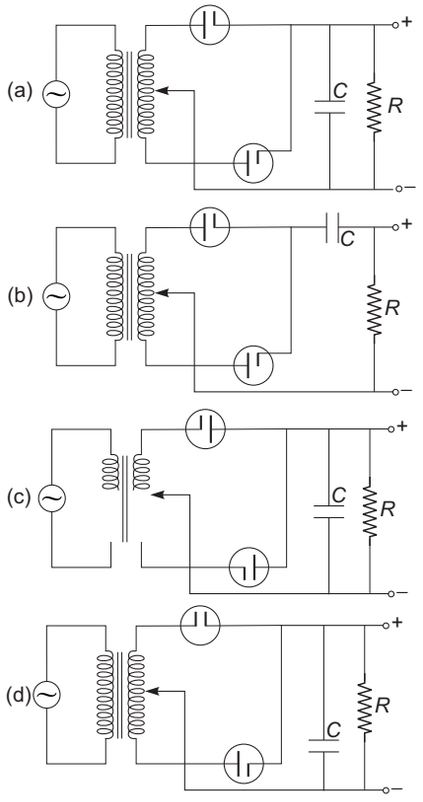
11. For a triode $r_p = 10\,\text{k}\Omega$ and $g_m = 3$ milli-mho. If load resistance is twice of plate resistance, then voltage gain will be
(a) 30 (b) 25
(c) 20 (d) 40

12. Which portion of plate characteristics of triode is used for amplification?
(a) Curved portion
(b) Straight portion
(c) Both (a) and (b)
(d) None of the above

13. For a triode $\mu = 25, r_p = 40\,\text{k}\Omega$, $R_L = 10\,\text{k}\Omega$ and input signal voltage is 0.5 V. Then output voltage is
(a) 1.25 V
(b) 2.5 V
(c) 5 V
(d) 10 V

14. In triode, amplification voltage gain depends on
(a) μ, R_L and input voltage
(b) r_p, R_L and μ
(c) μ, r_p and input voltage
(d) r_p, μ and g_m

15. The correct circuit for full wave rectifier containing a capacitor C and load resistance R_L is



16. In the space charge limited region for vacuum tube diode for plate voltage 400 V and 200 V, the plate currents I_{P_1} and I_{P_2} have ratio

(a) 1 : 2 (b) 2 : 1 (c) $1:2\sqrt{2}$ (d) $2\sqrt{2}:1$

17. For a triode valve $\mu = 50, \Delta V_g = 0.2$ V, then value of ΔV_p is

(a) 5 V (b) 10 V (c) 0.2 V (d) 50 V

18. When plate voltage in diode valve is increased from 100 V to 150 V and plate current increases from 7.5 mA to 12 mA, then dynamic plate resistance will be

(a) 10 kΩ (b) 11 kΩ (c) 15 kΩ (d) 11.1 kΩ

19. In a triode valve, grid is used to control the plate current, because

(a) grid has negative voltage
(b) grid has no voltage
(c) grid is nearer to cathode than plate
(d) None of the above

20. If in a diode valve, the plate potential becomes double, then plate current will be

(a) $2\sqrt{2}$ times (b) 8 times (c) $3\sqrt{3}$ times (d) 2 times

21. The amplification factor of a triode valve is 15. If the grid voltage is changed by 0.3 V, the change in plate voltage in order to keep the current constant (in volt) is

(a) 0.02 (b) 0.002
(c) 4.5 (d) 5.0

22. The relation between amplification factor (μ), plate resistance (r_p) and mutual conductance (g_m) of a triode valve is given by

(a) $\mu = r_p \times g_m$ (b) $r_p = \mu \times g_m$
(c) $g_m = \mu \times r_p$ (d) None of these

23. The amplification factor of a triode valve depends upon

(a) relative position of cathode, grid and plate
(b) temperature of the cathode
(c) temperature of the anode
(d) plate potential and grid

24. The grid of triode amplifier is maintained at negative potential, because

(a) it has distortion free point (b) it keeps noise below
(c) it consumes power (d) None of these

25. If control grid is made negative, then the plate current will

(a) increase
(b) remain constant
(c) decrease
(d) cannot say from given data

26. The material used for making thermionic cathode must have

(a) low work function and low melting point
(b) low work function and high melting point
(c) high work function and high melting point
(d) high work function and low melting point

27. For a triode $r_p = 10\,\text{k}\Omega$ and $g_m = 3$ milli-mho. If the load resistance is double of plate resistance, then the value of voltage gain will be

(a) 10 (b) 15
(c) 20 (d) 30

28. The slope of plate characteristic of a vacuum diode is 2×10^{-1} mAV^{-1}. The plate resistance of diode will be

(a) 50 Ω
(b) 50 kΩ
(c) 500 Ω
(d) 500 kΩ

29. For a given plate voltage, the plate current in a triode is maximum when the potential of

(a) the grid is positive and plate is negative
(b) the grid is positive and plate is positive
(c) the grid is zero and plant is positive
(d) the grid is negative and plate is positive

Answers

Level 1

Objective Problems

1. (c)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (c)
11. (a)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (c)
21. (a)	**22.** (c)	**23.** (b)	**24.** (c)	**25.** (c)	**26.** (d)	**27.** (a)	**28.** (d)	**29.** (c)	**30.** (a)
31. (c)	**32.** (b)	**33.** (a)	**34.** (c)	**35.** (c)	**36.** (b)	**37.** (b)	**38.** (b)	**39.** (b)	**40.** (c)
41. (a)	**42.** (d)	**43.** (c)	**44.** (a)	**45.** (b)	**46.** (b)	**47.** (b)	**48.** (c)	**49.** (a)	**50.** (d)
51. (c)	**52.** (b)	**53.** (b)	**54.** (d)	**55.** (c)	**56.** (d)	**57.** (a)			

Level 2

Only One Correct Option

1. (c)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (d)	**10.** (a)
11. (c)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (d)	**17.** (b)	**18.** (d)	**19.** (c)	**20.** (a)
21. (c)	**22.** (a)	**23.** (b)	**24.** (a)	**15.** (c)	**26.** (b)	**27.** (c)	**28.** (b)	**29.** (b)	

Solutions

Level 1 : Objective Problems

1. According to Richardson-Dushman equation, current density (J) (i.e. charge emitted per second per unit area) is given by

$$J = AT^2 e^{-\phi/kT}$$

So, $$J \propto T^2$$

2. Mutual conductance,

$$g_m = \frac{\Delta i_p}{\Delta V_g}$$

or $$3\times10^{-4} = \frac{\Delta i_p}{-1-(-3)}$$

$\Rightarrow$ $$\Delta i_p = 6\times10^{-4}\ \text{A}$$

$$= 0.6\ \text{mA}$$

3. Voltage gain (Amplification)

$$A_V = \frac{\mu R_L}{r_p + R_L}$$

and $$\mu = r_p \times g_m$$

So, $$r_p = \frac{42}{2\times10^{-3}}$$

$$= 21000\ \Omega$$

$\therefore$ $$A_V = \frac{42\times50\times10^3}{21000 + 50\times10^3}$$

$$= 29.57$$

4. $\mu = -\dfrac{\Delta V_p}{\Delta V_g}$

Also, $$\mu = r_p \times g_m$$

$\therefore$ $$7\times10^3 \times 2.5\times10^{-3} = -\frac{50}{\Delta V_g}$$

$\Rightarrow$ $$\Delta V_g = -2.86\ \text{V}$$

5. Voltage gain, $A_V = \dfrac{V_{\text{out}}}{V_{\text{in}}} = \dfrac{\mu R_L}{r_p + R_L}$

$\therefore$ $$\frac{V_{\text{out}}}{0.5} = \frac{25\times10\times10^3}{40\times10^3 + 10\times10^3}$$

$\Rightarrow$ $$V_{\text{out}} = 2.5\ \text{V}$$

6. Amplification factor, $\mu = -\dfrac{\Delta V_p}{\Delta V_g}$

$\therefore$ $$\Delta V_p = -\mu\Delta V_g$$

$$= -20\times(-0.2)$$

$$= 4\ \text{V}$$

7. Voltage gain, $A_V = \dfrac{\mu R_L}{r_p + R_L}$

and $$\mu = r_p \times g_m$$

$\therefore$ $$\mu = 10\times10^3 \times 3\times10^{-3}$$

$$= 30$$

$$A_V = \frac{\mu(2r_p)}{r_p + (2r_p)}$$

$$= \frac{2}{3}\mu = \frac{2}{3}\times30 = 20$$

8. Voltage gain, $A_V = \dfrac{\mu}{1 + (r_p / R_L)}$

$\because$ $$R_L = 1.5 r_p$$

$\therefore$ $$A_V = \frac{\mu}{1 + \frac{r_p}{1.5 r_p}} = \frac{3}{5}\mu = \frac{3}{5}\times20 = 12$$

9. Amplification factor,

$$\mu = \frac{\Delta V_p}{\Delta V_g} \quad \Rightarrow \quad \therefore \quad \Delta V_p = \mu \Delta V_g$$

$$= 15 \times 0.3 = 4.5 \text{ V}$$

10. Amplification factor,

$$\mu = \frac{\Delta V_P}{\Delta V_G} = \frac{(V_{P_1} - V_{P_2})}{(V_{G_1} - V_{G_2})} = \left(\frac{220 - 220}{0.5 - 1.3}\right) = 25$$

11. The ratio of small change in plate current (Δi_p) to the small change in grid voltage (ΔV_g) at constant plate voltage is known as transconductance or mutual conductance i.e.

$$g_m = \frac{\Delta i_p}{\Delta V_g} \text{ at constant } V_p$$

Its unit is mho or Ω^{-1} or siemen.

12. Voltage amplification,

$$A_V = \frac{\mu R_L}{r_p + R_L}$$

So, $$25 = \frac{\mu \times 50 \times 10^3}{r_p + 50 \times 10^3} \quad \ldots \text{(i)}$$

and $$30 = \frac{\mu \times 100 \times 10^3}{r_p + 100 \times 10^3} \quad \ldots \text{(ii)}$$

Solving Eqs. (i) and (ii),

$$r_p = 25 \text{ k}\Omega$$

13. Amplification factor, $\mu = \frac{\Delta V_p}{\Delta V_g}$

Multiplying and dividing the numerator and denominater on the RHS by Δi_p, we get

$$\mu = \frac{\Delta V_p}{\Delta V_g} \times \frac{\Delta i_p}{\Delta i_p} = \frac{\Delta V_p}{\Delta i_p} \times \frac{\Delta i_p}{\Delta V_g}$$

Hence, $\mu = r_p \times g_m$

14. Voltage amplification,

$$A_V = \frac{\mu R_L}{r_p + R_L} \quad \Rightarrow \quad \therefore \quad \frac{A_1}{A_2} = \frac{2+4}{4+4} = \frac{3}{4}$$

15. Amplification factor,

$$\mu = r_p \times g_m$$

$$\therefore \quad r_p = \frac{20}{10^{-3}} = 2 \times 10^4 \ \Omega$$

16. Amplification factor,

$$\mu = -\frac{\Delta V_p}{\Delta V_g}$$

$$\Delta V_p = -\mu \times \Delta V_g$$

$$= -50(-0.20) = 10 \text{ V}$$

17. The ratio of small change in plate current (ΔI_p) to the small change in grid voltage (ΔV_g) at constant plate voltage is known as transconductance or mutual conductance,

i.e. Mutual conductance, $g_m = \frac{\Delta I_p}{\Delta V_g}$ at constant V_p.

Its unit is mho or Ω^{-1} or siemen.

18. In full wave rectifier, we receive output corresponding to both the half cycle of the input AC cycle. In this rectifier, two diodes are used which conduct alternately.

20. $\mu = -\left(\frac{\Delta V_p}{\Delta V_g}\right) = -\left[\frac{225 - 200}{5.0 - 5.75}\right] = 33.3$

28. Slope of mutual characteristic $= \frac{\Delta i_p}{\Delta V_g}$

and slope of anode characteristic $= \frac{\Delta i_p}{\Delta V_p}$

Their ratio is, $\frac{\Delta V_p}{\Delta V_g}$ which is μ.

31. $\frac{I_1}{I_2} = \left(\frac{V_1}{V_2}\right)^{3/2} = \left(\frac{400}{200}\right)^{3/2} = 2\sqrt{2}$

35. The ratio of two slopes is the amplification factor.

$$= \frac{1.0}{0.02} = 50$$

37. $r_p = \frac{\mu}{g_m} = \frac{50}{25 \times 10^{-3}} = 25 \times 10^3 \ \Omega$

$$A = \frac{\mu}{1 + \frac{r_p}{R}} = \frac{50}{1 + \frac{25 \times 10^3}{25 \times 10^3}} = 25$$

39. $P = Vi$

$$\therefore \quad V = \frac{P}{i} = \frac{448 \times 10^{-3}}{1.4 \times 10^{16} \times 1.6 \times 10^{-19}} = 200 \text{ V}$$

40. $A = \frac{\mu}{1 + \frac{r_p}{R}} = \frac{36}{1 + \frac{10}{30}} = 27$

41. $\mu = \frac{\Delta V_p}{\Delta V_g}$ for same amount of increase in i_p

42. $\mu = \left(\frac{-\Delta V_p}{\Delta V_g}\right)$ for no change in i_p.

43. $\mu = \left(\frac{-\Delta V_p}{\Delta V_g}\right) = \frac{50}{2.5} = 20$

44. Energy of electrons (in eV) $= \frac{6.4 \times 10^{-19}}{1.6 \times 10^{-19}} = 4$ eV

$\therefore$ Cut-off voltage = 4 V

45. The electrons will retard by 2 eV, because the potential difference between cathode and grid is 2 V.

47. $A = \frac{\mu}{1 + \frac{r_p}{R}} = \frac{16}{1 + \frac{50}{30}} = 6$

$$\therefore \quad \Delta V_0 = A \, \Delta V_i = 15 \text{ V}$$

48. $A = A_1 \times A_2 \times A_3 = 10 \times 10 \times 10 = 1000$

49. $\frac{A_1}{A_2} = \frac{\mu}{\frac{1 + r_{p_1}/R}{1 + r_{p_2}/R}} = \frac{1 + r_{p_2}/R}{1 + r_{p_1}/R} = \frac{1 + 8/8}{1 + 4/8} = \frac{2}{3/2} = \frac{4}{3}$

50. $I \propto V^{3/2}$

52. $g_m = 3 \times 10^{-3} \ \Omega^{-1}$, $r_p = 14 \times 10^3 \ \Omega$

$$\therefore \quad \mu = r_p \times g_m = 42$$

$$A = \frac{42}{1 + \frac{14}{20}} = 24.7$$

53. $\mu = \left(\frac{-\Delta V_p}{\Delta V_g}\right) \quad \Rightarrow \quad \Delta V_p = l_c \times \Delta V_g = -15 \times 0.3 = -4.5 \text{ V}$

Level 2 : Only One Correct Option

1. The amplification factor of triode is

$$\mu = \frac{\delta V_p}{\delta V_g} = \frac{220-200}{-0.5-(-1.3)} = 25$$

2. $\mu = g_m \times r_p = 3\times10^{-3}\times3.3\times10^{3} = 9.9 \approx 10^{1}$

3. $r_p = \dfrac{\delta V}{\delta I}$

(where, δV is change in voltage and δI is change in current)

At saturation, $\delta I = 0$

Hence, $$r_p = \frac{\delta V}{0} = \infty$$

Therefore, the plate resistance will be infinity.

4. $V_g = \dfrac{V_p}{\mu} = \dfrac{300}{20} = 15$ V

Therefore, grid potential required to make plate curent to zero is 15 V.

5. $$g_m = \frac{\delta I_p}{\delta V_g} = \left(\frac{\delta I_p}{\delta V_p}\right)\times\left(\frac{\delta V_p}{\delta V_g}\right)$$

$$= \frac{(15-10)\times10^{-3}}{80-60}\cdot\frac{100-80}{-6-(-8)}$$

$$= \frac{5\times10^{-3}}{20}\cdot\frac{20}{2}$$

$$= 2.5\times10^{-3} \text{ mho}$$

9. The voltage amplification is given by

$$A = \frac{\mu R}{r_p + R} \quad \ldots(i)$$

where, R is the load resistance.

$\therefore$ $$30 = \frac{\mu\times50000}{r_p + 50000} \quad \ldots(ii)$$

$$34 = \frac{\mu\times85000}{r_p + 85000} \quad \ldots(iii)$$

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{30}{34} = \frac{10(r_p + 85000)}{17(r_p + 50000)}$$

or $$r_p = 20000\,\Omega$$

Substituting the value r_p in Eq. (ii), we get

$$\mu = 42$$

$\therefore$ $$g_m = \frac{\mu}{r_p} = \frac{42}{20000}$$

$$= 2.1\times10^{-3} \text{ mho}$$

11. $A = \dfrac{\mu R_L}{r_p + R_L}$ $\{\mu = r_p \times g_m = 10\times10^{3}\times3\times10^{-3} = 30\}$

$\therefore$ $$A = \frac{30\times2\times10\times10^{3}}{10\times10^{3} + 2\times10\times10^{3}} = 20$$

13. $A = \dfrac{\mu R_L}{r_p + R_L}$

$$= \frac{25\times10\times10^{3}}{40\times10^{3} + 10\times10^{3}} = 5$$

Now, $$\frac{V_{\text{out}}}{V_{\text{in}}} = A$$

$\therefore$ $$V_{\text{out}} = A\,V_{\text{in}}$$

$$= 5\times0.5 = 2.5 \text{ V}$$

16. $I_P \propto V_P^{3/2}$

$$\frac{I_{P_1}}{I_{P_2}} = \left(\frac{400}{200}\right)^{3/2} = \frac{2^{3/2}}{1}$$

$$= \frac{2\sqrt{2}}{1} = 2\sqrt{2}:1$$

17. $\mu = \dfrac{\Delta V_p}{\Delta V_g}$

$$50 = \frac{\Delta V_p}{0.2}$$

$\therefore$ $$\Delta V_p = 10 \text{ V}$$

18. $$\Delta r_p = \frac{150-100}{(12-7.5)\times10^{-3}}$$

$$= \frac{50\times10^{3}}{4.5} = 11.1 \text{ k}\Omega$$

21. $\mu = 15,\ \Delta V_g = 0.3$ V

$\therefore$ $$\mu = \frac{\Delta V_p}{\Delta V_g}$$

$\Rightarrow$ $$\Delta V_p = \mu\Delta V_g$$

$$= 15\times0.3$$

$$= 4.5 \text{ V}$$

27. Voltage gain, $A_v = \dfrac{\mu}{1 + \dfrac{r_p}{r_L}}$ and $\mu = r_p \times g_m$

$\Rightarrow$ $$\mu = 10\times10^{3}\times3\times10^{-3} = 30$$

$\therefore$ $$A_v = \frac{\mu}{1 + \dfrac{r_p}{2r_p}} = \frac{2}{3}\mu$$

$$= \frac{2}{3}\times30 = 20$$

30

Universe

30.1 Introduction

Astronomy is the branch of science which deals with the study of heavenly bodies of universe. The **geocentric theory** of planetry motion was put forward by **Ptolemy** in the first century AD. According to this theory the earth stood still and all the celestial bodies revolved around it. The geocentric theory was widely accepted for fifteen centuries, until **Copernicus (1473–1553)** proposed his **heliocentric theory** in which the sun was at rest and the planets including the earth went around it. Indian astronomer **Aryabhatt (498 AD)** attributed that the earth's rotation about its own axis is actually responsible for apparent circular motion of sun and occurance of day and night.

Chapter Snapshot

- Introduction
- The Solar System
- The Stars
- Big-Bang Theory of Universe
- The Milky Way
- Miscellaneous Points

30.2 The Solar System

Solar system is a family of nine planets, satellites, asteroids, comets, meteors, mateoroids and dust particles orbiting around the sun. Our solar system is dominated by the sun all other objects are bound to it by gravitation. Nearly 99.9% of the matter in the solar system is accounted for by the sun itself, the planets making up most of the rest. Let us discuss briefly the constituents of the solar system.

The Sun

The sun is a typical example of a star. Seventy percent of sun's mass is hydrogen, twenty eight percent helium and two percent heavier elements from lithium to uranium.

The temperature at the centre of sun is about 1.4×10^{6} K and that of the outer visible region is about 6000 K. The sun's mass is about 2×10^{30} kg which is more than 3×10^{5} times that of the earth.

We shall discuss the energy production in the sun later.

The Planets

There are total nine planets revolving around the sun. Six of them namely. Mercury, Venus, Earth, Mars, Jupiter and Saturn have been known from ancient times. The other three Uranus, Neptune and Pluto were discovered later, after the advent of telescope.

In the increasing order of their mean distance from sun the nine planets are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto. The order can be remembered as,

Meena **V**erma **E**lected **Ma**noj **J**oshi as **SUN** of **P**olice.

Some important points regarding planets are as under :

(a) **Mercury** Nearest, smallest and hottest planet of the solar system. Life is not possible on mercury.

(b) **Venus** Also called **morning star and the evening star** is brightest amongst all.

(c) **Earth** We all know about the earth. It is the only planet which contains suitable conditions for evolution and survival of life. It has only one natural satellite named moon.

(d) **Mars** Nearest planet to the earth. It is reddish. It has traces of O_2 but percentage of O_2 is not sufficient for evolution and survival of life.

(e) **Jupiter** Largest planet of the solar system having the maximum number of satellites.

(f) **Saturn** It has ring around it.

(g) **Uranus** It is the only planet which rotates from East to West on its axis.

(h) **Neptune** It has no special characteristic.

(i) **Pluto** Farthest, coldest, lightest and smallest planet of the solar system. It does not have any satellite.

Asteroids

These are small pieces of planet like material revolving around the sun mostly between the orbits of **Mars** and **Jupiter** are called asteroids.

Comets

The small pieces of rock like material surrounded by a large amount of substances such as water, ammonia and methane possessing head and tail while moving past the sun are called comets. A comet does not have any tail, when it is far away from the sun. As it approaches the sun, it begins to get elongated in the direction away from the sun due to radiation pressure. When the comet is near the sun, substances like water are vaporized due to the heat of the sun and radiation pressure forces the vapour away in the shape of tail. Thus, head of comet is made of heavy material like rock, while the tail is made of light material such as dust and gas. The comets have highly elliptical orbits. **Halley's comet** was discovered in 1757. It has a period of **76.2 years.**

Meteors and Meteoroids

It is also called **Shooting Star.** A small piece of planetary material (rock etc.), when enters the earth's atmosphere gets heated to very high temperature due to friction of air looking as bright lines of the fire and gets completely burnt before reaching the earth. Those meteors which are large enough to survive and manage to reach the earth are called **meteoroids.**

30.3 The Stars

Heavenly bodies that shine like sun are called stars. They have the energy of their own to shine. After sun, **Alpha Centauri** is nearest to the earth.

Birth of a Star

Dust particles, hydrogen and helium gas molecules present in the intersteller space first come together (at about $-173°C$) to form a cloud. Then they start contracting. As a result of compression heating of cloud takes place. When temperature of the core becomes about 10^7 K, the fusion of hydrogen atoms is initiated with the release of energy. This energy keeps the star shining for millions of years.

Death of a Star

A star lasts untill the hydrogen in the core of the star is exhausted. The core now starts contracting, it results in the rise in temperature of the star. As a result of rise in temperature, the outer layer of the star expand. Expansion of outer layer brings about cooling effect in them. The process continues till the temperature of outer layer falls enough to make the star appear red. It is then called **red giant.**

At this stage, a violent explosion called **nova** or **supernova** occurs in the star. Due to the explosion its outer layer are thrown into intersteller space leaving behind the core of the star. The core of the star may further end up into one of the following three steller dead materials.

White Dwarf

The core dies as white dwarf, if the original mass of the star was 1.4 solar masses. It was discovered by **S. Chandrasekhar** in 1930 and it is known as **Chandrasekhar limit.** The core is composed of protons and electrons. The core keeps on emitting heat and light for millions of years. As it cools steadily, its colour changes from white to yellow, then to red and finally it becomes black. It then becomes invisible forever as **black dwarf** and neither emits heat nor light.

Neutron Star

The core of the star finishes up as neutron star, if the mass of the star was between 1.5 to 5 solar masses. The compressed core is made only of neutrons. Neutron stars produce very high magnetic fields. A spinning neutron star emitting electromagnetic waves is called **Pulsar**.

Black Hole

If the original mass of the star is greater than 5 solar masses the core dies into black holes. The mass of a black hole is greater than the mass of the sun but its size is very small, therefore the gravitational pull of a black hole is so strong that even the photon or radiation emitted by it, cannot escape from its surface. Since, no radiation is received from a black hole, it cannot be seen through a telescope.

Source of Energy in Stars

When two or more light nuclei combine to form a heavier nucleus, the process is called **nuclear fusion.** In fusion, the mass of final nucleus is less than the combined rest mass of the original nuclei, loss of mass occurs, accompanied by loss of energy. Experiments have shown that nuclear fusion is the source of energy in most of the stars. Two of the main cycle for producing energy in stars are :

(i) Proton-proton cycle

(ii) Proton-carbon cycle

Proton-Proton Cycle

In sun and other stars where temperature is less than or equal to 10^7 K, fusion takes place dominantly by,

$$2\,(^1\mathrm{H} + {}^1\mathrm{H}) \longrightarrow 2(^2\mathrm{H} + e^+ + \nu) \quad Q = 2\,(0.42)\ \mathrm{MeV}$$

$$2\,(^2\mathrm{H} + {}^1\mathrm{H}) \longrightarrow 2\,(^3\mathrm{He} + \gamma) \quad Q = 2\,(5.49)\ \mathrm{MeV}$$

$$^3\mathrm{He} + {}^3\mathrm{He} \longrightarrow {}^4\mathrm{He} + 2\,{}^1\mathrm{H} \quad Q = 12.86\ \mathrm{MeV}$$

$$4\,{}^1\mathrm{H} \longrightarrow {}^4\mathrm{He} + 2e^+ + 2\nu + 2\gamma \quad Q \approx 24.7\mathrm{MeV}$$

As a result of this cycle, effectively four hydrogen nuclei combine to form a helium nucleus. A total 24.7 MeV energy is produced in this cycle. An additional 2 MeV is derived from the annihilation of the two positrons for a total of 26.7 MeV.

The sun is estimated to have been radiating energy for the last 4.5×10^9 yr and will continue to do so till the next 5×10^9 yr.

Proton-Carbon Cycle

In hotter stars, where the temperature is about 10^8 K another cycle known as proton-carbon cycle takes place.

$$^1\mathrm{H} + {}^{12}\mathrm{C} \longrightarrow {}^{13}\mathrm{N} + \gamma$$

$$^{13}\mathrm{N} \longrightarrow {}^{13}\mathrm{C} + e^+ + \nu$$

$$^1\mathrm{H} + {}^{13}\mathrm{C} \longrightarrow {}^{14}\mathrm{N} + \gamma$$

$$^1\mathrm{H} + {}^{14}\mathrm{N} \longrightarrow {}^{15}\mathrm{O} + \gamma$$

$$^{15}\mathrm{O} \longrightarrow {}^{15}\mathrm{N} + e^+ + \nu$$

$$^1\mathrm{H} + {}^{15}\mathrm{N} \longrightarrow {}^{12}\mathrm{C} + {}^4\mathrm{He}$$

The end result of this cycle is again the fusion of four hydrogen nuclei into a helium nucleus. Carbon nucleus acts only as a catalyst.

- It is clear that coulomb repulsion becomes more and more obstructive to fusion as Z increases. Thus, it needs still higher temperatures for heavier elements to fuse.

30.4 Big-Bang Theory of Universe

This theory about the origin of universe was given by **Le Maitre** and **Gammon**. This is the most accepted theory regarding the origin of universe. According to this theory, all the constituents of our universe were originally together as a single mass. A big explosion occurred at some instant and the single mass burst into a large number of fragments moving with different velocities.

Hubble's Law

The rate of expansion of universe (rate at which galaxies are receding from each other) is given by this law.

This law states that the rate of separation of any two galaxies in the universe is directly proportional to their separation. Thus, if we have two galaxies a distance R apart, their present relative velocity of separation is

$$\theta = HR$$

where, H is a constant called **Hubble's parameter**. Its value is $2.32 \times 10^{-18}\ \mathrm{s}^{-1}$.

We may define a time t_H equal to $\frac{R}{v}$ which corresponds to the time when two galaxies have reached a separation R. Thus,

$$t_H = \frac{R}{v} = \frac{1}{H} = 4.3 \times 10^{17}\ \mathrm{s} = 1.36 \times 10^{10}\ \mathrm{yr}$$

This time, which is called **Hubble's time** is an estimate of the order of magnitude of time that has elapsed since, the Big-Bang and thus the age of Universe.

The size of observable universe can be estimated from Hubble's time. The farthest distance, called **Hubble's distance**, that can be observed from the earth is,

$$d_H = ct_H = (3 \times 10^8\ \mathrm{m/s}) \times (4.3 \times 10^{17}\mathrm{s})$$

$$= 1.3 \times 10^{26}\ \mathrm{m}$$

30.5 The Milky Way

The Milky way or **Akash-ganga** is the name of the galaxy to which our solar system belongs. It is composed of a few hundred billion stars including our own sun. The systematic study of the Milky way was undertaken in the eighteenth century by **William Herschel,** the same Herschel who discovered the planet Uranus.

To the naked eye, Milky way looks like a stream of milk across the sky. Some of the important features of Milky way are given below :

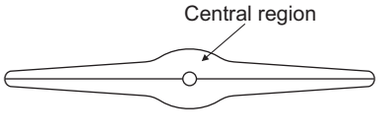


Fig. 30.1

(i) Milky way is a convex lens shaped disc with a thick centre and thinning outwards.

(ii) The Milky way contains about 150 billions of stars.

(iii) The space in the Milky way is filled with dust and gases called the interstellar matter. About 90% of the intersteller matter is in the form of hydrogen.

(iv) Some regions of the Milky way appear dark not because they are empty but because the intervening dust and gas obstructs the light from the stars behind them. These dark regions are called **dark nebulae.**

(v) Sometimes the regions of dust show a great variety of brightness. They shine in the light of the nearby stars and such bright regions of dust are called **bright nebulae. Orion** is the example of bright nebulae.

(vi) A remarkable feature of the Milky way is its rotation about its centre. This rotation is not rigid.

Our sun, alongwith the planetry system is revolving around the centre of the Milky way with a speed $v = 250$ km/s. It takes about 250 million years for the sun to go round once.

(vii) The mass of Milky way is estimated to be 150 billion solar masses or nearly 3×10^{41} kg. The intersteller matter forms nearly 1.5% of the total mass of the Milky way.

Finally, we see that we live in a galaxy, our Milky way, which is an immense collection of stars. However, as we shall see our galaxy is merely a single cosmic building block. It takes billions of such building blocks to make up the universe at large.

30.6 Miscellaneous Points

(a) Surface temperature of sun Energy emitted by sun per unit area per unit time

$$E = \sigma T^4 \qquad \text{(Stefan's law)}$$

$\therefore$ Total energy emitted by sun per second $= E \times 4\pi R^2$

Here, $R =$ radius of the earth

This energy is distributed in all directions.

If r be the distance of the earth from sun, then energy incident on earth's surface per unit time per unit area is $(E \times 4\pi R^2)/4\pi r^2$. This quantity is defined as **solar constant S.** Thus,

$$S = \frac{ER^2}{r^2} = \frac{\sigma T^4 R^2}{r^2}$$

We know the values of S, σ, R and r. Substituting these values in above equation, we get

$$T = 5790 \text{ K}$$

The value of solar constant is,

$$S = 2 \text{ cal/cm}^2\text{-s}$$

or $$1.388 \times 10^3 \text{ W/m}^2$$

(b) Mass and density of sun For circular orbits we can write,

$$\frac{GMm}{R^2} = mR\omega^2 = mR\left(\frac{2\pi}{T}\right)^2 \qquad \ldots\text{(i)}$$

Here, M is the mass of sun, m the mass of planet and R its radius of orbit.

From Eq. (i) we find,

$$M = \frac{4\pi^2 R^3}{GT^2}$$

If r is the radius of sun and ρ its density then,

$$\left(\frac{4}{3}\pi r^3 \rho\right) = M = \frac{4\pi^2 R^3}{GT^2}$$

$\therefore$ $$\rho = \frac{3\pi R^3}{GT^2 r^3}$$

Proceeding in the similar manner, we can also find mass and density of a planet which have satellites.

(c) Size of planet In the figure,

$d =$ diameter of planet

and $D =$ distance of planet from the earth

By measuring the angle α, we can find the diameter of the planet d. As,

$$\alpha \approx \frac{d}{D}$$

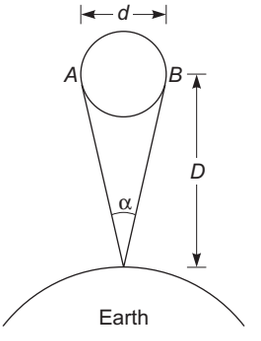


Fig. 30.2

or $$d = \alpha D$$

(d) Distance of planet from the earth For measuring distance of planet from the earth we use following two methods:

(i) Parallax method Planet O is observed from two points P_1 and P_2 on the earth. The distance between them b, is called **basis**. Angle subtended by planet at these two points is called **parallax angle** or **parallactic angle**. Thus,

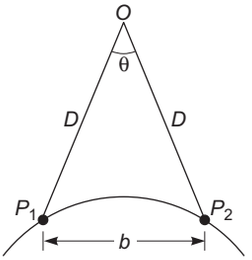


Fig. 30.3

$$\theta \approx \frac{b}{D} \quad \text{or} \quad D = \frac{b}{\theta}$$

(ii) Copernicus method For the planets having circular orbits, e.g. Mercury and Venus, the angle between EP and ES is called **planet's elongation**.

For circular orbits r_{ps} and r_{es} are the fixed distances. Only r_{pe} is a variable distance. As shown in figure, planet's elongation is maximum at 90°. From the figure,

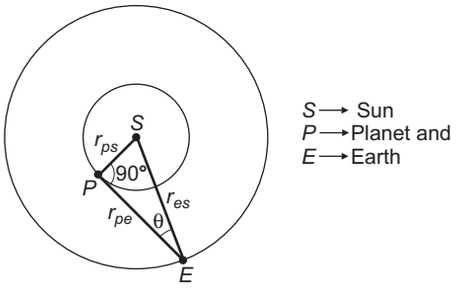


Fig. 30.4

$$\sin\theta = \frac{r_{ps}}{r_{es}}$$

But $r_{es} = 1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$

$\therefore$ $r_{ps} = \sin\theta$ (in AU)

Similarly, $r_{pe} = \cos\theta$ (in AU)

(e) Speed of a galaxy Every element has its characteristic spectra. The spectral lines of elements present in any galaxy can be identified by comparison with the spectral lines of the elements observed in the laboratories on the surface of the earth.

Let λ_0 and λ be the wavelength of a particular spectral line as observed on earth and as observed from some other galaxy. The spectral shift is measured by the quantity,

$$Z = \frac{\lambda - \lambda_0}{\lambda_0}$$

A positive Z ($\lambda > \lambda_0$) gives a **red shift.** All galaxies invariably showed red shift of spectral lines, which was the same for all lines of any particular galaxy. The red shift varied from galaxy to galaxy.

The shift in wavelength was first given by **Doppler.** According to him if speed of galaxy v is $<<$ speed of light c,

$$Z \approx \frac{v}{c} = \frac{\lambda - \lambda_0}{\lambda_0}$$

$\therefore$ $$v = c\left(\frac{\lambda - \lambda_0}{\lambda_0}\right)$$

The red shift of galaxies showed that they were rushing away from us. Indeed it demonstrated that we were living in an expanding universe.

(f) Estimation of mass of sun (using binary system) In a two body system two bodies revolve about their common centre of mass.

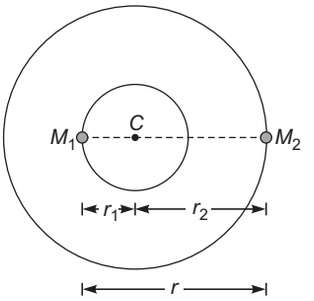


Fig. 30.5

From the known formulae regarding centre of mass we can have,

$$M_1 r_1 = M_2 r_2 \quad \text{and} \quad r_1 + r_2 = r$$

$\therefore$ $$r_1 = \left(\frac{M_2}{M_1 + M_2}\right) r \qquad \ldots(i)$$

and $$r_2 = \left(\frac{M_1}{M_1 + M_2}\right) r \qquad \ldots(ii)$$

The necessary centripetal force to M_2 is provided by the gravitational force of attraction between M_1 and M_2. Thus,

$$\frac{GM_1 M_2}{r^2} = M_2 r_2 \omega^2 = M_2 r_2 \left(\frac{2\pi}{T}\right)^2$$

$\therefore$ $$\frac{GM_1}{r^2} = \frac{4\pi^2 r_2}{T^2}$$

Substituting value of r_2 from Eq. (ii), we have

$$\frac{GM_1}{r^2} = \frac{4\pi^2}{T^2} \times \frac{M_1}{M_1 + M_2} r$$

$\therefore$ $$(M_1 + M_2) = \frac{4\pi^2}{T^2} \cdot \frac{r^3}{G} \qquad \ldots(iii)$$

The Eq. (iii) can be used to find the masses of the two stars of a binary system. By knowing $(M_1 + M_2)$ we can find M_1 and M_2 from Eqs. (i) and (ii).

(g) Brightness and luminosity of a star The brightness of a star is expressed through the system of **magnitude.**

"The magnitude of a star is the measure of its brightness, when observed from the earth." The Greek astronomer **Hipparcus,** divided the naked eye stars into six magnitude classes.

The stars in the first magnitude class are brightest and the stars in the sixth magnitude class are the faintest. First magnitude class star is about 100 times as bright as a sixth magnitude class star.

Decrease in magnitude number by one increases brightness by ratio $100^{1/5} \approx 2.512$.

In general,

$$\frac{\text{the brightness of star in } n^{\text{th}} \text{ magnetude class}}{\text{the brightness of star in } (n+m)^{\text{th}} \text{ magnitude class}} = (2.512)^m$$

If two stars have magnitudes m_1 and m_2 $(> m_1)$ and brightness l_1 and l_2 $(< l_1)$, then

$$\frac{l_1}{l_2} = 100^{(m_2 - m_1)/5}$$

Taking logarithm of both sides and solving, we get

$$(m_2 - m_1) = -2.5 \log^{(l_2/l_1)}$$

For a star of zero magnitude $m_1 = 0, l_1 = l_0, m_2 = m$ and $l_2 = l$.

Thus, $$m = -2.5 \log \frac{l}{l_0}$$

The star **Vega** is of zero magnitude and of brightness $l_0 = 2.52 \times 10^{-8}$ W/ m^2. Stars which are brighter than those of zero magnitude possess negative magnitude. Thus, a star having magnitude −5 will be 100 times more bright than a star of zero magnitude.

(h) Three important laws There are three more important laws which we have already discussed in the chapter of gravitation and heat in volume-I. Here, we are just writing down those three laws.

(i) Kepler's law $T^2 \propto r^3$

Here, T is the time period of a planet round the sun and r the semi-major axis of its elliptical path. In case of circular path, r is the radius.

(ii) Stefan's law $E = \sigma T^4$

Here, E is the energy radiated per unit area per unit time from the surface of a black body kept at absolute temperature T. In the formula σ is the **Stefan's constant** which has the value 5.67×10^{-8} W/ m^2-K^4.

(iii) Wien's displacement law $\lambda_m T =$ constant

Here, λ_m is the wavelength corresponding to maximum energy density. Constant has the value 2.898×10^{-3} m-K and is called **Wien's constant**, denoted by b.

Additional Examples

Example 1. *The moon is observed from two diametrically opposite points A and B on the earth. The angle* θ *subtended at the moon by the two directions of observation is* $1^\circ 54'$. *Find the distance of moon from the earth. Given, diameter of the earth* $\approx 1.2756 \times 10^7$ *m.*

Sol. See the parallax method,

$$D = \frac{b}{\theta} \quad \text{...(i)}$$

Here, b = diameter of the earth = 1.2756×10^7 m

and $$\theta = 1^\circ 54' = \left(1 + \frac{54}{60}\right)^\circ = \left(1 + \frac{54}{60}\right) \times \frac{\pi}{180} \text{ rad} = 3.3174 \times 10^{-2} \text{ rad}$$

Substituting in Eq. (i), we get

$$D = \frac{1.2756 \times 10^7}{3.3174 \times 10^{-2}} = 3.8452 \times 10^8 \text{ m}$$

Example 2. *In the case of Venus, the angle of maximum elongation* θ *(In Copernicus Method) is found to be* 47°. *Determine the distance between Venus and the Sun* r_{vs} *and the distance between Venus and the Earth* r_{ve}.

Sol. As we have shown in the theory,

$$r_{vs} = (\sin \theta)\,\text{AU} = (\sin 47^\circ)\,\text{AU} = 0.73\,\text{AU}$$

and $$r_{ve} = (\cos \theta)\,\text{AU} = (\cos 47^\circ)\,\text{AU} = 0.68\,\text{AU}$$

Since, 1 AU = 1.496×10^{11} m

The distance r_{vs} and r_{ve} can be expressed in meters also.

$$r_{vs} = 1.092 \times 10^{11} \text{ m} \quad \text{and} \quad r_{ve} = 1.017 \times 10^{11} \text{ m}$$

Example 3. *Taking the results of above example, find the orbital period of Venus.*

Sol. From Kepler's law, we have

$$\left(\frac{T_v}{T_e}\right)^2 = \left(\frac{r_v}{r_e}\right)^3 \quad \text{or} \quad T_v = T_e\left(\frac{r_v}{r_e}\right)^{3/2}$$

Here, $r_v = 0.73$ AU

and $r_e = 1$ AU

$$\therefore \quad T_v = (365 \text{ days})\left(\frac{0.73}{1}\right)^{3/2}$$

$$= 226 \text{ days}$$

Example 4. *Assuming that the earth's orbit around the sun is a circle of radius* $R = 1.496 \times 10^{11}$ *m, compute the mass of sun.*

(Given, $G = 6.668 \times 10^{-11}$ *N-m²/kg²)*

Sol. In article 30.6 (b), we have derived a formula

$$M = \frac{4\pi^2 R^3}{GT^2}$$

we have, $T = 365 \text{ days} = (3.156 \times 10^7)$ s,

then, $$M = \frac{4\pi^2 \ (1.496 \times 10^{11})^3}{(6.668 \times 10^{-11})\ (3.156 \times 10^7)^2}$$

$$= 1.989 \times 10^{30} \text{ kg}$$

Example 5. *Assuming that the earth re-emits all the radiation it receives from the sun, find the temperature of the earth assuming it to be perfectly black body.*

Sol. The amount of heat received by the earth from the sun is 1.388×10^3 W/m² (the solar constant S). If this amount is re-emitted, then it can be equated to σT^4.

$$\therefore \quad \sigma T^4 = S \quad \text{or} \quad T = \left(\frac{S}{\sigma}\right)^{\frac{1}{4}}$$

$$= \left(\frac{1.388 \times 10^3}{5.669 \times 10^{-8}}\right)^{\frac{1}{4}} = 395 \text{ K}$$

Example 6. *There are certain types of stars called variable stars which undergo periodic change in their light output. If such a star doubles its light output, how much does its magnitude change?*

Sol. Given $\frac{l_2}{l_1} = 2$

From the relation, $m = -2.5 \log\left(\frac{l}{l_0}\right)$

$$m_2 = -2.5 \log\left(\frac{l_2}{l_0}\right)$$

and $$m_1 = -2.5 \log\left(\frac{l_1}{l_0}\right)$$

$$\therefore \quad m_2 - m_1 = 2.5\left[\log\frac{l_1}{l_0} - \log\frac{l_2}{l_0}\right]$$

$$= 2.5 \log\left(\frac{l_1}{l_2}\right)$$

$$= 2.5 \log\left(\frac{1}{2}\right)$$

$$\therefore \quad m_2 - m_1 = -0.753$$

Example 7. *The K-line of singly ionized calcium has a wavelength of* 393.3 *nm as measured on the earth. In the spectrum of one of the observed galaxies, this spectrum line is located at* 401.8 *nm. Determine the speed with which this galaxy is moving away from us.*

Sol. Given, $\lambda_0 = 393.3$ nm

and $\lambda = 401.8$ nm

The red shift of the galaxy is given by,

$$Z = \frac{401.8 - 393.3}{393.3}$$

$$= 0.0216$$

The galaxy is then moving away from us with a speed,

$$v = Zc$$

$$= (0.0216) \times (3 \times 10^5 \text{ km/s})$$

$$= 6480 \text{ km/s}$$

Objective Problems

[Level 1]

1. Which one of the following nuclear reaction is a source of energy in the sun?

(a) ${}_4Be^9 + {}_2He^4 \rightarrow {}_6C^{12} + {}_0n^1$

(b) ${}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + {}_1H^1 + {}_1H^1$

(c) ${}_{56}Ba^{144} + {}_{36}Kr^{92} \rightarrow {}_{92}U^{235} + {}_0n^1$

(d) ${}_{26}Fe^{50} + {}_{48}Ca^{112} \rightarrow {}_{74}W^{161} + {}_0n^1$

2. Asteroids are

(a) small planets
(b) shooting stars
(c) found in a belt between Earth and Venus
(d) None of the above

3. What causes the Fraunhoffer lines in the solar spectrum?

(a) Absorption of radiation
(b) Emission of radiation
(c) Both absorption and emission of radiation
(d) Neither absorption nor emission of radiation

4. The maximum amount of radiation in the earth's atmosphere is of the type

(a) X-ray (b) γ-ray (c) ultraviolet (d) infrared

5. The expansion of galaxies is supported by

(a) neutron star (b) white dwarf star
(c) red shift (d) black holes

6. Time taken by the light to travel from sun to earth is approximately

(a) 8 s (b) 8 min
(c) 8 h (d) 8 ly

7. The theoretical limit to the size of the universe is about

(a) ten million year
(b) million light year
(c) hundred million light year
(d) ten thousand million light year

8. Basis of Hubble's law is

(a) Law of gravitation (b) Stefan's law
(c) Doppler effect (d) Wien's law

9. As we go up in the atmosphere, the heights of the various regions are in the order

(a) stratosphere > troposphere > ionosphere
(b) troposphere > ionosphere > stratosphere
(c) ionosphere > troposphere > stratosphere
(d) ionosphere > stratosphere > troposphere

10. The region of the atmosphere above troposphere is known as

(a) lithosphere (b) uppersphere
(c) ionosphere (d) stratosphere

11. If H is the Hubble constant, then the average life of universe will be

(a) $\frac{1}{\sqrt{H}}$ (b) $\sqrt{H}$ (c) $\frac{1}{H}$ (d) $\frac{1}{H^2}$

12. Milky way is the name of

(a) a planet (b) an asteroid
(c) a comet (d) a galaxy

13. Which one of the following statements regarding shooting stars is incorrect?

(a) They are meteors
(b) They are the debris which got separated from a comet
(c) They emit light due to extremely high temperature caused by friction while they are passing through the earth's atmosphere
(d) They are a kind of stars

14. Which of the following is the closest galaxy to the Milky way?

(a) Crab nebula (b) Andromeda nebula
(c) Magnetic cloud (d) 3C-73

15. The universe is

(a) contracting (b) expanding
(c) infinite (d) constant in size

16. Halley's comet was seen in India in 1986. It is expected to be visible again in the year

(a) 2052 (b) 2062
(c) 2072 (d) 2082

17. Which of the planet is brightest?

(a) Mercury (b) Venus
(c) Mars (d) Jupiter

18. The burnt pieces of comets that fall on Earth are called

(a) meteors (b) asteroids
(c) satellites (d) planets

19. The tail of a comet points

(a) towards the sun (b) away from the sun
(c) in arbitrary direction (d) away from the earth

20. The biggest planet in our solar system is

(a) Earth (b) Saturn (c) Jupiter (d) Uranus

21. Who disproved the geocentric theory of universe?

(a) Aristotle (b) Copernicus
(c) Issac Newton (d) Charls Darwin

22. CO_2 is found in which of the following pairs of planets?

(a) Earth and Mercury (b) Mercury and Saturn
(c) Venus and Saturn (d) Venus and Mars

23. Hubble's law is related with
(a) Comet (b) Black hole
(c) Speed of galaxy (d) Planetary motion

24. Albedo is
(a) absorptive power of a heavenly body
(b) transmittive power of a heavenly body
(c) refracting power of a heavenly body
(d) reflecting power of a heavenly body

25. The heating effect of atmosphere is due to?
(a) Ultraviolet rays (b) Infrared rays
(c) Carbon dioxide (d) Both (b) and (c)

26. Sun radiates energy continuously and maintains its brightness, because
(a) of fission of helium into hydrogen
(b) of burning of carbon in its core
(c) helium is converted into iron in its core
(d) of fusion of hydrogen nuclei into helium

27. Surface temperature of the sun is approximately
(a) 3000 K (b) 600 K
(c) 6000 K (d) 60000 K

28. When the original mass of a star is greater than 5 M (M = mass of sun), the death of a star gives rise to
(a) White dwarf (b) Neutron star
(c) Black hole (d) Nebula

29. Very small stars having diameter (1/50)th that of sun are called
(a) Dwarfs (b) White dwarfs
(c) Milky way (d) Neutron stars

30. Hubble's law states that the velocity with which the Milky way is moving away from the earth is proportional to
(a) square of distance of the Milky way from the earth
(b) distance of Milky way from the earth
(c) mass of Milky way
(d) product of mass of the Milky way and its distance from the earth

31. The most common stars like, the sun are
(a) Dwarfs (b) White dwarfs
(c) Milky way (d) Neutron stars

32. The death of a star results to a neutron star, if the original mass of star in terms of mass of sun (M) was
(a) less than $2M$ (b) between $1.5M$ and $5M$
(c) greater than $5M$ (d) exactly equal to M

33. Milky way is
(a) a planet of our solar system
(b) a sun
(c) one of the solar systems
(d) one of the solar galaxies of universe

34. Two stars P and Q are observed at night. Star P appears reddish white, star Q is white. From this we conclude
(a) temperature of Q is higher than that of P
(b) temperature of Q is lower than that of P
(c) star Q is at the same distance as that of star P
(d) star P is farther than star Q

35. Out of the following planets, the one which is called red planet is
(a) Mercury (b) Venus
(c) Jupiter (d) Mars

36. At present, which of the following theories is the most satisfactory about the origin of universe?
(a) Big-Bang theory (b) Pulsating theory
(c) Steady state theory (d) None of these

37. Which of the following planets have rings around it?
(a) Uranus (b) Mars
(c) Jupiter (d) Saturn

38. Black hole is
(a) hole in the ozone layer of atmosphere
(b) hole in the earth's centre
(c) highly dense matter available in the atmosphere
(d) hole in the troposphere

39. The tail of a comet points away from the sun due to
(a) centrifugal force
(b) electrical repulsion
(c) attraction of comet due to other planets
(d) radiation pressure

40. The group of small pieces of rock revolving round the sun between the orbits of Mars and Jupiter are called
(a) Meteors (b) Comets
(c) Meteorites (d) Asteroids

41. The name black hole is given, because
(a) it is part of space which has no matter
(b) it is completely made up of carbon
(c) its gravity is so high that it prevents even light to radiate into space
(d) it is a star which does not emit visible radiations

42. The solar family consists of
(a) the sun and the nine planets
(b) the sun, the planets and their satellites
(c) the sun, the planets and their satellites and other objects like asteroids, comets, meteors, etc.
(d) sun only

43. Satellites of Mars are
(a) Ceres (b) Fobos
(c) Dimos (d) (b) and (c) are correct

44. A group of faint and bright stars is called
(a) Galaxy (b) Comets
(c) Constellation (d) Asteroids

45. Venus appears brighter than other stars, because
(a) it is heavier than other planets
(b) its density is more than other planets
(c) it is nearer to the earth in comparison to other planets
(d) nuclear fusion takes place at its surface

46. Which of the following is not a part of solar system?
(a) Meteors (b) Comets (c) Quasars (d) Asteroids

47. The sun revolves about the axis of the galaxy at the speed nearest to
(a) 2500 kmh^{-1} (b) 2500 kms^{-1}
(c) 250 kmh^{-1} (d) 250 kms^{-1}

48. Which of the following are the hottest types of stars?
(a) *A*-type (b) *B*-type (c) *M*-type (d) *O*-type

49. The approximate ratio of diameter of sun to diameter of earth is
(a) 11 (b) 110
(c) 1100 (d) much more than 1100

50. The limit of observable universe is
(a) 2×10^{10} km (b) 2×10^{10} AU
(c) 2×10^{10} ly (d) 3×10^{10} ly

51. The age of universe is believed to be
(a) 1 billion year (b) 10 billion year
(c) 1-20 billion year (d) 1000 billion year

52. Heliocentric theory of the universe was first proposed by
(a) Ptolemy (b) Copernicus
(c) Galileo (d) Kepler

53. The solar constant on the surface of the earth is *S*. What will be its value on the surface of another planet which is about 5.3 AU away from sun?
(a) $\frac{S}{5.3}$ (b) $\frac{S}{(5.3)^2}$ (c) 5.3 *S* (d) $(5.3)^2 S$

54. The wavelength of maximum energy, released during an atomic explosion, was 2.93×10^{-10} m. Given that the Wien's constant is 2.93×10^{-3} mK, the maximum temperature attained must be of the order of
(a) 10^{-7} K (b) 10^{7} K (c) 10^{-13} K (d) 5.86×10^{7} K

55. A planet which is born sister of the earth is
(a) Mercury (b) Venus
(c) Mars (d) Jupiter

56. The hottest planet of solar system is
(a) Mars (b) Mercury
(c) Venus (d) Pluto

57. Which of the nine planets is nearest to sun?
(a) Venus (b) Mercury
(c) Mars (d) Jupiter

58. Albedo is maximum for
(a) Pluto (b) Venus
(c) Earth (d) Mercury

59. Which one of the following planet has the longest day?
(a) Venus (b) Mars
(c) Mercury (d) Earth

60. Which one of the following is known as Saptarishi?
(a) Orion (b) Ursa major
(c) Ursa minor (d) Scorpion

61. Smaller pieces of heavy stones and metals which on entering earth's atmosphere burns out are
(a) Comets
(b) Meteorites
(c) Asteroids
(d) All of the above

62. Great bear is a
(a) Star (b) Galaxy
(c) Constellation (d) Planet

63. The difference in the lengths of a mean solar day and a sidereal day is about
(a) 1 min (b) 4 min
(c) 15 min (d) 56 min

[Level 2]

Only One Correct Option

1. A spinning neutron star is known as a
(a) Black hole (b) Meteor
(c) Pulsar (d) Quasar

2. How many planets are there with the sun in our solar system?
(a) Indefinite number of planets
(b) Eleven planets
(c) Nine planets
(d) Seven planets

3. How does the red shift confirm that the universe is expanding?
(a) Wavelength of light emitted by galaxies appears to decrease
(b) Wavelength of light emitted by galaxies appears to be the same
(c) Wavelength of light emitted by galaxies appears to increase
(d) None of the above

4. Clusters consists of
(a) water vapours with star
(b) cloud on star
(c) small group of stars
(d) big group of stars

5. Black hole consists of
(a) upper surface of atmosphere
(b) ozone layer
(c) super dense planetary motion
(d) None of the above

6. The planet which is called twin sister of the earth is
(a) Pluto (b) Mars (c) Mercury (d) Venus

7. Meteors are
(a) small stars
(b) burnt pieces of comets that fall on earth
(c) comets without tail
(d) None of the above

8. A study of binary stars is most helpful in
(a) finding their distances
(b) finding their temperatures
(c) finding their masses
(d) varifying Newton's force law of gravitation

9. According to modern astronomers, into how many constellations the whole sky is divided
(a) 10^{11} (b) 88
(c) 880 (d) 5000

10. The temperature of sun can be determined with the use of
(a) Wein's law (b) Stefan's law
(c) Hubble's law (d) Kepler's law

11. An exploding star is called
(a) Nova (b) Supernova
(c) Black hole (d) Neutron star

12. The star whose mass is five times, the solar mass and which has a high g value is known as
(a) black hole (b) nebula
(c) neutron star (d) white dwarf

13. The surface temperature of sun is nearly
(a) 10^6 K (b) 10^8 K
(c) 10000 K (d) 5800 K

14. Hubble's law is expressed as (v = speed of recession, r = distance of galaxy and H = Hubble's constant)
(a) $v = \frac{H}{r^2}$ (b) $v = \frac{H}{r}$
(c) $v = Hr$ (d) $v = Hr^2$

15. The surface temperature is maximum for
(a) blue star
(b) yellow star
(c) green star
(d) red star

16. A satellite orbiting round the earth appears stationary when
(a) its time period is one day and is rotating in the same sense as that of earth
(b) its time period is one day and its rotation is normal to the direction of earth
(c) its time period is 12 h and it is rotating in the same direction as that of earth
(d) its time period is 12 h and it is rotating normal to direction of earth

17. The condition for a uniform spherical mass m of radius r to be a black hole is
(G = gravitational constant and g = acceleration due to gravity)
(a) $\left(\frac{2Gm}{r}\right)^{1/2} \le c$ (b) $\left(\frac{2gm}{r}\right)^{1/2} = c$
(c) $\left(\frac{2Gm}{r}\right)^{1/2} \ge c$ (d) $\left(\frac{gm}{r}\right)^{1/2} \ge c$

18. According to Hubble's law, the red-shift (Z) of a receding galaxy and its distance r from earth are related as?
(a) $Z \propto r$ (b) $Z \propto 1/r$ (c) $Z \propto 1/r^2$ (d) $Z \propto r^{3/2}$

19. According to the size, identify the correct decreasing order in
1. Original star 2. Red giant
3. White dwarf
(a) 1, 2, 3 (b) 2, 3, 1
(c) 3, 1, 2 (d) 2, 1, 3

20. How many times more, the mass of the original star is to be larger than that of the sun for the formation of Black hole ?
(a) 2 (b) 6
(c) 8 (d) 10

21. Top of the stratosphere has an electric field E (in units of Vm^{-1}) nearly equal to
(a) 0 (b) 10
(c) 100 (d) 1000

22. The surface charge density (in cm^{-2}) of the earth is about
(a) 10^{-9} (b) -10^9 (c) 10^9 (d) -10^{-9}

Assertion and Reason

Directions (Q. Nos. 1-8) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*
(a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** To an astronaut, the outer space appears black.

Reason In outer space, there is nothing to scatter light.

2. **Assertion** Moon is seen as it partly reflects the sun light falling on it.

Reason Moon is a satellite of earth. It does not emit light of its own.

3. **Assertion** Red shift confirms that the universe is expanding.

Reason Wavelength of red light is maximum in the visible region.

4. **Assertion** Our galaxy is called Milky way.

Reason Milky way gives the impression of steam of milk flowing across the sky.

5. **Assertion** A star which appears blue will be much hotter than the sun.

Reason It is based on Wien's law.

6. **Assertion** A pulsar emits radio signals at extremely regular intervals of time.

Reason A pulsar is high density neutron star.

7. **Assertion** There is no atmosphere on moon.

Reason Escape velocity at the surface of moon is low.

8. **Assertion** The comets do not obey Kepler's laws of planetary motion.

Reason The comets do not revolve in elliptical orbits.

Answers

Level 1

Objective Problems

1. (b)	**2.** (a)	**3.** (a)	**4.** (d)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (d)	**10.** (d)
11. (c)	**12.** (d)	**13.** (d)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (a)	**19.** (b)	**20.** (c)
21. (b)	**22.** (d)	**23.** (c)	**24.** (d)	**25.** (d)	**26.** (d)	**27.** (c)	**28.** (c)	**29.** (b)	**30.** (b)
31. (a)	**32.** (b)	**33.** (d)	**34.** (d)	**35.** (d)	**36.** (a)	**37.** (d)	**38.** (c)	**39.** (d)	**40.** (d)
41. (c)	**42.** (c)	**43.** (d)	**44.** (c)	**45.** (c)	**46.** (c)	**47.** (d)	**48.** (d)	**49.** (b)	**50.** (c)
51. (c)	**52.** (b)	**53.** (b)	**54.** (b)	**55.** (b)	**56.** (b)	**57.** (b)	**58.** (b)	**59.** (a)	**60.** (b)
61. (b)	**62.** (c)	**63.** (b)							

Level 2

Only One Correct Option

1. (c)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (a)
11. (b)	**12.** (a)	**13.** (d)	**14.** (c)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (b)
21. (b)	**22.** (d)								

Assertion and Reason

1. (a)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (a)

Solutions

Level 1 : Objective Problems

53. Solar constant is the energy incident per unit area per second at the earth's distance, area being normal to the sun's rays. Also, energy falling is inversely proportional to the square of distance from the source.

$$S' = \frac{S}{(5.3)^2}$$

54. According to Wien's law, $\lambda_m T = b$

or $$2.93 \times 10^{-10} \times T = 2.93 \times 10^{-3}$$

$$T = 10^7 \text{ K}$$

55. Venus is called the earth's sister.

58. Albedo is maximum for venus, because it reflects 85% of incident light. Its value of albedo is 0.85.

59. Venus has the longest day.

60. Ursa major is known as Saptarishi.

61. Smaller pieces of heavy stones and metals entering the earth's atmosphere burns out are Meteroites.

62. Great bear is a constellation, which is a group of some stars.

63. The difference in the length of mean solar day and a sideral day is about 4 min.

31

Theory of Relativity

31.1 Introduction

Maxwell in the year 1864, had established the presence of electromagnetic waves in space, travelling with the speed of light. In other words, he proved light to be an electromagnetic wave. Waves known till then (like sound wave etc.) all required a material medium for their propagation. It was supposed that there must also be a suitable medium to carry these electromagnetic waves which travelled even through empty space between the stars and the earth. This medium, though no one knew what it actually was, came to be referred to as **luminiferous ether**, prevailing all space empty or otherwise.

Further, to enable it to transmit light, a transverse wave motion, the ether had to be a rigid solid and in view of the tremendous velocity of light. It had to have a large shear modulus and yet all material objects, like the earth, planets and stars etc., were to continue in their regular courses through it without encountering the slightest resistance. But no one could seen its existence. On the other hand it was felt that perhaps it was the absolute space or the fundamental frame of reference Newton was looking for and in which his laws of motion would hold perfectly.

The idea of ether medium was such an inviting and exciting proposition that scientists started doing experiments to establish the existence of this elusive medium, ether and hence that of an absolute frame of reference. It was but natural that light waves should be used for this purpose, as ether was just a vehicle for them.

Michelson and Morley succeeded in performing, in the year 1887, one of their most famous experiments which gave a null result, striking at the very root of the ether hypothesis. It was 18 years later, in 1905, that the true and simple explanation for the negative result of Michelson and Morley's experiment was furnished by **Einstein**, namely that **the velocity of light in space is a universal constant**. In fact, he made this constancy of the velocity of light one of the basic postulates of his special theory of relativity.

Chapter Snapshot

- Introduction
- Einstein's Principle of Relativity
- Special Theory of Relativity
- Time Dilation
- Length Contraction
- Relativistic Momentum
- Relativistic Energy
- Relativistic Doppler Effect

31.2 Einstein's Principle of Relativity

From the negative results of the ether-drift experiments. Einstein was fully convinced that there was no such thing as an absolute or fixed frame of reference. He gave in the year 1905 his conclusions and revolutionary ideas into a theory called **Special theory of relativity**. Ten years later, in 1915, followed the second and the more complex and difficult part of it in the form of the **General theory of relativity**. The former deals with problems associated with unaccelerated frames of reference and the latter with those associated with accelerated ones.

We shall concern ourselves here mainly with the special theory of relativity, which is comparatively simpler.

31.3 Special Theory of Relativity

The two basic postulates on which the theory rests are :

1. **The principle of relativity** The laws of physics must be the same in all inertial frames.

2. **The constancy of the speed of light** The speed of light in vacuum has the same value $c = 3.00 \times 10^8$ m/s, in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

The first postulate asserts that all the laws of physics those dealing with mechanics, electricity, magnetism and optics etc., are the same in all reference frames moving with constant velocity relative to one another.

Let us now examine some of the relativistic effects. Quite a few of them, will appear to be surprisingly new and strange, because in view of the small relative motion between the frames of reference of which we have experience in our daily life. We do not ordinarily come across any perceptible relativistic phenomena. Let us consider a few important examples.

31.4 Time Dilation

Observers in different inertial frames always measure different time intervals between a pair of events. Consider a vehicle moving to the right with a speed v as shown in Fig. (a). A mirror is fixed to the ceiling of the vehicle and observer O' at rest in this system holds a laser a distance d below the mirror.

At some instant, the laser emits a pulse of light directed toward the mirror (event 1) and at some later time after reflecting from the mirror, the pulse arrives back at the laser (event 2). Observer O' carries a clock and uses it to measure the time interval Δt_p between these two events. Because the light pulse has a speed c.

$$\Delta t_p = \frac{2d}{c} = \text{proper time}$$

That proper time is the time interval between two events measured by an observer who sees the events occur at the same point in space.

Now consider the same pair of events as viewed by observer O in a second frame as shown in figure (b). According to this observer, the mirror and laser are moving to the right with a speed v and as a result the sequence of events appears entirely different. By the time the light from the laser reaches the mirror, the mirror has moved to the right a distance $\frac{v\,\Delta t}{2}$, where Δt is the time it takes the light to travel from O' to the mirror and back to O' as measured by O.

In other words, O concludes that because of the motion of the vehicle, if the light is to hit the mirror, it must leave the laser at an angle with respect to the vertical direction. Comparing figures (a) and (b), we see that the light must travel farther in (b) than in (a). According to the second postulate of the special theory of relativity, both observers must measure c for the speed of light. Because the light travels farther in the frame O, it follows that the time interval Δt measured by O is longer than the time interval Δt_p measured by O'. To obtain relationship between these two time intervals, it is convenient to the right angled triangle shown in figure (c).

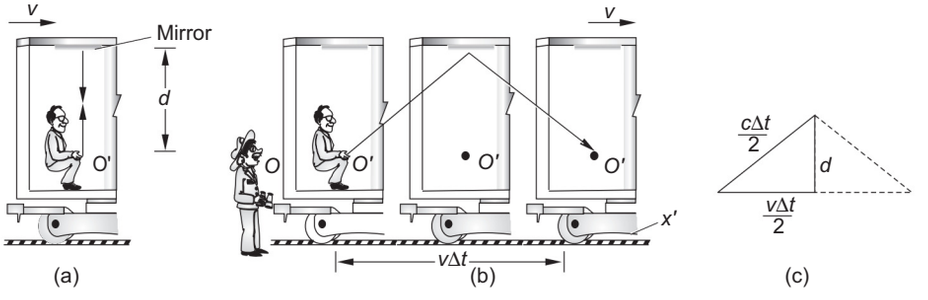


Fig. 31.1

From the Pythagorean theorem,

$$\left(\frac{c\,\Delta t}{2}\right)^2 = \left(\frac{v\,\Delta t}{2}\right)^2 + d^2$$

Solving for Δt, we get

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

Because $\Delta t_p = \frac{2d}{c}$

$$\therefore \quad \Delta t = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \cdot \Delta t_p$$

where, $\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$

Because γ is always greater than unity, this result says that, the time interval Δt measured by an observer moving with respect to a clock is longer than the time interval Δt_p measured by an observer at rest with respect to the clock.

This effect is known as **time dilation**.

Example 31.1 *Two events A and B occur at places separated by* 10^6 *km, B occuring* 5 *s after A.*

(a) Find the velocity of a frame in which these events occur at the same place.

(b) What is the time interval between the events in this frame?

Sol. (a) The frame should move a distance of 10^6 km in 5 s for both, the events to occur at the same place.

Hence, speed of frame,

$$v = \frac{\text{distance}}{\text{time}} = \frac{10^9 \text{ m}}{5\text{s}} = 2 \times 10^8 \text{ m/s}$$

(b) As the events *A* and *B* occur at the same place in the train frame, the time interval between the events measured in this frame is the proper interval. Thus this time interval is,

$$\Delta t_p = \Delta t \sqrt{1 - v^2/c^2} = 5\sqrt{1 - \left(\frac{2 \times 10^8}{3 \times 10^8}\right)^2} = 3.7 \text{ s}$$

The Twins Paradox

An interesting consequence of time dilation is the so called **twins paradox**. Consider a experiment involving a set of twins named Mohan and Sohan. When they are 20 yr old Mohan sets out a journey to a planet *X* located 10 light year from the earth. The speed of his spaceship is 0.5 *c* relative to the inertial frame of his brother Sohan. After reaching planet *X*, Mohan becomes home sick and immediately returns to the earth at the same speed 0.5 *c*.

Time taken from the ground frame,

$$t = 2\left(\frac{10}{0.5}\right) = 40 \text{ yr}$$

Upon his return, Mohan is shocked to discover that Sohan has aged 40 yr and is now 60 yr old. Mohan, on the other hand has aged only, $40\sqrt{1 - \left(\frac{0.5\,c}{c}\right)^2} = 34.6$ yr and is now 54.6 yr old. This phenomenon is called twins paradox.

31.5 Length Contraction

If you want to measure the length of a rod that is at rest with respect to you, you can note the positions of its end points on a long stationary scale and subtract one reading from the other. But if the rod is moving, you must note the positions of the end points simultaneously.

Let L_0 be the length of a rod that you measure when the rod is stationary. If instead there is a relative motion at speed *v* between you and the rod along the length of the rod, then with simultaneous measurement you obtain a length *L* given by

$$L = L_0\sqrt{1 - v^2/c^2} = \frac{L_0}{\gamma}$$

Because the factor γ is always greater than unity, if there is relative motion, *L* is less than L_0. The length L_0 of an object measured in the rest frame of the object is its **proper length** or **rest length**. Measurements of the length from any reference frame that is in relative motion parallel to that length are always less than the proper length.

That length contraction occurs only along the direction of relative motion.

Proof of Length Contraction

Length contraction is a direct consequence of time dilation. This time, Mohan seated on a train moving through a station and Sohan on the station platform, want to measure the length of the platform. Sohan using a tape, finds the length to be L_0, a proper length, because the platform is at rest with respect to him. Sohan also notes that Mohan, on the train, moves through this length in a time,

$$\Delta t = \frac{L_0}{v} \quad \text{or} \quad L_0 = v \cdot \Delta t \qquad \text{(Sohan)}$$

where, *v* is the speed of the train.

Here, Δt is not the proper time interval, because the two events that define it (Mohan passes the back of the platform and Mohan passes the front of the platform) occur at two different places and Sohan must use two synchronized clocks to measure the time interval Δt.

For Mohan, however, the platform is moving past him. He finds that the two events measured by Sohan occur at the same place in his reference frame.

He can time them with a single stationary clock, so the interval Δt_0 that he measures is a proper time interval. To him, the length L of the platform is given by,

$$L = v\,\Delta t_0 \qquad \text{(Mohan)}$$

From the previous two equations we have,

$$\frac{L}{L_0} = \frac{v\,\Delta t_0}{v\,\Delta t} = \frac{1}{\gamma} \quad \text{or} \quad L = \frac{L_0}{\gamma}$$

Example 31.2 *A rod lies parallel to the x-axis of reference frame S, moving along this axis at a speed of* 0.63 *c. Its rest length is* 1.70 *m. What will be its measured length in frame S?*

Sol. Given, $v = 0.63c$, $L_0 = 1.17$ m

$$\therefore \quad L = L_0\sqrt{1 - v^2/c^2}$$
$$= (1.70\text{ m})\sqrt{1 - (0.63)^2} = 1.32\text{ m}$$

31.6 Relativistic Momentum

The law of conservation of linear momentum states that when two isolated objects collide, their combined total momentum remains constant. Suppose that the collision is described in a reference frame S in which linear momentum is conserved. If we calculate the velocities in a second reference frame S' the linear momentum might not be conserved. However, because the laws of physics are the same in all inertial frames, linear momentum must be conserved in all frames.

In view of this condition, we must modify, the definition of linear momentum to satisfy the following conditions :

(i) Linear momentum p must be conserved in all collisions.

(ii) The relativistic value calculated for p must approach the classical value mu (where, u is the velocity of object) as u approaches zero.

For any particle, the correct relativistic equation for linear momentum that satisfies these conditions is,

$$p \equiv \frac{mu}{\sqrt{1 - \dfrac{u^2}{c^2}}} = \gamma\, mu$$

when $u << c$, $\gamma = \dfrac{1}{\sqrt{1 - u^2/c^2}}$ approaches unity and p approaches mu.

Example 31.3 *An unstable particle at rest breaks into two fragments of unequal mass. The mass of lighter fragment is* 2.50×10^{-28} *kg and that of the heavier fragment is* 1.67×10^{-27} *kg. If the lighter fragment has a speed of* 0.893 *c after the breakup, what is the speed of the heavier fragment?*

Sol. From conservation of linear momentum, since the initial momentum of the system is zero, final momentum should also be zero. Momentum of both the particles should be equal and opposite.

$$|p_1| = |p_2|$$

$$\therefore \quad \frac{m_1 u_1}{\sqrt{1 - \left(\dfrac{u_1^2}{c^2}\right)}} = \frac{m_2 u_2}{\sqrt{1 - \left(\dfrac{u_2^2}{c^2}\right)}}$$

Substituting $m_1 = 2.50 \times 10^{-28}$ kg

$m_2 = 1.67 \times 10^{-27}$ kg

and $u_1 = 0.893$ c

we get, $u_2 = 0.285$ c

31.7 Relativistic Energy

We have seen that the definition of linear momentum and the laws of motion require generalization to make them compatible with the principle of relativity. This implies that the definition of kinetic energy must also be modified. Before we study the relativistic energy, let us first discuss effect of relativity of mass.

Relativity of Mass

According to classical mechanics (Newton's mechanics) mass of an object is a constant quantity but in the light of relativistic ideas of time and space, it is to be noted that the mass is no longer constant. The mass of a body varies with its velocity according to the relation,

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Here, m_0 is the rest mass of the body.

Einstein's Mass-Energy Relation

Let us consider a particle of mass m moving with a velocity v, m_0 is the rest mass of the body, then it can be proved that the relativistic expression for kinetic energy is,

$$K = (m - m_0)\,c^2$$

This result shows that the kinetic energy of the body is equal to the relativistic increase in mass of the body over the rest mass multiplied by the square of velocity of light.

It means that even when the body is at rest, it possesses an amount of energy $m_0\,c^2$. Thus, $m_0\,c^2$ is called the **rest**

energy of the body. Thus, the total energy of the body is the sum of the kinetic energy and rest energy.

$$E = (m - m_0)\, c^2 + m_0\, c^2$$

or $$E = mc^2 = \frac{m_0\, c^2}{\sqrt{1 - v^2 / c^2}}$$

This is the famous Einstein's mass-energy relation.

Relativistic Relation between Kinetic Energy and Linear Momentum

Total relativistic energy is,

$$E = mc^2 = \frac{m_0\, c^2}{\sqrt{1 - v^2 / c^2}}$$

Now the linear momentum,

$$p = mv \quad \text{or} \quad v = p/m$$

$$\therefore \quad E = \frac{m_0\, c^2}{\sqrt{1 - \dfrac{p^2}{m^2\, c^2}}}$$

Simplifying this we obtain

$$E^2 = m_0^2 c^4 + p^2 c^2$$

This is the desired relation between E and p.

Note Points

- When the particle is at rest or $p=0$, $E = m_0c^2$ (rest mass energy).
- For particles that have zero mass, such as photons, we get $m_0 = 0$ and we see that

  $$E = pc$$

  This equation is an exact expression relating total energy and linear momentum.
- The mass m_0 of a particle is independent of its motion, m_0 must have the same value in all reference frames. On the other hand, because the total energy and linear momentum of a particle both depend on velocity these quantities depend on the reference frame in which they are measured.
- As m_0 is constant, we conclude that the quantity $E^2 - p^2c^2$ must have the same value in all reference frames.

Example 31.4 *Prove that the relativistic kinetic energy of a particle moving with a velocity $v << c$ is approximately equal to $1/2\, mv^2$.*

Sol. We know that,

$$K = (m - m_0)c^2$$

$$= \left[\frac{m_0}{\sqrt{1 - v^2/c^2}} - m_0\right]c^2$$

$$K = \left[m_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} - m_0\right]c^2$$

If $v << c$, then $\left(1 - \dfrac{v^2}{c^2}\right)^{-\frac{1}{2}} \approx 1 + \dfrac{v^2}{2c^2}$

$$\therefore \quad K \approx \left[m_0\left(1 + \frac{v^2}{2c^2}\right) - m_0\right]c^2$$

or $$K \approx \frac{1}{2} m_0 v^2$$

Example 31.5 *Because mass is a measure of energy, can we conclude that the mass of a compressed spring is greater than the mass of the same spring when it is not compressed.*

Sol. As we know elastic potential energy $\left(= \dfrac{1}{2} kx^2\right)$ is stored in a compressed spring.

According to special theory of relativity any change in the total energy of a system is equivalent to a change in the mass of the system. Therefore, the mass of a compressed (or stretched) spring is greater than the mass of the spring in its equilibrium position by an amount U/c^2.

31.8 Relativistic Doppler Effect

Another important consequence of time dilation is the shift in frequency found for light emitted by atoms in motion as opposed to light emitted by atoms at rest. This phenomenon is called the **Doppler effect**.

There is however, a difference between Doppler effect in sound and that in light. In sound, the Doppler effect depends not only upon the relative motion between the source and observer but also depends upon whether the source or the observer is moving. e.g. if the source is stationary and the observer is moving towards the source, then the Doppler effect (change in frequency) will be different from that when the observer is stationary and the source is moving with the same velocity toward the observer. The reason is that sound waves require a medium for their propagation and the velocity of sound is always with respect to the medium. Thus, for sound waves source in motion and observer in motion refer separately to motion with respect to the medium and represent physically different situations. Hence, they lead to different equations for the Doppler effect. Thus, Doppler effect in sound in **asymmetrical**.

Light waves however, can travel in vacuum without requiring any medium. Therefore, a source of light moving with velocity v toward an observer and an observer moving with velocity v toward the source of light are physically identical situations. Therefore, Doppler effect in light depends only upon the relative motion of the source and the observer, no matter which one is moving. Thus we can say that Doppler effect in light is **symmetrical**.

Expression for Apparent Frequency

Suppose, the frequency of light emitted by a source is f. It can be shown by theory of relativity that if either the source or the observer is moving with a velocity v towards the other (i.e. the distance between them is decreasing), then the apparent frequency is given by,

$$f' = f\sqrt{\frac{1+v/c}{1-v/c}}$$

As $f' > f$, the apparent frequency of light will be increased (or wavelength is decreased). So, that the spectral lines will be shifted towards the blue end of the spectrum (blue shift).

If however $v << c$, the factor $\frac{v^2}{c^2}$ is very small and the above expression becomes,

$$f' = f\left(1+\frac{v}{c}\right)$$

The apparent change in frequency is,

$$\Delta f = f' - f = \frac{v}{c}\cdot f \quad \text{or} \quad \Delta f = \frac{v}{c} f$$

As $\lambda = \frac{v}{f}$. Therefore, $\lambda' < \lambda$ and we can show that,

$$\Delta\lambda = \lambda - \lambda' = \frac{v}{c}\cdot\lambda \quad \text{or} \quad \Delta\lambda = \frac{v}{c}\lambda$$

If the distance between source and observer is increasing, then,

$$f' = f\sqrt{\frac{1-v/c}{1+v/c}}$$

or $f' < f$

and $\lambda' > \lambda$

Therefore, the spectral lines will be shifted towards red (called red shift).

Applications of Doppler Effect in Light

(i) Doppler effect is very useful in estimating, the speed of a star or a galaxy which are moving toward or away from earth. If the spectral lines are shifted towards the blue then the star is approaching the earth but if they are found shifted toward the red the star is receding from the earth. All galaxies for which measurements have been made show red shift, indicating that they are receding away from earth. This suggests that the universe is expanding.

(ii) Doppler effect can also be used in knowing the speeds of vehicles on the road by traffic police.

(iii) Doppler effect is used to measure the temperature of plasma.

Example 31.6 *A star is moving towards the earth with a velocity of* 4×10^4 *m/s. If the real wavelength of a line in the spectrum of the star be* 5920 *Å, determine the apparent wavelength of the line. (Speed of light* $=3\times10^8$ *m/s).*

Sol. As the star is moving towards the earth frequency will increase or wavelength will decrease. Change in wavelength is,

$$\Delta\lambda = \left(\frac{v}{c}\right)\lambda$$

$$= \left(\frac{4\times10^4}{3\times10^8}\right)\times 5920\ \text{Å}$$

$$= 0.8\ \text{Å}$$

Therefore, the apparent wavelength is,

$$\lambda' = \lambda - \Delta\lambda$$

$$= 5919.2\ \text{Å}$$

Additional Examples

Example 1. *At what speed would the relativistic value for the linear momentum of a particle be twice the classical values?*

Sol. p_c = classical momentum $= mv$

and p_r = relativistic momentum $= \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$

Given $p_r = 2p_c$

$$\therefore \quad \frac{mv}{\sqrt{1-v^2/c^2}} = 2mv$$

or $$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2}$$

$$\therefore \quad 1-\frac{v^2}{c^2} = \frac{1}{4}$$

Therefore, $$\frac{v^2}{c^2} = \frac{3}{4} \quad \text{or} \quad v = 0.866\ c$$

Example 2. *A train travelling at* $0.8\,c$ *takes* $5\,\mu s$ *to pass an observer on the platform.*

(a) What is the time interval measured in the train's frame?

(b) What is the length of the train according to observer on the train?

(c) On the platform.

Sol. (a) $\Delta t = \dfrac{\Delta t_p}{\sqrt{1 - v^2/c^2}} = \dfrac{(5\,\mu s)}{\sqrt{1 - \left(\dfrac{0.8\,c}{c}\right)^2}} = 8.33\,\mu s$

(b) $L_0 = (0.8c)\,(8.33\,\mu s) = (0.8 \times 3 \times 10^8)\,(8.33 \times 10^{-6})$

$= 2000 \text{ m} = 2 \text{ km}$

(c) $L = L_0\sqrt{1 - v^2/c^2} = (2 \text{ km})\sqrt{1 - \left(\dfrac{0.8\,c}{c}\right)^2}$

$= 1.2 \text{ km}$

Example 3. *According to classical physics, what is the potential difference needed to accelerate an electron from rest to* $0.9\,c$*?*

Sol. According to classical physics, kinetic energy in eV of the electron would be,

$$K = \frac{1}{2} \times \frac{(9.1 \times 10^{-31})\,(0.9 \times 3 \times 10^8)^2}{(1.6 \times 10^{-19})} \text{ eV}$$

$$K = 2.07 \times 10^5 \text{ eV}$$

$$= 207 \text{ keV}$$

$\therefore$ The required potential difference would be 207 kV.

Example 4. *Calculate the length and the orientation of the rod of length* 5m *in a reference frame moving with a velocity of* $0.6\,c$*, in a direction making an angle of* $30°$ *with the rod.*

Sol. The length of the moving rod is given by, $L_0\sqrt{1 - v^2/c^2}$, where L_0 is the proper length of the rod.

The rod has a velocity component in x-direction as well as in y-direction. The proper length of the rod in x-direction is,

$$(L_0)_x = 5\cos 30° = 2.5\sqrt{3} \text{ m}$$

and in y-direction it is,

$$(L_0)_y = 5 \sin 30° = 2.5 \text{ m}$$

$\therefore$ $$L_x = 2.5\sqrt{3}\sqrt{1 - \left(\frac{0.6\,c}{c}\right)^2} = 2\sqrt{3} \text{ m}$$

$$L_y = 2.5 \text{ m}$$

Length observed in the moving frame,

$$L = \sqrt{L_x^2 + L_y^2} = \sqrt{12 + 6.25} = 4.27 \text{ m}$$

The angle θ which the rod appears to make with x-axis is,

$$\theta = \tan^{-1}\left(\frac{L_y}{L_x}\right) = \tan^{-1}(0.72)$$

Example 5. *The total energy of a particle is twice its rest energy. Find its speed.*

Sol. Given that,

$$E = 2E_0 \;\Rightarrow\; \therefore \quad mc^2 = 2m_0c^2$$

or $$m = 2m_0 \;\Rightarrow\; \therefore \quad \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}} = 2m_0$$

Solving this equation, we get

$$v = 0.866\,c$$

- The speed does not depend on the mass of electron.

Example 6. *A particle of rest mass* m_0 *is kept at rest at the origin. A constant force F starts acting on it at* $t = 0$*. Find the speed of the particle at time t.*

Sol. From the equation of motion,

$$\frac{dP}{dt} = F$$

$$\int_0^P dP = \int_0^t F\,dt$$

$\therefore$ $$P = Ft \qquad (\text{as } F = \text{constant})$$

$\therefore$ $$\frac{m_0 v}{\sqrt{1 - v^2/c^2}} = Ft$$

Solving this equation for v, we get

$$v = \frac{F\,tc}{\sqrt{m_0^2c^2 + F^2t^2}}$$

Example 7. *An astronaut is approaching the moon. He sends out a radio signal of frequency* 5×10^9 *Hz and finds that the frequency shift in the echo received is* 10^3 *Hz. Find the speed of approach.*

Sol. Let f be the actual frequency of the radio signal and v the speed of approach of the astronaut to the moon. Then, the frequency received by the moon is,

$$f_1 = f\sqrt{\frac{1 + v/c}{1 - v/c}}$$

Again the frequency of the echo received by the astronaut is,

$$f_2 = f_1\sqrt{\frac{1 + v/c}{1 - v/c}}$$

From the above two equations, we can find

$$\frac{f_2}{f} = \frac{1 + v/c}{1 - v/c} = \frac{c + v}{c - v} \quad \text{or} \quad v = \left(\frac{f_2 - f}{f_2 + f}\right)c$$

Given, $$f_2 - f = 10^3 \text{ Hz}, \; f = 5 \times 10^9 \text{ Hz}$$

$\therefore$ $$v = (3 \times 10^8)\left(\frac{10^3}{5 \times 10^9 + 5 \times 10^9}\right)$$

$$= 30 \text{ m/s}$$

- $f_2 = f + 10^3 \text{ Hz} \approx f \; (\text{as } f >> 10^3 \text{ Hz})$

Objective Problems

[Level 1]

1. The velocity of a body of rest mass m_0 is $\frac{\sqrt{3}}{2}c$ (where, c is the velocity of light in vacuum). Then mass of this body is

(a) $(\sqrt{3}/2)m_0$ (b) $(1/2)m_0$ (c) $2m_0$ (d) $(2/\sqrt{3})m_0$

2. Special theory of relativity states that

(a) mass remains unaffected in any inertial frame
(b) velocity of light remains unaffected in any inertial frame
(c) time remains same in all inertial frames
(d) None of the above

3. A particle with rest mass m_0 is moving with speed c. The de-Broglie wavelength associated with it is

(a) zero (b) infinity (c) $h\nu/m_0c$ (d) m_0/ch

4. The total energy E of a sub atomic particle of rest mass m_0 moving at non-relativistic speed v is

(a) $E = m_0c^2$ (b) $E = \frac{1}{2}mv^2$

(c) $E = m_0c^2 + \frac{1}{2}m_0v^2$ (d) $E = m_0c^2 - \frac{1}{2}m_0v^2$

5. The magnitude of linear momentum of a particle moving at a relativistic speed v is proportional to

(a) v (b) $1 - v^2/c^2$

(c) $\sqrt{1 - v^2/c^2}$ (d) None of these

6. Two events take place simultaneously at points A and B as seen in the lab frame. They also occur simultaneously in frame moving with respect to the lab in a direction

(a) parallel to AB
(b) perpendicular to AB
(c) making an angle of 45° with AB
(d) making an angle of 135° with AB

7. An experimenter measures the length of a rod. In the cases listed, all motions are with respect to the lab and the measured length of the rod. In which of the cases the measured length will be minimum?

(a) The rod and the experimenter move with the same speed v in the same direction
(b) The rod and the experimenter move with the same speed v in opposite directions
(c) The rod moves at speed v but the experimenter stays at rest
(d) The rod stays at rest but the experimenter moves with the speed v

8. At what velocity must a particle move, so that its kinetic energy is equal to its rest energy?

(a) $\sqrt{3}c/4$ (b) $\sqrt{3}c/2$

(c) $2c/\sqrt{3}$ (d) $2\sqrt{2}c/3$

9. As the speed of a particle increases, its rest mass

(a) increases (b) decreases
(c) remains the same (d) changes

10. A particle of mass m and its antiparticle are annihilated in a nuclear reaction. The amount of energy released is

(a) zero (b) $\frac{1}{2}mc^2$ (c) mc^2 (d) $2mc^2$

11. Which of the following quantities related to an electron has a finite upper limit?

(a) Mass (b) Momentum
(c) Speed (d) Kinetic energy

12. An experimenter measures the length of a rod. Initially the experimenter and the rod are at rest with respect to the lab. Consider the following statements.

(A) If the rod starts moving parallel to its length but the observer stays at rest, the measured length will be reduced.
(B) If the rod stays at rest but the observer starts moving parallel to the measured length of the rod, the length will be reduced.

(a) A is true but B is false (b) B is true but A is false
(c) Both A and B are true (d) Both A and B are false

13. The length of a metre rod moving parallel to its length at a speed v is found to be 0.6 m. What is the value of v?

(a) $0.6\,c$ (b) $0.7\,c$ (c) $0.8\,c$ (d) $0.9\,c$

14. If the speed of a particle moving at a relativistic speed is doubled, its linear momentum will

(a) become double (b) become more than double
(c) remain equal (d) become less than double

15. The speed at which the mass of a body is twice its rest mass is approximately equal to

(a) 2.6×10^8 ms^{-1} (b) 2.2×10^8 ms^{-1}
(c) 1.8×10^8 ms^{-1} (d) 1.4×10^8 ms^{-1}

16. If a constant force acts on a particle, its acceleration will

(a) remain constant (b) gradually decrease
(c) gradually increase (d) be undefined

17. In the special theory of relativity we deal with events or phenomena in frames of reference which move with respect to each other with a constant

(a) speed (b) velocity
(c) momentum (d) acceleration

18. When a given amount of water is heated from 0°C to 100°C, its mass

(a) remains unchanged (b) decreases slightly
(c) increases slightly (d) increases substantially

19. A reference frame moves so fast that an observer travelling with it finds all lengths reduced to half. The time intervals measured by him will
(a) become half (b) become twice as long
(c) remain unchanged (d) None of these

20. How many calories of heat are produced, if one gram of substance is completely converted into heat energy?
(a) 2.1×10^{13} cal approximately
(b) 4.2×10^{13} cal approximately
(c) 2.1×10^{16} cal approximately
(d) 4.2×10^{16} cal approximately

21. The relativistic increase in mass was first experimentally detected in the case of electrons. The reason for this is that
(a) the relativistic increase in mass occurs only for subatomic particles
(b) the relativistic increase in mass occurs only for negatively charged particles
(c) the electrons can be imparted speed comparable with the speed of light in vacuum
(d) the relativistic increase in mass is sizeable in the case of electrons

22. Which one of the following particle parameters remains unchanged even at relativistic speeds?
(a) Charge (b) Mass
(c) Linear dimensions (d) Charge to mass ratio

23. If a train were to move with the velocity of light, its length would be
(a) infinite (b) zero
(c) unchanged (d) finite but indeterminant

24. The energy equivalent to 1 kg of matter in kilowatt hour is
(a) 2.5×10^{10} (b) 2.5×10^{12}
(c) 2.5×10^{14} (d) 2.5×10^{16}

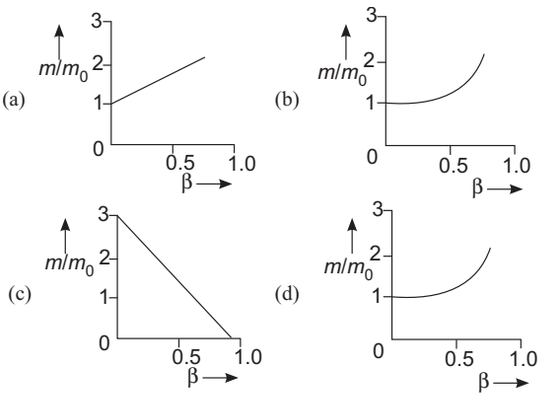
25. Which of the following groups of particles have zero rest mass?
(a) Photons and positrons (b) Neutrinos and mesons
(c) Mesons and neutrons (d) Photons and neutrinos

26. A particle which has zero rest mass and non-zero energy and momentum must travel with a speed
(a) equal to c (b) greater than c
(c) less than c (d) tending to infinity

27. An observer measures the length of a spaceship and finds that it is exactly one-third of its proper length. What is the speed of the spaceship relative to the observer?
(a) $\frac{2\sqrt{2}}{3}c$ (b) $\frac{2}{3}c$ (c) $\frac{\sqrt{2}}{3}c$ (d) c

28. The time interval between two events as measured by an observer A in a spaceship is 1s. What is the time interval between the same two events as measured by an observer B with respect to which the spaceship is moving with a velocity $\frac{2\sqrt{2}}{3}c$?
(a) 0.5 s (b) 1 s (c) 2 s (d) 3 s

29. The rest mass of a particle is m_0 and m is its mass when it moves with a velocity v. Which one of the graphs shown in figure represents the variation of m/m_0 with $\beta = v/c$, where c is the speed of light in vacuum?



30. Whose experimental work proved that the velocity of light is a universal and natural constant
(a) Michelson and Morley (b) Lorentz
(c) Maxwell (d) Einstein

31. The theory of relativity show that Newtonian mechanics is valid for
(a) very small velocities
(b) velocities upto velocity of light
(c) velocity equal to velocity of light
(d) velocity greater than the velocity of light

32. When a particle and its antiparticle are annihilated the energy released is E. What is the mass of each particle?
(a) E/c (b) $E/2c$ (c) E/c^2 (d) $E/2c^2$

Answers

Level 1

Objective Problems

1. (c)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (d)
11. (c)	**12.** (c)	**13.** (c)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (a)
21. (c)	**22.** (a)	**23.** (b)	**24.** (a)	**25.** (d)	**26.** (a)	**27.** (a)	**28.** (d)	**29.** (b)	**30.** (a)
31. (a)	**32.** (d)								

JEE Main & Advanced

Solved Paper

2015

Solved Paper 2015

JEE Main

Joint Entrance Examination

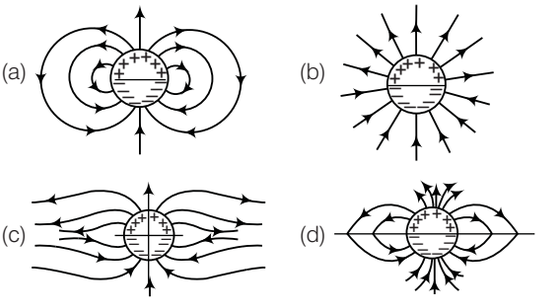
Physics

Instructions

- This test consists of 17 questions.
- Each question is allotted 4 marks for correct response.
- Candidates will be awarded marks as stated above for correct response of each question. 1 marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
- There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

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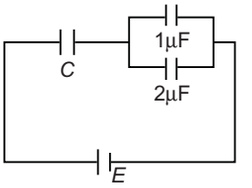
1. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in (figures are schematic and not drawn to scale)

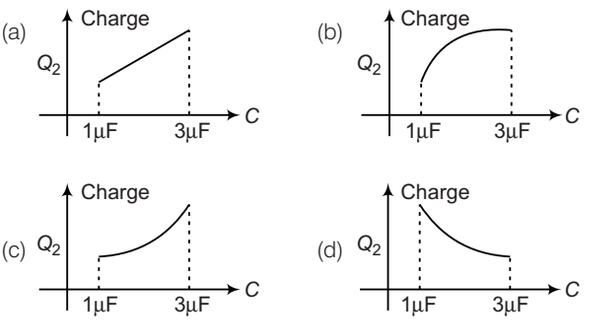


2. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere, the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3, and R_4 respectively. Then,
   (a) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
   (b) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
   (c) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
   (d) $2R < R_4$

3. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of C is given properly by

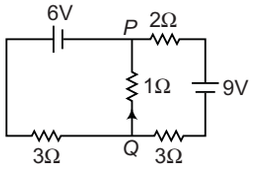
(figures are drawn schematically and are not to scale)





4. When 5V potential difference is applied across a wire of length 0.1m, the drift speed of electrons is $2.5 \times 10^{-4}\,\text{ms}^{-1}$. If the electron density in the wire is $8 \times 10^{28}\ \text{m}^{-3}$ the resistivity of the material is close to
   (a) $1.6 \times 10^{-8}\ \Omega\text{m}$
   (b) $1.6 \times 10^{-7}\ \Omega\text{m}$
   (c) $1.6 \times 10^{-6}\ \Omega\text{m}$
   (d) $1.6 \times 10^{-5}\ \Omega\text{m}$

5. In the circuit shown below, the current in the 1Ω resistor is

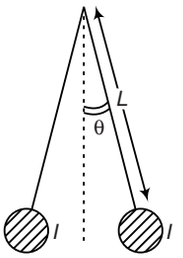


(a) 1.3 A, from P to Q (b) 0 A
(c) 0.13 A, from Q to P (d) 0.13 A, from P to Q

6. Two coaxial solenoids of different radii carry current I in the same direction. Let $\mathbf{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\mathbf{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then,

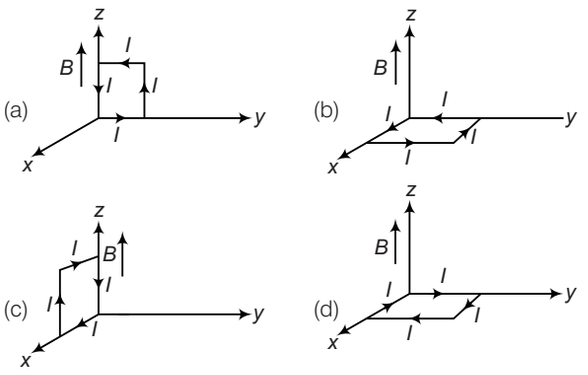
(a) $F_1 = F_2 = 0$
(b) F_1 is radially inwards and F_2 is radially outwards
(c) F_1 is radially inwards and $F_2 = 0$
(d) F_1 is radially outwards and $F_2 = 0$

7. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then, the value of I is (g = gravitational acceleration)



(a) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$ (b) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$
(c) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$ (d) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$

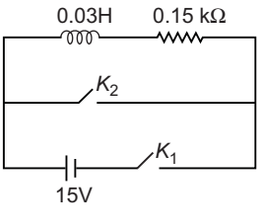
8. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below.



If there is a uniform magnetic field of 0.3 T in the positive z-direction in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

(a) (a) and (b) respectively (b) (a) and (c) respectively
(c) (b) and (d) respectively (d) (b) and (c) respectively

9. An inductor ($L = 0.03$ H) and a resistor ($R = 0.15$ kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ms, the current in the circuit will be ($e^5 \cong 150$)

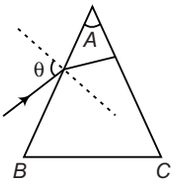


(a) 100 mA (b) 67 mA (c) 6.7 mA (d) 0.67 mA

10. A red LED emits light at 0.1 W uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is

(a) 1.73 V/m (b) 2.45 V/m (c) 5.48 V/m (d) 7.75 V/m

11. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided



(a) $\theta > \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
(b) $\theta < \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
(c) $\theta > \cos^{-1}\left[\mu\sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$
(d) $\theta < \cos^{-1}\left[\mu\sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

12. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens principle leads us to conclude that as it travels, the light beam

(a) becomes narrower
(b) goes horizontally without any deflection
(c) bends downwards
(d) bends upwards

13. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is
(a) 1 μm (b) 30 μm
(c) 100 μm (d) 300 μm

14. As an electron makes a transition from an excited state to the ground state of a hydrogen like atom/ion
(a) its kinetic energy increases but potential energy and total energy decrease
(b) kinetic energy, potential energy and total energy decrease
(c) kinetic energy decreases, potential energy increases but total energy remains same
(d) kinetic energy and total energy decrease but potential energy increases

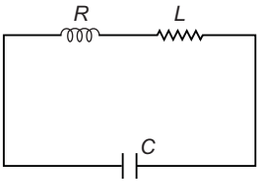
15. Match List-I (fundamental experiment) with List-II (its conclusion) and select the correct option from the choices given below the list.

List I	List II
A. Franck-Hertz experiment	1. Particle nature of light
B. Photo-electric experiment	2. Discrete energy levels of atom
C. Davisson-Germer experiment	3. Wave nature of electron
	4. Structure of atom

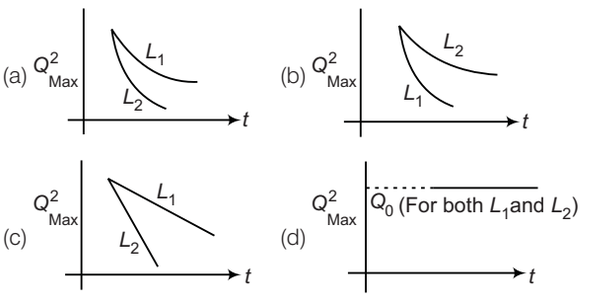
	A	B	C
(a)	1	4	3
(b)	2	4	3
(c)	2	1	3
(d)	4	3	2

16. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2MHz. The frequencies of the resultant signal is/are
(a) 2 MHz only
(b) 2005 kHz and 1995 kHz
(c) 2005 kHz 2000 kHz and 1995 kHz
(d) 2000 kHz and 1995 kHz

17. An *LCR* circuit is equivalent to a damped pendulum. In an *LCR* circuit, the capacitor is charged to Q_0 and then connected to the L and R as shown below.



If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L, then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)

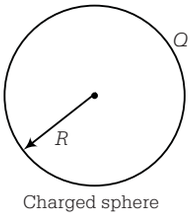


Answer *with* Explanations

1. ***(a)*** Field lines should originate from positive charge and terminate to negative charge. Thus, (b) and (c) are not possible.

Electric field lines cannot form corners as shown in (d).
Thus, correct option is (a).

2. ***(c, d)*** Potential at the surface of the charged sphere



Charged sphere

$$V_0 = \frac{KQ}{R}$$

$$V = \frac{KQ}{r}, r \geq R$$

$$= \frac{KQ}{2R^3}(3R^2 - r^2); r \leq R$$

$$V_{\text{centre}} = V_c = \frac{KQ}{2R^3} \times 3R^2$$

$$= \frac{3KQ}{2R} = \frac{3V_0}{2}$$

$$\Rightarrow \quad R_1 = 0$$

As potential decreases for outside points.
Thus, according to the question, we can write

$$V_{R_2} = \frac{5V_0}{4} = \frac{KQ}{2R^3}(3R^2 - R_2^2)$$

$$\frac{5V_0}{4} = \frac{V_0}{2R^2}(3R^2 - R_2^2)$$

or $$\frac{5}{2} = 3 - \left(\frac{R_2}{R}\right)^2$$

or $$\left(\frac{R_2}{R}\right)^2 = 3 - \frac{5}{2} = \frac{1}{2}$$

or $$R_2 = \frac{R}{\sqrt{2}}$$

Similarly,

$$V_{R_3} = \frac{3V_0}{4}$$

$$\Rightarrow \quad \frac{KQ}{R_3} = \frac{3}{4} \times \frac{KQ}{R}$$

or $$R_3 = \frac{4}{3} R$$

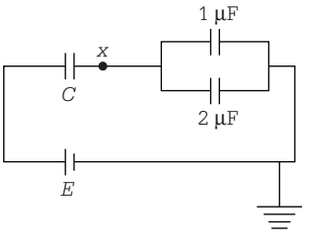
$$V_{R4} = \frac{KQ}{R_4} = \frac{V_0}{4}$$

$$\Rightarrow \quad \frac{KQ}{R_4} = \frac{1}{4} \times \frac{KQ}{R}$$

or $$R_4 = 4R$$

3. *(b)* ***Central Idea*** *Assume negative terminal of the battery as grounded* (0 V).

Suppose, potential of point x is V.



From the circuit diagram, we can write

$$Q_C = Q_1 + Q_2$$

or $$C(E - V) = 1 \times V + 2 \times V$$

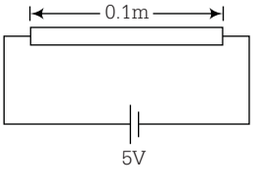
or $$V[C + 3] = CE \text{ or } V = \frac{CE}{3 + C}$$

$$\therefore \quad Q_2 = C_2 (V) = \frac{2\,CE}{3 + C} = \frac{2E}{1 + 3/C}$$

As C_1 varied from 1 μF to 3 μF, charge increases with decreasing slope.

Note As $C \to \infty, Q_2 \to 2E$ = constant

4. *(d)* According to the question



$$v_d = 2.5 \times 10^{-4} \text{ m/s}$$

$$\Rightarrow \quad n = 8 \times 10^{28} / \text{m}^3$$

we know that

$$J = nev_d \text{ or } I = nev_d A$$

where, symbols have their usual meaning.

$$\Rightarrow \quad \frac{V}{R} = nev_d A$$

or $$\frac{V}{\frac{\rho L}{A}} = nev_d A$$

or $$\frac{V}{\rho L} = nev_d$$

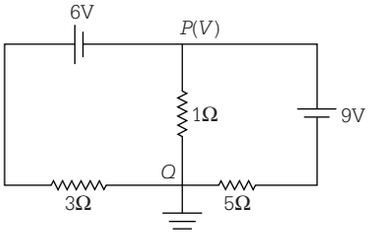
or $$\rho = \frac{V}{nev_d L}$$

$$= \frac{5}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4} \times 0.1}$$

or $$\rho = 1.6 \times 10^{-5} \ \Omega\text{m}$$

5. *(c)* ***Central Idea*** *Connect point Q to ground and apply KCL.*

Consider the grounded circuit as shown below.



Applying KCL of point Q we can write

Incoming current at Q = outgoing current from Q

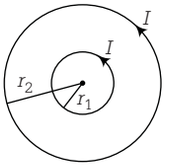
$$\Rightarrow \quad \frac{V + 6}{3} + \frac{V}{1} = \frac{9 - V}{5}$$

or $$V\left[\frac{1}{3} + \frac{1}{5} + 1\right] = \frac{9}{5} - 2 \quad \text{or } V\left[\frac{5 + 3 + 15}{15}\right] = \frac{9 - 10}{5}$$

or $$V\left[\frac{23}{15}\right] = \frac{-1}{5} \quad \text{or } V = \frac{-3}{23} = -0.13\,\text{V}$$

Thus, current in the 1 Ω resistance is 0.13 A, from Q to P.

6. *(a)* Consider the two coaxial solenoids. Due to one of the solenoids magnetic field at the centre of the other can be assumed to be constant.

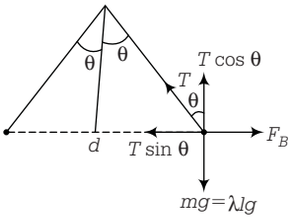


Due to symmetry, forces on upper and lower part of a solenoid will be equal and opposite and hence resultant is zero.

Therefore $F_1 = F_2 = 0$

7. *(b)* Consider free body diagram of the wire.

As the wires are in equilibrium, they must carry current in opposite direction .



Here, $F_B = \frac{\mu_0 I^2 l}{2\pi d}$, where l is length of each wire are d is separation between wires.

From figure, $d = 2L\sin\theta$

$$T = \cos\theta = mg = \lambda lg \quad \text{(in vertical direction)} \quad \text{....(i)}$$

$$T\sin\theta = F_B = \frac{\mu_0 I^2 l}{4\pi L \sin\theta} \quad \text{(In horizontal direction)} \quad \text{....(ii)}$$

From Eqs. (i) and (ii),

$$\frac{T\sin\theta}{T\cos\theta} = \frac{\mu_0 I^2 l}{4\pi L\sin\theta \times \lambda lg}$$

$$\therefore \quad I = \sqrt{\frac{4\pi\lambda Lg\sin^2\theta}{\mu_0\cos\theta}} = 2\sin\theta\sqrt{\frac{\pi\lambda Lg}{\mu_0\cos\theta}}$$

8. *(c) Since* B *is uniform only torque acts on a current carrying loop.*

As, $\tau = M \times B$

$\Rightarrow$ $|\tau| = |M||B|\sin\theta$

For orientation shown in (b) $\theta = 0^0$, $\tau = 0$ (stable equilibrium)

and for (d) $\theta = \pi$, $\tau = 0$ (unstable equilibrium)

9. *(d)* ***Central Idea*** *After long time inductor behave as short-circuit.*

At $t = 0$, the inductor behaves as short-circuited. The current

$$I_0 = \frac{E_0}{R} = \frac{15\,\text{V}}{0.15\,\text{k}\Omega} = 100\,\text{mA}$$

As K_2 is closed, current through the inductor starts decay, which is given at any time t as

$$I = I_0\, e^{\frac{-tR}{L}} = (100\,\text{mA})\, e^{\frac{-t\times 15000}{3}}$$

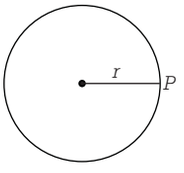
At $t = 1\,\text{ms}$

$$I = (100\,\text{mA})e^{-\frac{1\times10^{-3}\times15\times10^3}{3}}$$

$$I = (100\ \text{mA})\, e^{-5} = 0.6737\ \text{mA or } I = 0.67\ \text{mA}$$

10. *(b)* Consider the LED as a point source of light.

Let power of the LED is P.



Intensity at r from the source

$$I = \frac{P}{4\pi r^2} \qquad \ldots(i)$$

As we know that $I = \frac{1}{2}\varepsilon_0 E_0^2\, c$ ...(ii)

From Eqs. (i) and (ii), we can write

$$\frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0^2\, c$$

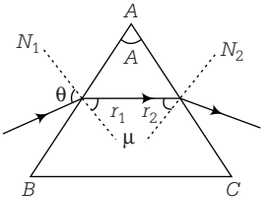
or $$E_0^2 = \frac{2P}{4\pi\varepsilon_0 r^2 c} = \frac{2\times0.1\times9\times10^9}{1\times3\times10^8}$$

or $$E_0^2 = 6$$

$\Rightarrow$ $$E_0 = \sqrt{6} = 2.45\,\text{V/m}$$

11. *(a)* ***Central Idea*** *The ray will get transmitted through face AC if* $i_{AC} < i_C$

Consider the ray diagram is shown below.



A ray of light incident on face AB at an angle θ.

r_1 = Angle of refraction on face AB

r_2 = Angle of incidence at face AC

For transmission of light through face AC

$$i_{AC} < i_C$$

or $$A - r_1 < i_C$$

or $$\sin(A - r_1) < \sin i_C$$

or $$\sin(A - r_1) < \frac{1}{\mu}$$

$$A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

or $$\sin r_1 > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

Now, applying Snell's law at the face AB

$$1\times\sin\theta = \mu\sin r_1$$

or $$\sin r_1 = \frac{\sin\theta}{\mu}$$

$\Rightarrow$ $$\frac{\sin\theta}{\mu} > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

or $$\theta > \sin^{-1}\left[\mu\sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$$

12. *(d)* According to Snell's law,

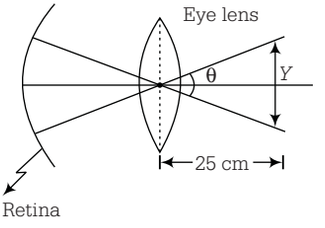
$$\mu\sin\theta = \text{Constant}$$

$\therefore$ $$\sin\theta \propto \frac{1}{\mu}$$

As μ increases, θ decreases.

Hence, beam will bend upward.

13. *(b)* We can write resolving angle of necked eye as



$$\theta = 1.22\frac{\lambda}{D}$$

Where, D is the diameter of eye lens.

or $$\frac{Y}{25\times10^{-2}} = \frac{1.22\times500\times10^{-9}}{0.25\times2\times10^{-2}}$$

$$Y = 30\times10^{-6}\ \text{m} = 30\,\mu\text{m}$$

14. *(a)* As we know that kinetic energy of an electron is

$$\text{KE} \propto \left(\frac{Z}{n}\right)^2$$

When the electron makes transition from an excited state to the ground state, then n decreases and KE increases. We know that PE is lowest for ground state. As TE = – KE. TE also decreases.

15. *(c)* (A) Franck-Hertz experiments is associated with discrete energy levels of atom.

(B) Photo-electric experiment is associated with particle nature of light.

(C) Davisson-Germer experiment is associated with wave nature of electron.

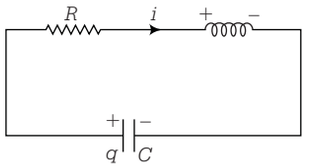
16. *(c)* ***Central Idea*** *Frequency associated with AM are* $f_c - f_m, f, f_c + f_m$

According to the question

$$f_c = 2\,\text{MHz} = 2000\ \text{kHz}$$
$$f_m = 5\ \text{kHz}$$

Thus, frequency of the resultant signal is/are carrier frequency $f_c = 2000$ kHz, LSB frequency $f_c - f_m = 2000\ \text{kHz} - 5\ \text{kHz} = 1995\ \text{kHz}$ and USB frequency $f_c + f_m = 2005$ kHz

17. *(a)* Considert the LCR circuit at any time t



Now, applying KVL

We have $\dfrac{q}{C} - iR - \dfrac{Ldi}{dt} = 0$

As current is decreasing with time we can write $i = -\dfrac{dq}{dt}$

$$\Rightarrow \quad \frac{q}{C} + \frac{dq}{dt}R + \frac{Ld^2q}{dt^2} = 0$$

or $$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$$

This equation is equivalent to that of a damped oscillator

Thus, we can write the solution as

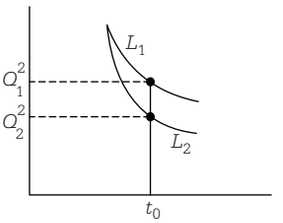
$$Q_{\max}(t) = Q_0 \cdot e^{-Rt/2L}$$

or $$Q^2_{\max} = Q_0^2\, e^{-\frac{Rt}{L}}$$

As $L_1 > L_2$ damping is faster for L_2

Aliter

Inductance is inertia of circuit. It means inductance opposes the flow of charge more inductance means decay of charge is slow. In option (a), in a given time to, $Q_1^2 > Q_2^2$.



So, $L_1 > L_2$.

Hence option (a) is correct.

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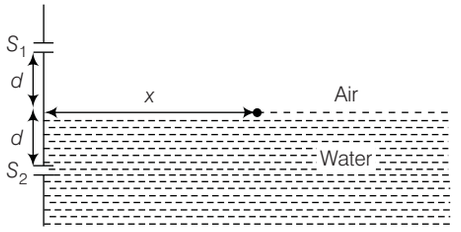
JEE Advanced

Paper 1

Section 1 (Maximum Marks : 20)

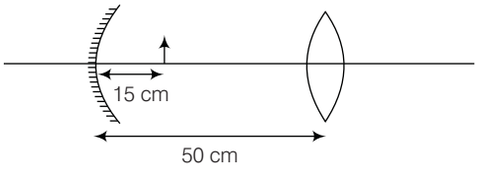
- This section contains FIVE questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in ORS.
- Marking scheme :
  + 4 If the bubble corresponding to the answer is darkened
  0 in all other cases

1. A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), $2d$ is the separation between the slits and m is an integer. The value of p is

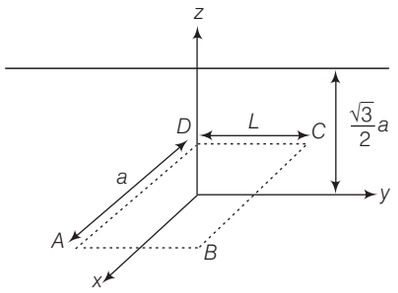


2. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1.

When the set-up is kept in a medium of refractive index $\frac{7}{6}$, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is



3. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface $ABCD$ lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is



4. Consider a hydrogen atom with its electro in the nth orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is
($hc = 1242$ eV nm)

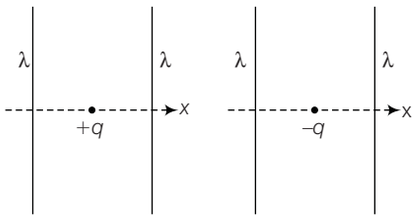
5. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel.

The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is

Section 2 (Maximum Marks : 20)

- This section contains FIVE questions.
- Each question has FOUR options (a), (b), (c) and (d), ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking scheme :
  + 4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
  0 If none of the bubbles is darkened.
  – 2 In all other cases.

6. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other.



In their resulting electric field, point charges q and $-q$ are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statements is/are

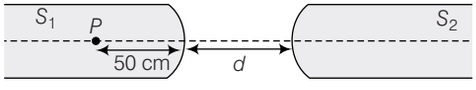
(a) both charges execute simple harmonic motion

(b) both charges will continue moving in the direction of their displacement

(c) charge + q executes simple harmonic motion while charge – q continues moving in the direction of its displacement

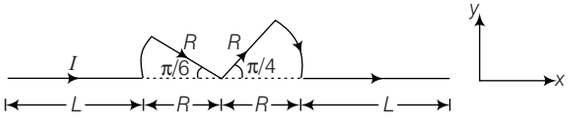
(d) charge – q executes simple harmonic motion while charge + q continues moving in the direction of its displacement

7. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is



(a) 60 cm (b) 70 cm (c) 80 cm (d) 90 cm

8. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field **B**. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statements is/are



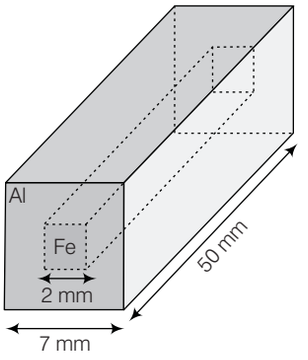
(a) if **B** is along $\hat{\mathbf{z}}$, $F \propto (L + R)$

(b) if **B** is along $\hat{\mathbf{x}}$, $F = 0$

(c) if **B** is along $\hat{\mathbf{y}}$, $F \propto (L + R)$

(d) if **B** is along $\hat{\mathbf{z}}$, $F = 0$

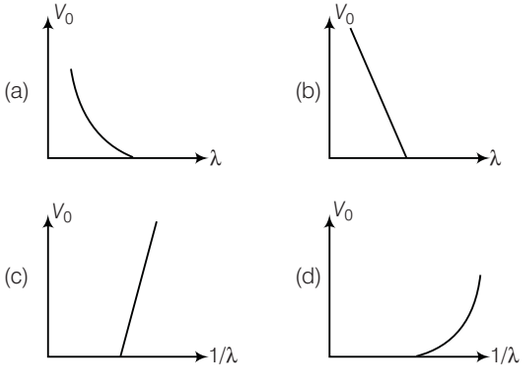
9. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8}\ \Omega$ m and $1.0 \times 10^{-7}\ \Omega$ m, respectively. The electrical resistance between the two faces P and Q of the composite bar is



(a) $\frac{2475}{64}\mu\Omega$ (b) $\frac{1875}{64}\mu\Omega$

(c) $\frac{1875}{49}\mu\Omega$ (d) $\frac{2475}{132}\mu\Omega$

10. For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and $\frac{1}{\lambda}$.



Section 3 (Maximum Marks : 8)

- This section contains ONE questions.
- Each question contains two columns, Column I and Column II.
- Column I has four entries (A), (B), (C) and (D).
- Column II has five entries (P), (Q), (R), (S) and (T)
- Match the entries in Column I with the entries in Column II.
- One or more entries in Column I may match with one or more entries in Column II.
- The ORS contains a 4×5 matrix whose layout will be similar to the one shown below :

(A) (P) (Q) (R) (S) (T)
(B) (P) (Q) (R) (S) (T)
(C) (P) (Q) (R) (S) (T)
(D) (P) (Q) (R) (S) (T)

- For each entry in Column I, darken the bubbles of all the matching entries. For example, if entry (A) in Column I matches with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- Marking schemes:

For each entry in Column I

+ 2 If only the bubble(s) corresponding to all the correct match(es) is (are) darkened.
0 If none of the bubbles is darkened.
– 1 In all other cases.

11. Match the nuclear processes given in Column I with the appropriate option(s) in Column II.

	Column I		Column II
A.	Nuclear fusion	P.	absorption of thermal neutrons by $^{235}_{92}U$
B.	Fission in a nuclear reactor	Q.	$^{60}_{27}Co$ nucleus
C.	β-decay	R.	Energy production in stars via hydrogen conversion to helium
D.	γ-ray emission	S.	Heavy water
		T.	Neutrino emission

Paper 2

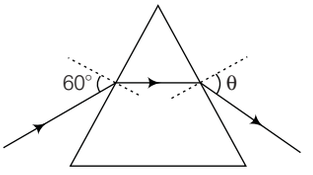
Section 1 (Maximum Marks : 16)

- This section contains FOUR questions.
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9 both inclusive.
- For each question, darken the bubble corresponding to the correct integer in ORS.
- Marking scheme :

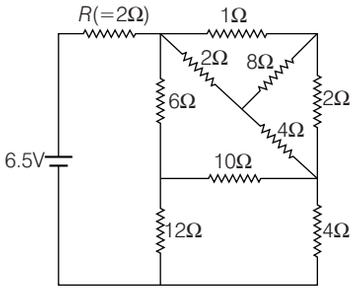
+4 If the bubble corresponding to the answer is darkened.
0 in all other cases.

1. For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where $N(t)$ is the number of nuclei at time t. Two radioactive source P(mean life τ) and Q (mean life 2τ) have the same activity at $t = 0$. Their rate of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is

2. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is



3. In the following circuit, the current through the resistor $R(=2\,\Omega)$ is I amperes. The value of I is



4. An electron in an excited state of Li^{2+} ion has angular momentum $\frac{3h}{2\pi}$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is

Section 2 (Maximum Marks : 12)

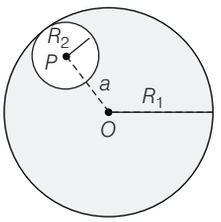
- This section contains THREE questions.
- Each question has FOUR options (a), (b), (c) and (d), ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking scheme :

  + 4 If only the bubble(s) corresponding to all the correct optin(s) is(are) darkened.

  0 If none of the bubbles is darkened.

  – 2 In all other cases.

5. Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance $OP = a = R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\mathbf{r}$ is $\mathbf{E}(\mathbf{r})$, then the correct statements is/are



(a) **E** is uniform, its magnitude is independent of R_2 but its direction depends on **r**

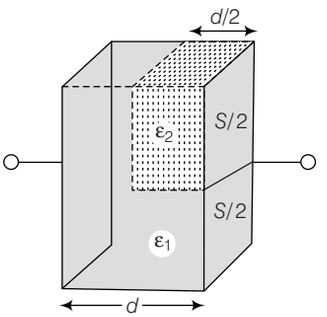
(b) **E** is uniform, its magnitude depends on R_2 and its direction depends on **r**

(c) **E** is uniform, its magnitude is independent of 'a' but its direction depends on **a**

(d) **E** is uniform and both its magnitude and direction depend on **a**

6. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2.

The ratio $\frac{C_2}{C_1}$ is



(a) $\frac{6}{5}$ (b) $\frac{5}{3}$

(c) $\frac{7}{5}$ (d) $\frac{7}{3}$

7. A fission reaction is given by ${}^{236}_{92}\mathrm{U} \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + x + y$, where x and y are two particles. Considering ${}^{236}_{92}\mathrm{U}$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_x(2 MeV) and K_y(2 MeV), respectively. Let the binding energies per nucleon of ${}^{236}_{92}\mathrm{U}$, ${}^{140}_{54}\mathrm{Xe}$ and ${}^{94}_{38}\mathrm{Sr}$ be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct options is/are

(a) $x = n,\ y = n,\ K_{\mathrm{Sr}} = 129\,\mathrm{MeV},\ K_{\mathrm{Xe}} = 86\,\mathrm{MeV}$

(b) $x = p,\ y = e^-,\ K_{\mathrm{Sr}} = 129\,\mathrm{MeV},\ K_{\mathrm{Xe}} = 86\,\mathrm{MeV}$

(c) $x = p,\ y = n,\ K_{\mathrm{Sr}} = 129\,\mathrm{MeV},\ K_{\mathrm{Xe}} = 86\,\mathrm{MeV}$

(d) $x = n,\ y = n,\ K_{\mathrm{Sr}} = 86\,\mathrm{MeV},\ K_{\mathrm{Xe}} = 129\,\mathrm{MeV}$

Section 3 (Maximum Marks : 16)

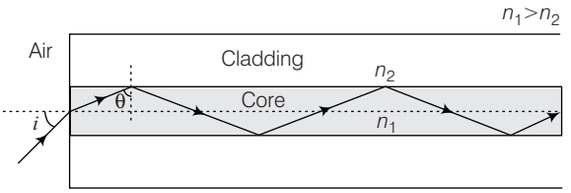
- This section contains TWO paragraphs.
- Based on each paragraph, there will be TWO questions.
- Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- Marking scheme:

+ 4 If only the bubble(s) corresponding to all the corect option(s) is(are) darkened.
0 If none of the bubbles is darkened.
– 2 In all other cases.

Paragraph I

Light guidance in an optical fibre can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as $\sin i_m$.

8. For two structures namely S_1 with $n_1 = \frac{\sqrt{45}}{4}$ and $n_2 = \frac{3}{2}$, and S_2 with $n_1 = \frac{8}{5}$ and $n_2 = \frac{7}{5}$ and taking the refractive index of water to be $\frac{4}{3}$ and that to air to be 1, the correct options is/are



(a) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(b) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water

(c) NA of S_1 placed in air is the same as that S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$

(d) NA of S_1 placed in air is the same as that of S_2 placed in water

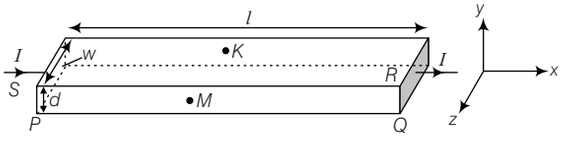
9. If two structures of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

(a) $\frac{NA_1 NA_2}{NA_1 + NA_2}$ (b) $NA_1 + NA_2$

(c) NA_1 (d) NA_2

Paragraph II

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are l, w and d, respectively. A uniform magnetic field **B** is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface $PQRS$ and appearance of equal and opposite charges on the face opposite to $PQRS$. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



10. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statements is/are

(a) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$

(b) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

(c) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$

(d) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

11. Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct options is/are

(a) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

(b) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

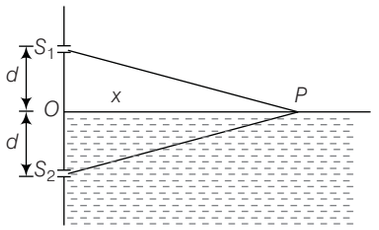
(c) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

(d) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

Answer with Explanations

Paper 1

1. (3)



$$\mu(S_2P) - S_1P = m\lambda$$

$$\Rightarrow \quad \mu\sqrt{d^2 + x^2} - \sqrt{d^2 + x^2} = m\lambda$$

$$\Rightarrow \quad (\mu - 1)\sqrt{d^2 + x^2} = m\lambda$$

$$\Rightarrow \quad \left(\frac{4}{3} - 1\right)\sqrt{d^2 + x^2} = m\lambda$$

or $$\sqrt{d^2 + x^2} = 3m\lambda$$

Squaring this equation we get,

$$x^2 = 9m^2\lambda^2 - d^2$$

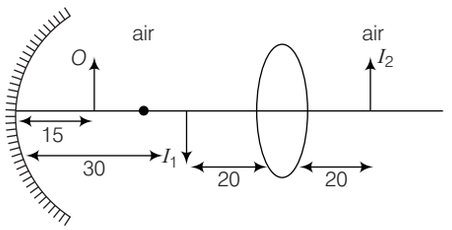
$$\Rightarrow \quad p^2 = 9 \quad \text{or} \quad p = 3$$

2. (7) Case I

Reflection from mirror

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \Rightarrow \quad \frac{1}{-10} = \frac{1}{v} + \frac{1}{-15}$$

$$\Rightarrow \quad v = -30$$



For lens

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

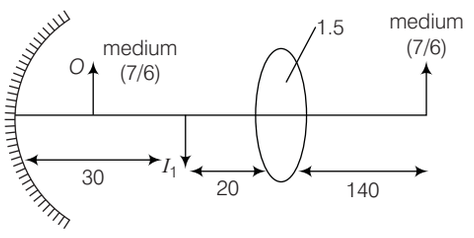
$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-20}$$

$$v = 20$$

$$|M_1| = \left|\frac{v_1}{u_1}\right|\left|\frac{v_2}{u_2}\right| = \left(\frac{30}{15}\right)\left(\frac{20}{20}\right) = 2 \times 1 = 2 \qquad \text{(in air)}$$

Case II For mirror, there is no change.

$$v = -30$$



For lens,

$$\frac{1}{f_{air}} = \left(\frac{3/2}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_{medium}} = \left(\frac{3/2}{7/6} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

with $$f_{air} = 10 \text{ cm}$$

We get $$\frac{1}{f_{medium}} = \frac{4}{70} \text{ cm}^{-1}$$

$$\frac{1}{v} - \frac{1}{-20} = \frac{4}{70}$$

$$\frac{1}{v} + \frac{1}{20} = \left(\frac{2}{7}\right)\left(\frac{2}{10}\right) = \frac{4}{70}$$

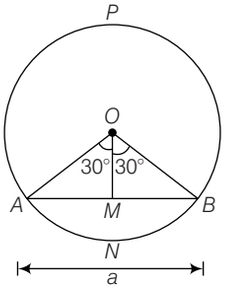
$$\frac{1}{v} = \frac{4}{70} - \frac{1}{20} \quad \Rightarrow \quad v = 140,$$

$$|M_2| = \left|\frac{v_1}{u_1}\right|\left|\frac{v_2}{u_2}\right| = \left(\frac{30}{15}\right)\left(\frac{140}{20}\right),$$

$$= (2)\left(\frac{140}{20}\right) = 14$$

$$\Rightarrow \quad \left|\frac{M_2}{M_1}\right| = \frac{14}{2} = 7$$

3. (6) *ANBP* is cross-section of a cylinder of length *L*. The line charge passes through the centre *O* and perpendicular to paper.



$$AM = \frac{a}{2}, MO = \frac{\sqrt{3}a}{2}$$

$$\therefore \quad \angle AOM = \tan^{-1}\left(\frac{AM}{OM}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

Electric flux passing from the whole cylinder

$$\phi_1 = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$\therefore$ Electric flux passing through *ABCD* plane surface (shown only *AB*) = Electric flux passing through cylindrical surface *ANB*

$$= \left(\frac{60°}{360°}\right)(\phi_1) = \frac{\lambda L}{6\varepsilon_0} \quad \Rightarrow \quad \therefore \quad n = 6$$

4. (2) Kinetic energy of ejected electron

= Energy of incident photon − energy required to ionize the electron from *n*th orbit (all in eV)

$$\therefore \qquad 10.4 = \frac{1242}{90} - |E_n|$$

$$= \frac{1242}{90} - \frac{13.6}{n^2} \text{ (as } E_n \propto \frac{1}{n^2} \text{ and } E_1 = -13.6 \text{ eV)}$$

Solving this equation we get

$$n = 2$$

5. (3) Let initial power available from the plant is P_0. After time $t = nT$ or n half lives, this will become $\left(\frac{1}{2}\right)^n P_0$. Now, it is given that,

$$\left(\frac{1}{2}\right)^n P_0 = 12.5\% \text{ of } P_0 = (0.125)\, P_0$$

Solving this equation we get,

$$n = 3$$

6. *(c)* At the shown position, net force on both charges is zero. Hence they are in equilibrium. But equilibrium of $+q$ is stable equilibrium. So, it will start oscillations when displaced from this position. These small oscillations are simple harmonic in nature. While equilibrium of $-q$ is unstable. So, it continues to move in the direction of its displacement.

7. *(b)* $R = 10$ cm

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ two times

$$\frac{1}{v} - \frac{1.5}{-50} = \frac{1 - 1.5}{-10}$$

$$\frac{1}{v} + \frac{1.5}{50} = \frac{0.5}{10}$$

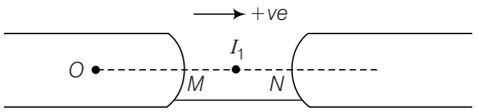
$$\frac{1}{v} = \frac{0.5}{10} - \frac{1.5}{50}$$

$$= \frac{2.5 - 1.5}{50}$$

$\Rightarrow$ $v = 50$

$MN = d$, $MI_1 = 50$ cm,

$NI_1 = (d - 50)$ cm

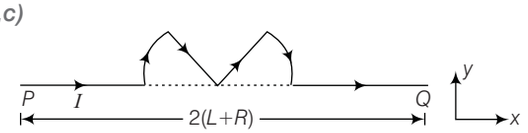


Again, $\frac{1.5}{\infty} - \frac{1}{-(d-50)} = \frac{1.5 - 1}{10}$

$$\frac{1}{d - 50} = \frac{1}{20}$$

$$d = 70$$

8. *(a,b,c)*



Force on the complete wire = force on straight wire PQ carrying a current I.

$$\mathbf{F} = I(\mathbf{PQ} \times \mathbf{B}) = I[\{2(L + R)\hat{\mathbf{i}}\} \times \mathbf{B}]$$

This force is zero if $\mathbf{B}$ is along $\hat{\mathbf{i}}$ direction or x-direction. If magnetic field is along $\hat{\mathbf{j}}$ direction or $\hat{\mathbf{k}}$ direction,

$$|\mathbf{F}| = F = (I)(2)(L + R)B \sin 90°$$

or $F = 2I(L + R)B$ or $F \propto (L + R)$

$\therefore$ Options (a), (b) and (c) are correct.

9. *(b)* $\frac{1}{R} = \frac{1}{R_{Al}} + \frac{1}{R_{Fe}} = \left(\frac{A_{Al}}{\rho_{Al}} + \frac{A_{Fe}}{\rho_{Fe}}\right)\frac{1}{\ell}$

$$= \left[\frac{(7^2 - 2^2)}{2.7} + \frac{2^2}{10}\right]\frac{10^{-6}}{10^{-8}} \times \frac{1}{50 \times 10^{-3}}$$

Solving we get,

$$R = \frac{1875}{64} \times 10^{-6}\ \Omega$$

$$= \frac{1875}{64}\ \mu\Omega$$

10. *(a, c)* $eV_0 = \frac{hc}{\lambda} - W$

$$V_0 = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \frac{W}{e}$$

V_0 versus $\frac{1}{\lambda}$ graph is in the form $y = mx - c$

Therefore option (c) is correct.

Clearly, V_0 versus λ graph is not a straight line but V_0 decreases with increase in λ and V_0 becomes zero when $\frac{hc}{\lambda} = W$.

i.e. $\lambda = \lambda_0$ (Threshold wavelength)

$\therefore$ Option (a) is also correct.

11. (A) $\rightarrow$ (R or RT); (B) $\rightarrow$ (P, S); (C) $\rightarrow$ (Q, T); (D) $\rightarrow$ (R,)

No solution is required.

Paper 2

1. (2) Let initial numbers are N_1 and N_2.

$$\frac{\lambda_1}{\lambda_2} = \frac{\tau_2}{\tau_1} = \frac{2\tau}{\tau} = 2 = \frac{T_2}{T_1} \qquad (T = \text{Half life})$$

$$A = \frac{-dN}{dt} = \lambda N$$

Initial activity is same

$\therefore$ $\lambda_1 N_1 = \lambda_2 N_2$ ...(i)

Activity at time t,

$A = \lambda N = \lambda N_0\, e^{-\lambda t}$ $\Rightarrow$ $A_1 = \lambda_1 N_1\, e^{-\lambda_1 t}$

$\Rightarrow$ $R_1 - = \frac{dA_1}{dt} = \lambda_1^2\, N_1\, e^{-\lambda_1 t}$

Similarly, $R_2 = \lambda_2^2\, N_2\, e^{-\lambda_2 t}$

After $t = 2\tau$

$$\lambda_1 t = \frac{1}{\tau_1}(t) = \frac{1}{\tau}(2\tau) = 2$$

$$\lambda_2 t = \frac{1}{\tau_2}(t) = 1 = \frac{1}{2\tau}(2\tau) = 1$$

$$\frac{R_P}{R_Q} = \frac{\lambda_1^2 N_1 e^{-\lambda_1 t}}{\lambda_2^2 N_2 e^{-\lambda_2 t}}$$

$$\frac{R_P}{R_Q} = \frac{\lambda_1}{\lambda_1}\left(\frac{e^{-2}}{e^{-1}}\right) = \frac{2}{e}$$

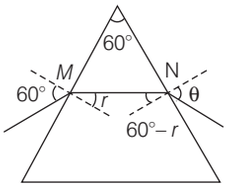
2. (2) Applying Snell's law at M and N,

$$\sin 60° = n \sin r \quad ...(i)$$

$$\sin \theta = n \sin (60 - r) \quad ...(ii)$$

Differentiating we get

$$\cos \theta \frac{d\theta}{dn} = -n \cos (60-r) \frac{dr}{dn} + \sin (60-r)$$



Differentiating Eq. (i),

$$n \cos r \frac{dr}{dn} + \sin r = 0$$

or $$\frac{dr}{dn} = -\frac{\sin r}{n \cos r} = \frac{-\tan r}{n}$$

$$\Rightarrow \quad \cos \theta \frac{d\theta}{dn} = -n \cos (60-r)\left(\frac{-\tan r}{n}\right) + \sin (60-r)$$

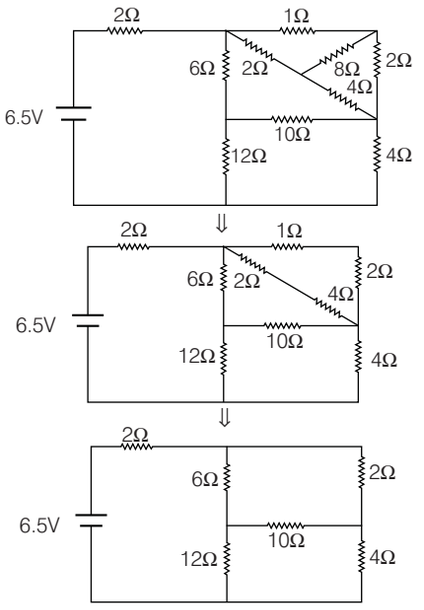
$$\frac{d\theta}{dn} = \frac{1}{\cos \theta}[\cos(60-r)\tan r + \sin (60-r)]$$

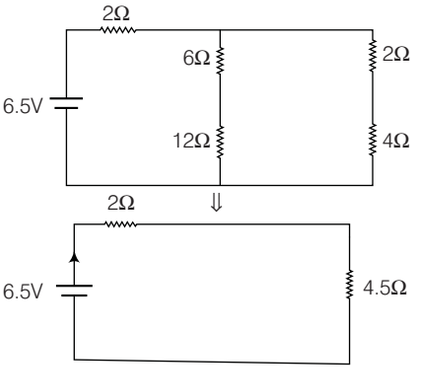
Form Eq. (i), $r = 30°$ for $n = \sqrt{3}$

$$\Rightarrow \quad \frac{d\theta}{dn} = \frac{1}{\cos 60}(\cos 30 \times \tan 30 + \sin 30)$$

$$= 2\left(\frac{1}{2} + \frac{1}{2}\right) = 2$$

3. (1)





4. (2) Angular momentum $= n\left(\frac{h}{2\pi}\right) = 3\left(\frac{h}{2\pi}\right)$

$\therefore \quad n = 3$

Now, $$r_n \propto \frac{n^2}{z}$$

$\therefore \quad r_3 = \frac{(3)^2}{3}(a_0) = 3a_0$

Now, $$mv_3 r_3 = 3\left(\frac{h}{2\pi}\right)$$

$\therefore \quad mv_3(3a_0) = 3\left(\frac{h}{2\pi}\right)$

or $$\frac{h}{mv_3} = 2\pi a_0$$

or $$\frac{h}{P_3} = 2\pi a_0 \qquad (P = mv)$$

or $$\lambda_3 = 2\pi a_0 \qquad \left(\lambda = \frac{h}{P}\right)$$

$\therefore$ Answer is 2.

5. *(d)* The sphere with cavity can be assumed as a complete sphere with positive charge of radius R_1 + another complete sphere with negative charge and radius R_2.

$E_+ \rightarrow \mathbf{E}$ due to total positive charge

$E_- \rightarrow \mathbf{E}$ due to total negative charge.

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_-$$

If we calculate it at P, then E_- comes out to be zero.

$\therefore \quad \mathbf{E} = \mathbf{E}_+$

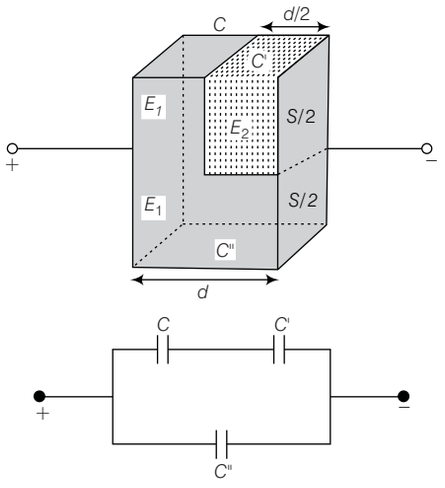
and $\mathbf{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{R_1^3}(OP)$, in the direction of OP.

Here, q is total positive charge on whole sphere.

It is in the direction of OP or **a**.

Now, inside the cavity electric field comes out to be uniform at any point. This is a standard result.

6. *(d)*



$$C_1 = \frac{\varepsilon_0 s}{d}, C = \frac{2\varepsilon_0 \frac{s}{2}}{\frac{d}{2}} = \frac{2\varepsilon_0 s}{d}$$

$$C' = \frac{4\varepsilon_0 \frac{s}{2}}{\frac{d}{2}} = \frac{4\varepsilon_0 s}{d}$$

and $$C'' = \frac{2\varepsilon_0 \frac{s}{2}}{d} = \frac{\varepsilon_0 s}{d}$$

$$C_2 = \frac{CC'}{C + C'} + C'' = \frac{4}{3}\frac{\varepsilon_0 s}{d} + \frac{\varepsilon_0 s}{d}$$

$$= \frac{7}{3}\frac{\varepsilon_0 s}{d}\frac{C_2}{C_1} = \frac{7}{3}$$

7. *(a)* From conservation laws of mass number and atomic number, we can say that $x = n,\ y = n$

$$(x = {}^{1}_{0}n,\ y = {}^{1}_{0}n)$$

$\therefore$ Only (a) and (d) options may be correct.

From conservation of momentum,

$$|P_{xe}| = |P_{sr}|$$

From $$K = \frac{P^2}{2m}$$

$\Rightarrow$ $$K \propto \frac{1}{m}$$

$$\frac{K_{sr}}{K_{xe}} = \frac{m_{xe}}{m_{sr}}$$

$\therefore$ $$K_{sr} = 129\,\text{MeV},\ K_{xe} = 86\text{MeV}$$

Note *There is no need of finding total energy released in the process.*

8. (a) $$\frac{4}{3}\sin i = \frac{\sqrt{45}}{4}\sin(90 - \theta_c) = \frac{\sqrt{45}}{4}\cos\theta_c$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

$\therefore$ $$\cos\theta_c = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$\Rightarrow$ $$\frac{4}{3}\sin i = \frac{\sqrt{45}}{4}\frac{3}{\sqrt{45}}$$

$$\sin i = \frac{9}{16}$$

In second case,

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{7}{8} \Rightarrow \cos\theta_c = \frac{\sqrt{15}}{8}$$

$$\frac{16}{3\sqrt{15}}\sin i = \frac{8}{5}\sin(90 - \theta_c)$$

Simplifying we get,

$$\sin i = \frac{9}{16}$$

(a) is correct.

Same approach can be adopted for other options. Correct answers are (a) and (c).

9. *(d)* (1) $\sin i_m = n_1 \sin(90 - \theta_c)$

$\Rightarrow$ $$\sin i_m = n_1 \cos\theta_c$$

$\Rightarrow$ $$NA = n_1\sqrt{1 - \sin^2\theta_c}$$

$$= n_1\sqrt{1 - \frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2}$$

Substituting the values we get,

$$NA_1 = \frac{3}{4} \text{ and } NA_2 = \frac{\sqrt{15}}{5} = \sqrt{\frac{3}{4}}$$

$$NA_2 < NA_1$$

Therefore, the numerical aperture of combined structure is equal to the lesser of the two numerical aperture, which is NA_2

10. $$F_B = Bev = Be\frac{I}{nAe} = \frac{BI}{nA}$$

$$F_e = eE$$

$$F_e = F_B$$

$$eE = \frac{BI}{nA} \Rightarrow E = \frac{B}{nAe}$$

$$V = Ed$$

$$= \frac{BI}{nAe}\cdot w = \frac{BIw}{n(wd)e} = \frac{BI}{ned}$$

$$\frac{V_1}{V_2} = \frac{d_2}{d_1}$$

$\Rightarrow$ if $w_1 = 2w_2$ and $d_1 = d_2$

$$V_1 = V_2$$

$\therefore$ Correct answers are (a) and (d).

11. $$V = \frac{BI}{ned}$$

$$\frac{V_1}{V_2} = \frac{B_1}{B_2} \times \frac{n_2}{n_1}$$

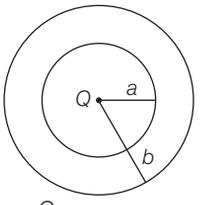
If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

$\therefore$ Correct answers are (a) and (c).

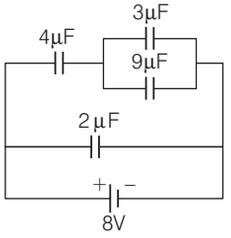
JEE Main 2016 (Solved)

1. The region between two concentric spheres of radii a and b, respectively (see the figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant is



(a) $\frac{Q}{2\pi a^2}$ (b) $\frac{Q}{2\pi(b^2 - a^2)}$ (c) $\frac{2Q}{\pi(a^2 - b^2)}$ (d) $\frac{2Q}{\pi a^2}$

2. A combination of capacitors is set-up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9μF capacitors), at a point distance 30 m from it, would equal to



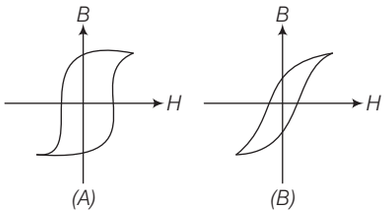
(a) 240 N/C (b) 360 N/C
(c) 420 N/C (d) 480 N/C

3. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by
(a) linear increase for Cu, linear increase for Si
(b) linear increase for Cu, exponential increase for Si
(c) linear increase for Cu, exponential decrease for Si
(d) linear decrease for Cu, linear decrease for Si

4. Two identical wires A and B, each of length l, carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16\sqrt{2}}$ (c) $\frac{\pi^2}{16}$ (d) $\frac{\pi^2}{8\sqrt{2}}$

5. Hysteresis loops for two magnetic materials A and B are as given below:



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then, it is proper to use
(a) A for electric generators and transformers
(b) A for electromagnets and B for electric generators
(c) A for transformers and B for electric generators
(d) B for electromagnets and transformers

6. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to
(a) 80 H (b) 0.08 H
(c) 0.044 H (d) 0.065 H

7. Arrange the following electromagnetic radiations per quantum in the order of increasing energy.
A. Blue light B. Yellow light
C. X-ray D. Radio wave
(a) D, B, A, C (b) A, B, D, C
(c) C, A, B, D (d) B, A, D, C

8. An observer looks at a distance tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears
(a) 10 times taller (b) 10 times nearer
(c) 20 times taller (d) 20 times nearer

9. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say $b_{\min}$) when

(a) $a = \frac{\lambda^2}{L}$ and $b_{\min} = \left(\frac{2\lambda^2}{L}\right)$ (b) $a = \sqrt{\lambda L}$ and $b_{\min} = \left(\frac{2\lambda^2}{L}\right)$
(c) $a = \sqrt{\lambda L}$ and $b_{\min} = \sqrt{4\lambda L}$ (d) $a = \frac{\lambda^2}{L}$ and $b_{\min} = \sqrt{4\lambda L}$

10. Radiation of wavelength λ is incident on a photocell. The fastest emitted electron has speed *v*. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be

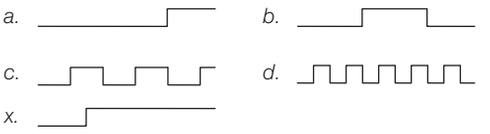
(a) $> v\left(\frac{4}{3}\right)^{1/2}$ (b) $< v\left(\frac{4}{3}\right)^{1/2}$

(c) $= v\left(\frac{4}{3}\right)^{1/2}$ (d) $= v\left(\frac{3}{4}\right)^{1/2}$

11. Half-lives of two radioactive elements *A* and *B* are 20 min and 40 min, respectively. Initially, the samples have equal number of nuclei. After 80 min, the ratio of decayed numbers of *A* and *B* nuclei will be

(a) 1 : 16 (b) 4 : 1 (c) 1 : 4 (d) 5 : 4

12. If *a, b, c and d* are inputs to a gate and *x* is its output, then, as per the following time graph, the gate is



(a) NOT (b) AND (c) OR (d) NAND

13. Choose the correct statement:

(a) In amplitude modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(b) In amplitude modulation, the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(c) In frequency modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(d) In frequency modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

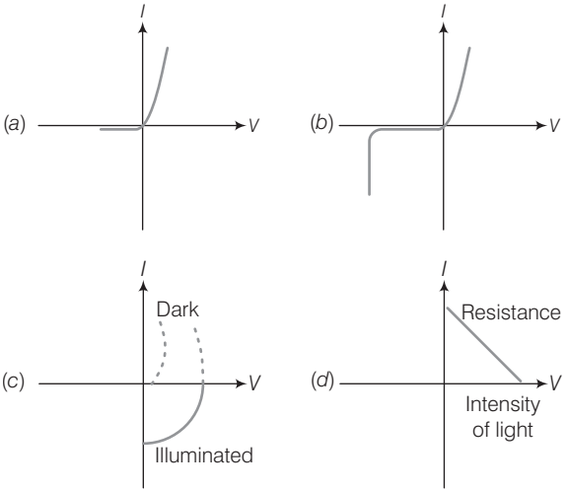
14. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. The value of the resistance which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is

(a) 0.01 Ω (b) 2 Ω (c) 0.1 Ω (d) 3 Ω

15. In an experiment for determination of refractive index of glass of a prism by *i*-δ, plot, it was found that a ray incident at an angle 35° suffers a deviation of 40° and that it emerges at an angle 79°. In that case, which of the following is closest to the maximum possible value of the refractive index?

(a) 1.5 (b) 1.6 (c) 1.7 (d) 1.8

16. Identify the semiconductor devices whose characteristics are as given below, in the order *(a),(b),(c),(d)*.



(a) Simple diode, Zener diode, Solar cell, Light dependent resistance

(b) Zener diode, Simple diode, Light dependent resistance, Solar cell

(c) Solar cell, Light dependent resistance, Zener diode, Simple diode

(d) Zener diode, Solar cell, Simple diode, Light dependent resistance

17. For a common-emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is

(a) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$ (b) $\alpha = \frac{\beta}{1-\beta}$

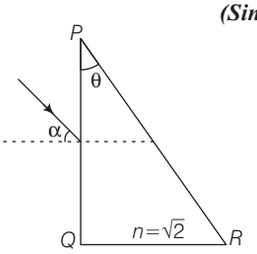
(c) $\alpha = \frac{\beta}{1+\beta}$ (d) $\alpha = \frac{\beta^2}{1+\beta^2}$

JEE Advanced 2016 (Solved)

Paper I

1. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n=\sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is

(Single Option Correct)



(a) 15° (b) 22.5° (c) 30° (d) 45°

2. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below:

λ (μm)	V_0(Volt)
0.3	2.0
0.4	1.0
0.5	0.4

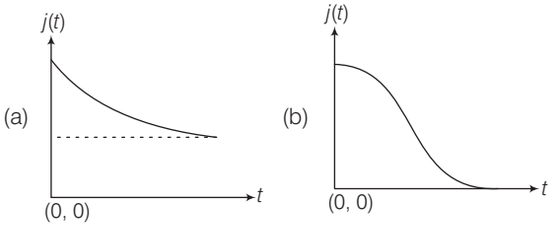
Given that $c=3\times10^8\,\text{ms}^{-1}$ and $e=1.6\times10^{-19}\,\text{C}$, Planck's constant (in units of J-s) found from such an experiment is)

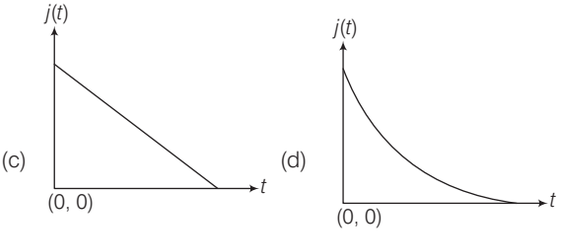
(Single Option Correct)

(a) 6.0×10^{-34} (b) 6.4×10^{-34} (c) 6.6×10^{-34} (d) 6.8×10^{-34}

3. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time $t=0$, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density $j(t)$ at any point in the material?

(Single Option Correct)





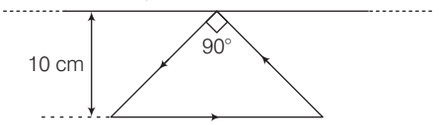
4. A plano-convex lens is made of material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is (are) true?

(One or More Than One Correct Option)

(a) The refractive index of the lens is 2.5
(b) The radius of curvature of the convex surface is 45 cm
(c) The faint image is erect and real
(d) The focal length of the lens is 20 cm

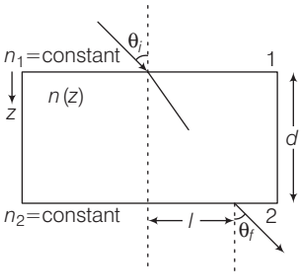
5. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of $10\,\text{As}^{-1}$. Which of the following statement(s) is (are) true?

(One or More Than One Correct Option)



(a) There is a repulsive force between the wire and the loop
(b) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire
(c) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt
(d) The induced current in the wire is in opposite direction to the current along the hypotenuse

6. A transparent slab of thickness d has a refractive index $n(z)$ that increases with z. Here, z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and $n_2\,(>n_1)$, as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l.



Which of the following statement(s) is (are) true?

(One or More Than One Correct Option)

(a) l is independent on $n(z)$ (b) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$

(c) $n_1 \sin\theta_i = n_2 \sin\theta_f$ (d) l is independent of n_2

7. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principle quantum number n, where $n >> 1$. Which of the following statement(s) is (are) true?

(One or More Than One Correct Option)

(a) Relative change in the radii of two consecutive orbitals does not depend on Z

(b) Relative change in the radii of two consecutive orbitals varies as $1/n$

(c) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(d) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

8. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is (are) true?

(One or More Than One Correct Option)

(a) The temperature distribution over the filament is uniform

(b) The resistance over small sections of the filament decreases with time

(c) The filament emits more light at higher band of frequencies before it breaks up

(d) The filament consumes less electrical power towards the end of the life of the bulb

9. A hydrogen atom in its ground state is irradiated by light of wavelength 970Å. Taking $hc/e = 1.237 \times 10^{-6}$ eVm and the ground state energy of hydrogen atom as -13.6 eV, the number of lines present in the emission spectrum is

(Single Digit Integer)

10. The isotopes $^{12}_{5}\text{B}$ having a mass 12.014 u undergoes β-decay to $^{12}_{6}\text{C}$. $^{12}_{6}\text{C}$ has an excited state of the nucleus ($^{12}_{6}\text{C}^*$) at 4.041 MeV above its ground state. If $^{12}_{5}\text{B}$ decays to $^{12}_{5}\text{C}^*$, the maximum kinetic energy of the β-particle in units of MeV is

(1u = 931.5 MeV/c^2, where c is the speed of light in vacuum) ***(Single Digit Integer)***

11. Two inductors L_1 (inductance 1mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time $t = 0$. The ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery is

(Single Digit Integer)

Paper II

1. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use? ***(Single Option Correct)***

(a) 64 (b) 90 (c) 108 (d) 120

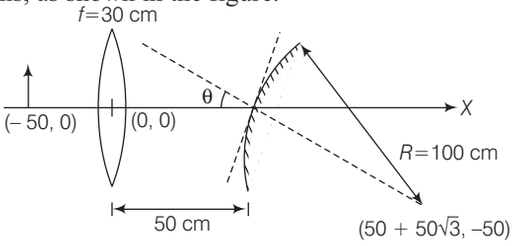
2. The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

The measured masses of the neutron, $^{1}_{1}\text{H}$, $^{15}_{7}\text{N}$ and $^{15}_{8}\text{O}$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the $^{15}_{7}\text{N}$ and $^{15}_{8}\text{O}$ nuclei are same, 1 u = 931.5 MeV/c^2 (c is the speed of light) and $e^2/(4\pi\varepsilon_0) = 1.44$ MeV fm. Assuming that the difference between the binding energies of $^{15}_{7}\text{N}$ and $^{15}_{8}\text{O}$ is purely due to the electrostatic energy, the radius of either of the nuclei is (1fm = 10^{-15} m) ***(Single Option Correct)***

(a) 2.85 fm (b) 3.03 fm (c) 3.42 fm (d) 3.80 fm

3. A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30°$ to the axis of the lens, as shown in the figure.



If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are

(Single Option Correct)

(a) $(125/3, 25/\sqrt{3})$ (b) $(50 - 25\sqrt{3}, 25)$

(c) (0, 0) (d) $(25, 25\sqrt{3})$

4. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_c < R/2$, which of the following statement(s) about anyone of the galvanometers is (are) true? ***(One or More Than One Option Correct)***

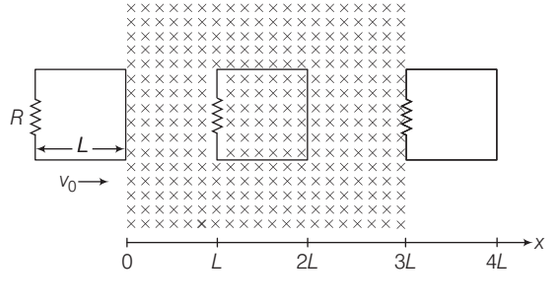
(a) The maximum voltage range is obtained when all the components are connected in series

(b) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer

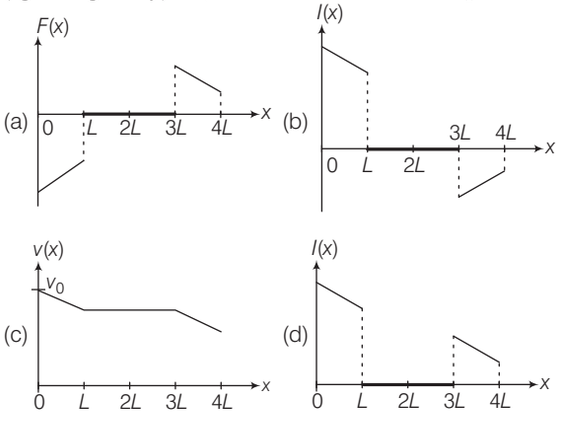
(c) The maximum current range is obtained when all the components are connected in parallel

(d) The maximum current range is obtained when the two galvanometers are connected in series, and the combination is connected in parallel with both the resistors

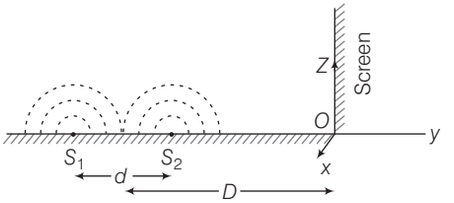
5. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper . At $t = 0$, the right edge of the loop enters a region of length $3L$ where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $v(x)$, $I(x)$ and $F(x)$ represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive.



Which of the following schematic plot(s) is (are) correct? (Ignore gravity) ***(One or More Than One Option Correct)***



6. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 mm. The student mistakenly placed the screen parallel to the x-z plane (for $z > 0$) at a distance $D = 3\,\text{m}$ from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the source $d = 0.6003$ mm. The origin O is at the intersection of the screen and the line joining S_1S_2.



Which of the following is (are) true of the intensity pattern on the screen? ***(One or More Than One Option Correct)***

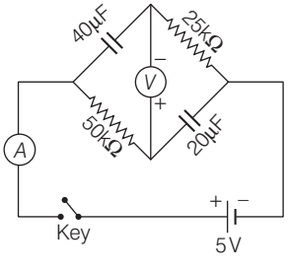
(a) Semi circular bright and dark bands centered at point O

(b) The region very close to the point O will be dark

(c) Straight bright and dark bands parallel to the x-axis

(d) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

7. In the circuit shown below, the key is pressed at time $t = 0$. Which of the following statement(s) is (are) true?



(One or More Than One Option Correct)

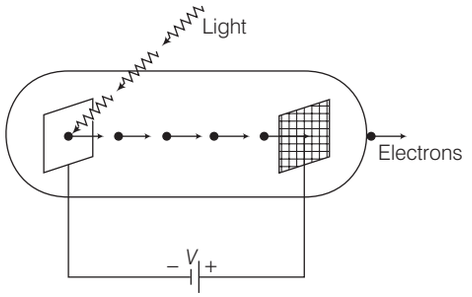
(a) The voltmeter display – 5 V as soon as the key is pressed and displays + 5 V after a long time

(b) The voltmeter will display 0 V at time $t = \ln 2$ seconds

(c) The current in the ammeter becomes $1/e$ of the initial value after 1 second

(d) The current in the ammeter becomes zero after a long time

8. Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statements(s) is (are) true?

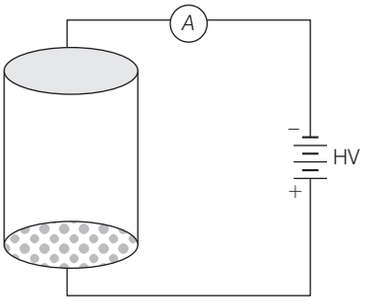


(One or More Than One Option Correct)

9. Which one of the following statements is correct?
   (a) The balls will execute simple harmonic motion between the two plates
   (b) The balls will bounce back to the bottom plate carrying the same charge they went up with
   (c) The balls will stick to the top plate and remain there
   (d) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

Paragraph 2

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r << h$. Now, a high voltage source (HV) connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charge, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to te soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)

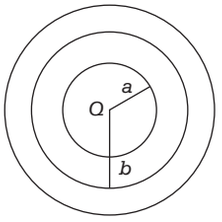


10. The average current in the steady state registered by the ammeter in the circuit will be
    (a) proportional to V_0^2
    (b) proportional to the potential V_0
    (c) zero
    (d) proportions to $V_0^{1/2}$

SOLUTIONS

JEE Main

1. ***(a)*** As, Gaussian surface at distance r from centre,



$$\frac{Q+\int_a^r \frac{A}{r}4\pi r^2 dr}{\varepsilon_0} = E4\pi r^2$$

$$E\,4\pi\varepsilon_0 r^2 = Q + A\frac{4\pi}{r^2}\left(\frac{r^2-a^2}{2}\right)$$

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r^2} + A\,2\pi\left(\frac{r^2-a^2}{r^2}\right)\right]$$

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r^2} + A2\pi - \frac{A2\pi a^2}{r^2}\right)$$

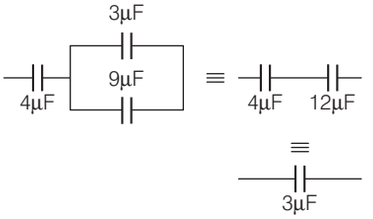
$$E = \frac{1}{4\pi\varepsilon_0}\times A\times 2\pi$$

At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant is

As, $\quad Q = 2\pi A a^2$

i.e. $\quad A = \dfrac{Q}{2\pi a^2}$

2. ***(c)*** Resultant circuit,



As, charge on $3\mu F = 3\mu F \times 8V = 24\mu C$

$\therefore$ Charge on $4\mu F$ = Charge on $12\,\mu F = 24\,\mu C$

Charge on $3\mu F = 3\mu F \times 2V = 6\mu C$

Charge on $9\mu F = 9\mu F \times 2V = 18\mu C$

Charge on $4\,\mu F$ + Charge on $9\mu F = (24 + 18)\mu C = 42\mu C$

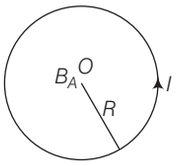
$\therefore$ Electric field at a point distant 30 m

$$= \frac{9 \times 10^3 \times 42 \times 10^{-6}}{30 \times 30} = 420 \text{N/C}$$

3. *(c)* As, we know Cu is conductor, so increase in temperature, resistance will increase. Then, Si is semiconductor, so with increase in temperature, resistance will decrease.

4. *(d)* Magnetic field in case of circle of radius R, we have

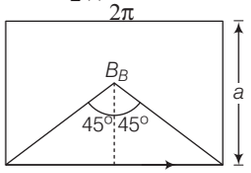
$$B_A = \frac{\mu_0 I}{2R}$$



As, $2\pi R = l$ (l is length of a wire)

$$R = \frac{l}{2\pi}$$

$$\Rightarrow \quad B_A = \frac{\mu_0 I}{2 \times \frac{l}{2\pi}} = \frac{\mu_0 I \pi}{l} \qquad ...(i)$$



Magnetic field in case of square of side a, we get

$$B_B = 4 \times \frac{\mu_0}{4\pi} \times \frac{I}{\left(\frac{a}{2}\right)} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad B_B = \frac{4 I \mu_0}{\pi a \sqrt{2}} = \frac{\mu_0 2\sqrt{2} I}{a\pi}$$

As, $\quad 4a = l, \; a = \frac{l}{4} \;\Rightarrow\; B_B = \frac{8\sqrt{2}\,\mu_0 I}{\pi l} \qquad ...(ii)$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

5. *(d)* Area of hysteresis loop is proportional to the net energy absorbed per unit volume by the material, as it is taken over a complete cycle of magnetisation.

For electromagnets and transformers, energy loss should be low.

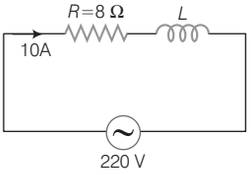
i.e. thin hysteresis curves.

Also, $|B| \to 0$ when $H = 0$ and $|H|$ should be small when $B \to 0$.

6. *(d)* Given, $\quad I = 10\,\text{A}, V = 80\,\text{V},$

$$R = \frac{V}{I} = \frac{80}{10} = 8\,\Omega \text{ and } \omega = 50\,\text{Hz}$$

For AC circuit, we have



$$I = \frac{V}{\sqrt{8^2 + X_L^2}} \;\Rightarrow\; 10 = \frac{220}{\sqrt{64 + X_L^2}} \;\Rightarrow\; \sqrt{64 + X_L^2} = 22$$

Squaring on both sides, we get

$$64 + X_L^2 = 484 \Rightarrow \; X_L^2 = 484 - 64 = 420$$

$$X_L = \sqrt{420} \Rightarrow \; 2\pi \times \omega L = \sqrt{420}$$

Series inductor on an arc lamp,

$$L = \frac{\sqrt{420}}{(2\pi \times 50)} = 0.065\,\text{H}$$

7. *(a)* As, we know energy liberated, $E = \frac{hc}{\lambda}$ i.e. $E \propto \frac{1}{\lambda}$

So, lesser the wavelength, than greater will be energy liberated by electromagnetic radiations per quantum.

As, order of wavelength is given by

X-ray, VIBGYOR, Radio waves

(C) (A) (B) (D)

$\therefore$ Order of electromagnetic radiations per quantum.

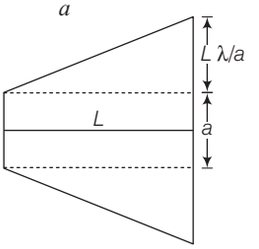
$\Rightarrow$ D < B < A < C

8. *(c)* Height of image depends upon the magnifying power to see a 20 times taller object, as the angular magnification should be 20 and we observe angular magnification. Option *(c)* would not be very correct as the telescope can be adjusted to form the image anywhere between infinity and least distance for distinct vision.

Suppose that the image is formed at infinity. Then, the observer will have to focus the eyes at infinity to observe the image. Hence, it is incorrect to say that the image will be appear nearer to the observer.

9. *(c)* In diffraction, first minima, we have

$$\sin\theta = \frac{\lambda}{a}$$



So, size of a spot

$$b = 2a + \frac{2L\lambda}{a} \qquad ...(i)$$

Then, minimum size of a spot, we get

$$\frac{\partial b}{\partial a} = 0 \;\Rightarrow\; 1 - \frac{L\lambda}{a^2} = 0 \Rightarrow \; a = \sqrt{\lambda L} \qquad ...(ii)$$

So, $\quad b_{\min} = 2\sqrt{\lambda L} + 2\sqrt{\lambda L}$

[by substituting the value of a from Eq. (ii) in Eq. (i)]

$$= 4\sqrt{\lambda L}$$

So, the radius of the spot, $\frac{4}{2}\sqrt{\lambda L} = \sqrt{4\lambda L}$

10. *(a)* According to the law of conservation of energy, i.e. Energy of a photon ($h\nu$) = Work function (ϕ) + Kinetic energy of the photoelectron $\left(\frac{1}{2} m v^2_{\max}\right)$

According to Einstein's photoelectric emission of light

i.e. $\quad E = (\text{KE})_{\max} + \phi$

As, $\frac{hc}{\lambda} = (\text{KE})_{\max} + \phi$

If the wavelength of radiation is changed to $\frac{3\lambda}{4}$, then

$\Rightarrow \quad \frac{4}{3}\frac{hc}{\lambda} = \left(\frac{4}{3}(\text{KE})_{\max} + \frac{\phi}{3}\right) + \phi$

For fastest emitted electron, $(\text{KE})_{\max} = \frac{1}{2}mv'^2 + \phi$

$\Rightarrow \quad \frac{1}{2}mv'^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{\phi}{3}$

i.e. $v' > v\left(\frac{4}{3}\right)^{1/2}$

11. *(d)* Given, 80 min = 4 half-lives of A = 2 half-lives of B.

Let the initial number of nuclei in each sample be N.

For radioactive element *A*,

N_A after 80 min $= \frac{N}{2^4}$

$\Rightarrow$ Number of A nuclides decayed $= N - \frac{N}{16} = \frac{15}{16}N$

For radioactive element *B*,

N_B after 80 min. $= \frac{N}{2^2} \Rightarrow$ Number of B nuclides decayed

$= N - \frac{N}{4} = \frac{3}{4}N$

$\therefore$ Ratio of decayed numbers of A and B nuclei will be

$\frac{(15/16)N}{(3/4)N} = \frac{5}{4}$

12. *(c)* Output of OR gate is 0 when all inputs are 0 and output is 1 when atleast one of the input is 1.

Observing output *x* It is 0 when all inputs are 0 and it is 1 when atleast one of the input is 1.

$\therefore$ The gate is OR.

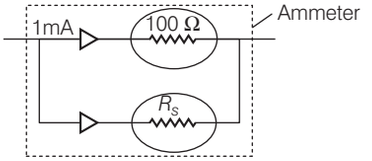
Alternative Method

OR Gate

a	b	c	d	x
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1

13. *(b)* As, we know, an amplitude modulated wave, the bandwidth is twice the frequency of modulating signal. Therefore, amplitude modulation (AM), the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

14. *(a)* Maximum voltage that can be applied across the galvanometer coil = $100\ \Omega \times 10^{-3}$ A = 0.1 V.



If R_s is the shunt resistance, then

$R_s \times 10\text{A} = 0.1\ \text{V}$

$\Rightarrow \quad R_s = 0.01\ \Omega.$

15. *(a)* If μ is refractive index of material of prism, then from Snell's law

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin(A + \delta_m)/2}{\sin A/2} \quad \ldots(\text{i})$$

where, A is angle of prism and δ_m is minimum deviation through prism.

Given, $i = 35°$, $\delta = 40°$, $e = 79°$.

So, angle of deviation by a glass prism,

$\delta = i + e - A \Rightarrow 40° = 35° + 79° - A$

i.e. Angle of prism $\Rightarrow A = 74°$.

Such that, $r_1 + r_2 = A = 74°$.

Let us put $\mu = 1.5$ in Eq. (i), we get

$$1.5 = \frac{\sin\left(\frac{A + \delta_{\min}}{2}\right)}{\sin A/2} \Rightarrow 1.5 = \frac{\sin\left(\frac{74° + \delta_{\min}}{2}\right)}{\sin 37°}$$

$\Rightarrow \quad 0.9 = \sin\left(37° + \frac{\delta_{\min}}{2}\right) \qquad (\because \sin 37° \approx 0.6)$

$\sin 64° = \sin\left(37° + \frac{\delta_{\min}}{2}\right) \qquad (\because \sin 64° = 0.9)$

$37° + \frac{\delta_{\min}}{2} = 64° \Rightarrow \delta_{\min} \approx 54°$

This angle is greater than 40° deviation angle already given. For greater μ, deviation will be even higher. Hence, μ of the given prism should be lesser than 1.5. Hence, the closest option will be 1.5.

16. *(a)* Zener diode works in breakdown region.

So, Simple diode $\rightarrow (a)$

Zener diode $\rightarrow (b)$

Solar cell $\rightarrow (c)$

Light dependent resistance $\rightarrow (d)$

17. *(a, c)* As, we know, In case of a common-emitter configuration,

DC current gain, $\alpha = \frac{I_c}{I_e}$.

Where, I_c is collector current and I_e is emitter current

and AC current gain, $\beta = \frac{I_c}{I_b}$.

Where, I_b is base current.

Also $I_e = I_b + I_c$

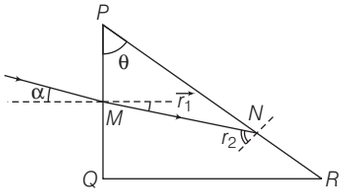
Dividing whole equation by I_c, we get

$\Rightarrow \frac{I_e}{I_c} = \frac{I_b}{I_c} + 1 \Rightarrow \frac{1}{\alpha} = \frac{1}{\beta} + 1 \Rightarrow \alpha = \frac{\beta}{1+\beta}$

JEE Advanced

Paper I

1. *(a)* According to question,



Applying Snell's law at M,

$$n = \frac{\sin\alpha}{\sin r_1} \Rightarrow \sqrt{2} = \frac{\sin 45^\circ}{\sin r_1}$$

$$\Rightarrow \quad \sin r_1 = \frac{\sin 45^\circ}{\sqrt{2}} = \frac{1/\sqrt{2}}{\sqrt{2}} = \frac{1}{2}$$

$$r_1 = 30^\circ$$

$$\sin\theta_c = \frac{1}{n} = \frac{1}{\sqrt{2}} \Rightarrow \theta_c = 45^\circ$$

Let us take $r_2 = \theta_c = 45^\circ$ for just satisfying the condition of TIR.
In ΔPNM,

$$\theta + 90^\circ + r_1 + 90^\circ - r_2 = 180^\circ$$

or $\theta = r_2 - r_1 = 45^\circ - 30^\circ = 15^\circ$

Note If $\alpha > 45^\circ$ (the given value). Then, $r_1 > 30^\circ$ (the obtained value)

$\therefore \quad r_2 > \theta_c$ (as $r_2 - r_1 = \theta$ or $r_2 = \theta + r_1$)

or TIR will take place. So, for taking TIR under all conditions α should be greater than 45° or this is the minimum value of α.

2. *(b)* $\frac{hc}{\lambda} - \phi = eV_0$ (ϕ = work function)

$$\frac{hc}{0.3\times10^{-6}} - \phi = 2e \quad \ldots(i)$$

$$\frac{hc}{0.4\times10^{-6}} - \phi = 1e \quad \ldots(ii)$$

Subtracting Eq. (ii) from Eq. (i)

$$hc\left(\frac{1}{0.3} - \frac{1}{0.4}\right)10^6 = e$$

$$hc\left(\frac{0.1}{0.12}\times10^6\right) = e$$

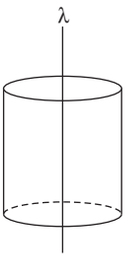
$$h = 0.64\times10^{-33} = 6.4\times10^{-34}\text{ J-s}$$

3. *(d)*

Suppose charger per unit length at any instant is λ.
Initial value of λ is suppose λ_0.
Electric field at a distance r at any instant is

$$E = \frac{\lambda}{2\pi\varepsilon r}$$

$$J = \sigma E = \sigma\frac{\lambda}{2\pi\varepsilon r}$$



$$i = \frac{dq}{dt} = J(A) = -J\sigma 2\pi rl$$

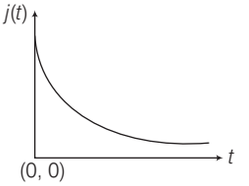
$$\frac{d\lambda l}{dt} = -\frac{\lambda}{2\pi\varepsilon r}\times\sigma 2\pi rl \qquad (q = \lambda l)$$

$$\int_{\lambda_0}^{\lambda}\frac{d\lambda}{\lambda} = -\frac{\sigma}{\varepsilon}\int_0^t dt \quad \Rightarrow \quad \lambda = \lambda_0 e^{-\frac{\sigma}{\varepsilon}t}$$

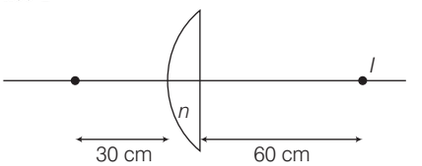
$$J = \frac{\sigma}{2\pi\varepsilon r}\lambda = \frac{\sigma\lambda_0}{2\pi\varepsilon r}e^{-\frac{\sigma}{\varepsilon}t} = J_0 e^{-\frac{\sigma}{\varepsilon}t}$$

Here, $J_0 = \frac{\sigma\lambda_0}{2\pi\varepsilon r}$

$\therefore$ $J(t)$ decreases exponentially as shown in figure below.



4. *(a,d)* **Case 1**



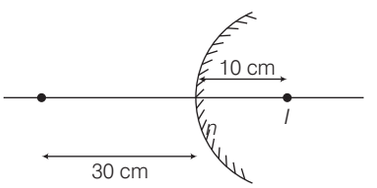
Using lens formula,

$$\frac{1}{60} + \frac{1}{30} = \frac{1}{f_1} \Rightarrow \frac{1}{f_1} = \frac{1}{60} + \frac{2}{60}$$

$$\Rightarrow \quad f_1 = 20\text{ cm}$$

Further, $\frac{1}{f_1} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right) \Rightarrow f_1 = \frac{R}{n-1} = +20\text{ cm}$

Case 2



Using mirror formula,

$$\frac{1}{10} - \frac{1}{30} = \frac{1}{f_2} \Rightarrow \frac{3}{30} - \frac{1}{30} = \frac{1}{f_2} = \frac{2}{30}$$

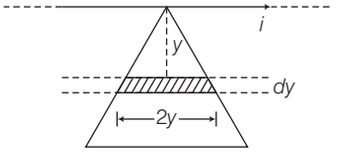
$$f_2 = 15 = \frac{R}{2} \Rightarrow R = 30$$

$$R = 30\text{ cm}$$

$$\frac{R}{n-1} = +20\text{ cm} = \frac{30}{n-1} \Rightarrow 2n - 2 = 3 \Rightarrow f_1 = +20\text{ cm}$$

Refractive index of lens is 2.5.
Radius of curvature of convex surface is 30 cm.
Faint image is erect and virtual. Focal length of lens is 20 cm.

5. *(a,c)* By reciprocity theorem of mutual induction, it can be assumed that current in infinite wire is varying at 10A/s and EMF is induced in triangular loop.



Flux of magnetic field through triangle loop, if current in infinite wire is ϕ, can be calculated as follows:

$$d\phi = \frac{\mu_0 i}{2\pi y}\cdot 2y\,dy \quad \Rightarrow \quad d\phi = \frac{\mu_0 i}{\pi}dy \quad \Rightarrow \quad \phi = \frac{\mu_0 i}{\pi}\left(\frac{l}{\sqrt{2}}\right)$$

$$\Rightarrow \qquad \text{EMF} = \left|\frac{d\phi}{dt}\right| = \frac{\mu_0}{\pi}\left(\frac{l}{\sqrt{2}}\right)\cdot\frac{di}{dt}$$

$$= \frac{\mu_0}{\pi}(10\,\text{cm})\left(10\frac{\text{A}}{\text{s}}\right) = \frac{\mu_0}{\pi}\ \text{volt}$$

If we assume the current in the wire towards right then as the flux in the loop increases we know that the induced current in the wire is counter clockwise. Hence, the current in the wire is towards right.

Field due to triangular loop at the location of infinite wire is into the paper. Hence, force on infinite wire is away from the loop.

By cylindrical symmetry about infinite wire, rotation of triangular loop will not cause any additional EMF.

6. *(a,c,d) From Snell's law,* $n\sin\theta$ = constant

$$\therefore \qquad n_1\sin\theta_i = n_2\sin\theta_f$$

Further, l will depend on n_1 and $n(z)$. But it will be independent of n_2.

7. *(a,b,d)* As radius $r \propto \frac{n^2}{z}$

$$\Rightarrow \qquad \frac{\Delta r}{r} = \frac{\left(\frac{n+1}{z}\right)^2 - \left(\frac{n}{z}\right)^2}{\left(\frac{n}{z}\right)^2} = \frac{2n+1}{n^2} \approx \frac{2}{n} \propto \frac{1}{n}$$

Energy $E \propto \frac{z^2}{n^2}$

$$\Rightarrow \qquad \frac{\Delta E}{E} = \frac{\frac{z^2}{n^2} - \frac{z^2}{(n+1)^2}}{\frac{z^2}{(n+1)^2}} = \frac{(n+1)^2 - n^2}{n^2\cdot(n+1)^2}\cdot(n+1)^2$$

$$\Rightarrow \qquad \frac{\Delta E}{E} = \frac{2n+1}{n^2} \simeq \frac{2n}{n^2} \propto \frac{1}{n}$$

Angular momentum $L = \frac{nh}{2\pi}$

$$\Rightarrow \qquad \frac{\Delta L}{L} = \frac{\frac{(n+1)h}{2\pi} - \frac{nh}{2\pi}}{\frac{nh}{2\pi}} = \frac{1}{n} \propto \frac{1}{n}$$

8. *(c,d)* Because of non-uniform evaporation at different section, area of cross-section would be different at different sections.

Region of highest evaporation rate would have rapidly reduced area and would become break up cross-section.

Resistance of the wire as whole increases with time.

Overall resistance increases hence power decreases.

$\left(P = \frac{V^2}{R} \text{ or } P \propto \frac{1}{R} \text{ as } V \text{ is constant}\right)$. At break up junction temperature would be highest, thus light of highest band frequency would be emitted at those cross-section.

9. (6) Energy of incident light (in eV)

$$E = \frac{12375}{970} = 12.7\,\text{eV}$$

After excitation, let the electron jumps to nth state, then

$$\frac{-13.6}{n^2} = -13.6 + 12.7$$

Solving this equation, we get

$$n = 4$$

$\therefore$ Total number of lines in emission spectrum,

$$= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

10. **(9)** ${}^{12}_{5}B \longrightarrow {}^{12}_{6}C + {}^{0}_{-1}e + \bar{\nu}$

Mass of ${}^{12}_{6}\text{C} = 12.000$ u (by definition of 1 a.m.u.)

Q-value of reaction,

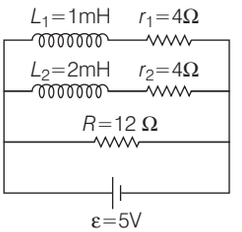
$Q = (M_B - M_C)\times c^2 = (12.014 - 12.000)\times 931.5$

$= 13.041$ MeV

4.041MeV of energy is taken by ${}^{12}_{6}\text{C}^*$

$\Rightarrow$ Maximum KE of β-particle is $(13.041 - 4.041) = 9$ MeV

11. (8)



$$I_{\max} = \frac{\varepsilon}{R} = \frac{5}{12}\text{A} \qquad \text{(Initially at } t = 0\text{)}$$

$$I_{\min} = \frac{\varepsilon}{R_{\text{eq}}} = \varepsilon\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{R}\right) \qquad \text{(finally in steady state)}$$

$$= 5\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{12}\right) = \frac{10}{3}\text{A}$$

$$\frac{I_{\max}}{I_{\min}} = 8$$

Paper II

1. *(c)* Using the relation, $R = R_0\left(\frac{1}{2}\right)^n$

Here, R is activity of radioactive substance, R_0 initial activity and n is number of half lives.

$$1 = 64\left(\frac{1}{2}\right)^n$$

Solving we get, $n = 6$

Now, $t = n(t_{12})$

$= 6(18 \text{ days}) = 108 \text{ days}$

2. *(c)* Electrostatic energy

$= \text{Binding energy of N} - \text{Binding energy of O}$

$= [[7M_H + 8M_n - M_N] - [8M_H + 7M_n - M_O]] \times C^2$

$= [-M_H + M_n + M_O - M_N]C^2$

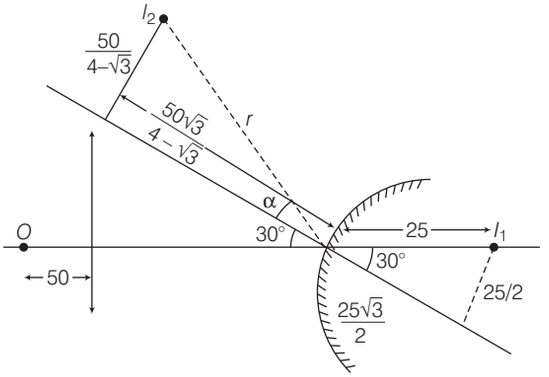
$= [-1.007825 + 1.008665 + 15.003065 - 15.000109] \times 93.15$

$= +3.5359 \text{ MeV}$

$$\Delta E = \frac{3}{5} \times \frac{1.44 \times 8 \times 7}{R} - \frac{3}{5} \times \frac{1.44 \times 7 \times 6}{R} = 3.5359 \text{ MeV}$$

$$R = \frac{3 \times 1.44 \times 14}{5 \times 3.5359} = 3.42 \text{ fm}$$

3. *(d)*



For Lens

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad v = \frac{uf}{u+f}$$

$$\Rightarrow \qquad v = \frac{(-50)(30)}{-50+30} = 75 \text{ cm}$$

For Mirror

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \Rightarrow \quad v = \frac{uf}{u-f}$$

$$\Rightarrow \qquad v = \frac{\left(\frac{25\sqrt{3}}{2}\right)(50)}{\frac{25\sqrt{3}}{2} - 50} = \frac{-50\sqrt{3}}{4-\sqrt{3}} \text{ cm}$$

$$m = -\frac{v}{u} = \frac{h_2}{h_1} \Rightarrow h_2 = -\left(\frac{\frac{-50\sqrt{3}}{4-\sqrt{3}}}{\frac{25\sqrt{3}}{2}}\right) \cdot \frac{25}{2}$$

$$h_2 = \frac{+50}{4-\sqrt{3}} \text{ cm}$$

The x-coordinate of the images

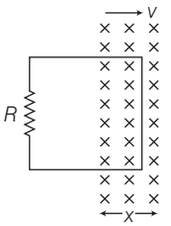
$= 50 - v\cos 30° + h_2 \cos 60° \approx 25 \text{ cm}$

The y-coordinate of the images

$= v \sin 30° + h_2 \sin 60° \approx 25\sqrt{3} \text{ cm}$

4. (a,c) For maximum range of voltage resistance should be maximum. So, all four should be connected in series. For maximum range of current, net resistance should be least. Therefore, all four should be connected in parallel.

5. *(b,c)*



When loop was entering ($x < L$)

$$\phi = BLx$$

$$e = -\frac{d\phi}{dt} = -BL\frac{dx}{dt}$$

$$|e| = BLv$$

$$i = \frac{e}{R} = \frac{BLv}{R} \qquad \text{(anticlockwise)}$$

$F = ilB$ (Left direction) $= \dfrac{B^2L^2v}{R}$ (in left direction)

$$\Rightarrow \qquad a = \frac{F}{m} = -\frac{B^2L^2v}{mR}$$

$$a = v\frac{dv}{dx}$$

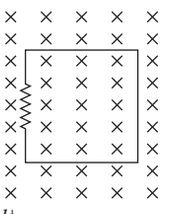
$$v\frac{dv}{dx} = -\frac{B^2L^2v}{mR}$$

$$\Rightarrow \qquad \int_{v_0}^{v} dv = -\frac{B^2L^2}{mR}\int_0^x dx$$

$\Rightarrow v = v_0 - \dfrac{B^2L^2v}{mR}x$ (straight line of negative slope for $x < L$)

$I = \dfrac{BL}{R}v \Rightarrow$ (I *vs* x will also be straight line of negative slope for $x < L$)

L* ≤ *x* ≤ 3*L

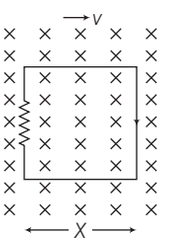


$$\frac{d\phi}{dt} = 0$$

$$e = 0, \; i = 0$$

$$F = 0$$

$$x > 4L$$



$$e = Blv$$

Force also will be in left direction.

$$i = \frac{BLv}{R} \text{ (clockwise)}, \; a = -\frac{B^2L^2v}{mR} = v\frac{dv}{dx}$$

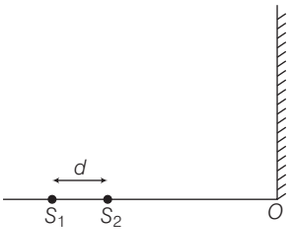
$$F = \frac{B^2L^2v}{R}\int_L^x -\frac{B^2L^2}{mR}dx = \int_{v_i}^{v_f} dv$$

$$\Rightarrow \quad -\frac{B^2L^2}{mR}(x-L)=v_f-v_i$$

$$v_f=v_i-\frac{B^2L^2}{mR}(x-L)\text{ (straight line of negative slope)}$$

$$I=\frac{BLv}{R}\rightarrow\text{(Clockwise) (straight line of negative slope)}$$

6. *(a,b)*



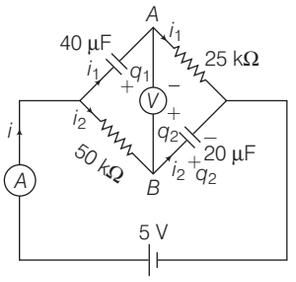
Path difference at point $O = d = 0.6003$ mm $= 600300$ nm.

This path difference is equal to $\left(1000\lambda+\frac{\lambda}{2}\right)$.

$\Rightarrow$ Minima is formed at point O.

Line S_1S_2 and screen are $\perp$ to each other so fringe pattern is circular (semi-circular because only half of screen is available)

7. *(a,b,c,d)*



Just after pressing key,

$$5-25000i_1=0$$

$$5-50000i_2=0\quad\text{(As charge in both capacitors)}$$

$\Rightarrow i_1=0.2$mA $\Rightarrow$ $i_2=0.1$ mA

and $V_B+25000i_1=V_A$

$\Rightarrow V_B-V_A=-5$V

After a long time, i_1 and $i_2=0$ (steady state)

$$\Rightarrow \quad 5-\frac{q_1}{40}=0 \Rightarrow q_1=200\mu C$$

and $$5-\frac{q_2}{20}=0 \quad\Rightarrow\quad q_2=100\mu C$$

$$V_B-\frac{q_2}{20}=V_A$$

$\Rightarrow V_B-V_A=+5$V

$\Rightarrow$ (a) is correct.

For capacitor 1, $q_1=200[1-e^{-t/1}]\mu C$

$$i_1=\frac{1}{5}e^{-t/1}\text{mA}$$

For capacitor 2, $q_2=100[1-e^{-t/1}]\mu C$

$$i_2=\frac{1}{10}e^{-t/1}\text{mA}$$

$$\Rightarrow \quad V_B-\frac{q_2}{20}+i_1\times 25=V_A$$

$$\Rightarrow \quad V_B-V_A=5[1-e^{-t}]-5e^{-t}=-5[1-2e^{-t}]$$

At $t=\ln 2$, $V_B-V_A=5[1-1]=0$

$\Rightarrow$ (b) is correct.

At $t=1$, $i=i_1+i_2=\frac{1}{5}e^{-1}+\frac{1}{10}e^{-1}=\frac{3}{10}\cdot\frac{1}{e}$

At $t=0$, $\quad i=i_1+i_2=\frac{1}{5}+\frac{1}{10}=\frac{3}{10}$

$\Rightarrow$ (c) is correct.

After a long time, $i_1=i_2=0 \Rightarrow$ (d) is correct.

8. *(d)* $K_{\max}=\frac{hc}{\lambda_{ph}}-\phi$

Kinetic energy of electron reaching the anode will be

$$K=\frac{hc}{\lambda_{ph}}-\phi+eV$$

Now, $$\lambda_e=\frac{h}{\sqrt{2mK}}=\frac{h}{\sqrt{2m\left(\frac{hc}{\lambda_{ph}}-\phi+eV\right)}}$$

If $eV >> \phi$, then

$$\lambda_e=\frac{h}{\sqrt{2m\left(\frac{hc}{\lambda_{ph}}+eV\right)}}$$

If $V_f=4V_i$, then $$(\lambda_e)_f\simeq\frac{(\lambda_e)_i}{2}$$

9. *(d)* Balls will gain positive charge and hence move towards negative plate.

On reaching negative plate, balls will attain negative charge

and come back to positive plate.

and so on, balls will keep oscillating.

But oscillation is not S.H.M.,

As fore on balls is not $\propto x$.

$\Rightarrow$ option (d) is correct.

10. *(a)* As the balls keep on carrying charge form one plate to another, current will keep on flowing even in steady state. When at bottom plate, if all balls attain charge q,

$$\frac{kq}{r}=V_0 \qquad \left(k=\frac{1}{4\pi\varepsilon_0}\right)$$

$$\Rightarrow \quad q=\frac{V_0r}{k}$$

Inside cylinder, electric field

$$E=[V_0-(-V_0)]h=2V_0h.$$

$\Rightarrow$ Acceleration of each ball,

$$a=\frac{qE}{m}=\frac{2hr}{k\,m}\cdot V_0^2$$

$\Rightarrow$ Time taken by balls to reach other plate,

$$t=\sqrt{\frac{2h}{a}}=\sqrt{\frac{2h.k\,m}{2hrV_0^2}}=\frac{1}{V_0}\sqrt{\frac{k\,m}{r}}$$

If there are n balls, then

Average current, $i_{av}=\frac{nq}{t}$

$$=n\times\frac{V_0r}{k}\times V_0\sqrt{\frac{r}{k\,m}}\Rightarrow i_{av}\propto V_0^2$$

JEE Main & Advanced/BITSAT/Kerala CEE/
KCET/AP & TS EAMCET/VIT/MHT CET

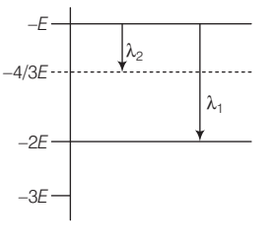
Solved Papers 2017

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AP & TS EAMCET/VIT/MHT CET

SOLVED PAPERS 2017

JEE Main

1. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1 / \lambda_2$ is given by



(a) $r = \frac{2}{3}$ (b) $r = \frac{3}{4}$

(c) $r = \frac{1}{3}$ (d) $r = \frac{4}{3}$

2. A radioactive nucleus with a half-life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. After sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by

(a) $t = T \frac{\log 1.3}{\log_e 2}$ (b) $t = T \log 1.3$

(c) $t = \frac{T}{\log 1.3}$ (d) $t = \frac{T \log_e 2}{2 \log 1.3}$

3. Which of the following statements is false?

(a) In a balanced Wheatstone bridge, if the cell and the galvanometer are exchanged, the null point is disturbed

(b) A rheostat can be used as a potential divider

(c) Kirchhoff's second law represents energy conservation

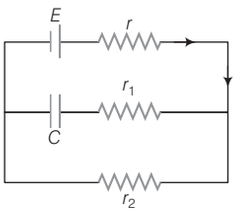
(d) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

4. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is

(a) 16 (b) 24

(c) 32 (d) 2

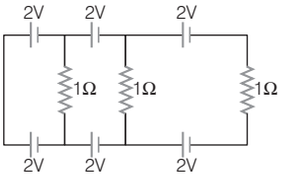
5. In the given circuit diagram, when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be



(a) $CE \frac{r_1}{(r_2 + r)}$ (b) $CE \frac{r_2}{(r + r_2)}$

(c) $CE \frac{r_1}{(r_1 + r)}$ (d) CE

6. In the below circuit, the current in each resistance is



(a) 0.25 A (b) 0.5 A (c) 0 A (d) 1 A

7. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave?

(a) ω_c (b) $\omega_m + \omega_c$ (c) $\omega_c - \omega_m$ (d) ω_m

8. In a common emitter amplifier circuit using an transistor, the phase difference between the input and the output voltages will be

(a) 90° (b) 135° (c) 180° (d) 45°

9. In a Young's double slit experiment, slits are separated by 0.5 mm and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide, is

(a) 7.8 mm (b) 9.75 mm (c) 15.6 mm (d) 1.56 mm

10. An electric dipole has a fixed dipole moment $\mathbf{p}$, which makes angle θ with respect to -axis. When subjected to an electric field $\mathbf{E}_1 = E\hat{\mathbf{i}}$, it experiences a torque $\mathbf{T}_1 = \tau\hat{\mathbf{k}}$. When subjected to another electric field $\mathbf{E}_2 = \sqrt{3}E_1\hat{\mathbf{j}}$, it experiences a torque $\mathbf{T}_2 = -\mathbf{T}_1$. The angle θ is

(a) 45° (b) 60° (c) 90° (d) 30°

11. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is

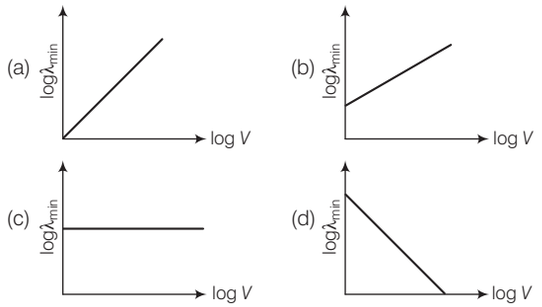
(a) virtual and at a distance of 40 cm from convergent lens

(b) real and at a distance of 40 cm from the divergent lens

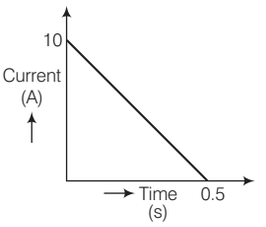
(c) real and at a distance of 6 cm from the convergent lens

(d) real and at a distance of 40 cm from convergent lens

12. An electron beam is accelerated by a potential difference V to hit a metallic target to produce -rays. It produces continuous as well as characteristic -rays. If λ_{min} is the smallest possible wavelength of -rays in the spectrum, the variation of log λ_{min} with log V is correctly represented in



13. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is



(a) 225 Wb (b) 250 Wb
(c) 275 Wb (d) 200 Wb

14. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0-10 V is

(a) 2.045×10^3 Ω (b) 2.535×10^3 Ω
(c) 4.005×10^3 Ω (d) 1.985×10^3 Ω

15. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is

(a) 8.89 s (b) 6.98 s (c) 8.76 s (d) 6.65 s

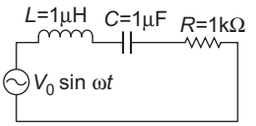
16. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is

(a) $\frac{\lambda_A}{\lambda_B} = 2$ (b) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$
(c) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ (d) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$

JEE Advanced

Paper 1

1. In the circuit shown, $L = 1\ \mu H$, $C = 1\ \mu F$ and $R = 1\ k\Omega$. They are connected in series with an AC source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?



(One or More Than One Correct Option)

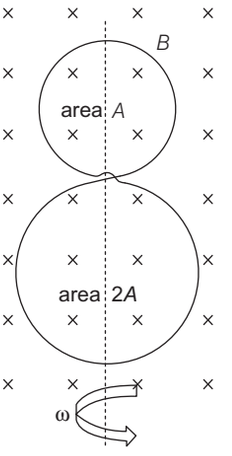
(a) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero
(b) The frequency at which the current will be in phase with the voltage is independent of R
(c) The current will be in phase with the voltage if $\omega = 10^4$ rads^{-1}
(d) At $\omega >> 10^6$ rads^{-1}, the circuit behaves like a capacitor

2. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

(One or More Than One Correct Option)

(a) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism
(b) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = \left(\frac{i_i}{2}\right)$
(c) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A \sqrt{4\cos^2 \frac{A}{2} - 1} - \cos A\right]$
(d) For this prism, the refractive index μ and the angle prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

3. A circular insulated copper wire loop is twisted to form two loops of area A and $2A$ as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field **B** points into the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



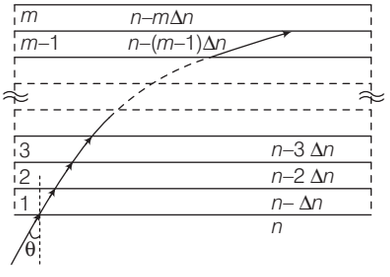
(One or More Than One Correct Option)

(a) the emf induced in the loop is proportional to the sum of the areas of the two loops
(b) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper
(c) The net emf induced due to both the loops is proportional to cos ωt
(d) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

4. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. v_i and v_f are respectively the initial and final potential energies of the electron. If $\frac{v_i}{v_f} = 6.25$, then the smallest possible n_f is ***(Single Digit Integer)***

5. ^{131}I is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4×10^5 Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 h, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in litres is approximately (you may use $e^2 \approx 1 + x$ for $|x| << 1$ and $\ln 2 \approx 0.7$). ***(Single Digit Integer)***

6. A monochromatic light is travelling in a medium of refractive index $n = 1.6$. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the mth slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)$th and mth slabs from the right side of the stack. What is the value of m?
(Single Digit Integer)



Matching Type Questions

Directions (Q.Nos. 7-9) Matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin $(x = 0,\ y = 0,\ z = 0)$ with a given initial velocity **v**. A uniform electric field **E** and a uniform magnetic field **B** exist everywhere. The velocity **v**, electric field **E** and magnetic field **B** are given in columns 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude.

Column 1	Column 2	Column 3
(I) Electron with $v = 2\frac{E_0}{B_0}\hat{x}$	(i) $E = E_0\hat{z}$	(P) $B = -B_0\hat{x}$
(II) Election with $v = \frac{E_0}{B_0}\hat{y}$	(ii) $E = -E_0\hat{y}$	(Q) $B = B_0\hat{x}$
(III) Proton with $v = 0$	(iii) $E = -E_0\hat{x}$	(R) $B = B_0\hat{y}$
(IV) Proton with $v = 2\frac{E_0}{B_0}\hat{x}$	(iv) $E = E_0\hat{x}$	(S) $B = B_0\hat{z}$

7. In which case would the particle move in a straight line along the negative direction of Y-axis (i.e. move along $-\hat{y}$)?
(a) (IV) (ii) (S) (b) (II) (iii) (Q) (c) (III), (ii) (R) (d) (III) (ii) (P)

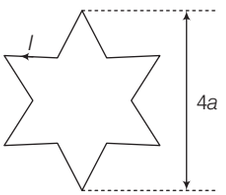
8. In which case will the particle move in a straight line with constant velocity?
(a) (II) (iii) (S) (b) (III) (iii) (P) (c) (IV) (i) (S) (d) (III) (ii) (R)

9. In which case will the particle describe a helical path with axis along the positive z-direction?
(a) (II) (ii) (R) (b) (III) (iii) (P) (c) (IV) (i) (S) (d) (IV) (ii) (R)

Paper 2

1. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength $\lambda \left(\lambda < \frac{hc}{\phi_0}\right)$. The fastest photoelectron has a de-Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d. Then, the ratio $\frac{\Delta\lambda_d}{\Delta\lambda}$ is proportional to ***(Single Option Correct)***
(a) $\frac{\lambda_d^2}{\lambda^2}$ (b) $\frac{\lambda_d}{\lambda}$ (c) $\frac{\lambda_d^3}{\lambda}$ (d) $\frac{\lambda_d^3}{\lambda^2}$

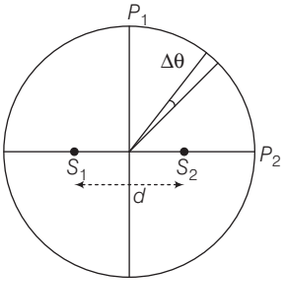
2. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is $4a$.
(Single Option Correct)



The magnitude of the magnetic field at the center of the loop is
(a) $\frac{\mu_0 I}{4\pi a} 6[\sqrt{3} - 1]$ (b) $\frac{\mu_0 I}{4\pi a} 6[\sqrt{3} + 1]$
(c) $\frac{\mu_0 I}{4\pi a} 3[\sqrt{3} - 1]$ (d) $\frac{\mu_0 I}{4\pi a} 3[2 - \sqrt{3}]$

3. Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the centre of the circle as shown. The sources are separated by a distance $d = 1.8$ mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$.

Which of the following options is/are correct?



(One or More Than One Correct Option)

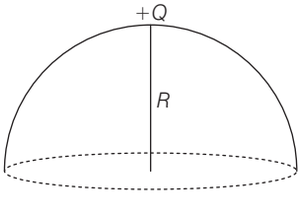
(a) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant

(b) A dark spot will be formed at the point P_2

(c) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000

(d) At P_2 the order of the fringe will be maximum

4. A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



(One or More Than One Correct Option)

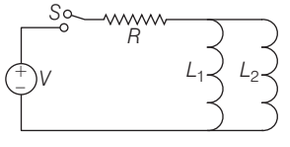
(a) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$.

(b) The component of the electric field normal to the flat surface is constant over the surface

(c) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$

(d) The circumference of the flat surface is an equipotential

5. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct?



(One or More Than One Correct Option)

(a) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

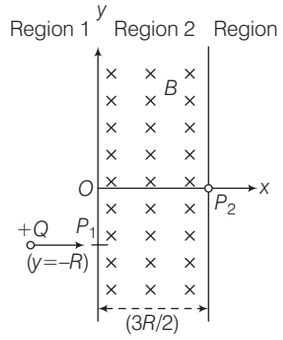
(b) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(c) The ratio of the currents through L_1 and L_2 is fixed at all times ($t > 0$)

(d) At $t = 0$, the current through the resistance R is $\frac{V}{R}$

6. A uniform magnetic field B exists in the region between $x = 0$ and $x = \frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge $+Q$ and momentum p directed along X-axis enters region 2 from region 1 at point P_1 ($y = -R$).

Which of the following option(s) is/are correct?



(One or More Than One Correct Option)

(a) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from Y-axis is $\frac{p}{\sqrt{2}}$

(b) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on X-axis

(c) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1

(d) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle

7. The instantaneous voltages at three terminals marked X, Y and Z are given by $V_X = V_0 \sin \omega t$,

$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$ and $V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$.

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

(One or More Than One Correct Option)

(a) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(b) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$

(c) independent of the choice of the two terminals

(d) $V_{XY}^{rms} = V_0$

Paragraph Based Questions (Q. Nos. 8-9)

Consider a simple RC circuit as shown in Figure 1.

Process 1 In the circuit the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (i.e. charging continues for time $T >> RC$). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_c.

Process 2 In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T >> RC$. Then, the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T >> RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in Figure 2.

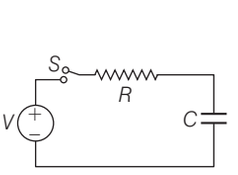


Figure 1

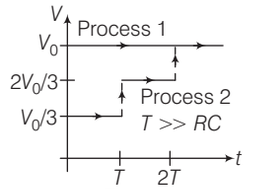


Figure 2

8. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by

(a) $E_C = E_D \ln 2$ (b) $E_C = E_D$ (c) $E_C = 2E_D$ (d) $E_C = \frac{1}{2}E_D$

9. In Process 2, total energy dissipated across the resistance E_D is

(a) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$ (b) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$

(c) $E_D = 3CV_0^2$ (d) $E_D = \frac{1}{2}CV_0^2$

BITSAT

1. The focal length of a thin convex lens for red and blue rays are 100 cm and 96.8 cm respectively. Then, the dispersive power of the material of the lens is

(a) 0.968 (b) 0.98 (c) 0.0325 (d) 0.325

2. Two metal plates having a potential difference of 800 V are 2 cm apart. It is found that a particle of mass 1.96×10^{-15} kg remains suspended in the region between the plates. The charge on the particle must be (e = elementary charge).

(a) $2e$ (b) $3e$ (c) $6e$ (d) $8e$

3. A positive charge q is projected in magnetic field of width $\frac{mv}{\sqrt{2}\,qB}$ with velocity v. Then, the time taken by charged particle to emerge from the magnetic field is

(a) $\frac{m}{\sqrt{2}\,qB}$ (b) $\frac{\pi m}{4qB}$ (c) $\frac{\pi m}{2qB}$ (d) $\frac{\pi m}{\sqrt{2}\cdot qB}$

4. In Young's double slit experiment, the slits are 2 mm apart and are illuminated by photons of two wavelengths $\lambda_1 = 12000$ Å and $\lambda_2 = 10000$ Å. At what minimum distance from the common central bright fringe on the screen 2m from the slit will a bright fringe from one interference pattern coincide with a bright fringe from the other?

(a) 8 mm (b) 6 mm (c) 4 mm (d) 3 mm

5. A conducting circular loop is placed in a uniform magnetic field, $B = 0.025$ T with its plane perpendicular to the loop. The radius of the loop is made to shrink at a constant rate of 1 mm/s. The induced emf when the radius is 2 cm, is

(a) $2\pi\mu$V (b) $\pi\mu$V

(c) $\frac{\pi}{2}\mu$V (d) 2 μV

6. The resistance of a wire at 20° C is 20Ω and 500 °C is 60 Ω. At which temperature, its resistance will be 25Ω?

(a) 50 °C (b) 60 °C (c) 70 °C (d) 80 °C

7. The de-Broglie wavelength of a proton (charge $= 1.6 \times 10^{-19}$ C, $m = 1.6 \times 10^{-27}$ kg) accelerated through a potential difference of 1 kV is

(a) 600 Å (b) 0.9×10^{-12} m

(c) 7 Å (d) 0.9 nm

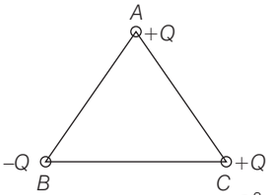
8. The total energy of an electron in the first excited state of hydrogen is about − 3.4 eV. Its kinetic energy in this state is

(a) − 3.4 eV (b) − 6.8 eV (c) 6.8 eV (d) 3.4 eV

9. A common emitter amplifier has a voltage gain of 50, an input impedance of 100Ω and an output impedance of 200Ω. The power gain of the amplifier is

(a) 500 (b) 1000 (c) 1250 (d) 50

10. Three charges are placed at the three vertices of an equilateral triangle of side 'a' as shown in the figure. The force experienced by the charge placed at the vertex A in a direction normal to BC is



(a) $\frac{Q^2}{4\pi\varepsilon_0 a^2}$ (b) $-\frac{Q^2}{4\pi\varepsilon_0 a^2}$

(c) zero (d) $\frac{Q^2}{2\pi\varepsilon_0 a^2}$

11. The activity of a radioactive sample is measured as N_0 counts per minute at $t = 0$ and $\frac{N_0}{e}$ counts per minute at $t = 5$ min. The time (in minutes) at which the activity reduces to half its value is

(a) $\log_e \frac{2}{5}$ (b) $\frac{5}{\log_e 2}$

(c) $5 \log_{10} 2$ (d) $5 \log_e 2$

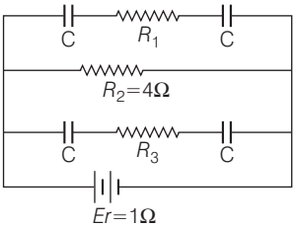
12. A plano-convex lens fits exactly into a plano-concave lens. Their plane surfaces are parallel to each other. If lenses are made of different materials of refractive indices μ_1 and μ_2, R is the radius of curvature of the curved surface of the lenses, then the focal length of combination is

(a) $\frac{R}{2(\mu_1 + \mu_2)}$ (b) $\frac{R}{2(\mu_1 - \mu_2)}$

(c) $\frac{R}{(\mu_1 - \mu_2)}$ (d) $\frac{2R}{\mu_1 - \mu_2}$

13. In the given circuit diagram,

$E = 5\,\text{V}$, $r = 1\Omega$, $R_2 = 4\Omega$, $R_1 = R_3 = 1\Omega$ and $C = 3\mu\text{F}$



Then, what will be the numerical value of charge on each plates of the capacitor.

(a) 24 μC (b) 12 μC

(c) 6 μC (d) 3 μC

14. Monochromatic radiation of wavelength λ is incident on a hydrogen sample in ground state. Hydrogen atom absorbs a friction of light and subsequently emits radiations of six different wavelengths. The wavelength λ is

(a) 97.5 nm (b) 121.6 nm

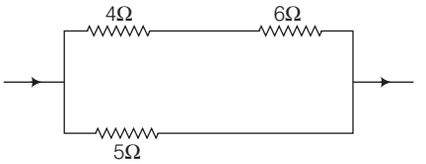
(c) 110.3 nm (d) 45.2 nm

15. A coil in the shape of an equilateral triangle of side l is suspended between the pole pieces of a permanent magnet such that B is in plane of the coil. If due to a current i in the triangle, a torque τ rests on it, the side l of the triangle is

(a) $2\left(\frac{\tau}{\sqrt{3}\,Bi}\right)^{\frac{1}{2}}$ (b) $\frac{2}{\sqrt{3}}\left(\frac{\tau}{Bi}\right)$

(c) $2\left(\frac{\tau}{Bi}\right)^{\frac{1}{2}}$ (d) $\frac{1}{\sqrt{3}}\left(\frac{\tau}{Bi}\right)$

16. In the circuit shown, the heat produced in 5Ω resistor is 10 cal s^{-1}. The heat produced per sec in 4Ω resistor will be



(a) 1 cal (b) 2 cal

(c) 3 cal (d) 4 cal

17. An α-particle after passing through potential difference of V volt collides with a nucleus. If the atomic number of the nucleus is Z, then distance of closest approach is

(a) $14.4 \cdot \frac{Z}{V}$ Å (b) $14.4 \frac{Z}{V}$ m

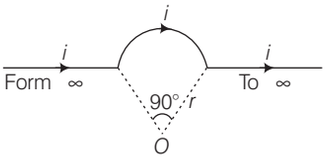
(c) $14.4 \frac{V}{Z}$ m (d) $14.4 \frac{V}{Z}$ Å

18. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between the plates is d. The space between the plates is now filled with two dielectrics constant $K_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $K_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now

(a) 1.8 pF (b) 45 pF

(c) 40.5 pF (d) 20.25 pF

19. What is the magnetic field at the centre of arc in the figure below?



(a) $\frac{\mu_0}{4\pi} \cdot \frac{2i}{r}[\sqrt{2} + \pi]$ (b) $\frac{\mu_0}{4\pi} \cdot \frac{2i}{r}\left[\sqrt{2} + \frac{\pi}{4}\right]$

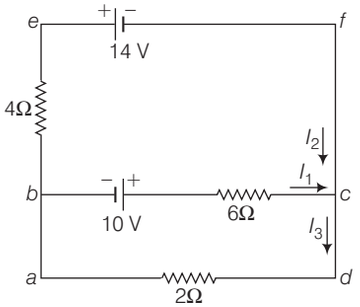
(c) $\frac{\mu_0}{4\pi} \cdot \frac{i}{r}[\sqrt{2} + \pi]$ (d) $\frac{\mu_0}{4\pi} \cdot \frac{i}{r}\left[\sqrt{2} + \frac{\pi}{4}\right]$

KERALA CEE

1. A copper wire with a cross-section area of 2×10^{-6} m^2 has a free electron density equal to 5×10^{22}/cm^3. If this wire carries a current of 16 A, the drift velocity of the electron is
(a) 1 m/s (b) 0.1 m/s (c) 0.01 m/s (d) 0.001 m/s (e) 0.0001 m/s

2. The resistance of the tungsten wire in the light bulb, which is 120V/75 W and powered by a 120 V direct current supply, is
(a) 0.37 Ω (b) 1.2 Ω (c) 2.66 Ω (d) 192 Ω (e) 9×10^3 Ω

3. The value of the current I_1, I_2 and I_3 flowing through the circuit given below is
(a) $I_1 = -3$A, $I_2 = 2$A, $I_3 = -1$A
(b) $I_1 = 2$A, $I_2 = -3$A, $I_3 = -1$A
(c) $I_1 = 3$A, $I_2 = -1$A, $I_3 = -2$A
(d) $I_1 = 1$A, $I_2 = -3$A, $I_3 = -2$A
(e) $I_1 = 2$A, $I_2 = -1$A, $I_3 = -3$A



4. A silver wire has temperature coefficient of resistivity 4×10^{-3}/°C and its resistance at 20°C is 10 Ω. Neglecting any change in dimensions due to the change in temperature, its resistance at 40°C is
(a) 0.8 Ω (b) 1.8 Ω (c) 9.2 Ω (d) 10.8 Ω (e) 11.6 Ω

5. A charge Q placed at the centre of a metallic spherical shell with inner and outer radii R_1 and R_2 respectively. The normal component of the electric field at any point on the Gaussian surface with radius between R_1 and R_2 will be
(a) Zero (b) $\frac{Q}{4\pi R_1^2}$ (c) $\frac{Q}{4\pi R_2^2}$
(d) $\frac{Q}{4\pi (R_1 - R_2)^2}$ (e) $\frac{Q}{4\pi (R_2 - R_1)^2}$

6. A sphere of radius R has a uniform volume charge density ρ. The magnitude of electric field at a distance r from the centre of the sphere, where $r > R$, is
(a) $\frac{\rho}{4\pi\varepsilon_0 r^2}$ (b) $\frac{\rho R^2}{\varepsilon_0 r^2}$ (c) $\frac{\rho R^3}{\varepsilon_0 r^2}$ (d) $\frac{\rho R^3}{3\varepsilon_0 r^2}$
(e) $\frac{\rho R^2}{4\varepsilon_0 r^2}$

7. Five equal point charges with $Q = 10$ nC are located at $x = 2, 4, 5, 10$ and 20 m. If $\varepsilon_0 = [10^{-9} / 36\pi]$ F/m, then the potential at the origin $(x = 0)$ is
(a) 9.9 V (b) 11.1 V (c) 90 V (d) 99 V (e) 111 V

8. Two infinitely long parallel plates of equal areas 6 cm^2 are separated by a distance of
1 cm. While one of the plates has a charge of + 10 nC and the other has − 10 nC. The magnitude of the electric field between the plates, if $\varepsilon_0 = \frac{10^{-9}}{36\pi}$ F/m is
(a) 0.6 π kV/m (b) 6 π kV/m (c) 600 π kV/m (d) 60 π V/m
(e) 6 π V/m

9. A proton moves with a speed of 5.0×10^6 m/s along the x-axis. It enters a region where there is a magnetic field of magnitude 2.0 Tesla directed at an angle of 30° to the x-axis and lying in the xy-plane. The magnitude of the magnetic force on the proton is
(a) 0.8×10^{-13} N (b) 1.6×10^{-13} N
(c) 8.0×10^{-13} N (d) 8.01×10^{-13} N
(e) 16×10^{-13} N

10. A long straight wire of radius R carries a steady current I_0, uniformly distributed throughout the cross-section of the wire. The magnetic field at a radial distance r from the centre of the wire, in the region $r > R$, is
(a) $\frac{\mu_0 I_0}{2\pi r}$ (b) $\frac{\mu_0 I_0}{2\pi R}$ (c) $\frac{\mu_0 I_0 R^2}{2\pi r}$ (d) $\frac{\mu_0 I_0 r^2}{2\pi R}$ (e) $\frac{\mu_0 I_0 r^2}{2\pi R^2}$

11. If the cyclotron oscillator frequency is
16 MHz, then what should be the operating magnetic field for accelerating the proton of mass 1.67×10^{-27} kg?
(a) 0.334 π T (b) 3.34 π T (c) 33.4 π T (d) 334 π T
(e) 3340 π T

12. The speed of light in vacuum is equal to
(a) $\mu_0\varepsilon_0$ (b) $\mu_0^2\varepsilon_0^2$ (c) $\sqrt{\mu_0\varepsilon_0}$ (d) $\frac{1}{\mu_0\varepsilon_0}$ (e) $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$

13. The direction of propagation of electromagnetic wave is along
(a) Electric field vector, E (b) Magnetic field vector, B
(c) E · B (d) E × B
(e) B × E

14. Assume that a radio station is about 200 km away at your location and the station operates 972 kHz. How long does it take for an electromagnetic signal to travel from the station to you and how many wave crests does it send out per second?
(a) 666 μs and 9.72×10^5 crests per second
(b) 666 μs and 972×10^5 crests per second
(c) 555 μs and 97.2×10^7 crests per second
(d) 555 μs and 0.972×10^5 crests per second
(e) 444 μs and 9×10^6 crests per second

15. What wavelength must electromagnetic radiation have if a photon in the beam has the same momentum as an electron moving with a speed 1.1×10^5 m/s (Planck's constant= 6.6×10^{-34} J-s, rest mass of electron = 9×10^{-31} kg?
(a) 2/3 nm (b) 20/3 nm (c) 4/3 nm (d) 40/3 mn (e) 3/20 nm

16. The electric field portion of an electromagnetic wave is given by (all variables in SI units)
$E = 10^{-4} \sin(6 \times 10^5 \ t - 0.01x)$. The frequency ($f$) and the speed ($\nu$) of electromagnetic wave are
(a) $f = 30/\pi$ kHz and $\nu = 1.5 \times 10^7$ m/s
(b) $f = 90/\pi$ kHz and $\nu = 6.0 \times 10^7$ m/s
(c) $f = 300/\pi$ kHz and $\nu = 6.0 \times 10^7$ m/s
(d) $f = 600/\pi$ kHz and $\nu = 7.5 \times 10^7$ m/s
(e) $f = 900/\pi$ kHz and $\nu = 8.0 \times 10^7$ m/s

17. Huygens' wave theory of light cannot explain
(a) Diffraction phenomena (b) interference phenomena
(c) Photoelectric effect (d) Polarization of light
(e) Propagation of light

18. An electron, a neutron and an alpha particle have same kinetic energy and their de-Broglie wavelength are λ_e, λ_n and λ_α respectively. Which statement is correct about their de-Broglie wavelengths?
(a) $\lambda_e > \lambda_n > \lambda_\alpha$ (b) $\lambda_e < \lambda_n > \lambda_\alpha$
(c) $\lambda_e < \lambda_n < \lambda_\alpha$ (d) $\lambda_e > \lambda_n < \lambda_\alpha$
(e) $\lambda_e = \lambda_n < \lambda_\alpha$

19. It takes 4.6 eV to remove one of the least tightly bound electrons from a metal surface. When monochromatic photons strike the metal surface, electrons having kinetic energy from zero to 2.2 eV are ejected. What is the energy of the incident photons?
(a) 2.4 eV (b) 2.2 eV
(c) 6.8 eV (d) 4.6 eV
(e) 5.8 eV

20. If copper and silicon pieces are heated, the resistance of
(a) each will increase
(b) each will decrease
(c) copper will increase and silicon will decrease
(d) copper will decrease and silicon will increase
(e) both does not change

21. In an insulator, band gap of the order of
(a) 0.1 eV (b) 1 eV (c) 5 eV (d) 100 eV (e) 1 MeV

22. For a P-N junction diode
(a) Forward current in mA and reverse current is in μA
(b) Forward current is in μA are reverse current is in mA
(c) Both forward and reverse currents are in μA
(d) Both forward and reverse currents are in mA
(e) No current flows in any direction

23. For a Zener diode
(a) both p and n regions are heavily doped
(b) p region is heavily doped but n region is lightly doped
(c) n region is heavily doped but p region is lightly doped
(d) both p and n regions are lightly doped
(e) depletion region is very thick

24. Speech signals is in the range of
(a) 3700 Hz to 7000 Å wavelength
(b) 20 Hz to 20 kHz frequency
(c) 300 Hz to 3100 Hz frequency
(d) 540 kHz to 1600 kHz frequency
(e) 88 MHz to 108 MHz frequency

25. Wavelength of the wave with 30 MHz frequency is
(a) 1 cm (b) 10 cm (c) 100 cm (d) 1000 cm
(e) 10000 cm

26. To transmit a signal of frequency ω_m, with a carrier frequency ω_c, in AM transmission, the bandwidth of the filter and amplifier is
(a) ω_m (b) $2\omega_m$ (c) ω_c (d) $\omega_c - \omega_m$ (e) $\omega_c + \omega_m$

27. If a magnet is dropped through a vertical hollow copper tube, then
(a) the time taken to reach the ground is longer than the time taken, if the tube was made out of plastic
(b) the magnet will get attracted and stick to the copper tube
(c) the time taken to reach the ground is longer than the time taken, if the tube was made out of stainless steel
(d) the time taken to reach the ground does not depend on the radius of the copper tube
(e) the magnet will be repelled away by the tube

28. Consider a circular wire loop of radius R spinning about a diametrical chord which is perpendicular to uniform magnetic field ($\mathrm{B} = \mathrm{B}_0\hat{k}$)
(a) The magnitude of the induced EMF in the loop is maximum when the plane of the loop is perpendicular to B
(b) Flux through the loop is maximum when the plane of the loop is perpendicular to B
(c) The direction of induced current remains same during the spinning motion of the loop
(d) EMF induced will be the same for a larger radius of the loop in the same field
(e) No EMF will be induced since magnetic field is constant

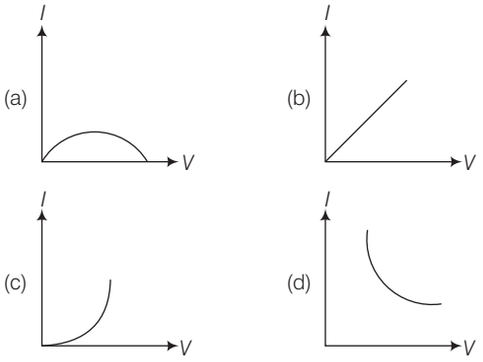
29. An electric motor which loaded has an effective resistance of 30Ω and an inductive reactance of 40 Ω. If the motor is powered by a source with maximum voltage of 420 V, the maximum current is
(a) 6 A (b) 8.4 A
(c) 10 A (d) 12 A
(e) 13 A

30. CO^- ion moving with kinetic energy of 20 keV dissociates into O^- and C which move along the parent ion direction. Assuming no energy is released during dissociation, the kinetic energy of the daughters $(K.E)_{O^-}$ and $(K.E)_C$ are related as
(a) $(K.E)_{O^-} = (K.E)_C$ (b) $(K.E)_{O^-} / (K.E)_C = 16/12$
(c) $(K.E)_{O^-} / (K.E)_C = 12/161$ (d) $(K.E)_{O^-} / (K.E)_C = 16/28$
(e) $(K.E)_{O^-} / (K.E)_C = 28/16$

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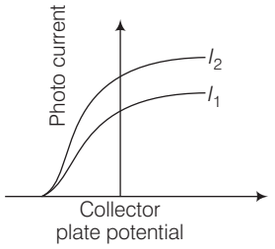
1. A coil of inductive reactance $1/\sqrt{3}\Omega$ and resistance $1\,\Omega$ is connected to a 200 V, 50 Hz AC supply. The time lag between maximum voltage and current is
(a) $\frac{1}{200}$ s (b) $\frac{1}{300}$ s (c) $\frac{1}{500}$ s (d) $\frac{1}{600}$ s

2. The particle emitted in the decay of $^{238}_{92}U$ to $^{234}_{92}U$
(a) 2α and 2β (b) $1\,\alpha$ and $1\,\beta$
(c) $1\,\alpha$ only (d) $1\,\alpha$ and $1\,\beta$

3. Of the following graphs, the one that correctly represents the *I-V* characteristics of a 'Ohmic device' is


4. The susceptibility of a ferromagnetic substance is
(a) Zero (b) > 1 (c) < 1 (d) >> 1

5. The working of magnetic braking of trains is based on
(a) eddy current
(b) pulsating current
(c) alternating current
(d) steady current

6. From the following graph of photo current against collector plate potential, for two different intensities of light I_1 and I_2, one can conclude


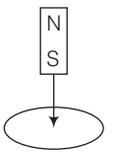
(a) $I_1 = I_2$
(b) $I_1 > I_2$
(c) $I_1 < I_2$
(d) Comparison is not possible

7. If **E** and **B** represent electric and magnetic field vectors of an electromagentic wave, the direction of propagation of the wave is along
(a) E × B (b) B × E
(c) E (d) B

8. Two point charges $A = +3$ nC and $B = +1$ nC are placed 5 cm apart in air. The work done to move charge B towards A by 1 cm is
(a) 2.0×10^{-7}J (b) 2.7×10^{-7} J
(c) 12.1×10^{-7}J (d) 1.35×10^{-7} J

9. A proton, a deuteron and an α-particle are projected perpendicular to the direction of a uniform magnetic field with same kinetic energy. The ratio of the radii of the circular paths described by them is
(a) $1 : \sqrt{2} : 1$ (b) $1 : \sqrt{2} : \sqrt{2}$ (c) $\sqrt{2} : \sqrt{2} : 1$ (d) $\sqrt{2} : 1 : 1$

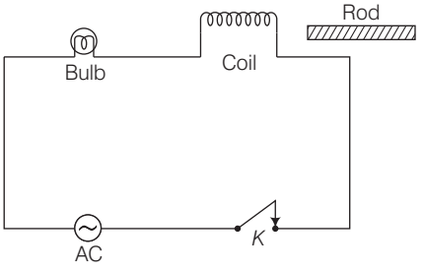
10. In the three parts of a transistor, 'Emitter' is of
(a) larger size and lightly doped
(b) moderate size and heavily doped
(c) thin size and heavily doped
(d) large size and moderately doped

11. A bar magnet is allowed to fall vertically through a copper coil placed in a horizontal plane. The magnet falls with a net acceleration, is


(a) Zero (b) = g
(c) < g (d) > g

12. A basic communication system consists of
(i) Transmitter (ii) Information sources
(iii) User of information (iv) Channel
(v) Receiver
(a) ii, i, iv, v and iii (b) i, ii, iii, iv and v
(c) ii, v, i, iv and iii (d) ii, iv, i, iii and v

13. A galvanometer of resistance 50 Ω is connected to a battery of 3 V along with a resistance of 2950 Ω in series shows full-scale deflection of 30 divisions. The additional series resistance required to reduce the deflection to 20 divisions is
(a) 2950 Ω (b) 1500 Ω
(c) 4440 Ω (d) 7400 Ω

14. In the A.C. circuit shown, keeping '*K*' pressed, if an iron rod inserted into the coil, the bulb in the circuit,


(a) glows less brightly
(b) glows with same brightness (as before the rod is inserted)
(c) gets damaged
(d) glows more brightly

15. A magnetic dipole of magnetic moment 6×10^{-2} A-m^2 and moment of inertia 12×10^{-6} kg-m^2 performs oscillations in a magnetic field of 2×10^{-2}T. The time taken by the dipole to complete 20 oscillations is ($\pi \simeq 3$)
(a) 18 s (b) 6 s (c) 36 s (d) 12 s

16. 4×10^{10} electrons are removed from a neutral metal sphere of diameter 20 cm placed in air. The magnitude of the electric field (in NC^{-1}) at a distance of 20 cm from its centre is
(a) 640 (b) 5760 (c) Zero (d) 1440

17. The mass defect of $^4_2$He is 0.03 u. The binding energy per nucleon of helium (in MeV) is
(a) 6.9825 (b) 27.93 (c) 2.793 (d) 69.825

18. A cylindrical conductor of diameter 0.1 mm carries a current of 90 mA. The current density (in Am^{-2}) is ($\pi \simeq 3$)
(a) 1.2×10^7 (b) 3×10^6 (c) 6×10^6 (d) 2.4×10^7

19. Which of the following properties is 'False' for a bar magnet?
(a) It doesn't produce magnetic field.
(b) Its like poles repel and unlike poles attract.
(c) Its poles cannot be separated.
(d) It points in North-South direction when suspended.

20. According to cartesian sign convention, in ray optics
(a) all distance are taken negative
(b) all distances in the direction of incident ray are taken positive
(c) all distances are taken positive
(d) al distances in the direction of incident ray are taken negative

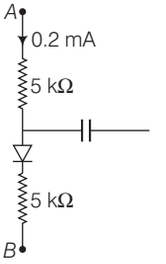
21. A linear object of height 10 cm is kept in front of a concave mirror of radius of curvature 15 cm, at a distance of 10 cm. The image formed is
(a) magnified and erect
(b) magnified and inverted
(c) diminished and erect
(d) diminished and inverted

22. The energy gap in case of which of the following is less than 3 eV?
(a) Germanium (b) Iron (c) Copper (d) Aluminium

23. The minimum value of effective capacitance that can be obtained by combining 3 capacitors of capacitances 1 pF, 2 pF and 4 pF
(a) $\frac{4}{7}$ pF (b) 1 pF (c) 2 pF (d) $\frac{7}{4}$ pF

24. Two spheres of electric charges + 2 nC and − 8 nC are placed at a distance d apart. If they are allowed to touch each other, what is the new distance between them to get a repulsive force of same magnitude as before?
(a) d (b) $\frac{d}{2}$ (c) $\frac{3d}{4}$ (d) $\frac{4d}{3}$

25. In the figure shown, if the diode forward voltage drop is 0.2 V, the voltage difference between A and B is



(a) 2.2 V (b) 1.3 V (c) 0 (d) 0.5 V

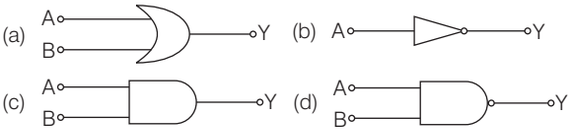
26. A straight wire of length 50 cm carrying a current of 2.5 A is suspended in mid-air by a uniform magnetic field of 0.5 T (as shown in figure). The mass of the wire is ($g = 10$ ms^{-2})



(a) 250 gm (b) 125 gm (c) 62.5 gm (d) 100 gm

27. Which of the following semi-conducting device is used as voltage regulator?
(a) LASER diode (b) Zener diode
(c) Solar cell (d) Photo diode

28. Which of the following logic gate is universal gate



29. A particle is dropped from a height H. The de-Broglie wavelength of the particle depends on height as
(a) $H^{-1/2}$ (b) H^0 (c) $H^{1/2}$ (d) H

30. Three point charges of + 2q, + 2q and − 4q are placed at the corners A, B and C of an equilateral triangle ABC of side x. The magnitude of the electric dipole moment of this system is
(a) $2qx$ (b) $3\sqrt{2}qx$ (c) $3qx$ (d) $2\sqrt{3}qx$

31. In a system of two crossed polarisers, it is found that the intensity of light from the second polariser is half from that of first polariser. The angle between their pass axes is
(a) 60° (b) 30° (c) 0° (d) 45°

32. A jet plane of wing span 20 m is travelling towards west at a speed of 400 ms^{-1}. If the earth's total magnetic field is 4×10^{-4} T and the dip angle is 30°, at that place, then the voltage differences developed across the ends of the wing is
(a) 0.8 V (b) 6.4 V
(c) 3.2 V (d) 1.6 V

33. The energy (in eV) required to excite an electron from $n = 2$ to $n = 4$ state in hydrogen atom is
(a) − 0.85 (b) + 4.25
(c) − 3.4 (d) + 2.55

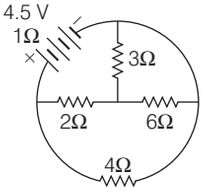
34. In a nuclear reactor, the function of the moderator is to decreases
(a) number of neutrons (b) speed of neutrons
(c) escape of neutrons (d) temperature of the reactor

35. The scientist who is credited with the discovery of 'nucleus' in an atom is
(a) Rutherford (b) Niels Bohr
(c) Balmer (d) J.J Thomson

36. A system of two capacitors of capacitance 2μF and 4 μF is connected in series across a potential difference of 6V. The electric charge and energy stored in a system are
(a) 36 μC and 108 μJ (b) 8 μC and 24 μJ
(c) 1 μC and 3 μJ (d) 10 μC and 30 μJ

37. In meter bridge experiment, with a standard resistance in the right gap and a resistance coil dipped in water (in a beaker) in the left gap, the balancing length obtained is 'ℓ'. If the temperature of water is increased, the new balancing
(a) < 1 (b) > 1 (c) = 0 (d) = 1

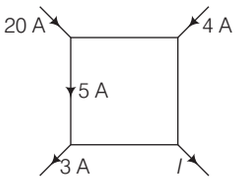
38. The power dissipated in 3Ω resistance in the following circuit is



(a) 0.25 W (b) 0.75 W (c) 1 W (d) 0.5 W

39. The magnetic field at the centre of a current carrying loop of radius 0.1 m is $5\sqrt{5}$ times that at a point along its axis. The distance of this point from the centre of the loop is
(a) 0.1 m (b) 0.2 m
(c) 0.05 m (d) 0.25 m

40. The value of I in the figure shown below is



(a) 21 A (b) 4 A
(c) 8 A (d) 19 A

41. In Young's double-slit experiment, if yellow light is replaced by blue light, the interference fringes becomes
(a) darker
(b) brighter
(c) wider
(d) narrower

42. The output of a step down transformers is measured to be 48 V when connected to a 12 W bulb. The value of peak current is
(a) $\frac{1}{\sqrt{2}}$ A (b) $\sqrt{2}$ A
(c) $\frac{1}{2\sqrt{2}}$ A (d) $\frac{1}{4}$ A

Andhra Pradesh EAMCET

1. An infinitely long thin straight wire has uniform linear charge density of $\frac{1}{3}\text{Cm}^{-1}$. Then the magnitude of the force acting on a charge 3 μC situated at a point of 18 cm away from the wire is
$\left(\frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{Nm}^2\text{C}^{-2}\right)$
(a) 2×10^5 N (b) 10^5 N (c) $\frac{1}{3}\times10^6$ N (d) 3×10^{11} N

2. An electrostatic paint sprayer has a metal sphere of diameter 18 cm and at a potential of 25 kV. The charge on the metal sphere is
(a) 0.25 μC (b) 2.5 μC
(c) 0.5 μC (d) 25 μC

3. Two long parallel conducting wires carrying currents are seperated by a distance 'x'. Work done per unit length in changing the distance between the wires is proportional to
(a) $\frac{1}{\log_e x}$ (b) $\frac{1}{x}$ (c) $\log_e x$ (d) x

4. A Zener diode voltage regulator operated in the range 120V-180V produces a constant supply of 110 V and 250 mA to the load. If the maximum current is equally shared between the load and the Zener diode, then the values of load resistance (R_L) and series resistance (R_S) are respectively
(a) $R_L = 280\,\Omega, R_S = 70\,\Omega$ (b) $R_L = 440\,\Omega, R_S = 140\,\Omega$
(c) $R_L = 70\,\Omega, R_S = 280\,\Omega$ (d) $R_L = 440\,\Omega, R_S = 1400\,\Omega$

5. Two point sources S_1 and S_2 are 24 cm apart. Where should a convex lens of focal length 9cm be placed in between them, so that the images of both sources are formed at the same place?
(a) 8 cm (b) 12 cm (c) 6 cm (d) 10 cm

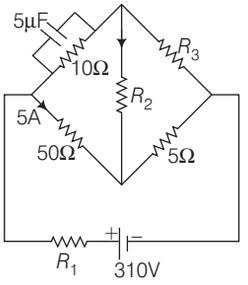
6. It is more difficult to push a magnet into a coil with more number of turns.
The emf induced in a coil opposes the motion of a magnet when it is moved towards the coil.
(a) A is false, R is true
(b) Both A and R are true. R is correct explanation of A
(c) A is true, R is false
(d) Both A and R are true. R is not correct explanation of A

7. In the determination of the internal resistance of a cell with a potentiometer, the error in the measurement of the balancing length is ± 1 mm. When the cell alone is connected in the circuit, the balancing length is obtained at 60 cm and when the cell is shunted with a resistance of 10 Ω ± 2%, the balancing length is obtained at 50 cm. The error in the determination of the internal resistance of the cell is ..... .
(a) 2.4% (b) 4.2% (c) 1.8% (d) 5.6 %

8. The half life of a stream of radioactive particles moving along a straight path with a constant kinetic energy of 4 eV is 1 minute. The percentage of particles which decay before travelling a distance of 3.6 km is (Mass of the radioactive particles = 3.2×10^{-21} kg and charge of the electron = 1.6×10^{-19} C).
(a) 87.5 (b) 175 (c) 37.5 (d) 75

9. The process of recovering the modulating signal from the modulated carrier wave is called
(a) amplification (b) detection
(c) rectification (d) noise

10. If the charge on the capacitor is 1 mC in the given circuit,

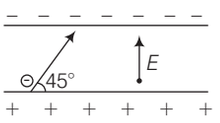


then $\frac{R_1R_2}{R_3}$ = ... Ω .
(a) 6 (b) 0.4 (c) 0.6 (d) 10

11. The amplitude of electric field in an electromagnetic wave is 60 Vm^{-1}. Then the amplitude of magnetic field is
(a) 2×10^{-7} T (b) 2×10^{7} T (c) 6×10^{7} T (d) 6×10^{-7} T

12. When a coil is connected to AC supply of frequency 50 Hz, a current of 4 A flows in it and it consumes 240 W power. If the potential difference across the coil is 100 V, then the inductance value of the coil is
(a) $L = (5\pi)$ H (b) $L = \frac{\pi}{5}$ H (c) $L = \frac{1}{5\pi}$ H (d) $L = \frac{1}{25\pi}$ H

13. The uniform electric field intensity between the two plates of a parallel plate capacitor is 1×10^{3} Vm^{-1} acting vertically upwards as shown in the figure.

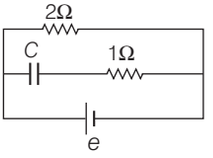


The plates are sufficiently long and have separation 2 cm. A particle of negative charge 1 μC and mass 2 g is projected at an angle 45° with the electric field from the lower plate with a velocity 'u'. The maximum velocity acquired by the particle, if it is not hit the upper plate is
(a) 2 ms^{-1} (b) 1 ms^{-1} (c) 0.1 ms^{-1} (d) 0.2 ms^{-1}

14. Both an electron and a photon have same de-Broglie wavelength of 1.2 Å. The ratio of their energies is nearly
(a) 1 : 100 (b) 1 : 10 (c) 1 : 1000 (d) 1 : 1

15. When the terminals of a cells are connected by a wire of resistance 4 Ω, the potential difference across the cell is 1.6 V. If a wire of the same resistance is connected in parallel with the first, the potential difference becomes 1.33 V. The emf and internal resistance of the cell are respectively
(a) 1 V, 1 Ω (b) 2 V, 1 Ω
(c) 1 V, 2 Ω (d) 2 V, 2 Ω

16. Two concentric coils of 20 turns each are placed in same plane. Their radii are 30 cm and 60 cm and carry 0.4 A and 0.6 A currents respectively in opposite directions. The magnetic induction at the centre in tesla is ..... .
(a) $\frac{8}{3}\mu_0$ (b) $\frac{2}{3}\mu_0$ (c) $\frac{5}{3}\mu_0$
(d) $\frac{10}{3}\mu_0$



17. In steady state, a capacitor of capacitance 2μF is charged to 4 μC, as shown in figure. If the internal resistance of the cell is 0.5 Ω, then the emf of the cell is
(a) 4 V (b) 5 V
(c) 2.5 V (d) 2 V

18. The radius of orbit of an electron and the speed of electron in the ground state of hydrogen atom are 5.5×10^{-11} m and 4×10^{6} ms^{-1} respectively. Then, the orbital period of this electron in the first excited state will be ..... .
(a) 6.908×10^{-16} s (b) 9.608×10^{-16} s
(c) 7.806×10^{-16} s (d) 8.9068×10^{-16} s

19. Two slits separated by 0.5 mm are illuminated by light of wavelength 500 nm. The screen is at a distance of 120 cm from the slits. The phase difference between the interfering waves at a point 3 mm on the screen from the central bright fringe is ..... .
(a) 5π (b) π (c) 3π (d) 7π

20. Match the following

List-I	List-II
(a) High retentivity	(i) Telephone diaphram
(b) High resistivity	(ii) Diamagnet
(c) Low coercivity	(iii) To decrease eddy current losses
(d) Negative susceptibility	(iv) Permanent magnet

The correct answer is
(a) a-(i), b-(iv), c-(iii), d-(ii) (b) a-(iv), b-(iii), c-(i), d-(ii)
(c) a-(i), b-(ii), c-(iii), d-(iv) (d) a-(iv), b-(ii), c-(i), d-(iii)

Telangana State EAMCET

1. A positive charge Q is placed on a conducting spherical shell with inner radius R_1 and outer radius R_2. A particle with charge q is placed at the center of the spherical cavity. The magnitude of the electric field at a point in the cavity, a distance r from center is

(a) zero (b) $\frac{Q}{4\pi\varepsilon_0 r^2}$ (c) $\frac{q}{4\pi\varepsilon_0 r^2}$ (d) $\frac{(q+Q)}{4\pi\varepsilon_0 r^2}$

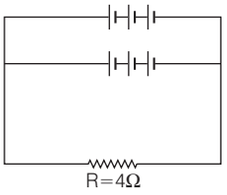
2. A coil having n turns and resistance $R\Omega$ is connected with a galvanometer of resistance $4R\Omega$. This combination is moved in time t seconds from a magnetic flux ϕ_1 Weber to ϕ_2 Weber. The induced current in the circuit is

(a) $\frac{\phi_2 - \phi_1}{5Rnt}$ (b) $-\frac{n(\phi_2 - \phi_1)}{5Rt}$ (c) $-\frac{(\phi_2 - \phi_1)}{Rnt}$ (d) $-\frac{n(\phi_2 - \phi_1)}{Rt}$

3. Consider a metal ball of radius r moving at a constant velocity v in a uniform magnetic field of induction $\overline{B}$. Assuming that the direction of velocity forms an angle α with the direction of $\overline{B}$, the maximum potential difference between points on the ball is

(a) $r|\overline{B}||\overline{v}|\sin\alpha$ (b) $|\overline{B}||\overline{v}|\sin\alpha$
(c) $2r|\overline{B}||\overline{v}|\sin\alpha$ (d) $2r|\overline{B}||\overline{v}|\cos\alpha$

4. Each of the six ideal batteries of emf 20 V is connected to an external resistance of 4Ω as shown in the figure. The current through the resistance is

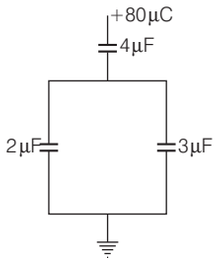


(a) 6 A (b) 3 A (c) 4 A (d) 5 A

5. The energy that should be added to an electron to reduce its de-Broglie wavelength from 1 nm to 0.5 nm is

(a) four-times the initial energy
(b) equal to the initial energy
(c) two-times the initial energy
(d) three-times the initial energy

6. In the given circuit, a charge of $+80\,\mu C$ is given to upper plate of a $4\mu F$ capacitor. At steady state, the charge on the upper plate of the $3\mu F$ capacitor is



(a) $60\,\mu C$ (b) $48\,\mu C$ (c) $80\,\mu C$ (d) $0\,\mu C$

7. Consider a solenoid carrying current supplied by a DC source with a constant emf containing iron core inside it. When the core is pulled out of the solenoid, the change in current will

(a) remain same (b) decrease
(c) increase (d) modulate

8. A parallel beam of light of intensity I_0 is incident on a coated glass plate. If 25% of the incident light is reflected from the upper surface and 50% of light is reflected from the lower surface of the glass plate, the ratio of maximum to minimum intensity in the interference region of the reflected light is

(a) $\left(\frac{\frac{1}{2}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$ (b) $\left(\frac{\frac{1}{4}+\sqrt{\frac{3}{8}}}{\frac{1}{2}-\sqrt{\frac{3}{8}}}\right)^2$ (c) $\frac{5}{8}$ (d) $\frac{8}{5}$

9. Which of the following is emitted, when $^{239}_{94}Pu$ decays into $^{235}_{92}U$?

(a) Gamma ray (b) Neutron
(c) Electron (d) Alpha particle

10. An AC generator 10 V (rms) at 200 rad/s is connected in series with a 50 Ω resistor, a 400mH inductor and a 200 μF capacitor. The rms voltage across the inductor is

(a) 2.5 V (b) 3.4 V (c) 6.7 V (d) 10.8 V

11. A wire has resistance of 3.1 Ω at 30°C and 4.5Ω at 100°C. The temperature coefficient of resistance of the wire is

(a) $0.0012°C^{-1}$ (b) $0.0024°C^{-1}$ (c) $0.0032°C^{-1}$ (d) $0.0064°C^{-1}$

12. An electron collides with a hydrogen atom in its ground state and excites it to $n = 3$ state. The energy given to the hydrogen atom in this inelastic collision (neglecting the recoil of hydrogen atom) is

(a) 10.2 eV (b) 12.1 eV (c) 12.5 eV (d) 13.6 eV

13. Consider a light source placed at a distance of 1.5 m along the axis facing the convex side of a spherical mirror of radius of curvature 1 m . The position (s'), nature and magnification (m) of the image are

(a) $s' = 0.375$ m, virtual, upright, $m = 0.25$
(b) $s' = 0.375$ m, real, inverted, $m = 0.25$
(c) $s' = 3.75$ m, virtual, inverted, $m = 2.5$
(d) $s' = 3.75$ m, real, upright, $m = 2.5$

14. A piece of copper and a piece of germanium are cooled from room temperature to 80K. Then, which one of the following is correct?

(a) Resistance of each will increase
(b) Resistance of each will decrease
(c) Resistance of copper will decrease while that of germanium will increase
(d) Resistance of copper will increase while that of germanium will decrease

15. A beam of light propagating at an angle α_1 from a medium 1 through to another medium 2 at an angle α_2. If the wavelength of light in medium 1 is λ_1, then the wavelength of light in medium 2, (λ_2), is

(a) $\frac{\sin\alpha_2}{\sin\alpha_1}\lambda_1$ (b) $\frac{\sin\alpha_1}{\sin\alpha_2}\lambda_2$ (c) $\left(\frac{\alpha_1}{\alpha_2}\right)\lambda_1$ (d) λ_1

16. An amplitude modulated signal consists of a message signal of frequency 1 KHz and peak voltage of 5V, modulating a carrier frequency of 1 MHz and peak voltage of 15 V. The correct description of this signal is

(a) $5\,[1 + 3\sin(2\pi 10^6 t)]\sin(2\pi 10^3 t)$

(b) $15\left[1 + \frac{1}{3}\sin(2\pi 10^3 t)\right]\sin(2\pi 10^6 t)$

(c) $[5 + 15\sin(2\pi 10^3 t)]\sin(2\pi 10^6 t)$

(d) $[15 + 5\sin(2\pi 10^6 t)]\sin(2\pi 10^3 t)$

17. A generator with a circular coil of 100 turns of area 2×10^{-2} m^2 is immersed in a 0.01 T magnetic field and rotated at a frequency of 50 Hz. The maximum emf which is produced during a cycle is

(a) 6.28 V (b) 3.44 V
(c) 10 V (d) 1.32 V

18. Which of the following statement is not true?

(a) the resistance of an intrinsic semiconductor decreases with increase in temperature
(b) doping pure Si with trivalent impurities gives p-type semiconductor
(c) the majority carriers in n-type semiconductors are holes
(d) a $p-n$ junction can act as a semiconductor diode

19. A current carrying wire in its neighbourhood produces

(a) electric field (b) electric and magnetic fields
(c) magnetic field (d) no field

VIT

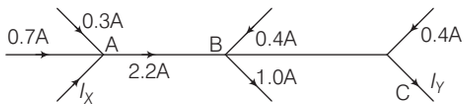
1. Charge Q is divided into two parts which are kept some distance apart. The force between them will be maximum, if the two parts are having the charge (e = electric charge)

(a) $Q/2$ each (b) $Q/4$ and $\frac{3Q}{4}$
(c) $Q/3$ and $\frac{3Q}{3}$ (d) e and $(Q-e)$

2. If a shunt of $1/10$ th of the coil resistance is applied to a moving coil galvanometer. Its sensitivity becomes

(a) 10 folds (b) $\frac{1}{10}$ folds (c) 11 folds (d) $\frac{1}{11}$ folds

3. If figure value of I_x and I_y are respectively



(a) 1A, 1A (b) 1.2, 1.2A
(c) 0.8A, 0.8A (d) 1A and 1.2A

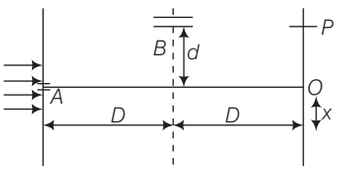
4. The period of oscillation of a magnet in a vibration magnetometer is 8 s. The period of oscillation of magnet whose magnetic moment is four time that of the first magnet is

(a) 1s (b) 4s (c) 8s (d) 16s

5. The refractive index of the material of a prism is $\sqrt{2}$ and its refracting angle is 30°. One of the refracting surface of the prism is made a mirror inwards. A beam of monochromatic light entering the prism from the other face will retrace its path after reflection from the mirrored surface if its angle of incidence on the prism is

(a) 45° (b) 60°
(c) 0° (d) 30°

6. Consider the arrangement as shown in figure. The distance D is large compared to d. Minimum value of d so that there is a dark fringe at O, is



(a) $\sqrt{\frac{\lambda D}{2}}$ (b) $\sqrt{\frac{2}{D\lambda}}$ (c) $\sqrt{\frac{D}{2\lambda}}$ (d) $\sqrt{\frac{2\lambda}{D}}$

7. Ratio of longest wavelength corresponding to Lyman and Balmar series in hydrogen spectrum is

(a) $\frac{5}{27}$ (b) $\frac{3}{23}$ (c) 7/29 (d) 9/31

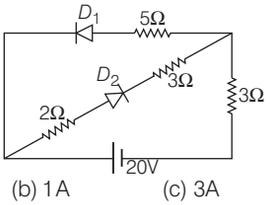
8. Hydrogen atom excites energy level from fundamental state to $n = 3$. Number of spectrum lines according to Bohr is

(a) 4 (b) 3 (c) 1 (d) 2

9. The half life of a radioactive isotope X is 50 yr. It decays to another element Y which is stable. The two elements X and Y are found to be the ratio of 1 : 15 in a sample of a given rock. The age of the rock was estimated to be

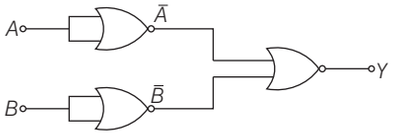
(a) 200 yr (b) 250 yr (c) 100 yr (d) 150 yr

10. In the given circuit, all diodes are ideal. The current through battery is



(a) 2A (b) 1A (c) 3A (d) 4 A

11. Identify the operation performed by the circuit given below



(a) NOT (b) AND (c) OR (d) NAND

12. There exists an electric field of 1N/C along Y-direction. The flux passing through the square of 1m placed in $X - Y$ plane inside the electric field is

(a) 1N-m^2/C (b) 10N-m^2/C (c) 2N-m^2/C (d) Zero

13. A uniform electric field and a uniform magnetic field are produced, pointed in the same direction. An electron is projected with its velocity pointing in the same direction.

(a) The electrons will turn to its left
(b) The electrons will turn to its right
(c) The electron velocity will increase in magnitude
(d) The electron velocity will decrease in magnitude

14. The magnetic flux through a circuit of resistance R changes by an amount $\Delta\phi$ in a time Δt. Then the total quantity of electric charge q that passes any point in the circuit during the time Δt is presented by

(a) $q = \frac{1}{R}\frac{\Delta\phi}{\Delta t}$ (b) $q = \frac{\Delta\phi}{R}$ (c) $q = \frac{\Delta\phi}{\Delta t}$ (d) $q = R\frac{\Delta\phi}{\Delta t}$

15. The internal resistance of a 2.1V cell which gives a current of 0.2A through a resistance of 10Ω is

(a) 0.2Ω (b) 0.5Ω (c) 0.8Ω (d) 1.0Ω

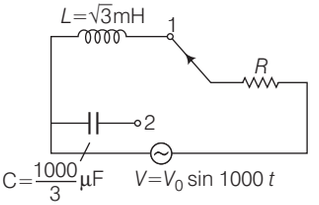
16. In a uniform magnetic field of induction B, a wire in the form of semicircle of radius r rotates about the diameter of the semicircle with angular frequency ω. If the total resistance of the semicircle is R, then mean power generated per rotation of the semicircular wire is

(a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$
(c) $\frac{(B\pi r\omega)^2}{2R}$ (d) None of these

17. A spherical drop of capacitance 1μF is broken into eight drops of equal radius. Then, the capacitance of each small drop is

(a) 1/2μF (b) $\frac{1}{4}$μF (c) $\frac{1}{8}$μF (d) 8μF

18. In the given AC circuit, when switch is at position 1, the source emf leads current by $\pi/6$. Now, if the switch is at position 2, then



(a) current leads source emf by $\pi/3$
(b) current leads source emf by $\pi/4$
(c) source emf lead current by $\pi/4$
(d) None of the above

19. A thin convergent glass lens ($\mu = 1.5$) has a power of + 5.0D. When this lens is immersed in a liquid of refractive index μ_l it acts as a divergence lens of focal length 100 cm. The value of μ_l should be

(a) 3/2 (b) 4/3 (c) 5/3 (d) 2

20. In Young's double slit experiment, one of the slit is covered with a transparent sheet of thickness $3{\cdot}6\times10^{-3}$ cm due to which position of central bright shifts to a position originally occupied by 30th bright fringe. The refractive index of the sheet if $\lambda = 6000$Å is

(a) 1.5 (b) 1.2
(c) 1.3 (d) 1.7

21. Energy E of a hydrogen atom with principle quantum number n is given by $E = \frac{13.6}{n^2}$ eV. The energy of a photon ejected when the electron jumps from $n = 3$ state to $n = 2$ state of hydrogen is approximately

(a) 1.5 eV (b) 0.85 eV
(c) 3.3 eV (d) 1.9 eV

22. Calculate the maximum energy possessed by a β-particle in following decay

$^{176}\text{Lu} \longrightarrow {}^{176}\text{Hf} + \beta^- + \bar{\nu}$ atomic mass of ^{176}Lu = 175.94269μ and of ^{176}Hf = 175.94142 μ

(a) 1.921 MeV (b) 2.923 MeV
(c) 1.182 MeV (d) 10 MeV

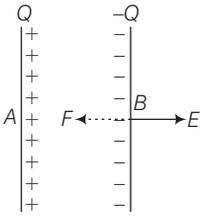
23. The activity of $_{79}\text{Au}^{200}$ of mass 2.0×10^{-8} kg is 60 curie. Find the disintegration constant of $_{79}\text{Au}^{200}$.

(a) $3.7\times10^{-5}\text{s}^{-1}$ (b) $4.9\times10^{-9}\text{s}^{-1}$
(c) $10\times10^{-9}\text{s}^{-1}$ (d) $9.1\times10^{-11}\text{s}^{-1}$

24. A transistor with $\alpha = 0.98$ is operated in common emitter circuit with a load resistance of 5K Ω. If the dynamic resistance of the base emitter junction is 70Ω. Calculate the voltage gain and power gain.

(a) $3500, 1.715\times10^5$ (b) $4000, 1.9\times10^6$
(c) $4000, 1.21\times10^9$ (d) $2000, 1.91\times10^{11}$

25. Find the force between the plates of a parallel plate air capacitor. Charge on capacitor is Q and area of each plate is A.



(a) $\frac{Q^2}{\varepsilon_0 A}$ (b) $\frac{Q^2}{\varepsilon_0}$ (c) $\frac{Q^2}{2A\varepsilon_0}$ (d) $\frac{Q^2}{2A}$

26. A current of 5 ampere is passing through a metallic wire of cross section area $4 \times 10^{-6} m^2$. If the density of charge carriers in the wire is $5 \times 10^{26} / m^3$ find the drift velocity of the electrons

(a) $\frac{1}{64}$ m/s (b) $\frac{2}{39}$ m/s (c) 64 m/s (d) 9 m/s

27. Inside a long solenoid wounded with 300 turns/metre, an iron rod is placed. The iron rod is 0.2 m long, 10 mm in diameter and of permeability 10^3. The magnetic moment of the rod, if 0.5 Å of current is passed through the rod is

(a) 2.356 SI unit (b) 1.335 SI unit
(c) 3.664 SI unit (d) 1.664 SI unit

28. A convex lens and a concave lens each having same focal length of 25 cm, are put in contact to form a combination of lenses. The power in dioptre of the combination of lenses is

(a) 25 (b) 50 (c) Infinite (d) zero

29. If prism angle $\alpha = 1^\circ$, $\mu = 1.54$, distance between screen and prism $b = 0.7$ m, distance between prism and source $a = 0.3$ m, $\lambda = 180\pi$ nm then in Fresnel biprism find the value of β (fringe width)

(a) 10^{-4} m (b) 10^{-3} mm
(c) $10^{-4} \times \pi$ m (d) $\pi \times 10^{-3}$ m

30. The wavelength λ of a photon and the de-Broglie wavelength of an electron have the same value. Find the ratio of energy of photon to the kinetic energy of electron in terms of mass m, speed of light c and Plank's constant h.

(a) $\frac{\lambda mc}{h}$ (b) $\frac{hmc}{\lambda}$ (c) $\frac{2hcm}{\lambda}$ (d) $\frac{2\lambda mc}{h}$

31. In an atom the two electrons move around the nucleus in circular orbits of radii R and $4R$. What is the ratio of the times taken by them to complete one revolution?

(a) 1 : 8 (b) 1 : 9
(c) 12 : 13 (d) 1 : 5

MHT CET

1. The frequencies for series limit of Balmer and Paschen series respectively are 'ν_1' and 'ν_3'. If frequency of first line of Balmer series is 'ν_2' then the relation between 'ν_1', 'ν_2' and 'ν_3' is

(a) $\nu_1 - \nu_2 = \nu_3$ (b) $\nu_1 + \nu_3 = \nu_2$
(c) $\nu_1 + \nu_2 = \nu_3$ (d) $\nu_1 - \nu_3 = 2\nu_1$

2. When three capacitors of equal capacities are connected in parallel and one of the same capacity is connected in series with its combination. The resultant capacity is 3.75 μF. The capacity of each capacitor is

(a) 5μF (b) 6μF
(c) 7μF (d) 8μF

3. Sensitivity of moving coil galvanometer is 'S'. If a shunt of $\left(\frac{1}{8}\right)$th of the resistance of galvanometer is connected to moving coil galvanometer, its sensitivity becomes

(a) $\frac{S}{3}$ (b) $\frac{S}{6}$ (c) $\frac{S}{9}$ (d) $\frac{S}{12}$

4. Two unknown resistances are connected in two gaps of a meter-bridge. The null point is obtained at 40cm from left end. A 30Ω resistance is connected in series with the smaller of the two resistances,
the null point shifts by 20cm to the right end. The value of smaller resistance in Ω is

(a) 12 (b) 24 (c) 36 (d) 48

5. In Fraunhofer diffraction pattern, slit width is 0.2 mm and screen is at 2m away from the lens. If wavelength of light used is 5000Å, then the distance between the first minimum on either side of the central maximum is (θ is small and measured in radian)

(a) 10^{-1}m (b) 10^{-2}m (c) 2×10^{-2}m (d) 2×10^{-1}m

6. In series circuit $R = 18\Omega$ and impedance is 33 Ω. An rms voltage 220V is applied across the circuit. The true power consumed in AC circuit is

(a) 220 W (b) 400 W (c) 600 W (d) 800 W

7. Two parallel plate air capacitors of same capacity are connected in series to a battery of emf . Then one of the capacitors is completely filled with dielectric material of constant . The change in the effective capacity of the series combination is

(a) $\frac{C}{2}\left[\frac{K-1}{K+1}\right]$ (b) $\frac{2}{C}\left[\frac{K-1}{K+1}\right]$ (c) $\frac{C}{2}\left[\frac{K+1}{K-1}\right]$ (d) $\frac{C}{2}\left[\frac{K-1}{K+1}\right]^2$

8. The polarising angle for transparent medium is 'θ' and 'v' is the speed of light in that medium. Then the relation between 'θ' and ' ' is (= velocity of light in air)

(a) $\theta = \tan^{-1}\left(\frac{v}{c}\right)$ (b) $\theta = \cot^{-1}\left(\frac{v}{c}\right)$
(c) $\theta = \sin^{-1}\left(\frac{v}{c}\right)$ (d) $\theta = \cos^{-1}\left(\frac{v}{c}\right)$

9. Two identical light waves having phase difference 'ϕ' propagate in same direction. When they superpose, the intensity of resultant wave is proportional to

(a) $\cos^2\phi$ (b) $\cos^2\frac{\phi}{2}$ (c) $\cos^2\frac{\phi}{3}$ (d) $\cos^2\frac{\phi}{4}$

10. For a transistor, α_{dc} and β_{dc} are the current ratios, then the value of $\frac{\beta_{dc} - \delta_{dc}}{\alpha_{dc} \cdot \beta_{dc}}$ is

(a) 1 (b) 1.5 (c) 2 (d) 2.5

11. A radioactive element has rate of disintegration 10,000 disintegrations per minute at a particular instant. After

four minutes it becomes 2500 disintegrations per minute. The decay constant per minute is

(a) 0.2 $\log_e 2$ (b) 0.5 $\log_e 2$ (c) 0.6 $\log_e 2$ (d) 0.8 $\log_e 2$

12. When the same monochromatic ray of light travels through glass slab and through water, the number of waves in glass slab of thickness 6 cm is same as in water column of height 7 cm. If refractive index of glass is 1.5, then refractive index of water is

(a) 1.258 (b) 1.269 (c) 1.286 (d) 1.310

13. If the electron in hydrogen atom jumps from second Bohr orbit to ground state and difference between energies of the two states is radiated in the form of photons. If the work function of the material is 4.2 eV, then stopping potential is

[Energy of electron in th orbit $= -\frac{13.6}{n^2}$ eV]

(a) 2 V (b) 4 V (c) 6 V (d) 8 V

14. The magnetic moment of electron due to orbital motion is proportional to (= principal quantum numbers)

(a) $\frac{1}{n^2}$ (b) $\frac{1}{n}$ (c) n^2 (d) n

15. Photodiode is a device

(a) which is always operated in reverse bias
(b) which is always operated in forward bias
(c) in which photo current is independent of intensity of incident radiation
(d) which may be operated in forward or reverse bias.

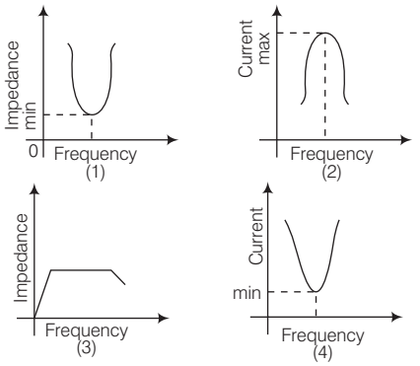
16. The ratio of binding energy of a satellite at rest on earth's surface to the binding energy of a satellite of same mass revolving around the earth at a height h above the earth's surface is (R = radius of the earth).

(a) $\frac{2(R+h)}{R}$ (b) $\frac{R+h}{2}$ (c) $\frac{R+h}{R}$ (d) $\frac{R}{R+h}$

17. On a photosensitive material, when frequency of incident radiation is increased by 30%, kinetic energy of emitted photoelectrons increases from 0.4 eV to 0.9 eV. The work function of the surface is

(a) 1 eV (b) 1.267 eV (c) 1.4 eV (d) 1.8 eV

18. Out of the following graphs, which graphs shows the correct relation (graphical representation) for parallel resonant circuit?



19. According to de-Broglie hypothesis, the wavelength associated with moving electron of mass ' ' is 'λ_e'. Using mass energy relation and Planck's quantum theory, the wavelength associated with photon is 'λ_p'. If the energy ( ) of electron and photon is same, then relation between 'λ_e' and 'λ_p' is

(a) $\lambda_p \propto \lambda_e$ (b) $\lambda_p \propto \lambda_e^2$ (c) $\lambda_p \propto \sqrt{\lambda_e}$ (d) $\lambda_p \propto \frac{1}{\lambda_e}$

20. A parallel plate air capacity ' ' farad, potential ' ' volt and energy ' ' joule. When the gap between the plates is completely filled with dielectric

(a) both V and E increase
(b) both V and E decrease
(c) V decrease, E increases
(d) V increases, E decrease

21. The resistivity of potentiometer wire is 40×10^{-8} ohm-metre and its area of cross-section is $8\times10^{-6}\text{m}^2$. If 0.2 ampere current is flowing through the wire, then the potential gradient of the wire is

(a) 10^{-1} V/m (b) 10^{-2} V/m (c) 10^{-3} V/m (d) 10^{-4} V/m

22. Two coils P and Q are kept near each other. When no current flows through coil and current increases in coil at the rate 10A/s, the emf in coil P is 15mV. When coil carries no current and current of 1.8A flows through coil , the magnetic flux linked with the coil is

(a) 1.4 m Wb (b) 2.2 mWb (c) 2.7 m Wb (d) 2.9 m Wb

23. In Young's double experiment, in air interference pattern second minimum is observed exactly in front of one slit. The distance between the two coherent source is ' ' and the distance between source and screen is ' '. The wavelength of light source used is

(a) $\frac{d^2}{D}$ (b) $\frac{d^2}{2D}$ (c) $\frac{d^2}{3D}$ (d) $\frac{d^2}{4D}$

24. In communication system, the process of superimposing a low frequency signal on a high frequency wave is known as

(a) repeater (b) attenuation
(c) modulation (d) demodulation

25. A bar magnet has length 3 cm, cross-sectional area 2 cm^3 and magnetic moment 3 Am^2. The intensity of magnetisation of bar magnet is

(a) 2×10^5 A/m (b) 3×10^5 A/m
(c) 4×10^5 A/m (d) 5×10^5 A/m

26. The magnetic flux near the axis and inside the air core solenoid of length 60cm carrying current ' ' is 1.57×10^{-6} Wb. Its magnetic moment will be (cross-sectional area of a solenoid is very small as compared to its length, $\mu_0 = 4\pi\times10^{-7}$ SI unit)

(a) 0.25 A (b) 0.50 A
(c) 0.75 A (d) 1 A

Answer *with* Explanations

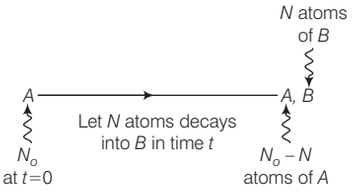
JEE Main

1. *(c)* We have, $\lambda = \frac{hc}{\Delta E}$

$\therefore$ So, ratio of wave lengths

$$\frac{\lambda_1}{\lambda_2} = \frac{hc/\Delta E_1}{hc/\Delta E_2} = \frac{\Delta E_2}{\Delta E_1} = \frac{\left(\frac{4}{3}E - E\right)}{2E - E} = \frac{1}{3}$$

2. *(a)* Decay scheme is ,



Given, $\frac{N_B}{N_A} = 0.3 = \frac{3}{10} \Rightarrow \frac{N_B}{N_A} = \frac{30}{100}$

So, $N_0 = 100 + 30 = 130$ atoms

By using $N = N_0 e^{-\lambda t}$

We have, $100 = 130\, e^{-\lambda t}$

$\Rightarrow \quad \frac{1}{1.3} = e^{-\lambda t} \Rightarrow \log 1.3 = \lambda t$

If T is half-life, then $\lambda = \frac{\log_e 2}{T}$

$\Rightarrow \log 1.3 = \frac{\log_e 2}{T} \cdot t \Rightarrow t = \frac{T \cdot \log (1.3)}{\log_e 2}$

3. *(a)* In a balanced Wheatstone bridge, there is no effect on position of null point, if we exchange the battery and galvanometer. So, option (a) is incorrect.

4. *(c)* As each capacitors cannot withstand more than 300 V, so there should be four capacitors in each row became in this condition 1 kV i.e. 1000 V will be divided by 4 (i.e. 250 not more than 300 V).

Now, equivalent capacitance of one row

$$= \frac{1}{4} \times 1\mu F = 0.25\mu F$$

$$\left[\because \text{in series combination, } C_{eq} = \frac{C}{n}\right]$$

Now, we need equivalent of 2 μF, so let we need n such rows

$\therefore \quad n \times 0.25 = 2\,\mu F \quad [\because$ in parallel combination $C_{eq} = nc]$

$$n = \frac{2}{0.25} = 8$$

$\therefore$ Total number of capacitors

= number of rows

× number of capacitors in each row

$= 8 \times 4 = 32$

5. *(b)* In steady state, no current flows through the capacitor. So, resistance r_p becomes ineffective.

So, the current in circuit,

$$I = \frac{E}{r + r_2 \text{ (Total Resistance)}}$$

$\because$ Potential drop across capacitor = Potential drop

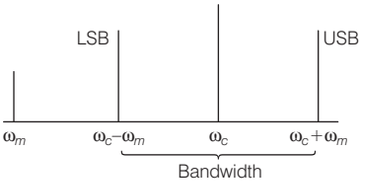
Across $r_2 = Ir_2 = \frac{Er_2}{r + r_2}$

$\therefore$ Stored charge of capacitor, $Q = CV$

$$= CE\frac{r_2}{r + r_2}$$

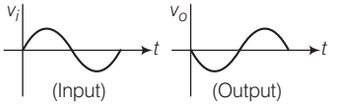
6. *(c)* Each resistance is converted with two cells combined in opposite direction, so potential drop across each resistor is zero. Hence, the current through each of resistor is zero.

7. *(d)* Frequency spectrum of modulated wave is



Clearly, ω_m is not included in the spectrum.

8. *(c)*



In a CE *npn* transistor amplifier output is 180° out of phase with input.

9. *(a)* Let n_1th fringe formed due to first wavelength and n_2th fringe formed due to second wavelength coincide i.e. their distance from common central maxima will be same

i.e. $y_{n_1} = y_{n_2}$

$$\Rightarrow \quad \frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2}$$

$$= \frac{520}{650} = \frac{4}{5}$$

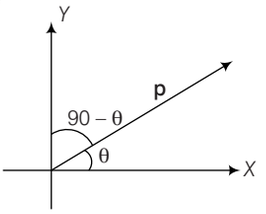
Hence, distance of the point of coincidence from the central maxima is

$$y = \frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$$

$$= \frac{4 \times 650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}}$$

$= 7.8$ mm

10. *(b)*



Torque applied on a dipole

$\tau = pE \sin \theta$

where θ = angle between axis of dipole and electric field.

For electric field $E_1 = E\hat{\mathbf{i}}$

it means field is directed along positive Xdirection, so angle between dipole and field will remain θ, therefore torque in this direction

$$E_1 = pE_1 \sin \theta$$

In electric field $E_2 = \sqrt{3}\, E\hat{\mathbf{j}}$, it means field is directed along positive Y-axis, so angle between dipole and field will be $90 - \theta$

Torque in this direction

$$T_2 = pE \sin (90 - \theta)$$

$$= p\sqrt{3}\, E_1 \cos \theta$$

According to question $\tau_2 = -\tau_1$

$\Rightarrow \quad |\tau_2| = |\tau_1|$

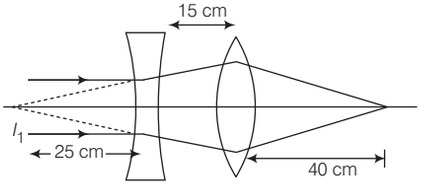
$\therefore \quad pE_1 \sin \theta = p\sqrt{3}\, E_1 \cos \theta$

$\tan\theta = \sqrt{3} \Rightarrow \tan \theta = \tan 60°$

$\therefore \qquad \theta = 60°$

11. *(d)* Focal length of diverging lens is 25 cm.

As the rays are coming parallel, so the image (I_1) will be formed at the focus of diverging lens i.e. at 25 cm towards left of diverging lens.



Now, the image (I_1) will work as object for converging lens.

For convergying lens, distance of object u

(i.e. distance of I_1) $= -(25 + 15)$

$= -40$ cm

$f = 20$ cm

$\therefore$ From len's formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{-40} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40}$$

$$\frac{1}{v} = \frac{1}{90} \Rightarrow v = 40 \text{ cm}$$

v is positive so image will be real and will form at right side of converging lens at 40 cm.

12. *(d)* $\lambda_{min} = \frac{hc}{eV}$

$$\log(\lambda_{min}) = \log\left(\frac{hc}{e}\right) - \log V$$

$$y = c - mx$$

So, the required graph is given in option (d).

13. *(b)* Induced constant, $I = \frac{e}{R}$

Here, e = induced emf $= \frac{d\phi}{dt}$

$$I = \frac{e}{R} = \left(\frac{d\phi}{dt}\right)\cdot\frac{1}{R}$$

$$d\phi = IRdt \Rightarrow \phi = \int IRdt$$

$\therefore$ Here, R is constant

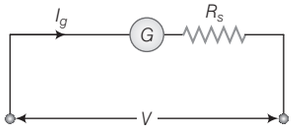
$\therefore \quad \phi = R\int Idt$

$\int I \cdot dt$ = Area under $I - t$ graph

$$= \frac{1}{2} \times 10 \times 0.5 = 2.5$$

$\therefore \phi = R \times 2.5 = 100 \times 2.5 = 250$ Wb.

14. *(d)* Suppose a resistance R_s is connected in series with galvanometer to convert it into voltmeter.



$$I_g (G + R_s) = V$$

$$\Rightarrow \quad R = \frac{V}{I_g} - G$$

$$\Rightarrow \quad R = 1985 = 1.985 \text{ k}\Omega$$

or $\quad R = 1.985 \times 10^3 \ \Omega$

15. *(d)* Time period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = 0.665 \text{ s}$$

Hence, time for 10 oscillations is $t = 6.65$ s.

16. *(a)* For elastic collision,

$p_{\text{before collision}} = p_{\text{after collision}}$.

$$mv = mv_A + \frac{m}{2}v_B$$

$$2v = 2v_A + v_B \qquad \cdots(i)$$

Now, coefficient of restitution,

$$e = \frac{v_B - v_A}{u_A - v_B}$$

Here, $u_B = 0$ (Particle at rest) and for elastic collisione = 1

$$\therefore \ 1 = \frac{v_B - v_A}{v} \Rightarrow v = v_B - v_A \qquad \ldots(ii)$$

From Eq. (i) and Eq. (ii)

$$v_A = \frac{v}{3} \text{ and } v_B = \frac{4v}{3}$$

Hence, $\frac{\lambda_A}{\lambda_B} = \frac{\left(\frac{h}{mV_A}\right)}{\left(\frac{h}{\frac{m}{2}.V_B}\right)} = \frac{V_B}{2V_A} = \frac{4/3}{2/3} = 2$

JEE Advanced

Paper 1

1. *(a,b)* At $\omega \approx 0$, $X_C = \frac{1}{\omega C} = \infty$.

Therefore, current is nearly zero.

Further at resonance frequency, current and voltage are in phase. This resonance frequency is given by,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6} \times 10^{-6}}} = 10^6 \text{ rad/s}$$

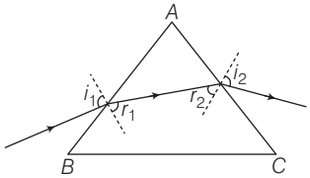
We can see that this frequency is independent of R.

Further, $X_L = \omega L$, $X_C = \frac{1}{\omega C}$

At, $\omega = \omega_r = 10^6$ rad/s, $X_L = X_C$.

For $\omega > \omega_r$, $X_L > X_C$. So, circuit is inductive.

2. *(a,b,c)* The minimum deviation produced by a prism



$\delta_m = 2i - A = A \Rightarrow i_1 = i_2 = A$

and $r_1 = r_2 = A/2 \Rightarrow r_1 = i_1/2$

Now, using Snell's law

$\sin A = \mu \sin A/2 \Rightarrow \mu = 2\cos(A/2)$

For this prism when the emergent ray at the second surface is tangential to the surface

$i_2 = \pi/2 \Rightarrow r_2 = \theta_c \Rightarrow r_1 = A - \theta_c$

so, $\sin i_1 = \mu \sin(A - \theta_c)$

so, $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$

For minimum deviation through isosceles prism, the ray inside the prism is parallel to the base of the prism if $\angle B = \angle C$.

But it is not necessarily parallel to the base if,

$\angle A = \angle B$ or $\angle A = \angle C$

3. *(b,d)* The net magnetic flux through the loops at time t is

$\phi = B(2A - A)\cos\omega t = BA\cos\omega t$

so, $\left|\frac{d\phi}{dt}\right| = B\omega A\sin\omega t$

$\therefore \quad \left|\frac{d\phi}{dt}\right|$ is maximum when

$\phi = \omega t = \pi/2$

The emf induced in the smaller loop,

$\varepsilon_{\text{smaller}} = -\frac{d}{dt}(BA\cos\omega t) = B\omega A\sin\omega t$

$\therefore$ Amplitude of maximum net emf induced in both the loops

= Amplitude of maximum emf induced in the smaller loop alone.

4. *(5)* Potential energy of hydrogen atom ($Z = 1$) in nth orbit (in eV)

$$\text{P, E} = -\frac{27.2}{n^2} \Rightarrow \frac{v_f}{v_i} = \frac{-\frac{27.2}{n_f^2}}{-\frac{27.2}{n_i^2}} = \frac{1}{6.25}$$

$$6.25 = \frac{n_f^2}{n_i^2} \Rightarrow \frac{n_f}{n_i} = 2.5 = \frac{5}{2}$$

Hence the answer is 5.

5. *(5)* $1^{131} \xrightarrow[T_{1/2} = 8 \text{ Days}]{} Xe^{131} + \beta$

$A_0 = 2.4 \times 10^5$ Bq $= \lambda N_0$

Let the volume is V, $t = 0$

$A_0 = \lambda N_0 \Rightarrow t = 11.5$ h $\Rightarrow A = \lambda N$

$$115 = \lambda\left(\frac{N}{V} \times 2.5\right)$$

$$115 = \frac{\lambda}{V} \times 2.5 \times (N_0 e^{-\lambda t})$$

$$115 = \frac{(N_0\lambda)}{V} \times (2.5) \times e^{-\frac{\ln 2}{8 \text{ day}}(11.5 \text{ h})}$$

$$115 = \frac{(2.4 \times 10^5)}{V} \times (2.5) \times e^{-1/24}$$

$$V = \frac{2.4 \times 10^5}{115} \times 2.5\left[1 - \frac{1}{24}\right]$$

$$= \frac{2.4 \times 10^5}{115} \times 2.5\left[\frac{23}{24}\right]$$

$$= \frac{10^5 \times 23 \times 25}{115 \times 10^2} = 5 \times 10^3 \text{ ml} = 5 \text{ L}$$

6. *(8)* But this value of refractive index is not possible.

$$1.6 \sin\theta = (n - m\Delta n)\sin 90°$$

$$1.6 \sin\theta = n - m\Delta n$$

$$1.6 \times \frac{1}{2} = 1.6 - m(0.1)$$

$$0.8 = 1.6 - m(0.1)$$

$$m \times 0.1 = 0.8 \Rightarrow m = 8$$

7. *(c)* For particle to move in negative y-direction, either its velocity must be in negative y-direction (if initial

velocity ≠ 0) and force should be parallel to velocity or it must experience a net force in negative y-direction only (if initial velocity = 0)

8. *(a)* $\mathbf{F}_{\text{net}} = \mathbf{F}_e + \mathbf{F}_B = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$

For particle to move in straight line with constant velocity, $\mathbf{F}_{\text{net}} = 0$

$\therefore \quad q\mathbf{E} + q\mathbf{v} \times \mathbf{B} = 0$

9. *(c)* For path to be helix with axis along positive z-direction, particle should experience a centripetal acceleration in xy-plane.

For the given set of options only option (c) satisfy the condition. Path is helical with increasing pitch.

Paper 2

1. *(d)* According to photoelectric effect equation

$$\text{KE}_{\max} = \frac{hc}{\lambda} - \phi_0$$

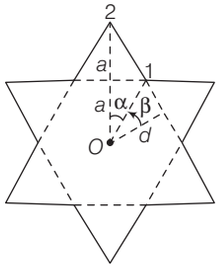
$$\frac{p^2}{2m} = \frac{hc}{\lambda} - \phi_0 \qquad [\text{KE} = p^2/2m]$$

$$\frac{(h/\lambda_d)^2}{2m} = \frac{hc}{\lambda} - \phi_0 \qquad [p = h/\lambda]$$

Assuming small changes, differentiating both sides,

$$\frac{h^2}{2m}\left(-\frac{2d\lambda_d}{\lambda_d^3}\right) = -\frac{hc}{\lambda^2}d\lambda \Rightarrow \frac{d\lambda_d}{d\lambda} \propto \frac{\lambda_d^3}{\lambda^2}$$

2. *(a)*



$$B_{12} = \frac{\mu_0 I}{4\pi d}[\sin\alpha + \sin\beta]$$

$$\alpha = 60° \text{ and } \beta = -30°$$

$$= \frac{\mu_0 I}{4\pi d}\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$$

$$B_{12} = \frac{\mu_0 I}{4\pi d}\left[\frac{\sqrt{3}-1}{2}\right]$$

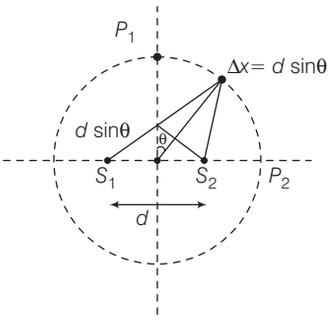
$$d = a$$

$$B_0 = 12\, B_{12}$$

$$= 12 \times \frac{\mu_0 I}{4\pi d}\left[\frac{\sqrt{3}-1}{2}\right]$$

$$= \frac{\mu_0 I}{4\pi a} 6\,[\sqrt{3} - 1]$$

3. *(c,d)*



$\lambda = 600$ nm

at $P_1 \quad \Delta x = 0$ at $P_2 \; \Delta x = 1.8$ mm $= n\lambda$

Number of maximas will be

$$n = \frac{\Delta x}{\lambda} = \frac{1.8 \text{ mm}}{600 \text{ nm}} = 3000$$

at $P_2 \quad \Delta x = 3000\lambda$

Hence, bright fringe will be formed.

At P_2, 3000 th maxima is formed.

For (a) option

$$\Delta x = d\sin\theta$$

$$d\Delta x = d\cos d\theta$$

$$R\lambda = d\cos\theta R d\theta$$

$$Rd\theta = \frac{R\lambda}{d\cos\theta}$$

As we move from P_1 to P_2

$\theta \uparrow \cos\theta \downarrow Rd\theta \uparrow$

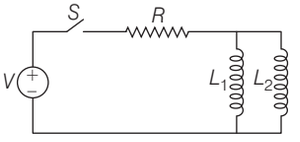
4. *(a,d)* (a) $\Omega = 2\pi(1 - \cos\theta)$; $\theta = 45°$

$$\phi = -\frac{\Omega}{4\pi} \times \frac{Q}{\varepsilon_0} = -\frac{2\pi(1-\cos\theta)}{4\pi}\frac{Q}{\varepsilon_0}$$

$$= -\frac{Q}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right)$$

(b) The component of the electric field perpendicular to the flat surface will decrease so we move away from the centre as the distance increases (magnitude of electric field decreases) as well as the angle between the normal and electric field will increase. Hence, the component of the electric field normal to the flat surface is not constant.

5. *(a,b,c)*



Since inductors are connected in parallel

$$V_{L_1} = V_{L_2} \Rightarrow L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt}$$

$$L_1 I_1 = L_2 I_2 \Rightarrow \frac{I_1}{I_2} = \frac{L_2}{L_1}$$

Current through resistor at any time t is given by

$$I = \frac{V}{R}(1 - e^{-\frac{RT}{L}}), \text{ where } L = \frac{L_1 L_2}{L_1 + L_2}$$

After long time $I = \dfrac{V}{R}$

$$I_1 + I_2 = I \qquad \ldots(\text{i})$$

$$L_1 I_1 = L_2 I_2 \qquad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

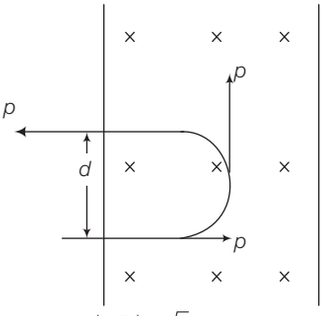
$$I_1 = \frac{V}{R}\frac{L_2}{L_1 + L_2}, \quad I_2 = \frac{V}{R}\frac{L_1}{L_1 + L_2}$$

(d) Value of current is zero at $t = 0$

Value of current is V/R at $t = \infty$

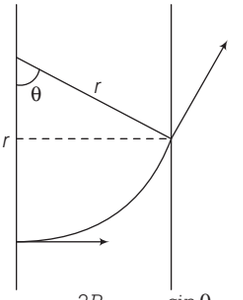
Hence option (d) is incorrect.

6. *(b,c)* (a)



$$|\Delta\mathbf{P}| = \sqrt{2}p$$

(b) $r(1 - \cos\theta) = R$

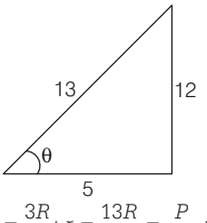


$$r\sin\theta = \frac{3R}{2} \Rightarrow \frac{\sin\theta}{1-\cos\theta} = \frac{3}{2}$$

$$\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \frac{3}{2} \Rightarrow \cot\frac{\theta}{2} = \frac{3}{2}$$

$$\Rightarrow \tan\frac{\theta}{2} = \frac{2}{3}$$

$$\Rightarrow \tan\theta = \frac{2\left(\frac{2}{3}\right)}{1 - \frac{4}{9}} = \frac{\frac{4}{3}}{\frac{5}{9}} = \frac{12}{5} \Rightarrow \sin\theta = \frac{12}{13}$$



$$r\left(\frac{12}{13}\right) = \frac{3R}{2};\; r = \frac{13R}{8} = \frac{P}{QB};\; B = \frac{8P}{13QR}$$

(c) $\frac{P}{QB} < \frac{3R}{2} \Rightarrow B > \frac{2P}{3QR}$

(d) $r = \frac{mv}{QB} \Rightarrow d = 2r = \frac{2mv}{QB} \Rightarrow d \propto m$

7. ***(b,c)***

$V_{XY} = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) - V_0 \sin \omega t$

$= V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) + V_0 \sin(\omega t + \pi)$

$\Rightarrow \quad \phi = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

$\Rightarrow \quad V_0' = 2V_0 \cos\left(\frac{\pi}{6}\right) = \sqrt{3}\, V_0$

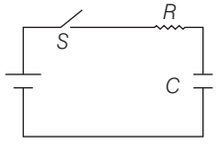
$\Rightarrow \quad V_{XY} = \sqrt{3}V_0 \sin(\omega t + \phi)$

$\Rightarrow (V_{XY})_{\text{rms}} = (V_{YZ})_{\text{rms}} = \sqrt{3}\frac{V_0}{\sqrt{2}}$

8. ***(b)*** When switch is closed for a very long time capacitor will get fully charged and charge on capacitor will be $q = CV$

Energy stored in capacitor,

$$E_C = \frac{1}{2}CV^2 \qquad \ldots(i)$$



Work done by a battery,

$W = Vq = VCV = CV^2$

Energy dissipated across resistance

E_D = (work done by battery) − (energy stored)

$E_D = CV^2 - \frac{1}{2}CV^2 = \frac{1}{2}CV^2 \qquad \ldots(ii)$

From Eqs. (i) and (ii)

$E_D = E_C$

9. ***(a)*** **For process (1)**

Charge on capacitor = $\frac{CV_0}{3}$

Energy stored in capacitor

$= \frac{1}{2}C\frac{V_0^2}{9} = \frac{CV_0^2}{18}$

Work done by battery

$= \frac{CV_0}{3} \times \frac{V}{3} = \frac{CV_0^2}{9}$

Heat loss = $\frac{CV_0^2}{9} - \frac{CV_0^2}{18} = \frac{CV_0^2}{18}$

Charge on capacitor = $\frac{2CV_0}{3}$

Extra charge flow through battery

$= \frac{CV_0}{3}$

Work done by battery

$= \frac{CV_0}{3} \cdot \frac{2V_0}{3} = \frac{2CV_0^2}{9}$

Final energy stored in capacitor

$= \frac{1}{2}C\left(\frac{2V_0}{3}\right)^2 = \frac{4CV_0^2}{18}$

Energy stored in process

$2 = \frac{4CV_0^2}{18} - \frac{CV_0^2}{18} = \frac{3CV_0^2}{18}$

Heat loss in process (2) = work done by battery in process (2) − energy store in capacitor process (2)

$= \frac{2CV_0^2}{9} - \frac{3CV_0^2}{18} = \frac{CV_0^2}{18}$

Charge on capacitor = CV_0

Extra charge flown through battery

$= CV_0 - \frac{2CV_0}{3} = \frac{CV_0}{3}$

Work done by battery in this process

$= \left(\frac{CV_0}{3}\right)(V_0) = \frac{CV_0^2}{3}$

Final energy stored in capacitor

$= \frac{1}{2}CV_0^2$

Energy stored in this process

$= \frac{1}{2}CV_0^2 - \frac{4CV_0^2}{18} = \frac{5CV_0^2}{18}$

Heat loss in process (3)

$= \frac{CV_0^2}{3} - \frac{5CV_0^2}{18} = \frac{CV_0^2}{18}$

Now, total heat loss (E_D)

$= \frac{CV_0^2}{18} + \frac{CV_0^2}{18} + \frac{CV_0^2}{18} = \frac{CV_0^2}{6}$

Final energy stored in capacitor

$= \frac{1}{2}CV_0^2$

So we can say that $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

BITSAT

1. ***(c)*** We have, $f_r - f_b = \omega f_y$

$\omega = \frac{f_r - f_b}{f_y}$

Here, $f_r = 100$ cm $\Rightarrow f_b = 96.8$ cm

$f_y = \sqrt{f_b \times f_r}$

$= \sqrt{96.8 \times 100} = 98$ cm

$\therefore$ Dispersive power

$\omega = \frac{f_r - f_b}{f_y} = \frac{100 - 96.8}{98} = \frac{3.2}{98} = 0.0325$

2. ***(b)*** $V = 800$ V

$E = \frac{V}{d} = \frac{800}{2 \times 10^{-2}}$ V/m $= 4 \times 10^4$ V/m

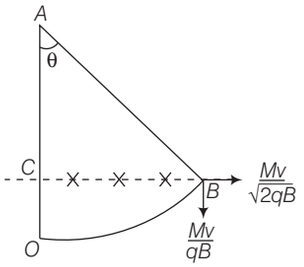
$qE = mg \Rightarrow q = \frac{mg}{E}$

$= \frac{1.96 \times 10^{-15} \times 9.8}{4 \times 10^4}$ coulomb

$= \frac{1.96 \times 10^{-15} \times 9.8}{4 \times 10^4 \times 1.6 \times 10^{-19}}$ elementary charges

$= 3e$

3. (b) From triangle ABC



$\sin\theta = \frac{l}{H} = \frac{\frac{mv}{\sqrt{2}\,qB}}{\frac{mv}{qB}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \sin\theta = \sin 45^\circ = \sin\frac{\pi}{4}$

Time period for angle 2π is $T = \frac{2\pi m}{qB}$

$\therefore$ Time by charged particle to emerge from magnetic field is $\frac{\pi m}{4qB}$.

4. ***(b)*** Given, $\lambda_1 = 12000$ Å

and $\lambda_2 = 10000$ Å; $D = 2$ cm

$d = 2$ mm $= 2 \times 10^{-3}$ m

We have, $\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} = \frac{12000}{10000} = \frac{6}{5}$

as $X = \frac{n_1\lambda_1 D}{d} \qquad (n_1 = 5)$

$= \frac{5 \times 12000 \times 10^{-10} \times 2}{2 \times 10^{-3}}$

$= 5 \times 1.2 \times 10^4 \times 10^{-10} \times 10^3 = 6$ mm

5. ***(b)*** Here, magnetic field, $B = 0.025$ T

$\therefore$ Magnetic flux $\phi = B \cdot A = B \cdot \pi r^2$

Induced emf $|e| = \frac{d\phi}{dt} = B\pi 2r \cdot \frac{dr}{dt}$

$= 0.025 \times \pi \times 2 \times 2 \times 10^{-2} \times 1 \times 10^{-3}$

$= \pi\mu$ V

6. ***(d)*** We have, $R_t = R_0(1 + \alpha t)$

Here, $20 = R_0(1 + 20\alpha)$

$60 = R_0(1 + 500\alpha)$

$\Rightarrow \frac{60}{20} = \frac{R_0(1 + 500\alpha)}{R_0(1 + 20\alpha)}, 3 = \frac{1 + 500\alpha}{1 + 20\alpha}$

$\Rightarrow \quad 3(1 + 20\alpha) = 1 + 500\alpha$

$\Rightarrow 3 + 60\alpha = 1 + 500\alpha \Rightarrow 440\alpha = 2$

$\Rightarrow \quad \alpha = \frac{2}{440} = \frac{1}{220}/°\text{C}$

Also given $R_t = 25°\text{ C}$, $t = ?$

Now, $\frac{20}{25} = \frac{R_0\left(1 + \frac{20}{220}\right)}{R_0\left(1 + \frac{t}{220}\right)}$

$4\left(1 + \frac{t}{220}\right) = 5\left(1 + \frac{20}{220}\right)$

On solving, $t = 80°\text{ C}$

7. *(b)* de-Broglie wavelength,

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2\,mqV}}$

$\Rightarrow \lambda = \frac{6.6\times 10^{-34}}{\sqrt{2\times 1.6\times 10^{-27}\times 1.6 \times 10^{-19}\times 1000}}$

$\Rightarrow \lambda = \frac{6.6\times 10^{-34}}{7.16\times 10^{-22}} = 0.9\times 10^{-12}\text{ m}$

8. (d) We have, Kinetic energy of electron,

$K = \frac{Ze^2}{8\pi\varepsilon_0 \cdot r}$

also, Potential energy of electron,

$U = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{r}$

$\therefore$ Total energy $E = K + U$

$= \frac{Ze^2}{8\pi\varepsilon_0 \cdot r} - \frac{Ze^2}{4\pi\varepsilon_0 \cdot r}$

$E = -\frac{Ze^2}{8\pi\varepsilon_0 r}$

or $K = -E = -(-3.4) = 3.4$ eV

9. *(c)* Voltage gain = $\beta \times$ impedance gain

$\Rightarrow \quad 50 = \beta \times \frac{200}{100} \Rightarrow \beta = 25$

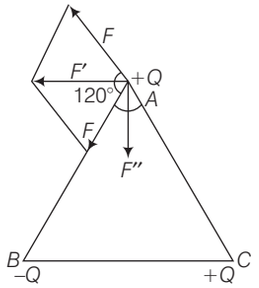
Also power gain = $\beta^2 \times$ impedance gain

$= (25)^2 \times \frac{200}{100} = 1250$

10. *(c)* Resultant force

$F' = \sqrt{F^2 + F^2 + 2FF\cos 120°} = F$

$\left[\because \cos 120° = \frac{-1}{2}\right]$



Now, from figure

$F = \sqrt{F'^2 + F''^2 + 2F'F''\cos 90°}$

Now, the force normal to BC at vertex A is

$F'' = \sqrt{F^2 - F'^2} = 0 \qquad (\because F' = F)$

11. *(d)* Fraction remains after n half-lives

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{\frac{t}{T}}$

Given, $N = \frac{N_0}{e} \Rightarrow \frac{N_0}{eN_0} = \left(\frac{1}{2}\right)^{\frac{5}{T}}$

$\Rightarrow \quad \frac{1}{e} = \left(\frac{1}{2}\right)^{\frac{5}{T}}$

Taking log on both sides, we get

$\log 1 - \log e = \frac{5}{T}\log\frac{1}{2}$

$\Rightarrow -1 = \frac{5}{T}(-\log 2) \Rightarrow T = 5\log_e 2$

Now, let t' be the time after which activity reduces to half

$\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{t'/5\log_e 2} \Rightarrow t' = 5\log_e 2$

12. *(c)* Focal length of combination

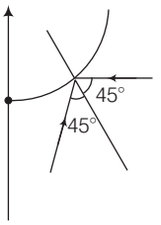
$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ ...(i)

Using lens maker's formula

$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = -\frac{(\mu_2 - 1)}{R}$

$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{\mu_1 - 1}{R}$

or $\frac{1}{f_1} = \frac{(\mu - 1)}{R} \Rightarrow \frac{1}{f_2} = -\frac{(\mu_2 - 1)}{R}$



Putting these values in Eq (i), we get

$\Rightarrow \quad \frac{1}{f} = \frac{(\mu_1 - 1)}{R} - \frac{(\mu_2 - 1)}{R}$

$\Rightarrow \quad \frac{1}{f} = \frac{(\mu_1 - 1 - \mu_2 + 1)}{R}$

$\Rightarrow \quad \frac{1}{f} = \frac{\mu_1 - \mu_2}{R} \Rightarrow f = \frac{R}{(\mu_1 - \mu_2)}$

13. *(c)* Current will flow only through the branch containing resistance R_2

$\therefore \qquad i = \frac{E}{R_2 + r} = \frac{5}{4 + 1} = 1\text{ A}$

Potential difference across R_2

$= 1 \times 4 = 4\text{ V}$

If q be the charge on each plate of capacitor, then

$\frac{q}{c} + \frac{q}{c} = 4 \quad \text{or} \quad \frac{2q}{c} = 4 \quad \text{or} \quad \frac{2q}{3\times 10^{-6}} = 4$

$q = 6\ \mu\text{C}$

14. *(a)* As H-atom emits 6 spectral lines

$\frac{n(n-1)}{2} = 6$

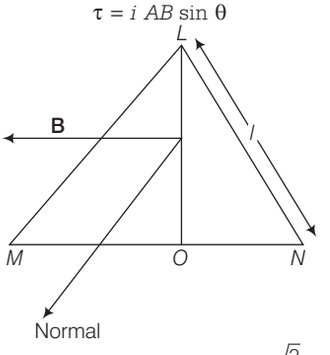
$\therefore \quad n = 4$

$\Delta E = E_4 - E_1 = 13.6 - 0.85 = 12.75$ eV

$l = \frac{1242\text{ nm}}{12.75} = 97.5\text{ nm}$

15. *(a)* Torque acting on equilateral triangle in a magnetic field B is

$\tau = i\,AB\sin\theta$



Area of triangle LMN, $A = \frac{\sqrt{3}}{2}l^2$ and $\theta = 90°$

Substituting the given values in expression for torque, we get

$\tau = i \times \frac{\sqrt{3}}{4}l^2 B\sin 90° \Rightarrow \tau = \frac{\sqrt{3}}{4}il^2 B$

$(\because \sin 90° = 1)$

Here, $l = 2\left(\frac{\tau}{\sqrt{3}\,Bi}\right)^{\frac{1}{2}}$

16. *(b)* $H = I^2 Rt$

$\therefore \qquad \frac{H_1}{H_2} = \left(\frac{I_1}{I_2}\right)^2 \times \left(\frac{R_1}{R_2}\right)$

or $\frac{10}{H_2} = \left[\frac{1}{\left(\frac{1}{2}\right)}\right] \times \frac{5}{4}$

$H_2 = 2\text{ cal}$

17. *(a)* Given KE of α-particle = 2 eV

$r = \frac{(Ze)(e)}{4\pi\varepsilon_0 \cdot (\text{KE})} = \frac{2Ze \times 9 \times 10^9}{2V}$

$\Rightarrow r = \frac{2 \times Z \times 1.6 \times 10^{-19} \times 9 \times 10^9}{2V}$

$= 14.4 \cdot \frac{Z}{V}$ Å

18. *(c)* The two capacitors formed are in parallel, hence capacitance of the combination

$$C = \frac{C_1C_2}{C_1 + C_2} \quad ...(i)$$

where, $C_1 = \dfrac{K_1\varepsilon_0 A}{\frac{d}{3}} \quad \left(\because C = \dfrac{\varepsilon_0 A}{d}\right)$ ...(ii)

$$C_2 = \frac{K_2\varepsilon_0 A}{\frac{2d}{3}} \quad ...(iii)$$

$$C_{eq} = \frac{\frac{3K_1\varepsilon_0 A}{d} \times \frac{K_2\varepsilon_0 A \cdot 3}{2d}}{\frac{3K_1\varepsilon_0 A}{d} + \frac{3K_2\varepsilon_0 A}{2d}}$$

$$= \frac{9K_1K_2\varepsilon_0^2A^2}{2d^2} \times \frac{d}{\varepsilon_0 A \times 18}$$

It is given

$2d/3$ K_1	$d/3$ K_2

$$\frac{\varepsilon_0 A}{d} = 9 \text{ pF}$$

Using given values

$$C_{eq} = 40.5 \text{ pF}$$

19. *(b)* Distance of straight path from O

$$= \frac{r}{\sqrt{2}}$$

Hence, $B = B_1 + B_2$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2i}{\frac{r}{\sqrt{2}}} + \frac{\mu_0}{4\pi}\frac{2\pi i}{r}\left(\frac{1}{4}\right)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2i}{r}\left[\sqrt{2} + \frac{\pi}{4}\right]$$

KERALA CEE

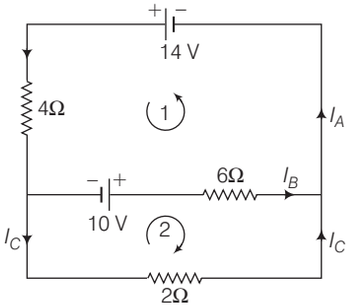
1. *(d)* Drift velocity, $V_d = \dfrac{I}{neA}$

$$v_d = \frac{16}{5 \times 10^{28} \times 1.6 \times 10^{-19} \times 2 \times 10^{-6}}$$

$$= \frac{1}{10^3} = 0.001 \text{ m/s}$$

2. *(d)*

3. *(b)*



Applying Kirchoff's law

$10 - 6I_B + 14 - 4I_A = 0$ ...(loop 1)

$$24 = 6I_B + 4I_A$$

$$12 = 3I_B + 2I_A \quad ...(i)$$

$I_A = I_B + I_C$ ...(junction)

$10 - 6I_B + 2I_C = 0$ ...(loop 2)

$$10 = 6I_B - 2I_C$$

$$10 = 6I_B - 2(I_A - I_B) \quad ...(ii)$$

On solving Eqs. (i) and (ii), we get

$I_B = 2A$, $I_A = 3A$, $I_C = 1A$

Now as per given circuit,

$I_1 = I_B = 2A$, $I_2 = -I_A = -3A$

$I_3 = -I_C = -1A$

4. *(d)*

5. *(a)*

6. *(d)* $E\ (r > R)$

$$E = \frac{\rho}{4\pi \in_0 r^2} = \frac{\rho R^3}{\left(\frac{4}{3}\pi R^3\right)3\varepsilon_0 r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2}$$

7. *(d)* Given, $Q = 10\ nC\ 10 \times 10^{-9}$ C

The potential point $x = 0$, will be the sum of potential produced by the charges placed at$x = 2, 4, 5, 10$ and 20 meter.

Potential, $V = \dfrac{k.q}{r}$

The resultant potential,

$V = V_1 + V_2 + V_3 + V_4 + V_5$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{10} + \frac{1}{20}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{10 + 5 + 4 + 2 + 1}{20}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{10 \times 10^{-9} \times 22}{20}$$

$$= \frac{1 \times 36\pi}{4\pi \times 10^{-9}} \times \frac{10 \times 10^{-9} \times 22}{20}$$

$$= 99 \text{ V}$$

8. *(b)* We know that,

$$E = \frac{q}{\varepsilon_0 A} = \frac{10^{-8} \times 36\pi}{10^{-9} \times 6 \times 10^{-4}} \text{ V/m}$$

$$= 6\pi \times 10^3 = 6\pi \text{ kV/m}$$

9. *(c)* When the proton enters in magnitude field, it experiences Lorentz force

$F = q(v \times B) \Rightarrow |F| = qvB\sin\theta$

$F = 1.6 \times 10^{-19} \times 5 \times 10^6 \times 2 \times \sin 30°$

$$= 8.0 \times 10^{-13} \text{ N}$$

10. *(a)*

11. *(a)* Given,

frequency of cyclotron oscillator = 16 MHz

$$\nu = 16 \times 10^6 \text{ Hz}$$

Mass of proton, $m_p = 1.67 \times 10^{-27}$

The cyclotron frequency, $\nu = \dfrac{Bq}{2\pi m}$

Where B = Required magnetic field

$$B = \frac{\nu \times 2\pi m}{q}$$

$$= \frac{16 \times 10^6 \times 2 \times \pi \times 1.67 \times 10^{-27}}{1.6 \times 10^{-19}}$$

$$= 0.334\ \pi \text{ T}$$

12. *(e)*

13. *(d)*

14. *(a)* The speed of radio waves is equal to electromagnetic wave speed i.e. 3×10^8 m/s

Given, the distance of radio station, $s = 200 \text{ km} = 200 \times 10^3$ m

Time taken by radio wave to his toner

$$t = \frac{s}{c} = \frac{200 \times 10^3}{3 \times 10^8} = 666 \times 10^{-6}$$

$$= 666\ \mu\text{s}$$

The wave emitted send out per second = 972 kHz

$$= 972 \times 10^3 \text{ Hz}$$

$$= 9.72 \times 10^5 \text{ crest per second}$$

15. *(b)* From de-Broglie waves

$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34}}{9.9 \times 10^{-26}} \text{ m} = \frac{2}{3} \times 10^{-8} \text{ m}$$

$$= \frac{20}{3} \times 10^{-9} \text{ m} = \frac{20}{3} \text{ nm}$$

16. *(c)* $\omega = 6 \times 10^5 = 2\pi f$

$$f = \frac{6}{2\pi} \times 10^5 = \frac{3}{\pi} \times 10^5 \text{ Hz}$$

$$= \frac{300}{\pi} \times 10^3 \text{ Hz} = \frac{300}{\pi} \text{ kHz}$$

Wave speed $= \dfrac{\omega}{k} = \dfrac{6 \times 10^5}{0.01}$

$$= 6 \times 10^7 \text{ m/s}$$

17. *(c)*

18. *(a)* According to de-Broglie theory, the wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mk}}$$

$$\lambda \propto \frac{1}{\sqrt{m}} \text{ (for same kinetic energy)}$$

The wavelength is inversely proportional to the root of mass of the particle.

As $m_e < m_n < m_\alpha$

So, $\lambda_e > \lambda_n > \lambda_\alpha$,

19. *(c)* According to question, the work function of the metal surface is $W = 4.6$ eV

and K.E of ejected electron = 2.2 eV

Let the energy of incident photon is E.

i.e. $E = W + K_e$

K_e = kinetic energy of ejected electrons

$E = 4.6\text{ eV} + 2.2\text{ eV} = 6.8\text{ eV}$

20. *(c)* **21.** *(c)* **22.** *(a)*

23. *(a)* **24.** *(c)*

25. *(d)* Given,

frequency $\nu = 30\text{ MHz} = 30 \times 10^6$ Hz

We know that, $c = \nu\lambda$

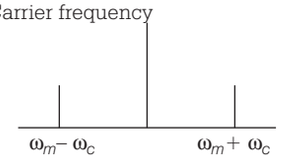
$$\lambda = \frac{c}{\nu}$$

$c = 3 \times 10^8$ m/s

$$\lambda = \frac{3 \times 10^8}{30 \times 10^6} = 10\text{ m} = 1000\text{ cm}$$

26. *(b)* Given, Transmitted frequency of signal = ω_m

Carrier frequency



The band width of the filter and amplifier.

$= (\omega_m - \omega_e) + (\omega_m + \omega_e) = 2\,\omega_m$

27. *(d)*

28. *(a)* Given, $\mathrm{B} = B_0\hat{\mathrm{k}}$

The induced e.m.f,

$$e = -\frac{d\phi}{dt} = -\frac{d(BA)}{dt}$$

$$= -B\frac{dA}{dt}$$

$\because \quad \phi = (B.A)$

So, the flux linked with the circular loop of radius will be maximum, when its plane is perpendicular to the magnetic field, then the induced e.m.f will be maximum.

29. *(b)* Given,

Effective resistance, $R = 30\,\Omega$

inductive reactance, $X_L = 40\ \Omega$

The effective impedance

$$Z = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{((30)^2 + (40)^2}$$

$$= \sqrt{900 + 1600}$$

$$= \sqrt{2500} = 50\ \Omega$$

The maximum current,

$$I = \frac{V}{Z} = \frac{420}{50} = 8.4\text{ A}$$

30. *(c)* As no energy is released, lighter particle carries more K.E

$$\therefore \quad \text{K.E} \propto \frac{1}{\text{mass}}$$

$\therefore$ Molar mass of carbon = 12

Molar mass of oxygen = 16

Thus, they are related as

$$\frac{K_O}{K_C} = \frac{\text{Molar mass of carbon}}{\text{Molar mass of oxygen}} = \frac{12}{16}$$

KCET

1. *(d)* Given,

Inductive reactance of coil $(X_L) = \frac{1}{\sqrt{3}}\,\Omega$

Resistance $(R) = 1\Omega$

We know that,

$$\tan\phi = \frac{\omega L}{R} \Rightarrow \tan\phi = \frac{X_L}{R}$$

$$\tan\phi = \frac{1/\sqrt{3}}{1} \Rightarrow \phi = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\phi = 30° \Rightarrow \omega t = \frac{\pi}{6}$$

$$t = \frac{\pi}{6\omega} \Rightarrow t = \frac{\pi}{6 \times (2\pi f)} \Rightarrow t = \frac{1}{12f}$$

Here, $f = 50$ Hz, $t = \frac{1}{12 \times 50} = \frac{1}{600}$ s

2. *(*)* (No option is matched)

$$^{238}_{92}\mathrm{U} \xrightarrow{\alpha} {}^{234}_{90}\mathrm{U} \xrightarrow{\beta^{-1}} {}^{234}_{91}\mathrm{U} \xrightarrow{\beta^{-1}} {}^{234}_{92}\mathrm{U}$$

Number of α-particle = 1

Number of β-particle = 2

3. *(b)*

4. *(d)*

5. *(a)*

6. *(c)*

7. *(a)*

8. *(d)* Given, $A = +3\text{ nC} = 3 \times 10^{-9}$ C

$B = +1\text{ nC} = 1 \times 10^{-9}$ C

Distance, $r_1 = 5$ cm

$= 0.05\text{ m} = 5 \times 10^{-2}$ m

Work done $(W) = U_B - U_A$

$$= \frac{kq_1q_2}{r_2} - \frac{kq_1q_2}{r_1}$$

Here, $r_2 = r_1 - 1$

$r_2 = 5 - 1 = 4$ cm $= 0.04$ m

$= 4 \times 10^{-2}$ m

$$= kq_1q_2\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= 9 \times 10^9 \times 3 \times 10^{-9} \times 1 \times 10^{-9}$$

$$\left[\frac{1}{4 \times 10^{-2}} - \frac{1}{5 \times 10^{-2}}\right]$$

$$= \frac{27 \times 10^{-9} \times 1}{5 \times 4 \times 10^{-2}} = \frac{27}{20} \times 10^{-7}$$

$= 1.35 \times 10^{-7}$ J

9. *(a)* According to the question, magnetic field is uniform and the kinetic energy for each particle are same.

We know that,

$$r = \frac{mv}{qB} = \frac{\sqrt{2m(\text{K.E})}}{qB}$$

$$\Rightarrow \quad r \propto \frac{m}{q}$$

$$r_p : r_d : r_\alpha = \sqrt{\frac{m}{q}} : \sqrt{\frac{2m}{q}} : \sqrt{\frac{m}{q}}$$

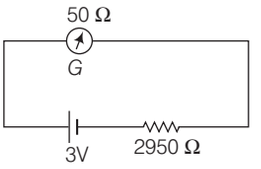
$$= 1 : \sqrt{2} : 1$$

10. *(b)* **11.** *(c)* **12.** *(a)*

13. *(c)* Current flowing in galvanometer,

$$I = \frac{3}{(50 + 2950)}$$

$$I = 10^{-3}\text{A}$$



14. *(a)*

15. *(d)* Given,

Magnetic moment

$(M) = 6 \times 10^{-2}$ A-m^2

Moment of inertia

$(I) = 1.2 \times 10^{-6}$ kg-m^2

Magnetic field $(B) = 2 \times 10^{-2}$ T

We know that,

$$\text{Time } (t) = 2\pi\sqrt{\frac{I}{MB}}$$

$$= 2\pi\sqrt{\frac{1.2 \times 10^{-6}}{6. \times 10^{-2} \times 2 \times 10^{-2}}}$$

$$= 2\pi\sqrt{\frac{1.2 \times 10^{-2}}{12}}$$

$$= 2\pi \times 10^{-1}$$

For 20 oscillation,

Time $(t) = 20 \times 2\pi \times 10^{-1} \approx 12$ s

16. *(d)* Given,

Number of electrons $(n) = 4 \times 10^{10}$

Diameter of sphere $(D) = 20$ cm

Radius of sphere $(r) = \frac{20}{2} = 10$ cm

$= 0.01$ m

According to question,

$$E_{\text{outside}} = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{ne}{4\pi\varepsilon_0 r^2}$$

Here, $r = 20 \times 10^{-2}$ m

$$= \frac{4 \times 10^{10} \times 1.6 \times 10^{-19} \times 9 \times 10^9}{(20 \times 10^{-2})^2}$$

$$= \frac{4 \times 1.6 \times 9}{20 \times 20 \times 10^{-4}}$$

$$= \frac{4 \times 1.6 \times 9 \times 10^4}{20 \times 20}$$

$$= 1.6 \times 9 \times 10^2$$

$$= 1.44 \times 10^2 = 1440 \text{ NC}^{-1}$$

17. *(a)*

18. *(a)* Given,

Diameter of cylindrical conductor $(D) = 0.1$ mm

Radius $(r) = \frac{D}{2} = \frac{0.1}{2}$ mm

Current $(I) = 90$ mA $= 90 \times 10^{-3}$ A

We know that,

Current density $(J) = \frac{I}{A}$

$$= \frac{I}{\pi r^2} = \frac{90 \times 10^{-3}}{\frac{22}{7} \times \left(\frac{0.1 \times 10^{-3}}{2}\right)^2}$$

$$= \frac{90 \times 10^{-3}}{3.14 \times \left(\frac{0.1 \times 10^{-3}}{2}\right)^2}$$

$$= 12000 \times 10^3 = 1.2 \times 10^7 \text{ A/m}^2$$

19. *(a)*

20. *(b)*

21. *(b)* Given,

Height of object $(h_o) = 10$ cm

Radius of curvature $(R) = 15$ cm

Distance of object $(u) = -10$ cm

We know that,

$$f = \frac{R}{2} = \frac{15}{2} \text{ cm}$$

According to mirror formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{-\frac{15}{2}} = \frac{1}{-10} + \frac{1}{v}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{-2}{15} + \frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{-4+3}{30}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{-1}{30}$$

$$\Rightarrow \quad v = -30 \text{ cm}$$

Magnification $(m) = \frac{h_i}{h_o} = \frac{-v}{u}$

$$= \frac{h_i}{10} = \frac{-(-30)}{-10}$$

$$\Rightarrow \quad h_i = -3$$

Hence, image will be magnified and inverted.

22. *(a)*

23. *(a)* Now, the equivalent capacitance,

$$\frac{1}{C_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{4}$$

$$\frac{1}{C_{eq}} = \frac{4+2+1}{4}$$

$$\frac{1}{C_{eq}} = \frac{7}{4}$$

$$C_{eq} = \frac{4}{7} \text{ pF}$$

24. *(c)* In the first condition,

Given , $q_1 = +2$ nC $= 2 \times 10^{-9}$ C

$q_2 = -8$ nC

$= -8 \times 10^{-9}$C

$$F = \frac{kq_1q_2}{r^2}$$

$$F = \frac{k(2 \times 10^{-9}) \times (8 \times 10^{-9})}{d^2}$$

$$F = \frac{k\,(16 \times 10^{-18})}{d^2} \quad \ldots \text{(i)}$$

In the second condition,

$$F_1 = \frac{k\,q_1q_2}{d'^2}$$

After touching of sphere to each other the total charge $= 2 - 8 = -6$ nC

Charge on each sphere $= 3$ nC

$= 3 \times 10^{-9}$ C

$$F_1 = \frac{k(3 \times 10^{-9})(3 \times 10^{-9})}{(d')^2}$$

$$F_1 = \frac{k(9 \times 10^{-18})}{(d')^2} \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{k\,(16 \times 10^{-18})}{d^2} = \frac{k\,(9 \times 10^{-18})}{(d')^2}$$

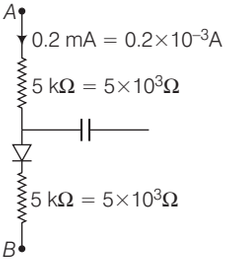
$$\frac{d^2}{d'^2} = \frac{16 \times 10^{-18}}{9 \times 10^{-18}}$$

$$\frac{d^2}{d'^2} = \frac{16}{9} \Rightarrow \frac{d}{d'} = \sqrt{\frac{16}{9}}$$

$$\frac{d}{d'} = \frac{4}{3} \Rightarrow d = \frac{4}{3}\,d'$$

$$d' = \frac{3}{4}d$$

25. *(a)*



The voltage drop across 5 kΩ resistance is given by

$V = 0.2 \times 10^{-3} \times 5 \times 10^3 \Rightarrow V = 1.0$ V

Applying the KVL (Kirchoff's Voltage Law)

$V_{AB} = 1 + 0.2 + 1$

[$\because$ Voltage drop across diode = 0.2 V]

$V_{AB} = 2.2$ V

26. *(c)*



Given,

length of wire $(l) = 50$ cm $= 50 \times 10^{-2}$ m

current $(I) = 2.5$ A

magnetic field $(B) = 0.5$ T

$g = 10$ ms^{-2}

For the balance condition,

$$F_B = mg \Rightarrow BIl = mg$$

$$m = \frac{BIl}{g}$$

$$= \frac{0.5 \times 2.5 \times 50 \times 10^{-2}}{10} = 62.5 \text{ g}$$

27. *(b)*

28. *(d)*

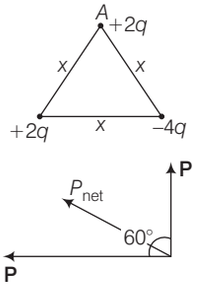
29. *(a)* de-Broglie wavelength $(\lambda) = \frac{h}{mv}$

$$\lambda = \frac{h}{m\sqrt{2gH}}$$

Hence, $\lambda \propto H^{-1/2}$

30. *(d)* According to question,

The magnitude of electric dipole moment



$$P_{net} = \sqrt{p^2 + p^2 + 2p\ p\cos 60°}$$

$$= \sqrt{2p^2 + 2p^2 \times \frac{1}{2}}$$

$$= \sqrt{3} \cdot p$$

We know that, $p = 2q \cdot x$ [$\because p = q \cdot l$]

$= \sqrt{3} \cdot 2qx = 2\sqrt{3}\,qx$

31. *(d)* According to Malus law,

$$I = I_0 \cos^2 \theta$$

Here, $I = \frac{I_0}{2}$

Now, $\frac{I_0}{2} = I_0 \cos^2\theta$

$\Rightarrow \quad \cos^2\theta = \frac{1}{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta = \cos^{-1}\frac{1}{\sqrt{2}} = 45°$

32. *(d)* Given,

Speed $(v) = 400\ \text{ms}^{-1}$

Magnetic field $(B) = 4 \times 10^{-4}$ T

Dip angle $\theta = 30°$

We know that,

Voltage difference $= Blv\sin\theta$

$= 4 \times 10^{-4} \times 20 \times 400 \times \sin 30°$

$= 32000 \times 10^{-4} \times \sin 30$

$= 32000 \times 10^{-4} \times \frac{1}{2} = 1.6$ V

33. *(d)* We know that, $E_n = \frac{13.6}{n^2}$

According to question, the required energy is

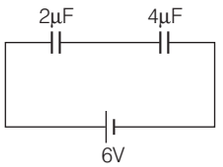
$$E = E_2 - E_4 = \frac{13.6}{(2)^2} - \frac{13.6}{4^2} = \frac{13.6}{4} - \frac{13.6}{16}$$

$$= \frac{(13.6 \times 4) - 13.6}{16} = \frac{40.8}{16} = +2.55$$

34. *(b)*

35. *(a)*

36. *(b)* According to the question, we can draw the following diagram.



The capacitor are connected in series combination,

So, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{4}$

$\Rightarrow \quad C_{eq} = \frac{4}{3}\ \mu\text{F}$

We know that,

$$Q = CV = \frac{4}{3} \times 6 = 8\ \mu\text{C}$$

Energy, $E = \frac{1}{2}CV^2 = \frac{1}{2} \times \frac{4}{3} \times (6)^2$

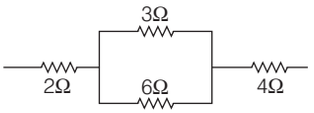
$= 24\ \mu\text{J}$

37. *(b)* According to the question,

$$\frac{R_{\text{unkown}}}{R_{\text{standard}}} = \frac{l}{(1-l)}$$

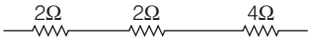
If temperature increases than resistance increases. Hence the new balancing is greater than 1.

38. *(b)* The equivalent circuit of the given circuit will be



From circuit, $\frac{1}{R} = \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{3}{6}\Omega$

$R_1 = 2\Omega$



$R_2 = R_2 + R_2 + R_4$

$= 2 + 2 + 4 = 8\Omega$

The internal resistance of battery $= 1\Omega$

So, the equivalent resistance of circuit

$= 8\Omega + 1\Omega = 9\Omega$

The current in the circuit

$$I = \frac{V}{R} = \frac{4.5}{9} = \frac{1}{2}\text{A}.$$

The power dissipated in 3Ω resistance

$$P = I^2 \times R = \left(\frac{1}{2}\right)^2 \times 3$$

$$= \frac{1}{4} \times 3 = 0.75\ \text{W}.$$

39. *(b)* We know that,

$$\frac{\mathbf{B}_{\text{centre}}}{\mathbf{B}_{\text{axis}}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}$$

Given that, $\mathbf{B}_{\text{centre}} = 5\sqrt{5}\ \mathbf{B}_{\text{axis}}$

$$\frac{\mathbf{B}_{\text{centre}}}{\mathbf{B}_{\text{axis}}} = 5\sqrt{5}$$

$\therefore \qquad 5\sqrt{5} = \left[1 + \frac{x^2}{(0.1)^2}\right]^{3/2}$

On squaring both sides, we get

$$25 \times 5 = \left[1 + \frac{x^2}{(0.1)^2}\right]^3$$

$$\sqrt[3]{125} = 1 + \frac{x^2}{(0.1)^2}$$

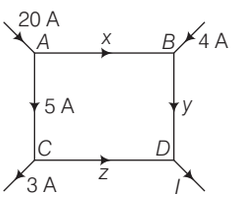
$\Rightarrow \quad 0.01 + x^2 = 0.05$

$\Rightarrow \quad x^2 = 0.05 - 0.01$

$\Rightarrow \quad x^2 = 0.04$

$\Rightarrow \quad x = 0.2$ m

40. *(a)*



At junction A, $\quad 20 = 5 + x$

$x = 20 - 5 = 15$ A

At junction B,

$4 + x = y$

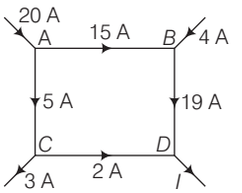
$4 + 15 = y$

$y = 19$ A

At junction C, $\quad = 5 = 3 + z$

$z = 5 - 3 = 2$ A

Now, network is shown by following diagram



At junction D,

$I = 19 + 2$

$I = 21$ A

41. *(d)*

42. *(c)* Given,

Output of step down transformer $(V_s) = 48$ V

Power associated with secondary coil $(P_s) = 12$ W

For secondary coil,

$$I_s = \frac{P_s}{V_s} = \frac{12}{48} = \frac{1}{4} = 0.25\ \text{A}$$

Amplitude of current $(I_0) = I_s\sqrt{2}$

$$= 0.25(\sqrt{2}) = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}\ \text{A}$$

AP EAMCET

1. *(b)* Given,

Linear charge density, $\lambda = \frac{1}{3}\ \text{Cm}^{-1}$

Charge $\quad q = 3\mu\text{C} = 3 \times 10^{-6}\text{C}$

Distance $\quad r = 18\ \text{cm} = 18 \times 10^{-2}\text{m}$

Intensity of the electric field due to infinitely long time at distance r is

$$E = \frac{\lambda}{2\pi\varepsilon_0 \cdot r} = \frac{2\lambda}{4\pi\varepsilon_0 \cdot r} = \frac{9 \times 10^9 \times 2 \times 1}{3 \times 18 \times 10^{-2}}$$

$$= \frac{1}{3} \times 10^{11}\ \text{N/C}$$

Force on charge of $3\ \mu\text{C}$

$$F = qE = 3 \times 10^{-6} \times \frac{1}{3} \times 10^{11} = 10^5\text{N}$$

2. *(a)* Given

Radius of sphere, $r = \frac{18}{2} = 9$ cm

$= 9 \times 10^{-2}$ m

Potential, $\quad V = 25\ \text{kV} = 25 \times 10^3\text{V}$

$$V = \frac{kq}{r}$$

Charge, $q = \dfrac{V \times r}{k}$

$= \dfrac{25 \times 10^3 \times 9 \times 10^{-2}}{9 \times 10^9}$

$= 25 \times 10^{-8}$ C $= 0.25 \times 10^{-6}$C $= 0.25\ \mu$C

3. ***(c)*** We know that

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi x} \quad \ldots(i)$$

When conducting wire is displaced by $(x)\,dx$ distance, then work done per unit length is;

$$dW = F \,.\, dx$$

$$dW = \frac{\mu_0 I_1 I_2}{2\pi x}\, dx$$

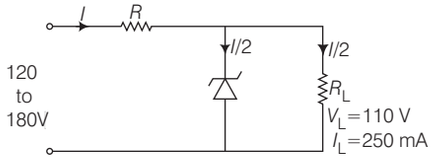
[From Eq. (i)]

$$\int dW = \int \frac{\mu_0 I_1 I_2}{2\pi x}\, dx$$

$$W = \frac{\mu_0 I_1 I_2}{2\pi} \int \frac{dx}{x} \Rightarrow W = \left(\frac{\mu_0 I_1 I_2}{2\pi}\right) \log_e x$$

$$\Rightarrow \qquad W \propto \log_e x$$

4. ***(b)***



$$R_L = \frac{V_L}{I_L} = \frac{110}{250 \times 10^{-3}} = 440\,\Omega$$

As current is equally shared by Zener diode and resistance R_2

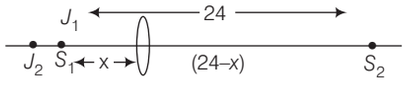
Then,

$$I = 2 \times 250 = 500 \text{ mA} = 0.5 \text{ A}$$

So, $R_S = \dfrac{V_S}{I} = \dfrac{(180 - 110)}{0.5}$

$$= \frac{70}{0.5} = 140\,\Omega$$

5. ***(b)***



The given condition will be satisfied only if one source (S_1) is placed on one side such that $u < f$ (i.e. it lies under the focus. The other source (S_2) is placed on the other side of the lens such that $u > f$ (i.e. it lies beyond the focus)

If S_1 is the object for the lens, then

$$\frac{1}{f} = \frac{1}{-y} - \frac{1}{-x}$$

$$\Rightarrow \qquad \frac{1}{y} = \frac{1}{x} - \frac{1}{-f} \quad \ldots(i)$$

If S_2 is the object for the lens, then

$$\frac{1}{f} = \frac{1}{+y} - \frac{1}{-(24 - x)}$$

$$\Rightarrow \qquad \frac{1}{-y} = \frac{1}{f} - \frac{1}{(24 - x)} \quad \ldots(ii)$$

From the Eqs (i) and (ii), we get

$$\frac{1}{x} - \frac{1}{f} = \frac{1}{f} - \frac{1}{(24 - x)}$$

$$\frac{1}{x} + \frac{1}{(24 - x)} = \frac{2}{f} = \frac{2}{7}$$

$$x^2 - 24x + 108 = 0$$

$x = 18$ cm and $x = 6$ cm

So, the correct answer will be 6 cm.

6. ***(b)*** (A) It is more difficult to push a magnet into a coil with large number of turns because of repulsion caused by emf induced.

(B) The emf induced in a coil always opposes the motion of a magnet when it is moved towards the cell. It was experimentally verified by lens.

So both *A* and *R* are true. *R* is correct explanation of *A*.

7. ***(b)*** $r = R\left(\dfrac{l_1}{l_2} - 1\right)$

$$\therefore \ \frac{\Delta r}{r} \times 100 = \frac{\Delta R}{R} \times 100 + \frac{\Delta\left(\frac{l_1}{l_2}\right)}{\left(\frac{l_1}{l_2}\right)} \times 100$$

$$= 4 + 0.2 = 4.2\%$$

8. ***(a)*** Given, KE = 4 eV

$$m = 3.2 \times 10^{-21} \text{ kg}$$

$$e = 1.6 \times 10^{-19}\text{C}$$

$$\text{KE} = \frac{1}{2} mv^2$$

$$\Rightarrow \qquad v^2 = \frac{2 \times \text{KE}}{m}$$

$$= \frac{2 \times 4 \times 1.6 \times 10^{-19}}{3.2 \times 10^{-21}}$$

$$= \frac{2 \times 4}{2 \times 10^{-2}}$$

$v^2 = 400 \quad \Rightarrow \quad v = 20$ m/s

Time to travel a distance of 3.6 km is

$$t = \frac{D}{v} = \frac{3.6 \times 1000}{20} \text{ s} = \frac{3.6 \times 1000}{20 \times 60} \text{ min}$$

$= 3$ min $= 3$ half-time

In three half life times, the number of particles decayed

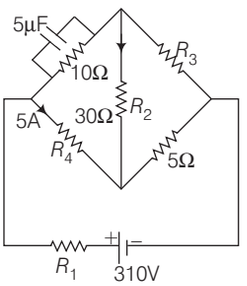
$$\approx 87.5\%$$

$$= \frac{N_0 - N}{N_0} \times 100 \text{ ; where } N = \frac{N_0}{8}$$

$$= \frac{7}{8} \times 100 = 87.5\%$$

9. ***(b)***

10. ***(c)***



Charge on cap = 1 mC

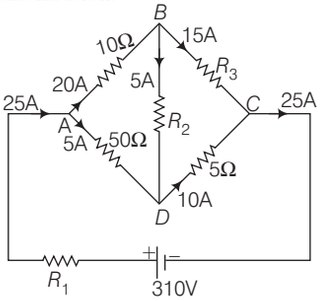
$$V_{\text{capacitor}} = \frac{Q}{C} = \frac{1 \times 10^{-3}}{5 \times 10^{-6}} = \frac{1000}{5}$$

$= 200$ V

So, (current throught (R_5),

$$I = \frac{V}{R} = \frac{200}{10} = 20 \text{ A}$$

Hence, we have following current distribution.



In loop *BCDB*,

$$-15R_3 + 10 \times 5 + 5R_2 = 0 \quad \ldots(i)$$

In loop *ABCDA*,

$$-200 - 15\,R_3 + 50 + 250 = 0$$

$$15R_3 = 100$$

$$R_3 = \frac{100}{15}\,\Omega$$

From Eq. (i), we get

$$5R_2 = 50$$

$$R_2 = 10\ \Omega.$$

and from loop *ADCA*,

$$-250 - 50 + 310 - 25\,R_1 = 0$$

$$25R_1 = 10 \Rightarrow R_1 = \frac{10}{25}\,\Omega$$

So, $\dfrac{R_1 \times R_2}{R_3} = \dfrac{\frac{10}{25} \times 10}{\left(\frac{100}{15}\right)} = \dfrac{15}{25} = \dfrac{3}{5} = 0.6$

11. ***(a)*** Given, $E_0 = 60$ V/m, $B_0 = ?$

The speed of e.m. wave

$$c = \frac{E_0}{B_0}$$

$$B_0 = \frac{E_0}{c} \qquad [\because c = 3 \times 10^8 \text{ m/s}]$$

$$= \frac{60}{3 \times 10^8} = 20 \times 10^{-8} = 2 \times 10^{-7} \text{ T}$$

12. *(c)* $I^2R = 240$ W, $I = 4$ A

$\therefore \quad R = \dfrac{240}{16} = 15\,\Omega$

Now, for the coil $(L\text{-}R)$ circuit,

$$V = IZ = I\sqrt{X_L^2 + R^2}$$

$$\Rightarrow \quad \frac{V^2}{I^2} = X_L^2 + R^2$$

or $$X_L^2 = \frac{100 \times 100}{4 \times 4} - 225$$

$$= 625 - 225 = 400$$

$\therefore \quad X_L = 20 = L\omega$

$\therefore \quad L = \dfrac{20}{2\pi f} = \dfrac{20}{2\pi \times 50} = \dfrac{1}{5\pi}$ H.

13. *(d)*

14. *(a)* The de-Broglie wavelength of electron

$$\lambda_e = \frac{h}{\sqrt{mK_e}}$$

K_e = Kinetic energy of electron

$$\lambda_e^2 = \frac{h^2}{\lambda^2 \times 2\, m_e}$$

De-Broglie wavelength of photon is λ_p

Kinetic energy of photon,

$$K_P = h\nu = \frac{hc}{\lambda_P}$$

$\therefore \quad \dfrac{K_e}{K_p} = \dfrac{h^2 / \lambda_e^2 . 2m_e}{hc / \lambda_p}$

$$\lambda_e = \lambda_P$$

$$\frac{K_e}{K_P} = \frac{\lambda_P}{hc} \times \frac{h^2}{\lambda_e^2 \times 2m_e}$$

we know taht $m_e = 9.1 \times 10^{-31}$ kg

$h = 6.66 \times 10^{-34}$ J- sec

$\lambda_e = 1.2\,\text{Å} = 1.2 \times 10^{-10}$m

and $c = 3 \times 10^8$ m/s

Then, $\dfrac{K_e}{K_P} = \dfrac{h}{c\,\lambda_e \times 2\, m_e}$

$$= \frac{6.66 \times 10^{-34}}{3 \times 10^8 \times 1.2 \times 10^{-10} \times 2 \times 9.1 \times 10^{-31}}$$

$$\cong \frac{1}{100}$$

$\therefore \quad K_e : K_P :: 1 : 100$

15. *(b)* $E = V + Ir$

Given, $V = 1.6$ V

$R = 4\,\Omega$

$$I_1 = \frac{1.6}{4} = 0.4 \text{ A}$$

$$E = 1.6 + 0.4r \qquad \ldots(i)$$

When a resistance of 4Ω is connected in parallel, the resultant resistance

$$\frac{1}{R'} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}$$

$$R' = 2\Omega$$

$$I_2 = \frac{1.33}{2} = 0.66 \text{ A}$$

So, $E = 1.33 + 0.66r \qquad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$1.6 + 0.4r = 1.33 + 0.66r$$

$$1.6 - 1.33 = 0.66r - 0.4r$$

$$0 \cdot 27 = 0 \cdot 26r$$

$$r = \frac{0.27}{0.26} = 1$$

Substituting of r in Eq. (i), we get

$$E = 1.6 + 0.4 \times 1 = 2 \text{ V}$$

16. *(d)*

17. *(c)* $V_{cap} = \dfrac{Q}{C} = \dfrac{4 \times 10^{-6}}{2 \times 10^{-6}} = \mathbf{2\ v}$

Now $V_{cap} = V$ (across 2Ω resistor)

$\therefore \quad I = \dfrac{V}{R} = \dfrac{2}{2} = 1$ A

Now, using the relation

$$E = V + Ir$$

$$= 2 + 1 \times 0.5 = 2.5 \text{ V}$$

18. *(a)* Given, in ground state the radius and velocity of electron in hydrogen atom

$$r_1 = 5.5 \times 10^{-11} \text{ m}$$

$$v_1 = 4 \times 10^6 \text{ m/s}$$

In first excited state

$$r_n \propto \frac{n^2}{Z} \quad \Rightarrow \quad r_n = 5.5 \times 10^{-11} \times 2^2$$

$$= 22.0 \times 10^{-11}\text{m/s}$$

The speed of electron

$$v_n \propto \frac{Z}{n}$$

$$= \frac{4 \times 10^6}{2} = 2 \times 10^6 \text{ m/s}$$

The time period of revolution

$$T = \frac{2\pi r}{v_n} = \frac{2 \times 3.14 \times 22.0 \times 10^{-11}}{2 \times 10^6}$$

$$= 6.908 \times 10^{-16} \text{ s}$$

19. (a) The fringe width

$$\beta = \frac{\lambda d}{D} = \frac{120 \times 10^{-2} \times 0.5 \times 10^{-9}}{500 \times 10^{-9}}$$

$$= 12 \times 10^{-4}\,\text{m}$$

Number of fringe in 3×10^{-3}m

distance on screen

$$= \frac{3 \times 10^{-3}}{12 \times 10^{-4}} = 2.5 = 3 \text{ rd fringe}$$

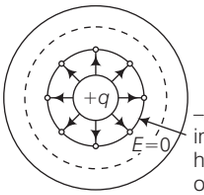
The phase difference

$$= 2\pi + 2\pi + \pi = 5\pi$$

20. *(b)*

TS EAMCET

1. *(b)*



Total charge on shell = $+Q$

Here, total charge on shell is $+Q$. So, the $(-Q)$ is on the inner surface of shell.

Hence, electric field inside the conductor = 0

According the Gauss's law,

$$\oint E ds = \frac{Q}{\varepsilon_0} \Rightarrow E \oint ds = \frac{Q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

Electric field, $E = \dfrac{Q}{4\pi r^2 \varepsilon_0}$

2. *(b)* Given,

Number of turns in coil = n

We know that,

Current $(i) = \dfrac{e}{R'} = \dfrac{-\Delta\phi}{R'\Delta t} \quad \left[\because e = \dfrac{-\Delta\phi}{\Delta t}\right]$

$$R' = R + 4R$$

$$i = -\frac{(\phi_2 - \phi_1)}{(R + 4R)\,\Delta t}$$

$$i = \frac{(\phi_2 - \phi_1)}{5R(\Delta t)}$$

Here, coil have n number of turns.

Hence, $\quad i = -\dfrac{n(\phi_2 - \phi_1)}{5R\,(\Delta t)}$

Induced current, $i = -\dfrac{n(\phi_2 - \phi_1)}{5Rt}$

3. *(c)* Given,

Radius of metal ball = r

Velocity = v

Angle between direction of velocity and magnetic field $(B) = \alpha$

We know that, $e = Bl\ v \sin\alpha$

Here, $l = 2r$

$$e = 2r|B||v| \sin\alpha$$

4. *(*)* Resistance = 4Ω

The equivalent emf of the combination

$$E_{eq} = nE = 3 \times 20 = 60$$

The cell have no internal resistance.

$\therefore$ Main current flowing through the load

$$i = \frac{nE}{R} = \frac{60}{4} = 15A$$

5. *(d)* We know that,

We have, $\lambda = \frac{h}{\sqrt{2mk}}$

$\therefore$ Kinetic energy $(KE) \propto \frac{1}{\lambda^2}$

Hence,

$$\frac{KE_2}{KE_1} = \left(\frac{\lambda_1}{\lambda_2}\right)^2$$

Here, $\lambda_1 = 1$ nm

$\lambda_2 = 0.5$ nm

$$\frac{KE_2}{KE_1} = \left(\frac{1}{0.5}\right)^2$$

$$KE_2 = 4KE_1$$

Hence, the kinetic energy is increases three times.

$$\Delta KE = 3KE_1$$

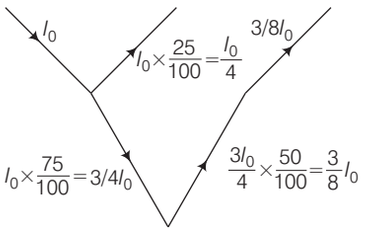
6. *(b)* The net charge on upper plates both capacitor ($2\mu F$ and $3\mu F$) will be $+80\ \mu C$, so charge on upper plate of $3\mu C$ capacitor will be

$$q = \left(\frac{3}{3+2}\right)(80\ \mu C)$$

$$q = \frac{3}{5} \times 80 = 48\ \mu C$$

7. *(a)*

8. *(a)* According to question, we can draw the following diagram



$$I_1 = \frac{I_0}{4} \Rightarrow I_2 = \frac{3}{8} I_0$$

We know that,

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = \left(\frac{\frac{1}{2} + \sqrt{\frac{3}{8}}}{\frac{1}{2} - \sqrt{\frac{3}{8}}}\right)^2$$

9. *(d)*

10. *(d)* $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{(50)^2 + (80 - 25)^2}$

$= \sqrt{(50)^2 - (55)^2}$

$= \sqrt{2500 + 3025}$

$Z = \sqrt{5525} = 74.3\ \Omega$

$I = \frac{E}{Z} = \frac{10}{74.3} = 0.13459$ A

$E_L = IX_L = 0.13459 \times 80$

$= 10.76$ V or 10.8 V

11. *(*)* Temperature coefficient of resistance,

$$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

$$= \frac{4.5 - 3.1}{(3.1)(100) - (4.5 \times 30)}$$

$$= \frac{1.4}{310 - 135} = \frac{1.4}{175} = 0.008°C^{-1}$$

12. *(b)* According to the question,

Energy given to hydrogen atom = Difference in energy level from $n_1 = 1$ to $n_2 = 3$ states

$$= 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = 13.6\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$= 13.6\left(1 - \frac{1}{9}\right) = 13.6 \times \frac{8}{9}$$

$= 12.08 \approx 12.1$ eV

13. *(a)* Given,

Distance of light source $(u) = -1.5$ m $= -\frac{-3}{2}$ m

Radius of curvature of mirror $(R) = 1$ m

We know that,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \qquad \left[\because f = \frac{R}{2}\right]$$

$$\therefore \quad \frac{2}{R} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{2}{1} = \frac{1}{v} + \frac{1}{\frac{-3}{2}} \quad \Rightarrow \quad \frac{2}{1} = \frac{1}{v} - \frac{2}{3}$$

$$\frac{1}{v} = 2 + \frac{2}{3} \quad \Rightarrow \quad \frac{1}{v} = \frac{6+2}{3}$$

$$\frac{1}{v} = \frac{8}{3} \quad \Rightarrow \quad v = \frac{3}{8} = 0.375 \text{ m}$$

The image is virtual.

Magnification

$$(m) = \frac{-v}{u} = \frac{\frac{-3}{8}}{\frac{-3}{2}} = \frac{1}{4} = 0.25$$

14. *(c)*

15. *(a)* We know that,

$$\lambda = \frac{\lambda_0}{\mu}$$

According to the question,

$$\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2}$$

$$\lambda_2 = \lambda_1 \frac{\sin\alpha_2}{\sin\alpha_1}$$

16. *(b)* Given,

Frequency of message signal

$(f_m) = 1$ KHz

$= 1 \times 10^3$ Hz

Peak voltage of message signal

$(E_m) = 5$ V

Carrier frequency $(f_c) = 1$ MHz

$= 1 \times 10^6$ Hz

Peak voltage of carrier $(E_c) = 15$ V

We know that,

The equation of amplitude modulated wave is given by

$$e(t) = E_c\left[1 + \frac{E_m}{E_c}\sin\omega_m t\right]\sin\omega_c t$$

$$= E_c\left[1 + \frac{E_m}{E_c}\sin(2\pi f_m)t\right]\sin 2\pi f_c t$$

$$= 15\left[1 + \frac{5}{15}\sin(2\pi \times 10^3)t\right] \sin 2\pi \times 10^6 t$$

$$= 15\left[1 + \frac{1}{3}\sin(2\pi\ 10^3 t)\right]\sin(2\pi\ 10^6 t)$$

17. *(a)* $e = NBA\omega = NBA(2\pi f)$

$$= 100 \times 0.01 \times 2 \times 10^{-2} \times 2 \times \frac{22}{7} \times 50$$

$= 6.28$ V

18. *(c)*

19. *(c)*

VIT

1. *(a)* On differentiating force between two charges q and $Q - q$ is

$F = \frac{kq(Q-q)}{r^2}$ w.r.t q, we get

$$\frac{dF}{dq} = \frac{d}{dq}\left[\frac{kq}{r^2}(Q - q)\right]$$

$$\Rightarrow \quad \frac{dF}{dq} = \frac{k}{r^2}\frac{d}{dq}[Qq - q^2]$$

$$\Rightarrow \quad \frac{dF}{dq} = \frac{k}{r^2}[Q - 2q] \qquad \ldots(i)$$

But we know that, when force is maximum then

$$\frac{dF}{dq} = 0 \qquad \ldots(ii)$$

Then, from eq (ii), we have

$$\frac{k}{r^2}[Q - 2q] = 0$$

$$\Rightarrow \quad Q - 2q = 0$$

$$\Rightarrow \quad Q = 2q$$

$$\Rightarrow \quad q = Q/2$$

So the other part $= Q - \frac{Q}{2} = Q/2$

Hence the each part have the same charge $Q/2$.

2. *(d)* Sensitivity of galvanometer is directly proportional to the current through it.

When a shunt resistance $S = \left(\frac{G}{10}\right)$ is applied to galvanometer of resistance G, then current through galvanometer, I_g can be written as

$$I_g G = (I - I_g)S$$

$$= \frac{(I - I_g)\,G}{10}$$

or $$I_g = \frac{I - I_g}{10}$$

$$\Rightarrow \quad 10 I_g = I - I_g$$

$$\Rightarrow \quad 11 I_g = I \Rightarrow I_g = \frac{I}{11}$$

3. ***(b)*** At node A, $I_x + 0.7 + 0.3 = 2.2$

$I_x = 1.2$ A

At the node B, 2.2 A enters the node while the other three currents leave the node.

The unknown current at B is

$2.2 - (1.0 + 0.4) = 0.8$A

Then at the node C, $0.8 + 0.4 = I_y$

i.e., $I_y = 1.2$ A

4. ***(b)*** $T = 2\pi(I / P_m B)^{1/2}$. So when P_m becomes 4 times T becomes half

$$\therefore \qquad T = \frac{8}{2} = 4\,\text{sec}$$

5. ***(a)***

6. ***(a)*** Path difference $= AB + BO - 2D$

$$\Rightarrow \quad 2\sqrt{(D^2 + d^2)} - 2D = \lambda/2$$

$$\Rightarrow \qquad 2\sqrt{D^2 + d^2} = \frac{\lambda}{2} + 2D$$

$$\Rightarrow \quad 4(D^2 + d^2) = \frac{\lambda^2}{4} + 4D^2 + 2\lambda D$$

Eliminating $\frac{\lambda^2}{4}$ as $\lambda << D$

Hence, $$d = \sqrt{\frac{\lambda D}{2}}$$

7. ***(a)*** Wavelength for Lyman series

$$\lambda = \frac{1}{R\left(1 - \frac{1}{4}\right)} = \frac{4}{3R}$$

and wavelength for Balmer series

$$\lambda = \frac{1}{R\left(\frac{1}{4} - \frac{1}{9}\right)}$$

$$= \frac{1}{R\left(\frac{5}{36}\right)} = \frac{36}{5R}$$

$$\therefore \qquad \frac{\lambda_L}{\lambda_B} = \frac{4}{3R} \times \frac{5R}{36} = \frac{5}{27}$$

$$\lambda_L : \lambda_B = 5 : 27$$

8. ***(b)*** Number of spectral lines

$$N_E = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

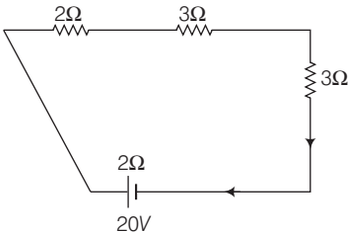
9. ***(a)*** We know that

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

$$\Rightarrow \qquad \frac{1}{16} = \left(\frac{1}{2}\right)^{t/50} \Rightarrow t = 4 \times 50$$

$$\Rightarrow \qquad t = 200 \text{ yrs}$$

10. ***(a)*** Since D_1 is in reverse biased, so current through 5Ω resistance is zero but D_2 is forward bias. So D_2 behaves as resistor of zero resistance. The equivalent circuit is



$$I = \frac{20}{10} \Rightarrow I = 2\text{A}$$

11. (b) The output of the circuit is

$$Y = \overline{\overline{A} + \overline{B}} = \overline{\overline{A}} \cdot \overline{\overline{B}}$$

$$= A \cdot B \; (\because \overline{\overline{A}} = A \text{ and } \overline{\overline{B}} = B)$$

which is the output of an AND gate.

12. ***(d)***

13. ***(d)***

14. ***(b)***

15. ***(b)***

16. ***(b)*** We know that

$$\phi = BA\cos\theta = \frac{1}{2} B\pi r^2 \cos\omega t$$

$$(\because A = 1/2\,\pi r^2)$$

$$e_{\text{induced}} = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{1}{2} B\pi r^2 \cos\omega t\right)$$

$$= \frac{1}{2} B\pi r^2 \omega \sin\omega t$$

$$P = \frac{e^2_{\text{induced}}}{R} = \frac{B^2\pi^2 r^4 \omega^2 \sin^2\omega t}{4R}$$

Hence, $$P_{\text{mean}} = <P> = \frac{B^2\pi^2 r^4 \omega^2}{4R} \times \frac{1}{2}$$

$$= \frac{(B\pi r^2 \omega)^2}{8.R}$$

17. ***(a)*** Let R and r be the radii of bigger and each smaller drop respectively.

$$\therefore \qquad \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

$$R = 2r \qquad \ldots \text{(i)}$$

The capacitance of smaller spherical drop is

$$C = 4\pi\varepsilon_0 r \qquad \ldots \text{(ii)}$$

The capacitance of bigger drop is

$$C' = 4\pi\varepsilon_0 R \qquad (\because R = 2r)$$

$$= 2 \times 4\pi\varepsilon_0 r = 2C \text{ [from eq. (ii)]}$$

$\because$ Capacitance of each small drop

$$C = \frac{C'}{2} = \frac{1}{2}\mu\text{F} \qquad (\because C' = 1\mu\text{F})$$

18. ***(b)*** In position I.

$$\tan\frac{\pi}{6} = \frac{\omega L}{R} = \frac{1000(\sqrt{3} \times 10^{-3})}{R}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{R} \Rightarrow \; R = 3\Omega$$

In position II, $\tan\phi = \frac{X_C}{R} = \frac{\left(\frac{1}{\omega_C}\right)}{R}$

$$\tan\phi = \frac{1}{1000 \times \frac{1000}{3} \times 10^{-6}} = 1$$

$$\Rightarrow \; \phi = \tan^{-1}(1) \; \Rightarrow \phi = \frac{\pi}{4}$$

In second position the circuit is capacitance, therefore current leads the source emf.

19. ***(c)*** When the lens is in air, we have

$$P_a = \frac{1}{f} = \frac{\mu_g - \mu_a}{\mu_a}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

When the lens is in liquid, we have

$$P_l = \frac{1}{f_l} = \frac{\mu_g - \mu_l}{\mu_l}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Here $P_a = 5$, $P_l = -1$, $\mu_0 = 1, \mu_g = 1{\cdot}5$

This gives $\mu_l = 5/3$

20. ***(a)*** The position of 30th bright fringe

$$Y_{30} = \frac{30(\lambda D)}{d}$$

Position shift of central bright fringe is

$$\Delta Y = \frac{30(\lambda D)}{d}$$

$$\Rightarrow \qquad \Delta Y = \frac{D}{d}(\mu - 1)t$$

$$\Rightarrow \qquad \frac{30\lambda D}{d} = \frac{D}{d}(\mu - 1)t$$

$$\Rightarrow \qquad (\mu - 1) = \frac{30\lambda}{t}$$

$$= \frac{30 \times (6000 \times 10^{-10})}{3.6 \times 10^{-5}}$$

$$\Rightarrow (\mu - 1) = 0.5 \Rightarrow \mu = 0.5 + 1 = 1.5$$

21. ***(d)*** Energy of ejected photon

$$E = Rhc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$= 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\text{eV}$$

$$= 13.6 \times \frac{5}{36} = 1.9 \text{ eV}$$

22. ***(c)*** Kinetic energy available for β-particle and antineutrino is

$Q = m(Lu^{176}) - m(Hf^{176})$

$= (175.94269 - 175.94142)\,u$

$= 0.00127$ amu $= 0.00127 \times 931$ MeV

$= 1.182$ MeV

As β-rays have continuous spectrum;

So maximum energy shared by β-particle

$= 1.182$ MeV

23. *(a)* Number of nuclei of $_{79}Au^{200}$ in 2.0×10^{-8} kg

$$= \frac{6.02 \times 10^{25}}{200} \times 2.0 \times 10^{-8} = 6.02 \times 10^{16}$$

Activity $A = 60$ curie

$= 60 \times 3{\cdot}7 \times 10^{11}$

disintegrations/sec

Activity $A = \lambda N$

$$\Rightarrow \quad \lambda = \frac{A}{N} = \frac{60 \times 3.7 \times 10^{10}}{6.02 \times 10^{16}}$$

$$= 3{\cdot}7 \times 10^{-5}\,\text{sec}^{-1}$$

24. *(a)*

25. *(c)* The electric field due to plate A in the vicinity of plate B is

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

$\therefore$ Force on plate B.

$F = QE$ (towards plate A)

$$= Q\left(\frac{Q}{2A\varepsilon_0}\right) = \frac{Q^2}{2A\varepsilon_0} \text{ (attraction)}$$

26. *(a)*

27. *(a)* Intensity of magnetisation, I

$$= \frac{B}{\mu_0} - H = \frac{\mu H}{\mu_0} - H \quad (\because B = \mu H)$$

$= \mu_r H - H = (\mu_r - 1)\,H$

For a long solenoid, also $H = ni$

where n is number of turns/metre

$\therefore \quad I = (1000 - 1) \times 300 \times 0.5$

$\approx 0.15 \times 10^6$ SI unit

The magnetic moment of the rod

$= I \times V = I \times \pi r^2 L$

$= 0.15 \times 10^6 \times 3.142 \times (5 \times 10^{-3})^2 \times 0{\cdot}2$

$= 2.356$ SI units

28. *(d)* Focal length of combination of lenses place in contact is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

For convex lens, $f_1 = 25$ cm

For concave lens, $f_2 = -25$ cm

Hence $\frac{1}{F} = \frac{1}{25} + \left(-\frac{1}{25}\right) = \frac{1}{25} - \frac{1}{25} = 0$

$\therefore \quad F = \frac{1}{0} = \infty$

Hence power of combination,

$P = \frac{1}{F} = 0$

29. *(a)* By using

$$\beta = \frac{(a + b)\lambda}{2a(\mu - 1)\alpha}$$

$$= \frac{(0.3 + 0.7) \times 180\pi \times 10^{-9}}{2 \times 0.3(0.54 - 1) \times \left(\frac{\pi}{180}\right)}$$

$= 10^{-4}$ m

30. *(d)* **31.** *(a)*

MHT CET

1. *(a)* As we know, $v = n\lambda$,

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{n}{v} \Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \nu = Rc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\therefore \quad \nu_2 = Rc\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = Rc\left(\frac{1}{4} - \frac{1}{9}\right) \quad \ldots(i)$$

$$\nu_1 = Rc\left(\frac{1}{2^2}\right) = \frac{Rc}{4}$$

$$\nu_3 = Rc\left(\frac{1}{3^2}\right) = \frac{Rc}{9}$$

$$\Rightarrow \quad \nu_1 - \nu_3 = Rc\left(\frac{1}{4} - \frac{1}{9}\right) \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

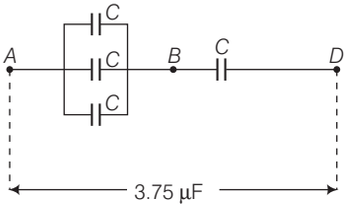
$\nu_1 - \nu_3 = \nu_2 \Rightarrow \nu_1 - \nu_2 = \nu_3$

2. *(a)* Net capacitance between A and B is

$C' = C + C + C = 3C$

Net capacitance between A and D is

$$C_{eq} = \frac{3C \times C}{3C + C} \Rightarrow C_{eq} = \frac{3C^2}{4C} \Rightarrow C_{eq} = \frac{3C}{4}$$



$$\therefore \quad C = \frac{4C_{eq}}{3} = \frac{4 \times 3.75}{3}$$

$\therefore \quad C = 4 \times 1.25 = 5.00\,\mu F$

3. *(c)* Given,

$$S = \frac{G}{8} \quad \ldots(i)$$

As, $$S = \frac{G}{n-1} \quad \ldots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$\frac{G}{8} = \frac{G}{n-1} \quad \therefore \quad 8 = n - 1$$

$\therefore \quad n = 8 + 1 = 9$

Since, the range of galvanometer is increased by nine times, therefore its sensitivity reduces to $\frac{1}{9}$.

$$\therefore \quad S' = \frac{S}{9}$$

4. *(b)* In first case,

$l_x = 40$ cm, $l_R = 60$ cm

$$\frac{x}{R} = \frac{l_x}{l_R} = \frac{40}{60} = \frac{2}{3} \quad \ldots(i)$$

In second case,

$$\therefore \quad \frac{x+30}{R} = \frac{60}{40} = \frac{3}{2} \Rightarrow R = \frac{2(x+30)}{3} \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{\mu}{2\left(\frac{x+30}{3}\right)} = \frac{2}{3} \Rightarrow \frac{3x}{2(x+30)} = \frac{2}{3}$$

$9x = 4x + 120 \Rightarrow 5x = 120 \Rightarrow x = 24\,\Omega$

5. *(b)* Given, $a = 0.2 \times 10^{-3}$ m, $D = 2$ m

$\lambda = 5 \times 10^{-7}$ m

As, $$y = \frac{\lambda D}{a} = \frac{5 \times 10^{-7} \times 2}{0.2 \times 10^{-3}}$$

$$\therefore \quad y = \frac{5 \times 10^{-7}}{10^{-4}} \Rightarrow y = 5 \times 10^{-3}\,\text{m}$$

Distance between 1st minima on either side

$= 5 \times 10^{-3} + 5 \times 10^{-3}$

$= 10 \times 10^{-3} = 10^{-2}$ m

6. *(d)* Given, $R = 18\Omega$, $Z = 33\Omega$, $V_{rms} = 220$ V

The power consumed in an AC circuit is given as,

$$P = e_{rms} \cdot i_{rms} \cdot \cos\phi = e_{rms} \cdot \frac{e_{rms}}{Z} \cdot \frac{R}{Z}$$

$$= \frac{220 \times 220 \times 18}{33 \times 33} = 20 \times 20 \times 2 = 800\,\text{W}$$

7. *(a)* Initial effective capacity of the series combination is

$$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \quad \Rightarrow \quad C_1 = \frac{C}{2}$$

Effective capacity of the series combination with dielectric material is

$$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{KC} \Rightarrow \frac{1}{C_2} = \frac{1}{C}\left[1 + \frac{1}{K}\right]$$

$$\therefore \quad C_2 = \frac{C}{\left(1 + \frac{1}{K}\right)} = \frac{CK}{(K+1)}$$

The change in the effective capacitance is

$$\Delta C = C_2 - C_1$$

$$= \frac{CK}{(K+1)} - \frac{C}{2} = C\left[\frac{K}{K+1} - \frac{1}{2}\right]$$

$$= C\left[\frac{2K - K - 1}{2(K+1)}\right] = \frac{C}{2}\left[\frac{K-1}{K+1}\right]$$

8. *(b)*

9. *(b)* For two coherent sources, the resultant intensity is given as,

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$\therefore$ For two identical light waves,

$$I_1 = I_2 = I \Rightarrow I_R = 4I\cos^2\frac{\phi}{2}$$

$$\therefore \quad I_R \propto \cos^2\frac{\phi}{2}$$

10. *(a)* From the relation,

$$\beta_{dc} = \frac{\alpha_{dc}}{1-\alpha_{dc}} \Rightarrow (1-\alpha_{dc}) = \frac{\alpha_{dc}}{\beta_{dc}} \quad \ldots(i)$$

Also, $$\frac{\beta_{dc}-\alpha_{dc}}{\alpha_{dc}\cdot\beta_{dc}} = \frac{\beta_{dc}\left(1-\frac{\alpha_{dc}}{\beta_{dc}}\right)}{\alpha_{dc}\cdot\beta_{dc}}$$

Using relation (i), we get

$$= \frac{1-\frac{\alpha_{dc}}{\beta_{dc}}}{\alpha_{dc}} = \frac{1-(1-\alpha_{dc})}{\alpha_{dc}} = \frac{1-1+\alpha_{dc}}{\alpha_{dc}} = 1$$

11. *(b)* Given, $N_0 = 10{,}000$ disintegeration/min

$$N = 2500$$ disintegeration/min

$$t = 4 \text{ min}$$

From the radioactive decay law, we have

$$\frac{N}{N_0} = e^{-\lambda t} \Rightarrow \frac{2500}{10000} = e^{-\lambda\times 4}$$

$$\Rightarrow \quad \frac{1}{4} = e^{-4\lambda} \Rightarrow e^{4\lambda} = 4$$

$$\Rightarrow \quad 4\lambda = \log_e 4 \Rightarrow 4\lambda = \log_e 2^2$$

$$\Rightarrow \quad 4\lambda = 2\log_e 2 \Rightarrow \lambda = \frac{2}{4}\log_e 2$$

$$\therefore \quad \lambda = 0.5\log_e 2$$

12. *(c)*

13. *(c)* The energy difference between two states is given as,

$$\Delta E = E_2 - E_1 = \frac{-13.6}{2^2} - \left(\frac{-13.6}{1^2}\right)$$

$$\Rightarrow \quad \Delta E = \frac{13.6}{1^2} - \frac{13.6}{2^2}$$

$$\Rightarrow \quad \Delta E = 13.6\left[\frac{4-1}{4}\right] = 13.6\times\frac{3}{4}$$

$$\therefore \quad \Delta E = 10.2 \text{ eV}$$

Since, the energy is radiated in form of photons, so we have

energy of photons $= h\nu = 10.2$ eV

From Einstein's photoelectric equation, we have

$$h\nu = \phi_0 + eV_s$$

Substituting the given values,

$\therefore 10.2\text{eV} = 4.2\text{eV} + eV_s$

$\Rightarrow \quad 6\text{ eV} = eV_s \quad \therefore \quad V_s = 6$ eV

14. *(d)* **15.** *(a)*

16. *(a)* Binding energy on the surface of the earth is

$$E_1 = \frac{GMm}{R} \quad \ldots(i)$$

Binding energy of revolving satellite at a height h is

$$E_2 = \frac{GMm}{2(R+h)} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\therefore \quad \frac{E_1}{E_2} = \frac{2(R+h)}{R}$$

17. *(b)* According to the Einstein's equation,

$$KE_{max} = h\nu_o - \phi_o$$

where, KE_{max} = maximum kinetic energy and ϕ_0 = work function.

Initially, $\quad h\nu = 0.4 + \phi_0 \quad \ldots(i)$

When the frequency of incident radiation is increased by 30%, then

$$1.3h\nu = 0.9 + \phi_0 \quad \ldots(ii)$$

Solving Eqs. (i) and (ii), we get

$$0.3\phi_o = 0.9 - 1.3(0.4)$$

$$\therefore \quad \phi_o = \frac{0.38}{0.3} = 1.267\text{ eV}$$

18. *(d)* In a parallel resonant circuit, at resonating frequency, the current would be minimum.

This is correctly depicted in the graph (4).

19. *(a)* The energy of photon is given as,

$$E_p = \frac{hc}{\lambda_p} \Rightarrow \lambda_p = \frac{hc}{E_p} \quad \ldots(i)$$

Energy of an moving electron is given as,

$$E_e = mc^2 = pc$$

$$\Rightarrow \quad p = \frac{E_p}{c}$$

$$\therefore \quad \lambda_e = \frac{h}{p} = \frac{hc}{E_e} \quad \ldots(ii)$$

Given, $E_p = E_e$.

Therefore, from Eqs. (i) and (ii), we get

$$\lambda_p \propto \lambda_e.$$

20. *(b)*

21. *(b)* Given, $\rho = 40\times10^{-8}\ \Omega$m

$$A = 8\times10^{-6}\text{ m}^2;\ I = 0.2\text{ A}$$

As, resistance is given as,

$$R = \frac{pl}{A}$$

$$\frac{R}{l} = \frac{\rho}{A} = \frac{40\times10^{-8}}{8\times10^{-6}} = 5\times10^{-2}$$

The potential gradient of the were is

$$\frac{V}{l} = \frac{IR}{l} = 0.2\times5\times10^{-2} = 10^{-2}\text{ V/m}$$

22. *(c)* Given, $\frac{dI_\theta}{dt} = 10$ A/s

$e_p = 15$ mA $= 15\times10^{-3}$ V

$I_p = 1.8$ A

Induced emf in coil P is given as,

$$|e_p| = M\cdot\frac{dI_Q}{dt}$$

Substituting the given values, we get

$$15\times10^{-3} = M\times10$$

$$\Rightarrow \quad M = 15\times10^{-4}\text{ H}$$

Magnetic fluxed linked with coil Q is givne as,

$$\phi_Q = MI_p$$
$$= 15\times10^{-4}\times1.8$$
$$= 27.0\times10^{-4}$$
$$= 2.7\times10^{-3} = 2.7\text{ mWb}$$

23. *(c)* Position of a minima in YDSE is given as,

$$y = \frac{(2n-1)\lambda D}{d} \quad \ldots(i)$$

Here, $y = \frac{d}{2}$, $n = 2$.

Substituting the values in Eq. (i), we get

$$\Rightarrow \quad \frac{d}{2} = \frac{D}{d}\left(\frac{2\times2-1}{2}\right)\lambda$$

$$\Rightarrow \quad \frac{d}{2} = \frac{D}{d}\frac{3}{2}\lambda$$

$$\Rightarrow \quad \lambda = \frac{d^2}{3D}$$

24. *(c)*

25. *(d)* Intensity of magnetisation $= \frac{M}{L\times A}$

$$= \frac{3\text{ A-m}^2}{3\times2\times10^{-6}\text{ m}^3}$$
$$= \frac{1}{2}\times10^6\text{ A/m}$$
$$= 5\times10^5\text{ A/m}$$

26. *(c)* Given, $L = 60$ cm,

$$\phi = 1.57\times10^{-6}\text{ Wb}$$

Magnetic induction inside the solenoid is

$$B = \frac{\mu_0 NI}{L} \quad \ldots(i)$$

Magnetic flux, $\quad \phi = BA$

$$= \frac{\mu_0 NI\cdot A}{L} \quad [\because \text{from Eq. (i)}]$$

Magnetic moment $= NIA = \frac{\phi L}{\mu_0}$

Substituting the given values, we get

$$M = \frac{1.57\times10^{-6}\times0.6}{4\times3.14\times10^{-7}} = 0.75\text{ A}$$

Engineering Entrances Solved Papers 2018

JEE Main & Advanced/BITSAT/KCET/Andhra Pradesh & Telangana State (EAMCET)/VIT/MHT CET

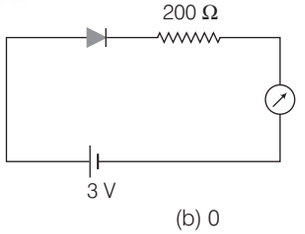
JEE Main

1. Three concentric metal shells A, B and C of respective radii a, b and c $(a < b < c)$ have surface charge densities $+\sigma, -\sigma$ and $+\sigma$, respectively. The potential of shell B is

(a) $\frac{\sigma}{\varepsilon_0}\left(\frac{b^2 - c^2}{c} + a\right)$ (b) $\frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{a} + c\right)$

(c) $\frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$ (d) $\frac{\sigma}{\varepsilon_0}\left(\frac{b^2 - c^2}{b} + a\right)$

2. The reading of the ammeter for a silicon diode in the given circuit is



(a) 13.5 mA (b) 0

(c) 15 mA (d) 11.5 mA

3. Unpolarised light of intensity I passes through an ideal polariser A. Another identical polariser B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$.

Now, another identical polariser C is placed between A and B. The intensity beyond B is now found to be $\frac{1}{8}$. The angle between polariser A and C is

(a) 60° (b) 0°

(c) 30° (d) 45°

4. For an circuit driven with voltage of amplitude V_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, the current exhibits resonance. The quality factor Q is given by

(a) $\frac{CR}{\omega_0}$ (b) $\frac{\omega_0 L}{R}$

(c) $\frac{\omega_0 R}{L}$ (d) $\frac{R}{\omega_0 C}$

5. Two batteries with emf 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω, respectively. The voltage across the load lies between

(a) 11.7 V and 11.8 V (b) 11.6 V and 11.7 V

(c) 11.5 V and 11.6 V (d) 11.4 V and 11.5 V

6. In an AC circuit, the instantaneous emf and current are given by

$$e = 100 \sin 30\, t,\ i = 20 \sin\left(30\, t - \frac{\pi}{4}\right)$$

In one cycle of AC, the average power consumed by the circuit and the wattless current are respectively,

(a) 50 , 0 (b) 50 , 10 (c) $\frac{1000}{\sqrt{2}}$, 10 (d) $\frac{50}{\sqrt{2}}$, 0

7. An EM wave from air enters a medium. The electric fields are

$\mathbf{E}_1 = E_{01}\hat{\mathbf{x}} \cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$ in air and

$\mathbf{E}_2 = E_{02}\hat{\mathbf{x}} \cos[k(2z - ct)]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic.

If ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

(a) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$ (b) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$ (c) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$ (d) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$

8. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilised for transmission. How many telephonic channels can be transmitted simultaneously, if each channel requires a bandwidth of 5 kHz?

(a) 2×10^6 (b) 2×10^3 (c) 2×10^4 (d) 2×10^5

9. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = 5/3$ is inserted between the plates, the magnitude of the induced charge will be

(a) 0.9 nC (b) 1.2 nC (c) 0.3 nC (d) 2.4 nC

10. The dipole moment of a circular loop carrying a current I is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2.

The ratio $\frac{B_1}{B_2}$ is

(a) $\frac{1}{\sqrt{2}}$ (b) 2

(c) $\sqrt{3}$ (d) $\sqrt{2}$

11. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de-Broglie wavelength of the electron in the nth state and the ground state, respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the nth state to the ground state. For large n, (A, B are constants)

(a) $\Lambda_n^2 \approx \lambda$ (b) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$

(c) $\Lambda_n \approx A + B\lambda_n^2$ (d) $\Lambda_n^2 \approx A + B\lambda_n^2$

12. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively, in a uniform magnetic field B. The relation between r_e, r_p, r_α is

(a) $r_e < r_\alpha < r_p$ (b) $r_\alpha > r_e = r_p$

(c) $r_e < r_p = r_\alpha$ (d) $r_e < r_p < r_\alpha$

13. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances?

(a) 990 Ω (b) 910 Ω (c) 900 Ω (d) 550 Ω

14. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. What is the internal resistance of the cell?

(a) 1 Ω (b) 2 Ω (c) 1.5 Ω (d) 2.5 Ω

15. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is

(a) $\nu_L / 25$ (b) $16\,\nu_L$ (c) $\frac{\nu_L}{16}$ (d) $\frac{\nu_L}{20}$

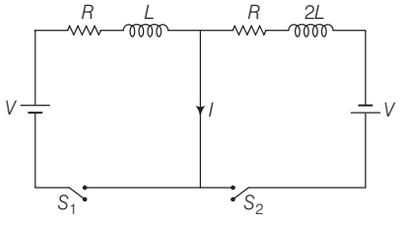
16. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.)

(a) 100 μm (b) 25 μm (c) 50 μm (d) 75 μm

JEE Advanced

Paper 1

1. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time $t = \tau$. Which of the following statement(s) is/are true ?



(One or More than One Option Correct)

(a) $I_{max} = \frac{V}{2R}$ (b) $I_{max} = \frac{V}{4R}$ (c) $\tau = \frac{L}{R}\ln 2$ (d) $\tau = \frac{2L}{R}\ln 2$

2. Two infinitely long straight wires lie in the xy-plane along the lines $x = \pm R$. The wire located at $x = +R$ carries a constant current I_1 and the wire located at $x = -R$ carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the xy-plane.

This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive, if it is in the $+\hat{\mathbf{j}}$-direction. Which of the following statements regarding the magnetic field $\mathbf{B}$ is (are) true?

(One or More than One Option Correct)

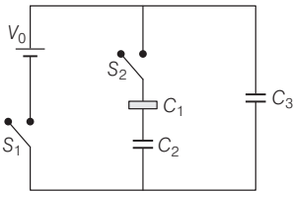
(a) If $I_1 = I_2$, then B cannot be equal to zero at the origin (0,0,0)

(b) If $I_1 > 0$ and $I_2 < 0$, then B can be equal to zero at the origin (0,0,0)

(c) If $I_1 < 0$ and $I_2 > 0$, then B can be equal to zero at the origin (0,0,0)

(d) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$

3. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ε_r. The cell electromotive force (emf) $V_0 = 8$V. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5μC. The value of ε_r =.....



(Numerical Value Based Question)

4. Sunlight of intensity 1.3 kWm^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is ..... .

(Numerical Value Based Question)

Paragraph (Q. Nos. 5-6)

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [**E**] and [**B**] stand for dimensions of electric and magnetic fields respectively, while $[\varepsilon_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space, respectively. [L] and [T] are dimensions of length and time, respectively. All the quantities are given in SI units. *(Paragraph Based Question)*

5. The relation between [E] and [B] is

(a) [E] = [B] [L] [T] (b) [E] = [B] $[L]^{-1}[T]$
(c) [E] = [B][L]$[T]^{-1}$ (d) [E] = [B]$[L]^{-1}[T]^{-1}$

6. The relation between $[\varepsilon_0]$ and $[\mu_0]$ is

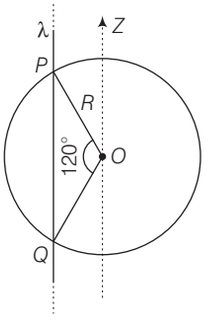
(a) $[\mu_0] = [\varepsilon_0][L]^2[T]^{-2}$ (b) $[\mu_0] = [\varepsilon_0][L]^{-2}[T]^2$
(c) $[\mu_0] = [\varepsilon_0]^{-1}[L]^2[T]^{-2}$ (d) $[\mu_0] = [\varepsilon_0]^{-1}[L]^{-2}[T]^2$

Paper 2

7. An infinitely long thin non-conducting wire is parallel to the *Z*-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius *R* in such a way that the arc *PQ* subtends an angle 120° at the centre *O* of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true?

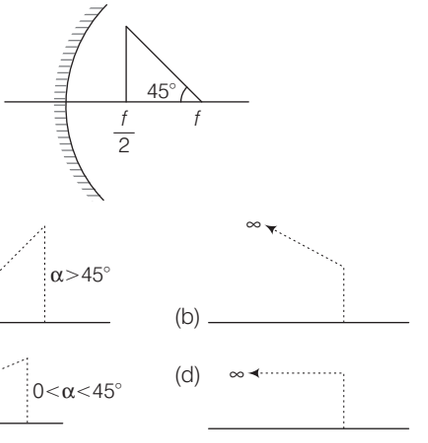
(One or More than One Option Correct)

(a) The electric flux through the shell is $\sqrt{3}\, R\lambda / \varepsilon_0$
(b) The *z*-component of the electric field is zero at all the points on the surface of the shell
(c) The electric flux through the shell is $\sqrt{2}\, R\lambda / \varepsilon_0$
(d) The electric field is normal to the surface of the shell at all points



8. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length *f* as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale)

(One or More than One Option Correct)



9. In a radioactive decay chain, $^{232}_{90}$Th nucleus decays to $^{212}_{82}$Pb nucleus. Let N_α and N_β be the number of α and β- particles respectively, emitted in this decay process. Which of the following statement(s) is/are true?

(One or More than One Option Correct)

(a) $N_\alpha = 5$ (b) $N_\alpha = 6$
(c) $N_\beta = 2$ (d) $N_\beta = 4$

10. A particle of mass 10^{-3}kg and charge 1.0 C is initially at rest. At time $t = 0$, the particle comes under the influence of an electric field $\mathbf{E}(t) = E_0 \sin \omega t\, \hat{\mathbf{i}}$, where $E_0 = 1.0\,\text{NC}^{-1}$ and $\omega = 10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then, the maximum speed in m s^{-1}, attained by the particle at subsequent times is ..... .

(Numerical value Based question)

11. A moving coil galvanometer has 50 turns and each turn has an area $2 \times 10^{-4}\,m^2$. The magnetic field produced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} N - m rad^{-1}. When a current flows through the galvanometer, a full scale deflection occurs, if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 – 1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance in ohms, is ..... .

(Numerical Value Based Question)

12. In a photoelectric experiment, a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency, so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%.

A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of n is ..... . (Take, mass of the electron, $m_e = 9 \times 10^{-31}$ kg and eV = 1.6×10^{-19} J)

(Numerical Value Based Question)

13. Consider a hydrogen-like ionised atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the $n = 2$ to $n = 1$ transition has energy 74.8 eV higher than the photon emitted in the $n = 3$ to $n = 2$ transition. The ionisation energy of the hydrogen atom is 13.6 eV. The value of Z is ..... .

(Numerical Value Based Question)

14. The electric field E is measured at a point $P(0, 0, d)$ generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

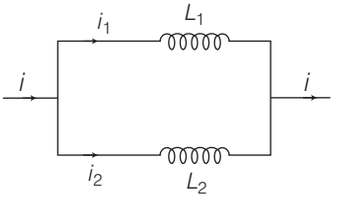
List-I		List-II
P.	E is independent of d	1. A point charge Q at the origin
Q.	$E \propto \frac{1}{d}$	2. A small dipole with point charges Q at (0, 0, l) and $-Q$ at (0, 0, – 1). (Take, $2l << d$)
R.	$E \propto \frac{1}{d^2}$	3. An infinite line charge coincident with the X-axis, with uniform linear charge density λ.
S.	$E \propto \frac{1}{d^3}$	4. Two infinite wires carrying a uniform linear charge density parallel to the X- axis. The one along ($y = 0$, $z = l$) has a charge density + λ and the one along ($y = 0$, $z = -l$) has a charge density – λ. (Take, $2l << d$).
		5. Infinite plane charge coincident with the xy-plane with uniform surface charge density.

(Matching Type Question)

(a) P → 5; Q → 3,4; R → 1; S → 2
(b) P → 5; Q → 3; R → 1, 4; S → 2
(c) P → 5; Q → 3; R → 1, 2; S → 4
(d) P → 4; Q → 2, 3; R → 1; S → 5

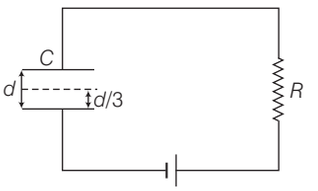
BITSAT

1. Two inductors L_1 and L_2 are connected in parallel and a time varying current flows as shown in figure. The ratio of current $\frac{i_1}{i_2}$ at any time t is



(a) $\frac{L_2}{L_1}$ (b) $\frac{L_1}{L_2}$ (c) $\frac{L_2^2}{(L_1 + L_2)^2}$ (d) $\frac{L_1^2}{(L_1 + L_2)^2}$

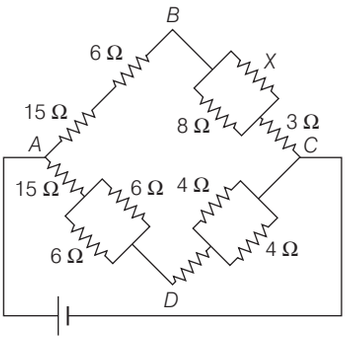
2. A parallel plate capacitor with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$, the level of liquid is $\frac{d}{3}$, initially.



Suppose, the liquid level decreases at a constant speed v, the time constant as a function of time is

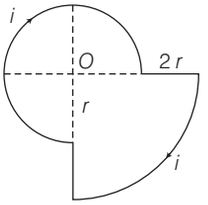
(a) $\frac{6\varepsilon_0 R}{5d + 3vt}$ (b) $\frac{(15d + 9vt)\varepsilon_0 R}{2d^3 - 3dvt - gv^2t^2}$
(c) $\frac{6\varepsilon_0 R}{5d - 3vt}$ (d) $\frac{(15d - 9vt)\varepsilon_0 R}{2d^3 + 3dvt - 9v^2t^2}$

3. In the circuit given below, the value of resistance X when the potential difference between the points B and D is zero, will be



(a) 9 Ω (b) 8 Ω (c) 4 Ω (d) 6 Ω

4. As current i flowing through the loop as shown in figure. The magnetic field at the centre is



(a) $\frac{7\mu_0 i}{12r}$ acting downwards (b) $\frac{5\mu_0 i}{12r}$ acting upwards

(c) $\frac{7\mu_0 i}{12r}$ acting upwards (d) $\frac{5\mu_0 i}{12r}$ acting downwards

5. The ratio of angular momentum L to the atomic dipole moment μ_l for hydrogen-like atoms and ions is

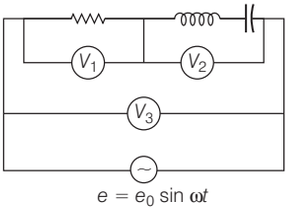
(a) always constant and is equal to the ratio of mass to the charge of electron

(b) always constant and is equal to twice the ratio of mass to the charge of electron to the principal quantum number n

(c) proportional to the principal quantum number n

(d) proportional to $\frac{1}{n^2}$

6. A resistor R, an inductor L, a capacitor C and voltmeters V_1, V_2 and V_3 are connected to an oscillator in the circuit as shown in the adjoining diagram. When the frequency of the oscillation is increased, then at the resonant frequency, the voltmeter reading is zero in the case of



(a) voltmeter V_1 only (b) voltmeter V_2 only

(c) voltmeter V_3 only (d) All the three voltmeters

7. At a certain temperature, the number density of charge carriers in a semiconductor is n. When an electric field is applied to it, the charge carriers drift with an average speed v. If the temperature of the semiconductor is raised, then

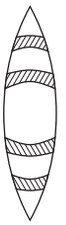
(a) n will increase but v will decrease

(b) n will decrease but v will increase

(c) Both n and v will increase

(d) Both n and v will decrease

8. A convex lens shown in the figure is made up of two types of transparent materials. A point sources of light is placed on its principal axis. If reflections from the boundaries between layers are ignored, the lens will form



(a) only one image (b) two images

(c) infinite images (d) no image at all

9. Some amount of a radioactive substance (half-life $= 10$ days) is spread inside a room and consequently the level of radiation becomes 50 times the permissible level for normal occupancy of the room. After how many days will the room be safe for occupation?

(a) 20 days (b) 34.8 days

(c) 56.4 days (d) 62.9 days

10. The frequency and the intensity of incident beam of light falling on the surface of a photoelectric material is increased by a factor of two. This will

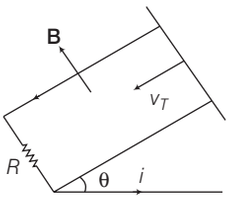
(a) increase the maximum kinetic energy of the ejected photoelectrons by a factor of more than two and would increase the photoelectric current by a factor of two.

(b) increase the maximum kinetic energy of the photo electrons and would increase the photoelectric current both by a factor of two.

(c) increase the maximum kinetic energy of the photoelectrons by a factor of two and will have no effect on the magnitude of the photoelectric current produced.

(d) not produce any effect on the kinetic energy of the emitted photoelectrons but will increase the photoelectric current by a factor of two.

11. A copper rod of mass m slides under gravity on two smooth parallel rails distance apart and set an angle θ to the horizontal. At the bottom, the rails are joined by a resistance R in figure. There is a uniform magnetic field B prependicular to the plane of the rails. The terminal velocity of rod is



(a) $\frac{mgR\tan\theta}{B^2l^2}$ (b) $\frac{mgR\cot\theta}{B^2l^2}$ (c) $\frac{mgR\sin\theta}{B^2l^2}$ (d) $\frac{mgR\cos\theta}{B^2l^2}$

12. When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.50)$ eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then choose the correct statement(s).

(a) The work function of A is 1.50 eV

(b) The work function of B is 4.0 eV

(c) $T_A = 3.2$ eV

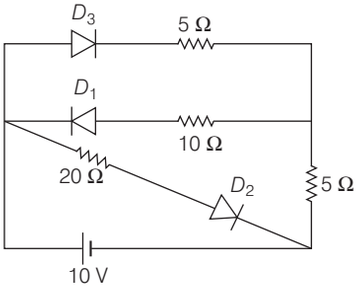
(d) All of the above

13. A planoconvex lens has thickness of 4 cm. When placed on a horizontal table, with the curved surface in contact with it, the apparent depth of the bottom most point of the lens is found to be 3 cm.

If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face is found to be $\frac{25}{8}$ cm. The focal length of the lens is (assume thickness of lens to be negligible)

(a) 85 cm (b) 59 cm
(c) 75 cm (d) 7.5 cm

14. In the given circuit,



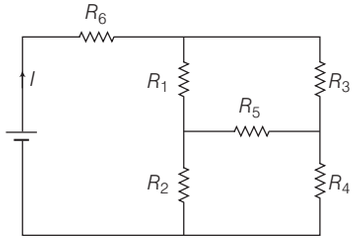
The current through battery is

(a) 0.5 A (b) 1 A (c) 1.5 A (d) 2.5 A

15. To increase the current sensitivity of a moving coil galvanometer by 50%, its resistance is increased, so that new resistance becomes twice its initial resistance, by what factor does its voltage sensitivity change?

(a) Increases by 15% (b) Decreases by 15%
(c) Increases by 25% (d) Decreases by 25%

16. In the given circuit, it is observe that the current is independent of the value of resistance R_6. Then, the resistance value must satisfy



(a) $\frac{1}{R_5}+\frac{1}{R_6}=\frac{1}{R_1+R_2}\cdot\frac{1}{R_3+R_4}$ (b) $R_1R_4 = R_2R_3$
(c) $R_1R_2R_5 = R_3R_4R_6$ (d) $R_1R_3 = R_2R_4 = R_5R_6$

KCET

1. The half-life of tritium is 12.5 years. What mass of tritium of initial mass 64 mg will remain undecayed after 50 years?

(a) 32 mg (b) 8 mg (c) 16 mg (d) 4 mg

2. In a CE amplifier, the input AC signal to be amplified is applied across

(a) forward biased emitter-base junction
(b) reverse biased collector-base junction
(c) reverse biased emitter-base junction
(d) forward biased collector-base junction

3. If $A = 1$ and $B = 0$, then in terms of Boolean algebra, $A + \overline{B}$ equal to

(a) B (b) $\overline{B}$ (c) A (d) $\overline{A}$

4. The density of electron-hole pair in a pure germanium is 3×10^{16} m^{-3} at room temperature. On doping with aluminium, the hole density increases to 4.5×10^{22} m^{-3}. Now, the electron density (in m^{-3}) in doped germanium will be

(a) 1×10^{10} (b) 2×10^{10} (c) 0.5×10^{10} (d) 4×10^{10}

5. The DC common emitter current gain of an *n-p-n* transistor is 50. The potential difference applied across the collector and emitter of a transistor used in CE configuration is, $V_{CE} = 2$ V. If the collector resistance $R_C = 4$ kΩ, the base current (I_B) and the collector current (I_C) are

(a) $I_B = 10\,\mu A$, $I_C = 0.5$ mA (b) $I_B = 0.5\,\mu A$, $I_C = 10$ mA
(c) $I_B = 5\,\mu A$, $I_C = 1$ mA (d) $I_B = 1\,\mu A$, $I_C = 0.5$ mA

6. The dimensions of the ratio of magnetic flux (ϕ) and permeability (μ) are

(a) $[M^0L^1T^0A^1]$ (b) $[M^0L^{-3}T^0A^1]$ (c) $[M^0L^1T^1A^{-1}]$ (d) $[M^0L^2T^0A^1]$

7. The magnitude of point charge due to which the electric field 30 cm away has the magnitude 2 NC^{-1} will be

(a) 2×10^{-11} C (b) 3×10^{-11} C
(c) 5×10^{-11} C (d) 9×10^{-11} C

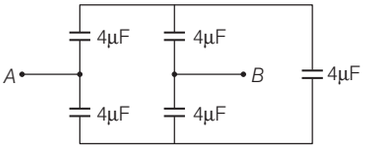
8. A mass of 1 kg carrying a charge of 2 C is accelerated through a potential of 1 V. The velocity acquired by it is

(a) $\sqrt{2}$ ms^{-1} (b) 2 ms^{-1} (c) $\frac{1}{\sqrt{2}}$ ms^{-1} (d) $\frac{1}{2}$ ms^{-1}

9. The force of repulsion between two identical positive charges when kept with a separation r in air is F. Half the gap between the two charges is filled by a dielectric slab of dielectric constant = 4. Then, the new force of repulsion between those two charges becomes

(a) $\frac{F}{3}$ (b) $\frac{F}{2}$ (c) $\frac{F}{4}$ (d) $\frac{4F}{9}$

10. For the arrangement of capacitors as shown in the circuit, the effective capacitance between the points and is (capacitance of each capacitor is 4 μF)



(a) 4 μF (b) 2 μF (c) 1 μF (d) 8 μF

11. Two capacitors of 3 μF and 6 μF are connected in series and a potential difference of 900 V is applied across the combination. They are then disconnected and reconnected in parallel. The potential difference across the combination is

(a) zero (b) 100 V (c) 200 V (d) 400 V

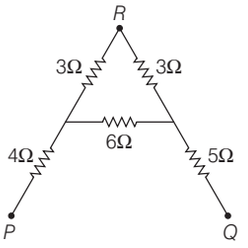
12. Ohm's law is applicable to

(a) diode (b) transistor (c) electrolyte (d) conductor

13. If the last band on the carbon resistor is absent, then the tolerance is

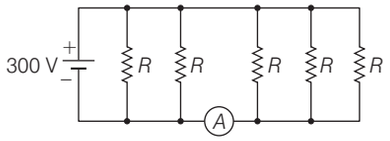
(a) 5% (b) 20%
(c) 10% (d) 15%

14. The effective resistance between and for the following network is



(a) $\frac{1}{12}\ \Omega$ (b) 21Ω (c) $12\,\Omega$ (d) $\frac{1}{21}\ \Omega$

15. Five identical resistors each of resistance $R = 1500\Omega$ are connected to a 300 V battery as shown in the circuit. The reading of the ideal ammeter is

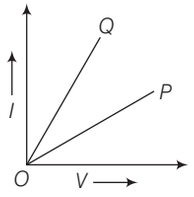


(a) $\frac{1}{5}$ A (b) $\frac{3}{5}$ A (c) $\frac{2}{5}$ A (d) $\frac{4}{5}$ A

16. Two cells of internal resistances r_1 and r_2 and of same emf are connected in series, across a resistor of resistance R. If the terminal potential difference across the cells of internal resistance r_1 is zero, then the value of R is

(a) $R = 2\,(r_1 + r_2)$ (b) $R = r_2 - r_1$
(c) $R = r_1 - r_2$ (d) $R = 2\,(r_1 - r_2)$

17. The graphs for two different electrical appliances and are shown in the diagram. If R_P and R_Q be the resistances of the devices, then



(a) $R_P = R_Q$ (b) $R_P > R_Q$ (c) $R_P < R_Q$ (d) $R_P = \frac{R_Q}{2}$

18. The correct Biot-Savart law in vector form is

(a) $dB = \frac{\mu_0}{4\pi} \cdot \frac{I(dl \times r)}{r^2}$ (b) $dB = \frac{\mu_0}{4\pi} \cdot \frac{I(dl \times r)}{r^3}$

(c) $dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{r^2}$ (d) $dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{r^3}$

19. An electron is moving in a circle of radius r in a uniform magnetic field B. Suddenly, the field is reduced to $\frac{B}{2}$. The radius of the circular path now becomes

(a) $\frac{r}{2}$ (b) $2r$ (c) $\frac{r}{4}$ (d) $4r$

20. A charge q is accelerated through a potential difference . It is then passed normally through a uniform magnetic field, where it moves in a circle of radius r. The potential difference required to move it in a circle of radius $2r$ is

(a) 2 V (b) 4 V (c) 1 V (d) 3 V

21. A cyclotron's oscillator frequency is 10 MHz and the operating magnetic field is 0.66 T. If the radius of its dees is 60 cm, then the kinetic energy of the proton beam produced by the accelerator is

(a) 9 MeV (b) 10 MeV (c) 7 MeV (d) 11 MeV

22. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance, respectively. A magnet when brought close to them will

(a) attract all three of them
(b) attract N_1 strongly, N_2 weakly and repel N_3 weakly
(c) attract N_1 strongly but repel N_2 and N_3 weakly
(d) attract N_1 and N_2 strongly but repel N_3

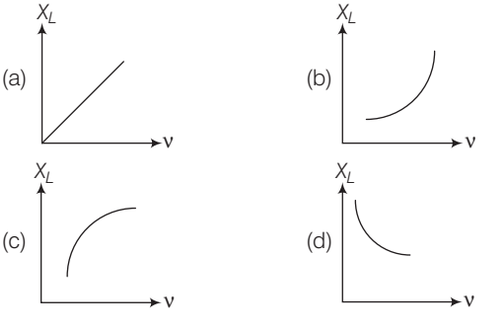
23. The strength of the earth's magnetic field is

(a) constant everywhere (b) zero everywhere
(c) having very high value
(d) varying from place to place on the Earth's surface

24. A jet plane having a wing-span of 25 m is travelling horizontally towards East with a speed of 3600 km/h. If the earth's magnetic field at the location is 4×10^{-4} T and the angle of dip is 30°, then the potential difference between the ends of the wing is

(a) 4 V (b) 5 V (c) 2 V (d) 2.5 V

25. Which of the following, represents the variation of inductive reactance (X_L) with the frequency of voltage sources (ν)?



26. The magnetic flux linked with a coil varies as $\phi = 3t^2 + 4t + 9$. The magnitude of the emf induced at $t = 2$ s is
(a) 8 V (b) 16 V (c) 32 V (d) 64 V

27. A 100 W bulb is connected to an AC source of 220 V, 50 Hz. Then, the current flowing through the bulb is
(a) $\frac{5}{11}$ A (b) $\frac{1}{2}$ A (c) 2 A (d) $\frac{3}{4}$ A

28. In the series circuit, the power dissipation is through
(a) *R* (b) *L*
(c) *C* (d) Both *L* and *C*

29. In Karnataka, the normal domestic power supply AC is 220 V, 50 Hz. Here, 220 V and 50 Hz refer to
(a) peak value of voltage and frequency
(b) rms value of voltage and frequency
(c) mean value of voltage and frequency
(d) peak value of voltage and angular frequency

30. A step-up transformer operates on a 230 V line and a load current of 2 A. The ratio of primary and secondary windings is 1 : 25. Then, the current in the primary is
(a) 25 A (b) 50 A (c) 15 A (d) 12.5 A

31. The number of photons falling per second on a completely darkened plate to produce a force of 6.62×10^{-5} N is *n*. If the wavelength of the light falling is 5×10^{-7}m, then n = ... $\times 10^{22}$. (Take, $h = 6.62 \times 10^{-34}$ J-s)
(a) 1 (b) 5 (c) 0.2 (d) 3.3

32. An object is placed at the principal focus of a convex mirror. The image will be at
(a) centre of curvature (b) principal focus
(c) infinity (d) no image will be formed

33. An object is placed at a distance of 20 cm from the pole of a concave mirror of focal length 10 cm. The distance of the image formed is
(a) + 20 cm (b) + 10 cm (c) – 20 cm (d) – 10 cm

34. A candle placed 25 cm from a lens forms an image on a screen placed 75 cm on the other side of the lens. The focal length and type of the lens should be
(a) + 18.75 cm and convex lens
(b) – 18.75 cm and concave lens
(c) + 20.25 cm and convex lens
(d) – 20.25 cm and convex lens

35. A plane wavefront of wavelength λ is incident on a single slit of width *a*. The angular width of principal maximum is
(a) $\frac{\lambda}{a}$ (b) $\frac{2\lambda}{a}$ (c) $\frac{a}{\lambda}$ (d) $\frac{a}{2\lambda}$

36. In a Fraunhofer diffraction at a single slit, if yellow light illuminating that slit is replaced by blue light, then diffraction bands
(a) remain unchanged (b) become wider
(c) disappear (d) become narrower

37. In Young's double slit experiment, two wavelengths $\lambda_1 = 780$ nm and $\lambda_2 = 520$ nm are used to obtain interference fringes. If the th bright band due to λ_1 coincides with $(n+1)$th bright band due to λ_2, then the value of *n* is
(a) 4 (b) 3 (c) 2 (d) 6

38. In Young's double slit experiment, slits are separated by 2 mm and the screen is placed at a distance of 1.2 m from the slits. Light consisting of two wavelengths 6500 Å and 5200 Å are used to obtain interference fringes. Then, the separation between the fourth bright fringes of two different patterns produced by the two wavelengths is
(a) 0.312 mm (b) 0.123 mm (c) 0.213 mm (d) 0.412 mm

39. The maximum kinetic energy of emitted photoelectrons depends on
(a) intensity of incident radiation
(b) frequency of incident radiation
(c) speed of incident radiation
(d) number of photons in the incident radiation

40. A proton and an α-particle are accelerated through the same potential difference . The ratio of their de-Broglie wavelength is
(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\sqrt{3}$ (d) $2\sqrt{3}$

41. The total energy of an electron revolving in the second orbit of hydrogen atom is
(a) – 13.6 eV (b) – 1.51 eV (c) – 3.4 eV (d) zero

42. The period of revolution of an electron in the ground state of hydrogen atom is *T*. The period of revolution of the electron in the first excited state is
(a) 2 *T* (b) 4*T* (c) 6 *T* (d) 8 *T*

AP EAMCET

1. Two resistance 60.36 Ω and 30.09 Ω are connected in parallel. The equivalent resistance is
(a) 20 ± 0.08 Ω (b) 20 ± 0.06 Ω
(c) 20 ± 0.03 Ω (d) 20 ± 0.10 Ω

2. A light ray incidents normally on one surface of an equilateral prism. The angle of deviation of the light ray is (refractive index of the material of the prism = $\sqrt{2}$)
(a) 60° (b) 30° (c) 0° (d) 120°

3. Two polaroids are placed in the path of unpolarised light beam of intensity I_0 such that no light is emitted from the second polaroid. If a third polaroid whose polarisation axis makes an angle θ with that of the first polaroid is placed between the polaroids, then intensity of light emerging from the last polaroid is

(a) $\left(\frac{I_0}{8}\right)\sin^2 2\theta$ (b) $\left(\frac{I_0}{4}\right)\sin^2 2\theta$

(c) $\left(\frac{I_0}{2}\right)\cos^2\theta$ (d) $I_0\cos^2\theta$

4. Two points charges are kept in air with a separation between them. The force between them is F_1, if half of the space between the charges is filled with a dielectric constant 4 and the force between them is F_2. If $\frac{1}{3}$rd of the space between the charges is filled with dielectric of dielectric constant 9. Then, $\frac{F_1}{F_2}$ is

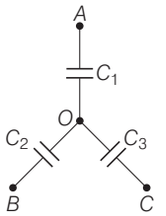
(a) $\frac{27}{64}$ (b) $\frac{16}{81}$ (c) $\frac{81}{64}$ (d) $\frac{100}{81}$

5. A parallel plate capacitor has a capacity 80×10^{-6} F, when air is present between its plates. The space between the plates is filled with a dielectric slab of dielectric constant 20. The capacitor is now connected to a battery of 30V by wires. The dielectric slab is then removed. Then, the charge passing through the wire is

(a) 12×10^{-3}C (b) 25.3×10^{-3}C

(c) 120×10^{-3}C (d) 45.6×10^{-3}C

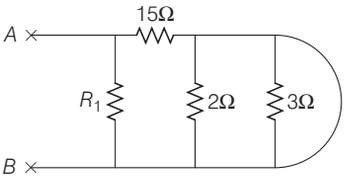
6. Three uncharged capacitors of capacities C_1, C_2 and C_3 are connected as shown in the figure. A, B and C are at potentials V_1, V_2 and V_3, respectively, then the potential at O is



(a) $\frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3}$ (b) $\frac{C_1V_1 + C_2V_2 - C_3V_3}{C_1 + C_2 + C_3}$

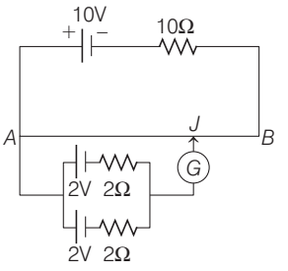
(c) $\frac{C_1V_1 - C_2V_2 - C_3V_3}{C_1 + C_2 + C_3}$ (d) zero

7. The equivalent resistance between A and B is 6 Ω. The value of R_1 is



(a) 20 Ω (b) 10 Ω (c) 5 Ω (d) 25 Ω

8. A battery of emf 10 V is connected to a uniform wire AB of 1m length and having a resistance of 10 Ω in series with a 10Ω resistor as shown in the figure. Two cells of emf 2V and 3V having internal resistance 2 Ω and 3 Ω respectively are connected as shown in the figure. If the galvanometer shows null deflection at point J on the wire, then the distance of point J from the point B is



(a) 48 cm (b) 50 cm (c) 52 cm (d) 54 cm

9. Two infinitely long wires carry currents 4A and 3A placed along X-axis and Y-axis, respectively. Magnetic field at a point $P(0, 0, d)$ m will be …… T.

(a) $\frac{4\mu_0}{2\pi d}$ (b) $\frac{3\mu_0}{2\pi d}$ (c) $\frac{7\mu_0}{2\pi d}$ (d) $\frac{5\mu_0}{2\pi d}$

10. Two moving coil galvanometer, X and Y have coils with resistance 10 Ω and 14 Ω cross-sectional areas $4.8\times10^{-3}\,m^2$ and $2.4\times10^{-3}\,m^2$, number of turns 30 and 45, respectively. They are placed in magnetic field of 0.25 T and 0.50 T, respectively. Then, the ratio of their current sensitivities and the ratio of their voltage sensitivities are respectively

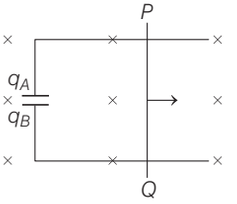
(a) 2 : 3, 14 : 15 (b) 5 : 7, 2 : 1

(c) 2 :13, 1 : 2 (d) 14 : 15, 2 : 9

11. Two short bar magnets each of magnetic moment of $9Am^2$ are placed such that one is at $x = -3$ cm and the other at $y = -3$ cm. If their magnetic moments are directed along positive and negative x-directions respectively, then the resultant magnetic field at the origin is

(a) 100T (b) 10T (c) 0.1T (d) 0.001T

12. A conducting rod PQ of length 1 m is moving with a uniform speed 2 ms^{-1} in a uniform magnetic field of 4 T which is directed into the paper. A capacitor of capacity 10 μF is connected as shown in the figure. Then, the charge on the plates of the capacitor are



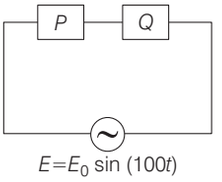
(a) $q_A = +80\,\mu C, q_B = -80\,\mu C$

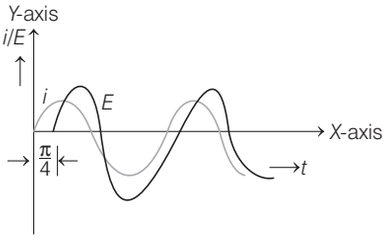
(b) $q_A = -80\,\mu C, q_B = +80\,\mu C$

(c) $q_A = +1.25\,\mu C, q_B = 1.25\,\mu C$

(d) $q_A = -1.25\,\mu C, q_B = +1.25\,\mu C$

13. For the AC circuit shown below, phase difference between emf and current is $\frac{\pi}{4}$ rad as shown in the graph. If the impedance of the circuit is 1414 Ω, then the values of P and Q are





(a) 1 kΩ, 10 μF (b) 1 kΩ, 1 μF
(c) 1 kΩ, 10 mH (d) 1 kΩ, 1 mH

14. In a plane electromagnetic wave, the electric field oscillates with a frequency 2×10^{10} s^{-1} and amplitude 40 Vm^{-1}, then the energy density due to electric field is (Take, $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1})

(a) 1.52×10^{-9} Jm^{-3} (b) 2.54×10^{-19} Jm^{-3}
(c) 3.54×10^{-9} Jm^{-3} (d) 4.56×10^{-9} Jm^{-3}

15. Photons of frequencies equal to the frequencies of H_β and H_∞ lines of hydrogen incident on a photosensitive plate, whose threshold frequency is equal to the frequency of H_α line of hydrogen. The ratio of the maximum kinetic energies of the emitted electrons is

(a) 7 : 16 (b) 3 : 4 (c) 8 : 27 (d) 5 : 36

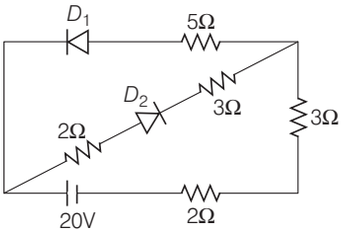
16. Hydrogen atom is in its n th energy state. If de-Broglie wavelength of the electron is λ, then

(a) $\lambda \propto \frac{1}{n^2}$ (b) $\lambda \propto \frac{1}{n}$ (c) $\lambda \propto n^2$ (d) $\lambda \propto n$

17. If 200 MeV of energy is released in the fission of one nucleus of $^{235}_{92}U$, then the number of nuclei that must undergo fission to release an energy of 1000 J is

(a) 3.125×10^{13} (b) 6.25×10^{13}
(c) 12.5×10^{13} (d) 3.125×10^{14}

18. If the diodes are ideal in the circuit given below, then the current through the cell is



(a) 4 A (b) 1.5 A
(c) 2 A (d) 3 A

19. If a message signal of frequency 10 kHz and peak voltage 12 V is used to modulate a carrier wave of frequency 1 MHz, the modulation index is 0.6. To make the modulation index 0.75, the carrier peak voltage should be

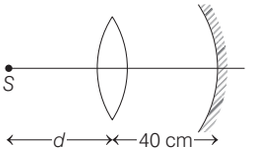
(a) decreased by 25% (b) increased by 25%
(c) decreased by 20% (d) increased by 20%

TS EAMCET

1. Where should an object be placed on the axis of a convex lens of focal length 8 cm, so as to achieve magnification of –4? (Distances are measured from optic centre of the lens)

(a) –6 cm (b) –10 cm (c) –12 cm (d) –9 cm

2. A converging mirror is placed on the right hand side of a converging lens as shown in the figure. The focal length of the mirror and the lens are 20 cm and 15 cm, respectively. The separation between the lens and the mirror is 40 cm and their principal axis coincide. A point source is placed on the principal axis at a distance d to the left of the lens. If the final beam comes out parallel to the principal axis, then the value of d is



(a) 4 cm (b) 8 cm (c) 12 cm (d) 16 cm

3. Interference fringes are obtained in a Young's double slit experiment using beam of light consisting two wavelengths 500 nm and 600 nm. Bright fringes due to both wavelengths coincide at 2.5 mm from the central maximum. If the separation between the slits is 3 mm, then the distance between the screen and plane of the slits is

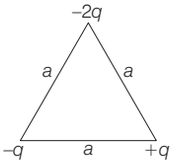
(a) 1.2 m (b) 2.8 m
(c) 2.5 m (d) 3.2 m

4. A point charge of 50 μC is placed in the XY-plane at a location with radius vector $\mathbf{r}_0 = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ m. The electric field strength and its magnitude at a point with radius vector $\mathbf{r} = 8\hat{\mathbf{i}} - 5\hat{\mathbf{j}}$ m is

(Take, $\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2/N\text{-}m^2$)

(a) 4.5 kV/m (b) 45 kV/m
(c) 0.45 kV/m (d) 450 kV/m

5. The work done to assemble the three charges in a configuration as shown in the figure below is



(a) $\frac{-3q^2}{4\pi\varepsilon_0 a}$ (b) $\frac{-2q^2}{4\pi\varepsilon_0 a}$ (c) $\frac{-q^2}{4\pi\varepsilon_0 a}$ (d) 0

6. Consider the following two circuits

[A] 20 bulbs are connected in series to a power supply line.

[B] 20 bulbs identical to [A] are connected in a parallel circuit to an identical power supply line.

Identify which of the following is not true?

(a) If one bulb in [A] blows out, all others will stop glowing
(b) Bulbs in [A] glow brighter, since the current flowing in [A] is higher
(c) If one bulb in [B] blows, other bulbs will still glow
(d) Bulbs in [B] have the highest voltage across each bulb

7. A tapered bar of length L and end diameters D_1 and D_2 is made of a material of electrical resistivity ρ. The electrical resistance of the bar is

(a) $\frac{4\rho L}{\pi(D_1 + D_2)^2}$ (b) $\frac{4\rho L}{\pi(D_1 - D_2)^2}$
(c) $\frac{\rho\pi\sqrt{D_1 D_2}}{4L^2}$ (d) $\frac{4\rho L}{\pi D_1 D_2}$

8. Two circular loops L_1 and L_2 of wire carrying equal and opposite currents are placed parallel to each other with a common axis. The radius of loop L_1 is R_1 and that of L_2 is R_2. The distance between the centres of the loops is $\sqrt{3}R_1$. The magnetic field at the centre of L_2 shall be zero, if

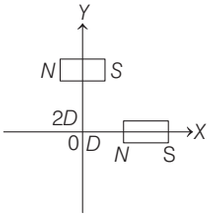
(a) $R_2 = 4R_1$ (b) $R_2 = 2R_1$
(c) $R_2 = \sqrt{2}R_1$ (d) $R_2 = 8R_1$

9. A non-conducting thin disc of radius R rotates about its axis with an angular velocity ω. The surface charge density on the disc varies with the distance r from the centre as $\sigma(r) = \sigma_0\left[1 + \left(\frac{r}{R}\right)^{\beta}\right]$, where σ_0 and β are constants. If the magnetic induction at the center is $B = \left(\frac{9}{10}\right)\mu_0\sigma_0\omega R$, the value of β is

(a) $\frac{1}{4}$ (b) 4 (c) $\frac{1}{2}$ (d) 2

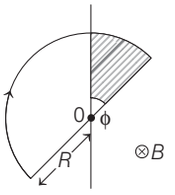
10. A bar magnet of magnetic moment M is placed at a distance D with its axis along positive X-axis. Likewise, second bar manget of magnetic moment M is placed at a distance $2D$ on positive Y-axis and perpendicular to it as shown in the figure.

The magnitude of magnetic field at the origin is $|\mathbf{B}| = \alpha\left[\frac{\mu_0}{4\pi}\frac{M}{D^3}\right]$. The value of α must be (assume $D >> l$, where l is the length of magnets)



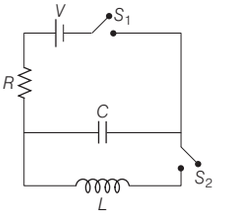
(a) 2 (b) $\frac{15}{8}$ (c) $\frac{17}{8}$ (d) $\frac{9}{8}$

11. A wire loop enclosing a semi-circle of radius R is located on the boundary of a uniform magnetic field of induction **B**. At time $t = 0$, the loop is set into rotation with angular velocity angular ω about its axis 0, coinciding with a line vector **B** on the boundary as shown in the figure. The emf induced in the loop is



(a) $\frac{BR^2}{2}\omega$ (b) $BR\omega$ (c) $BR^2\omega$ (d) $\frac{BR^2}{2\omega}$

12. In the circuit given below, the capacitor C is charged by closing the switch S_1 and opening the switch S_2. After charging, the switch S_1 is opened and S_2 is closed, then the maximum current in the circuit



(a) $V\sqrt{\frac{L}{C}}$ (b) $V\sqrt{\frac{C}{L}}$ (c) $\frac{V}{2\pi}\sqrt{\frac{L}{C}}$ (d) $2\pi V\sqrt{\frac{L}{C}}$

13. A 100 W electric bulb produces electromagnetic radiation with electric field amplitude of $\frac{2V}{m}$ at a distance of 10 m. Assuming it as a point source, estimate the efficiency of the bulb.

(a) 4.9% (b) 2.5% (c) 6.6% (d) 19.7%

14. At an incident radiation frequency of ν_1, which is greater than the threshold frequency, the stoopping potential for a certain metal is V_1. At frequency $2\nu_1$, the stopping potential is $3V_1$. If the stopping potential at frequency $4\nu_1$ is nV_1, then n is

(a) 2 (b) 3 (c) 6 (d) 7

15. The de-Broglie wavelength of an electron of kinetic energy 9 eV is (Take, $h = 4 \times 10^{-15}$ eV-s, $c = 3 \times 10^{10}$ cm/s and the mass m_e of electron as $m_e c^2 = 0.5$ MeV)

(a) 4×10^{-8} cm (b) 3×10^{-8} cm
(c) 4×10^{-7} cm (d) 3×10^{-7} cm

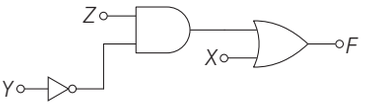
16. An active nucleus decays to $\left(\frac{1}{3}\right)$rd in 20 h. The fraction of original activity remaining after 80 h is

(a) $\frac{1}{16}$ (b) $\frac{1}{81}$ (c) $\frac{1}{36}$ (d) $\frac{1}{54}$

17. Consider an amplifier circuit, where in a transistor is used in common emitter mode. The change in collector current and base current respectively are 4 mA and 20 μA when a signal of 40 mV is added to the base-emitter voltage. If the load resistance is 10 kΩ, then power (gain in watt) in the circuit is

(a) 1×10^4 (b) 2×10^7 (c) 8×10^5 (d) 1×10^6

18. The output F of the logic circuit given below is



(a) $X + \bar{Y} \cdot Z$ (b) $(Y + Z) \cdot X$
(c) $(\bar{Y} + Z) + X$ (d) $X + \bar{Y} + Z$

19. A carrier wave of peak voltage 20 V is used to transmit a message signal. For getting a modulation index of 60%, the peak voltage of the modulating signal is

(a) 6 V (b) 8 V (c) 12 V (d) 33.3 V

VIT

1. Two equal charges $(+Q, +Q)$ are separated by a distance d. A positive third charge is placed on a perpendicular bisector of the line joining these two charges, at a distance x, will experience maximum force when

(a) $x = \frac{d}{\sqrt{2}}$ (b) $x = \frac{d}{2}$ (c) $x = \frac{d}{2\sqrt{2}}$ (d) $x = \frac{d}{2\sqrt{3}}$

2. A moving coil galvanometer has 150 equal division. It's current sensitivity is 10 division per milliampere and voltage sensitivity is 2 division per millivolt. If each division reads 1 V, the resistance in ohm's needed to be connected in series with coil will be

(a) 10^3 (b) 10^5 (c) 99995 (d) 9995

3. A fuse wire with a radius of 1 mm blows at 1.5 A. If the fuse wire of the same material should blow at 3.0 A. The radius of the wire must be

(a) 2 mm (b) $\sqrt{2}$ mm (c) 0.5 mm (d) 8.0 mm

4. The angle of dip at a place is 60°. A magnetic needle oscillates in a horizontal plane at this place with period T. The same needle will oscillate in a vertical plane coinciding with the magnetic meridian with period.

(a) T (b) $2T$ (c) $\frac{T}{2}$ (d) $\frac{T}{\sqrt{2}}$

5. The focal length of a double convex lens in air is 20 cm. When we put this lens in a liquid, it disappears. What must be the refractive index of the liquid? (Take, $R_1 = R_2 = 10$ cm)

(a) $\frac{4}{3}$ (b) $\frac{4}{5}$ (c) $\frac{5}{4}$ (d) $\frac{3}{4}$

6. Red light of wavelength 5400 Å from a distant source falls on a slit 0.80 mm wide. Calculate the distance between first two dark bands on each side of central bright band in the diffraction pattern observed on a screen place 1.4 m from the slit.

(a) 1.89 mm (b) 4 mm (c) 1 mm (d) 3 mm

7. A photon of energy 12.4 eV is completely absorbed by an electron of a hydrogen atom initially in the ground state, so that it can excited.

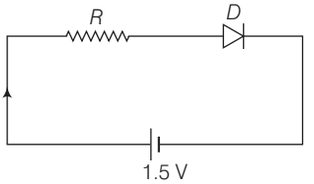
Then, the principal quantum number of the excited state is

(a) $n = 1$ (b) $n = 3$
(c) $n = 4$ (d) $n = \infty$

8. The half-life of radioactive radon is 3.8 days. The time at the end of which $\frac{1}{20}$th of the radon sample will remain undecayed in

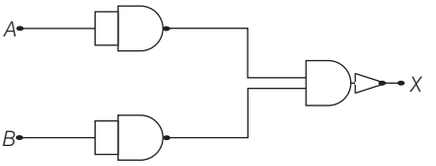
(a) 3.81 days (b) 16.28 days
(c) 33 days (d) 76 days

9. The diode has a constant potential drop of 0.5 V across it and a maximum power rating of 100 mW. What resistance must be connected in series with this diode, so that current in circuit is maximum?



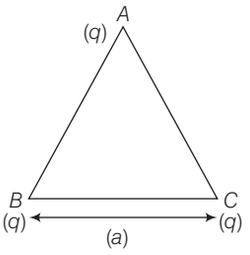
(a) 200 Ω (b) 6.67 Ω (c) 5 Ω (d) 1.5 Ω

10. The combination of the gates shown below yields



(a) OR gate (b) NOT gate
(c) XOR gate (d) NAND gate

11. Three equal charges q are placed at the corners of an equilateral triangle of side a. The potential energy of this system of charge system is

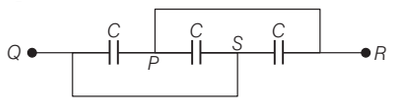


(a) $\frac{3kq^2}{a}$ (b) $\frac{8kq^2}{a}$ (c) $\frac{5kq^2}{2a}$ (d) $\frac{3kq^2}{2a}$

12. The electric potential at a point (x, y) in the XY-plane is given by, $V=-Kxy$. The electric field intensity of a distance r from the origin varies as

(a) r^2 (b) r (c) $2r$ (d) $2r^3$

13. Three capacitors, each of capacitance $C=3$ μF are connected as shown in figure. The equivalent capacitance between points P and S is



(a) 1 μF (b) 3 μF (c) 6 μF (d) 9 μF

14. Two cells of same emf E but of different internal resistances r_1 and r_2 are connected in series with an external resistor of resistance . The potential drop across the first cell is found to be zero. The external resistance is

(a) r_1+r_2 (b) r_1-r_2 (c) r_2-r_1 (d) r_1r_2

15. An equal current i is flowing in three infinitely long wires along positive x, y and z direction. The magnitude of field at a point $(0,0,-a)$ would be

(a) $\frac{\mu_0 i}{2\pi a}(\hat{j}-\hat{i})$ (b) $\frac{\mu_0 i}{2\pi a}(\hat{i}+\hat{j})$

(c) $\frac{\mu_0 i}{2\pi a}(\hat{i}-\hat{j})$ (d) $\frac{\mu_0 i}{2\pi a}(\hat{i}+\hat{j}+\hat{k})$

16. Electric field varies through a coil of circular cross-section $A=2$ cm^2 at the rate of 2 N/C per second. The induced emf in the coil is

(a) 4×10^{-4} T (b) 2×10^{-4} T

(c) 10^{-9} T (d) zero

17. In a series circuit, $X_L=3R$. Now, a capacitor with $X_C=R$ is connected in series in this circuit. Then, ratio of new and old power factor is

(a) 1 (b) 2 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

18. The refractive index of a prism for a monochromatic wave is $\sqrt{2}$ and its refracting angle is 60°. For minimum deviation, the angle of incidence will be

(a) 30° (b) 45° (c) 60° (d) 75°

19. What is the angular spread between central maximum and first order maximum of the diffraction pattern due to a single slit of width 0.20 mm, when light of wavelength 5460 Å is incident on it normally?

(a) 9×10^{-4} rad (b) 8.19×10^{-3} rad

(c) 6×10^{-5} rad (d) 8.65×10^{-4} rad

20. Ratio of the wavelength of first line of Lyman series and first line of Balmer series is

(a) 1:3 (b) 27:5

(c) 5:27 (d) 4:9

21. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2:1. The ratio of their nuclear sizes will be

(a) $2^{\frac{1}{3}}:1$ (b) $1:3^{\frac{1}{2}}$

(c) $3^{\frac{1}{2}}:1$ (d) $1:2^{\frac{1}{3}}$

22. The binding energy of deuteron ${}_1^2\mathrm{H}$ is 1.112 MeV per nucleon and α-particle ${}_2^4\mathrm{He}$ has a binding energy of 7.047 MeV per nucleon. Then, in the fusion reaction

$${}_1^2\mathrm{H}+{}_1^2\mathrm{H}\longrightarrow {}_2^4\mathrm{He}+Q$$

The energy Q released is

(a) 1 MeV (b) 11.9 MeV

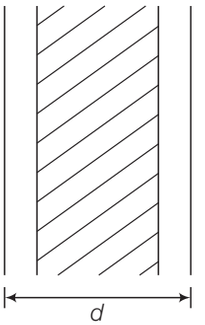
(c) 23.8 MeV (d) 931 MeV

23. In a common emitter amplifier, the low resistance of the output circuit is 500 times the resistance of the input circuit. If $\alpha = 0.98$, then what is the voltage gain and the power gain?

(a) 23500, 1300500 (b) 24500, 1200500

(c) 22505, 1300500 (d) 23500, 1200500

24. A copper plate of thickness b is placed inside a parallel plate capacitor of plate distance d and area A as shown in figure. The capacitance of capacitor is



(a) $\frac{A\varepsilon_0}{d}$ (b) $\frac{A\varepsilon_0}{b}$

(c) $\frac{A\varepsilon_0}{d-b}$ (d) ∞

25. The temperature coefficient of resistance for a wire is $0.00125°\ C^{-1}$. At 300 K, its resistance is 1 Ω. The temperature at which the resistance becomes 1.5 Ω is

(a) 450 K (b) 714 K

(c) 454 K (d) 900 K

26. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is . It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be

(a) nB (b) n^2B (c) $2nB$ (d) $2n^2B$

27. When a glass slab is placed on cross made on a sheet, the cross appears to be raised by 1 cm. The thickness of the glass is 3 cm. The critical angle for glass is

(a) $\sin^{-1}(0.33)$ (b) $\sin^{-1}(0.5)$

(c) $\sin^{-1}(0.67)$ (d) $\sin^{-1}\left(\frac{\sqrt{3}}{7}\right)$

28. What is the minimum distance between two points objects with microscope with cone angle of light falling equal to 60°?

(Take, $\lambda = 600$ nm and $\mu_{air} = 1$)

(a) 1.67×10^{-6} m (b) 1.8×10^{-6} m

(c) 1.5×10^{-4} m (d) 7.3×10^{-7} m

29. Monochromatic radiation of wavelength λ is incident on a hydrogen sample which is in ground state. Hydrogen atoms absorb the light and subsequently emit radiation of ten different wavelengths. The value of λ is

(a) 95 nm (b) 103 nm (c) 73 nm (d) 88 nm

30. The threshold wavelength for a metal having work function W_0 is λ_0. What is the threshold wavelength for a metal whose work function is $\frac{W_0}{2}$?

(a) $4\lambda_0$ (b) $2\lambda_0$ (c) $\frac{\lambda_0}{2}$ (d) $\frac{\lambda_0}{4}$

MHT CET

1. A transistor is used as a common emitter amplifier with a load resistance $2\,k\Omega$. The input resistance is 150Ω. Base current is changed by $20\,\mu A$ which results in a change in collector current by 1.5 mA. The voltage gain of the amplifier is

(a) 900 (b) 1000 (c) 1100 (d) 1200

2. A circular coil carrying current I has radius R and magnetic field at the centre is B. At what distance from the centre along the axis of the same coil, the magnetic field will be $\frac{B}{8}$?

(a) $R\sqrt{2}$ (b) $R\sqrt{3}$ (c) $2R$ (d) $3R$

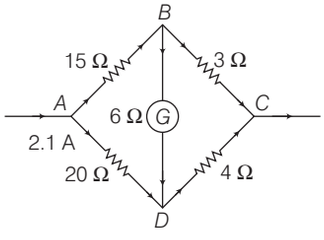
3. Two light waves of intensities I_1 and I_2 having same frequency pass through same medium at a time in same direction and interfere. The sum of the minimum and maximum intensities is

(a) $(I_1 + I_2)$ (b) $2(I_1 + I_2)$ (c) $(\sqrt{I_1} + \sqrt{I_2})$ (d) $(\sqrt{I_1} - \sqrt{I_2})$

4. An alternating voltage $e = 200\sqrt{2}\sin(100\,t)$ volt is connected to $1\,\mu F$ capacitor through AC ammeter. The reading of ammeter is

(a) 5 mA (b) 10 mA (c) 15 mA (d) 20 mA

5. In the following network, the current flowing through 15Ω resistance is



(a) 0.8 A (b) 1.0 A (c) 1.2 A (d) 1.4 A

6. The angle made by incident ray of light with the reflecting surface is called

(a) glancing angle (b) angle of incidence

(c) angle of deviation (d) angle of refraction

7. If numerical aperture of a microscope is increased, then its

(a) resolving power remains constant

(b) resolving power becomes zero

(c) limit of resolution is decreased

(d) limit of resolution is increased

8. In amplitude modulation,

(a) amplitude remains constant but frequency changes

(b) both amplitude and frequency do not change

(c) both amplitude and frequency change

(d) amplitude of the carrier wave changes according to the information signal

9. If M_z = magnetisation of a paramagnetic sample, B = external magnetic field, T = absolute temperature, C = curie constant, then according to Curie's law in magnetism, the correct relation is

(a) $M_z = \frac{T}{CB}$ (b) $M_z = \frac{CB}{T}$

(c) $C = \frac{M_z B}{T}$ (d) $C = \frac{T^2}{M_z B}$

10. An electron of stationary hydrogen atom jumps from 4th energy level to ground level. The velocity that the photon acquired as a result of electron transition will be (where, h = Planck's constant, R = Rydberg's constant, m = mass of photon)

(a) $\frac{9\,Rh}{16m}$ (b) $\frac{11\,hR}{16\,m}$

(c) $\frac{13\,hR}{16m}$ (d) $\frac{15\,hR}{16m}$

11. An ideal transformer converts 220 V AC to 3.3 kV AC to transmit a power of 4.4 kW. If primary coil has 600 turns, then alternating current in secondary coil is

(a) $\frac{1}{3}$A (b) $\frac{4}{3}$A
(c) $\frac{5}{3}$A (d) $\frac{7}{3}$A

12. A conducting wire has length L_1 and diameter d_1. After stretching the same wire length becomes L_2 and diameter d_2. The ratio of resistances before and after stretching is

(a) $d_2^4 : d_1^4$ (b) $d_1^4 : d_2^4$ (c) $d_2^2 : d_1^2$ (d) $d_1^2 : d_2^2$

13. With forward biased mode, the junction diode

(a) is one in which width of depletion layer increases
(b) is one in which potential barrier increases
(c) acts as closed switch
(d) acts as open switch

14. An alternating electric field of frequency ν is applied across the dees (radius R) of a cyclotron to accelerate protons (mass m).The operating magnetic field B used and KE of the proton beam produced by it are respectively (where, e = charge on proton)

(a) $\frac{2\pi m\nu}{e}, 2\pi^2 m\nu^2 R^2$ (b) $\frac{2\pi^2 m\nu}{e^2}, 4\pi^2 m\nu^2 R^2$
(c) $\frac{\pi m\nu}{e}, \pi^2 m\nu^2 R^2$ (d) $\frac{2\pi^2 m^2\nu^2}{e}, 2\pi^2 m\nu^2 R^2$

15. A ray of light is incident normally on a glass slab of thickness 5 cm and refractive index 1.6. The time taken to travel by a ray from source to surface of slab is same as to travel through glass slab. The distance of source from the surface is

(a) 4 cm (b) 8 cm (c) 12 cm (d) 16 cm

16. Magnetic susceptibility for a paramagnetic and diamagnetic materials is respectively,

(a) small, positive and small, positive
(b) large, positive and small, negative
(c) small, positive and small, negative
(d) large, negative and large, positive

17. In a photocell, frequency of incident radiation is increased by keeping other factors constant ($\nu > \nu_0$), the stopping potential

(a) decreases
(b) increases
(c) becomes zero
(d) first decreases and then increases

18. The expression for electric field intensity at a point outside uniformly charged thin plane sheet is (where, d is the distance of point from plane sheet)

(a) independent of d
(b) directly proportional to $\sqrt{d}$
(c) directly proportional to d
(d) directly proportional to $\frac{1}{\sqrt{d}}$

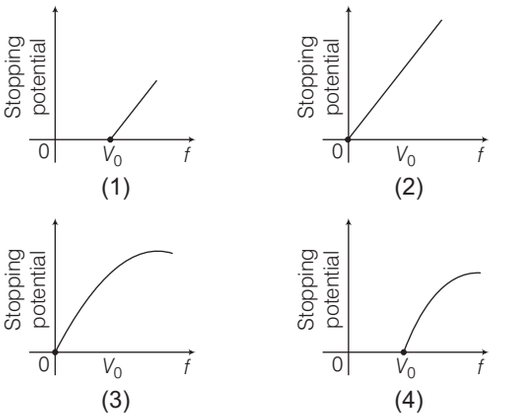
19. The deflection in galvanometer falls to $\left(\frac{1}{4}\right)$th when it is shunted by 3Ω. If additional shunt of 2Ω is connected to earlier shunt in parallel, the deflection in galvanometer falls to

(a) $\frac{1}{2}$th (b) $\left(\frac{1}{3}\right)$rd (c) $\left(\frac{1}{4}\right)$th (d) $\left(\frac{1}{8.5}\right)$th

20. A series combination of N_1 capacitors (each of capacity C_1) is charged to potential difference $3V$. Another parallel combination of N_2 capacitors (each of capacity C_2) is charged to potential difference V. The total energy stored in both the combinations is same. The value of C_1 in terms of C_2 is

(a) $\frac{C_2 N_1 N_2}{9}$ (b) $\frac{C_2 N_1^2 N_2^2}{9}$ (c) $\frac{C_2 N_1}{9N_2}$ (d) $\frac{C_2 N_2}{9N_1}$

21. Following graphs show the variation of stopping potential corresponding to the frequency of incident radiation (F) for a given metal. The correct variation is shown in graph (where, ν_0 = threshold frequency)



(a) (1) (b) (2) (c) (3) (d) (4)

22. In compound microscope, the focal length and aperture of the objective used is respectively,

(a) large and large (b) large and small
(c) short and large (d) short and small

23. The energy of an electron having de-Broglie wavelength λ is (where, h = Planck's constant, m = mass of electron)

(a) $\frac{h}{2m\lambda}$ (b) $\frac{h^2}{2m\lambda^2}$ (c) $\frac{h^2}{2m^2\lambda^2}$ (d) $\frac{h^2}{2m^2\lambda}$

24. The ratio of magnetic fields due to a bar magnet at the two axial points P_1 and P_2 which are separated from each other by 10 cm is 25 : 2. Point P_1 is situated at 10 cm from the centre of the magnet. Magnetic length of the bar magnet is (points P_1 and P_2 are on the same side of magnet and distance of P_2 from the centre is greater than distance of P_1 from the centre of magnet)

(a) 5 cm (b) 10 cm
(c) 15 cm (d) 20 cm

Answer *with* Explanations

JEE Main

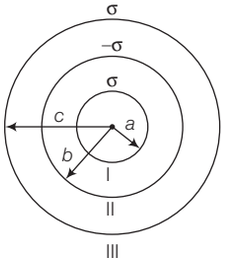
1. *(c)* $q_{\mathrm{I}} = (4\pi a^2)(\sigma), q_{\mathrm{II}} = 4\pi b^2 (-\sigma), q_{\mathrm{III}} = 4\pi c^2(\sigma)$

(Potential due to I) + (Potential due to II) + (Potential due to III)

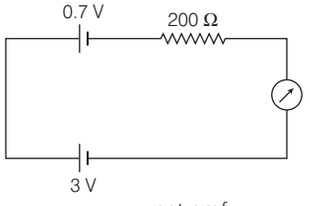
$$V_B = \frac{K4\pi a^2\sigma}{b} + \frac{K4\pi b^2}{b}(-\sigma) + \frac{K4\pi c^2(\sigma)}{c}$$

Substituting $K = \frac{1}{4\pi\varepsilon_0}$, we get

$$V_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$



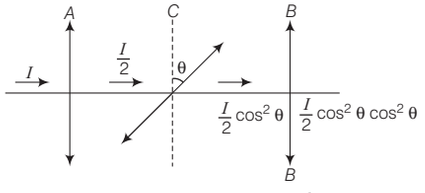
2. *(d)* For silicon diode, potential barrier is 0.7 V. Therefore, the given circuit is as shown in figure.



$$I = \frac{\text{net emf}}{\text{net resistance}}$$

$$= \frac{3 - 0.7}{200}\text{A} = 11.5\text{ mA}$$

3. *(d)*



Using the relation $I = I_0\cos^2\theta$ we have

$$\frac{I}{2}(\cos^2\theta)^2 = \frac{I}{8}$$

$$\Rightarrow \quad \cos^2\theta = \frac{1}{2}$$

or $$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = 45°$$

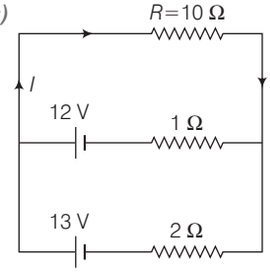
4. *(b)* $\therefore Q = \frac{\omega_1}{\omega_0 - \omega_2}$

Here, $\omega_1 - \omega_2 = \text{bandwidth} = \frac{R}{L}$

Substituting the values, we get

$$Q = \frac{\omega_0 L}{R}$$

5. *(c)*



$$E_{\text{eq}} = \frac{\Sigma E / r}{\Sigma(1/r)} = \frac{\frac{12}{1} + \frac{13}{2}}{\frac{1}{1} + \frac{1}{2}} = \frac{37}{3}\text{ V}$$

$$r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2} = \frac{2}{3}\Omega$$

$$\Rightarrow \qquad I = \frac{\frac{37}{3}}{\frac{2}{3} + 10} = \frac{37}{32}\text{A}$$

Voltage across load

$$= IR = \left(\frac{37}{32}\right)(10) = 11.56\text{ V}$$

So, the voltage across the load lies between 11.5V and 11.6V.

6. *(c)* $\langle P\rangle = e_{\text{rms}}\, i_{\text{rms}}\cos\phi = \left(\frac{e_0}{\sqrt{2}}\right)\left(\frac{i_0}{\sqrt{2}}\right)\cos\phi$

$$= \frac{e_0 i_0}{2}\cos\phi = \frac{(100)(20)\cos\frac{\pi}{4}}{2} = \frac{1000}{\sqrt{2}}\text{ W}$$

Wattless current

$$= I_{\text{rms}}\sin\phi = \frac{20}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = 10\text{ A}$$

7. *(d)* $v = \frac{\text{coefficient of } t}{\text{coefficient of } z} = \frac{1}{\sqrt{\varepsilon\mu}}$

where, $\varepsilon = \varepsilon_0\,\varepsilon_r$ and $\mu = \mu_0\mu_r$.

$$\because \quad \frac{v_{\text{air}}}{v_{\text{med}}} = \frac{c}{c/2} = 2 = \frac{\sqrt{\mu_0\,\varepsilon_0\,\mu_{r_2}\,\varepsilon_{r_2}}}{\sqrt{\mu_0\,\varepsilon_0\,\mu_{r_1}\,\varepsilon_{r_1}}}$$

$$\Rightarrow \quad \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$$

Note *Medium is non-magnetic.*

$$\therefore \qquad \mu_{r_1} = \mu_{r_2}$$

8. *(d)* $N = \frac{1}{10}\cdot\frac{(10\text{ kHz})}{(5\text{kHz})}$

$$= \frac{10^9}{5\times10^3} = \frac{10^6}{5} = 2\times10^5$$

9. *(b)* $Q = KC_0V \quad |Q_{\text{in}}| = \left|Q\left(1 - \frac{1}{K}\right)\right|$

$$= (90\times10^{-12})(20)\left(\frac{5}{3}\right)\left(1 - \frac{3}{5}\right)\text{C}$$

$$= 1200\times10^{-12}\text{ C} = 1.2\text{ nC}$$

10. *(d)* $m = I\times\pi R^2$

$$2m = I\times\pi(R')^2$$

$$\Rightarrow \quad R' = \sqrt{2}\,R$$

$$B = \frac{\mu_0 I}{2\pi R} \Rightarrow B \propto \frac{1}{R} \Rightarrow \frac{B_1}{B_2} = \frac{R'}{R} = \sqrt{2}$$

11. *(b)* $2\pi r = n\lambda_n$

$$\lambda_n = \frac{2\pi r}{n} = \frac{2\pi r_0 n^2}{n} = 2\pi r_0 n \qquad \ldots(\text{i})$$

$$\frac{1}{\Lambda_n} = R\left\{\frac{1}{1^2} - \frac{1}{n^2}\right\}$$

$$\Lambda_n = \frac{1}{R}\left\{1 + \frac{1}{n^2 - 1}\right\}$$

$$\Lambda_n = \frac{1}{R}\left\{1 + \frac{1}{n^2}\right\} \qquad (n >> 1)\ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$\Lambda_n = \frac{1}{R}\left\{1 + \frac{4\pi^2 r_0^2}{\lambda_n^2}\right\} \approx A + \frac{B}{\lambda_n^2}$$

12. *(c)* For circular path in magnetic field,

$$r = \frac{\sqrt{2mK}}{qB}$$

where, $K = \text{kinetic energy}$.

$$\Rightarrow \qquad r \propto \frac{\sqrt{m}}{q}$$

Further,

	e	p	α
m	1/1836	1	4
q	$-e$	$+e$	$2e$

$$\therefore \qquad r_p = r_\alpha > r_e$$

13. *(d)* $\frac{R_1}{R_2} = \frac{l}{100 - l} \qquad \ldots(\text{i})$

and $\frac{R_2}{R_1} = \frac{l - 10}{100 - (l - 10)} \qquad \ldots(\text{ii})$

From Eqs. (i) and (ii), we get

$$l = 55\text{cm}$$

Substituting in Eq. (i), we get

$$\frac{R_1}{R_2} = \frac{55}{45} \qquad \ldots(\text{iii})$$

Further, $R_1 + R_2 = 100\ \Omega$ ...(iv)

Solving Eqs. (iii) and (iv), we get

$R_1 = 550\ \Omega$

14. *(c)* Using formula, $r = R\left(\frac{l_1}{l_2} - 1\right)$

$= 5\left(\frac{52}{40} - 1\right) = 5 \times \frac{12}{40} = 1.5\,\Omega$

15. *(a)* Lyman series ends at $n = 1$.

$\therefore$ Series limit frequency of the Lyman series is given by

$\nu_L = RcZ^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) \Rightarrow \nu_L = RcZ^2$

Pfund series ends at $n = 5$.

$\therefore$ Series limit frequency of the Pfund series,

$\nu_P = RcZ^2\left(\frac{1}{5^2} - \frac{1}{\infty^2}\right) = \frac{RcZ^2}{25}$

or $\nu_p = \frac{\nu_L}{25}$

16. *(b)* In single slit diffraction pattern, $\lambda = b\sin\theta$

At $\theta = 30°$,

$\lambda = \frac{b}{2} = \frac{1\times10^{-6}}{2} = 5\times10^{-7}\,\text{m}$

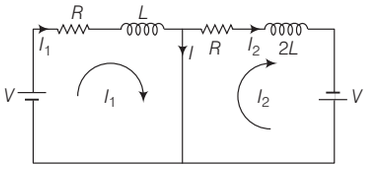
In YDSE, fringe width,

$\omega = \frac{\lambda D}{d} \Rightarrow d = \frac{\lambda D}{\omega} = \frac{5\times10^{-7}\times0.5}{1\times10^{-2}}$

$d = 25\times10^{-6}\ \text{m} = 25\mu\text{m}$

JEE Advanced

1. *(b, d)* $I_1 = \frac{V}{R}\left(1 - e^{-\frac{tR}{L}}\right)$



$I_2 = \frac{V}{R}\left(1 - e^{-\frac{tR}{2L}}\right)$

From principle of superposition,

$I = I_1 - I_2$

$\Rightarrow \quad I = \frac{V}{R}e^{-\frac{tR}{2L}}\left(1 - e^{-\frac{tR}{2L}}\right)$ ...(i)

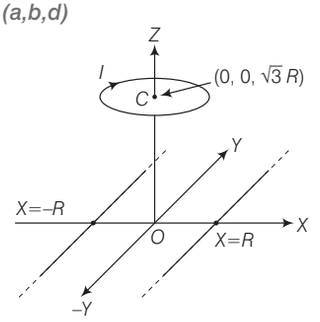
I is maximum when $\frac{dI}{dt} = 0$, which gives

$e^{-\frac{tR}{2L}} = \frac{1}{2}$ or $t = \frac{2L}{R}\ln 2$

Substituting this time in Eq. (i), we get

$I_{\max} = \frac{V}{4R}$

2. *(a,b,d)*

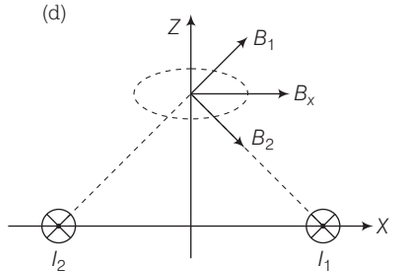


(a) At origin, $\mathbf{B} = 0$ due to two wires, if $I_1 = I_2$, hence $(\mathbf{B}_{\text{net}})$ at origin is equal to $\mathbf{B}$ due to ring. which is non-zero.

(b) If $I_1 > 0$ and $I_2 < 0$, $\mathbf{B}$ at origin due to wires will be along $+\hat{\mathbf{k}}$. Direction of $\mathbf{B}$ due to ring is along $-\hat{\mathbf{k}}$ direction and hence $\mathbf{B}$ can be zero at origin.

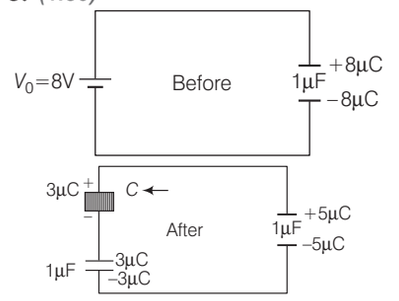
(c) If $I_1 < 0$ and $I_2 > 0$, $\mathbf{B}$ at origin due to wires is along $-\hat{\mathbf{k}}$ and also along $-\hat{\mathbf{k}}$ due to ring, hence B cannot be zero.

(d)



At centre of ring, $\mathbf{B}$ due to wires is along X-axis. Hence, z-component is only because of ring which $\mathbf{B} = \frac{\mu_0 i}{2R}(-\hat{\mathbf{k}})$.

3. *(1.50)*



$C = \varepsilon_r C_1 = (\varepsilon_r)\,\mu\text{F}$

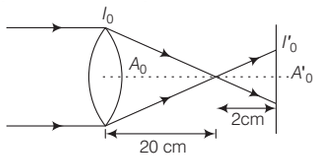
Applying loop rule,

$\frac{5}{1} - \frac{3}{\varepsilon_r} - \frac{3}{1} = 0$

$\Rightarrow \quad \frac{3}{\varepsilon_r} = 2$

$\varepsilon_r = 1.50$

4. *(130.0 kW / m²)*



$\therefore \quad \frac{A_0'}{A_0} = \left(\frac{2}{20}\right)^2 = \frac{1}{100}$

$\Rightarrow \quad A_0' = \frac{A_0}{100}$

$P = I_0 A_0 = I_0' A_0'$

$\Rightarrow \quad I_0' = \frac{I_0 A_0}{\frac{A_0}{100}}$

$\Rightarrow \quad = 100 I_0 = 130\ \text{kW/m}^2$

5. *(c)* In terms of dimension, $F_e = F_m$

$\Rightarrow \quad qE = qvB$ or $E = vB$

$[\text{E}] = [\text{B}][\text{L}][\text{T}]^{-1}$

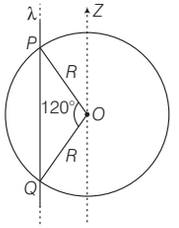
6. *(d)* $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$c^2 = \frac{1}{\mu_0\varepsilon_0}$

$\mu_0 = \varepsilon_0^{-1} \cdot c^{-2}$

$[\mu_0] = [\varepsilon_0]^{-1}[\text{L}^{-2}]\,[\text{T}^{2}]$

7. *(a, b)* $PQ = (2)\,R\sin 60°$



$= (2R)\frac{\sqrt{3}}{2} = (\sqrt{3}R)$

$q_{\text{enclosed}} = \lambda\,(\sqrt{3}R)$

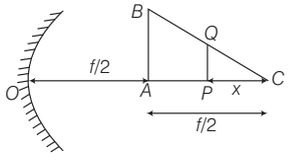
We have, $\phi = \frac{q_{\text{enclosed}}}{\varepsilon_0}$

$\Rightarrow \quad \phi = \left(\frac{\sqrt{3}\,\lambda R}{\varepsilon_0}\right)$

Also, electric field is perpendicular to wire, so Z-component will be zero.

8. *(d)* Image of point A,

$\frac{PQ}{x} = \frac{AB}{f/2} \Rightarrow PQ = \frac{2(AB)x}{f}$



For A,

$$\frac{1}{v}+\frac{1}{[-(f/2)]}=\frac{1}{-f}$$

$$\Rightarrow \quad v=f$$

$$\Rightarrow \quad \frac{I_{AB}}{AB}=-\frac{v}{u}=-\frac{f}{\left(-\frac{f}{2}\right)}$$

$$\Rightarrow \quad I_{AB}=2\,AB$$

For height of PQ,

$$\frac{1}{v}+\frac{1}{-[(f-x)]}=\frac{1}{-f}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{(f-x)}-\frac{1}{f}$$

$$\Rightarrow \quad v=\frac{f(f-x)}{x}$$

$$\Rightarrow \frac{I_{PQ}}{PQ}=-\frac{v}{u}=\frac{f(f-x)}{x[(f-x)]}=\left(\frac{f}{x}\right)$$

$$\Rightarrow I_{PQ}=\frac{f}{x}PQ=\left(\frac{f}{x}\right)\left(\frac{2\,(AB)x}{f}\right)$$

$$\left[\because PQ=\frac{2(AB)x}{f}\right]$$

$$I_{PQ}=2AB$$

(Size of image is independent of x. So, final image will be of same height terminating at infinity)

9. *(a, c)* $^{232}_{90}\text{Th}$ is converting into $^{212}_{82}\text{Pb}$.

Change in mass number $(A)=20$

$\therefore$ Number of α-particle emitted

$$=\frac{20}{4}=5$$

Due to 5 α-particles, Z will change by 10 units.

Since, given change is 8, therefore number of β-particles emitted is 2.

10. *(2)* $\therefore \quad a=\frac{F}{m}=\frac{qE}{m}=10^3\sin(10^3t)$

$$\frac{dv}{dt}=10^3\sin(10^3t)$$

$$\Rightarrow \quad \int_0^v dv=\int_0^t 10^3\sin(10^3t)\,dt$$

$$\therefore \quad v=\frac{10^3}{10^3}[1-\cos(10^3t)]$$

Velocity will be maximum when $\cos(10^3t)=-1$.

$$v_{\max}=2\text{ m/s}$$

11. *(5.55)* Given, $N=50$,

$A=2\times10^{-4}\text{ m}^2$, $C=10^{-4}$, $R=50\,\Omega$,

$B=0.02$ T, $\theta=0.2$ rad

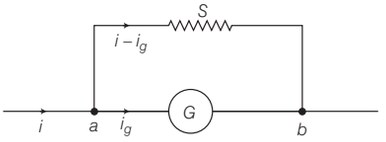
$\therefore \quad Ni_gAB=C\theta$

$$\Rightarrow i_g=\frac{C\,\theta}{N\,AB}=\frac{10^{-4}\times0.2}{50\times2\times10^{-4}\times0.02}$$

$$=0.1\text{ A}$$

$\therefore \quad V_{ab}=i_g\times G=(i-i_g)S$

$$0.1\times50=(1-0.1)\times S$$



$$5=0.9\times S$$

$$\therefore \quad S=\frac{50}{9}\ \Omega=5.55\ \Omega$$

12. *(24)* $\therefore$ Power $=nhf$

(where, n = number of photons incident per second)

Since, KE = 0, hf = work-funcition W

$$200=nW=n\,[6.25\times1.6\times10^{-19}]$$

$$\Rightarrow \quad n=\frac{200}{1.6\times10^{-19}\times6.25}$$

As photon is just above threshold frequency $\text{KE}_{\max}$ is zero and they are accelerated by potential difference of 500 V.

$$\therefore \quad \text{KE}_f=q\Delta V$$

$$\frac{P^2}{2m}=q\Delta V$$

$$\Rightarrow \quad P=\sqrt{2\,mq\,\Delta V}$$

Since, efficiency is 100%, number of electrons emitted per second = number of photons incident per second.

As, photon is completely absorbed, force exerted

$$=n(mV)=nP=n\sqrt{2mq\Delta V}$$

$$=\frac{200}{6.25\times1.6\times10^{-19}}\times$$

$$\sqrt{2\,(9\times10^{-31})\times1.6\times10^{-19}\times500}$$

$$=24$$

13. *(3)* $\Delta E_{2-1}=13.6\times Z^2\left[1-\frac{1}{4}\right]$

$$=13.6\times Z^2\left[\frac{3}{4}\right]$$

$$\Delta E_{3-2}=13.6\times Z^2\left[\frac{1}{4}-\frac{1}{9}\right]$$

$$=13.6\times Z^2\left[\frac{5}{36}\right]$$

$$\therefore \quad \Delta E_2=\Delta E_{3-2}+74.8$$

$$13.6\times Z^2\left[\frac{3}{4}\right]=13.6\times Z^2\left[\frac{5}{36}\right]+74.8$$

$$13.6\times Z^2\left[\frac{3}{4}-\frac{5}{36}\right]=74.8$$

$$Z^2=9$$

$$\therefore \quad Z=3$$

14. *(b)* List-II

(1) $E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{d^2} \quad \Rightarrow \quad E\propto\frac{1}{d^2}$

(2) $E_{\text{axis}}=\frac{1}{4\pi\varepsilon_0}\frac{2Q(2l)}{d^3}\Rightarrow \quad E\propto\frac{1}{d^3}$

(3) $E=\frac{\lambda}{2\pi\varepsilon_0 d} \quad \Rightarrow \quad E\propto\frac{1}{d}$

(4) $E=\frac{\lambda}{2\pi\varepsilon_0(d-l)}-\frac{\lambda}{2\pi\varepsilon_0(d+l)}$

$$=\frac{\lambda(2l)}{2\pi\varepsilon_0 d^2}$$

$$\Rightarrow \quad E\propto\frac{1}{d^2}$$

(5) $E=\frac{\sigma}{2\varepsilon_0}\Rightarrow E$ is independent of d

BITSAT

1. *(a)* As the inductors are in parallel, induced emf across the two inductors is the same, i.e.

$$e_1=e_2$$

$$L_1\left(\frac{di_1}{dt}\right)=L_2\left(\frac{di_2}{dt}\right)$$

On integrating both sides, we get

$$L_1\int\frac{di_1}{dt}=L_2\int\frac{di_2}{dt}$$

$$L_1i_i=L_2i_2$$

$$\Rightarrow \quad \frac{i_1}{i_2}=\frac{L_2}{L_1}$$

2. *(a)* We have, the time constant,

$$\tau=RC' \qquad \ldots(i)$$

Now, $C'=\frac{C_1C_2}{C_1+C_2}$

$$=\frac{\left(\frac{A\varepsilon_0}{d-x}\right)\left(\frac{KA\,\varepsilon_0}{x}\right)}{\frac{A\varepsilon_0}{d-x}+\frac{KA\varepsilon_0}{x}}$$

$$\therefore \quad C'=\frac{KA\varepsilon_0}{x+K(d-x)}$$

Putting the value of C' in Eq. (i), we get

$$\tau=\frac{R\,K\,A\varepsilon_0}{\frac{d}{3}-vt+k\left(d-\frac{d}{3}+vt\right)}$$

$$\left[\because x=\frac{d}{3}-vt\right]$$

Given, $A=1$ and $K=2$

$\therefore$

$$\tau=\frac{3\times2\,\varepsilon_0\times R}{d-3vt+6d-2d+6vt}$$

$$=\frac{6\,R\varepsilon_0}{5d+3vt}$$

3. *(b)* $P = 15 + 6 = 21\ \Omega$,

$$Q = \frac{8X}{8+X} + 3$$

$$R = 15 + \frac{6\times 6}{6+6} = 18\ \Omega,$$

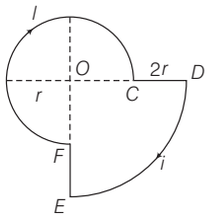
$$S = 4 + \frac{4\times 4}{4+4} = 6\ \Omega$$

As, $\frac{P}{Q} = \frac{R}{S}$, so $Q = \frac{PS}{R} = \frac{21\times 6}{18} = 7\ \Omega$

$$3 + \frac{8X}{8+X} = 7 \Rightarrow X = 8\Omega$$

4. *(d)* The angle, subtended by arc DE at O is $\frac{\pi}{2}$ and FC at $O = \frac{3\pi}{2}$ the effective magnetic field at O is

$$B = B_{DE} + B_{FC}$$

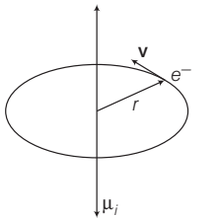


$$= \frac{\mu_0 i}{4\pi 3r}\times\frac{\pi}{2} + \frac{\mu_0 i}{4\pi r}\times\left(\frac{3\pi}{2}\right)$$

$$= \frac{\mu_0 i}{8r}\left[\frac{1}{3}+3\right]$$

$$= \frac{5\mu_0 i}{12r} \text{ acting downwards}$$

5. *(a)* The electron moving with a speed v in the circular Bohr orbit of radius r constitutes a current of magnitude.



$i = \frac{e}{T}$, where T is the orbital period of the electron.

$$T = 2\pi r / v$$

$$i = \frac{ev}{2\pi r}$$

By definition of atomic dipole moment μ_l, its magnitude is given by

$$\mu_l = iA = \frac{ev}{2\pi r}\cdot\pi r^2 = \frac{evr}{2}$$

$\therefore$ Angular momentum,
$L = mvr$

$$\therefore \quad \frac{L}{\mu_l} = \frac{2mvr}{evr} = \frac{2m}{e} = \text{constant}$$

6. *(b)* At resonance, $X_L = X_C$

or $$\omega L = \frac{1}{\omega C}$$

Voltage across the series L-C combination,

$$V_2 = i\,(X_L - X_C) = 0$$

7. *(a)* On raising the temperature of the semiconductor, covalent bonds start breaking up and thus more charge carriers release. This will increase the value of n and as a result the rate of collision of charge carriers will increase. This will decrease the drift speed v.

8. *(b)* Since, the lens is made up of two kinds of transparent material, it has two refractive indices for the incident beam of light. Hence, there will be two focal lengths of the lens and therefore two images will be observed.

9. *(c)* Since, the initial activity is 50 times the activity for safe occupancy, therefore $R_0 = 50\,R$, where $R = \lambda N$.

Since, $R \propto N$

$$\frac{R}{R_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T}$$

or $$\left(\frac{1}{2}\right)^{t/10} = \frac{1}{50}, (2)^{t/10} = 50$$

$$\Rightarrow \quad \frac{t}{10}\log_{10}2 = \log_{10}50$$

$$\Rightarrow \quad t = \frac{10\log_{10}50}{\log_{10}^2}$$

$$= \frac{10\times 1.699}{0.301} = 56.4 \text{ days}$$

10. *(a)* Using Einstein's photoelectric equations,

$$h\nu = \phi + eV_s$$

$$eV_s = h\nu - \phi \qquad \ldots (i)$$

Thus, when the frequency of incident light radiation (photons) ν increases to 2ν, the stopping potential V_s changes to V_s'.

$$eV_s' = h(2\nu) - \phi \qquad \ldots (ii)$$

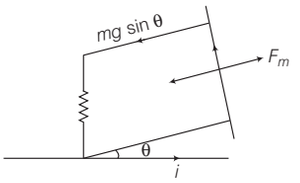
On dividing Eq. (ii) by Eq. (ii), we get

$$\frac{V_s'}{V_s} = \frac{2h\nu - \phi}{h\nu - \phi} = \frac{2h\nu - 2\phi + \phi}{h\nu - \phi}$$

$$= \frac{2(h\nu - \phi) + \phi}{h\nu - \phi} = 2 + \frac{\phi}{h\nu - \phi}$$

$$\therefore \quad \frac{V_s'}{V_s} > 2 \Rightarrow V_s' > 2V_s$$

11. *(c)* Terminal velocity of the rod is attained when magnetic force on the rod (Bil) balances the component of weight of the rod ($mg\sin\theta$), as shown in figure.



So, $Bil = mg\sin\theta$

$$B\left(\frac{e}{R}\right)l = mg\sin\theta$$

$$\frac{Blev}{R} = mg\sin\theta$$

$$\frac{Bl(Blv_T)}{R} = mg\sin\theta$$

$$v_T = \frac{mg\sin\theta R}{B^2 l^2}$$

12. *(b)* From Einstein photoelectric equation,

$$E = \phi_0 + \text{KE}_{\max}$$

For metal A,

$$4 = \phi_A + T_A \qquad \ldots (i)$$

For metal B,

$$4.5 = \phi_B + (T_A - 1.5) \qquad \ldots (ii)$$

From Eqs. (i) and (ii), we get

$$\phi_B - \phi_A = 2$$

Now, according to de-Broglie hypothesis,

$$\lambda_A = \frac{h}{mv} = \frac{h}{\sqrt{2mT_A}}$$

Similarly, $\lambda_B = \frac{h}{\sqrt{2mT_B}}$

$$\therefore \quad \frac{\lambda_A}{\lambda_B} = \sqrt{\frac{T_B}{T_A}} = \sqrt{\frac{T_A - 1.5}{T_A}}$$

$$= \left(1 - \frac{1.5}{T_A}\right)^{1/2}$$

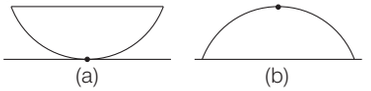
$$\left(\frac{1}{2}\right)^2 = 1 - \frac{1.5}{T_A}$$

On solving, $T_A = 2.0$ eV

So, $\phi_A = 4 - T_A = 4 - 2 = 2.0$ eV

$\phi_B = 6 - T_A = 6 - 2 = 4.0$ eV

13. *(c)*



In first case refraction of the rays, takes place from a plane surface, so we can use $d_{\text{app}} = \frac{d_{\text{actual}}}{\mu}$

$$3 = \frac{4}{\mu} \Rightarrow \mu = \frac{4}{3}$$

where, d_{app} and d_{actual} = apparent and real depth, respectively.

Now, in second case, as from Fig. (b), refraction takes place from a spherical surface, so

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or $$\frac{1}{(-25/8)} - \frac{4/3}{(-4)} = \frac{1 - 4/3}{-R}$$

or $$\frac{1}{3R} = \frac{1}{3} - \frac{8}{25} = \frac{1}{75}$$

$$R = 25 \text{ cm}$$

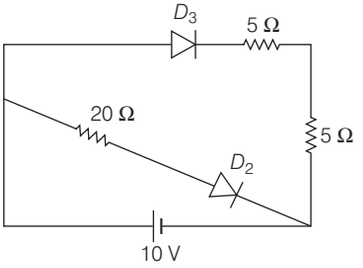
Now, using lens' maker's formula, to calculate focal length

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{\infty} - \frac{1}{(-25)}\right) = \frac{1}{75}$$

$$f = 75 \text{ cm}$$

14. *(c)* In the given circuit, diode D_1 is reverse biased, so it will not conduct but D_2, D_3 are forward biased, so they will conduct, hence corresponding equivalent circuit. For the given circuit is



Now, the equivalent resistance of the circuit is

$$R_{eq} = \frac{(5+5)\times 20}{(5+5)+20} + \frac{20}{3}\ \Omega$$

Current through battery,

$$I = \frac{10 \text{ V}}{\frac{20}{3}\ \Omega} = 1.5 \text{ A}$$

15. *(d)* New current sensitivity,

$$I_s' = I_s + \frac{50}{100} I_s$$

$$= \frac{150}{100} I_s = \frac{3}{2} I_s$$

New resistance, $R' = 2R$

Initial voltage sensitivity, $V_s = \frac{I_s}{R}$

Now, new voltage sensitivity,

$$V_s' = \frac{I_s'}{R'} = \frac{\frac{3}{2} I_s}{2R} = \frac{3}{4} V_s$$

% decrease in voltage sensitivity

$$= \frac{V_s - V_s'}{V_s} \times 100$$

$$= \left(1 - \frac{V_s'}{V_s}\right) \times 100$$

$$= \left(1 - \frac{3}{4}\right) \times 100 = 25\%$$

16. *(b)* From the given circuit, we can say that I is independent of resistance R_5, so no current flows through R_5. This required that the junction of R_1 and R_2 is at the same potential as the junction of R_3 and R_4
($\therefore$ Wheatstone bridge condition)

So, $$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

or $$R_1 R_4 = R_3 R_2$$

KCET

1. *(d)* Given, $T_{1/2} = 12.5$ years

Initial mass, $N_0 = 64$ mg

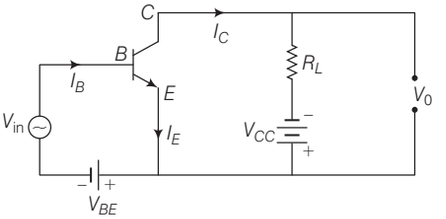
Number of half lives, $n = \frac{50}{12.5} = 4$

Now, $$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad \frac{N}{64} = \left(\frac{1}{2}\right)^4$$

$$\Rightarrow \quad N = 64 \times \frac{1}{16} = 4 \text{ mg}$$

2. *(a)* The circuit diagram for CE amplifier is given below



Input signal is applied across base-emitter junction.

3. *(c)* $A = 1, B = 0 \Rightarrow \overline{B} = 1$

$\therefore \quad A + \overline{B} = 1 + 1 = 1 = A$

4. *(b)* $n_i = 3 \times 10^{16}\ \text{m}^{-3}$

$n_h = 4.5 \times 10^{22}\ \text{m}^{-3}$

$n_e = ?$

$n_e n_h = (n_i)^2$

$$\Rightarrow \quad n_e = \frac{(n_i)^2}{n_h} = \frac{(3\times 10^{16})^2}{4.5\times 10^{22}}$$

$$= \frac{9\times 10^{32}}{4.5\times 10^{22}} = 2\times 10^{10}$$

5. *(a)* Given, $\beta = 50$, $V_{CE} = 2$ V,

$R_C = 4\ \text{k}\Omega$

$$I_C = \frac{V_{CE}}{R_C} = \frac{2}{4\times 10^3} = 5\times 10^{-4}$$

$$I_C = 0.5 \text{ mA}$$

DC current gain, $\beta = \frac{I_C}{I_B}$

$$I_B = \frac{I_C}{\beta} = \frac{0.5\times 10^{-3}}{50} = 10^{-5} \text{ A}$$

Hence, $I_B = 10\ \mu\text{A}$

6. *(a)* Dimensions of magnetic flux,

$$\phi = [\text{ML}^2\text{T}^{-2}\text{A}^{-1}]$$

Dimensions of permeability,

$\mu = [\text{MLT}^{-2}\text{A}^{-2}]$

$$\therefore \quad \frac{[\phi]}{[\mu]} = \frac{[\text{ML}^2\text{T}^{-2}\text{A}^{-1}]}{[\text{MLT}^{-2}\text{A}^{-2}]}$$

$$= [\text{M}^0\text{L}^1\text{T}^0\text{A}^1]$$

7. *(a)* Given, $E = 2$ N/C,

$r = 30 \text{ cm} = 0.3 \text{ m}$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} = 9\times 10^9 \frac{q}{r^2}$$

$$\Rightarrow q = \frac{E r^2}{9\times 10^9} = \frac{2\times(0.3)^2}{9\times 10^9} = 2\times 10^{-11} \text{ C}$$

8. *(b)* Given, $m = 1$ kg, $q = 2$ C, $V = 1$ V

$$qV = \frac{1}{2} m v^2$$

$$\Rightarrow \quad 2\times 1 = \frac{1}{2}\times 1 \times v^2$$

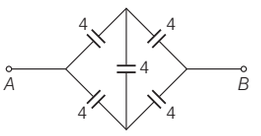
$$v = 2 \text{ ms}^{-1}$$

9. *(c)* $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r^2}$

When dielectric is kept, then

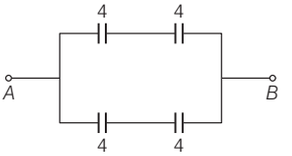
$$F' = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{q^2}{r^2} = \frac{F}{K} = \frac{F}{4}$$

10. *(a)* The circuit can be redrawn as



This is a balanced Wheatstone bridge.

So, the circuit can be redrawn as



11. *(b)* When the two capacitors 3 μF and 6 μF are connected in series, then

$$\frac{1}{C_s} = \frac{1}{3} + \frac{1}{6}$$

$$C_s = 2\ \mu\text{F} = 2\times 10^{-6} \text{ F}$$

Hence, the charge on capacitors,

$$C_s V_1 = q_1 = 2 \times 900 \times 10^{-6}$$

$$= 1800 \times 10^{-6} \text{ C}$$

Now, when connected in parallel,

$$C_p = 3 + 6 = 9 \ \mu\text{F} = 9 \times 10^{-6} \text{ F}$$

When they are reconnected in parallel,

$$q_2 = 1800 \times 10^{-6} = C_p V = 9 \times 10^{-6} \times V$$

$$\Rightarrow \quad V = \frac{1800 \times 10^{-6}}{9 \times 10^{-6}} = 200 \text{ V}$$

Hence, V_1, $V_2 = 100$ V each

12. *(d)* Ohm's law is obeyed only by conductors, i.e. metals and not semiconductors like diode, transistors, etc.

13. *(b)* The value of tolerance of a carbon resistor is given below
Gold = 5%, Silver = 10%,
No colour = 20%

14. *(c)* In the given circuit diagram,
$3\,\Omega$ and $3\,\Omega$ are in series, so

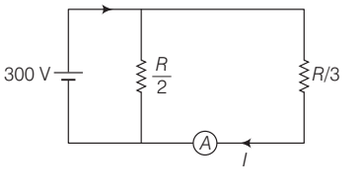
$$3 + 3 = 6\,\Omega$$

$6\,\Omega$ and $6\,\Omega$ are in parallel, so

$$\frac{6 \times 6}{6 + 6} = 3\,\Omega$$

This $3\,\Omega$ is in series with $4\ \Omega$ and $5\ \Omega$, so $4 + 3 + 5 = 12\,\Omega$

15. *(b)* The circuit can be reduced as follows



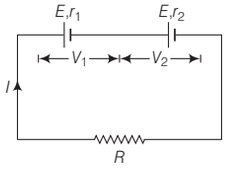
Reading of ammeter = I

$$= \frac{300}{R/3} = \frac{900}{R}$$

As, $R = 1500\ \Omega$,

$$I = \frac{3}{5} \text{ A}$$

16. *(c)* Given, $V_1 = 0$



$$V_1 + V_2 = IR$$

$$\Rightarrow \quad V_2 = IR$$

$$E - Ir_2 = IR$$

But $V_1 = E - Ir_1$

i.e. $E = Ir_1$ (as $V_1 = 0$)

$$Ir_1 = Ir_2 = IR$$

or $R = r_1 - r_2$

17. *(b)* The slope of V-I graph
= resistance.

The graph is plotted with I on Y-axis and V on X-axis, hence the slope $= \frac{1}{R}$.

So, if R_P and R_Q be the resistances of the devices, then $R_P > R_Q$.

18. *(b)* Biot-Savart law is given by

$$d\text{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\, d\text{l} \times \text{r}}{r^3}$$

19. *(b)* As an electron moving in a circle of radius r in a uniform magnetic field B, with velocity u,

Hence, $v = \frac{qBr}{m}$

Now, for the first case when magnetic field is B_1,

$$v_1 = \frac{qB_1 r}{m}$$

$$\Rightarrow \quad r_1 = \frac{mv}{qB_1} \qquad \ldots(\text{i})$$

Now, when the field is suddenly reduced to half the value, then

$$B_2 = \frac{B_1}{2}$$

Hence, $r_2 = \frac{mv}{qB_2}$ [from Eq. (i)]

$$= \frac{2mv}{qB_2} = 2r_1$$

20. *(a)*

$$qE = q\, v\, B$$

$$\frac{V}{r} = vB$$

$$V = r\, v\, B$$

$$V' = (2r)v\, B = 2\, V$$

21. *(c)* $\text{KE} = \frac{q^2 B^2 R^2}{2\, m}$

Given, $f = 10$ MHz $= 10^7$ Hz, $B = 0.66$ T
$R = 60$ cm $= 0.6$ m

$$f = \frac{qB}{2\pi m}$$

$$\therefore \quad \text{KE} = \frac{qB}{2\pi m} \times \pi \times qBR^2$$

$$= f \times \pi \times qBR^2$$

$$= 10^7 \times 3.14 \times 1.6 \times 10^{-19} \times 0.66 \times 0.6^2 \text{ J}$$

$$= 7 \text{ MeV}$$

22. *(b)* Ferromagnetic substance are strongly magnetised. Paramagnetic substance is weakly magnetised. Dimagnetic substance repels.

23. *(d)* To a first approximation, the Earth's magnetic field resembles a huge bar magnet. The field lines emerges from the southern half to northern half.

The physical quantities, i.e. magnetic elements determine the strength of magentic field. As magnetic declination, inclination or angle of dip and horizontal components of earth's magnetic field change from place to place hence strength of the earth's magnetic field varies from place to place.

As angle of dip (δ) of a place changes its strength also changes.

24. *(b)* Given, $B = 4 \times 10^{-4}$ T, $l = 25$ m,
$v = 3600$ km/h $= 1000$ m/s, $\theta = 30^\circ$

$$\therefore \quad E = B\, lv \sin\theta$$

$$= 4 \times 10^{-4} \times 25 \times 1000 \times \sin 30 = 5 \text{ V}$$

25. *(a)* $X_L = \omega L = 2\pi\nu L$

$$\therefore \quad X_L \propto \nu$$

26. *(b)* $\varepsilon = \frac{d\phi}{dt} = 6t + 4$

At $t = 2$, $\varepsilon = 6 \times 2 + 4 = 16$ V

27. *(a)* Given, $P = 100$ W, $V = 220$ V

$$\therefore \quad I = \frac{P}{V} = \frac{100}{220} = \frac{5}{11} \text{ A}$$

28. *(a)* In a series L-C-R circuit, the average power $P = V / \cos\ \phi$. This power dissipated only in the resistor.

29. *(b)* If a AC power supply is rated as 220 V, 50 Hz, then 220 V represents the rms value of voltage and 50 Hz is the frequency of the AC.

30. *(b)* Given, $\frac{N_P}{N_S} = \frac{1}{25} = \frac{I_S}{I_P}$

$$\Rightarrow \quad I_P = I_S \times 25 = 2 \times 25 = 50 \text{ A}$$

31. *(b)* As, $P = n \cdot \frac{hc}{\lambda}$ or $F \times c = n \cdot \frac{hc}{\lambda}$

[as power (P) = force × speed]

$$= F \times c$$

or $n = \frac{F\lambda}{h} = \frac{6.62 \times 10^{-5} \times 5 \times 10^{-7}}{6.62 \times 10^{-34}}$

$$= 5 \times 10^{22} = 5$$

32. *(c)* The focus of a convex mirror is behind the mirror. Image is formed only when object is kept infront of mirror.
Image will be formed at infinity.

33. *(c)* Given, $f = 10$ cm, $u = -20$ cm

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} + \frac{1}{20} = \frac{-1}{20}$$

$$\therefore \quad v = -20 \text{ cm}$$

34. *(a)* Given, $u = -25$ cm, $v = +75$ cm, $f = ?$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{75} - \frac{1}{(-25)} = \frac{4}{75}$$

$$f = \frac{75}{4} = 18.75 \text{ cm}$$

$\because f$ is positive, it is a convex lens.

35. *(b)* Angular width $= \frac{2\lambda}{a}$

36. *(d)* Bandwidth $\propto \lambda$

$\lambda_{blue} < \lambda_{yellow} \Rightarrow$ Bandwidth becomes narrower when yellow light is replaced by blue light.

37. *(c)* $\frac{nD\lambda_1}{d} = \frac{(n+1)D\lambda_2}{d}$

$$n\lambda_1 = (n+1)\lambda_2$$

$$\Rightarrow 780n = (n+1)520$$

$$n = \frac{520}{260}$$

$$\Rightarrow n = 2$$

38. *(a)* $y_4 = \frac{4D\lambda}{d}$

Separation between the fourth bright fringes

$$= \frac{4D\lambda_1}{d} - \frac{4D\lambda_2}{d}$$

$$\frac{4D}{d}(\lambda_1 - \lambda_2)$$

$$= \frac{4 \times 1.2}{2 \times 10^{-3}}(6500 - 5200) \times 10^{-10}$$

$$= 3120 \times 10^{-7}$$

$$= 0.312 \text{ mm}$$

39. *(b)* $\text{KE} = \frac{1}{2}mv^2_{max} = h(\nu - \nu_0)$

$\text{KE}_{max} = h\nu - h\nu_0$

$\therefore \text{KE}_{max}$ depends on frequency of incident radiation.

40. *(b)* $\lambda = \frac{h}{\sqrt{2meV}}$

As V is same.

$$\lambda \propto \frac{1}{\sqrt{me}}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{\sqrt{m_\alpha e_\alpha}}{\sqrt{m_p e_p}} = \frac{\sqrt{4m_p \times 2e_p}}{\sqrt{m_p e_p}}$$

$$= 2\sqrt{2}$$

41. *(c)* $E = \frac{-13.6}{n^2}$ eV

For $n = 2$,

$E = \frac{-13.6}{2^2} = -3.4$ eV

42. *(a)* $T = \frac{nh\nu}{e^2}$

$\Rightarrow T \propto n$

In ground state, $T_1 = T$

In first excited state, $T_2 = 2T$

AP EAMCET

1. *(a)* Equivalent resistance is

$$R_{parallel} = \frac{R_1R_2}{R_1 + R_2} = \frac{60 \times 30}{60 + 30} = 20\,\Omega$$

and tolerance value is

$$\Delta R_P = R_P\left\{\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} - \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}\right\}$$

$$= 20\left\{\frac{0.36}{60} + \frac{0.09}{30} - \frac{0.36 + 0.09}{90}\right\}$$

$$= 20\{0.006 + 0.003 - 0.005\} = 0.085\Omega$$

So, resistance in parallel is

$R_P = 20 \pm 0.08$

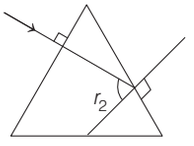
2. *(a)* Given, $i = 0°$

So, $\frac{\sin i}{\sin r_1} = \sqrt{2}$

$$\frac{\sin 0°}{\sin r_1} = \sqrt{2}$$

$$\sin r_1 = 0$$

or $r_1 = 0$



Now, $\frac{\sin r_2}{\sin e} = \frac{1}{\mu} \Rightarrow \sin e = \sqrt{2}\sin r_2$

But, $r_1 + r_2 = 60°$

So, $\sin e = \sqrt{2}\sin 60° = \sqrt{2} \times \sqrt{3}/2 > 1$

So, light is incidenting at more than critical angle and totally internally reflected.

$\therefore$ Deviation angle $= (i + e) - (r_1 + r_2)$

$= 0 - 60° = 60°$ (in magnitude)

3. *(a)* When unpolarised light passes through first polariser, it becomes polarised and its intensity becomes half.

$\therefore$ After first polariser, intensity

$I_1 = I_0/2$

After second polariser, intensity

$I_2 = \frac{I_0}{2}\cos^2\theta$ (Malus law)

After third polariser, intensity

$$I_3 = \frac{I_0}{2}\cos^2\theta\cos^2(90° - \theta)$$

(because this is at 90° angle from first polariser)

$$\Rightarrow I_3 = \frac{I_0}{2}\cos^2\theta\sin^2\theta = \frac{I_0}{8}(2\sin\theta\cos\theta)^2$$

$$= \frac{I_0}{8}\sin^2 2\theta$$

4. *(d)* When dielectric of thickness t is introduced in two charges at distance r, the effective force between the charges is given by

$$F = \frac{q_1q_2}{4\pi\varepsilon_0[r - t + t\sqrt{K}]^2}$$

where, K = dielectric constant of medium

In first case, $t = r/2$ and $K = 4$

$$\therefore F_1 = \frac{q_1q_2}{4\pi\varepsilon_0\left[r - r/2 + \frac{r}{2}\sqrt{4}\right]^2}$$

$$= \frac{q_1q_2}{4\pi\varepsilon_0\frac{9}{4}r^2} = \frac{q_1q_2}{9\pi\varepsilon_0 r^2}$$

In second case, $t = r/3$ and $K = 9$

$$\therefore F_2 = q_1q_2/4\pi\varepsilon_0\left[r - \frac{r}{3} + \frac{r}{3}\sqrt{3}\right]^2$$

$$= \frac{q_1q_2}{4\pi\varepsilon_0\left(\frac{25}{9}\right)r^2}$$

$$\therefore \frac{F_1}{F_2} = \frac{q_1q_2}{9\pi\varepsilon_0 r^2} \times \frac{4\pi\varepsilon_0\left(\frac{25}{9}\right)r^2}{q_1q_2} = \frac{100}{81}$$

5. *(d)* $C_{air} = 80\,\mu\text{F}$

$C_{dielectric} = \varepsilon_r C_{air} = 1600\,\mu\text{F}$

Charge stored in the presence of air,

$q_{air} = C_{air} \times V = 80 \times 30 \times 10^{-6}$

$= 2400\,\mu\text{C}$

Charge stored in presence of dielectric medium,

$q_d = C_{dielectric} \times V = 1600 \times 30 \times 10^{-6}$

$= 48000\,\mu\text{C}$

When dielectric is removed, effective charge remained is

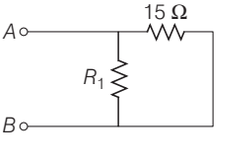
$$q = q_d - q_{air}$$

$$= (48 - 2.4) \times 10^{-3}\text{C}$$

$$q = 45.6 \times 10^{-3}\text{C}$$

6. *(a)* Total charge in the capacitors (accumulated at point O)

$Q_{total} = C_1V_1 + C_2V_2 + C_3V_3$



Total capacitance of the combination,

$C_{total} = C_1 + C_2 + C_3$ (w.r.t. point O)

So, potential at O,

$$V = \frac{Q_{total}}{C_{total}} = \frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3}$$

7. *(b)* Here, 2Ω and 3Ω resistances are short circuited, so circuit can be reduced as

$$R_{AB} = \frac{15R_1}{15 + R_1} = 6$$

$$\Rightarrow 15R_1 - 6R_1 = 15 \times 6$$

$$\Rightarrow 9R_1 = 90$$

$$\Rightarrow R_1 = 10\ \Omega$$

8. *(c)* Given $r = 10\,\Omega$, $R = 10\,\Omega$, $E = 10$ V and $L = 100$ cm.

Now, voltage drop on the wire,

$$E' = \frac{E}{(r + R)} \times R$$

$$\Rightarrow \quad E' = \frac{10}{20} \times 10 = 5\,R$$

So, potential gradient on wire,

$$x = E'/L = \frac{5}{100}\ \text{V/cm} \qquad \ldots (i)$$

Effective emf of combination in secondary circuit,

$$\frac{V}{r_{\text{eff}}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

Here, $$\frac{1}{r_{\text{eff}}} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$\Rightarrow \qquad V = \frac{6}{5}(1 + 1) = \frac{12}{5}$$

Now, at balancing point (from point A) is

$$l = V/x = \frac{12}{5} \times \frac{100}{5}$$

$$l = 48\ \text{cm}$$

So, length from point B is

$$100 - 48 = 52\ \text{cm}$$

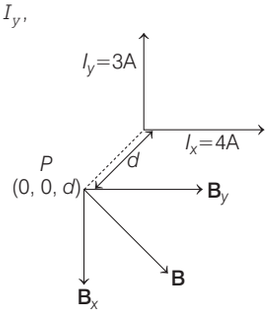
9. *(d)* Magnetic field at point P due to I_x,

$$\mathbf{B}_x = \frac{\mu_0 I_x}{2\pi d}$$

($-$ y-direction)

or $$\mathbf{B}_x = -\frac{\mu_0 \cdot 4}{2\pi d}\,\hat{\mathbf{j}}$$

Similarly, magnetic field at point P due to I_y,



$$\mathbf{B}_y = \frac{\mu_0 I_y}{2\pi d}$$

(X-direction)

$$\Rightarrow \qquad \mathbf{B}_y = \frac{\mu_0 \cdot 3}{2\pi d}\,a\,\hat{\mathbf{i}}$$

$\therefore$ Resultant magnetic field,

$$B = \sqrt{B_x^2 + B_y^2}$$

$$= \frac{\mu_0}{2\pi d}\sqrt{4^2 + 3^2}$$

$$B = \frac{5\mu_0}{2\pi d}\ (\text{at} - 45° \text{ from } X\text{-axis})$$

10. *(a)* There is no information regarding spring constant.

So, let's assume their spring constants are same.

Current sensitivity, $I_S = \dfrac{N\,BA}{k}$

So, $$\frac{I_{S_1}}{I_{S_2}} = \frac{N_1\,B_1\,A_1\,k_2}{N_2\,B_2\,A_2\,k_1}$$

$$= \frac{30 \times 0.25 \times 4.8 \times 10^{-3}}{45 \times 0.5 \times 2.4 \times 10^{-3}} \times \left(\frac{1}{1}\right)$$

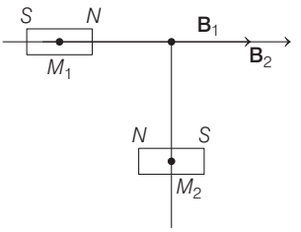
$$\frac{I_{S_1}}{I_{S_2}} = 2/3$$

Voltage sensitivity, $V_S = NBA/Rk$

So, $$\frac{V_{S_1}}{V_{S_2}} = \frac{N_1 B_1 A_1 k_2 R_2}{N_2 B_2 A_2 k_1 R_1}$$

$$= \frac{2}{3} \times \left(\frac{R_2}{R_1}\right) = \frac{2}{3} \times \frac{14}{10} = \frac{14}{15}$$

11. *(c)* At origin, both magnetic fields will be in same direction.



Magnetic field due to M_1 (axial point),

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{2M}{r^3}$$

$$= 10^{-7} \times \frac{2 \times 9}{27 \times 10^{-6}}$$

$$B_1 = \frac{2}{3} \times 10^{-1}\,\text{T}$$

Magnetic field due to M_2 (equatorial point),

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{M}{r^3} = 10^{-7} \times \frac{9}{27 \times 10^{-6}}$$

$$= \frac{1}{3} \times 10^{-1}\,\text{T}$$

As both B_1 and B_2 points in same direction, so effective field,

$$B = \left(\frac{1}{3} + \frac{2}{3}\right) \times 10^{-1}\,\text{T}$$

$$\Rightarrow \quad B = 10^{-1}\,\text{T} = 0.1\ \text{T}$$

12. *(a)* Charge on each plate should be

$$q = CV$$

where, $$V = vBl$$

$$\Rightarrow \qquad q = C(vBl)$$

$$= 10 \times 10^{-6} \times 2 \times 4 \times 1$$

$$q = 80\ \mu\text{C}$$

So, charges on plates are $\pm$ 80 μC.

13. *(a)* In the shown figure, current is ahead of voltage, so its a RC circuit, so P is a resistor and Q is a capacitor.

Now, in RC circuit,

$$Z = \sqrt{R^2 + X_C^2}$$

For simple solution,

Given, $$Z = 1414 = 1000 \times 1.414$$

$$= 1000\sqrt{2}$$

$$= 1000\sqrt{1 + 1}$$

$$= \sqrt{(1000)^2 + (1000)^2}$$

$$\Rightarrow \qquad X_C = 1000\,R = 1000\ \Omega$$

Now, $$X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C}$$

$$\Rightarrow \qquad C = \frac{1}{100 \times 1000} = 10\ \mu\text{F}$$

14. *(c)* Energy density due to electric field is

$$U = \frac{1}{2}\varepsilon_0 E^2$$

Here, $$E = \frac{E_0}{\sqrt{2}} = \frac{40}{\sqrt{2}}$$

$$\therefore U = \frac{1}{2} \times 8.85 \times 10^{-12} \times \frac{40 \times 40}{2}$$

$$= 3.54 \times 10^{-9}\ \text{J/m}^3$$

15. *(a)* Energy of photons of

$$\text{H}_\alpha \text{ line} = \Delta E(3 \to 2) = 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$= \frac{13.6 \times 5}{36} = 1.89\ \text{eV}$$

Energy of photons of H_β line

$$= \Delta E(4 \to 2)$$

$$= 13.6\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$= \frac{12 \times 13.6}{64} = 2.55\ \text{eV}$$

Energy of photons of H_∞ line

$$= \Delta E(\infty \to 2)$$

$$= \frac{13.6}{4} = 3.4\ \text{eV}$$

Ratio of kinetic energy of emitted photons

$$= \frac{2.55 - 1.89}{3.4 - 1.89} = \frac{0.66}{1.51} \approx \frac{0.7}{1.6} \approx \frac{7}{16}$$

16. *(d)* Angular momentum of electron in n th orbit of hydrogen is

$$L = mvr = \frac{nh}{2\pi}$$

de-Broglie wavelength,

$$\lambda = \frac{h}{mv} \Rightarrow \frac{mv}{h} = \frac{1}{\lambda}$$

$$\Rightarrow \quad \frac{r}{\lambda} = \frac{n}{2\pi} \Rightarrow \lambda = \frac{2\pi r}{n}$$

Now, $r \propto n^2$ (Bohr radius, $r = 0.529 \times n^2$ Å)

$\Rightarrow \quad \lambda \propto n$

17. *(a)* Number of nuclei

$$= \frac{E_{\text{total}}}{E_1} = \frac{1000\text{J}}{200\text{MeV}}$$

$$= \frac{1000\text{ J}}{200 \times 10^6 \times 1.6 \times 10^{-19}\text{J}}$$

$\Rightarrow \quad N = \dfrac{10^{14}}{3.2}$

$\Rightarrow \quad N = 3.125 \times 10^{13}$

18. *(c)* Diode, D_1 = reverse biased (OFF) and diode, D_2 = forward biased (ON). Because diodes are ideal, so voltage drop across D_2 is zero.

$\therefore$ Effective resistance,

$R = 3 + 2 + 3 + 2 = 10\,\Omega$

Current through the cell,

$I = V / R = 20 / 10 = 2\text{A}$

19. *(c)* Modulation index, $M = \dfrac{V_{\text{peak, signal}}}{V_{\text{peak, carrier}}}$

Let's write $V_{\text{peak, carrier}} = V$

In first case, $0.6 = \dfrac{12}{V}$

$\Rightarrow \quad V = 20\text{ V}$

In second case, $0.75 = \dfrac{12}{V'}$

$\Rightarrow \quad V' = \dfrac{12}{0.75} = 16\text{ V}$

Change in peak voltage of carrier wave

$\Delta V = 20 - 16 = 4\text{V}$

% change $= \dfrac{4}{20} \times 100\% = 20\%$

(decrement)

TS EAMCET

1. *(b)* We know that, lens formula,

$$\frac{1}{f} = \frac{1}{u} - \frac{1}{v}$$

Magnification, $m = -\dfrac{v}{u} = -4$

$\Rightarrow \quad v = +4u$

$\therefore \quad \dfrac{1}{+8} = \dfrac{1}{-u} - \dfrac{1}{4u}$

($-u$, because of sign convention)

$\Rightarrow \quad \dfrac{5}{4u} = -\dfrac{1}{8}$

$\Rightarrow \quad u = \dfrac{-40}{4} = -10\text{ cm}$

2. *(c)* Image of lens will act as object for mirror.

Here, $v_1 + u_2 = 40$ cm

Lens formula, $\dfrac{1}{f_1} = \dfrac{1}{u_1} - \dfrac{1}{v_1}$

Mirror formula, $\dfrac{1}{f_2} = \dfrac{1}{u_2} + \dfrac{1}{v_2}$

For final image, $v_2 = \infty$

So, $\dfrac{1}{f_2} = \dfrac{1}{u_2}$

$\Rightarrow \quad u_2 = 15\text{ cm}$

Now, $v_1 = 40 - 15 = 25$ cm

So, value of u_1 should be

$$\frac{1}{20} = \frac{1}{u_1} - \frac{1}{25}$$

$\Rightarrow \quad \dfrac{1}{u_1} = \dfrac{1}{20} + \dfrac{1}{25}$

$\Rightarrow \quad u_1 = 100 / 9$ cm

$\Rightarrow \quad u_1 = 11.11$ cm

$\simeq 12$ cm

(most near by answer)

3. *(c)* Position of bright fringe on screen is given by

$x = \dfrac{n\lambda D}{d}$ [given, $x = 2.5$ mm]

For coinciding case, $n_1\lambda_1 = n_2\lambda_2$

$\therefore$ $n = 6$ for wavelength of 500 nm and $n = 5$ for wavelength of 600 nm.

Using any one of those,

$$2.5 \times 10^{-3} = \frac{6 \times 500 \times 10^{-9} \times D}{3 \times 10^{-3}}$$

$\Rightarrow \quad D = 2.5\text{ m}$

4. *(a)* Distance between point charge and observation point is

$\mathrm{x} = \mathbf{r} - \mathbf{r}_0 = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$

Magnitude of $x = x = \sqrt{6^2 + 8^2}$

$\Rightarrow \quad x = 10\text{ m}$

$\therefore$ Electric field, $E = \dfrac{Kq}{x^2}$

$$= \frac{9 \times 10^9 \times 50 \times 10^{-6}}{10 \times 10} = 4.5\text{ kV/m}$$

5. *(c)* Work done,

$$W = U_{12} + U_{23} + U_{13}$$

$$= \frac{kq_1q_2}{r_{12}} + \frac{kq_2kq_3}{r_{23}} + \frac{kq_1q_3}{r_{13}}$$

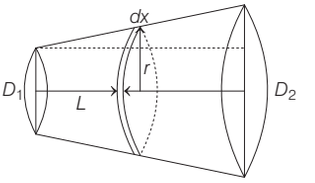
$$= \frac{1}{4\pi\varepsilon_0 a}[-2q^2 + 2q^2 - q^2]$$

$$= -\frac{q^2}{4\pi\varepsilon_0 a}$$

6. *(b)* Statement B is not true because $R_{\text{eff}} = 20\,R$ (Let's assume R is resistance of one bulb)

So, $i = \dfrac{V}{R_{\text{eff}}} = \dfrac{V}{20\,R}$ which will be minimum.

7. *(d)* Similarity of triangles,



$$\frac{r - a}{x} = \frac{b - a}{l}$$

(here, $a = D_1 / 2$, $b = D_2 / 2$)

$\Rightarrow \quad r = a + \left(\dfrac{b - a}{L}\right)x$

Resistance of elemental disc is

$$dR = \frac{\rho dx}{\pi r^2} = \frac{\rho dx}{\pi\left[a + \left(\dfrac{b - a}{L}\right)x\right]^2}$$

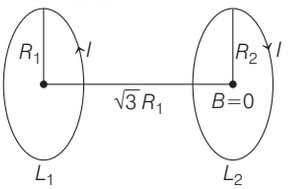
Total resistance of tapered wire is

$$R = \int dR = \int_0^l \frac{\rho dx}{\pi\left[a + \left(\dfrac{b - a}{L}\right)x\right]^2}$$

$$\Rightarrow R = \frac{\rho}{\pi}\left[-\frac{1}{\left[a + \left(\dfrac{b - a}{L}\right)x\right]}\frac{L}{(b - a)}\right]_0^L$$

$$= \frac{\rho}{\pi} \times \frac{L}{b - a}\left[\frac{1}{a} - \frac{1}{b}\right] = \frac{\rho L}{\pi\, ab} = \frac{4\,\rho L}{\pi\, D_1 D_2}$$

8. *(d)* Magnetic field at centre of L_2 will be zero when



$B_{L_1} = B_{L_2}$ (magnitude)

$$\frac{\mu_0 I R_1^2}{2(R_1^2 + x^2)^{3/2}} = \frac{\mu_0 I}{2R_2}$$

Here, $x = \sqrt{3}R_1$

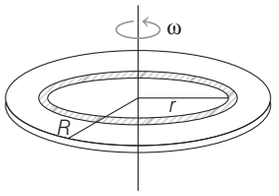
$\therefore \quad \dfrac{R_1^2}{2(R_1^2 + 3R_1^2)^{3/2}} = \dfrac{1}{2R_2}$

$\Rightarrow \quad \dfrac{R_1^2}{8R_1^3} = \dfrac{1}{R_2}$

$\Rightarrow \quad R_2 = 8R_1$

9. *(a)* Total charge on the disc, $Q = \sigma(\pi R^2)$

Period of motion, $T = 2\pi/\omega$



So, current in the ring, $dI = \frac{dQ}{T}$

$\because \quad Q = \sigma \pi R^2$

$\Rightarrow \quad dQ = 2\sigma \pi r dr$

$\therefore \quad dI = \frac{2\sigma \pi r dr}{2\pi} \times \omega = \sigma \omega r\, dr$

Magnetic field due to ring is

$$dB = \frac{\mu_0 dI}{2r}$$

So, magnetic field due to disk is

$$B = \int dB = \int_0^R \frac{\mu_0}{2r} \cdot \sigma \omega r\, dr$$

On putting value of σ, we get

$$B = \frac{\mu_0 \sigma_0 \omega}{2} \int_0^R \left[1 + \left(\frac{r}{R}\right)^{\beta}\right] dr$$

$$= \frac{\mu_0 \sigma_0 \omega}{2}\left[R + \frac{R^{\beta+1}}{R^{\beta}(1+\beta)}\right] = \frac{\mu_0 \sigma_0 \omega}{2} \times \left(\frac{2+\beta}{1+\beta}\right) R$$

Now,

$$B = \frac{9}{10}\mu_0 \sigma_0 \omega R = \frac{\mu_0 \sigma_0 \omega R}{2} \times \frac{(2+\beta)}{(1+\beta)}$$

$\Rightarrow \quad 5\beta + 10 = 9\beta + 9$

$\Rightarrow \quad \beta = \frac{1}{4}$

10. *(b)* Magnetic field at axial point ($d >> l$)

$$\mathbf{B}_1 = \frac{2\mu_0 M}{4\pi d^3} \quad \text{(from } S \text{ to } N\text{)}$$

$$= \frac{2\mu_0 M}{4\pi D^3} \quad (\because d = D)$$

Magnetic field at equatorial line ($d >> l$)

$$\mathbf{B}_2 = \frac{2\mu_0 M}{4\pi d^3} \quad \text{(from } N \text{ to } S\text{)}$$

Now, for equatorial point $d = 2D$

$$\therefore \quad \mathbf{B}_2 = \frac{1}{8}\frac{\mu_0 M}{4\pi D^3} \quad \text{(from } N \text{ to } S\text{)}$$

So, resultant magnetic field,

$$\mathbf{B} = \mathbf{B}_1 - \mathbf{B}_2 \quad \text{(from } S \text{ to } N\text{)}$$

$$= \frac{\mu_0 M}{4\pi D^3}\left(2 - \frac{1}{8}\right)$$

$$\mathbf{B} = \frac{15}{8}\frac{\mu_0 M}{4\pi D^3} \quad \text{(in magnitude)}$$

On comparing with given formula, we get

$$\alpha = 15/8$$

11. *(a)* Emf $= \frac{d}{dt}\phi_B$

$$= \left(\frac{d}{dt}A\right) \cdot B$$

$$= \left(\frac{\pi R^2}{2\pi/\omega}\right) B = \frac{1}{2} B R^2 \omega$$

12. *(b)* When switch S_1 is closed and S_2 is open, capacitor C will be fully charged. After charging, when S_1 is opened and S_2 is closed, this L-C circuit will act as tank circuit.

In tank circuit,

$$\frac{1}{2}CV^2 = \frac{1}{2}LI^2 \quad \text{(total energy)}$$

$\Rightarrow \quad I^2 = \frac{C}{L}V^2$

$\Rightarrow \quad I = V\sqrt{C/L}$

13. *(c)* Intensity of radiations,

$$I = \frac{\text{Power}}{\text{Area}} \times \text{Efficiency}$$

$$= \frac{100}{4 \times \pi \times (10)^3} \times \eta$$

Now, half of intensity is provided by electric field and another half by magnetic field.

So, $\frac{1}{2}I = \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 c = \frac{1}{2}\varepsilon_0 \frac{E_{\max}^2}{2} c$

$[\because E_{\text{rms}} = E_{\max}/\sqrt{2}]$

$$\Rightarrow \quad \frac{100}{4\pi \times 10^3}\eta = \frac{1}{2} \times 8.85 \times 10^{-12} \times \frac{4}{2} \times 3 \times 10^8$$

$$\Rightarrow \quad \eta = \frac{8.85 \times 2 \times 3 \times 1256 \times 10^{-4}}{100}$$

$$\Rightarrow \quad \eta = \frac{6.66}{100}$$

or $\quad \eta = 6.6\%$

14. *(d)* Photoelectric effect, $h\nu = h\nu_0 + eV$ (Einstein's equation)

$$\Rightarrow \quad V = \frac{h}{e}(\nu - \nu_0)$$

Given, $\quad V_1 = \frac{h}{e}(\nu_1 - \nu_0)$ and

$$3V_1 = \frac{h}{e}(2\nu_1 - \nu_0)$$

$$\Rightarrow \quad \frac{3V_1}{V_1} = \frac{2\nu_1 - \nu_0}{\nu_1 - \nu_0} \Rightarrow \nu_1 = 2\nu_0$$

$$\therefore \quad V_1 = \frac{h}{e}(2\nu_0 - \nu_0) = \frac{h}{e}\nu_0$$

At frequency $4\nu_1$, $nV_1 = \frac{h}{e}(4\nu_1 - \nu_0)$

$$= \frac{h}{e}(8\nu_0 - \nu_0) = 7 \times \frac{h}{e}\nu_0$$

$$\Rightarrow \quad nV_1 = 7V_1 \Rightarrow n = 7$$

15. *(a)* For electron, $pc = \sqrt{2\text{KE}m_0c^2}$

$$\Rightarrow \quad pc = \sqrt{2 \times 9\,\text{eV} \times 0.58 \times 10^6}$$

$$\Rightarrow \quad pc = 3 \times 10^3 \text{ eV}$$

Now, de-Broglie wavelength,

$$\lambda = \frac{hc}{pc} = \frac{4 \times 10^{-15} \times 3 \times 10^{10}}{3 \times 10^3}$$

$$= 4 \times 10^{-8}\text{cm} \quad (\because c = 3 \times 10^{10} \text{ cm/s})$$

16. *(b)* Decay equation, $N = N_0 \exp(-\lambda t)$

Given, $\quad N_1 = N_0/3,\ N_2 = ?$

$t_1 = 20$ h $\quad t_2 = 20$ h

So, $\quad N_1 = N_0 \exp(-\lambda t_1)$

$$\Rightarrow \quad \frac{1}{3} = e^{(-20\,\lambda)}$$

$$\Rightarrow \quad e^{20\lambda} = 3$$

Now, $\quad N_2 = N_0 \exp(-\lambda t_2)$

$$\Rightarrow \quad \frac{N_0}{N_2} = e^{80\,\lambda}$$

$$\Rightarrow \quad \frac{N_0}{N_2} = (e^{20\lambda})^4 = (3)^4$$

$$\Rightarrow \quad N_2 = \frac{1}{(3)^4}N_0 = \frac{N_0}{81}$$

17. *(b)* $R_L = 10 \times 10^3\,\Omega$, $\Delta I_c = 4 \times 10^{-3}$ A, $\Delta I_B = 20 \times 10^{-6}$ A, $V_{BE} = 40 \times 10^{-3}$ V

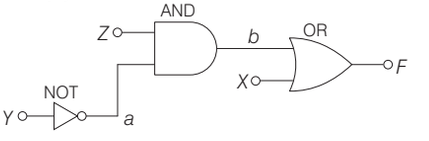
Power gain $= \beta^2 R_L / R_B$

$$= \left(\frac{\Delta I_c}{\Delta I_B}\right)^2 \cdot \frac{R_L}{R_B} = \left(\frac{\Delta I_c}{\Delta I_B}\right)^2 \times \frac{R_L}{(V_{BE}/\Delta I_B)}$$

$$= \left(\frac{40 \times 10^{-3}}{20 \times 10^{-6}}\right)^2 \times \frac{10 \times 10^3}{40 \times 10^{-3}} \times 20 \times 10^{-6}$$

$\Rightarrow \quad A_P = 2 \times 10^{-7}$ W

18. *(a)*



Let's name intermediate states as a and b as shown in figure.

Now, $\quad a = \overline{Y}$

$b = Za = z\overline{y}$

$F = b + X = Z\overline{Y} + X$

19. *(c)* Modulation index,

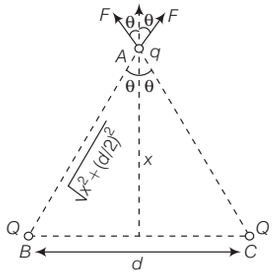
$$m = \frac{\text{Peak value of } m(t)}{A} = \frac{M}{A}$$

$\therefore \quad M = mA$

$$= 20 \times 60\% = 20 \times \frac{60}{100} = 12\text{ V}$$

VIT

1. *(c)* Suppose third charge is $+q$ and is placed at point A as shown in the figure below,



So, net force on charge q is

$$F_{net} = 2F\cos\theta$$

As, $$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{\left(x^2+\frac{d^2}{4}\right)}$$

and $$\cos\theta = \frac{x}{\sqrt{x^2+\frac{d^2}{4}}}$$

$\therefore$

$$F_{net} = 2\times\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{\left(x^2+\frac{d^2}{4}\right)}\times\frac{x}{\left(x^2+\frac{d^2}{4}\right)^{\frac{1}{2}}}$$

$$= \frac{2Qqx}{4\pi\varepsilon_0\left(x^2+\frac{d^2}{4}\right)^{\frac{3}{2}}}$$

This force experienced by the charge 'q' would be maximum at a distance x only when,

$$\frac{dF_{net}}{dx} = 0$$

$$\Rightarrow \left[\left(x^2+\frac{d^2}{4}\right)^{-\frac{3}{2}} - 3x^2\left(x^2+\frac{d^2}{4}\right)^{-\frac{5}{2}}\right] = 0$$

or $$x = \pm\frac{d}{2\sqrt{2}}$$

As distance can never be negative, so $x = -\frac{d}{2\sqrt{2}}$ will be neglected.

Thus, $x = \frac{d}{2\sqrt{2}}$

2. *(d)* Full scale deflection current $= \frac{150}{10}$ mA $= 15$ mA

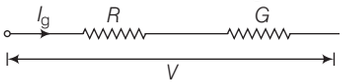
Full scale deflection voltage $= \frac{150}{2}$ mV $= 75$ mV

Galvanometer resistance, $G = \frac{75}{15} = 5\,\Omega$

Required full scale deflection voltage,

$$V = 1\times 150 = 150\text{ V}$$

Let resistance to be connected in series is as shown in the figure below,



then $V = I_g(R+G)$

$$150 = 15\times 10^{-3}(R+5)$$

$$10^4 = R+5$$

$$R = 10000 - 5 = 9995$$

3. *(a)* Fuse melts due to heating effect of current

$\therefore$ Heat produced in a fuse wire is given as

$$H = I^2R$$

$$\Rightarrow \quad I_1^2R_1 = I_2^2R_2$$

$$I_1^2\rho\frac{l_1}{A_1} = I_2^2\rho\cdot\frac{l_2}{A_2}$$

$\because$ The wires are made from same material, therefore their resistivity (ρ) would be same.

$$I_1^2l_1r_2^2 = I_2^2l_2r_1^2$$

Taking $l_1 = l_2 \Rightarrow I_1^2r_2^2 = I_2^2r_1^2$

$$r_2^2 = \left(\frac{I_2}{I_1}\cdot r_1\right)^2 \quad r_2^2 = \left(\frac{3}{1.5}\times 1\right)^2$$

$$r_2^2 = 4 \qquad r_2 = 2\text{ mm}$$

4. *(d)* When needle oscillates in horizontal plane, then its time period,

$$T = 2\pi\sqrt{\frac{I}{mB_H}} \qquad ...(i)$$

When needle oscillates in vertical plane, i.e. it oscillates in the earth's magnetic field B.

Hence, $$T' = 2\pi\sqrt{\frac{I}{mB}} \qquad ...(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{T'}{T} = \sqrt{\frac{B_H}{B}} \qquad \frac{T'}{T} = \sqrt{\frac{B\cos\theta}{B}}$$

$$\frac{T'}{T} = \sqrt{\cos 60^\circ} \Rightarrow T' = \frac{T}{\sqrt{2}}$$

5. *(c)* The lens will disappear only if the refractive index of the lens and the liquid is same.

So, $$\frac{1}{f} = \left(\frac{\mu_l}{\mu_{air}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{20} = (\mu_l - 1)\left[\frac{1}{10} - \frac{1}{(-10)}\right]$$

$$\frac{1}{20} = (\mu_l - 1)\left[\frac{1}{10} + \frac{1}{10}\right]$$

$$\frac{1}{20} = (\mu_l - 1)\left[\frac{2}{10}\right]$$

$$\frac{1}{4} = \mu_l - 1 \Rightarrow \mu_l = 1 + \frac{1}{4} = \frac{5}{4}$$

6. *(a)* Here, Wavelength, $\lambda = 5400$ Å

$$= 5.4\times 10^{-7}\text{ m}$$

$$a = 0.80\text{ mm} = 8\times 10^{-4}\text{ m}$$

$$D = 1.4\text{ m}$$

$\therefore$ Distance between first two dark bands on each side of central maximum is the width of central maximum $= 2x = \frac{2\lambda D}{a}$

$$= \frac{2\times 5.4\times 10^{-7}\times 1.4}{8\times 10^{-4}}$$

$$= 1.89\times 10^{-3}\text{ m}$$

$$= 1.89\text{ mm}$$

7. *(b)* Let the electron absorbing the photon energy reaches to the excited state n. Then, using law of energy conservation, energy of electron in the nth orbit = ground state energy + energy absorbed by the electon

$$\Rightarrow \quad -\frac{13.6}{n^2} = \frac{-13.6}{1^2} + 12.4$$

$$-\frac{13.6}{n^2} = -1.2$$

$$n^2 = \frac{13.6}{1.2} = 11.34 \simeq 12$$

$$n \simeq 3.36 \simeq 3$$

8. *(b)* Given, $T_{1/2} = 3.8$days

$\frac{N}{N_0} = \frac{1}{20}$, where N is the number of nuclei present at time t and N_0 is the number of nuclie present at $t = 0$.

As we know,

$$T_{1/2} = \frac{0.693}{\lambda}$$

where, λ is decay constant.

$$\Rightarrow \quad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.8} = 0.184$$

According to Rutherford and Soddy's law,

$$N = N_0e^{-\lambda t}$$

$$\Rightarrow \quad \frac{N}{N_0} = e^{-\lambda t}$$

or $$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\Rightarrow \quad 2.303\log_{10}\left(\frac{1}{20}\right) = -0.184\times t$$

$$\Rightarrow \quad \frac{2.303\times 1.3010}{0.184} = t$$

$$t = 16.28\text{ days}$$

9. *(c)* Maximum current passing in the circuit is

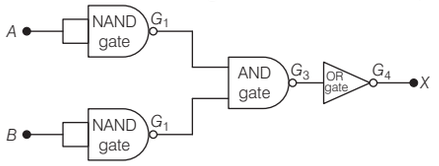
$$I = \frac{P}{V} = \frac{100\times 10^{-3}}{0.5} = 0.2\text{ A}$$

Thus, the value of the resistance of the resistor connected in series with the diode is

$$R = \frac{\text{Net voltage drop across the resistor}}{\text{Current in the circuit}} = \frac{1.5 - 0.5}{0.2}$$

$$R = \frac{1}{0.2} = 5\,\Omega$$

10. *(a)* According to the given combination of gates as shown below,



Output of gate $G_1 = \overline{A \cdot A} = \overline{A}$

Output of gate $G_2 = \overline{B \cdot B} = \overline{B}$

Output of gate $G_3 = \overline{A} \cdot \overline{B}$

Output of gate $G_4 = \overline{A \cdot B}$

Using de-Morgan's rule,

$$\overline{A \cdot B} = \overline{\overline{A}} + \overline{\overline{B}} = A + B \quad [\because \overline{\overline{A}} = A]$$

This output is same as the output of or gate $= A + B$

11. *(a)* Assume that all the charges are at infinity initially. Then, work done on putting charge q at corner A, $W_1 = q(V_A) = 0$

Because, potential at A is zero in absence of charges.

Work done in putting q at corner B in presence of charge, at A

$$W_2 = \left(\frac{kq}{a} - 0\right) q = \frac{kq^2}{a}$$

and $W_3 =$ work done in placing q at C

$$= \left(\frac{2kq}{a}\right) q$$

So, potential energy of the system of charges is

$$= W_1 + W_2 + W_3 = \frac{3kq^2}{a}$$

12. *(b)* Coordinate of the point are (x, y).

$\therefore$ Distance of point from origin,

$$r = \sqrt{x^2 + y^2}$$

Given, $V = -Kxy$

$$\Rightarrow \quad E_X = -\frac{dV}{dx} = -\frac{d}{dx}(-Kxy) = Ky$$

$$E_Y = -\frac{dV}{dy} = -\frac{d}{dy}(-Kxy) = Kx$$

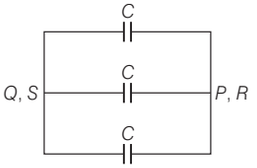
Net electric field intensity,

$$E = \sqrt{E_X^2 + E_Y^2} = K\sqrt{y^2 + x^2}$$

or $E = Kr$

$\therefore$ $E \propto r$

13. *(d)* The three capacitors can be rearranged as shown in the below circuit

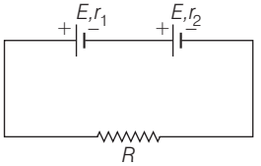


As, the three capacitors are in parallel. Thus, equivalent capacitance between Q, S and P, R is

$$C_{eq} = C + C + C = 3C$$

$$= 3 \times 3 = 9\,\mu\text{F} \quad [\because C = 3\,\mu\text{F (given)}]$$

14. *(b)* The combination of the cells and resistor as given in the question is shown in the figure below. As both cells are in series in the circuit.



Net emf, $E = E_1 + E_2 = E + E = 2E$

Net resistance, $R_{eq} = r_1 + r_2 + R$

$\therefore$ Current, $i = \dfrac{2E}{r_1 + r_2 + R}$

As terminal potential drop across 1st cell is zero, hence

$$V_1 = E - ir_1$$

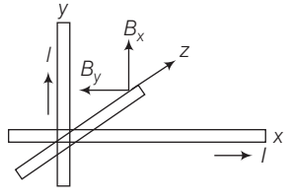
$$0 = E - \frac{2E}{(r_1 + r_2 + R)} r_1$$

$$E = \frac{2Er_1}{r_1 + r_2 + R}$$

$$r_1 + r_2 + R = 2r_1$$

$$R = (r_1 - r_2)$$

15. *(a)* The arrangement of the infinitely long wires is shown in the figure below.



Since, the point lies on the Z-axis. Thus, the magnetic field due to current along Z-axis will be zero. However, magnetic field due to wire on X-axis is given as,

$$\mathbf{B}_x = \frac{\mu_0 i}{2\pi a}(\hat{\mathbf{j}})$$

Magnetic field due to wire an Y-axis is given as,

$$\mathbf{B}_y = \frac{\mu_0 i}{2\pi a}(\hat{\mathbf{i}})$$

$\therefore$ Net magnetic field,

$$\mathbf{B}_{net} = \mathbf{B}_x + \mathbf{B}_y = \frac{\mu_0 i}{2\pi a}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$$

16. *(d)* An emf is induced in a coil due to the change in net magnetic field lines passing through it, but not by the change in electric field lines associated with it. However, in the given question, the change in magnetic field lines through the coil has not been mentioned. Thus, there will be zero induced emf.

17. *(c)* Initially impedance of L-R circuit is

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{10}\,R$$

$\Rightarrow$ Power factor, $\cos\phi_1 = \dfrac{R}{Z} = \dfrac{1}{\sqrt{10}}$

Now, when the capacitance is connected, the new impedance of circuit will be

$$Z = \sqrt{(X_L - X_C)^2 + R^2} = \sqrt{5}\,R$$

$$\Rightarrow \quad \cos\phi_2 = \frac{R}{Z} = \frac{1}{\sqrt{5}}$$

Ratio, $\dfrac{\cos\phi_1}{\cos\phi_2} = \dfrac{\frac{1}{\sqrt{10}}}{\frac{1}{\sqrt{5}}} = \dfrac{\sqrt{5}}{\sqrt{10}} = \dfrac{1}{\sqrt{2}}$

18. *(b)* Given, $\mu = \sqrt{2}$, $A = 60°$

We know that, for a prism,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\sqrt{2} = \frac{\sin\left(\frac{60° + \delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$\frac{1}{\sqrt{2}} = \sin\left(\frac{60° + \delta_m}{2}\right)$$

$$\Rightarrow \quad \sin 45° = \sin\left(\frac{60° + \delta_m}{2}\right)$$

$$\Rightarrow \quad \delta_m = 30°$$

Also,

$$i = \frac{A + \delta_m}{2} = \frac{60° + 30°}{2} = \frac{90°}{2} = 45°$$

19. *(b)* For nth order maxima,

$$a\sin\theta = (2n + 1)\frac{\lambda}{2}$$

$\therefore$ For 1st order maxima,

$$= (2 \times 1 + 1)\frac{\lambda}{2} = \frac{3\lambda}{2}$$

where, θ is the half angular spread between 1st order maxima and central maxima.

When θ is small, then we can write

$$\sin\theta \approx \theta$$

$$\Rightarrow \quad a\theta = \frac{3\lambda}{2} \text{ or } \theta = \frac{3\lambda}{2a}$$

Substituting the given values, we get

$$= \frac{3 \times 5460 \times 10^{-10}}{2 \times 0.20 \times 10^{-3}}$$

$$= 4.095 \times 10^{-3} \text{ rad}$$

Total angular speed,

$$2\theta = (4.095 \times 10^{-3} \text{ rad}) \times 2$$

$$= 8.19 \times 10^{-3} \text{ rad}$$

20. *(c)* For Lyman series,

$$\frac{1}{\lambda_{L_1}} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

For $n = 2$,

$$\frac{1}{\lambda_{L_1}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4} \quad \text{...(i)}$$

For Balmer series,

$$\frac{1}{\lambda_B} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

For $n = 3$,

$$\frac{1}{\lambda_{B_1}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\lambda_{L_1}}{\lambda_{B_1}} = \frac{5}{27}$$

21. *(d)* According to the law of conservation of momentum,

initial momentum = final momentum

Since, the nucleus is initially at rest. Thus, if after disintegeration, the nuclear parts have mass m_1 and m_2 and moves with velocity v_1 and v_2, respectively. Then, the relation between the magnitudes of momentum of these parts will be

$$m_1 v_1 = m_2 v_2 \text{ or } \frac{m_1}{m_2} = \frac{v_2}{v_1}$$

But mass of a nucleus is

$$m = \frac{4}{3}\pi r^3 \rho \text{ or } m \propto r^3$$

$$\therefore \quad \frac{m_1}{m_2} = \frac{r_1^3}{r_2^3} = \frac{v_2}{v_1}$$

$$\frac{r_1}{r_2} = \left(\frac{1}{2}\right)^{\frac{1}{3}} \qquad \left[\text{given, } \frac{v_1}{v_2} = \frac{2}{1}\right]$$

or $\quad r_1 : r_2 = 1 : 2^{\frac{1}{3}}$

22. *(c)* Given,

Binding energy of deutron, ${}_1H^2 = 1.112$ MeV per nucleon

$\therefore$ Total binding energy of initial constituents of the given reaction

$$= 2 \times 2 \times 1.112 = 4.448 \text{ MeV}$$

Binding energy of α-particle (${}_2He^4$)

$$= 7.047 \text{ MeV per nucleon}$$

$\therefore$ Total binding energy of final constituents of the given reaction

$$= 4 \times 7.047 = 28.188 \text{ MeV}$$

Thus, energy released,

$$\theta = 28.188 - 4.448$$

$$= 23.74 \text{ MeV}$$

$$\simeq 23.8 \text{ MeV}$$

23. *(b)* Given, $\alpha = 0.98$

and $\frac{R_{out}}{R_{in}} = 500$

As, current gain is

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1 - 0.98} = 49$$

$$\text{Voltage gain} = \beta\left(\frac{R_{out}}{R_{in}}\right)$$

$$= 49 \times 500 = 24500$$

$$\text{Power gain} = (\beta)^2 \left(\frac{R_{out}}{R_{in}}\right)$$

$$= (49)^2 (500) = 1200500$$

24. *(d)* Since, no field exist inside copper. So, the dielectric constant of the copper plate would be infinite. Therefore, capacitance

$$C = KC_0 = \infty C_0 = \infty$$

25. *(b)* As, $\quad R_{t_2} = R_0(1 + \alpha t_2)$

and $\quad R_{t_1} = R_0(1 + \alpha t_1)$

where, $\quad R_{t_2}$ = resistance of wire at t Kelvin,

R_0 = resistance of wire at 273 K(0°C)

and $\quad R_{t_1}$ = resistane of wire at 300 K(27°C)

$$\therefore \quad \frac{R_{t_2}}{R_{t_1}} = \frac{1 + \alpha t_1}{1 + \alpha t_2}$$

$$\Rightarrow \quad \frac{1.5}{1} = \frac{1 + 0.00125 \times t_2}{1 + 0.00125 \times 27}$$

On solving, we get

$$t_2 = 440.5° \text{ C}$$

$$= 440.5 + 273$$

$$= 713.5 \text{ K}$$

$$\approx 714 \text{ K}$$

26. *(b)* The magnetic field at the centre of circular coil,

$$B = \frac{\mu_0 I}{2r}$$

where, r = radius of circle $= \frac{l}{2\pi}$ $(\because l = 2\pi r)$

$$B = \frac{\mu_0 I}{2} \times \frac{2\pi}{l} = \frac{\mu_0 I \pi}{l} \quad \text{...(i)}$$

When wire of length l bents into a circular loop of n turns, then

$$l = n \times 2\pi r'$$

$$r' = \frac{l}{n \times 2\pi}$$

Thus, new magnetic field

$$B' = \frac{\mu_0 n I}{2r'} = \frac{\mu_0 n I}{2} \times \frac{n \times 2\pi}{l}$$

$$= \frac{\mu_0 I \pi}{l} \times n^2 = n^2 B \quad \text{[from Eq. (i)]}$$

27. *(c)* Apparent shift is given as

$$x = d\left(1 - \frac{1}{\mu}\right)$$

where, d is the actual thickness of the object in the denser medium of refractive index μ.

Here, $\quad x = 1$ cm

and $\quad d = 3$ cm

$$\Rightarrow \quad 1 = 3\left(1 - \frac{1}{\mu}\right)$$

$$\Rightarrow \quad 1 - \frac{1}{\mu} = \frac{1}{3} \text{ or } \frac{1}{\mu} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\mu = \frac{3}{2}$$

As, critical angle, $\quad i_C = \sin^{-1}\left(\frac{1}{\mu}\right)$

or $\quad i_C = \sin^{-1}\left(\frac{2}{3}\right)$

$$i_C = \sin^{-1}(0.67)$$

28. *(d)* Given, $\quad 2\theta = 60°$,

$$\Rightarrow \quad \theta = 30°$$

Wavelength, $\quad \lambda = 600 \text{ nm}$

$$= 6 \times 10^{-7} \text{ m}$$

$$\mu = 1$$

$$\therefore \text{Minimum distance, } \frac{1}{d} = \frac{2\mu \sin\theta}{1.22\lambda}$$

$$= \frac{2 \times 1 \times \sin 30°}{1.22 \times 6 \times 10^{-7}}$$

$$d = 1.22 \times 6 \times 10^{-7}$$

$$= 7.3 \times 10^{-7} \text{ m}$$

29. *(a)* Number of spectral lines obtained due to the transition of electron from n^{th} orbit to a lower orbit is given as

$$N = \frac{n(n-1)}{2}$$

Here, $N = 10$

$$\Rightarrow \quad 10 = \frac{n(n-1)}{2}$$

or $n = 5$

Further, $E_n = \frac{E_1}{n^2}$

$$\therefore \quad E_5 = \frac{-13.6}{5^2} = -0.544 \text{ eV}$$

The energy needed to take a hydrogen atom from its ground state to $n = 5$.

So, required energy

$$= E_5 - E_1$$
$$= -0.54 - (-13.6)$$
$$= 13.06 \text{ eV}$$

The corresponding wavelength of light,

$$= \frac{hc}{\Delta E}$$
$$= \frac{1240 \text{ eV} - \text{nm}}{13.06 \text{ eV}} \approx 95 \text{ nm}$$

30. *(b)* As work function, $W_0 = \frac{hc}{\lambda_0}$

where, λ_0 is the threshold wavelength

$$W_0 \propto \frac{1}{\lambda_0}$$

$$\therefore \quad \frac{W_0}{W'_0} = \frac{\lambda'_0}{\lambda_0}$$

Here, $W_0' = \frac{W_0}{2}$

$$\Rightarrow \quad \frac{W_0}{\frac{W_0}{2}} = \frac{\lambda'_0}{\lambda} \quad \text{or} \quad \lambda'_0 = 2\lambda_0$$

MHT CET

1. *(b)* Given, load resistance $(R_e) = 2\,\text{k}\Omega$

$$= 2 \times 10^3 \ \Omega$$

Input resistance $(R_i) = 150 \ \Omega$

Change in collector current $(\Delta I_C) = 1.5 \text{ mA} = 1.5 \times 10^{-3}$ A

Change in base current $(\Delta I_B) = 20\,\mu\text{A}$

$$= 20 \times 10^{-6} \text{ A}$$

We know that,

$$\text{voltage gain} = \frac{V_o}{V_i} = \frac{R_o \times \Delta I_C}{R_i \times \Delta I_B}$$

$$= \frac{2 \times 10^3 \times 1.5 \times 10^{-3}}{150 \times 20 \times 10^{-6}}$$

$$= \frac{3 \times 10^6}{150 \times 20} = 1000$$

2. *(b)* According to the question, magnetic field at the centre of circular coil is given by

$$B_{\text{centre}} = \frac{\mu_0 Ni}{2R} \quad \ldots(\text{i})$$

$$B_{\text{axis}} = \frac{\mu_0}{4\pi} \frac{2\pi NiR^2}{(x^2 + R^2)^{3/2}}$$

$$\frac{B}{8} = \frac{\mu_0 NiR^2}{2(x^2 + R^2)^{3/2}}$$

From Eq. (i), we get

$$\frac{\mu_0 Ni}{8 \times 2R} = \frac{\mu_0 NiR^2}{2(x^2 + R^2)^{3/2}}$$

$$\frac{1}{8R} = \frac{1}{(x^2 + R^2)^{3/2}}$$

$$\frac{1}{8R^3} = \frac{1}{(x^2 + R^2)^{3/2}}$$

$$8R^3 = (x^2 + R^2)^{3/2}$$

(on taking cube root)

$$2R = (x^2 + R^2)^{1/2}$$ (on taking square root)

$$4R^2 = (x^2 + R^2)$$

$$4R^2 - R^2 = x^2$$

$$3R^2 = x^2$$

$$x = R\sqrt{3}$$

3. *(b)* We know that,

$$I_{\max} = (a_1 + a_2)^2 \quad \ldots(\text{i})$$

$$I_{\min} = (a_1 - a_2)^2 \quad \ldots(\text{ii})$$

On adding Eqs. (i) and (ii), we get

$$I_{\max} + I_{\min} = (a_1 + a_2)^2 + (a_1 - a_2)^2$$

$$I_{\max} + I_{\min} = a_1^2 + a_2^2 + 2a_1a_2 + a_1^2 + a_2^2 - 2a_1a_2$$

$$I_{\max} + I_{\min} = 2(a_1^2 + a_2^2)$$

But $I \propto a^2$

Therefore,

$$I_{\max} + I_{\min} = 2(I_1 + I_2)$$

4. *(d)* Given,

$$e = 200\sqrt{2}\sin(100t) \quad \ldots(\text{i})$$

Capacitance of capacitor $(C) = 1\,\mu\text{F}$

$$= 1 \times 10^{-6} \text{ F}$$

The standard equation of voltage of AC is given by

$$V = V_0 \sin\omega t \quad \ldots(\text{ii})$$

On comparing Eqs. (i) and (ii), we get

$$V_0 = 200\sqrt{2}$$

$$\omega = 100$$

We know that,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C}$$

But $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$

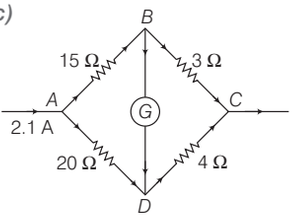
$$I_{\text{rms}} = \frac{V_0}{\sqrt{2}\,X_C}$$

$$I_{\text{rms}} = \frac{V_0 \omega C}{\sqrt{2}} \left(\because X_C = \frac{1}{\omega C}\right)$$

$$I_{\text{rms}} = \frac{200 \times \sqrt{2} \times 100 \times 1 \times 10^{-6}}{\sqrt{2}}$$

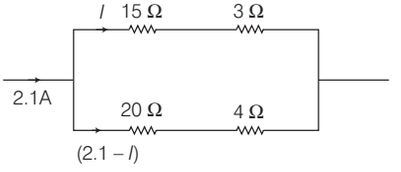
$$I_{\text{rms}} = 20 \times 10^{-3} \text{ A} = 20 \text{ mA}$$

5. *(c)*



Above circuit is a balanced Wheatstone bridge.

Hence, the above circuit is drawn in the following manner



From the above circuit,

$$18I = 24(2.1 - I)$$

$$18I = 50.4 - 24I$$

$$24I + 18I = 50.4$$

$$42I = 50.4$$

$$I = \frac{50.4}{42}$$

$$\therefore \quad I = 1.2 \text{ A}$$

6. *(a)* The angle made by incident ray of light with the reflecting surface is called glancing angle.

7. *(c)* Resolving limit of microscope is given by

$$\text{RL} = \frac{\lambda}{2\mu\sin\alpha}$$

where, $\mu\sin\alpha$ is called numerical aperture [NA]

and λ = wavelength of light.

$$\therefore \quad \text{RL} = \frac{\lambda}{2(\text{NA})}$$

Hence, if numerical aperture of a microscope is increased, then its limit of resolution is decreased.

8. *(d)* In amplitude modulation, amplitude of the carrier wave changes according to the information signal.

9. *(b)* Here,

M_z = magnetization of a paramagnetic sample

B = external magnetic field

T = absolute temperature

C = Curie constant

Curie's law in magnetism states that in paramagnetic material, the material's magnetization is directly proportional to the applied field, but when temperature increases the relation is reversed, i.e. the magnetization becomes inversely proportional to the temperature. Mathematically, it is written as

$$M_z = \frac{C \times B}{T}$$

10. *(d)* Given,

$$n_1 = 1,\ n_2 = 4$$

We know that,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = R\left[\frac{1}{(1)^2} - \frac{1}{(4)^2}\right]$$

$$\frac{1}{\lambda} = R\left[1 - \frac{1}{16}\right]$$

$$\frac{1}{\lambda} = R\left[\frac{16-1}{16}\right]$$

$$\frac{1}{\lambda} = \frac{15R}{16}$$

$$\lambda = \frac{16}{15R} \quad \ldots(i)$$

But, $p = \frac{h}{\lambda}$

$$mv = \frac{h}{\lambda}$$

$$v = \frac{h}{m\lambda}$$

$$v = \frac{h}{m} \times \frac{15R}{16} \quad \text{[from Eq. (i)]}$$

$$\therefore \quad v = \frac{15hR}{16m}$$

11. *(b)* Given, input voltage $(V_P) = 220$ V

Output voltage $(V_S) = 3.3 \times 10^3$ V

Power $(P) = 4.4$ kW $= 4.4 \times 10^3$ W

Number of turns in primary coil = 600

We know that,

$$P = V_P \times I_P$$

$$\Rightarrow \quad I_P = \frac{P}{V_P}$$

or $$I_P = \frac{4.4 \times 10^3}{220} = 20 \text{ A}$$

For an ideal transformer,

$$\frac{V_S}{V_P} = \frac{I_P}{I_S}$$

$$I_S = I_P \times \frac{V_P}{V_S}$$

$$I_S = \frac{20 \times 220}{3.3 \times 10^3}$$

or $$I_S = \frac{44}{33} = \frac{4}{3} \text{ A}$$

12. *(a)* Resistance of wire is given by

$$R = \rho\frac{l}{A}$$

where, l is the length of conductor and A is the cross-sectional area.

According to the question,

$$R_1 = \rho\frac{l_1}{A_1} \quad \ldots(i)$$

In the second condition,

$$R_2 = \rho\frac{l_2}{A_2} \quad \ldots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{R_1}{R_2} = \frac{l_1}{A_1} \times \frac{A_2}{l_2}$$

$$\frac{R_1}{R_2} = \frac{A_2}{A_1} \times \frac{l_1}{l_2}$$

$$\frac{R_1}{R_2} = \frac{\pi\left(\frac{d_2}{2}\right)^2}{\pi\left(\frac{d_1}{2}\right)^2} \times \frac{l_1}{l_2}$$

$$\frac{R_1}{R_2} = \frac{(d_2)^2}{(d_1)^2} \times \frac{l_1}{l_2} \quad \ldots(iii)$$

Volume of wire remains constant.

Hence, $V_1 = V_2$

$$A_1 l_1 = A_2 l_2$$

$$\Rightarrow \quad \pi\left(\frac{d_1}{2}\right)^2 l_1 = \pi\left(\frac{d_2}{2}\right)^2 l_2$$

$$\Rightarrow \quad \frac{\pi d_1^2}{4} l_1 = \frac{\pi d_2^2}{4} l_2$$

$$\frac{l_1}{l_2} = \frac{(d_2)^2}{(d_1)^2} \quad \ldots(iv)$$

Substituting the value of Eq. (iv) in Eq. (iii), we get

$$\frac{R_1}{R_2} = \frac{(d_2)^2}{(d_1)^2} \times \frac{(d_2)^2}{(d_1)^2}$$

$$\frac{R_1}{R_2} = \frac{(d_2)^4}{(d_1)^4}$$

13. *(c)* In the forward biased mode, the *p-n* junction diode acts as closed switch.

14. *(a)* In cyclotron frequency, $\nu = \frac{eB}{2\pi m}$

or $$B = \frac{2\pi m \cdot \nu}{e}$$

Also, relation for radius (R) is

$$R = \frac{mv}{eB} \quad \text{or} \quad v = \frac{eBR}{m}$$

$$\therefore \quad \text{KE} = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m \times \frac{e^2B^2R^2}{m^2}$$

$$= \frac{e^2B^2R^2}{2m}$$

$$= \frac{e^2R^2}{2m} \times \frac{4\pi^2m^2\nu^2}{e^2}$$

$$= 2\pi^2mv^2R^2$$

15. *(b)* Time taken by light to cross the glass slab,

$$t = \frac{\text{Distance}}{\text{Speed of light in glass}}$$

$$= \frac{5 \times 10^{-2}}{\frac{3}{1.6} \times 10^8} \text{ s}$$

$$= 2.67 \times 10^{-10} \text{ s}$$

In same time distance travelled by light in air

$$= c \times t$$

$$= 3 \times 10^8 \times 2.67 \times 10^{-10}$$

$$= 8.02 \times 10^{-2} \text{ m}$$

$$\approx 8 \text{ cm}$$

16. *(c)* The magnetic susceptibility of a magnetic substance is defined as the ratio of intensity of magnetisation (I) to magnetic intensity (H), i.e. $\chi = \frac{I}{H}$

For paramagnetic material, χ is positive and small $(0 < \chi < \varepsilon)$, where ε is small positive number.

For diamagnetic meterial, χ is negative and small $(-1 \le \chi \le 0)$.

17. *(b)* We know that,

$$eV_0 = h\nu - h\nu_0$$

or $$eV_0 = h(\nu - \nu_0)$$

or $$V_0 = \frac{h}{e}(\nu - \nu_0)$$

Here, V_0 = stopping potential

and V = frequency of incident radiation.

Therefore, stopping potential is directly proportional to the frequency of incident radiation. Hence, stopping potential V_0 is increased.

18. *(a)* We know that,

the electric field intensity outside the sheet is given by

$$E_{\text{outside}} = \frac{\sigma}{2\varepsilon_0}$$

Therefore, the electric field intensity is independent of d.

19. *(d)* In galvanometer,

$$\frac{I_g}{I} = \frac{S}{S+G}$$

where, S is shunt resistance, G is resistance of galvanometer,

$$\Rightarrow \quad \frac{1}{4} = \frac{3}{3+G} \quad \text{or} \quad G = 9\,\Omega$$

In new condition,

$$\frac{I_g}{I} = \frac{\frac{6}{5}}{\frac{6}{5}+9}$$

$$\left[\text{Effective shunt resistance} = \frac{3\times 2}{3+2} = \frac{6}{5}\,\Omega\right]$$

$$= \left(\frac{1}{8.5}\right)\text{th}$$

20. *(a)* In the first condition,

$$C_{\text{eq}} = \frac{C_1}{N_1}$$

Potential difference $(V) = 3V$

$$\therefore \quad \text{Energy stored } (E_1) = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\left(\frac{C_1}{N_1}\right)(3V)^2$$

$$= \frac{9}{2}\frac{C_1}{N_1}V^2 \qquad \ldots(\text{i})$$

In the second condition,

$C_{\text{eq}} = N_2C_2$, potential difference $= V$

$$\text{Energy stored } (E_2) = \frac{1}{2}CV^2$$

$$= \frac{1}{2}N_2C_2V^2 \qquad \ldots(\text{ii})$$

According to the question,

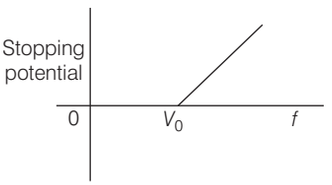
$$E_1 = E_2$$

From Eqs. (i) and (ii), we get

$$\frac{9}{2}\frac{C_1}{N_1}V^2 = \frac{1}{2}N_2C_2V^2$$

$$C_1 = C_2\frac{N_2N_1}{9}$$

21. *(a)* The graph showing the variation of stopping potential with the frequency is shown below.



22. *(d)* In compound microscope, the focal length and aperture of objective used is respectively, short and small.

23. *(b)* We know that,

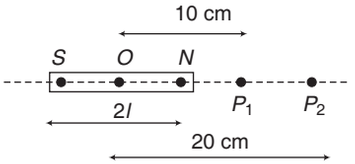
$$\lambda = \frac{h}{\sqrt{2m(\text{KE})}}$$

On taking square on both sides, we get

$$\lambda^2 = \frac{h^2}{2m(\text{KE})}$$

$$\text{KE} = \frac{h^2}{2m\lambda^2}$$

24. *(b)* Let magnetic field at P_1 is B_1 and at P_2 is B_2



$$\therefore \quad \frac{B_1}{B_2} = \frac{\frac{x_1}{(x_1^2 - l^2)^2}}{\frac{x_2}{(x_2^2 - l^2)^2}} = \frac{(x_2^2 - l^2)^2}{(x_1^2 - l^2)^2} \times \frac{x_1}{x_2}$$

$$\Rightarrow \quad \frac{25}{2} = \frac{10}{20}\left[\frac{x_2^2 - l^2}{x_1^2 - l^2}\right]^2$$

or $$\left[\frac{x_2^2 - l^2}{x_1^2 - l^2}\right] = 5$$

or $$x_2^2 - l^2 = 5x_1^2 - 5l^2$$

or $$4l^2 = 5x_1^2 - x_2^2$$

$$= 5\times(10)^2 - (20)^2$$

$$= 500 - 400 = 100$$

$$\Rightarrow \quad l^2 = 25$$

$$\Rightarrow \quad l = 5\text{ cm}$$

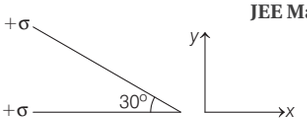
$$\therefore \quad 2l = 10\text{ cm}$$

Engineering Entrance Questions 2019-20

JEE Main & Adv., BITSAT, AP & TS EAMCET, MHTCET, WBJEE

Electrostatics

1. Two infinite planes each with uniform surface charged density $+\sigma$ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by **JEE Main 2020**



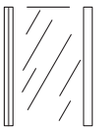
(a) $\frac{\sigma}{2\varepsilon_0}\left[(1+\sqrt{3})\hat{\mathbf{y}}-\frac{\hat{\mathbf{x}}}{2}\right]$

(b) $\frac{\sigma}{2\varepsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{\mathbf{y}}-\frac{\hat{\mathbf{x}}}{2}\right]$

(c) $\frac{\sigma}{2\varepsilon_0}\left[(1+\sqrt{3})\hat{\mathbf{y}}+\frac{\hat{\mathbf{x}}}{2}\right]$

(d) $\frac{\sigma}{\varepsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{\mathbf{y}}+\frac{\hat{\mathbf{x}}}{2}\right]$

2. A parallel plate capacitor has plates of area A separated by distance d between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$, where x is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression **JEE Main 2020**



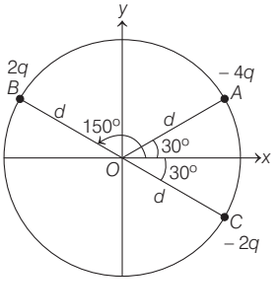
(a) $\frac{AK\varepsilon_0}{d}(1+\alpha d)$

(b) $\frac{A\varepsilon_0 K}{d}\left(1+\left(\frac{\alpha d}{2}\right)^2\right)$

(c) $\frac{AK\varepsilon_0}{d}\left(1+\frac{\alpha d}{2}\right)$

(d) $\frac{A\varepsilon_0 K}{d}\left(1+\frac{\alpha^2 d^2}{2}\right)$

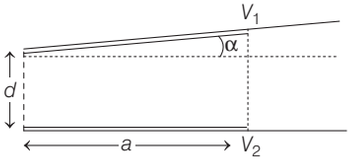
3. Three charged particles A, B and C with charges $-4q$, $2q$ and $-2q$ are present on the circumference of a circle of radius d. The charged particles A, C and centre O of teh circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is **JEE Main 2020**



(a) $\frac{2\sqrt{3}q}{\pi\varepsilon_0 d^2}$ (b) $\frac{\sqrt{3}q}{4\pi\varepsilon_0 d^2}$

(c) $\frac{3\sqrt{3}q}{4\pi\varepsilon_0 d^2}$ (d) $\frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}$

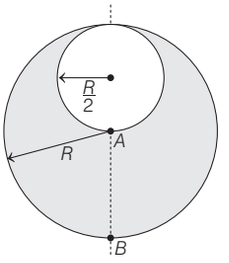
4. A capacitor is made of two square plates each of side a making a very small angle α between them, as shown in figure. The capacitance will be close to **JEE Main 2020**



(a) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{4d}\right)$ (b) $\frac{\varepsilon_0 a^2}{d}\left(1+\frac{\alpha a}{d}\right)$

(c) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{\alpha a}{2d}\right)$ (d) $\frac{\varepsilon_0 a^2}{d}\left(1-\frac{3\alpha a}{2d}\right)$

5. Consider a sphere of radius R which carries a uniform charge density ρ. If a sphere of radius $\frac{R}{2}$ is carved out of it, as shown in the figure the ratio $\frac{|\mathbf{E}_A|}{|\mathbf{E}_B|}$ of magnitude of electrical field $\mathbf{E}_A$ and $\mathbf{E}_B$ respectively, at points A and B due to the remaining portion is **JEE Main 2020**



(a) $\frac{21}{34}$ (b) $\frac{18}{54}$ (c) $\frac{17}{54}$ (d) $\frac{18}{34}$

6. A parallel plate capacitor with plates of area 1 m^2 each, are at a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge on each plate is

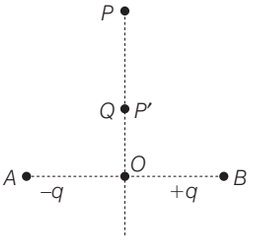
$\left(\text{Take, } \varepsilon_0 = 8.85\times10^{-12}\frac{C^2}{N-m^2}\right)$

JEE Main 2019

(a) 9.85×10^{-10} C (b) 8.85×10^{-10} C

(c) 7.85×10^{-10} C (d) 6.85×10^{-10} C

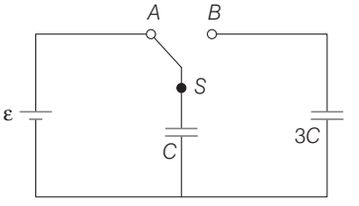
7. Charges $-q$ and $+q$ located at A and B, respectively, constitute an electric dipole. Distance $AB = 2a$, O is the mid point of the dipole and OP is perpendicular to Ab. A charge Q is placed at P, where $OP = y$ and $y >> 2a$. The charge Q experience an electrostatic force F. **JEE Main 2019**



If Q is now moved along the equatorial line to P' such that $OP' = \left(\frac{y}{3}\right)$, the force on Q will be close to $\left(\frac{y}{3} >> 2at\right)$ **JEE Main 2019**

(a) $\frac{F}{3}$ (b) $3F$ (c) $9F$ (d) $27F$

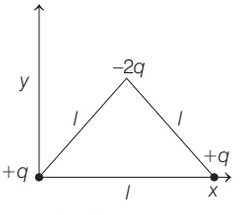
8. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is **JEE Main 2019**



(a) $\frac{3}{4}\cdot\frac{Q^2}{C}$ (b) $\frac{5}{8}\cdot\frac{Q^2}{C}$

(c) $\frac{1}{8}\cdot\frac{Q^2}{C}$ (d) $\frac{3}{8}\cdot\frac{Q^2}{C}$

9. Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle as shown in the figure. **JEE Main 2019**

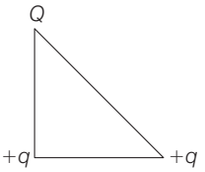


(a) $\sqrt{3}\ ql\ \frac{\hat{\mathbf{j}}-\hat{\mathbf{i}}}{\sqrt{2}}$ (b) $2ql\ \hat{\mathbf{j}}$

(c) $-\sqrt{3}\ ql\ \hat{\mathbf{j}}$ (d) $(ql)\ \frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$

10. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right angle isosceles triangle as shown below.

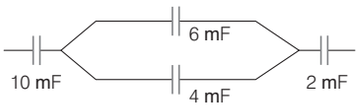
The net electrostatic energy of the configuration is zero, if the value of Q is **JEE Main 2019**



(a) $-2q$ (b) $\frac{-q}{1+\sqrt{2}}$

(c) $+q$ (d) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

11. In the figure shown below, the charge on the left plate of the 10 μF capacitor is − 30 μC. The charge on the right plate of the 6 μF capacitor is **JEE Main 2019**



(a) + 12 μC (b) + 18 μC
(c) − 12 μC (d) − 18 μC

12. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4R^2(\alpha >> 1)$ is made on the shell without affecting the rest of the shell. Which one of the following statements is correct? **JEE Advanced 2019**

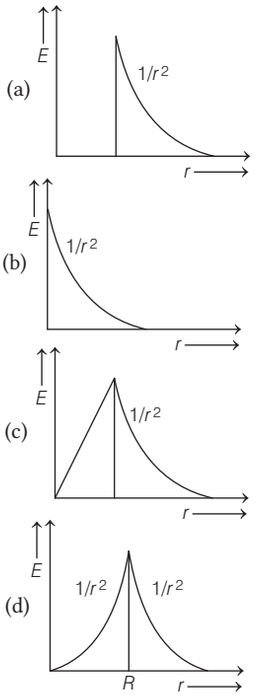
(a) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}R$ from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

(b) The potential at the center of the shell is reduced by $2\alpha V_0$.

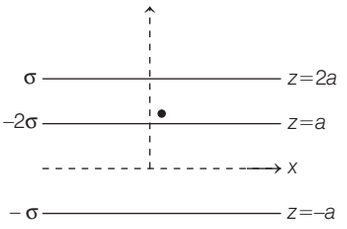
(c) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$.

(d) The magnitude of electric field at a point, located on a line passing through the hole and shell's center, on a distance $2R$ from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$.

13. In the following figures, which represents the variation of the electric field with distance from the centre of a uniformly charged non-conducting spheres of radius R? **BITSAT 2019**



14. Three infinitely long charged sheets are placed as shown in the figure. The electric force acting on a charge $-q$ placed at the point P is (σ = surface charge density, ε_0 = permittivity of the free space) **AP EAMCET 2019**



(a) $+\frac{2\sigma}{\varepsilon_0}\hat{\mathbf{k}}$ (b) $-\frac{2\sigma}{\varepsilon_0}\hat{\mathbf{k}}$

(c) $+\frac{4\sigma}{\varepsilon_0}\hat{\mathbf{k}}$ (d) $-\frac{4\sigma}{\varepsilon_0}\hat{\mathbf{k}}$

15. An infinite line of charge with uniform line charge density of 1 C/m is palced along the y-axis. A charge of 1C is placed on the x-axis at a distance of $d = 3$ m from the origin. At what distance r from the origin on the x-axis, the total electric field is zero. (Assume, $0 < r < d$) **TS EAMCET 2019**

(a) 1 m (b) 2 m
(c) 2.5 m (d) 1.75 m

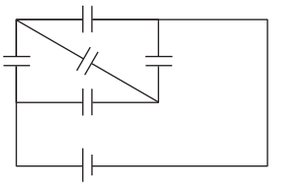
16. A capacitor of capacitance 4 μF is charged to a potential difference of 6 V with a battery. The battery is removed and in its place another capacitor of capacitance is 8 μF introduced and the circuit is closed. The potential difference attained by each of the capacitors in V is **TS EAMCET 2019**

(a) 2 (b) 4 (c) 6 (d) 8

17. In a parallel plate air capacitor the distance between plates is reduced to one fourth and the space between them is filled with a dielectric medium of constant 2. If the initial capacity of the capacitor is 4μF. then its new capacity is **MHT CET 2019**

(a) 32 μF (b) 18 μF (c) 8 μF (d) 44 μF

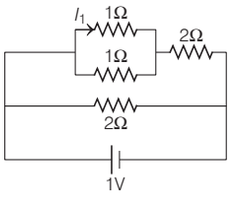
18. Five identical capacitors, of capacitance 20μF each, are connected to a battery of 150 V, in a combination as shown in the diagram. What is the total amount of charge stored? **WB JEE 2019**



(a) 15×10^{-3}C (b) 12×10^{-3}C
(c) 10×10^{-3}C (d) 3×10^{-3}C

Current Electricity

1. The current I_1 (in ampere) flowing through 1Ω resistor in the following circuit is **JEE Main 2020**



(a) 0.25 (b) 0.4 (c) 0.5 (d) 0.2

2. In a building, there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kW. The voltage of electric main is 220 V. The minimum fuse capacity (rated value) of the building will be **JEE Main 2020**

(a) 25 A (b) 10 A (c) 20 A (d) 15 A

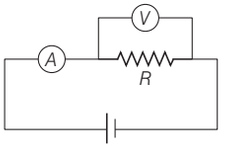
3. The length of a potentiometer wire is 1200 cm and it carries a current of 60 mA. For a cell of emf 5 V and internal resistance of 20 Ω, the null point on it is found to be at 1000 cm. The resistance of whole wire is **JEE Main 2020**

(a) 80 Ω (b) 100 Ω
(c) 60 Ω (d) 120 Ω

4. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is **JEE Main 2019**

(a) 11×10^{-4} W (b) 11×10^{-5} W
(c) 11×10^{5} W (d) 11×10^{-3} W

5. The actual value of resistance R, shown in the figure is 30 Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is **JEE Main 2019**

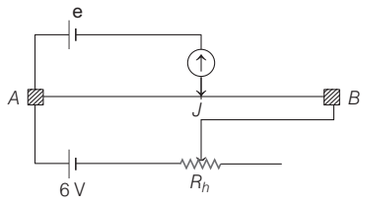


(a) 600 Ω (b) 570 Ω (c) 350 Ω (d) 35 Ω

6. An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and resistance 5 Ω. The value of R to give a potential difference of 5 mV across 10 cm of potentiometer wire is **JEE Main 2019**

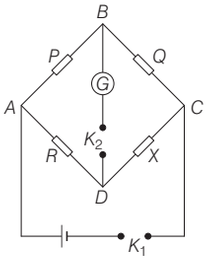
(a) 395 Ω (b) 495 Ω (c) 490 Ω (d) 480 Ω

7. The resistance of the meter bridge AB in given figure is 4 Ω. With a cell of emf $\varepsilon = 0.5$ V and rheostat resistance $R_h = 2\,\Omega$. The null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$, the same null point J is found for $R_h = 6\,\Omega$. The emf ε_2 is **JEE Main 2019**



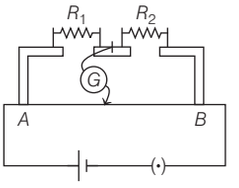
(a) 0.6 V (b) 0.3 V (c) 0.5 V (d) 0.4 V

8. In a Wheatstone bridge (see figure), resistances P and Q are approximately equal. When $R = 400\,\Omega$, the bridge is balanced. On interchanging P and Q, the value of R for balance is 405 Ω. The value of X is close to **JEE Main 2019**



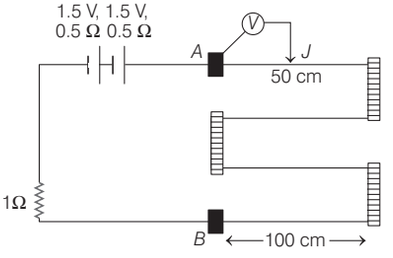
(a) 404.5 Ω (b) 401.5 Ω
(c) 402.5 Ω (d) 403.5 Ω

9. In the experimental set up of meter bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a 10 Ω resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1 + 10)$ Ω such that the null point shifts back to its initial position is **JEE Main 2019**



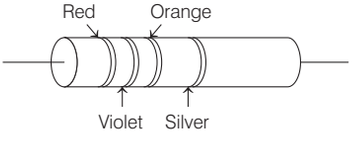
(a) 60 Ω (b) 20 Ω (c) 30 Ω (d) 40 Ω

10. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is $r = 0.01$ Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be **JEE Main 2019**



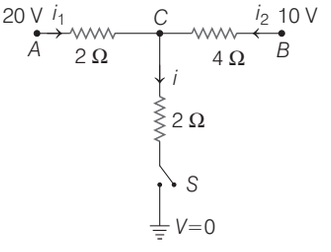
(a) 0.20 V (b) 0.75 V (c) 0.25 V (d) 0.50 V

11. A resistance is shown in the figure. Its value and tolerance are given respectively by **JEE Main 2019**



(a) 270 Ω, 5% (b) 27 k Ω, 20%
(c) 27 k Ω, 10% (d) 270 k Ω, 10%

12. When the switch S in the circuit shown is closed, then the value of current i will be **JEE Main 2019**

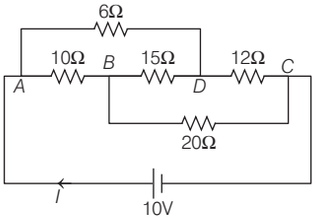


(a) 4A (b) 3A (c) 2A (d) 5A

13. A current of 5 A passes through a copper conductor (resistivity $= 1.7 \times 10^{-8}$ Ω-m) of radius of cross-section 5 mm. Find the mobility of the charges, if their drift velocity is 1.1×10^{-3} m/s. **JEE Main 2019**

(a) 1.5 m^2 / V-s (b) 1.3 m^2 / V-s
(c) 1.0 m^2 / V-s (d) 1.8 m^2 / V-s

14. The value of current I as shown in the given circuit diagram is **BITSAT 2019**



(a) 2 A (b) 1.5 A (c) 0.9 A (d) 0.2 A

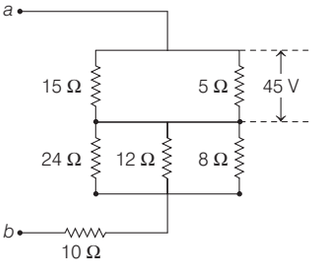
15. Four resistors A, B, C and D form a Wheatstone bridge as shown in the figure. The bridge is balanced when $C = 100$ Ω. If A and B are interchanged, the bridge balances for $C = 121$ Ω. The value of D is **AP EAMCET 2019**

(a) 10 Ω (b) 100 Ω
(c) 110 Ω (d) 120 Ω

16. Estimate the magnitude of current that passes through a wire, if 0.1 mol of electrons flow through it in 40 min. (Assume, Avagadro's number $= 6.0 \times 10^{23}$) **TS EAMCET 2019**

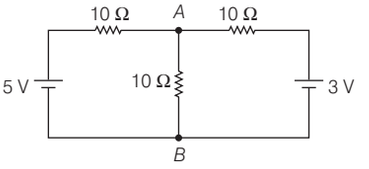
(a) 4 A (b) 9 A
(c) 12 A (d) 14 A

17. Find the potential difference between a and b, as shown in the below circuit, **TS EAMCET 2019**



(a) 165 V (b) 198 V (c) 213 V (d) 224 V

18. Calculate the voltage across AB terminals in the given circuit, **TS EAMCET 2019**



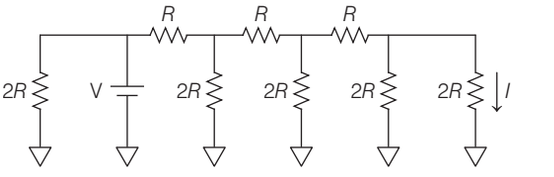
(a) $\frac{3}{8}$ V (b) $\frac{8}{3}$ V
(c) $\frac{3}{2}$ V (d) $\frac{2}{3}$ V

19. A potentiometer wire has length L. For given cell of emf E, the balancing length is $\frac{L}{3}$ from the positive end of the wire. If the length of potentiometer wire is increased by 50%, then for the same cell, the balance point is obtained at length **MHT CET 2019**

(a) $\frac{L}{2}$ from positive end (b) $\frac{L}{5}$ from positive end

(c) $\frac{L}{3}$ from positive en (d) $\frac{L}{4}$ from positive end

20. What is the current I shown in the given circuit? **WB JEE 2019**



(a) $\frac{V}{2R}$ (b) $\frac{V}{R}$ (c) $\frac{V}{16R}$ (d) $\frac{V}{8R}$

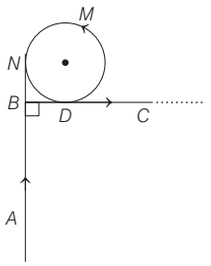
Magnetic Effects of Current

1. A particle of mass m and charge q has an initial velocity $\mathbf{v} = u_0\hat{\mathbf{j}}$. If an electric field $\mathbf{E} = E_0\hat{\mathbf{i}}$ and magnetic field $\mathbf{B} = B_0\hat{\mathbf{i}}$ act on the particle, its speed will double after a time **JEE Main 2020**

(a) $\frac{3mv_0}{qE_0}$ (b) $\frac{\sqrt{3}mv_0}{qE_0}$

(c) $\frac{2mv_0}{qE_0}$ (d) $\frac{\sqrt{2}mv_0}{qE_0}$

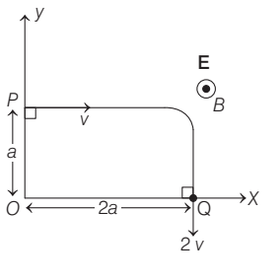
2. A very long wire $ABDMNDC$ is shown in figure carrying current I. AB and BC parts are straight, long and at right angle. At D wire forms a circular turn $DMND$ of radius R. AB, BC parts are tangential to circular turn at N and D. Magnetic field at the centre of circle is **JEE Main 2020**



(a) $\frac{\mu_0 I}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right)$ (b) $\frac{\mu_0 I}{2R}$

(c) $\frac{\mu_0 I}{2\pi R}(\pi + 1)$ (d) $\frac{\mu_0 I}{2\pi R}\left(\pi - \frac{1}{\sqrt{2}}\right)$

3. A charged particle of mass m and charge q moving under the influence of uniform electric field $E\hat{\mathbf{i}}$ and a uniform magnetic field $B\hat{\mathbf{k}}$ follows a trajectory from point P to Q as shown in the figure. The velocities at P and Q are respectively, $v\hat{\mathbf{i}}$ and $-2v\hat{\mathbf{j}}$. Then, which of the following statements (A, B, C, D) are the correct? (Trajectory shown is schematic and not to scale) **JEE Main 2020**



1. $E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$.
2. Rate of work done by the electric field at P is $\frac{3}{4}\left(\frac{mv^3}{a}\right)$.
3. Rate of work done by both the fields at Q is zero.
4. The difference between the magnitude of angular momentum of the particle at P and Q is $2mav$.

(a) (1), (3),(4) (b) (1), (2), (3)

(c) (1), (2), (3), (4) (d) (2), (3), (4)

4. A long, straight wire of radius a carries a current distributed uniformly over its cross-section. The ratio of the magnetic fields due to the wire at distance $\frac{a}{3}$ and $2a$ respectively, from the axis of the wire is **JEE Main 2019**

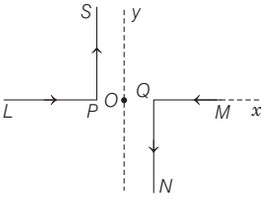
(a) $\frac{3}{2}$ (b) 2 (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

5. A galvanometer whose resistance is 50 Ω, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of **JEE Main 2019**

(a) 250 Ω (b) 6200 Ω

(c) 200 Ω (d) 6250 Ω

6. As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the X-axis, while segments PS and QN are parallel to the Y-axis. If $OP = OQ = 4$ cm and the magnitude of the magnetic field at O is 10^{-4} T and the two wires carry equal currents (see figure), the magnitude of the current in each wire



and the direction of the magnetic field at O will be

(Take, $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}) **JEE Main 2019**

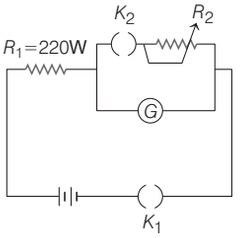
(a) 40 A, perpendicular out of the page

(b) 20 A, perpendicular into the page

(c) 20 A, perpendicular out of the page

(d) 40 A, perpendicular into the page

7. The galvanometer deflection, when key K_1 is closed but K_2 is open equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to 5 Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance of the galvanometer is given by (neglect the internal resistance of battery) : **JEE Main 2019**



(a) 22 Ω (b) 5 Ω (c) 25 Ω (d) 12 Ω

8. A proton and an α-particle (with their masses in the ratio of 1 : 4 and charges in the ratio of 1 : 2) are accelerated from rest through a potential difference V. If a uniform magnetic field B is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be **JEE Main 2019**

(a) 1 : $\sqrt{2}$ (b) 1 : $\sqrt{3}$ (c) 1 : 3 (d) 1 : 2

9. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately **JEE Main 2019**

(a) 10^{-3} (b) 10^{-4} (c) 10^{-1} (d) 10^{-2}

10. A rectangular coil (dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3A in the clockwise direction, is kept centred at the origin and in the X-Z

plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is **JEE Main 2019**

(a) 0.27 N-m (b) 0.38 N-m
(c) 0.42 N-m (d) 0.55 N-m

11. A rigid square loop of side a and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be **JEE Main 2019**

(a) repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$
(b) attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$
(c) zero
(d) repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$

12. A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0-5 V. Therefore, the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0.10 mA is **JEE Main 2019**

(a) 100 Ω (b) 500 Ω
(c) 200 Ω (d) 10 Ω

13. Two electrons are moving with speed of 5×10^5 m/s parallel to each other, then the electrostatic and magnetic force between them is **BITSAT 2019**

(a) 5×10^5
(b) 2.5×10^4
(c) 3.6×10^5
(d) 4.4×10^3

14. An electron accelerated through a potential difference V, passes through a uniform transverse magnetic field and experiences a force F. If the accelerating potential is increased to $2V$, the electron in the same magnetic field will experience a force. **AP EAMCET 2019**

(a) F (b) $\frac{F}{2}$ (c) $\sqrt{2}\,F$ (d) $2F$

15. A coil in the shape of an equilateral triangle of side 2 cm is suspended from a vertex such that it hangs in a vertical plane between the poles of a permanent magnet producing a horizontal magnetic field of 100×10^{-3} T. The magnetic field is parallel to the plane of the coil. For the moment of couple acting on the coil to be $2\sqrt{3} \times 10^{-5}$Nm, the current to be passed through the coil is **AP EAMCET 2019**

(a) 0.5 A (b) 1 A (c) 2 A (d) 4 A

16. A magnetic field of 5×10^{-5} T is produced at a perpendicular distance of 0.2 m from a long straight wire carrying electric current. If the permeability of free space is $4\pi \times 10^{-7}$ T m/A. The current passing through the wire in A is **TS EAMCET 2019**

(a) 45 (b) 40 (c) 50 (d) 30

17. A long wire carries a current of 18 A kept along the axis of a long solenoid of radius 1 cm. The field due to the solenoid is 8.0×10^{-3}T.

The magnitude of the resultant field at a point 0.6 mm from the solenoid axis is (Assume, $\mu_0 = 4\pi \times 10^{-7}$ Tm/A) **TS EAMCET 2019**

(a) 6×10^{-3} T (b) 6×10^{-4} T
(c) $2\sqrt{7} \times 10^{-3}$ T (d) 10×10^{-3} T

18. A circular coil of wire consisting of 100 turns each of radius 9 cm carries a current of 0.4 A. The magnitude of the magnetic field at the centre of coil is $[\mu_0 = 12.56 \times 10^{-7}$ SI Unit] **MHT CET 2019**

(a) 2.4×10^{-11} T
(b) 2.79×10^{-5} T
(c) 2.79×10^{-4} T
(d) 2.79×10^{-3} T

19. An electric current 'I' enters and leaves a uniform circular wire of radius r through diametrically opposite points. A particle carrying a charge q moves along the axis of the circular wire with speed v. What is the magnetic force experienced by the particle when it passes through the centre of the circle? **WB JEE 2019**

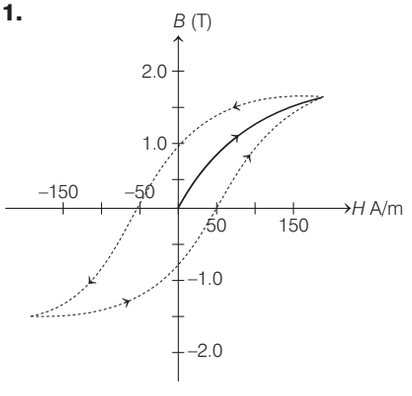
(a) $qv\frac{\mu_0 i}{a}$ (b) $qv\frac{\mu_0 i}{2a}$
(c) $qv\frac{\mu_0 i}{2\pi a}$ (d) Zero

20. A current 'I' is flowing along an infinite, straight wire, in the positive Z-direction and the same current is flowing along a similar parallel wire 5 m apart, in the negative Z-direction. A point P is at a perpendicular distance 3m from the first wire and 4m from the second. What will be magnitude of the magnetic field **B** of P? **WB JEE 2019**

(a) $\frac{5}{12}(\mu_0 I)$ (b) $\frac{7}{24}(\mu_0 I)$
(c) $\frac{5}{24}\frac{(\mu_0 I)}{\pi}$ (d) $\frac{25}{288}(\mu_0 I)$

Magnetism

1.



The figure gives experimentally measured B *versus* H variation in a ferromagnetic material. The retentivity, coercivity and saturation respectively of the material are **JEE Main 2020**

(a) 1.0 T, 50 A/m and 1.5 T
(b) 150 A/m, 1.0 T and 1.5 T
(c) 1.5 T, 50 A/m and 1.0 T
(d) 1.5 T, 50 A/m and 1.0 T

2 A small circular loop of conducting wire has radius a and carries current i. It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and released, it starts performing simple harmonic motion of time period T. If the mass of the loop is m, then **JEE Main 2020**

(a) $T = \sqrt{\frac{\pi m}{iB}}$ (b) $T = \sqrt{\frac{2m}{iB}}$
(c) $T = \sqrt{\frac{2\pi m}{iB}}$ (d) $T = \sqrt{\frac{\pi m}{2iB}}$

3 A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder.

They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then **JEE Main 2019**

(a) $T_h = 0.5\ T_c$ (b) $T_h = T_c$
(c) $T_h = 2\ T_c$ (d) $T_h = 1.5\ T_c$

4 At some location on earth, the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 A-m is suspended from its mid point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is **JEE Main 2019**

(a) 6.5×10^{-5} N (b) 3.6×10^{-5} N
(c) 1.3×10^{-5} N (d) 1.8×10^{-5} N

5 A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^{3} A/m is applied. Its magnetic susceptibility is **JEE Main 2019**

(a) 3.3×10^{-4} (b) 3.3×10^{-2}
(c) 4.3×10^{-2} (d) 2.3×10^{-2}

6 A magnetic compass needle oscillates 30 times per minute at a place, where the dip is 45° and 40 times per minute, where the dip is 30°. If B_1 and B_2 are respectively, the total magnetic field due to the earth at the two places, then the ratio $\frac{B_1}{B_2}$ is best given by **JEE Main 2019**

(a) 1.8 (b) 0.7
(c) 3.6 (d) 2.2

7 A proton is revolving on a circular path of radius 2 mm with frequency 10 Hz. Magnetic dipole moment associated with proton is **BITSAT 2019**

(a) 2×10^{-24} A-m^2 (b) 4×10^{-24} A-m^2
(c) 3×10^{-20} A-m^2 (d) 6×10^{-20} A-m^2

8 A metal rod is subjected to cycles of magnetisation at the rate of 42 Hz. Density of the metal is 6×10^{3} kg m^{-3} and its specific heat capacity is 0.1×10^{-3} cal kg^{-1}°C^{-1}. If the area of its *B-H* loop corresponds to energy density of 10^{-2} Jm^{-3}, then the rise in its temperature in one minute is **AP EAMCET 2019**

(a) 5°C (b) 10°C (c) 15°C (d) 20°C

9 A toroid has an iron core with an internal magnetic field of 10 π mT, when the current in the winding of 1500 turns per meter is 10 A. Determine the field due to magnetisation ($\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}). **TS EAMCET 2019**

(a) (4 π) mT (b) (10 π) mT
(c) $\left(\frac{8}{\pi}\right)$ mT (d) $\left(\frac{\pi}{4}\right)$ mT

10 The magnetization of bar magnet of length 5 cm, cross sectional area 2 cm^2 and net magnetic moment 1 Am^2 is **MHT CET 2019**

(a) 3×10^{5} A/m
(b) 4×10^{5} A/m
(c) 2×10^{5} A/m
(d) 1×10^{5} A/m

Electromagnetic Induction

1. Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The magnetic flux through the area of the circular coil area is given by ϕ_0. Which of the following option is correct? **JEE Main 2020**

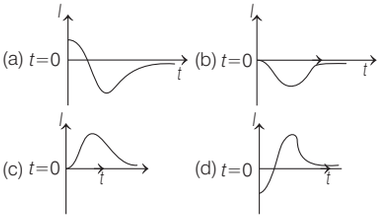
(a) $\phi_i > \phi_0$ (b) $\phi_i < \phi_0$
(c) $\phi_i = \phi_0$ (d) $\phi_i = -\phi_0$

2. The self-induced emf of a coil is 25 V. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is **JEE Main 2020**

(a) 437.5 J (b) 740 J (c) 637.5 J (d) 540 J

3. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t} (k > 0)$, as a function of time ($t \geq 0$). Counter clockwise current is taken to be positive.

A circular conducting coil of radius $2R$ is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by **JEE Main 2019**



4. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to **JEE Main 2020**

(a) $1/L$ (b) L^2 (c) L (d) $1/L^2$

5. A conducting circular loop is made of a thin wire has area 3.5×10^{-3}m^2 and resistance 10 Ω. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4\text{T})\sin(0.5\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during $t = 0$ s and $t = 10$ ms is close to **JEE Main 2019**

(a) 6 mC (b) 21 mC (c) 7 mC (d) 14 mC

6. There are two long coaxial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2, respectively. The ratio of mutual inductance to the self-inductance of the inner coil is **JEE Main 2019**

(a) $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$ (b) $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$
(c) $\frac{n_2}{n_1}$ (d) $\frac{n_1}{n_2}$

7. A coil is placed in a time varying magnetic field. The power dissipated due to current induced in the coil is P_1. If the number of turns is doubled and radius of the wire is halved, the power dissipated is P_2. Then $P_1 : P_2$ is **JEE Main 2019**

(a) 1 : 4 (b) 3 : 2 (c) 2 : 1 (d) 4 : 1

8. A metal disc of radius, $a = 10$ cm rotates with a constant angular speed of $\omega = 200$ rad/s about its axis. The potential difference between the centre and the rim of the disc under a uniform magnetic field, $B = 5$ mT directed perpendicular to the disc, is **AP EAMCET 2019**

(a) 2 mV (b) 5 mV
(c) 10 mV (d) 15 mV

9. Consider a current in a circuit falls from 6.0 A to 1.0 A in 0.2 s. If an average emf of 150 V is induced by the circuit, then the self inductance of the circuit is **TS EAMCET 2019**

(a) 2 H (b) 6 H
(c) 4 H (d) 8 H

10. The magnetic field of an electromagnetic- wave obeys the relation in a certain region is $B = 10^{-12}\sin(5\times10^{6}t)$ T, where t is the time. Then, the induced emf, in a 300 turns in coil of area 20 cm^2 oriented perpendicular to the field is **TS EAMCET 2019**

(a) $-2\times10^{-5}\cos(5\times10^{6}t)$ V
(b) $-3\times10^{-6}\cos(5\times10^{6}t)$ V
(c) $-2.5\times10^{-6}\cos(5\times10^{6}t)$ V
(d) $-3.3\times10^{-6}\cos(5\times10^{6}t)$ V

Alternating Current

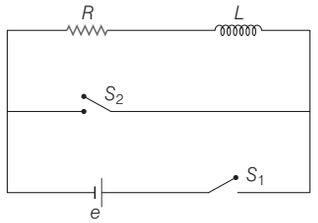
1. In L-C circuit, the inductance $L = 40$ mH and capacitance $C = 100\mu$F. If a voltage $V(t) = 10\sin(314t)$ is applied to the circuit, the current in the circuit is given as **JEE Main 2020**

(a) $0.52\cos 314t$ (b) $0.52\sin 314t$
(c) $10\cos 314t$ (d) $5.2\cos 314t$

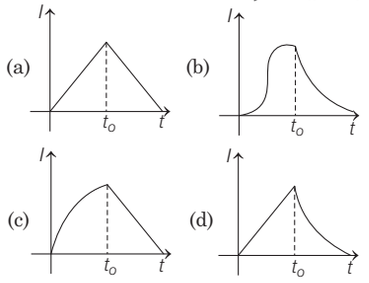
2. An emf of 20 V is applied at time $t = 0$ to a circuit containing in series 10 mH inductor and 5Ω resistor. The ratio of the currents at time $t = \infty$ and at $t = 40$ s is close to (Take, $e^2 = 7.389$) **JEE Main 2020**

(a) 1.06 (b) 1.15 (c) 1.46 (d) 0.84

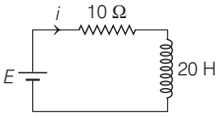
3. In the circuit shown,



The switch S_1 is closed at time $t = 0$ and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behaviour of the current I as a function of time 't' is given by **JEE Main 2019**

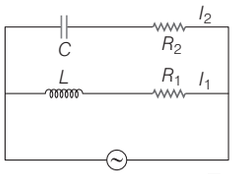


4. A 20 H inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is **JEE Main 2019**



(a) $\frac{2}{\ln 2}$ (b) $\frac{1}{2}\ln 2$
(c) $2\ln 2$ (d) $\ln 2$

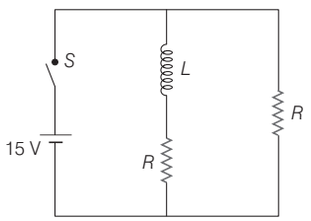
5.



In the above circuit, $C = \frac{\sqrt{3}}{2}$ μF, $R_2 = 20$ Ω, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10$ Ω. Current in L - R_1 path is I_1 and in C - R_2 path is I_2.The voltage of AC source is given by $V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is **JEE Main 2019**

(a) 30° (b) 60° (c) 0° (d) 90°

6. In the figure shown, a circuit contains two identical resistors with resistance $R = 5\,\Omega$ and an inductance with $L = 2$ mH. An ideal battery of 15 V is connected in the circuit.



What will be the current through the battery long after the switch is closed? **JEE Advanced 2019**

(a) 6 A (b) 3 A
(c) 5.5 A (d) 7.5 A

7. A charged capacitor of capacitance C is discharging through a resistor of resistance R

At what time the charge on the capacitor is equal to one half of its initial value? **BITSAT 2019**

(a) $\frac{RC}{2}$ (b) $\frac{RC}{\log_e 2}$
(c) $RC\log_e 2$ (d) $\frac{1}{RC}\log_e 2$

11. Two coils have a mutual inductance of 0.01H. The current in the first coil changes according to equation, $I = 5\sin 200\pi t$. The maximum value of *emf* induced in the second coil is **MHT CET 2019**

(a) 10 π V (b) 0.1 π V
(c) π V (d) 0.01 π V

8. When an inductor of inductance $\frac{6}{\pi}$ H, a capacitor of capacitance $\frac{50}{\pi}$μF and resistor of resistance R are connected in series with an AC supply of rms voltage 220 V and frequency 50 Hz, the rms current through the circuit is 440 mA. Match the inductive reactance, X_L the capacitive reactance, X_C the resistance R and the impedance Z of the circuit given in List-I with the corresponding values given in List-II. **AP EAMCET 2019**

	List- I		List- II
(A)	X_L	(i)	200 Ω
(B)	X_C	(ii)	300 Ω
(C)	R	(iii)	500 Ω
(D)	Z	(iv)	600 Ω

A B C D A B C D
(a)(iv) (ii) (i) (iii) (b)(iv)(iii)(i) (ii)
(c)(iv) (i) (ii) (iii) (d) (i) (iv)(iii)(ii)

9. An inductor coil is connected to a capacitor and an AC source of rms voltage 8V in series. The rms current in the circuit is 16 A and is in phase with emf. If this inductor coil is connected to 6 V DC battery, the magnitude of steady current is **AP EAMCET 2019**

(a) 8 A (b) 10 A (c) 12 A (d) 16 A

10. *LCR* circuit, the resonance frequency of circuit increases two times of the initial circuit by changing C and C' and R from 100 Ω to 400 Ω, while the inductance was kept the same. The ratio C / C', is **TS EAMCET 2019**

(a) 2 (b) 8 (c) 16 (d) 4

11. An alternating voltage is given by $E = 100\sin\left(\omega + \frac{\pi}{6}\right)$ V. The voltage will be maximum for the first time when is [T = periodic time) **MHT CET 2019**

(a) $\frac{T}{12}$ (b) $\frac{T}{2}$
(c) $\frac{T}{6}$ (d) $\frac{T}{3}$

MHT CET2019

(a) 3.6 A (b) 5.0 A (c) 2.5 A (d) 2.8 A

Geometric Optics

1. A vessel of depth $2h$ is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}$. The liquids are immiscible.The apparent depth of the inner surface of the bottom of vessel will be **JEE Main 2020**

(a) $\frac{h}{\sqrt{2}}$ (b) $\frac{3}{4}h\sqrt{2}$

(c) $\frac{h}{3\sqrt{2}}$ (d) $\frac{h}{2(\sqrt{2}+1)}$

2. A thin lens made of glass (refractive index = 1.5) of focal length $f = 16$ cm in immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_1, then the ratio f_1 / f is closest to the integer **JEE Main 2020**

(a) 1 (b) 5
(c) 9 (d) 17

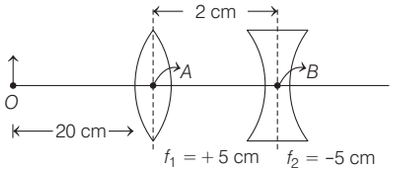
3. The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength,will be **JEE Main 2020**

(a) 45° (b) 60° (c) 15° (d) 30°

4. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus. **JEE Main 2019**

(a) 4.0 cm (b) 2 cm
(c) 3.1 cm (d) 1 cm

5. What is the position and nature of image formed by lens combination shown in figure? (where, f_1 and f_2 are focal lengths) **JEE Main 2019**



(a) $\frac{20}{3}$ cm from point B at right, real
(b) 70 cm from point B at right, real
(c) 40 cm from point B at right, real
(d) 70 cm from point B at left, virtual

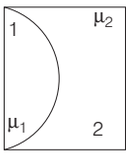
6. The wavelength of the carrier waves in a modern optical fibre communication network is close to **JEE Main 2019**

(a) 2400 nm (b) 1500 nm
(c) 600 nm (d) 900 nm

7. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and $x_2 (x_1 > x_2)$ from the lens. The ratio of x_1 and x_2 is **JEE Main 2019**

(a) 5 : 3 (b) 2 : 1
(c) 4 : 3 (d) 3 : 1

8. One plano-convex and one plano-concave lens of same radius of curvature R but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is



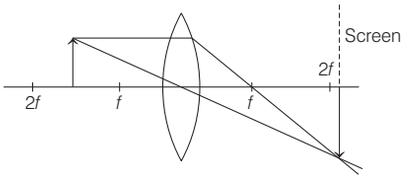
(a) $\frac{2R}{\mu_1 - \mu_2}$ (b) $\frac{R}{2-(\mu_1 - \mu_2)}$

(c) $\frac{R}{2(\mu_1 - \mu_2)}$ (d) $\frac{R}{\mu_1 - \mu_2}$

9. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is **JEE Main 2019**

(a) 0.16 m (b) 1.60 m
(c) 0.32 m (d) 0.24 m

10. Formation of real image using a biconvex lens is shown below. If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen? **JEE Main 2019**

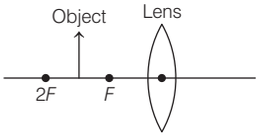


(a) No change
(b) Magnified image
(c) Image disappears
(d) Erect real image

11. A convex lens of focal length 25 cm produces images of the same magnification 2, when an object is kept at two positions x_1 and $x_2 (x_1 > x_2)$ from the lens. The ratio of x_2 and x_1 is **BITSAT 2019**

(a) 2 : 1 (b) 1 : 3
(c) 1 : 2 (d) 3 : 1

12. In the shown situation, if middle portion of the lens is painted black, then **BITSAT 2019**



(a) complete image of the object will form with low intensity
(b) complete image of the object will form with high intensity
(c) incomplete image of the object will form with low intensity
(d) incomplete image of the object will form with high intensity

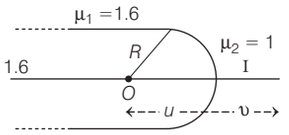
13. An object is placed 0.l m infront of a convex lens of focal length 20 cm made of a material of refractive index 1.5. The surface of the lens away from the object is silvered. If the radius of curvature of the silvered surface is 22 cm, then the distance of the final image from the silvered surface is **AP EAMCET 2019**

(a) 10 cm (b) 11 cm
(c) 12 cm (d) 13 cm

14. An object is fixed at the bottom of a vessel and water is filled in the vessel upto a height of 10 cm. A plane mirror is placed at a height of 7 cm from the surface of water in such a way that its reflecting surface faces the water. The distance of the image from the mirror is (Refractive index of water, $n = 1.33$) **AP EAMCET 2019**

(a) 7.5 cm (b) 7 cm
(c) 14.5 cm (d) 21.8 cm

15. A point object O is placed on the axis of a cylindrical piece of glass of refractive index 1.6 as shown in the figure. One surface of the glass piece is convex with radius of curvature 3 mm. The point appeared to be at 5 mm on the axis when viewed along the axis and from right side of convex surface. The distance of the point object from the convex surface is : **TS EAMCET 2019**



(a) 4 mm (b) 6 mm
(c) 3 mm (d) 2.5 mm

16. A thin hollow prism of refracting angle 3°, filled with water gives a deviation of 1°. The refractive index of water is **MHT CET 2019**

(a) 1.59 (b) 1.33
(c) 1.46 (d) 1.51

17. The luminous border that surrounds the profile of a mountain just before sun rises behind it, is an example of **MHT CET 2019**

(a) dispersion
(b) total internal reflection
(c) interference
(d) diffraction

18. When light enters glass from vacuum, then the wavelength of light **MHT CET 2019**

(a) decreases (b) becomes zero
(c) remains same (d) increases

19. The magnifying power of a telescope is nine. When it is adjusted for parallel rays, the distance between the objective and eyepiece is 20 cm. The focal length of objective and eyepiece are respectively **MHT CET 2019**

(a) 10 cm, 10 cm (b) 18 cm, 2 cm
(c) 15 cm, 5 cm (d) 11 cm, 9 cm

20. A point object is placed on the axis of a thin convex lens of focal length 0.05 m at a distance of 0.2 m from the lens and its image is formed on the axis. If the object is now made to oscillate along the axis with a small amplitude of A cm, then what is the amplitude of oscillation of the image? **WB JEE 2019**

[you may assume, $\frac{1}{1+x} \approx 1-x$, where $x << 1$]

(a) $\frac{4A}{9} \times 10^{-2}$ m (b) $\frac{5A}{9} \times 10^{-2}$ m
(c) $\frac{A}{3} \times 10^{-2}$ m (d) $\frac{A}{9} \times 10^{-2}$ m

Interference and Diffraction of Light

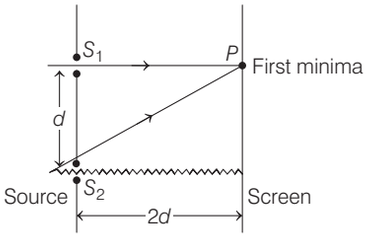
1. The aperture of a telescope is 5m. The separation between the moon and the earth is 4×10^5 km. With light of wavelength of 5500 Å, the minimum separation between objects on the surface of moon, so that they are just resolved, is close to **JEE Main 2020**

(a) 600 m (b) 20 m
(c) 60 m (d) 200 m

2. In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is **JEE Main 2020**

(a) 4.9 mm (b) 3.9 mm
(c) 6.9 mm (d) 5.9 mm

3. Consider a Young's double slit experiment as shown in figure.



What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1) ? **JEE Main 2019**

(a) $\frac{\lambda}{2(5-\sqrt{2})}$ (b) $\frac{\lambda}{(5-\sqrt{2})}$
(c) $\frac{\lambda}{2(\sqrt{5}-2)}$ (d) $\frac{\lambda}{(\sqrt{5}-2)}$

4. In a Young's double slit experiment, the path difference at a certain point on the screen between two interfering waves is $\frac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to **JEE Main 2019**

(a) 0.80 (b) 0.74 (c) 0.94 (d) 0.85

5. In an interference experiment, the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be **JEE Main 2019**

(a) 2 (b) 18 (c) 4 (d) 9x

6. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio **JEE Main 2019**

(a) 16 : 9 (b) 5 : 3
(c) 25 : 9 (d) 4 : 1

7. A system of three polarisers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60° to the pass axis of P_3. When a beam of unpolarised light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarisers is I. The ratio (I_0 / I) equals (nearly)

(a) 5.33 (b) 16.00
(c) 10.67 (d) 1.80 **JEE Main 2019**

8. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used) **JEE Main 2019**

(a) $\frac{2\lambda}{(\mu-1)}$ (b) $\frac{\lambda}{2(\mu-1)}$
(c) $\frac{\lambda}{(\mu-1)}$ (d) $\frac{\lambda}{(2\mu-1)}$

9. Two monochromatic light waves are travelling with same frequency and constant phase different. If both waves interfere, then **BITSAT 2019**

(a) there is a gain of energy
(b) there is a loss of energy
(c) the energy is redistributed and the distribution remains constant in time
(d) the energy is redistributed and the distribution changes with time

10. The angular deviation of 5th order dark fringe is 12° in a single slit experiment. If the width of the slit is 9 μm then the wavelength of the incident light is **AP EAMCET 2019**

(a) 4862 Å (b) 5892 Å
(c) 6022 Å (d) 3768 Å

11. When light of an unknown polarisation is examined with a polaroid, it is found to exhibit maximum intensity I_0 along y-axis and minimum intensity $\frac{2I_0}{3}$ along x-axis. The intensity transmited through a polaroid with pass axis at 45° to y-axis (in x-y plane) is **AP EAMCET 2019**

(a) $\frac{5}{8}I_0$ (b) $\frac{I_0}{2}$
(c) $\frac{5}{6}I_0$ (d) $\frac{I_0}{4}$

12. In a Young's double slit experiment, mth order and nth order of bright fringes are formed at point P on a distant screen, if monochromatic source of wavelength 400 nm and 600 nm are used respectively. The minimum value of m and n are respectively, **TS EAMCET 2019**

(a) 4,6 (b) 3,2
(c) 2,3 (d) 4,2

13. The limit of resolution of a telescope is 2.5×10^{-7} rad. If the telescope is used to detect light of wavelength 500 nm coming from a star, the diameter of the objective lens used by telescope is **TS EAMCET 2019**

(a) 244 cm (b) 258 cm
(c) 228 cm (d) 264 cm

14. If a star emitting yellow light is accelerated towards earth, then to an observer on earth it will appear

(a) becoming orange. **MHT CET 2019**
(b) shining yellow.
(c) gradually changing to blue.
(d) gradually changing to red.

15. Light of wavelength 'λ' is incident on a single slit of width 'a' and the distance between slit and screen is 'D'. In diffraction pattern, if slit width is equal to the width of the central maximum then 'D' is equal to **MHT CET 2019**

(a) $\frac{a}{2\lambda}$ (b) $\frac{a^2}{2\lambda}$
(c) $\frac{a}{\lambda}$ (d) $\frac{a^2}{\lambda}$

16. The phenomenon of interference is based on **MHT CET 2019**

(a) conservation of momentum
(b) quantum nature of light
(c) conservation of energy
(d) conservation of charge

17. In Young's experiment for the interference of light, the separation between the slits is d and the distance of the screen from the slits is D. If D is increased by 0.5% and d is decreased by 0.3% then for the light of a given wavelength, which one of the following is true? **WB JEE 2019**

"The fringe width .....

(a) increases by 0.8% (b) decreases by 0.8%
(c) increases by 0.2% (d) decreases by 0.2%

Modern Physics

1. An electron (mass m) with initial velocity $\mathbf{v} = v_0\hat{\mathbf{i}} + v_0\hat{\mathbf{j}}$ is in an electric field $\mathbf{E} = -E_0\hat{\mathbf{k}}$. If λ_0 is initial de-Broglie wavelength of electron, then its de Broglie wavelength at time t is given by **JEE Main 2020**

(a) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}}$ (b) $\frac{\lambda_0}{\sqrt{2+\frac{e^2E_0^2t^2}{m^2v_0^2}}}$

(c) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2t^2}{2m^2v_0^2}}}$ (d) $\frac{\lambda_0\sqrt{2}}{\sqrt{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}}$

2. If the magnetic field in a plane electromagnetic wave is given by $\mathbf{B}=3\times10^{-8}\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{\mathbf{j}}\,\text{T}$, then what will be expression for electric field? **JEE Main 2020**

(a) $\mathbf{E}=60\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{\mathbf{k}}\,\text{V/m}$

(b) $\mathbf{E} = 3\times10^{-8}\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{\mathbf{i}}\,\text{V/m}$

(c) $\mathbf{E} = 3\times10^{-8}\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{\mathbf{j}}\,\text{V/m}$

(d) $\mathbf{E}=9\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{\mathbf{k}}\,\text{V/m}$

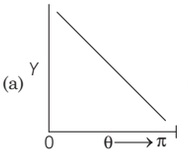
3. Radiation with wavelength 6561Å falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of 3×10^{-4}T. If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal is close to **JEE Main 2020**

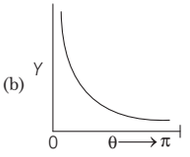
(a) 0.8 eV (b) 1.1 eV
(c) 1.8 eV (d) 1.6 eV

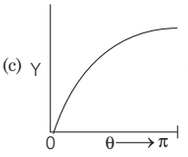
4. The energy required to ionise a hydrogen like ion in its ground state is 9 Rydbergs. What is the wavelength of the radiation emitted when the electron in this ion jumps from the second excited state to the ground state ? **JEE Main 2020**

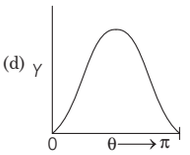
(a) 8.6 nm (b) 24.2 nm
(c) 11.4 nm (d) 35.8 nm

5. The graph which depicts the results of Rutherford gold foil experiment with α-particles is θ : Scattering angle
Y : Number of scattered α-particles detected
(plots are schematic and not to scale) **JEE Main 2020**

(a) 

(b) 

(c) 

(d) 

6. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric fields and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by **JEE Main 2020**

(a) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$ (b) $(\sqrt{n}, \sqrt{n})$
(c) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ (d) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

7. If the de-Broglie wavelength of an electron is equal to 10^{-3} times, the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to
(Take, speed of light = 3×10^8 m/s, Planck's constant = 6.63×10^{-34} J-s and mass of electron = 9.1×10^{-31} kg) **JEE Main 2020**

(a) 1.45×10^6 m/s (b) 1.8×10^6 m/s
(c) 1.1×10^6 m/s (d) 1.7×10^6 m/s

8. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state in terms of Bohr radius a_0 will be (Take $hc = 12500$ eV-Å) **JEE Main 2020**

(a) $4a_0$ (b) $9a_0$ (c) $16a_0$ (d) $25a_0$

9. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ n-m and then by light of wavelength $\lambda_2 = 540$ n-m. It is found that the maximum speed of the photoelectrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to
(energy of photon = $\frac{1240}{\lambda(\text{in n-m})}$ eV) **JEE Main 2020**

(a) 5.6 (b) 2.5 (c) 1.8 (d) 1.4

10. The electric field of light wave is given as

$$\mathbf{E} = 10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14}t\right)\hat{\mathbf{x}}\ \text{NC}^{-1}.$$

This light falls on a metal plate of work function 2eV. The stopping potential of the photoelectrons is

Given, E (in eV) $= \dfrac{12375}{\lambda(\text{in Å})}$

JEE Main 2020

(a) 0.48 V (b) 0.72 V
(c) 2.0 V (d) 2.48 V

11. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2nd Balmer line (n = 4 to n = 2) will be **JEE Main 2020**

(a) 889.2 nm (b) 388.9 nm
(c) 642.7 nm (d) 488.9 nm

12. Two particles move at right angle to each other. Their de-Broglie wavelengths are λ_1 and λ_2, respectively. The particles suffer perfectly inelastic collision. The de-Broglie wavelength λ of the final particle, is given by **JEE Main 2019**

(a) $\dfrac{1}{\lambda^2} = \dfrac{1}{\lambda_1^2} + \dfrac{1}{\lambda_2^2}$ (b) $\lambda = \sqrt{\lambda_1\lambda_2}$
(c) $\lambda = \dfrac{\lambda_1 + \lambda_2}{2}$ (d) $\dfrac{2}{\lambda} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$

13. A nucleus A, with a finite de-Broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same directions as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelengths λ_B and λ_C of B and C respectively **JEE Main 2019**

(a) $2\lambda_A, \lambda_A$ (b) $\dfrac{\lambda_A}{2}, \lambda_A$
(c) $\lambda_A, 2\lambda_A$ (d) $\lambda_A, \dfrac{\lambda_A}{2}$

14. In a radioactive sample, $^{40}_{19}$K nuclei either decay into stable $^{40}_{20}$Ca nuclei with decay constant 4.5×10^{-10} per year or into stable $^{40}_{18}$Ar nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable $^{40}_{20}$Ca and $^{40}_{18}$Ar nuclei are produced by the $^{40}_{19}$K nuclei only. In time $t\times10^9$ years, if the ratio of the sum of stable $^{40}_{20}$Ca and $^{40}_{18}$Ar nuclei to the radioactive $^{40}_{19}$K nuclei is 99, the value of t will be [Given: ln 10 = 2.3] **JEE Advanced 2019**

(a) 9.2 (b) 1.15
(c) 4.6 (d) 2.3

15. A radioactive sample of half life 23.1 days is disintegrating continuously. The percentage decay of its in 15th to 16th days will be [Take, $e^{0.03} = 1.03$] **JEE Advanced 2019**

(a) 5% (b) 1% (c) 2.9% (d) 3.5%

16. An electron revolves in a circular orbit of radius r with angular speed ω. The magnetic field at the centre of electron orbit is **BITSAT 2019**

a. $\dfrac{\mu_0 e\omega}{\pi r}$ b. $\dfrac{\mu_0 e\omega}{4\pi r}$ c. $\dfrac{\mu_0 e^2\omega}{4\pi r}$ d. $\dfrac{\mu_0 e\omega}{2\pi r}$

17. An electron of charge e and mass m moving with an initial velocity $v_0\hat{\mathbf{i}}$ is subjected to all electric field $E_0\hat{\mathbf{j}}$. The de-Broglie wavelength of the electron at a time t is
(Initial de-Broglie wavelength of the electron = λ_0) **BITSAT 2019**

(a) λ_0 (b) $\lambda_0\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}$
(c) $\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E_0^2t^2}{m^2v_0^2}}}$ (d) $\dfrac{\lambda_0}{\left(1+\dfrac{e^2E_0^2t^2}{mv_0^2}\right)}$

18. Match the following List-I with List-II in connection with Bohr's atomic model.

	List I		List II
(A)	Speed of revolution of electron	(i)	$\dfrac{1}{4\pi\in_0}\dfrac{2\pi Ze^2}{nh}$
(B)	Kinetic energy	(ii)	$-\left(\dfrac{1}{4\pi\in_0}\right)^2\dfrac{2\pi^2me^4Z^2}{n^2h^2}$
(C)	Total energy	(iii)	$\left(\dfrac{1}{4\pi\in_0}\right)^2\dfrac{2\pi^2me^4Z^2}{n^2h^2}$
(D)	Frequency	(iv)	$\left(\dfrac{1}{4\pi\in_0}\right)^2\dfrac{4\pi^2Z^2e^4m}{n^3h^3}$

The correct answer is **AP EAMCET 2019**

	A	B	C	D		A	B	C	D
(a)	i	iii	ii	iv	(b)	ii	iv	iii	i
(c)	iii	i	iv	ii	(d)	iii	i	ii	iv

19. Half-life of a radioactive substance is 18 minutes. The time interval between its 20% decay and 80% decay in minutes is **AP EAMCET 2019**

(a) 6 (b) 9 (c) 18 (d) 36

20. The half life of neutron is 693 seconds. What fraction of neutrons will decay when a beam of neutrons, having kinetic energy of 0.084 eV, travells a distance of 1 km?
(mass of neutron = 1.68×10^{-27} kg, and ln 2 = 0.693)

TS EAMCET 2019

(a) 60×10^{-5} (b) 15×10^{-5} (c) 25×10^{-5} (d) 50×10^{-5}

21. In a hydrogen atom, an electron of charge e revolves in a orbit of radius r with speed v. Then, magnetic moment associated with electron is **MHT CET 2019**

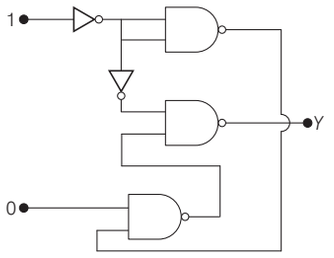
(a) $\dfrac{evr}{2}$ (b) $2\,evr$ (c) evr (d) $\dfrac{evr}{3}$

22. A parent nucleus X undergoes α-decay with a half-life of 75000 yrs. The daughter nucleus Y undergoes β-decay with a half-life of 9 months. In a particular sample, it is found that the rate of emission of β-particles is nearly constant (over several months) at 10^7/h. What will be the number of α-particles emitted in an hour? **WB JEE 2019**

(a) 10^2 (b) 10^7 (c) 10^{12} (d) 10^{14}

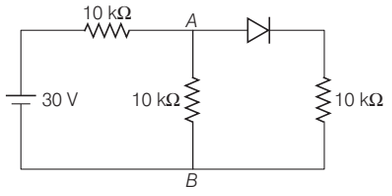
Solids and Semiconductor Devices

1. In the given circuit, value of Y is **JEE Main 2020**



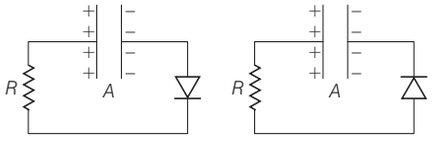
(a) toggles between 0 and 1
(b) 1
(c) 0
(d) will not execute

2. In the figure, potential difference between A and B is **JEE Main 2020**



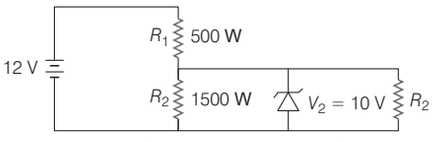
(a) zero (b) 10 V
(c) 5 V (d) 15 V

3. Two identical capacitors A and B, charged to the same potential $5V$ are connected in two different circuits as shown below at time $t = 0$. If the charge on capacitors A and B at time $t = CR$ is Q_A and Q_B respectively, then (here e is the base of natural logarithm) **JEE Main 2020**



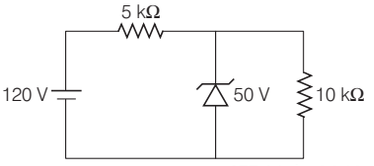
(a) $Q_A = VC, Q_B = \frac{VC}{e}$
(b) $Q_A = \frac{VC}{e}, Q_B = \frac{CV}{2}$
(c) $Q_A = \frac{CV}{2}, Q_B = \frac{VC}{e}$
(d) $Q_A = VC, Q_B = CV$

4. In the given circuit, the current through zener diode is close to **JEE Main 2020**



(a) 6.0 mA (b) 6.7 mA
(c) 0 (d) 4.0 mA

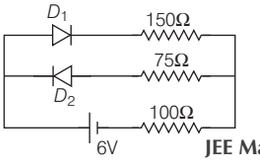
5. For the circuit shown below, the current through the Zener diode is



JEE Main 2019

(a) 14 mA (b) zero (c) 5 mA (d) 9 mA

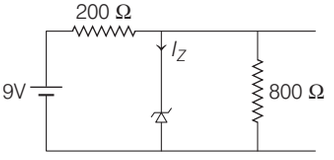
6. The circuit shown below contains two ideal diodes, each with a forward resistance of 50 Ω. If the battery voltage is 6 V, the current through the 100 Ω resistance (in ampere) is



JEE Main 2019

(a) 0.027 (b) 0.020
(c) 0.030 (d) 0.036

7. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit. **JEE Main 2019**



The current I_z through the Zener is
(a) 10 mA (b) 17 mA
(c) 15 mA (d) 7 mA

8. In a transistor, the value of α varies between $\frac{20}{21}$ and $\frac{100}{101}$. Then the value of β varies between **JEE Main 2019**

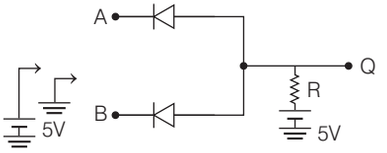
(a) 1 and 10 (b) 0.95 and 0.99
(c) 20 and 100 (d) 200 and 300

9. For a given truth table A, B and C are inputs and Y is the output, then the functional form of the circuit is **TS EAMCET 2019**

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

(a) $\overline{A}$ (b) $\overline{B}$
(c) $\overline{A} + BC$ (d) $A + B + C$

10. Two diodes are connected in the following fashion. Provision is made to connect either +5 V or ground (0 V) to the points A to B. The output Q will act as **TS EAMCET 2019**

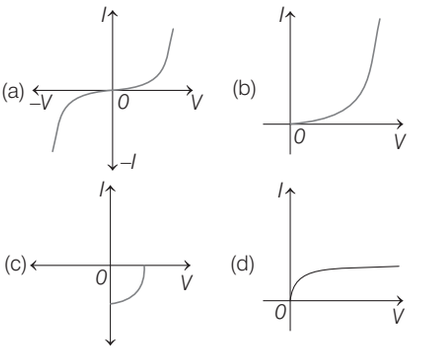


(a) OR gate
(b) AND gate
(c) XOR gate
(d) NAND gate

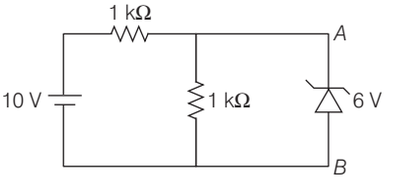
11. For a transistor, the current ratio 'β_{dc}' is defined as the ratio of **MHT CET 2019**

(a) collector current to emitter current.
(b) collector current to base current.
(c) base current to collector current.
(d) emitter current to collector current.

12. V-I characterstics of LED is shown correctly by graph **MHT CET 2019**



13. In the circuit shown, what will be the current through the 6V zener? **WB JEE 2019**



(a) 6 mA, from A to B (b) 2 mA, from A to B
(c) 2 mA, from B to A (d) Zero

14. Each of the two inputs A and B can assume values either 0 or 1. Then which of the following will be equal to $\overline{A} \cdot \overline{B}$? **WB JEE 2019**

(a) $A + B$ (b) $\overline{A + B}$
(c) $\overline{A \cdot B}$ (d) $\overline{A} + \overline{B}$

Basics of Communications

1. An amplitude modulates signal is given by $v(t) = 10[1 + 0.3\cos(2.2 \times 10^4 t)]$ $\sin(5.5 \times 10^5 t)$.

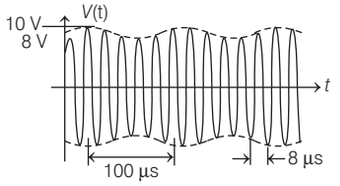
Here, t is in seconds. The sideband frequencies (in kHz) are $\left(\text{Take, } \pi = \frac{22}{7}\right)$ **JEE Main 2019**

(a) 892.5 and 857.5 (b) 89.25 and 85.75
(c) 178.5 and 171.5 (d) 1785 and 1715

2. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license, what broadcast frequency will you allot? **JEE Main 2019**

(a) 2000 kHz (b) 2250 kHz
(c) 2900 kHz (d) 2750 kHz

3. An amplitude modulated signal is plotted below



Which one of the following best describes the above signal? **JEE Main 2019**

(a) $[1 + 9\sin(2\pi \times 10^4 t)]\sin(2.5\pi \times 10^5 t)$V
(b) $[9 + \sin(2\pi \times 10^4 t)]\sin(2.5\pi \times 10^5 t)$V
(c) $[9 + \sin(4\pi \times 10^4 t)]\sin(5\pi \times 10^5 t)$V
(d) $[9 + \sin(2.5\pi \times 10^5 t)]\sin(2\pi \times 10^4 t)$V

4. A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index? **JEE Main 2019**

(a) 0.4 (b) 0.5 (c) 0.6 (d) 0.3

5. A signal $A\cos\omega t$ is transmitted using $v_0 \sin\omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is **JEE Main 2019**

(a) $(v_0 \sin\omega_0 t + A\cos\omega t$
(b) $(v_0 + A)\cos\omega t \sin\omega_0 t$
(c) $v_0 \sin[\omega_0 (1 + 0.01A\sin\omega t)t]$
(d) $v_0 \sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$

6. Carrier waves are used for communicating signals over long distance, because **TS EAMCET 2019**

(a) carrier waves can be generated very easily.
(b) low frequencies can not be easily modulated by the carrier waves.
(c) low frequencies can be transmitted over long distances.
(d) carrier waves generated at higher frequencies can be transmitted over long distances.

7. The transmitting antenna placed at the top of a tower has a height of 45 m from the ground. The distance between receiving and transmitting antennas is 40 km and the radius of earth is 6400 km. The minimum height (in m) at which the receiving antenna is to be placed for satisfactory communication in LOS made, is **TS EAMCET 2019**

(a) 5 (b) 15 (c) 20 (d) 25

8. In amplitude modulation, the amplitude of carrier wave is A_c and that of the modulating signal is A_m. In practice, the ratio of A_m to A_c is kept less than or equal to one, to avoid **MHT CET 2019**

(a) distortion (b) attenuation
(c) fading (d) noise

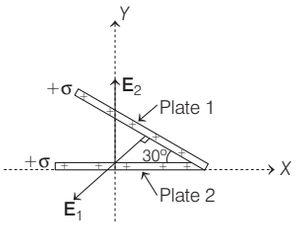
9. In frequency modulated wave **MHT CET 2019**

(a) frequency varies with time
(b) both frequency and amplitude vary with time
(c) amplitude varies with time
(d) both frequency and amplitude are constant

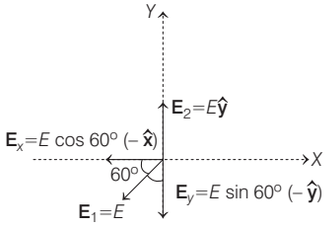
Solutions with Explanation

Electrostatics

1 *(b)* Electric field of an infinite plate is perpendicular to the plane of plate and its magnitude is $E = \frac{\sigma}{2\varepsilon_0}$. We are given with two positively charged plates with a set of coordinate axes as shown in the figure.



From geometry of figure, net electric field in region between plates is resultant of fields of both plates,



Now, field of plate 1 can be resolved along X and Y-axes as shown in above figure.

Now, $\mathbf{E}_{net} = \mathbf{E}_1 + \mathbf{E}_2$

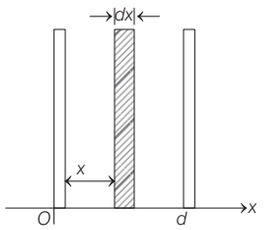
$= E\cos 60°(-\hat{\mathbf{x}}) + E\sin 60°(-\hat{\mathbf{y}}) + E\hat{\mathbf{y}}$

Here, $E = \frac{\sigma}{2\varepsilon_0}$ and $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are unit vectors along X and Y-axes.

$$= \frac{\sigma}{2\varepsilon_0}\left(-\frac{\hat{\mathbf{x}}}{2} - \frac{\sqrt{3}}{2}\hat{\mathbf{y}} + \hat{\mathbf{y}}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\left(1 - \frac{\sqrt{3}}{2}\right)\hat{\mathbf{y}} - \frac{\hat{\mathbf{x}}}{2}\right]$$

2 *(c)* Consider a dielectric slab of differentiable thickness dx at a distance x from left hand side plate.



Capacity of this differential thickness capacitor,

$$dC = \frac{\varepsilon_0 KA}{dx} = \frac{\varepsilon_0 K(1+\alpha x)A}{dx}$$

Total capacity of system is equivalent capacitance of series combination of these differential capacitors.

As in series, $\frac{1}{C_{eq}} = \sum_{i=1}^{n}\frac{1}{C_i}$

So, $$\frac{1}{C_{eq}} = \int_0^d \frac{1}{dC} = \int_0^d \frac{dx}{\varepsilon_0 K(1+\alpha x)A}$$

or $$\frac{1}{C_{eq}} = \frac{1}{\varepsilon_0 AK}\int_0^d \frac{dx}{1+\alpha x}$$

$$= \frac{1}{\varepsilon_0 A\alpha K}[\ln(1+\alpha x)]_0^d$$

$$= \frac{1}{\varepsilon_0 A\alpha K}[\ln(1+d\alpha) - \ln(1)]$$

$$= \frac{1}{\varepsilon_0 A\alpha K}\ln(1+d\alpha)$$

Now, we use expansion,

$\ln(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3}$ ....., we get

$$\frac{1}{C_{eq}} = \frac{1}{\varepsilon_0 A\alpha K}\left(\alpha d - \frac{(\alpha d)^2}{2} + \frac{(\alpha d)^3}{3} \ldots\ldots\right)$$

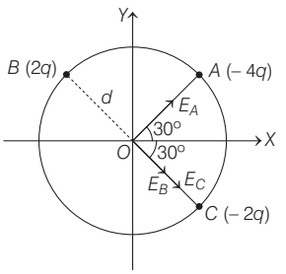
$$= \frac{\alpha d}{\varepsilon_0 A\alpha K}\left(1 - \frac{\alpha d}{2} + \frac{(\alpha d)^2}{3}\ldots\right)$$

or $$C_{eq} = \frac{\varepsilon_0 AK}{d}\left(1 - \frac{\alpha d}{2}\right)^{-1}$$

From binomial approximation $(1+x)^n = 1 + nx$,

we have $$C_{eq} = \frac{\varepsilon_0 AK}{d}\left(1 + \frac{\alpha d}{2}\right)$$

3 *(d)* Fields of charges at A, B and C are as shown below.



Magnitude of resultant field component directed along positive X-axis,

$E_x = E_A\cos 30° + E_B\cos 30° + E_C\cos 30°$

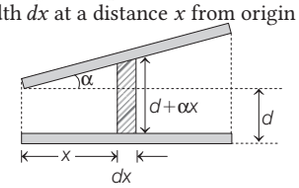
$= (E_A + E_B + E_C)\cos 30°$

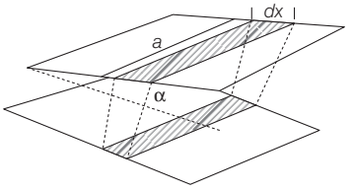
$$= \left(\frac{k(4q)}{d^2} + \frac{k(2q)}{d^2} + \frac{k(2q)}{d^2}\right)\times\frac{\sqrt{3}}{2}$$

$$= \frac{1}{4\pi\varepsilon_0}\times\frac{8q}{d^2}\times\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad E_x = \frac{\sqrt{3}\,q}{\pi\varepsilon_0 d^2}$$

4 *(c)* For given capacitor, consider a small width dx at a distance x from origin O.





For this elemental capacitor, we have

Area of each plate $= a \times dx$

Distance between plates $= d + \alpha x$

Capacity of this elemental capacitor,

$$dC = \frac{\varepsilon_0 a\,dx}{d+\alpha x}$$

Total capacity of arrangement is sum of capacitances of all such elemental capacitors because they all are in parallel.

Hence,

$$C_{system} = \int_0^a dC = \int_0^a \frac{\varepsilon_0 a}{d+\alpha x}\cdot dx$$

$$= \frac{\varepsilon_0 a}{\alpha}\ln\left(1 + \frac{\alpha a}{d}\right)$$

As, for small values of $\frac{a\alpha}{d}$,

$$\ln\left(1+\frac{a\alpha}{d}\right) \approx \frac{a\alpha}{d} - \frac{1}{2}\left(\frac{a\alpha}{d}\right)^2$$

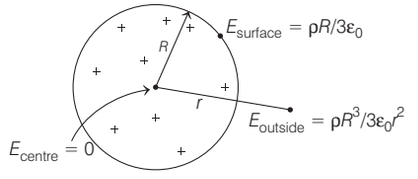
So, $$C_{system} = \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{a\alpha}{2d}\right)$$

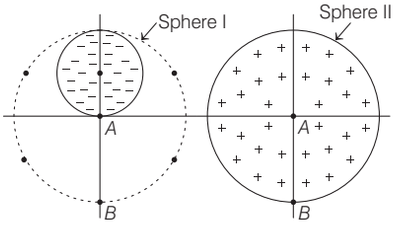
5 *(d)* Electric field at points A and B can be viewed as a superposition of

(i) electric field due to complete solid sphere of radius R and charge density ρ.

(ii) electric field due to sphere of radius $\frac{R}{2}$ and charge density $-\rho$.

Now, we use standard result of electric field due to a solid sphere which are





Now, electric field due to sphere I at points A and B are

$$E_{1A} = \frac{\rho\left(\frac{R}{2}\right)}{3\varepsilon_0} = \frac{\rho R}{6\varepsilon_0}$$

$$E_{IB} = \frac{\rho\left(\frac{R}{2}\right)^3}{3\varepsilon_0\left(\frac{3R}{2}\right)^2} = \frac{\rho R}{54\varepsilon_0}$$

and similarly due to sphere II,

$$E_{IIA} = 0 \Rightarrow E_{IIB} = \frac{\rho R}{3\varepsilon_0}$$

So, net field magnitude at point A,

$$E_A = E_{IA} + E_{IIA} = \frac{\rho R}{6\varepsilon_0} + 0 = \frac{\rho R}{6\varepsilon_0}$$

and net field magnitude to point B,

$$E_B = \frac{\rho R}{3\varepsilon_0} - \frac{\rho R}{54\varepsilon_0} = \frac{17}{54}\frac{\rho R}{\varepsilon_0}$$

Hence, ratio

$$\frac{E_A}{E_B} = \frac{\rho R}{6\varepsilon_0} \times \frac{54\varepsilon_0}{17\rho R} = \frac{18}{34}$$

6 *(b)* Since, $Q = CV = \frac{\varepsilon_0 A}{d} \cdot Ed = \varepsilon_0 AE$

Here, $A = 1\ \text{m}^2$, $E = 100\ \text{N/C}$

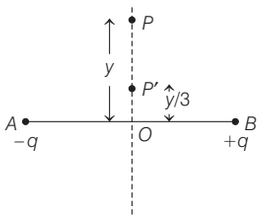
and $\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{\text{C}^2}{\text{N} - \text{m}^2}$

So, by substituting given values, we get

$Q = 8.85 \times 10^{-12} \times 1 \times 100 = 8.85 \times 10^{-10}$ C

7 *(d)* Electric field on the equatorial line of a dipole

$$E = \frac{2kP}{(r^2 + a^2)^{3/2}} \quad \text{... (i)}$$



In first case $r = y$

$$\Rightarrow \quad E_1 = \frac{2kP}{(y^2 + a^2)^{3/2}}$$

$$E_1 = \frac{2kP}{y^3} \quad \text{... (ii)}$$

$\because$ Here, $y^2 >> a^2$

So, force on the charge q

$$F = QE_1 = \frac{2kPQ}{y^3} \quad \text{... (iii)}$$

In second case $r = y/3$

From Eq. (i), electric field at point P' will be

$$E_2 = \frac{2kP}{\left[\left(\frac{y}{3}\right)^2 + a^2\right]^{3/2}} = \frac{2kP}{(y/3)^3}$$

$\because$ Again, $\frac{y}{3} >> a$

$$\Rightarrow \quad E_2 = 27 \times \frac{2kP}{y^3}$$

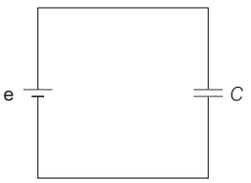
Force on charge

$$F' = QE_2 = 27 \times \frac{2kPQ}{y^3} \quad \text{... (iv)}$$

From Eqs. (iii) and (iv), we get

$$F' = 27\ F$$

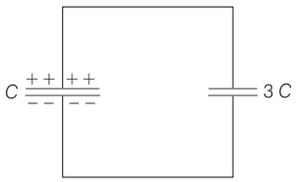
8 *(d)* In position 'A' of switch, we have a capacitor joined with battery.



So, energy stored in position

$$U_1 = \frac{1}{2} C \varepsilon^2$$

When switch is turned to position B, we have a charged capacitor joined to an uncharged capacitor.



Common potential in steady state will be

$$V = \frac{\text{total charge}}{\text{total capacity}} = \frac{C\varepsilon}{4C} = \frac{\varepsilon}{4}$$

Now, energy stored will be

$$U_2 = \frac{1}{2}(C_{eq})(V_{common})^2 = \frac{1}{2} 4C \times \left(\frac{\varepsilon}{4}\right)^2$$

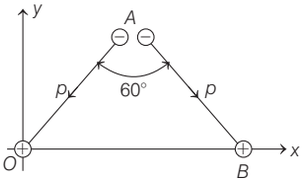
$$= \frac{1}{8} C\varepsilon^2$$

So, energy dissipated is

$$\Delta U = U_1 - U_2 = \frac{1}{2} C\varepsilon^2 - \frac{1}{8} C\varepsilon^2$$

$$= \frac{3}{8} C\varepsilon^2 = \frac{3}{8} C\left(\frac{Q}{C}\right)^2 = \frac{3}{8} \cdot \frac{Q^2}{C}$$

9 *(c)* Given system is equivalent to



Now, magnitude of resultant

$$p_{net} = \sqrt{p^2 + p^2 + 2p \cdot p \cos 60°}$$

$$= \sqrt{3}p = \sqrt{3}ql$$

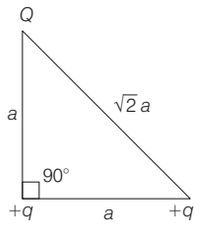
As, resultant is directed along negative y-direction

$$p_{net} = -\sqrt{3}p\hat{\mathbf{j}} = -\sqrt{3}ql\,\hat{\mathbf{j}}$$

10 *(d)* Electrostatic energy

$$U = \frac{K\ q_1 q_2}{r}$$

In accordance to the principle of superposition,



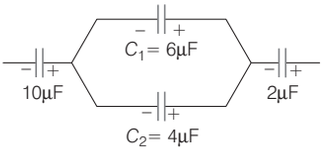
$$U = \frac{Kq^2}{a} + \frac{KQq}{a} + \frac{KQq}{\sqrt{2}a}$$

It is given that, $U = 0$

$$\therefore \quad \frac{Kq}{a}\left[q + Q + \frac{Q}{\sqrt{2}}\right] = 0$$

$$\Rightarrow \quad Q = \frac{-\sqrt{2} \times q}{(\sqrt{2} + 1)}$$

11 *(b)* Applying the concept of charge conservation since, 6 μF and 4 μF are in parallel, so total charge on this combination will be 30 μC.

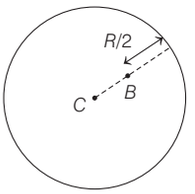


$\therefore$ Charge on 6 μF, capacitor

$$= \left(\frac{C_1}{C_1 + C_2}\right) q = \frac{6}{6 + 4} \times 30 = 18\ \mu\text{C}$$

Since, the charge has been asked on the right plate of the capacitor. Thus, it would be + 18 μC.

12 *(a)*



Given, V at surface of the sphere

$$V_0 = \frac{KQ}{R}$$

Here, $K = \frac{1}{4\pi\varepsilon_0} = \text{constant}$

V at point C,

$$V_C = \frac{KQ}{R} - \frac{K\alpha Q}{R} = V_0(1 - \alpha)$$

V at point B,

$$V_B = \frac{KQ}{R} - \frac{K(\alpha Q)}{R/2} = V_0(1 - 2\alpha)$$

$$\therefore \quad \frac{V_C}{V_B} = \frac{1 - \alpha}{1 - 2\alpha}$$

13 *(c)* The electric field intensity at a point lying outside the sphere (non-conducting) is

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} \text{ i.e. } E \propto \frac{1}{r^2} \quad \ldots(\text{i})$$

The electric field intensity at the surface ($r = R$),

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} \Rightarrow E \propto \frac{1}{R^2}$$

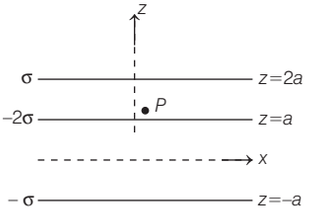
where, R being the radius of sphere,

The electric field intensity inside the sphere is

$$E = \frac{qr}{4\pi\varepsilon_0 R^3} \Rightarrow E \propto r$$

At the centre of sphere, $r = 0, \therefore E = 0$

14 *(b)* According to the question,



Given, surface charge density = σ

permittivity of the free space = ε_0

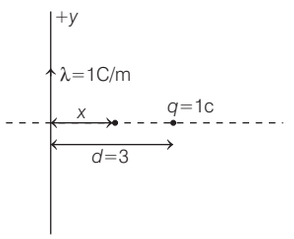
Now, the electric force acting on a charge $-q$ at a place the point P,

$$\therefore \quad \mathbf{F}_p = -\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{k}} - \frac{2\sigma}{2\varepsilon_0}\hat{\mathbf{k}} - \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{k}}$$

$$\text{or} \quad = -\frac{4\sigma}{2\varepsilon_0}\hat{\mathbf{k}} \quad \text{or} \quad \mathbf{F}_p = \frac{-2\sigma}{\varepsilon_0}\hat{\mathbf{k}}$$

15 *(b)* According to question (figure),

For electric field to be zero at x, there should be



$$E_{\text{line charge}} = E_{\text{point charge}} \quad \ldots(\text{i})$$

As we know, for line charge

$$E_{\text{line charge}} = \frac{2k\lambda}{x} = \frac{2k}{x} \quad \ldots(\text{ii})$$

Similarly, for point charge,

$$E_{\text{point charge}} = \frac{kq}{(3-x)^2} = \frac{k}{(3-x)^2} \quad \ldots(\text{iii})$$

From Eqs. (i), (ii) and (iii), we get

$$\frac{2k}{x} = \frac{k}{(3-x)^2} \Rightarrow \frac{2}{x} = \frac{1}{(3-x)^2}$$

$$\Rightarrow \quad 2x^2 - 13x + 18 = 0$$

This is a quadratic equation, which have solution

$$x_1, x_2 = \frac{13 \pm \sqrt{13^2 - 144}}{4}$$

So, $x_1 = 4.5$ m (not possible, as $0 < x < 3$)

and $x_2 = 2$ m.

Hence, the correct answer is 2 m, which matches option (b).

16 *(a)* Since, we know that in a series capacitor system, the charge stored by both the capacitors is equal.

Hence, $Q = C_{eq}V$

where, $C_{eq} = \frac{4\times 8}{4+8} = \frac{8}{3}\mu\text{F}$

$$\Rightarrow \quad Q = \frac{8}{3}\times 6 = 16\,\mu\text{C}$$

Hence, $V_1 = \frac{Q}{C_1} = \frac{16}{4} = 4\text{V}$

and $V_2 = \frac{Q}{C_2} = \frac{16}{8} = 2\text{ V}$

Hence, the potential difference,

$$\Delta V = V_1 - V_2$$

$$\Rightarrow \quad \Delta V = 4 - 2 = 2\text{ V}$$

Hence, the correct option is (a).

17 *(a)* The capacitance of a parallel plate air capacitor is given by

$$C_0 = \frac{\varepsilon_0 A}{d} \quad \ldots(\text{i})$$

When the distance between plates is reduced and a dielectric slab is introduced, then the capacitance becomes

$$C = \frac{K\,\varepsilon_0 A}{d_1} \quad \ldots(\text{ii})$$

Here, $K = 2, d_1 = \frac{d}{4}$

and $C_0 = 4\mu\text{F} = 4\times 10^{-6}$ F

From Eq. (ii), we get

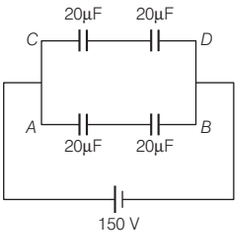
$$C = 4\left(\frac{K\,\varepsilon_0 A}{d_1}\right) = 4K\left(\frac{\varepsilon_0 A}{d}\right)$$

$$= 4KC_0 \quad \ldots(\text{iii}) \text{ [from Eq. (i)]}$$

Substituting given values in Eq. (iii), we get

$$C = 4\times 2\times 4 = 32\,\mu\text{F}$$

18 *(d)* The given circuit shows a balanced Wheatstone bridge. Now, the circuit becomes



Equivalent capacitance the branch AB is given by

$$\frac{1}{C_{AB}} = \frac{1}{20} + \frac{1}{20} = \frac{2}{20} = \frac{1}{10}$$

$$C_{AB} = 10\ \mu\text{F}$$

Similarly, in branch CD,

$$C_{CD} = 10\ \mu\text{F}$$

Now, C_{AB} and C_{CD} are connected in parallel combination.

Hence, the equivalent capacitance of the circuit,

$$C_{\text{eq}} = 10 + 10 = 20\ \mu\text{F}$$

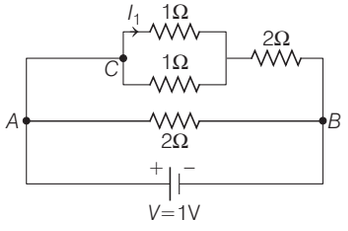
We know that, charge $Q = C_{\text{eq}}V$

$$Q = 20\times 10^{-6}\times 150\text{ V}$$

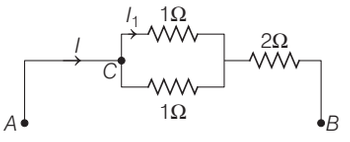
$$Q = 3\times 10^{-3}\text{C}$$

Current Electricity

1 *(d)* Given circuit is



In given circuit $V_{AB} = 1$ V, so upper branch of circuit is as shown below figure.



Equivalent resistance of upper branch,

$$R_{\text{eq}} = (1\Omega \,||\, 1\Omega) + 2\Omega$$

$$= \frac{1}{2} + 2 = \frac{5}{2}\Omega$$

So, current in upper branch,

$$I = \frac{V}{R} = \frac{V_{AB}}{R_{\text{eq}}} = \frac{1}{5/2} = \frac{2}{5}\text{A}$$

At point C, this current is equally divided into two parts.

So, $I_1 = \frac{1}{2}\left(\frac{2}{5}\right) = 0.2\,\text{A}$

2 *(c)* Power of 15 bulbs (each of 45 W),

$$P_1 = 15\times 45 = 675\text{ W}$$

Power of 15 bulbs (each of 100 W),

$$P_2 = 15\times 100 = 1500\text{ W}$$

Power of 15 fans (each of 10 W),

$$P_3 = 15\times 10 = 150\text{ W}$$

Power of 2 heaters (each of 1 kW),

$$P_4 = 2\times 1000 = 2000\text{ W}$$

Total power usage of building,

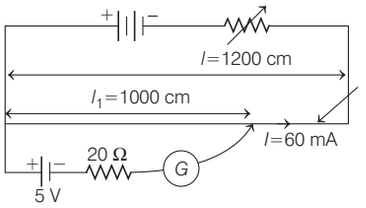
$$P = P_1 + P_2 + P_3 + P_4 = 4325\,\text{W}$$

Using $P = VI$, current drawn from mains supply of 220 V,

$$I = \frac{P}{V} = \frac{4325}{220} = 19.66\,\text{A}$$

$\therefore$ Minimum fuse capacity required is 20 A.

3 *(b)* The given situation is as shown below.



At null point, a length of 1000 cm balances emf of 5 V.

So, potential drop across 1000 cm of wire is equal to emf of cell,

$$E = l_1 \cdot x \cdot I \Rightarrow 5 = 1000 \times x \times 60 \times 10^{-3}$$

where, x = resistance per centimetre length of potentiometer wire.

$$\Rightarrow x = \frac{5}{1000 \times 60 \times 10^{-3}} \text{ or } x = \frac{1}{12}\ \Omega\ \text{cm}^{-1}$$

So, resistance of 1200 cm length of wire,

$$R = l \cdot x = 1200 \times \frac{1}{12} = 100\ \Omega$$

4 *(b)* Power dissipated by any resistor R,

$$P = I^2 R \qquad \ldots (i)$$

Given $I = 2\ \text{mA} = 2 \times 10^{-3}\ \text{A}$ and

$$P = 4.4\ \text{W}$$

Using Eq. (i), we get

$$4.4 = (2 \times 10^{-3})^2 \times R$$

or $$R = \frac{4.4}{4 \times 10^{-6}} \cdot \Omega \qquad \ldots(ii)$$

When this resistance R is connected with 11 V. supply then power dissipated is

$$P = \frac{V^2}{R} \text{ or } P = \frac{(11)^2}{4.4} \times 4 \times 10^{-6}$$

[$\because$ Using Eq. (ii)]

$$\Rightarrow \qquad P = \frac{11 \times 11 \times 4 \times 10^{-6}}{44 \times 10^{-1}}\ \text{W}$$

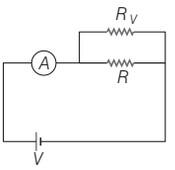
or $$P = 11 \times 10^{-5}\ \text{W}$$

5 *(b)* Measured value of R = 5% less than actual value of R.

i.e., $$R' = 30 - (5\%\ \text{of}\ 30) = 30 - \frac{5}{100} \times 30$$

$$\Rightarrow \qquad R' = 28.5\ \Omega \qquad \ldots (i)$$

Now, let us assume that internal resistance of voltmeter R_V. Replacing voltmeter with its internal resistance, we get following circuit.



It is clear that the measured value, R' should be equal to parallel combination of R and R_V. Mathematically,

$$R' = \frac{RR_V}{R + R_V} = 28.5\ \Omega$$

Given, $$R = 30\ \Omega$$

$$\Rightarrow \qquad \frac{30R_V}{30 + R_V} = 28.5$$

$$\Rightarrow \qquad 30R_V = (28.5 \times 30) + 28.5\,R_V$$

$$\Rightarrow \qquad 1.5R_V = 28.5 \times 30$$

$$\Rightarrow \qquad R_V = \frac{28.5 \times 30}{1.5} = 19 \times 30$$

or $$R_V = 570\ \Omega$$

6 *(a)* Given, potential difference of 5 mV is across 10 m length of potentiometer wire. So potential drop per unit length is

$$= \frac{5 \times 10^{-3}}{10 \times 10^{-2}} = 5 \times 10^{-2} \left(\frac{\text{V}}{\text{m}}\right)$$

Hence, potential drop across 1 m length

$$V_{AB} = 5 \times 10^{-2} \left(\frac{\text{V}}{\text{m}}\right) \times 1 = 5 \times 10^{-2}\ \text{V}$$

Now, potential drop across resistance R is

$$V_R = 4 - 5 \times 10^{-2} = \frac{395}{100}\ V$$

Now, circuit current is $i = \dfrac{V}{R_{\text{total}}} = \dfrac{4}{R + 5}$

Hence, for resistance R, using $V_R = iR$, we get

$$\frac{395}{100} = \frac{4}{R + 5} \times R$$

$$395\ (R + 5) = 400R \Rightarrow R = 395\ \Omega$$

7 *(b)* Let length of null point 'J' be 'x' and length of the potentiometer wire be 'L'.

In first case, current in the circuit

$$I_1 = \frac{6}{4 + 2} = 1\ \text{A}$$

$\therefore$ Potential gradient $= I \times R = \dfrac{1 \times 4}{L}$

$\Rightarrow$ Potential difference in part 'AJ'

$$= \frac{1 \times 4}{L} \times x = \varepsilon_1$$

Given, $\varepsilon_1 = 0.5 = \dfrac{4x}{L}$ or $\dfrac{x}{L} = \dfrac{1}{8}$ ...(i)

In second case, current in the circuit

$$I_2 = \frac{6}{4 + 6} = 0.6\ \text{A}$$

$\therefore$ Potential gradient $= \dfrac{0.6 \times 4}{L}$

$\Rightarrow$ Potential difference in part 'AJ'

$$= \frac{0.6 \times 4}{L} \times x = \varepsilon_2$$

$$\Rightarrow \qquad \varepsilon_2 = \frac{0.6 \times 4}{L} \times \frac{L}{8}$$

[using Eq. (i)]

$$\Rightarrow \qquad \varepsilon_2 = 0.3\ \text{V}$$

8 *(c)* For a balanced Wheatstone bridge,

$$\frac{P}{R} = \frac{Q}{X}$$

In first case when $R = 400\ \Omega$,

$$\Rightarrow \qquad \frac{P}{400\ \Omega} = \frac{Q}{X}$$

$$\Rightarrow \qquad P = \frac{400 \times Q}{X} \qquad \ldots(i)$$

In second case, P and Q are interchanged and $R = 405\ \Omega$

$$\therefore \qquad \frac{Q}{R} = \frac{P}{X} \qquad \Rightarrow \frac{Q}{405} = \frac{P}{X} \qquad \ldots(ii)$$

Substituting the value of P from Eq. (i) in Eq. (ii), we get

$$\frac{Q}{405} = \frac{Q \times 400}{X^2}$$

$$\Rightarrow \qquad X^2 = 400 \times 405$$

$$\Rightarrow \qquad X = \sqrt{400 \times 405} = 402.5$$

The value of X is close to 402.5 Ω.

9 *(a)* For meter bridge, if balancing length is l cm, then in first case, $\dfrac{R_1}{R_2} = \dfrac{l}{(100 - l)}$

It is given that, $l = 40$ cm;

So; $$\frac{R_1}{40} = \frac{R_2}{100 - 40} \text{ or } \frac{R_1}{R_2} = \frac{2}{3} \qquad \ldots(i)$$

In second case, $R_1' = R_1 + 10$, and balancing length is now 50 cm then

$$\frac{R_1 + 10}{50} = \frac{R_2}{(100 - 50)}$$

or $$R_1 + 10 = R_2 \qquad \ldots(ii)$$

Substituting value of R_2 from (ii) to (i) we get, or $\dfrac{R_1}{10 + R_1} = \dfrac{2}{3}$

$$\Rightarrow \qquad 3R_1 = 20 + 2R_1$$

$$\Rightarrow \qquad R_1 = 20\Omega \text{ and } R_2 = 30\Omega$$

Let us assume the parallel connected resistance is x.

Then equivalent resistance is $\dfrac{x\,(R_1 + 10)}{x + R_1 + 100}$

So, this combination should be again equal to R_1.

$$\frac{(R_1 + 10)x}{R_1 + 10 + x} = R_1$$

$$\Rightarrow \qquad \frac{30x}{30 + x} = 20 \Rightarrow \text{or } 30x = 600 + 20x$$

or $$x = 60\Omega$$

10 *(c)* In given potentiometer, resistance per unit length is $x = 0.01\ \Omega\text{cm}^{-1}$.

Length of potentiometer wire is $L = 400$ cm

Net resistance of the wire AB is

R_{AB} = resistance per unit length $\times$ length of AB

$$= 0.01 \times 400$$

$\Rightarrow \quad R_{AB} = 4\ \Omega$

Net internal resistance of the cells

$r = 0.5 + 0.5 = 1\Omega$

$\therefore$ Current in given potentiometer circuit is

$$I = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{\text{Net emf}}{r + R + R_{AB}}$$

$$= \frac{3}{1 + 1 + 4} = 0.5 \text{ A}$$

Reading of voltmeter when the jockey is at 50 cm (l') from one end A,

$V = IR = I(xl') = 0.5 \times 0.01 \times 50 = 0.25$ V

11 *(c)*

12 *(d)* When the switch 'S' is closed in the circuit,

Then the current at point C is

$\therefore \quad i_1 + i_2 = i$

$$\Rightarrow \quad \frac{V_A - V_C}{2} + \frac{V_B - V_C}{4} = \frac{V_C - V}{2}$$

($\because$ using Ohm's law, $V = iR$)

Where V_A, V_B and V_C one potential at point A, B, and C respectively.

or $\quad \frac{20 - V_C}{2} + \frac{10 - V_C}{4} = \frac{V_C - 0}{2}$

$\Rightarrow \quad 20 - V_C + (10 - V_C)2 = V_C$

$40 = V_C + 3V_C$

$40 = 4V_C$ or $V_C = 10$ V

$\therefore$ The current, $i = \frac{V_C}{2} = \frac{10}{2} = 5$A

13 *(c)* Given, $I = 5$A,

$\rho = 1.7 \times 10^{-8}\ \Omega$-m, $r = 5$ mm $= 5 \times 10^{-3}$ m,

and $v_d = 1.1 \times 10^{-3}$ m/s

Mobility of charges in a conductor is given by

$$\mu = \frac{v_d}{E} \qquad \ldots (i)$$

and resistivity is given by

$$\rho = \frac{E}{j} = \frac{E}{I/A} \qquad (\because j = \sigma E = \frac{1}{\rho} \times E)$$

$$\Rightarrow \quad \rho = \frac{EA}{I} \quad \Rightarrow E = \frac{\rho I}{A} \qquad \ldots (ii)$$

From Eqs. (i) and (ii), we get

$$\mu = \frac{v_d A}{\rho I}$$

Substituting the given values, we get

$$= \frac{1.1 \times 10^{-3} \times \pi \times (5 \times 10^{-3})^2}{1.7 \times 10^{-8} \times 5}$$

$$= \frac{86.35 \times 10^{-9}}{8.5 \times 10^{-8}} = 10.1 \times 10^{-1}$$

$\Rightarrow \quad \mu \approx 1\ \text{m}^2/\text{V-s}$

14 *(c)* it is a balanced Wheatstone

hence, $\frac{R_{AB}}{R_{BC}} = \frac{R_{AD}}{R_{DC}} = \frac{1}{2}$, bridge, therefore current flowing in arm $BD = 0$

$\therefore \quad R_{eq} = (10 + 20)||(6 + 12) = 30||18$

$$= \frac{30 \times 18}{30 + 18} = 11.25\ \Omega$$

$\therefore \quad I = \frac{V}{R_{eq}} = \frac{10}{11.25}$

$= 0.89$ A $\simeq 0.9$ A

15 *(c)* According to the question,

First case when, $C = 100\ \Omega$

Now, By balanced wheat stone bridge,

$$\frac{R_A}{R_B} = \frac{100}{D} \qquad \ldots(i)$$

Second case when, $C = 121\ \Omega$

$$\frac{R_B}{R_A} = \frac{121}{D} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$\therefore \quad D = \sqrt{100 \times 121}$

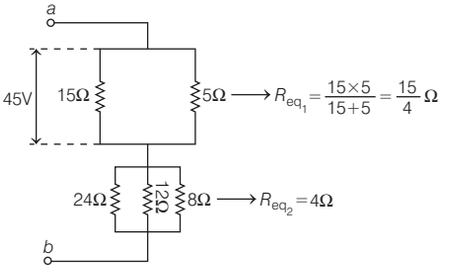
$D = 110\ \Omega$

16 *(a)* As, current,

$$I = \frac{nQ}{t} = \frac{0.1 \times 6 \times 10^{23} \times 1.6 \times 10^{-19}}{2400}$$

$\Rightarrow \quad I = 4$ A

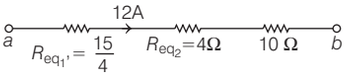
17 *(c)* The circuit can be redrawn as,



45 V is drop across the branch one, so current through it,

$$I_1 = \frac{45}{R_{eq_1}} = \frac{45}{\frac{15}{4}} = 12 \text{ A}$$

Now, above circuit can be redrawn as,



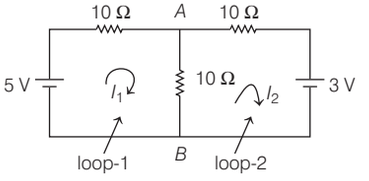
Since, in a series circuit, the current flows will be same,

hence, $V_{ab} = I_{ab} R_{eq}(ab) = 12\left(\frac{15}{4} + 4 + 10\right)$

$\Rightarrow \quad V_{ab} = 213$ V

Hence, the correct option is (c).

18 *(b)* The circuit can be redrawn as.



By KVL in loop (1),

$$-5 + 10I_1 + 10(I_1 - I_2) = 0$$

$$20I_1 - 10I_2 = 5 \qquad \ldots (i)$$

By KVL in loop (2),

$$10I_2 + 3 + 10(I_2 - I_1) = 0$$

$$-10I_1 + 20I_2 = -3$$

$$-20I_1 + 40I_2 = -6 \qquad \ldots (ii)$$

Solving Eqs. (i) and (ii), we get

$$30I_2 = -1$$

$$I_2 = \frac{-1}{30} \text{ A}$$

From Eq. (i), $20I_1 - 10\left(\frac{-1}{30}\right) = 5 \Rightarrow$

$I_1 = \frac{7}{30}$ A

$\therefore$ Current in branch AB, $I_{AB} = I_1 - I_2$

$$= \frac{7}{30} - \left(\frac{-1}{30}\right)$$

$$I_{AB} = \frac{8}{30} \text{ A}$$

$\therefore$ Voltage across a terminal AB,

$$V_{AB} = I_{AB} \times R_{AB}$$

$$= \frac{8}{30} \times 10 = \frac{8}{3} \text{V}$$

19 *(a)* For cell of emf E balancing length

$= \frac{L}{3} = \frac{1}{3} \times L$ (length of wire)

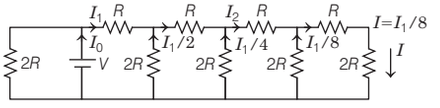
Increased length of wire $= L + \frac{L}{2} = \frac{3L}{2}$

Since, the cell is the same, therefore the new balancing length $= \frac{1}{3}$(new increased length)

$$= \frac{1}{3} \times \frac{3}{2} L = \frac{L}{2}$$

Therefore, for the same cell the balance point will be as length $\frac{L}{2}$ from positive end.

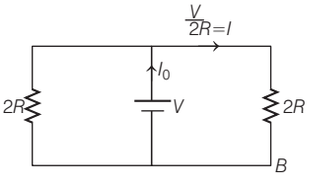
20 *(c)* According to the question,



Total resistance of the given circuit

$$R_{eq} = 2R$$

Now, circuit



$\therefore \quad I = \frac{V/2R}{8} \qquad (\because I_1 = V/2R)$

So, current I in the circuit

$$I = \frac{V}{16R} \qquad \left(\because I = \frac{I_1}{8}\right)$$

Magnetic Effects of Current

1 *(b)* Acceleration produced by electric field,

$$\mathbf{a} = \frac{qE_0}{m}\hat{\mathbf{i}}$$

After time t, velocity of the particle,

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\Rightarrow \quad \mathbf{v} = v_0\hat{\mathbf{j}} + \left(\frac{qE_0 t}{m}\right)\hat{\mathbf{i}} \qquad [\because u = \mathbf{v} = v_0\hat{\mathbf{j}}]$$

So, speed of the particle,

$$|\mathbf{v}| = \sqrt{v_0^2 + \left(\frac{qE_0 t}{m}\right)^2}$$

Now, given $|\mathbf{v}| = 2v_0$ after time t, so

$$2v_0 = \sqrt{v_0^2 + \left(\frac{qE_0 t}{m}\right)^2}$$

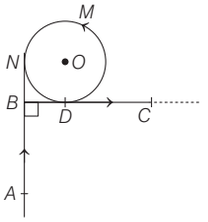
$$\Rightarrow \qquad 4v_0^2 = v_0^2 + \left(\frac{qE_0 t}{m}\right)^2$$

$$\Rightarrow \qquad 3v_0^2 = \left(\frac{qE_0 t}{m}\right)^2$$

So, time interval, $t = \dfrac{\sqrt{3}\,mv_0}{qE_0}$.

Here, we must note that no change in magnitude of velocity is caused by a perpendicular magnetic field. So, we are not taking effect of magnetic field while calculating change in speed.

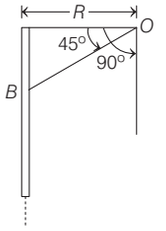
2 *(a)* Given arrangement consists of two current carrying wires and one current loop as shown in the figure below.



For wire *ABN*, magnetic field at *O* is given by

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{I}{R}(\sin\phi_1 + \sin\phi_2)$$

Here, $\phi_1 = +90°$ and $\phi_2 = -45°$.

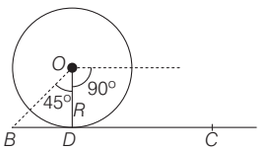


Hence, magnetic field of wire,

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{I}{R}[\sin 90° + \sin(-45°)]$$

$$= \frac{\mu_0}{4\pi R}\left(1 - \frac{1}{\sqrt{2}}\right)\otimes$$

Also, field of wire *BDC* at *O*,



$$B_2 = \frac{\mu_0 I}{4\pi R}(\sin\phi_1 + \sin\phi_2)$$

Here, $\phi_1 = 45°$ and $\phi_2 = 90°$.

$$\therefore \qquad B_2 = \frac{\mu_0 I}{4\pi R}\left(\frac{1}{\sqrt{2}} + 1\right)\odot$$

Magnetic field of circular coil at *O*,

$$B_3 = \frac{\mu_0 I}{2R}\odot$$

Hence, net field at *O*,

$$B_{\text{net}} = B_1 \otimes + B_2 \odot + B_3 \odot$$

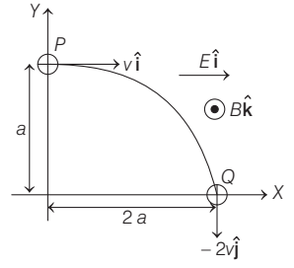
$$= \frac{\mu_0 I}{4\pi R}\left(1 - \frac{1}{\sqrt{2}}\right) - \frac{\mu_0 I}{4\pi R}\left(\frac{1}{\sqrt{2}} + 1\right) - \frac{\mu_0 I}{2R}$$

$$= \frac{\mu_0 I}{2R}\left(\frac{1}{2\pi} - \frac{1}{\sqrt{2}\cdot 2\pi} - \frac{1}{\sqrt{2}\cdot 2\pi} - \frac{1}{2\pi} - 1\right)$$

$$= \frac{\mu_0 I}{2\pi R}\left(\pi + \frac{1}{\sqrt{2}}\right)$$

Negative sign is taken for outward field.

3 *(b)* Charged particle follows trajectory as shown in the figure.



As magnetic field is perpendicular to the velocity of charged particle, work done by magnetic field is zero.

So, work done by electric field = change in kinetic energy of charged particle.

$$\Rightarrow \qquad \mathbf{F}\cdot ds = \frac{1}{2}m(-2v\hat{\mathbf{j}})^2 - \frac{1}{2}m(v\hat{\mathbf{i}})^2$$

$$\Rightarrow \qquad (qE\hat{\mathbf{i}})\,(2a\hat{\mathbf{i}} - a\hat{\mathbf{j}}) = 2mv^2 - \frac{1}{2}mv^2$$

$$\Rightarrow \qquad 2qaE = \frac{3}{2}mv^2$$

$$\Rightarrow \qquad E = \frac{3}{4}\frac{mv^2}{qa} \text{ (statement 1 is correct)}$$

Rate of work done

$$= \text{Power} = \mathbf{F}\cdot\mathbf{v} = qE\hat{\mathbf{i}}\cdot v\hat{\mathbf{i}}$$

$$= qEv = q\cdot\frac{3}{4}\frac{mv^2}{qa}\cdot v = \frac{3}{4}\frac{mv^3}{a}$$

(statement 2 is correct)

At point *Q*, both electric field and magnetic field are perpendicular to velocity of charged particle, so work done by both $\mathbf{E}$ and $\mathbf{B}$ at *Q* is zero. (statement 3 is correct)

Angular momentum about origin at $P = mva\hat{\mathbf{i}}$.

Angular momentum about origin at

$$Q = -4mva\hat{\mathbf{j}}.$$

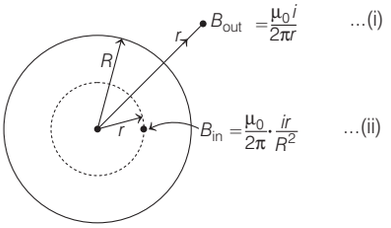
Difference between magnitudes of angular momentum at *P* and *Q*,

$$\Delta L = L_Q - L_P = 4mva - mva$$

$$= 3mva \text{ (statement 4 is incorrect)}$$

Thus, option (b) is correct.

4 *(c)* By Ampere's circuital rule, magnetic field inside and outside a wire carrying a uniformly distributed current is,



In given case, radius of wire is *a*. So, magnetic field due to wire at a distance $\frac{a}{3}$ from centre,

$$B_1 = \frac{\mu_0}{2\pi}\frac{i\left(\frac{a}{3}\right)}{a^2} = \frac{\mu_0 i}{6\pi a} \qquad \text{[from Eq. (ii)]}$$

and magnetic field at a distance $2a$ from centre,

$$B_2 = \frac{\mu_0 i}{2\pi(2a)} = \frac{\mu_0 i}{4\pi a} \qquad \text{[from Eq. (i)]}$$

Ratio of fields will be

$$\frac{B_1}{B_2} = \frac{\left(\frac{\mu_0 i}{6\pi a}\right)}{\left(\frac{\mu_0 i}{4\pi a}\right)} = \frac{2}{3}$$

5 *(c)* Given,

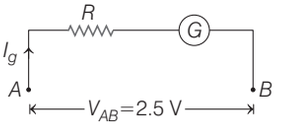
1 division = 4×10^{-4} A

So, current for full-scale deflection = I_g

= Number of divisions × Current for 1 division

$$\Rightarrow \qquad I_g = 25 \times 4 \times 10^{-4} = 1 \times 10^{-2} \text{ A}$$

Now, let a resistance of *R* is put in series with galvanometer to make it a voltmeter of range 2.5 V.



Then, $I_g(R+G) = V_{AB}$

$1\times10^{-2}\ (R+50) = 2.5$

[$\because$ Given, $G = 50\ \Omega$, $V_{AB} = 2.5$ V]

$\therefore \quad R = 250 - 50 = 200\ \Omega$

6 *(b)* Since there is no magnetic field along axis of a current carrying wire.

Hence, $B_O = B_{PS} + B_{QN} + B_{PL} + B_{QM}$

$$= \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4\pi r} + 0 + 0 = \frac{\mu_0 i}{2\pi r}$$

So, $\quad i = \dfrac{2\pi r B_0}{\mu_0}$

Here, $r = OP = OQ = 4$ cm and $B_O = 10^{-4}$ T.

Substituting values, we get

$$\Rightarrow \quad i = \frac{2\pi\times4\times10^{-2}\times10^{-4}}{4\pi\times10^{-7}}$$

$\Rightarrow i = 20$ A, Also, magnetic field points perpendicular into the plane of paper.

7 *(a)* For a galvanometer, $i_g \propto \theta$ or $i_g = C\theta$

Now, K_1 is closed and K_2 is opened, circuit resistance is

$R_{eq} = R_1 + R_g$

Hence, galvanometer current is

i.e., $\quad i_g = \dfrac{V}{R_1 + R_g}$

As deflection is given θ_0, we have

$$i_g = \frac{V}{R_1 + R_g} = C\theta_0 \qquad \ldots(i)$$

When both keys K_1 and K_2 are closed, circuit resistance.

$$R_{eq} = R_1 + \left(\frac{R_2 \times R_g}{R_2 + R_g}\right)$$

Current through galvanometer will be

$$i_{g_2} = \frac{V}{\left(R_1 + \dfrac{R_2 R_g}{R_2 + R_g}\right)} \times \frac{R_2}{(R_2 + R_g)}$$

$$= \frac{VR_2}{R_1R_2 + R_1R_g + R_2R_g} = C\cdot\frac{\theta_0}{5} \ldots(ii)$$

Now, dividing Eq. (i) by Eq. (ii), we get

$$\frac{R_1R_2 + R_1R_g + R_2R_g}{R_2\ (R_1 + R_g)} = 5$$

Substituting $R_1 = 220\ \Omega$ and $R_2 = 5\ \Omega$, we get

$$\frac{1100 + 220\ R_g + 5\ R_g}{5\ (220 + R_g)} = 5$$

$$1100 + 225\ R_g = 5500 + 25R_g$$

$$\Rightarrow \quad 200\ R_g = 4400$$

$$\therefore \quad R_g = 22\ \Omega$$

8 *(a)* Radius of path of charged particle q in a uniform magnetic field B of mass 'm' moving with velocity v is

$$r = \frac{mv}{Bq} = \frac{m\sqrt{(2qV/m)}}{Bq}$$

Hence, $r \propto \dfrac{\sqrt{m}}{\sqrt{q}}$

so, required ratio is

$$\frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha}} \times \sqrt{\frac{q_\alpha}{q_p}}$$

$$= \sqrt{\frac{1}{4}} \times \sqrt{\frac{2}{1}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

9 *(a)* As, torque on coil,

$$\tau = \mathbf{M}\times\mathbf{B} - NIAB\sin\alpha$$

Since, plane of the coil i parallel to the field

$\therefore \quad \alpha = 90° \Rightarrow \tau = NIBA$

Torque of trosion band, $T = k\theta$

$$\therefore \quad BINA = k\theta \text{ or } B = \frac{k\theta}{INA} \qquad \ldots(i)$$

Substituting values given in question in Eq. (i), we get

$$B = \frac{10^{-6}\times22}{1\times10^{-3}\times175\times7\times180\times10^{-4}}$$

$$= 0.998\times10^{-3} \simeq 10^{-3}\ \text{T}$$

10 *(a)* Given,

Area of the rectangular coil,

$A = 5\ \text{cm}\times 2.5\ \text{cm}$

$$\Rightarrow \quad A = 12.5\ \text{cm}^2 = 12.5\times10^{-4}\ \text{m}^2$$

$N = 100$ turns

$I = 3$ A

$B = 1$ T and

Angle between the magnetic field and area vector of the coil, $\theta = 45°$

Then the net torque experienced by the coil is,

$\tau = \text{M}\times\text{B} = NI(\mathbf{A}\times\mathbf{B}) = NIAB\sin\theta$

Substituting the given values, we get

$\tau = 100\times3\times12.5\times10^{-4}\times1\times\sin45°$

$\tau = 0.707\times100\times3\times12.5\times10^{-4}$ N-m

$= 2.651\times10^{-1}$ N-m

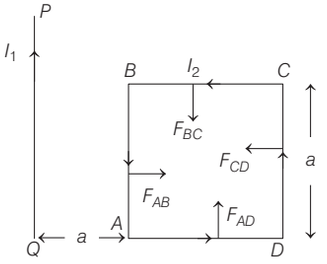
≈ 0.27 N-m

11 *(d)* Force on a wire 1 in which current I_1 is flowing due to another wire 2 which are separated by a distance r is given as

$$\mathbf{F} = I_1(l\times\mathbf{B}_2)$$

or $\quad F = \dfrac{\mu_0 I_1 I_2}{2\pi r}\cdot l\sin\theta \quad \left[\because \mathbf{B}_2 = \dfrac{\mu_0 I_2}{2\pi r}\right]$

Thus, the given square loop can be drawn as shown below



$$F_{AB} = \frac{\mu_0 I_1}{2\pi a}\cdot I_2 a \text{ (away from wire } PQ)$$

$$F_{BC} = F_{AD} = 0 \qquad [\because \theta = 0°]$$

$$F_{CD} = \frac{\mu_0 I_1}{2\pi(2a)}\cdot I_2 a$$

$$= \frac{\mu_0 I_1}{4\pi a}\cdot I_2 a \text{ (towards the wire } PQ)$$

$$\therefore \quad F_{net} = F_{AB} - F_{CD}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} - \frac{\mu_0 I_1 I_2}{4\pi} \quad \text{(away from wire)}$$

$$= \frac{\mu_0 I_1 I_2}{4\pi} \quad \text{(repulsive in nature)}$$

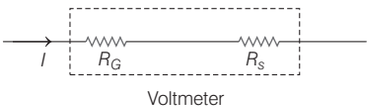
12 *(*)* Given data,

$I = 10^{-4}$ A,

$R_S = 2\ \text{M}\Omega = 2\times10^6\ \Omega$,

$V_{max} = 5$ V

Let internal resistance of galvanometer is R_G.



Then,

$$I\times R_S + I\times R_G = V_{max}$$

$$\Rightarrow \quad 2\times10^6\times10^{-4} + 10^{-4}\times R_G = 5$$

$$\Rightarrow \quad 10^{-4}\ R_G = 5 - 200 = -195$$

or $\quad R_G = -195\times10^4\ \Omega$

Resistance cannot be negative.

$\therefore$ No option is correct.

13 *(c)* Given, speed of electrons,

$v = 5\times10^5$ m/s

Electrostatic force between two electrons,

$$F_e = k.\frac{e^2}{r^2} \qquad \ldots(i)$$

Magnetic field produced due to moving electron with velocity v,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{ev}{r^2} \qquad \ldots(ii)$$

Magnetic force on proton

$$F_m = Bev = \frac{\mu_0}{4\pi}\cdot\frac{ev}{r^2}\cdot ev \qquad \text{(from Eq. (ii))}$$

$$F_m = \frac{\mu_0}{4\pi}\cdot\frac{e^2v^2}{r^2} \qquad \ldots(iii)$$

$\therefore$ From Eqs. (i) and (iii), we have

$$\frac{F_e}{F_m} = \frac{ke^2/r^2}{\dfrac{\mu_0 e^2 v^2}{4\pi.r^2}} = \frac{k\,.\,4\pi}{\mu_0 v^2}$$

$$= \frac{9\times10^9\times4\times3.14}{4\pi\times10^{-7}\ (5\times10^5)^2} = 3.6\times10^5$$

14 *(c)* $\because$ Magnetic force, $F = q(\mathbf{v}\times\mathbf{B})$

$= qvB\sin\theta$

For uniform transverse magnetic field, $\theta = 90°$

So, $\quad F = qvB = q\left(\sqrt{\dfrac{2qV}{m_e}}\right)B$

when electron accelerated through a potential difference,

$$F = e\left(\sqrt{\frac{2eV}{m_e}}\right)B \qquad ...(i)$$

When electron accelerated through a potential difference $2V$.

$$F' = e\left(\sqrt{\frac{2e(2V)}{m_e}}\right)B$$

From Eqs. (i), we get $F' = \sqrt{2}\,F$

15 *(c)* Given, $B = 100 \times 10^{-3}$ T and

$\tau = 2\sqrt{3} \times 10^{-5}$ Nm

Now, torque $\tau = MB = IAB$

Putting the given values in above expression,

$2\sqrt{3} \times 10^{-5} = I \cdot \sqrt{3} \times 10^{-4} \times 100 \times 10^{-3}$

or $I = 2$ A.

16 *(c)* Given, magnetic field, $B = 5 \times 10^{-5}$ T

$r = 0.2$ m

and $\mu_0 = 4\pi \times 10^{-7}$ mT/A

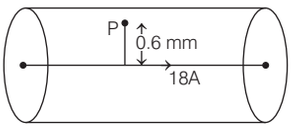
Magnetic field produced by long straight current (I) carrying wire is given by

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}$$

$$\Rightarrow \quad 5 \times 10^{-5} = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{I}{0.2}$$

$$\Rightarrow I = \frac{5 \times 10^{-5} \times 2\pi \times 0.2}{4\pi \times 10^{-7}} = 0.5 \times 10^{2} = 50 \text{ A}$$

17 *(d)* Given, current in a long wire, $I = 18$ A



magnetic field due to a solenoid,

$$B_1 = 8 \times 10^{-3} \text{ T}$$

and $r = 0.6$ mm $= 0.6 \times 10^{-3}$ m

Radius of solenoid, $R = 1$ cm $= 1 \times 10^{-2}$ m

Magnetic field due to current carrying wire at a point, P

$$B_2 = \frac{\mu_0}{2\pi} \cdot \frac{I}{r} = 2 \times 10^{-7} \times \frac{18}{0.6 \times 10^{-3}}$$

$$= 6 \times 10^{-3} \text{ T (in upward direction)}$$

Since, current carrying wire is placed along the axis of solenoid, hence magnetic field produced by wire and magnetic field due to solenoid at point

P, both are perpendicular to each other, Hence, the magnitude of the resultant magnetic field,

$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \sqrt{(8 \times 10^{-3})^2 + (6 \times 10^{-3})^2}$$

$$= 10 \times 10^{-3} \text{ T}$$

18 *(c)* Given,

The magnitude of the magnetic field at the centre of a coil of n turns

$$B = \frac{\mu_0 nI}{2r}$$

$$\Rightarrow \qquad B = \frac{12.56 \times 10^{-7} \times 100 \times 0.4}{2 \times 10^{-1} \times 9 \times 10^{-2}}$$

$$= 2.79 \times 10^{-4} \text{ T}$$

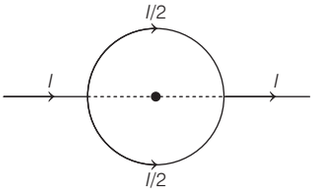
19 *(d)* Given, electric current in circular wire $= I$

Radius of wire $= r$

Charge on particle $= q$

Speed of the particle $= v$

The given loop can be as shown in the figure below.

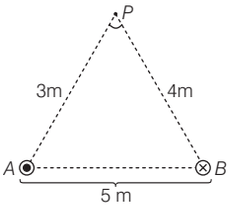


$\because$ Net magnetic field at the centre $= 0$

So, magnetic force $F_{\text{magnetic}} = q(v \times B) = 0$

So, the magnetic force experienced by the particle when it passes through the centre is $F = 0$

20 *(c)* According to the question,



Magnetic field due to first wire (A) is

$$B_1 = \frac{\mu_0 \times I}{2\pi \times 3}$$

Magnetic field due to second wire (B) is

$$B_2 = \frac{\mu_0 \times I}{2\pi \times 4}$$

Net magnitude of magnetic field

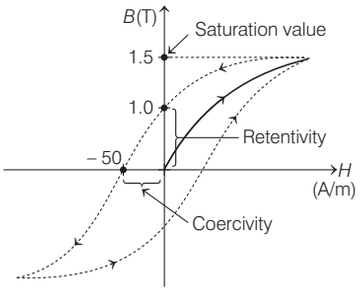
$$B = \sqrt{B_1^2 + B_2^2}$$

$$B = \frac{\mu_0 I}{2\pi}\sqrt{\frac{1}{9} + \frac{1}{16}} = \frac{\mu_0 I \times 5}{2\pi \times 3 \times 4}$$

Magnetic field $B = \frac{5}{24} \times \frac{\mu_0 I}{\pi}$

Magnetism

1 *(a)* We can obtain the values of retentivity (the remaining value of magnetic field B when $H = 0$), coercivity (value of H when $B = 0$) and saturation value of induced magnetic field ($B_{\max}$) as shown in the figure.



Clearly, retentivity = 1 T,

coercivity = 50 Am^{-1}

and saturation value = 1.5 T

2. *(c)* Time period of oscillation of a magnetic dipole in a uniform magnetic field,

$$T = 2\pi\sqrt{\frac{I}{\mu B}}$$

where, μ = magnetic dipole.

Moment of dipole = Current × Area

$$= \pi a^2 i$$

I = moment of inertia of a coil $= \frac{ma^2}{2}$

$$\text{So, } T = 2\pi\sqrt{\frac{\frac{ma^2}{2}}{\pi a^2 iB}} = 2\pi\sqrt{\frac{m}{2\pi iB}} = \sqrt{\frac{2\pi m}{iB}}$$

3 *(b)* The time-period of magnet

$$T = 2\pi\sqrt{\frac{I}{MB}} \qquad ... (i)$$

For the hoop, let us assume its moment of inertia I_h and magnetic moment M_h, then its time period will be

$$T_h = 2\pi\sqrt{\frac{I_h}{M_h B}} \qquad ... (ii)$$

Similarly, for solid cylinder, time period is,

$$T_c = 2\pi\sqrt{\frac{I_c}{M_c B}} \qquad ... (iii)$$

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{T_h}{T_c} = \sqrt{\frac{I_h M_c}{M_h I_c}} \qquad ... (iv)$$

Now, it is given that,

$$M_h = 2M_c \quad \text{or} \quad T_h = 2T_c$$

4 *(a)* We know that, horizontal and vertical components of earth's magnetic field (B_H and B_V) are related as

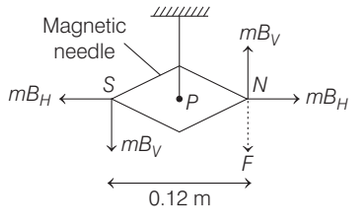
$$\frac{B_V}{B_H} = \tan\theta$$

Here, $\theta = 45°$ and $B_H = 18 \times 10^{-6}$ T

$\Rightarrow B_V = B_H \tan 45°$

$\Rightarrow B_V = B_H = 18 \times 10^{-6}$ T $\quad (\because \tan 45° = 1)$

Now, when the external force F is applied, so as to keep the needle stays in horizontal position is shown below,



Taking torque at point P, we get

$$mB_V \times 2l = Fl$$

$$\therefore \quad F = 2 \times mB_V$$

Substituting the given values, we get

$$= 2 \times 1.8 \times 18 \times 10^{-6}$$

$$= 6.48 \times 10^{-5} = 6.5 \times 10^{-5} \text{ N}$$

5 *(a)* Given, side of cube = 1 cm = 10^{-2} m

$\therefore$ Volume, $V = 10^{-6}\,\text{m}^3$

Dipole moment, $M = 20 \times 10^{-6}$ J/T

Applied magnetic intensity,

$$H = 60 \times 10^3 \text{ A/m}$$

Intensity of magnetisation

$$I = \frac{M}{V} = \frac{20 \times 10^{-6}}{10^{-6}} = 20 \text{ A/m}$$

Now, magnetic susceptibility χ is

$$\chi = \frac{\text{Intensity of magnetisation}}{\text{Applied magnetic intensity}} = \frac{I}{H}$$

$$= \frac{20}{60 \times 10^3}$$

$$\Rightarrow \quad \chi = \frac{1}{3} \times 10^{-3}$$

$$= 3.3 \times 10^{-4}$$

6 *(b)* Given, at first place, angle of dip, $\theta_1 = 45°$

Time period, $T_1 = \frac{60}{30} = 2\,\text{s}$

At second place, angle of dip, $\theta_2 = 30°$

Time period, $T_2 = \frac{60}{40} = \frac{3}{2}\,\text{s}$

Now, at first place,

$$B_{H_1} = B_1 \cos\theta_1 = B_1 \cos 45° = \frac{B_1}{\sqrt{2}} \quad \ldots(\text{i})$$

and at second place,

$$B_{H_2} = B_2 \cos\theta_2 = B_2 \cos 30° = \frac{\sqrt{3}}{2} B_2 \quad \ldots(\text{ii})$$

Also, time period of a magnetic needle is given by

$$T = 2\pi \sqrt{\frac{I}{MB_H}} \quad \ldots(\text{iii})$$

$$\therefore \quad T \propto \sqrt{\frac{1}{B_H}} \text{ or } \frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}} \quad \ldots(\text{iv})$$

By putting the values from Eqs. (i) and (ii) into Eq. (iv), we get

$$\frac{2}{3} = \sqrt{\frac{\frac{\sqrt{3}}{2} B_2}{\frac{B_1}{\sqrt{2}}}} \text{ or } \left(\frac{4}{3}\right)^2 = \frac{\sqrt{3} \times \sqrt{2}\ B_2}{2B_1}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{\sqrt{3} \times \sqrt{2}}{2} \times \frac{9}{16} \Rightarrow \frac{B_1}{B_2} = \frac{9\sqrt{3}}{16\sqrt{2}}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{9 \times 1.732}{16 \times 1.414} = \frac{15.588}{22.624}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = 0.689 \approx 0.7 \text{ T}$$

7 *(a)* Given, charge on proton, $q = e$

$$= 1.6 \times 10^{-19} \text{ C}$$

Radius, $r = 2\,\text{mm} = 2 \times 10^{-3}$ cm

Frequency, $f = 10$ Hz

Current associated by proton is given by

$$I = \frac{q}{t} \qquad [T \rightarrow \text{time period}]$$

$$= q f \qquad \left[\because T = \frac{1}{f}\right]$$

$$= 1.6 \times 10^{-19} \times 10 = 1.6 \times 10^{-18} \text{ A}$$

$\therefore$ Magnetic dipole moment, $M = IA$

$$= I \cdot \pi r^2$$

$$= 1.6 \times 10^{-18} \times 3.14 \times (2 \times 10^{-3})^2$$

$$= 2 \times 10^{-24} \text{ A-m}^2$$

8. *(b)* Energy of area of *B-H* loop,

$$\Delta Q = ms\ (\Delta\theta)$$

$$\Rightarrow \quad 10^{-2} \times 42 \times 60 = 6 \times 10^3 \times 0.1 \times 10^{-3} \times 4.2 \times \Delta\theta$$

$$\Rightarrow \quad \Delta\theta = \frac{10^{-2} \times 42 \times 60}{6 \times 10^3 \times 0.1 \times 10^{-3} \times 4.2}$$

$$\Rightarrow \quad \Delta\theta = 10°\text{C}$$

9. *(a)* Given, internal magnetic field, $B_{\text{net}} = 10\pi$ mT

current, flowing through winding, $I = 10$ A and number of turns per unit length, $n = 1500$

Since, magnetic field of toroid,

$$B_{\text{toroid}} = \mu_0 nI$$

$$= 4\pi \times 10^{-7} \times 1500 \times 10 = 6\pi \text{ mT}$$

Hence, the field due to magnetisation,

$$B_m = B_{\text{net}} - B_{\text{toroid}} = 10\pi - 6\pi$$

$$\Rightarrow \quad B_m = 4\pi \text{ mT}$$

Hence, the correct option is (a).

10. (d) Given, $l = 5\,\text{cm} = 5 \times 10^{-2}$ m,

$$a = 2\,\text{cm}^2 = 2 \times 10^{-4}\,\text{m}^2$$

and $M = 1\ \text{Am}^{-2}$

The magnetisation of the bar magnet is given by

$$I = \frac{M}{V}$$

where, V = volume of bar magnet = $a \times l$

$$\Rightarrow \quad I = \frac{M}{a \times l}$$

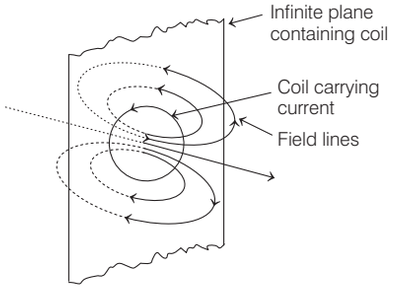
Substituting given values, we get

$$I = \frac{1}{2 \times 10^{-4} \times 5 \times 10^{-2}} = \frac{1}{10^{-5}}$$

$$= 1 \times 10^5 \text{ A/m}$$

Electromagnetic Induction

1. *(d)* We are given with following situation,



From the diagram of field lines, we can observe that whatever be the number of field lines emitted from coil, all of them goes back into the infinite plane only.

So, magnetic flux emanating from coil is equal and opposite to the flux linked with infinite plane. So, $\phi_i = -\phi_o$

2. *(a)* Energy stored in an inductor of inductance L and current I is given by

$$E = \frac{1}{2} LI^2$$

When current is being changed from I_1 to I_2, change in energy will be

$$\Delta E = E_2 - E_1 = \frac{1}{2} LI_2^2 - \frac{1}{2} LI_1^2 \quad \ldots(\text{i})$$

As I_1 and I_2 are given, we need to find value of L. Now, induced emf in a coil is

$$\varepsilon = L \frac{dI}{dt}$$

Here, $\varepsilon = 25$ V,

$$dI = I_2 - I_1 = (25 - 10) = 15 \text{ A and } dt = 1 \text{ s}$$

$$\Rightarrow \quad 25 = L \times \frac{15}{1} \text{ or } L = \frac{25}{15} = 5/3 \text{ H}$$

Putting values of L, I_1 and I_2 in Eq. (i), we get

$$\Delta E = \frac{1}{2} \times \frac{5}{3} \times [25^2 - 10^2] = \frac{1}{2} \times \frac{5}{3} \times 525$$

$$\Delta E = 437.5 \text{ J}$$

3. *(d)* Magnetic flux associated with the outer coil is

$$\phi_{\text{outer}} = \mu_0 \pi NR \cdot I = \mu_0 N\pi R(kte^{-\alpha t}) = Cte^{-\alpha t}$$

where,

$$C = \mu_0 N\pi Rk = \text{constant}$$

Induced emf,

$$e = \frac{-d\phi_{outer}}{dt} = Ce^{-\alpha t} + (-\alpha C t e^{-\alpha t})$$

$$= Ce^{-\alpha t}(1 - \alpha t)$$

$\therefore$ Induced current, $I = \dfrac{e}{\text{Resistance}}$

$\Rightarrow$ At $t = 0$, $I = -$ ve

$\therefore$ The correct graph representing this condition is given in option (d).

4. *(a)* Self inductance L_{sol} of a solenoid is given by

$L_{sol} = \mu_0 n^2 \pi r^2 L$

(Here, $n = N / L$ and L = length of solenoid)

or $L_{sol} = \dfrac{\mu_0 N^2 \pi r^2}{L}$

Clearly, $L_{sol} \propto \dfrac{1}{L}$

($\because$ All other parameters are fixed)

5. *(d)* Since, the magnetic field is dependent on time, so the net charge flowing through the loop will be given as

$$Q = \frac{\text{change in magnetic flux, } \Delta\phi_B}{\text{resistance, } R}$$

As, $\Delta\phi_B = B\,A = BA\cos\theta$

where, A is the surface area of the loop and 'θ' is an angle between B and A.

Here, $\theta = 0$

$\Rightarrow \Delta\phi_B = BA$

$\therefore$ For the time interval,

$t = 0$ ms to $t = 10$ ms,

$$Q = \frac{\Delta\phi_B}{R} = \frac{A}{R}(B_{f \text{ at } 0.01 \text{ s}} - B_{i \text{ at } 0 \text{ s}})$$

Substituting the given values, we get

$$Q = \frac{3.5 \times 10^{-3}}{10}[0.4\sin(0.5\pi) - 0.4\sin 0]$$

$$= 14\text{mC}$$

6. *(c)* Mutual inductance for a coaxial solenoid of radius r_1 and r_2 and number of turns n_1 and n_2, respectively is given as,

$M = \mu_0 n_1 n_2 \pi r_1^2 l$ (for internal coil of radius r_1)

Self inductance for the internal coil,

$$L = \mu_0 n_1^2 \pi r_1^2 l$$

$$\therefore \quad \frac{M}{L} = \frac{n_1 n_2}{n_1^2} = \frac{n_2}{n_1}$$

7. *(a)* According to the question,

radius of wire become $\dfrac{r}{2}$, so its length will be $4l$ and its resistance will become $16R$.

The number of turns is doubled in a coil, so its radius should be doubled to accomodate the length of wire. The area of coil will become 4 times.

$$\Rightarrow \quad V_2 = 8V_1 \qquad \ldots(i)$$

Now, current induced in the coil is P,

$$\therefore \quad P_1 = \frac{V_1^2}{R}$$

and power dissipated, $P_2 = \dfrac{V_2^2}{16R}$ ...(ii)

From Eqs. (i) and (ii), we get

$$P_2 = \frac{(8V_1)^2}{16R} \Rightarrow P_2 = \frac{64V_1^2}{16R}$$

$$\therefore \quad P_2 = \frac{4V_1^2}{R}$$

Then the ratio, $P_1 : P_2 = \dfrac{V_1^2}{R} : \dfrac{4V_1^2}{R}$ or $P_1 : P_2 = 1 : 4$

8. *(b)* ***Key Idea*** Induced emf due to a rotating metallic disc in uniform magnetic field B is given by expression,

$$e = \frac{1}{2}B\omega a^2$$

where, ω = angular speed and a = radius of the disc.

Given, radius of metallic disc $a = 10$ cm, angular speed of metallic disc, $\omega = 200$ rad/sec and magnetic field $B = 5$ mT

Now, putting the values in above expression,

$$e = \frac{1}{2}B\omega a^2$$

$$= \frac{1}{2} \times 200 \times 5 \times 10^{-3} \times (10 \times 10^{-2})^2$$

$$\Rightarrow \quad e = 5 \text{ mV}$$

Hence, the correct option is (b).

9. *(b)* Given , $I_1 = 6$ A, $I_2 = 1$ A, $\Delta t = 0.2$ s and $e = 150$ V

Average emf induced in a inductor,

$$e = L\frac{(I_1 - I_2)}{\Delta t}$$

Self inductance,

$$L = \frac{e \cdot \Delta t}{I_1 - I_2} = \frac{150 \times 0.2}{6 - 1} = \frac{15 \times 2}{5}$$

$$\Rightarrow \quad L = 6 \text{ H}$$

Hence, the correct option is (b).

10. (*b*) Given, $B = 10^{-12}\sin(5 \times 10^6 t)$ T,

$N = 300$ turns and area,

$A = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$

Hence, the emf induced

$$e = -N\frac{d\phi}{dt} = -N\frac{d(BA)}{dt} = -AN\frac{dB}{dt}$$

$$\Rightarrow e = -20 \times 10^{-4} \times 300 \times \frac{d}{dt}(10^{-12}\sin 5 \times 10^6 t)$$

$$\Rightarrow e = 6000 \times 10^{-4} \times 10^{-12} \times (\cos 5 \times 10^6 t) \times 5 \times 10^6$$

$$\Rightarrow e = -3 \times 10^{-6}\cos(5 \times 10^6 t) \text{ V}$$

Hence, the correct option is (b).

11. *(a)* Given, mutual inductance in the coil, $M = 0.01$ H and current in coil, $I = 5\sin 200\ \pi t$.

Emf induced in secondary coil is given,

$$e = M\frac{dI}{dt}$$

putting the value in above relation,

$$e = 0.01 \times \frac{d}{dt}(5\sin 200\pi t)$$

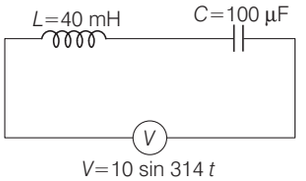
$$= 0.01 \times 5 \times 200\,\pi\cos 200\pi t$$

$$= 10\pi\cos 200\pi t$$

$$\therefore e_{max} = 10\pi \text{V} \qquad [\because \cos 200\ \pi t = 1]$$

Alternating Current

1 (*a*)



Impedance of given ciruit,

$$Z = \sqrt{(X_C - X_L)^2} = X_C - X_L$$

$$= \frac{1}{\omega C} - \omega L$$

$$= \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3}$$

$$= 19.28\,\Omega$$

As $X_C > X_L$, circuit is captive, hence current in circuit leads emf by $\dfrac{\pi}{2}$ radians.

Current in circuit is given by

$$I = I_{max}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= \frac{V_{max}}{Z}\cos\omega t = \frac{10}{19.28} \times \cos 314t$$

or $I = 0.52\cos 314t$

2 *(a)* In an *L*-*R* circuit, current growth occurs as

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right) \qquad \ldots(i)$$

where, I = instantaneous current,

I_0 = maximum current

= current at $t = \infty = \dfrac{E}{R}$,

R = resistance of circuit,

L = inductance of circuit

and t = instantaneous time.

Here, $R = 5\,\Omega$, $L = 10$ mH

$= 10 \times 10^{-3}$ H,

$E = 20$ V, $t = 40$ s

So, substituting these values in Eq. (i), we get

$$\frac{I}{I_0} = \left(1 - e^{-\frac{5}{10 \times 10^{-3}} \times 40}\right) = 1 - e^{-2 \times 10^4}$$

$$\Rightarrow \quad \frac{I_0}{I} = \frac{1}{1 - e^{-2 \times 10^4}} \approx 1$$

$$\therefore \quad \frac{I_0}{I} \approx 1.06 \text{ (nearest option)}$$

3 *(b)* Initially in the given *RL* circuit with a source, when S_1 is closed and S_2 is open at $t \le t_0$.

$$I_1 = \frac{V}{R}\left[1 - \exp\left(\frac{-R}{L}t\right)\right]$$

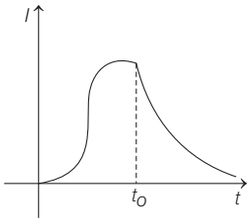
In this case, inductor L is charging.

When switch S_2 is closed and S_1 is open (after $t > t_0$), the inductor will be discharged through resistor.

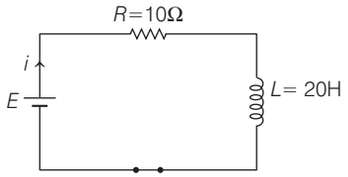
In this case ($t > t_0$),

$$I_2 = \frac{V}{R}\exp\left[-\frac{R}{L}(t - t_0)\right]$$

Thus, the variation of I with t approximately is shown below



4 *(c)* Given circuit is a series *L*-*R* circuit



In an *L*-*R* circuit, current increases as

$$i = \frac{E}{R}\left(1 - e^{\frac{-R}{L}t}\right)$$

Now, energy stored in inductor is

$$U_L = \frac{1}{2}Li^2$$

where, L = self inductance of the coil

and energy dissipated by resistor is

$$U_R = i^2 R$$

Given, rate of energy stored in inductor is equal to the rate of energy dissipation in resistor. So, after differentiating, we get

$$iL\frac{di}{dt} = i^2 R \Rightarrow \frac{di}{dt} = \frac{R}{L}i$$

$$\Rightarrow \quad \frac{E}{R}\cdot\frac{R}{L}e^{-\frac{R}{L}t} = \frac{R}{L}\cdot\frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow \quad 2e^{-\frac{R}{L}t} = 1 \Rightarrow t = 2 \ln 2$$

5 *(a)* Phase difference between I_2 and V, i.e. $C - R_2$ circuit is given by

$$\tan\phi = \frac{X_C}{R_2} \quad \Rightarrow \tan\phi = \frac{1}{C\omega R_2}$$

Substituting the given values, we get

$$\tan\phi = \frac{1}{\frac{\sqrt{3}}{2} \times 10^{-6} \times 100 \times 20} = \frac{10^3}{\sqrt{3}}$$

$\therefore \phi_1$, is nearly 90°.

Phase difference between I_1 and V, i.e. in $L - R_1$ circuit is given by

$$\tan\phi_2 = -\frac{X_L}{R_1} = -\frac{L\omega}{R}$$

Substituting the given values, we get

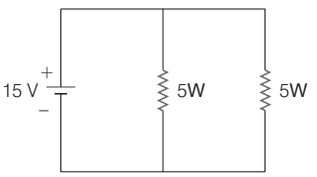
$$\tan\phi_2 = -\frac{\frac{\sqrt{3}}{10} \times 100}{10} = -\sqrt{3}$$

As, $\phi_2 = 120°$

Now, phase difference between I_1 and I_2 is

$$\Delta\phi = \phi_2 - \phi_1 = 120° - 90° = 30°$$

6 *(a)* After a sufficiently long time, in steady state, resistance offered by inductor is zero. So, circuit is reduced to



$\therefore$ Current in circuit is

$$I = \frac{E}{R_{eq}} = \frac{15}{\left(\frac{5 \times 5}{5 + 5}\right)} = \frac{15 \times 2}{5} = 6 \text{ A}$$

7 (*c*) The charge on the capacitor at time t in a discharging circuit is given by $q = q_0 e^{-t/s}$

where, $\tau = RC$ = time constant

$q = q_0 e^{t/RC}$

when, $q = \frac{q_0}{2}$ then $\frac{q_0}{2} = q_0 e^{-t/RC}$

$\frac{1}{2} = e^{-t/RC}$

Taking natural log on both side

$$\log_e\left(\frac{1}{2}\right) = \log_e e^{-t/RC}$$

$$-\log_e 2 = -\frac{t}{RC}\log_e e \Rightarrow t = RC\log_e 2$$

8 (*c*) Given, inductance of inductor, $L = \frac{6}{\pi}$ H,

capacitance of capacitor, $C = \frac{50}{\pi}\mu$F

supply voltage, $V_{rms} = 220$ V,

supply frequency, $f = 50$ Hz

and supply current, $I_{rms} = 440$ mA

Now, (A) inductive reactance, $X_L = \omega L$

$$X_L = 2\pi f \times L \qquad (\because \omega = 2\pi f)$$

$$= 2\pi \times 50 \times \frac{6}{\pi} = 600\,\Omega$$

$$X_L = 600\ \Omega$$

(B) Capacitance reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$X_C = \frac{\pi \times 10^6}{2\pi \times 50 \times 50}$$

$$\Rightarrow \quad X_C = \frac{10^6}{2500 \times 2} \Rightarrow X_C = 200\,\Omega$$

(C) Resistance, R

$\therefore$ Impedance of *LCR* circuit,

$$Z^2 = R^2 + (X_L - X_C)^2$$

Putting the given values, we get

$$(500)^2 = R^2 + (600 - 200)^2$$

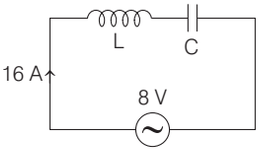
(D) Impedance,

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{220}{440} \times 10^3 \Rightarrow Z = 500\ \Omega$$

9 (*c*) Given,

rms voltage of AC source $V_{rms} = 8$ V and the rms current in the circuit, $I_{rms} = 16$A

According to the question,



$\therefore$ Impedance of the circuit,

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{8}{16}$$

$$Z = 0.5\ \Omega$$

When the inductor coil is connected to a 6V DC battery then $V'_{rms} = 6$ V $= V_{DC}$

$\therefore$ Magnitude of steady current,

$$I' = \frac{V'_{rms}}{Z} = \frac{6}{0.5} = 12 \text{ A}$$

10 (*d*) Given,

in series *LCR* circuit,

initial capacitance, $C_1 = C$

final value of capacitance, $C_2 = C'$

inductance, $L_1 = L_2 = L$

and resistance, $R_1 = 100\,\Omega$

$R_2 = 400\,\Omega$

$\therefore$ Resonance frequency,

$$f_1 = \frac{1}{2\pi\sqrt{L_1 C_1}} \Rightarrow f_1 = \frac{1}{2\pi\sqrt{LC}}$$

and $f_2 = \frac{1}{2\pi\sqrt{L_2 C_2}} \Rightarrow f_2 = \frac{1}{2\pi\sqrt{L.C'}}$

Given, $f_2 = 2f_1$

$$\Rightarrow \quad \frac{1}{2\pi\sqrt{LC'}} = \frac{2}{2\pi\sqrt{LC}}$$

$$\frac{1}{\sqrt{C'}} = \frac{2}{\sqrt{C}} \Rightarrow \frac{1}{C'} = \frac{4}{C}$$

$$\Rightarrow \quad \frac{C}{C'} = 4$$

11 *(c)* Given, alternating voltage is

$$E = 100\sin\left(\omega t + \frac{\pi}{6}\right)\text{V}$$

The voltage will be maximum, when

$$\sin\left(\omega t + \frac{\pi}{6}\right) = 1$$

$$\Rightarrow \sin\left(\omega t + \frac{\pi}{6}\right) = \sin\frac{\pi}{2}$$

$$\Rightarrow \quad \omega t + \frac{\pi}{6} = \frac{\pi}{2}$$

$$\Rightarrow \qquad t = \frac{T}{6}\left[\because \omega = \frac{2\pi}{T}\right]$$

Thus, the voltage will be maximum for the first time when $t = \frac{T}{6}$

12 (*b*) Given, input voltage supplied to transformer,

$V_1 = 220$ V

Current at output circuit of transformer,

$i_2 = 2$ A

$V_2 = 440$ V

$$\frac{\text{Output power } (P_2)}{\text{Input power } (P_1)} = 0.8 \Rightarrow \frac{V_2 i_2}{V_1 i_1} = 0.8$$

$$\frac{440 \times 2}{220 \times i_1} = 0.8$$

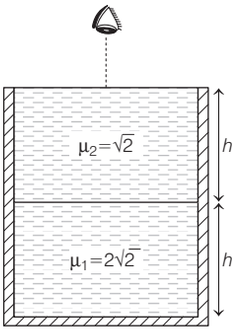
$$\frac{4}{i_1} = 0.8$$

$$\Rightarrow \qquad i_1 = \frac{4}{0.8} = 5 \text{ A}$$

Hence, current drawn by the primary windings is 5 A.

Geometric Optics

1 ***(b)***



When a vessel contains immiscible liquids, then apparent depth will be

$$d_{\text{apparent}} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \ldots$$

In given case,

$$d_{\text{apparent}} = \frac{h}{\mu_1} + \frac{h}{\mu_2} = \frac{h}{2\sqrt{2}} + \frac{h}{\sqrt{2}} = \frac{3h}{2\sqrt{2}}$$

or $\quad d_{\text{apparent}} = \frac{3}{4}h\sqrt{2}$

2 *(c)* Focal length of lens of refractive index n_2 placed in a medium of refractive index n_1 is

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a glass-lens in air, focal length,

$$\frac{1}{f_{\text{air}}} = \frac{1}{f} = \left(\frac{n_g}{n_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots (i)$$

When lens is dipped in a liquid, its focal length,

$$\frac{1}{f_{\text{liquid}}} = \frac{1}{f_1} = \left(\frac{n_g}{n_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots (ii)$$

From Eqs. (i) and (ii), we have

$$\frac{f_1}{f} = \frac{\left(\frac{n_g}{n_a} - 1\right)}{\left(\frac{n_g}{n_l} - 1\right)}$$

As $f = 16$ cm, $n_g = 1.5$, $n_l = 1.42$, we have

$$\frac{f_1}{f} = \frac{\left(\frac{1.5}{1} - 1\right)}{\left(\frac{1.5}{1.42} - 1\right)} \Rightarrow \frac{f_1}{f} = \left(\frac{\frac{3}{2} - 1}{\frac{150}{142} - 1}\right) = \left(\frac{\frac{1}{2}}{\frac{4}{71}}\right)$$

$$= \frac{71}{8} \approx 8.9$$

So, $\frac{f_1}{f}$ is closest to integer 9.

3 *(d)* Speed of light in a medium is given by

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \times \frac{1}{\sqrt{\varepsilon_r \mu_r}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

Here, $\varepsilon_r = 3$ and $\mu_r = \frac{4}{3}$

$$\therefore \qquad \frac{v}{c} = \frac{1}{\sqrt{\varepsilon_r \mu_r}} = \frac{1}{\sqrt{3 \times \frac{4}{3}}} = \frac{1}{2}$$

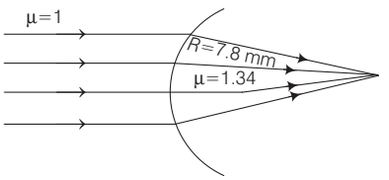
So, refractive index of medium w.r.t. vacuum or air,

$$n = \frac{c}{v} = 2$$

Hence, critical angle,

$$i_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\frac{1}{2} = 30°$$

4 *(c)* The given condition is shown in the figure below



where, a parallel beam of light is coming from air ($\mu = 1$) to a spherical surface (eye) of refractive index 1.34.

Radius of curvature of this surface is 7.8 mm.

From the image formation formula for spherical surface, i.e. relation between object, image and radius of curvature.

$$\frac{\mu_r}{v} - \frac{\mu_i}{u} = \frac{\mu_r - \mu_i}{R} \quad \ldots (i)$$

Given, $\mu_r = 1.34, \mu_i = 1, u = \infty$ (– ve) and $R = 7.8$

Substituting the given values, we get

$$\frac{1.34}{v} + \frac{1}{\infty} = \frac{1.34 - 1}{7.8}$$

$$\Rightarrow \qquad v = \frac{4}{3} \times 3 \times 7.8 \text{ mm}$$

($\because$ approximately $1.34 = 4/3$ and $0.34 = 1/3$)

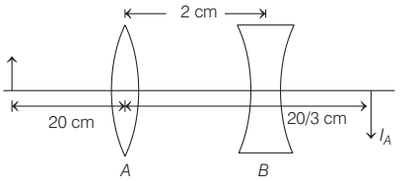
$\Rightarrow \quad v = 31.2$ mm or 3.12 cm

5 (*b*) For lens A, object distance, $u = -20$ cm

Focal length, $f = +5$ cm

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

We have, $\quad \frac{1}{v} = \frac{1}{5} - \frac{1}{20} \Rightarrow v = \frac{20}{3}$ cm



For lens *B*, image of *A* is object for *B*.

$$\therefore \qquad u = \frac{20}{3} - 2 = +\frac{14}{3} \text{ cm}$$

$f = -5$ cm

Now, from lens formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{-5} + \frac{3}{14} \Rightarrow v = 70 \text{ cm}$$

Hence, image is on right of lens *B* and is real in nature.

6 *(b)* In optical fibre communication network, the signals are transmitted by laser light operating in range of 1310 nm-1550 nm. So, the closest value is 1500 nm.

7 *(d)* Since, image formed by a convex lens can be real or virtual in nature.

Thus, $m = +2$ or -2.

First let us take image to be real in nature, then

$m = -2 = \frac{v}{u}$, where v is image distance and u is object distance.

$$\Rightarrow \qquad v = -2x_1 \quad [\text{Taking } u = x_1]$$

Now, by using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \qquad \frac{1}{-2x_1} - \frac{1}{x_1} = \frac{1}{20}$$

$$\Rightarrow \qquad x_1 = -30 \text{ cm}$$

Now, let us take image to be virtual in nature, then

$$m = 2 = \frac{v}{u} \Rightarrow v = 2x_2 \quad [\text{Taking } u = x_2]$$

Again, by using lens equation,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{2x_2}-\frac{1}{x_2}=\frac{1}{20}$$

$$\Rightarrow \quad x_2=-10 \text{ cm}$$

So, the ratio of x_1 and x_2 is

$$\frac{x_1}{x_2}=\frac{-30}{-10}=3:1$$

8 *(d)* Focal length of a lens is given as

$$\frac{1}{f}=(\mu_1-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

∴Focal length of plano-convex lens, i.e. lens 1,

$R_1=\infty$ and $R=-R$

$$\Rightarrow \quad \frac{1}{f_1}=\frac{\mu_1-1}{R} \text{ or } f_1=\frac{R}{(\mu_1-1)} \ldots \text{(i)}$$

Similarly, focal length of plano-concave lens, i.e. lens 2,

$R_1=-R$ and $R_2=\infty$

$$\Rightarrow \frac{1}{f_2}=-\frac{(\mu_2-1)}{R} \text{ or } f_2=\frac{-R}{(\mu_2-1)} \ldots \text{(ii)}$$

From Eqs. (i) and (ii), net focal length is

$$\frac{1}{f}=\frac{\mu_1-1}{R}-\frac{\mu_2-1}{R}=\frac{\mu_1-\mu_2}{R}$$

$$\Rightarrow \quad f=\frac{R}{\mu_1-\mu_2}$$

9 *(c)* Given, focal length of concave mirror,

$$f=-0.4 \text{ m}$$

Magnification = 5

We know that, magnification produced by a mirror, $m=-\dfrac{\text{image distance}}{\text{object distance}}$

$$\Rightarrow \quad \frac{v}{u}=-5 \text{ or } v=-5u$$

Using mirror formula,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Substituting the given values in the above equation, we get

$$\frac{1}{-5u}+\frac{1}{u}=-\frac{1}{0.4}$$

$$\Rightarrow \quad \frac{4}{5u}=-\frac{1}{0.4}$$

$$\Rightarrow \quad u=-\frac{1.6}{5}=-0.32 \text{ m}$$

10 *(c)* When given set up is immersed in water, focal length of lens increases,

Note

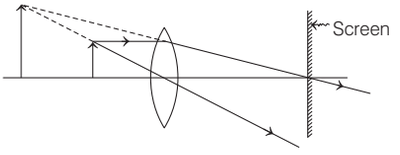
$$\frac{f_{\text{liquid}}}{f_{\text{air}}}=\frac{n_{ga}-1}{n_{gl}-1}=\frac{n_{ga}-1}{\left(\frac{n_{ga}}{n_{la}}-1\right)}$$

Now, $n_{ga}=\frac{3}{2}$ and $n_{la}=\frac{4}{3}$

$$\therefore f_{\text{liquid}}=f_{\text{air}}\left(\frac{\frac{1}{2}}{\frac{1}{8}}\right)=4f_{\text{air}}$$

As focal length increases there is no focussing of image on screen

∴ image will disappear



Actually a virtual image is formed on same side of object.

11 *(b)* For the given convex lens, focal length, $f=25$ cm

Magnification of lens in term of u and f is given by

$$m=\frac{f}{f+u}=\frac{25}{25+u} \quad \ldots \text{(i)}$$

When the image is real, then $m=-2$ and $u=-x_1$

$$\therefore \text{From Eq. (i)}, -2=\frac{25}{25-x_1}\Rightarrow x_1=\frac{75}{2}$$

Again, when the image is virtual, then $m=2$ and $u=-x_2$

$$\therefore \text{ From Eq. (i)}, 2=\frac{25}{25-x_2}\Rightarrow x_2=\frac{25}{2}$$

Hence, the ratio of x_2 and x_1 is

$$\therefore \quad \frac{x_2}{x_1}=\frac{25/2}{75/2}=\frac{1}{3}$$

12 (*a*) If middle portion of lens is painted black, then less number of refracted rays will intersect, therefore intensity of the corresponding image of object will be less. Though, the rays coming from object refracted with small portion of lens, but complete image of object will be formed but of lesser intensity.

13 (*b*) Given, focal length of convex lens,

$$f=20 \text{ cm}$$

object distance, $u=-0.1 \text{ m}=-10 \text{ cm}$

refractive index of material, $\mu=1.5$

radius of curvature of silvered surface, $R=22$ cm and focal length of concave mirror,

$$f_m=\frac{-R}{2}=-11 \text{ cm}$$

Therefore, the power of the mirror,

$$P_m=-\frac{1}{f_m}\Rightarrow P_m=\frac{1}{11}\text{D} \quad \ldots\text{(i)}$$

Further, focal length of the lens is 20 cm.

So, the power of lens, $P_l=\frac{1}{20}\text{D}$ ...(ii)

When the light after passing through the lens will be reflected back by concave mirror through convex lens again then , image will formed.

Now, the power,

P = power of mirror + power of lens + power of lens

$$P=P_m+P_l+P_l \quad \ldots\text{(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$P=\frac{1}{11}+\frac{1}{20}+\frac{1}{20}=\frac{21}{110}$$

The focal length of equivalent mirror,

$$f=-\frac{110}{21} \text{ cm}$$

Now, the object in front of a convex lens,

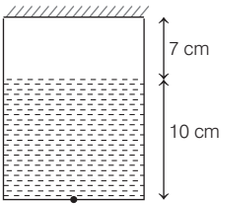
$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

$$\Rightarrow \quad \frac{-21}{110}=\frac{1}{v}-\frac{1}{10}$$

$$v=-11 \text{ cm}$$

Hence, the distance of final image from the silvered surface is 11cm.

14 (*c*) According to the question,



the apparent distance of object from the mirror, distance of image from the mirror, d

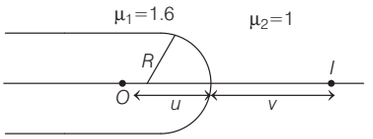
= 7 cm + apparent depth

$$\because \text{Apparent depth}=\frac{\text{Real depth}}{\mu}$$

$$=\frac{10}{1.33} \quad (\because \mu=1.33, \text{ given})$$

Hence, $d=7 \text{ cm}+\frac{10}{1.33}=14.5 \text{ cm}$

15. (*a*) A point object O is placed on the axis of a cylindrical glass as shown in the figure,



For a curved spherical surface,

$$\frac{\mu_2}{v}+\frac{\mu_1}{u}=\frac{\mu_1-\mu_2}{R}$$

where, $v=5$ mm, $R=-3$ mm, $\mu_1=1.6$ and $\mu_2=1$

Putting the given values in the above expression, we get

$$\frac{1}{5}+\frac{1.6}{u}=\frac{1.6-1}{-3}$$

$$\Rightarrow \quad u=-\frac{1.6\times 5}{2}=-4 \text{ mm}$$

Hence, the correct option is (a).

16. (*b*) Angle of deviation of the light ray produced by prism is,

$$\delta=A(\mu-1) \quad \ldots\text{(i)}$$

where, δ = angle of deviation

A = angle of prims and

μ = refractive index of the prism

Given, $A=3°$, $\delta=1°$ and $\mu=?$

Substituting the given values in Eq. (i), we get

$$1^\circ = 3^\circ(\mu - 1) \Rightarrow \mu = \frac{4}{3} = 1.33$$

Hence, the refractive index of the prism is 1.33.

17 (*d*)

18 (*a*)

19 (*b*) For final image at infinity, magnifying power of a telescope is given by

$$m = \frac{f_o}{f_e} = 9$$

where, m = magnification,

f_o = focal length of objective

and f_e = focal length of eyepiece

$$\Rightarrow \quad f_o = 9 f_e \qquad \ldots(i)$$

Also, distance between objective and eyepiece

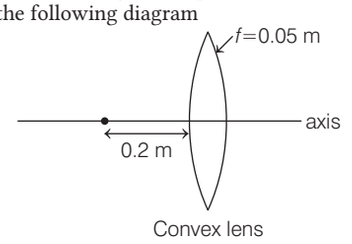
$$= f_o + f_e = 20 \text{ (given)}$$

$$\Rightarrow \quad 9 f_e + f_e = 20$$

$$\Rightarrow \quad f_e = 2 \text{ cm}$$

$$f_o = 9 f_e = 18 \text{ cm}$$

20. (*d*) According to the question, we can draw the following diagram



Given, $u = -0.2$ m and $f = 0.05$ m

As we know that,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \qquad \ldots(i)$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{0.05} - \frac{1}{0.2}$$

$$\frac{1}{v} = \frac{100}{5} - \frac{10}{2}$$

$$\frac{1}{v} = 20 - 5 \quad \Rightarrow v = \frac{1}{15} \text{ m}$$

Now, differentiating eq. (i), we get

$$-\frac{dv}{v^2} = -\frac{du}{u^2} \quad \therefore \quad dv = du\,\frac{v^2}{u^2}$$

$$A_{\max} = A \times \left(\frac{1}{15}\right)^2 \times \left(\frac{1}{(-0.2)}\right)^2$$

$$A_{\max} = A \times \frac{1}{225} \times 25$$

$$A_{\max} = \frac{A}{9}$$

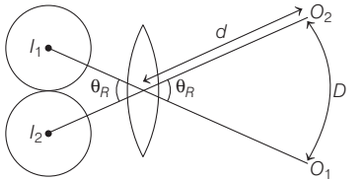
Here A is in cm.

Hence, $A_{\max} = \frac{A}{9} \times 10^{-2}$ m

Interference and Diffraction of Light

1 *(c)* From Rayleigh's criteria, two objects will be just resolved when their angular separation, $\theta_R = 1.22\,\frac{\lambda}{a}$

where, λ = wavelength and a = aperture size.



Two objects are just resolved when their central maximas does not overlap.

From diagram, distance between objects in object plane,

$$D = \theta_R \cdot d = 1.22\,\frac{\lambda d}{a}$$

Here, $a = 5$ m, $d = 4 \times 10^5$ km $= 4 \times 10^8$ m,

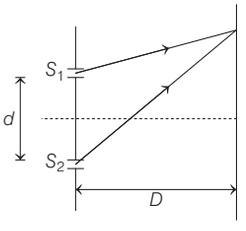
$$\lambda = 5500 \text{ Å} = 5500 \times 10^{-10} \text{ m}$$

So, $$D = \frac{(1.22 \times 5500 \times 10^{-10} \times 4 \times 10^8)}{5}$$

$$\approx 54 \text{ m}$$

From options, minimum separation distance is 60 m.

2 *(d)* In YDSE, separation between successive bright fringes is width of one dark fringe.



As, fringe width in YDSE,

$$\beta = \frac{\lambda D}{d} \qquad \ldots(i)$$

Here, $\lambda = 589$ nm $= 589 \times 10^{-9}$ m,

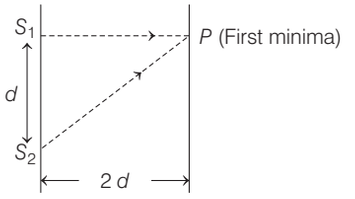
$D = 1.5$ m, $d = 0.15$ mm $= 0.15 \times 10^{-3}$ m

Substituting the given values in Eq. (i), we get

$$\beta = \frac{589 \times 10^{-9} \times 1.5}{0.15 \times 10^{-3}} = 589 \times 10^{-5} \text{ m}$$

$$= 5.89 \text{ mm or } 5.9 \text{ mm}$$

3 *(c)* In the given case, figure for first minima will be as shown below



We know that condition for minima in Young's double slit experiment is path difference,

$$\Delta x = (2n - 1)\,\lambda / 2$$

For first minima, $n = 1$

$$\Rightarrow \quad \Delta x = \lambda / 2 \qquad \ldots(i)$$

Path difference between the rays coming from virtual sources S_1 and S_2 at point 'P' will be

$$\Delta x = S_2P - S_1P \qquad \ldots(ii)$$

From triangle S_1S_2P,

$$S_1P = 2d \qquad \ldots(iii)$$

and

$$(S_2P^2) = (S_1S_2)^2 + (S_1P)^2 = d^2 + (2d)^2$$

$$\Rightarrow \quad (S_2P^2) = 5d^2 \text{ or } S_2P = \sqrt{5}\,d \qquad \ldots(iv)$$

Substituting the values from Eqs. (iii) and (iv) in Eq. (ii), we get

$$\Delta x = \sqrt{5}\,d - 2d \qquad \ldots(v)$$

From Eqs. (i) and (v), we get

$$\sqrt{5}\,d - 2d = \lambda / 2 \Rightarrow d = \frac{\lambda}{2(\sqrt{5} - 2)}$$

4 *(d)* Let intensity of each wave is I_0.

Then, intensity at the centre of bright fringe will be $4I_0$.

Given, path difference, $\Delta x = \lambda / 8$

$$\therefore \text{ Phase difference, } \phi = \Delta x \times \frac{2\pi}{\lambda}$$

$$\Rightarrow \quad \phi = \frac{\lambda}{8} \times \frac{2\pi}{\lambda} \text{ or } \phi = \pi/4$$

Intensity of light at this point,

$$I' = I_0 + I_0 + 2I_0 \cos(\pi/4)$$

$$= 2I_0 + \sqrt{2}\,I_0 = 3.41\,I_0$$

Now, $$\frac{I'}{4I_0} = \frac{3.41}{4} = 0.85$$

5 *(c)* In Young's double slit experiment, ratio of maxima and minima intensity is given by

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$$

$$= \left(\frac{\sqrt{I_1 / I_2} + 1}{\sqrt{I_1 / I_2} - 1}\right)^2$$

As, intensity $(I) \propto [\text{amplitude } (a)]^2$

$$\therefore \quad \frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

So, $\frac{I_{max}}{I_{min}} = \left(\frac{\frac{1}{3}+1}{\frac{1}{3}-1}\right)^2 = 4:1$

6 *(c)* Let the intensity of two coherent sources be I_1 and I_2, respectively.

It is given that,

$$\frac{\text{maximum intensity}}{\text{minimum intensity}} = \frac{I_{max}}{I_{min}} = \frac{16}{1}$$

Since, we know

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

and $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

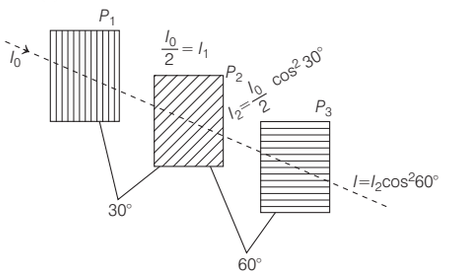
$\therefore$ We can write,

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = 16$$

$$\Rightarrow \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{\sqrt{16}}{1} = \frac{4}{1}$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{25}{9}$$

7 (*c*)



When unpolarised light pass through polaroid P_1, intensity obtained is

$$I_1 = \frac{I_0}{2}$$

where, I_0 = intensity of incident light.

Now, this transmitted light is polarised and it pass through polariser P_2. So, intensity I_2 transmitted is obtained by Malus law.

$$\Rightarrow \quad I_2 = I_1 \cos^2\theta$$

As angle of pass axis of P_1 and P_3 is 90° and angle of pass axis of P_2 and P_3 is 60°, so angle between pass axis of P_1 and P_2 is $(90° - 60°) = 30°$. So,

$$I_2 = \frac{I_0}{2}\cos^2 30° = \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8} I_0$$

When this light pass through third polariser P_3, intensity I transmitted is again obtained by Malus law.

So, $I = I_2 \cos^2 60°$

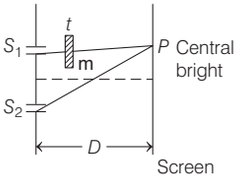
$$= \left(\frac{3}{8} I_0\right) \cos^2 60° = \frac{3}{32} I_0$$

So, ratio $\frac{I_0}{I} = \frac{32}{3} = 10.67$

8 *(c)* As we know,

Path difference introduced by thin film,

$$\Delta = (\mu - 1)t \qquad \ldots(i)$$



and if fringe pattern shifts by one frings width, then path difference,

$$\Delta = 1 \times \lambda = \lambda \qquad \ldots(ii)$$

So, from Eqs. (i) and (ii), we get

$$(\mu - 1)t = \lambda$$

$$\Rightarrow \quad t = \frac{\lambda}{\mu - 1}$$

9 (*c*)

10 *(d)* Given, angular deviation of 5th order dark fringe, $\theta = 12°$,

Width of the slit, $d = 9\ \mu m$

Now, angular deviation of nth fringe $= n \cdot \frac{\lambda}{d}$

$$\therefore \quad 12 \times \frac{\pi}{180} = 5 \times \frac{\lambda}{9 \times 10^{-6}}$$

or $$\lambda = \frac{12 \times \pi}{180} \times \frac{9 \times 10^{-6}}{5}\ m$$

or $$\lambda = 3768\ Å$$

So, the wave length of incident light is $\lambda = 3768$ Å.

11 *(c)* Given, maximum intensity along y-axis,

$$I_Y = I_0$$

and minimum intensity along x-axis,

$$I_X = \frac{2I_0}{3}$$

The intensity transmitted, through a polaroid with pass axis 45° to y-axis (i.e., 45° to x-axis also) is given by Brewster's law,

$$I = I_x \cos^2 45° + I_y \cos^2 45°$$

$$= I_0 \cos^2 45° + \frac{2I_0}{3}\cos^2 45° = \frac{5I_0}{6}$$

12 *(b)* Let m^{th} bright fringe of wavelength λ_m and n^{th} bright fringe of wave length λ_n formed at a distance x_n from central maxima at same point P, i.e., they coincide, therefore, $x_m = x_n$.

$$\frac{m\lambda_m D}{d} = \frac{n\lambda_n D}{d} \Rightarrow \frac{m}{n} = \frac{\lambda_n}{\lambda_m}$$

Putting the given values, we get

$$= \frac{600 \times 10^{-9}}{400 \times 10^{-9}}$$

[Given, $\lambda_m = 400 \times 10^{-9}$ m, $\lambda_n = 600 \times 10^{-9}$ m]

$$\therefore \quad \frac{m}{n} = \frac{3}{2}$$

So, least integral values of m and n satisfying above requirement are $m = 3$ and $n = 2$

13 *(a)* Given, limit of resolution, $\alpha = 2.5 \times 10^{-7}$ rad. and wavelength, $\lambda = 500$ nm

As, limit of resolution of a telescope,

$$\alpha = \frac{1.22\lambda}{a}$$

So,

$$a = \frac{1.22\lambda}{\alpha} = \frac{1.22 \times 500 \times 10^{-9}}{2.5 \times 10^{-7}}$$

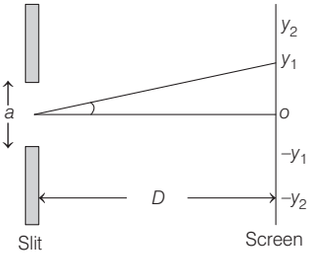
$$\Rightarrow \quad a = 2.44\ m$$

$$= 244\ cm$$

Hence, the correct option is (a)..

14 (*c*) Due to the doppler's effect of light, when an object moves closer to source the light moves to the blue end of the spectrum, as its wavelength get shorter. (or frequency increases) Thus, if a star emitting yellow light and is accelerating towards an observer on earth, it will appear that its colour is gradually changing to blue. So, option (c) is correct.

15 *(b)* The diffraction pattern due to a single slit is shown below



The width of central maximum is given by

$$2y = \frac{2D\lambda}{a} \qquad \ldots(i)$$

where, λ = wavelength of incident light.

Here, $a = 2y$, then from Eq. (i), we get

$$a = \frac{2D\lambda}{a}$$

$$\Rightarrow \quad D = \frac{a^2}{2\lambda}$$

16 *(c)*

17 *(c)* As we know that,

fringe width, $\beta = \frac{\lambda D}{d}$

where, λ = wavelength of light

D = Distance of the screen from the slits

d = separation between the slits

Now, $\frac{\Delta\beta}{\beta} \times 100 = \frac{\Delta D}{D} \times 100 + \frac{\Delta d}{d} \times 100$

According to the question,

$$\frac{\Delta\beta}{\beta} \times 100 = 0.5 + (-0.3) = 0.2\%$$

Hence, the fringe width increases by 0.2%.

Modern Physics

1 *(c)* Initial velocity of electron,

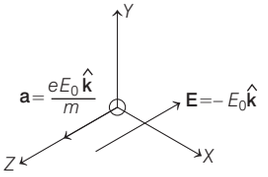
$\mathbf{v} = v_0\hat{\mathbf{i}} + v_0\hat{\mathbf{j}} \Rightarrow |\mathbf{u}| = \sqrt{v_0^2 + v_0^2} = v_0\sqrt{2}$

Initial de Broglie wavelength associated with electron,

$$\lambda_0 = \frac{h}{p} = \frac{h}{mv_0\sqrt{2}} \quad \ldots(i)$$

After t seconds velocity of electron,

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$



or $\mathbf{v} = (v_0\hat{\mathbf{i}} + v_0\hat{\mathbf{j}}) + \frac{eE_0}{m}\cdot t\cdot\hat{\mathbf{k}}$

Note that electron is accelerated opposite to the field.

Magnitude of velocity after time t,

$$|\mathbf{v}| = \sqrt{v_0^2 + v_0^2 + \left(\frac{eE_0}{m}t\right)^2}$$

$$= \sqrt{2v_0^2 + \frac{e^2E_0^2t^2}{m^2}} = v_0\sqrt{2 + \frac{e^2E_0^2t^2}{m^2v_0^2}}$$

∴de Broglie wavelength of electron at time t,

$$\lambda = \frac{h}{p} = \frac{h}{m|\mathbf{v}|} \Rightarrow \lambda = \frac{h}{mv_0\sqrt{2 + \frac{e^2E_0^2t^2}{m^2v_0^2}}}$$

$$= \frac{\lambda_0\sqrt{2}}{\sqrt{2 + \frac{e^2E_0^2t^2}{m^2v_0^2}}} = \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{2m^2v_0^2}}}$$

2 *(d)* An electromagnetic wave is a travelling wave in which electric vector $\mathbf{E}$ is perpendicular to magnetic vector $\mathbf{B}$ and both of these vectors are also perpendicular to the direction of propagation of electromagnetic wave.

So, it is given that

$$\mathbf{B} = 3\times10^{-8}\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{\mathbf{j}}\ \text{T}$$

$$= B_0\sin(kx + \omega t)\hat{\mathbf{j}}\,\text{T}$$

Here, $k = 1.6\times10^3$ units

and $\omega = 48\times10^{10}$ units

Then, it can be implied that electromagnetic wave is travelling along $-X$-axis (this is obtained from phase $kx + \omega t$) and its magnetic vector $\mathbf{B}$ is directed along $+Y$-axis (this is indicated by unit vector $\hat{\mathbf{j}}$).

Hence, electric field vector is along $+Z$-axis and its magnitude is given by

$E_0 = c\cdot B_0 = 3\times10^8\times3\times10^{-8} = 9$ V/m

∴ Expression for $\mathbf{E} = 9\sin(kx + \omega t)\hat{\mathbf{k}}$

$= 9\sin(1.6\times10^3 x + 48\times10^{10}t)\hat{\mathbf{k}}$ V/m

3 *(b)* Radius of largest circular path of electron in perpendicular magnetic field,

$$r = \frac{mv_{\max}}{Be}$$

∵ Velocity of most energetic electron emitted from metal surface,

$$v_{\max} = \frac{B\,e\,r}{m}$$

Kinetic energy with which electron is emitted,

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}m\cdot\frac{B^2e^2r^2}{m^2}$$

$$\Rightarrow \quad K_{\max} = \frac{1}{2}\left(\frac{B^2e^2r^2}{m}\right)\text{joule}$$

Kinetic energy of emitted electron (in electron volts),

$$K_{\max}\ (\text{eV}) = \frac{K_{\max}\ (\text{in joule})}{e}$$

$$\therefore \quad K_{\max}\ (\text{eV}) = \frac{1}{2}\left(\frac{B^2er^2}{m}\right)$$

Here given,

$B = 3\times10^{-4}$ T, $e = 1.6\times10^{-19}$ C,

$r = 10\,\text{mm} = 10^{-2}$ m and $m = 9.1\times10^{-31}$ kg

So,

$$K_{\max} = \frac{(3\times10^{-4})^2\times(1.6\times10^{-19})\times(10^{-2})^2}{2\times(9.1\times10^{-31})}$$

$$= \frac{9\times1.6}{2\times9.1}\times10^{31-31} \approx 0.8\ \text{eV}$$

∴ Energy of incident photon $= hf = \frac{hc}{\lambda}$

$$= \frac{12400\ \text{eV-Å}}{6561\ \text{Å}} \approx 1.9\,\text{eV}$$

(**Note** $hc = 1240$ eV- nm $= 12400$ eV-Å)

So, work function of metal,

ϕ_0 = incident energy − kinetic energy of emitted electron

$= 1.9 - 0.8 = 1.1\,\text{eV}$

4 *(c)* As, ionisation energy of ground state

$= -$ (ground state energy)

$\Rightarrow 9\times13.6 = -E_1$ (∵ 1 Rydberg = 13.6 eV)

or $E_1 = -9\times13.6\,\text{eV}$

and energy of nth state for the given atom will be

$$E_n = \frac{E_1}{n^2} = \frac{-9\times13.6}{n^2}\,\text{eV}$$

In transition from second excited state $(n = 3)$ to ground state $(n = 1)$, energy released will be

$$\Delta E = E(n=3) - E(n=1)$$

$$= \left(\frac{-9\times13.6}{3^2}\right) - \left(\frac{-9\times13.6}{1^2}\right)$$

or $\Delta E = 9\times13.6\left(1 - \frac{1}{9}\right) = 8\times13.6\,\text{eV}$

So, wavelength of radiation emitted will be

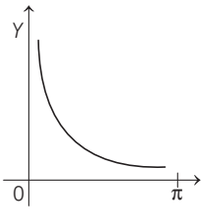
$$\lambda = \frac{hc}{\Delta E} = \frac{1240(\text{eV- nm})}{\Delta E(\text{eV})}$$

$$\Rightarrow \quad \lambda = \frac{1240}{8\times13.6}$$

$= 11.39$ nm ≈ 11.4 nm

5 *(b)* In Rutherford's experiment, number of particles scattered at large angles is very less and most of the particles are scattered at small angles.

Hence, graph of Y = number of α- particles and θ = scattering angle is as shown in the figure below.



6 *(d)* We know that,

$$\left(\frac{E_0}{B_0}\right)_{\text{air/vacuum}} = c \quad \text{and} \quad \left(\frac{E}{B}\right)_{\text{medium}} = v$$

Also, $n = \frac{c}{v}$

$$\frac{E_0/B_0}{E/B} = \frac{c}{v} = n \Rightarrow \frac{E_0/E}{B_0/B} = n$$

This is possible only if $\frac{E_0}{E} = \sqrt{n}$ and $\frac{B_0}{B} = \frac{1}{\sqrt{n}}$.

Also, refractive index of medium is 'n' and is given as

$$\frac{c}{v} = n \quad \text{or} \quad \frac{v}{c} = \frac{1}{n} \quad \ldots(iv)$$

∴ From Eqs. (iii) and (iv), we get

$$n = \sqrt{K} \ \text{or}\ K = n^2 \quad \ldots(v)$$

The intensity of a EM wave is given as,

$$I = \frac{1}{2}\varepsilon_0E_0^2c$$

and in the medium, it is given as

$$I' = \frac{1}{2}K\varepsilon_0E^2v$$

It is given that, $I = I'$

$$\Rightarrow \quad \frac{1}{2}\varepsilon_0E_0^2c = \frac{1}{2}K\varepsilon_0E^2v$$

or $\left(\frac{E_0}{E}\right)^2 = \frac{Kv}{c}$ …(vi)

From Eqs. (iv), (v) and (vi), we get

$$\left(\frac{E_0}{E}\right)^2 = \frac{n^2}{n} = n$$

or $E_0/E = \sqrt{n}$

Similarly, $\frac{1}{2}\cdot\frac{B_0^2}{\mu_0}c = \frac{1}{2}\cdot\frac{B^2}{\mu_0}v$

$$\Rightarrow \quad \frac{B_0}{B} = \frac{1}{\sqrt{n}}$$

7 *(a)* Wavelength of the given photon is given as,

$$\lambda_p = \frac{c}{\nu_p} = \frac{3\times10^8}{6\times10^{14}}\ \text{m}$$
$$= 5\times10^{-7}\ \text{m} \quad ...(i)$$

As, it is given that, de-Broglie wavelength of the electron is

$$\lambda_e = 10^{-3}\times\lambda_p \quad [\because \text{using Eq. (i)}]$$
$$= 5\times10^{-10}\ \text{m}$$

Also, the de-Broglie wavelength of an electron is given as,

$$\lambda_e = \frac{h}{p} = \frac{h}{mv_e} \Rightarrow v_e = \frac{h}{\lambda_e m_e}$$

Substituting the given values, we get

$$= \frac{6.63\times10^{-34}}{5\times10^{-10}\times9.1\times10^{-31}}\ \text{m/s}$$
$$= 1.45\times10^6\ \text{m/s}$$

8 *(c)* We know that net change in energy of a photon in a transition with wavelength λ is $\Delta E = hc/\lambda$.

Here, $hc = 12500$ eV Å and $\lambda = 980$ Å

$$\Delta E = 12500/980 = 12.76\ \text{eV}$$
$$\Rightarrow \quad E_n - E_1 = 12.76\ \text{eV}$$

Since, the energy associated with an electron in n^{th} Bohr's orbit is given as,

$$E_n = \frac{-13.6}{n^2}\ \text{eV} \quad ...(i)$$
$$\Rightarrow \quad E_n = E_1 + 12.76\ \text{eV} \quad ...(ii)$$
$$= \frac{-13.6}{(1)^2} + 12.76 = -0.84$$

Putting this value in Eq. (i)

$$\Rightarrow \quad n^2 = \frac{-13.6}{-0.84} = 16 \Rightarrow n = 4$$

and radius of nth orbit, $r_n = n^2 a_0$

$$\Rightarrow \quad r_n = 16\,a_0$$

9 *(*)* Let maximum speed of photo electrons in first case is v_1 and maximum speed of photo electrons in second case is v_2

Assumption I if we assume difference in maximum speed in two cases is 2 then $v_1 = v$ and $v_2 = 3v$

According to Einstein's photo electron equation

Energy of incident photon = work function + KE

i.e. $$\frac{hc}{\lambda} = \phi_0 + \frac{1}{2}mv^2$$

Where $hc = 1240$ eV, λ is wavelength of light incident, ϕ_0 is work function and v is speed of photo electrons.

When $\lambda_1 = 350$ nm

$$\therefore \quad \frac{hc}{350} = \phi_0 + \frac{1}{2}mv^2$$

or $$\frac{hc}{350} - \phi_0 = \frac{1}{2}mv^2 \quad ...(i)$$

when $\lambda_2 = 540$ nm

$$\therefore \quad \frac{hc}{540} = \phi_0 + \frac{1}{2}m(3v^2)$$

$$\therefore \quad \frac{hc}{540} - \phi_0 = \left(\frac{1}{2}mv^2\right)\times9 \quad ...(ii)$$

Now, we divide Eq. (i) by Eq. (ii), we get

$$\frac{\frac{hc}{350} - \phi_0}{\frac{hc}{540} - \phi_0} = \frac{\frac{1}{2}mv^2}{\left(\frac{1}{2}mv^2\right)\times9} = \frac{1}{9}$$

or $$9\left(\frac{hc}{350} - \phi_0\right) = \frac{hc}{540} - \phi_0$$

or $$8\phi_0 = hc\left[\frac{9}{350} - \frac{1}{540}\right]$$

or $$\phi_0 = \frac{1}{8}\times1240\left[\frac{9\times540 - 350}{350\times540}\right] = 3.7\ \text{eV}.$$

No option given is correct.

10 *(a)* Given,

$$\mathbf{E} = 10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14}t\right)\hat{\mathbf{x}}\ \text{NC}^{-1}$$

By comparing it with the general equation of electric field of light, i.e.

$$E = E_0\cos(kx - \omega t)\,\hat{\mathbf{x}}, \quad \text{we get} \quad k = \frac{2\pi}{5\times10^{-7}} = 2\pi/\lambda$$

(from definition, $k = 2\pi/\lambda$)

$$\Rightarrow \quad \lambda = 5\times10^{-7}\ \text{m} = 5000\ \text{Å} \quad ...(i)$$

Or

The value of λ can also be calculated as, after comparing the given equation of $\mathbf{E}$ with standard equation, we get

$$\omega = 6\times10^{14}\times2\pi$$
$$\Rightarrow \quad \nu = 6\times10^{14} \quad [\because 2\pi\nu = \omega]$$

As, $$c = \nu\lambda$$

$$\Rightarrow \lambda = \frac{c}{\nu} = \frac{3\times10^8}{6\times10^{14}} = 5\times10^{-7}\ \text{m} = 5000\ \text{Å}$$

According to Einstein's equation for photoelectric effect, i.e.

$$\frac{hc}{\lambda} - \phi = (\text{KE})_{\max} = eV_0 \quad ...(ii)$$

For photon, substituting the given values,

$$E = \frac{hc}{\lambda} = \frac{12375\ \text{eV}}{\lambda} \quad [\text{given}]$$

or $$\frac{hc}{\lambda} = \frac{12375}{5000}\ \text{eV} \quad [\text{using Eq. (i)}] \quad ...(iii)$$

Now, substituting the values from Eq. (iii) in Eq. (ii), we get

$$\frac{12375}{5000}\ \text{eV} - 2\ \text{eV} = eV_0$$
$$\Rightarrow \quad 2.475\ \text{eV} - 2\ \text{eV} = eV_0$$
or $$V_0 = 2.475\ \text{V} - 2\ \text{V}$$
$$= 0.475\ \text{V} \Rightarrow V_0 \approx 0.48\ \text{V}$$

11. *(d)* Expression for the energy of the hydrogenic electron states for atoms of atomic number Z is given by

$$E = h\nu = \frac{Z^2me^4}{8h^2E_0^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right] \text{ (Here, } m < n\text{)}$$

or $$\frac{hc}{\lambda} = \frac{Z^2me^4}{8h^2E_0^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda} \propto \left(\frac{1}{m^2} - \frac{1}{n^2}\right)Z^2$$

For first case,

$$\lambda = 660\ \text{nm},\ m = 2 \text{ and } n = 3$$

$$\therefore \quad \frac{1}{660\ \text{nm}} \propto \left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right]Z^2$$

$$\Rightarrow \frac{1}{660\ \text{nm}} \propto \left(\frac{1}{4} - \frac{1}{9}\right)Z^2 \text{ or } \frac{5}{36}Z^2 \quad ...(i)$$

For second case, transition is from $n = 4$ to $n = 2$, i.e. $m = 2$ and $n = 4$

$$\therefore \quad \frac{1}{\lambda} \propto \left(\frac{1}{(2)^2} - \frac{1}{(4)^2}\right)Z^2$$

$$\Rightarrow \quad \frac{1}{\lambda} \propto \left(\frac{1}{4} - \frac{1}{16}\right)Z^2$$

or $$\frac{1}{\lambda} \propto \frac{3}{16}Z^2 \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{\lambda}{660\,\text{nm}} = \frac{5}{36}\times\frac{16}{3}$$

$$\Rightarrow \lambda = \frac{80}{108}\times660\ \text{nm} = 488.9\,\text{nm}$$

12 *(a)* Given, de-Broglie wavelengths for particles are λ_1 and λ_2.

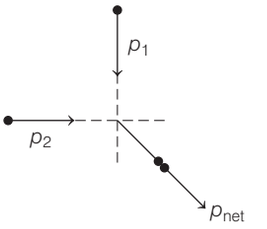
So, $$\lambda_1 = \frac{h}{p_1} \text{ and } \lambda_2 = \frac{h}{p_2}$$

and momentum of particles are

$$p_1 = \frac{h}{\lambda_1} \text{ and } p_2 = \frac{h}{\lambda_2}$$

Given that, particles are moving perpendicular to each other and collide inelastically.

So, they move as a single particle.



So, by conservation of momentum and vector addition law, net momentum after collision,

$$p_{\text{net}} = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos90^\circ}$$
$$= \sqrt{p_1^2 + p_2^2}$$

Since, $$p_1 = \frac{h}{\lambda_1} \text{ and } p_2 = \frac{h}{\lambda_2}$$

So, $$p_{\text{net}} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}} \quad ...(i)$$

Let the de-Broglie wavelength after the collision is λ_{net}, then

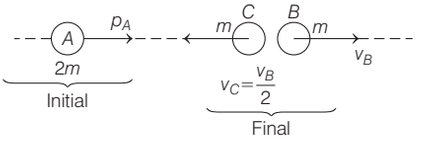
$$p_{net} = \frac{h}{\lambda_{net}} \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{h}{\lambda_{net}} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}} \Rightarrow \frac{1}{\lambda_{net}^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

13 *(b)* Let m be the mass of nuclei B and C.

So, the given situation can be shown in the figure below



Now, according to the conservation of linear momentum,

Initial momentum = Final momentum

$$\Rightarrow p_A = p_B + p_C \text{ or } 2mv_A = mv_B + mv_C$$

$$2mv_A = mv_B - \frac{mv_B}{2}$$

$$\Rightarrow \quad 2v_A = \frac{1}{2}v_B \Rightarrow v_B = 4v_A \quad ...(i)$$

and $$v_C = \frac{v_B}{2} = 2v_A \quad ...(ii)$$

So, momentum of B and C respectively, can now be given as

$$p_B = m_B v_B = m4v_A = 2(2mv_A)$$

[using Eq. (i)]

or $$p_B = 2p_A \quad ...(iii)$$

and $$p_C = m_C v_C = m2v_A$$ [using Eq. (ii)]

or $$p_C = p_A \quad ...(iv)$$

From the relation of de-Broglie wavelength,

i.e. $\lambda = \frac{h}{p}$

where, p is momentum and h is Planck's constant.

So, for A, $\lambda_A = \frac{h}{p_A}$ or $p_A = \frac{h}{\lambda_A}$ ...(v)

For B, $\lambda_B = \frac{h}{p_B} = \frac{h}{2p_A}$ [using Eq. (iii)]

From Eq. (v), λ_B can be written as

$$\lambda_B = \frac{h}{2 \times \frac{h}{\lambda_A}} = \frac{\lambda_A}{2}$$

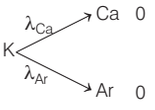
Similarly, for C, $\lambda_C = \frac{h}{p_C} = \frac{h}{p_A}$

[using Eq. (iv)]

Similarly, from Eq. (v), λ_C can be written as

$$\lambda_C = \frac{h}{\frac{h}{\lambda_A}} = \lambda_A$$

14. *(a)*



at $t = 0$ dissipated energy,

$$\frac{dN}{dt} = -(\lambda_1 + \lambda_2) \times N$$

$$\Rightarrow \quad \frac{dN}{N} = -(\lambda_1 + \lambda_2)\, dt$$

Integration on both sides, we get

$$\log_e\left(\frac{N}{N_0}\right) = -(\lambda_1 + \lambda_2)t$$

$$2.3 \times \log_{10}\left(\frac{N_0}{N_0/100}\right) = 5 \times 10^{-10} t$$

$$\frac{2.303 \times 2}{5 \times 10^{-10}} = t$$

$$2.303 \times 0.4 \times 10^{10} = t \Rightarrow t = 9.2 \times 10^9 \text{ Yr}$$

15 *(c)* Given, half life $(T_{1/2}) = 23.1$ days

If N_0 be the initial amount, then active nuclei at $t = 15$ days

$$N_1 = N_0 e^{-15\lambda} \quad ... (i)$$

Active nuclei at $t = 16$ days

$$N_2 = N_0 e^{-16\lambda} \quad ...(ii)$$

Disintegration constant,

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{23.1} = 0.03$$

$\therefore$% decay in 15th to 16th days

$$= \frac{N_1 - N_2}{N_1} \times 100$$

$$= \frac{N_0 e^{-15\lambda} - N_0 e^{-16\lambda}}{N_0 e^{-15\lambda}} \times 100$$

$$= (1 - e^{-\lambda}) \times 100 = (1 - e^{-0.03}) \times 100$$

$$= \left(1 - \frac{1}{e^{0.03}}\right) \times 100$$

$$= \left(1 - \frac{1}{1.03}\right) \times 100 = 2.9\%$$

16 *(b)* Magnetic field at the centre of circular loop,

$$B = \frac{\mu_0 I}{2r} \quad ... (i)$$

Period of revolution of electron, where ω = angular velocity of electron

$$T = \frac{2\pi}{\omega}$$

where, ω = angular velocity of electron

Current produced due to movement of electron,

$$I = \frac{e}{T} = \frac{e}{\frac{2\pi}{\omega}} = \frac{e\omega}{2\pi}$$

On substituting the value of I in Eq. (i), we get

$$B = \frac{\mu_0}{2r} \cdot \frac{e\omega}{2\pi} \quad \Rightarrow B = \frac{\mu_0 e\omega}{4\pi r}$$

17 *(c)* $\because$ de-Broglie relation of a charged particles,

$$\lambda = \frac{h}{mv}$$

Velocity of charged particle at time t,

$$\mathbf{v} = v_0\hat{\mathbf{i}} + \frac{eE_0}{m} t\,\hat{\mathbf{j}} \text{ or } |v| = \sqrt{v_0^2 + \left(\frac{eE_0}{m} t\right)^2}$$

Hence, $\lambda = \frac{h}{m\sqrt{v_0^2 + \left(\frac{eE_0}{m}t\right)^2}}$

or $$\lambda = \frac{h}{mv_0\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}}$$

or $$\lambda = \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}} \quad \left(\because \lambda_0 = \frac{h}{mv_0}\right)$$

18 *(a)* (A) Speed of revolution of electron,

$$= \frac{1}{4\pi\varepsilon_0} \frac{2\pi Ze^2}{nh}$$

(B) Kinetic energy $= \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{2\pi^2 me^4 Z^2}{n^2h^2}$

(C) Total Energy

$$= -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{2\pi^2 me^4 Z^2}{n^2h^2}$$

(D) Frequency $= \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{4\pi^2 Z^2 e^4 m}{n^3h^3}$

Hence, A $\rightarrow$ i; B $\rightarrow$ iii; C $\rightarrow$ ii; D $\rightarrow$ iv.

19 *(d)* After n half-life, the numbers of atom left undecayed is given by,

$$N = N_0\left(\frac{1}{2}\right)^n$$

$\because$ $$n = \frac{t}{T_{\frac{1}{2}}} \text{ or } N = N_0\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

For 20% decay of radioactive substance,

$$N = N_0 - \frac{20}{100}N_0 = 0.8\, N_0$$

or $$0.8\, N_0 = N_0\left(\frac{1}{2}\right)^{t_{1/18}} \quad ...(i)$$

For 80% decay of radioactive substance,

$N = N_0 - \frac{80}{100}N_0 = 0.2\, N_0$ or

$$0.2\, N_0 = N_0\left(\frac{1}{2}\right)^{t_{2/18}} \quad ...(ii)$$

From Eqs. (i) and (ii), we get $\Rightarrow 4 = \left(\frac{1}{2}\right)^{\frac{t_1 - t_2}{18}}$

Taking log on the both sides, we get

$$\log 4 = \log\left(\frac{1}{2}\right)^{\frac{t_1 - t_2}{18}}$$

$$\Rightarrow \quad \log 2^2 = \log 2^{\left(\frac{t_2 - t_1}{18}\right)}$$

$$\Rightarrow \quad 2 = \frac{t_2 - t_1}{18} \quad \Rightarrow t_2 - t_1 = 36 \text{ min}$$

20 (*) Given, the half-life of neutron, $t_{1/2} = 693$ sec

Kinetic energy of neutron = 0.084 eV

$$\frac{1}{2}mv^2 = 0.084 \times 1.6 \times 10^{-19}$$

$$v^2 = \frac{2 \times 0.084 \times 1.6 \times 10^{-19}}{m}$$

$$= \frac{2 \times 0.084 \times 1.6 \times 10^{-19}}{1.68 \times 10^{-27}} = 0.16 \times 10^8$$

$$v = 0.4 \times 10^4 \text{ m/s}$$

Time taken by neutron to travelled in 1 km,

$$t = \frac{1000}{0.4 \times 10^4} \Rightarrow t = 0.25 \text{ s}$$

By Radioactive Decay's law,

$$N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{N_0}{N} = 2^n$$

Taking log on the both sides, we get

$$\ln \frac{N_0}{N} = \ln 2^n = n \ln 2 = \frac{t}{t_{1/2}} \ln 2$$

$$= \frac{0.25 \times 0.693}{693} \quad [\because \ln 2 = 0.693]$$

$$= 0.25 \times 10^{-5}$$

21 (*a*) Magnetic moment of revolving electron,

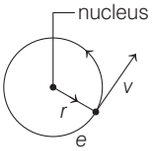
$$\mu_e = i \times A$$

where, i = current and A = area,

$$\therefore \quad \mu_e = \frac{e}{T} \times \pi r^2$$

$$\left(\text{As current, } i = \frac{\text{charge}}{\text{time}} = \frac{e}{T}\right)$$

where, T = time period of revolution and r = radius of orbit.



$$\mu_e = \frac{e}{2\pi r} \times v \times \pi r^2$$

$$\left(\because \text{Time period } (T) = \frac{\text{distance}}{\text{speed}}\right)$$

where, v = velocity of revolving electron

$$\Rightarrow \quad \mu_e = \frac{evr}{2}$$

Hence, the magnetic moment of revolving electron is $\frac{evr}{2}$.

22 (*b*) According to the question,

$$X \xrightarrow{\alpha\text{-particle}} Y \xrightarrow{\beta\text{-particle}} Z(\text{say})$$

According to Rutherford Soddy law of radioactive decay, the rate of decay of radioactive along at any instant is proportional to the number of atoms present at that instant

$$\frac{dN}{dt} = -\lambda N$$

Since, λ = constant (decay constant) and

N = number of atoms

Rate of decay of Y particle is given as,

$$\lambda_X N_X - \lambda_Y N_Y = \frac{dN_Y}{dt} = 0$$

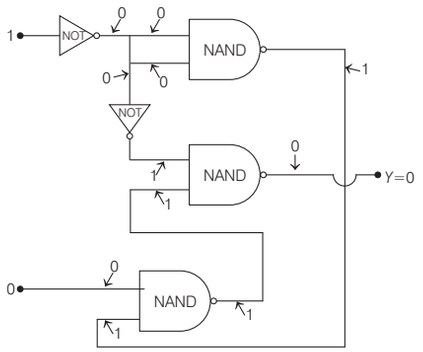
[$\because$ Decay rate for β-particle become constant after some time]

Given, rate of emission of β-particle = 10^7/h

$$\therefore \lambda_X N_X = \lambda_Y N_Y = 10^7/\text{h}$$

Solids and Semiconductor Devices

1. (*c*) We are showing intermediate outputs on the circuit as given below.

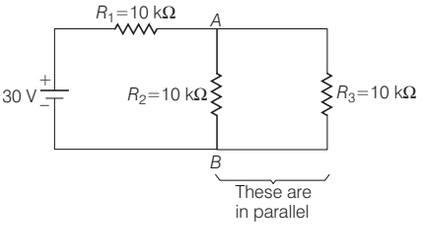


Output of above circuit at given inputs is zero. To calculate output, we uses truth tables of NAND and NOT gates.

NAND gate		
A	***B***	***Y***
0	0	1
1	0	1
0	1	1
1	1	0

NOT gate	
Y_{in}	Y_{out}
0	1
1	0

2. (*b*) In ideal condition, a forward biased *p-n* junction acts like a conductive path of zero resistance. So, given circuit is equivalent to circuit shown below.



Total resistance of circuit,

R = (R_3 in parallel with R_2) and these are in series with R_1

$$= \frac{10 \times 10}{10 + 10} + 10 = 15 \text{ k}\Omega$$

So, total current in the circuit,

$$I = \frac{V}{R} = \frac{30}{15 \text{ k}\Omega} = 2 \times 10^{-3} \text{ A}$$

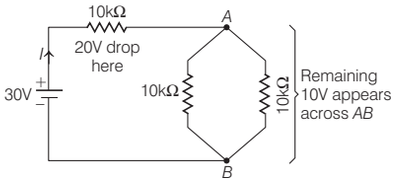
Now, potential drop across R_1, (10 kΩ),

$$V_{R_1} = IR_1 = 2 \times 10^{-3} \times 10 \times 10^3 = 20 \text{ V}$$

Hence, remaining potential appears across parallel combination of R_2 and R_3,

$$V_{AB} = 30 - 20 = 10 \text{ V}$$

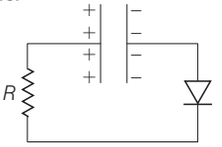
This can be seen in the figure below.



3. (*a*) Charge on capacitor after charging with a battery of potential V,

$$Q = CV$$

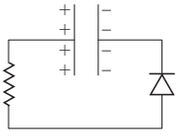
Now, capacitor A is connected as shown in the figure.



As diode is in reverse bias, so capacitor charge remains same after time $t = CR$.

$$\therefore \quad Q_A = CV$$

In case B, capacitor is connected as shown in the figure.



Here, diode is in forward bias, so capacitor discharges and its charge decays as

$$Q_B = Q.e^{\frac{-t}{RC}}$$

where, $t = RC$

4. (*c*) $\therefore$ Current in $R_1 = \frac{2}{500} = 4\text{mA} = I_1$ (say)

Current in each

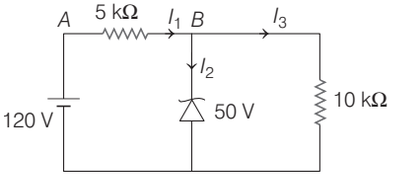
$$R_2 = \frac{10}{750} = \frac{2}{150} = 13.33 \text{ mA} = I_2 \text{ (say)}$$

$$\Rightarrow \quad I_1 < I_2 \text{ which is not possible.}$$

So, Zener diode will never reach to its breakdown.

$\therefore$ Current flowing through a reverse biased Zener diode = 0.

5. (*c*) In the circuit, let the current in branches is as shown in figure below



By Kirchhoff's node law,

$$I_1 = I_2 + I_3 \quad \text{... (i)}$$

Now, when diode conducts, voltage difference between points A and B will be

$$V_{AB} = 120 - 50 = 70 \text{ V}$$

So, current $I_1 = \dfrac{V_{AB}}{5 \text{ k}\Omega} = \dfrac{70}{5 \times 10^3}$

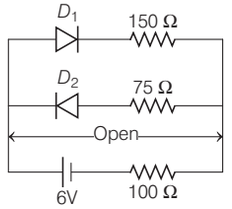
$$I_1 = 14 \text{ mA} \quad \text{...(ii)}$$

Since, diode and 10 kΩ resistor are in parallel combination, so voltage across 10 k Ω resistor will be 50 V only.

$$\Rightarrow \quad I_3 = \frac{50}{10 \text{ k}\Omega} = \frac{50}{10 \times 10^3}$$

$$\Rightarrow \quad I_3 = 5 \text{ mA} \quad \text{... (iii)}$$

6. (*b*) In this circuit, D_1 is forward biased and D_2 is reversed biased.



Resistance of D_1 is 50 Ω.

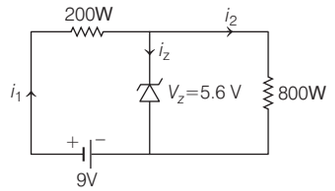
∴ Net resistance of the circuit,

$$R_{\text{net}} = 50 + 150 + 100 = 300\,\Omega$$

∴ Current through the 100 Ω resistance

$$= \frac{V}{R_{\text{net}}} = \frac{6}{300} = 0.020 \text{ A}$$

7. (*a*) Given circuit is Zener diode circuit



where,potential drop across 800 Ω resistance = potential drop across Zener diode = 5.6 V

So, current, $i_2 = \dfrac{V}{R} = \dfrac{5.6}{800} = 7\text{mA}$

Now, potential drop across 200 Ω resistance

$$= 9 - 5.6 = 3.4 \text{ V}$$

Current, $i_1 = \dfrac{V}{R} = \dfrac{3.4}{200} = 17 \text{ mA}$

So, current, $i_z = i_1 - i_2$

$$= 17 - 7 = 10 \text{ mA}$$

8. (*c*) Given, in a transistor current gain,

$$\alpha = \frac{20}{21} \text{ and } \frac{100}{101}$$

Now, common emitter transistor

DC current gain, $\beta = \dfrac{\alpha}{1-\alpha}$

Case I If $\alpha = \dfrac{20}{21}$, then $\beta = \dfrac{\frac{20}{21}}{1 - \frac{20}{21}} = 20$

Case II $\alpha = \dfrac{100}{101}$, then $\beta = \dfrac{\frac{100}{101}}{1 - \frac{100}{101}} = 100$

∴ Value of β varies between, $\beta \in [20, 100]$.

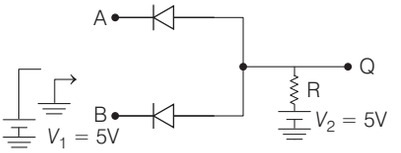
9. (*b*) The given truth table is

A	B	C	$Y = \overline{B}$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

From above truth table, it is clear that the function from of the circuit is $\overline{B}$ i.e., $y = \overline{B}$.

10. (*b*) From circuit diagram, when both or either terminal of A and B are at low (0V) then, 5V of

supply from input appears across resistor R in opposite sense of voltage V_2 which eliminated the voltage across R. Therefore, the final output across R is zero (low).



When both the terminal of A and B at 5V (high), then both diodes become in reverse bias, hence no voltage appears across due to input voltage, therefore the voltage across R appears as 5 V high due to V_2.

This situation is obtained only is AND gate, hence output Q will be act as AND gate.

11. (*b*) The current amplification factor of a transistor for direct current (DC) is the ratio of collector current (I_C) to the base current (I_B) in common emitter configuration i.e.,

$$B_{DC} = \frac{I_C}{I_B}$$

However, the ratio of collector current to the emitter current gives the value of current amplification factor in common base configuration, i.e.,

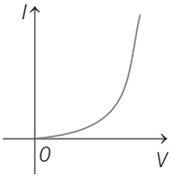
$$\alpha_{DC} = \frac{I_C}{I_E}$$

12. (*b*) Since, an LED is forward biased diode, so its characteristics are those of forward bias, i.e.,

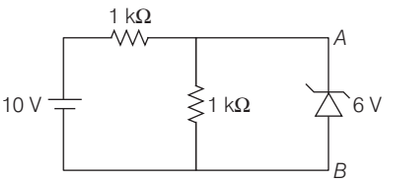
Here, option (a) is characteristics of a diode in forward and reverse bias.

Option (c) → characteristics of solar cell.

Option (d) → reverse bias characteristics 1 drawn in first quadrant can be photodiode or Zener diode.



13. (*d*) According to the question,



Input voltage $V_S = 10\text{V}$

Source resistance $R_S = 1\text{k}\Omega$

Zener diode voltage $V_Z = 6\text{V}$

∵ Breakdown voltage of zener diode is = 6V, and the potential difference across the zener diode 5V.

∴ Current flow in zener diode $I_Z = 0$

14. (*b*) By using the de-Morgan's law

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

Hence, option (b) is the correct.

Basics of Communications

1. (*b*) $v(t)=10\,[1+0.3\cos(2.2\times10^4 t)]\,[\sin(5.5\times10^5 t)]$

Upper band angular frequency

$$\omega_v = (2.2\times10^4 + 5.5\times10^5)\ \text{rad/s}$$
$$= 572\times10^3\ \text{rad/s}$$

Similarly, lower band angular frequency.

$$\omega_L = (5.5\times10^5 - 2.2\times10^4)\ \text{rad/s}$$
$$= 528\times10^3\ \text{rad/s}$$

$\therefore$ Side band frequency are,

$$f_u = \frac{\omega_u}{2\pi} = \frac{572}{2\pi}\ \text{kHz} \simeq 91\ \text{kHz}$$

and $$f_L = \frac{\omega_L}{2\pi} = \frac{528}{2\pi}\ \text{kHz} \simeq 84\ \text{kHz}$$

Thus, the approximate side band frequencies are 89.25 and 85.75.

2. (*a*) For a given carrier wave of frequency f_c with modulation frequency f_m, the bandwidth is calculated by

$$f_{\text{upper}} = f_c + f_m$$
$$f_{\text{lower}} = f_c - f_m \quad \ldots(i)$$

To avoid overlapping of bandwidths, next broadcast frequencies can be

$$f_1 = f_c \pm 2f_m,$$
$$f_2 = f_c \pm 3f_m$$

So, next immediate available broadcast frequency is

$$f_1 = f_c + 2f_m \text{ and } f_1' = f_c - 2f_m$$

Given, $f_m = 250$ kHz

and also that f_m is 10% of f_c, i.e. $f_c = 2500$ kHz.

So, $f_1 = 2500 + (2\times250) = 3000$ kHz

and $f'_1 = 2500 - (2\times250) = 2000$ kHz

3. (*b*) Equation of an amplitude modulated wave is given by the relation,

$$C_m = (A_c + A_m\sin\omega_m t)\cdot\sin\omega_c\cdot t \quad \ldots(i)$$

For the given graph,

maximum amplitude, $A_c + A_m = 10$ ...(ii)

and minimum amplitude, $A_c - A_m = 8$ ...(iii)

From Eqs. (ii) and (iii), we get

$$A_c = 9\text{V and } A_m = 1\text{V} \quad \ldots(iv)$$

$\because$ For angular frequency of message signal and carrier wave, we use a relation

$$\omega_c = \frac{2\pi}{T_c} = \frac{2\pi}{8\times10^{-6}}$$

(as from given graph, $T_c = 8\times10^{-6}$ s)

$$= 2.5\pi\times10^5\ \text{s}^{-1} \quad \ldots(v)$$

and $$\omega_m = \frac{2\pi}{T_m} = \frac{2\pi}{100\times10^{-6}}$$

(as from given graph, $T_m = 100\times10^{-6}$ s)

$$= 2\pi\times10^4\ \text{s}^{-1}$$

When we put values of A_c, A_m, ω_c and ω_m in Eq. (i), we get

$$C_m = [9 + \sin(2\pi\times10^4 t)]\sin(2.5\pi\times10^5 t)\ \text{V}$$

4. (*c*) Modulation index is given by

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$$
$$= \frac{160 - 40}{160 + 40}$$
$$= \frac{120}{200} = 0.6$$

5. (*d*) Given, modulating signal,

$$A_m = A\cos\omega t$$

Carrier wave,

$$A_c = v_0\sin\omega_0 t$$

In amplitude modulation, modulated wave is given by

$$Y_m = [A_0 + A_m]\sin\omega_0 t$$

where, A_0 is amplitude of the carrier wave (given as v_0)

$$\therefore\ Y_m = [v_0 + A\cos\omega t]\sin\omega_0 t$$
$$= v_0\sin\omega_0 t + A\sin\omega_0 t\cos\omega t$$
$$= v_0\sin\omega_0 t + \frac{A}{2}$$
$$[\sin(\omega_0 + \omega)t + \sin(\omega_0 - \omega)t]$$
$$= v_0\sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$$

6. (*d*) Low frequency signals have very low energy, hence, they can not transmitted over long distance without modulation. In modulation, characteristics of carrier wave is varies according to the amplitude of low frequency (messages signal). Carrier waves are high frequency signal which are used to transmit a low frequency signal over a long distance by modulation.

7. (*c*) **Key Idea** *The maximum line of right distance d_m between two transmitting antenna of height h_T and receiving antenna of height h_R above the earth is given by,*

$$d_m = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

As, given that,height of transmitting antenna , $h_T = 45\times10^{-3}$ km, distance between receiving and transmitting antenna, $d_m = 40$ km and radius of eath, $R = 6400$ km

Let height of receiving antenna, $h_R = x$

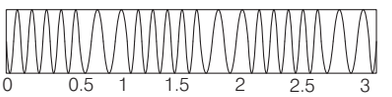
By putting the values in above expression,

$$40 = \sqrt{2\times6400\times45\times10^{-3}} + \sqrt{2\times6400\times x\times10^{-3}}$$
$$40 = 24 + \sqrt{2\times6.4\times x}$$
$$x = \frac{(40-24)^2}{2\times6.4} = 20\ \text{m}$$

Hence, the correct option is (c).

8. (*a*) Modulation index, $\mu = \frac{A_m}{A_c} \le 1$ is carried by the carrier wave, i.e. there is no signal distortion.

9. (*a*) In frequency modulated wave, only frequency varies with time, while the amplitude remains constant. This wave can be shown as



It is used in FM broadcasting, telemetry and RADAR etc.